KOREA 2025

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Re: Visiting the Essence of Mathematics Education in the Era of Digital Transformation

JUL. 18 FRI ~ 22 TUE, 2025

Seoul National University, Siheung Campus

Proceeding Vol. 3

- Topic Study Groups

Co-Host

대한수학교육학회
The Korean Society of Educational Studies in Mathematics

한국수학교육학회
The Korean Society of Mathematical Education

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

CONTENTS

iii Welcome Message
iv Program at a Glance
vi Scientific Program
xiv Contents
1 Topic Study Groups

Welcome Message

It is with great pleasure that we welcome you to the 9th ICMI-East Asia Regional Conference on Mathematics Education (EARCOME 9), held from July 18 to 22, 2025, at Seoul National University's Siheung Campus in South Korea.

EARCOME has become a cornerstone for mathematics education in East Asia, bringing together researchers, educators, and practitioners from across the region and beyond. Its origin can be traced to the South East Asia Conferences on Mathematics Education (SEACME), which began in 1978. Since the first official EARCOME in 1998, the conference has evolved into a triennial gathering—except during ICME years—fostering collaboration, scholarly exchange, and innovation in mathematics education.

The theme for EARCOME 9, "RE: Visiting the Essence of Mathematics Education in the Era of Digital Transformation," invites us to reflect deeply on foundational principles in mathematics education amid the profound shifts driven by digital technology. In this transformative age, questions of purpose, practice, and pedagogy are more urgent than ever.

As the International Program Committee Chair and Co-Chair, we are honored to support a program that reflects the diverse voices and vibrant scholarship across East Asia and beyond. This year's conference is co-hosted by The Korean Society of Educational Studies in Mathematics (KSESM) and The Korean Society of Mathematics Education (KSME). It includes an exceptional lineup of plenary lectures, invited presentations, topic study groups, and practice-oriented sessions for practitioners.

We extend our deepest gratitude to the Local Organizing Committee, co-chaired by Ho Kyoung Ko (Ajou University), GwiSoo Na (Cheongju National University of Education), and Jinho Kim (Daegu National University of Education), whose dedication and leadership have been instrumental in realizing this event. We also acknowledge the many LOC members and volunteers who have contributed their time and expertise to make EARCOME 9 a meaningful and enriching experience.

We are especially grateful for the generous support of the National Research Foundation of Korea (NRF), funded by the Ministry of Education (MOE), under Grant No. NRF-2025S1A8A4A01014320. Their support has been crucial in making this conference possible.

We warmly welcome you to EARCOME 9 and invite you to engage fully in the discussions, share your insights, and help shape a forward-looking, inclusive vision for mathematics education. We hope this conference will not only deepen your academic inquiry but also strengthen our shared commitment to building a collaborative and culturally grounded mathematics education agenda in East Asia.

Oh Nam Kwon
Chair, International Program Committee
Seoul National University, South Korea

Berinderjeet Kaur
Co-Chair, International Program Committee
National Institute of Education, Singapore

	July 18(Fri)	July 19(Sat)	July 20(Sun)
08:30~09:00		Networking	Networking
09:00~09:30		Plenary Panel Discussion 1 (CH)	Plenary Lecture 2 (CH)
09:30~10:00			
10:00~10:30		Coffee Break	Coffee Break
10:30~11:00		TSG1 (#109) / TSG2-1 (#CHI-A) / TSG2-2 (#CHI-C) / TSG3 (#102) / TSG4-1 (#603) / TSG4-2 (#604) / TSG5 (#806) / TSG6-1 (#605) / TSG6-2 (#606) / TSG7 (#806-1) / TSG8 (#103) / TSG9 (#406) / TSG10 (#609)	TSG1 (#109) / TSG2-1 (#CHI-A) / TSG2-2 (#CHI-C) / TSG3 (#102) / TSG4-1 (#603) / TSG4-2 (#604) / TSG5 (#806) / TSG6-1 (#605) / TSG6-2 (#606) / TSG7 (#806-1) / TSG8 (#103) / TSG9 (#406) / TSG10 (#609)
11:00~11:30			
11:30~12:00			
12:00~12:30			
12:30~13:00		Lunch/Networking	Lunch/Networking
13:00~13:30	Registration		
13:30~14:00			
14:00~14:30		Invited Lecture 1-1 (CH) / Invited Lecture 1-2 (#109) / Invited Lecture 1-3 (#806) / Invited Lecture 1-4 (#806-1)	Invited Lecture 2-1 (CH) / Invited Lecture 2-2 (#109) / Invited Lecture 2-3 (#806) / Invited Lecture 2-4 (#806-1)
14:30~15:00			
15:00~15:30		Poster Presentation 1 (1F Lobby)	Poster Presentation 2 (1F Lobby)
15:30~16:00			
16:00~16:30		Coffee Break	Coffee Break
16:30~17:00	Opening Ceremony (CH)	WG1-1 (#109) / WG1-2 (#606) / WG1-3 (#605) / WG1-4 (#806-1) / SSG1-1 (CH) / SSG1-2 (#806) / SSG1-3 (#603) / SSG1-4 (#604)	WG2-1 (#109) / WG2-2 (#806-1) / WG2-3 (#605) / WG2-4 (#606) / SSG2-1 (CH) / SSG2-2 (#806) / SSG2-3 (#603) / SSG2-4 (#604)
17:00~17:30	Plenary Lecture 1 (CH)		
17:30~18:00			
18:00~20:00	Welcome Reception (2F Lobby)		

Exhibition

*** TSG** | Topic Study Group
*** WG** | Working Group
*** SSG** | Special Sharing Group

***CH** | Convention Hall

Program at a Glance

KOREA2025 EARCOME9
The 9th ICMI-East Asia Regional Conference on Mathematics Education

	July 21(Mon)																July 22(Tue)			
08:30~09:00	Networking																Networking			
09:00~09:30	Exhibition	Plenary Lecture 3 (CH)															Exhibition	Invited Lecture 3-1 (CH)	Invited Lecture 3-2 (#109)	Invited Lecture 3-3 (#806)
09:30~10:00																				
10:00~10:30		Coffee Break																Coffee Break		
10:30~11:00		TSG 1 (#109)	TSG 2-1 (#CHIA)	TSG 2-2 (#CHIC)	TSG 3 (#102)	TSG 4-1 (#603)	TSG 4-2 (#604)	TSG 5 (#806)	TSG 6-1 (#605)	TSG 6-2 (#606)	TSG 7 (#806-1)	TSG 8 (#103)	TSG 9 (#406)	TSG 10 (#609)				Plenary Lecture 4 (CH)		
11:00~11:30																				
11:30~12:00																		Closing (CH)		
12:00~12:30																				
12:30~13:00		Lunch/Networking **IPC & AB Meeting (#102)**																		
13:00~13:30																				
13:30~14:00																				
14:00~14:30		Plenary Panel Discussion 2 (CH)																		
14:30~15:00																				
15:00~15:30		Coffee Break																		
15:30~16:00		WG 3-1 (#109)	WG 3-2 (#806-1)	WG 3-3 (#605)	WG 3-4 (#606)	SSG 3-1 (CH)					SSG 3-2 (#806)									
16:00~16:30																				
16:30~17:00																				
17:00~17:30																				
17:30~18:00																				
18:00~20:00		Conference Banquet																		

Topic Study Groups

Topic Study Groups 1

The Essence of Mathematics Education in Curriculum and Materials

Chair: Ji-Eun Lee (Oakland University)
Co-Chair: Rakhi Banerjee (Azim Premji University), Hui-Yu Hsu (National Tsing Hua University)
Corrdinator: Miin Lim (Seoul National University of Education)

July 19, Saturday	
10:30	**PRESERVICE TEACHERS' BELIEFS AND VIEWS ON CURRICULUM AND TEACHER GUIDE COMPONENTS** Ji-Eun Lee (Oakland University)
10:40	**DO DIGITAL CURRICULUM RESOURCES SUPPLEMENT THE EDUCATIVE FEATURES OF TEACHER GUIDEBOOKS?** MinYoung Oh (Graduate School of Korea National University of Education)
10:50	**A GUIDING FRAMEWORK FOR DESIGNING GRAPHING TASKS FOR STUDENT LEARNING** Hwa Young Lee (Texas State University)
11:00	**THE IMPACT OF TEACHING MATERIAL FORMATS ON TWO-STEP PROBLEM SOLVING: A COGNITIVE LOAD PERSPECTIVE** Yen-Ting Lai (National Changhua University of Education)
11:10	**EQUITY IN THE KOREAN ELEMENTARY MATHEMATICS CURRICULUM** MinKyung Kim (Graduate School of Korea National University of Education)
11:30	**A COMPARISON OF THE CATEGORIES OF NON TEXT ELEMENTS IN HIGH SCHOOL TEXTBOOKS - A CASE STUDY OF PEP A VERSION AND A-LEVEL VERSION** Xiaolin Yuan (East China Normal University)
11:40	**A COMPARATIVE STUDY OF DERIVATIVE CONTENT IN HIGH SCHOOL MATHEMATICS TEXTBOOKS** Chunxia Qi (Beijing Normal University)
11:50	**A COMPARATIVE STUDY ON THE PRAXEOLOGICAL ANALYSIS OF ELEMENTARY SCHOOL MATHEMATICS TEXTBOOKS IN KOREA AND FINLAND: FOCUSING ON SYMMETRY AND TRANSFORMATION** Jaehyeon Park (Seoul National University)
12:00	**DE/COLONIALITY IN MATHEMATICS CURRICULUMS OF THE PHILIPPINES AND INDIA** Harita Raval (Stockholm University)
12:10	**A STUDY OF PROOF TASKS IN SECONDARY SCHOOL MATHEMATICS TEXTBOOKS IN SINGAPORE** Weng Kin Ho (National Institute of Education)

July 20, Sunday	
10:30	**A STUDY ON EXPLORING THE CONTEXTUAL FEATURES OF QUESTIONS IN MATHEMATICS TEXTBOOK** Yi-An Cho (National Academy for Educational Research)
10:40	**ACTION RESEARCH OF IMPLEMENTING A PROBLEM-BASED LEARNING MODEL INTO THE CURRICULUM OF TEACHING MODELS FOR ELEMENTARY MATHEMATICS GIFTED STUDENTS** Yuan Yuan (National Taichung University of Education)

10:50	**FROM CLASSROOM TO REAL LIFE:** **EMBRACING MATHEMATICAL MODELLING IN HONG KONG'S CURRICULUM—INSIGHTS FROM SINGAPORE** Ka Shing Chui (The Education University of Hong Kong)
11:00	**EFFECTIVENESS OF THE TEACHING STRATEGY USING A CONFLICT MAP IN LESSONS ON MULTIPLICATION WITH DECIMAL FRACTIONS** Tadayuki Kishimoto (University of Toyama)
only proceedings	**MATHEMATICAL MODELING IN HONG KONG PRIMARY MATHEMATICS EDUCATION:** **A CONTENT ANALYSIS OF SCHOOL CURRICULUM** Hange Yun (The Education University of Hong Kong)
11:30	**AN ANALYSIS OF THE TYPES OF CONNECTIONS BETWEEN THE KOREAN MATHEMATICS COLLEGE SCHOLASTIC ABILITY TEST AND EBS-LINKED MATHEMATICS TEXTBOOKS** Minsik Woo (Seoul National University)
11:40	**A METHOD OF TEXTBOOK ANALYSIS FROM A PRAXEOLOGICAL PERSPECTIVE: THE CASE OF STATISTICS** Seitaro Hanasaki (Graduate School of Humanities and Social Sciences in Hiroshima University)
11:50	**PREALGEBRAIC CONCEPTS IN SOLVING NUMBER SENTENCES: A TEXTBOOK ANALYSIS** Maria Alva Aberin (Ateneo de Manila University)
12:00	**THE HIGHER-ORDER THINKING SKILLS (HOTS) CONTENT ANALYSIS ON MATHEMATICS TEXTBOOKS OF SENIOR HIGH SCHOOL** Haili Liang (Beijing Normal University)
12:10	**DEVELOPING AND VALIDATING IB-BASED MATHEMATICS ASSESSMENT TASKS:** **FOCUSING ON STUDENT THINKING AND RELIABILITY** Yoonjoo Cho (Seoul National University)

July 21, Monday	
10:30	**FROM COUNTABLE TO UNCOUNTABLE: DEVELOPING TEACHING MATERIALS FOR IRRATIONAL NUMBERS** Toshikazu Ikeda (Yokohama National University)
10:40	**INSIGHTS FROM GERMAN SECONDARY STATISTICAL EDUCATION ON THE DEVELOPMENT OF RISK LITERACY** Che-Yu Hsu (National Central University)
10:50	**CONTEXT IN STATISTICAL TASKS:** **INSIGHTS FROM LOWER SECONDARY MATHEMATICS TEXTBOOKS IN THAILAND** Dhanachat Anuniwat (Hiroshima University)
11:00	**EXPLORING DIGITAL LITERACY IN THE 2022 REVISED KOREAN MATHEMATICS CURRICULUM AND TEXTBOOKS** Jungeun Yoon (Seoul National University)
11:10	**A DECOLONIZED CURRICULUM AND CHILDREN'S BUNDLE-NUMBERS WITH UNITS MAY REACH THE UN DEVELOPMENT NUMERACY GOAL** Allan Tarp (MATHeCADEMY.net)
11:30	**GRADE 10 CALCULUS: WHY AND HOW** Wei-Chang Shann (National Central University)

11:40	**TRENDS AND PERSPECTIVE IN RESEARCH ON DIGITAL MATHEMATICS TEXTBOOKS: INSIGHTS FROM A SYSTEMATIC REVIEW** Hyosup Shim (Seoul National University)
11:50	**SOME LANGUAGE ISSUES IN DEFINING A NEW MATHEMATICAL CONCEPT: THE CASE OF JAPANESE SECONDARY SCHOOL STUDENTS** Shogo Murata (Nippon Sport Science University)
12:00	**DEVELOPMENT OF TEACHING MATERIALS ON THE LOCUS OF THE CIRCUMCENTER OF A REGULAR POLYGON** Yuki Osawa (Kaichi Mirai Junior and Senior High School)
12:10	**GEOMETRIC CONSTRUCTION IN TAIWANESE AND JAPANESE JUNIOR HIGH SCHOOL TEXTBOOKS** Yi-Wen Su (University of Taipei)

Topic Study Groups 2

The Essence of Mathematics Education in Classroom Practice

Chair: Hino Keiko (Utsunomiya University)
Co-Chair: Leong Yew Hoong (National Institute of Education), Kotaro Komatsu (University of Tsukuba)
Corrdinator: Jin Sunwoo (Gwangju National University of Education)

Topic Study Groups 2-1	
July 19, Saturday	
10:30	**TEACHING AND LEARNING OF SOLID GEOMETRY IN PRIMARY EDUCATION IN NICARAGUA: A DIDACTIC PROPOSAL BASED ON THE IMPLEMENTATION OF THE VAN HIELE MODEL AND THE PROBLEM-SOLVING APPROACH** Douglas Jared Martínez Cruz (Naruto University of Education)
10:50	**FOURTH-GRADE STUDENTS' JUSTIFICATIONS OF THE PROPERTIES OF EVEN AND ODD NUMBERS** Hae Song Oh (Graduate School of Korea National University of Education)
11:10	**A STUDY ON MANIPULATIVE ACTIVITIES FOR THE UNDERSTANDING SUBSUMPTION RELATIONS AMONG OF FIGURES : THROUGH CLASSES ON QUADRILATERALS FOR SIXTH-GRADE ELEMENTARY SCHOOL STUDENTS** Daisuke Itagaki (Graduate Student at the Graduate School of Education, Shimane University)
11:30	**USING TEACHING WITH VARIATION IN AFRICAN MATHS CLASSROOMS: A PARTICIPATORY CASE STUDY** Xumai Lin (University of Macau)
11:50	**HOW TEACHERS' INSTRUCTION INFLUENCES STUDENTS' UNDERSTANDING IN THE AREA OF A CIRCLE** Yi-Lun Tsai (National Pingtung University)
12:10	**IMPACT OF LESSONS EMPHASIZING A RELATIONAL UNDERSTANDING OF THE EQUAL SIGN: FOCUSED ON SAMENESS AND SUBSTITUTION MEANINGS** Juhyeon Kim (Graduate School of Korea National University of Education)

July 20, Sunday	
10:30	**TEACHING WHOLE-NUMBER MULTIPLICATION, COMBINING CHILDREN'S LEARNING TRAJECTORY OF PROPORTIONAL REASONING: A CLASSROOM-BASED STUDY** Hino Keiko (Utsunomiya University)
10:50	**AN ANALYSIS OF STUDENT' FOCUS ON THE RELATIONSHIP BETWEEN ORIGINAL SIZE AND FRACTIONS: A CASE STUDY OF 2ND GRADE LESSONS ON "FRACTION"** Yuri Masutani (Shimane University)
11:10	**THE USE OF MATHEMATICAL AND NATURAL LANGUAGE BY ELEMENTARY SCHOOL TEACHERS IN DIVISION LESSONS** Yun Hsia Pai (National Tsing Hua University)
11:30	**CURRICULUM ALIGHMENT THROUGH MULTILITERACY IN MATHEMATICS EDUCATION: INSIGHTS FROM FINLAND** Eunji Kim (Tampere University)
11:50	**ENHANCING MATHEMATICAL CREATIVITY USING THE CONCEPTS OF INCUBATION BREAK AND THE FIVE LEGS OF CREATIVITY** Rhett Anthony C. Latonio (Ateneo de Manila University)
12:10	**MANAGING THE TRANSITIONS IN CONCRETE-PICTORIAL-ABSTRACT** Yew Hoong Leong (National Institute of Education, Nanyang Technological University)

July 21, Monday	
10:30	**ANALYSIS OF TYPES OF DEALING WITH THE CONTEXT OF TASK IN SIXTH GRADE STUDENTS' MATHEMATICAL MODELING** Nahee Kim (Seoul National University of Education)
10:50	**DESIGNING MODELLING TASKS TO ENHANCE STATISTICAL THINKING: INTEGRATING STUDENTS' EXPERIENCES OF HITTING THE LITTLE TOE WITH SCIENTIFIC PROBLEMS** Daiki Fujiwara (Naruto University of Education)
11:10	**A PROBABILITY MODELING APPROACH TO PROMOTE STUDENTS' CONNECTIONS ACROSS THREE PERSPECTIVES OF PROBABILITY** Minh Kiem Tran (Hue University of Education)
11:30	**MATHEMATICS CLASSROOM MANAGEMENT IN EL SALVADOR ELEMENTARY SCHOOLS AND ITS RELATIONSHIP WITH STUDENTS' ATTITUDE AND ACADEMIC PERFORMANCE** Félix Abraham Guevara (Naruto University of Education)

Topic Study Groups 2-2	
July 19, Saturday	
10:30	**EXPLORING THE POTENTIAL OF CHATGPT IN MATHEMATICS EDUCATION: A STUDY ON EXPLANATIONS ON SUPERSCRIPT (-1) IN PEDAGOGY AND MATHEMATICS** Xiaoying Chen (China West Normal University)
10:50	**NARRATIVES OF A HIGH SCHOOL TEACHER IN A MATHEMATICS CLASS** Mayumi Kawamura (Oita University)

11:10	**EXPLORING THE CURRENT STATE OF GIFTED EDUCATION IN MATHEMATICS IN JAPAN: A CASE STUDY OF CLASSROOM PRACTICES IN A SUPER SCIENCE HIGH SCHOOL** Natsuki Uchikubo (University of Tsukuba)
11:30	**PROPOSAL FOR WIDE-AREA DEVELOPMENT AND CLASSROOM PRACTICE USING ICT TEACHING MATERIALS THAT CAN BE CONSIDERED FROM DAILY LIFE** Satoru Sakanashi (Tokyo Metropolitan Minato Municipal Odaiba-Gakuen Koyo Junior High School)
11:50	**EMBRACING STUDENTS' DIVERSITY IN MATHEMATICS THROUGH DIFFERENTITATED INSTRUCTION AND OPEN APPROACH: A FRAMEWORK DEVELOPMENT** Yoga Dwi Windy Kusuma Ningtyas (Hiroshima University)

July 20, Sunday	
10:30	**GRADE 10 STUDENTS CONCEPTIONS OF PROOF - AN EXPLORATORY STUDY** Berinderjeet Kaur (National Institute of Education)
10:50	**THE IMPACT OF PROBLEM POSING ON STUDENT'S MATHEMATICAL INQUIRY SKILLS : AN EMPIRICAL INVESTIGATION** Jung Sook Park (Jungwha High School)
11:10	**PROBLEM-POSING PRACTICES: A CASE STUDY OF A FILIPINO SCIENCE HIGH SCHOOL MATHEMATICS TEACHER** Lester Hao (Ateneo de Manila University)
11:30	**THE ROLE OF REFLECTION CARD IN FACILITATING APPROPRIATION PROCESSES IN MATHEMATICS EDUCATION** Shinya Nakao (Kansai University of Welfare Sciences)
11:50	**USING STUDENT'S SELF-ASSESSMENT OF THEIR INTERPRETED PREPARATION FOR CLASS: A PILOT OF USING THE MASTU EVALUATION** Amanda Jo Meiners (Northwest Missouri State University)

July 21, Monday	
10:30	**RESULTS OF TEACHER QUESTION ANALYSIS IN A MATHEMATICS RESEARCH LESSON** Ganbaatar Tumurbaatar (Mongolian National University of Education)
10:50	**TEACHERS INTERPRETING STUDENT WORK IN GEOMETRY: USING CURRICULAR TEXT TO SUPPORT INSTRUCTIONAL EXCHANGE** Soobin Jeon (University of Michigan)
11:10	**A STUDY ON ASPECTS OF GRASPING QUANTITATIVE RELATIONS IN WORD PROBLEMS INVOLVING INVERSE CALCULATION : FOCUSING ON THE PROCESS OF DRAWING THE DIAGRAMS** Maoko Nakahata (Shimane University)
11:30	**STREAMING IN SECONDARY MATHEMATICS: A VIEW FROM AUSTRALIA** Elena Prieto-Rodriguez (University of Newcastle)

Topic Study Groups 3

The Essence of Mathematics Education in Assessment and Evaluation

Chair: Xinrong Yang (University of Macau)
Co-Chair: Zhenzhen Miao (Jiangxi Normal University), Younggon Bae (University of Texas Rio Grande Valley)
Corrdinator: SungHwan Hwang (Chuncheon National University of Education)

July 19, Saturday	
10:30	**THE RELATIONSHIP BETWEEN MIDDLE SCHOOL STUDENTS' MATHEMATICS SELF-EFFICACY, MATHEMATICS ANXIETY, AND MATHEMATICAL LITERACY** Yi-Jhen Li (National University of Tainan)
10:50	**AN EXPLORATORY STUDY OF MIDDLE SCHOOL STUDENTS' PERFORMANCE AND TYPICAL ERRORS IN MODELING TASKS** Qianhui Zhao (East China Normal University)
11:10	**EXAMINING MASTERY OF COGNITIVE ATTRIBUTES IN MATHEMATICS USING SOUTH KOREAN NATIONAL ASSESSMENT OF EDUCATIONAL ACHIEVEMENT DATA** Jihyun Hwang (Korea National University of Education)
11:30	**FROM ICT USE TO CREATIVE THINKING: THE MEDIATION OF ATTITUDES TOWARDS CREATIVE THINKING AMONG EAST ASIAN STUDENTS** Si Chen (Beijing Normal University)
11:50	**FACTORS PREDICTING EAST ASIAN STUDENTS' PERCEIVED QUALITY OF MATHEMATICS INSTRUCTION: A MACHINE LEARNING APPROACH** Xu Wei (Beijing Normal University)
12:10	**CREATIVITY IN MATH ASSESSMENTS** Yujin Lee (Kangwon National University)

July 20, Sunday	
10:30	**A VALIDATION STUDY MEASURING INTERDISCIPLINARY MATHEMATICAL PROBLEM SOLVING ABILITY** Zhongtian Ji (Beijing Normal University)
10:50	**THE EFFECTS OF TECHNOLOGICAL INNOVATION IN NATIONAL MATHEMATICS ASSESSMENT** Chang-Geun Song (Cheongju National University of Education)
11:10	**STUDENTS' SELF-REFLECTION IN A GEOMETRY LESSON: PERSPECTIVES AND QUALITY** Satoshi Kusaka (Naruto University of Education)
11:30	**COMMUNICATION SKILLS PERFORMANCE IN QUANTITATIVE RELATED LITERACY AMONG ELEMENTARY SCHOOL STUDENTS** Yun-Chi Tsou (Southern Illinois University Carbondale)
11:50	**IN BUNDLENUMBER-MATH YOU JUST ASK THE BUNDLEBUNDLE BOARD** Allan Tarp (MATHeCADEMY.net)

10:30	**A STUDY ON PERSPECTIVES FOR EVALUATING VISUAL REPRESENTATIONS IN MATHEMATICS LESSONS** Naoya Miwa (University of Tsukuba)
10:50	**THE ROLE OF ORIGINAL PROBLEMS IN MATHEMATICAL PROBLEM POSING ASSESSMENTS** Ziyi Jin (Hiroshima University)
11:10	**SCAFFOLDED INTERVIEW TASKS BUILD ON WRITTEN TASKS TO REVEAL ALGEBRAIC THINKING** Catherine Pearn (The University of Melbourne)
11:30	**ENHANCING ADAPTIVE MATHEMATICS ASSESSMENT THROUGH GENERATIVE AI: INTEGRATING THE COGNITIVE DISTRACTOR GENERATION MODEL (CDGM) INTO THE GENERATIVE AI-BASED MULTI-STAGE ASSESSMENT (GAMSA) FRAMEWORK** Jin Min Chung (University of Iowa)

Topic Study Groups 4

The Essence of Mathematics Teacher Education

Chair: Takeshi Miyakawa (Waseda University)
Co-Chair: Maitree Inprasitha (Khon Kaen University), Qiaoping Zhang (The Education University of Hong Kong)
Corrdinator: Soo Jin Lee (Korea National University of Education)

Topic Study Groups 4-1

July 19, Saturday

10:30	**EXPLORING LESSON STUDY TEAM COLLABORATION IN TEACHING POSITIVE AND NEGATIVE NUMBERS TO GRADE 7 STUDENTS** Noppasorn Boonsena (Khon Kaen University)
10:45	**COMPARISON OF LESSON PLANNING PROCESSES BY PRE-SERVICE MATHEMATICS TEACHERS: FROM THE PERSPECTIVE OF DESIGN RESEARCH** Hayato Hanazono (Miyagi University of Education)
11:00	**COLLABORATIVE RESEARCH LEARNING: COMMUNITIES OF PRACTICE AMONG PRE-SERVICE AND IN-SERVICE MATHEMATICS TEACHERS** Mailizar Mailizar (Syiah Kuala University)
11:30	**TRANSFORMING LESSON REFLECTION MEETING USING 'LEARNER OBSERVATION SHEET'** Naoki Yachimoto (Hokkaido University of Education)
11:45	**SUSTAINING LESSON STUDY AS A PROFESSIONAL DEVELOPMENT MODEL: FROM THE TEACHERS' PERSPECTIVE** Edna Callanta (University of the Philippines National Institute for Science and Mathematics Education Development)
12:00	**TEACHERS' EXPERIENCE OF REFLECTIVE PRACTICE IN THE OPEN CLASS** Narumon Changsri (Khon Kaen University)

July 20, Sunday	
10:30	**AN ANALYSIS OF MATHEMATICS TEACHER'S SUPPORT IN HIGH SCHOOLERS' INQUIRY: FOCUSING ON THE MOMENT OF SUPERVISIONS IN SRP** Toshihiko Ogawa (Waseda University)
10:45	**TOWARDS REFASHIONING TEACHERS' TASK DESIGN IN TRIGONOMETRY: A PROFESSIONAL DEVELOPMENT MODEL** Jake Garnace (Ateneo de Manila University and University of Northern Philippines)
11:00	**REFLECTING IN-SERVICE ELEMENTARY TEACHERS' MATHEMATICAL KNOWLEDGE FROM TEACHER PROFESSIONAL EDUCATION PROGRAM** Zetra Hainul Putra (Universitas Riau)
11:15	**THE DEVELOPMENT OF DIDACTIC PRAXEOLOGIES IN LESSON DESIGN THROUGH SOCIAL RESOURCE INTERACTIONS: EVIDENCE FROM TWO NOVICE MATHEMATICS TEACHERS** Xingfeng Huang (Shnaghai Normal University)
11:40	**A STUDY ON INSTRUCTIONAL REPRESENTATIONS IN DIFFERENT TEACHING SCENARIOS: INTRODUCING TEXT MINING TECHNIQUES** Ling-Yi Huang (National Taipei University of Education)
11:55	**THE ROLE OF PROFESSIONAL DEVELOPMENT IN ENHANCING MATHEMATICAL CREATIVITY: A CASE STUDY OF NOVICE TEACHERS** Yujin Lee (Kangwon National University)
12:10	**INTERVENTIONS TO DEVELOP PRE-SERVICE MATHEMATICS TEACHERS' COMPETENCE IN DIFFERENTIATED INSTRUCTION: A SCOPING REVIEW** Iden Rainal Ihsan (National Taiwan Normal University)

July 21, Monday	
10:30	**TEACHERS' INTENTION TO IMPLEMENT BLENDED LEARNING IN ALGEBRA: INSIGHTS FROM A PILOT STUDY** Nursaadah Jailani (The National University of Malaysia)
10:45	**INNOVATION IN MATHEMATICS TEACHER EDUCATION THROUGH A STEAM-BASED GLOBAL PROFESSIONAL LEARNING COMMUNITY MODEL** Jungin Heo (Samjung High school)
11:00	**DEVELOPMENT OF A TEACHER TRAINING MODEL FOR ENHANCING AI DIGITAL TEXTBOOK UTILIZATION COMPETENCY** Seo Hyun Ahn (University of Ajou)
11:30	**"AM I BEING TOO UNHELPFUL?" MATHEMATICS TEACHER DECISION-MAKING & RESPONSIBILITY** Samantha A. Marshall (North Carolina State University)
11:45	**ANALYZING STUDENT REFLECTIONS IN TEACHING PRACTICUM TO ENHANCE MATHEMATICS INSTRUCTION COURSES** Hideyo Makishita (Yamato University)
12:00	**INVESTIGATING BEST PRACTICES OF HIGH SCHOOL MATH TEACHERS** Hyunkyoung Yoon (California Polytechnic University, Pomona)

Topic Study Groups 4-2	
July 19, Saturday	
10:30	**INVESTIGATING PRESERVICE TEACHERS' CURRICULAR NOTICING ABILITY THROUGH TEXTBOOK ANNOTATION ACTIVITIES** Chunxia Qi (Beijing Normal University)
10:45	**MATHEMATICS TEACHERS' TEXTBOOK UNDERSTANDING: A MEDIATION EFFECT ANALYSIS** Qinqiong Zhang (Fujian Normal University)
11:15	**SUPPORTING EARLY CAREER TEACHERS' NOTICING: A FACILITATOR'S INTENT IN LESSON STUDY** Hyomin Kang (University of Tsukuba)
11:30	**VISUAL PERCEPTION OF EXPERT AND NOVICE MATHEMATICS TEACHERS ACROSS STUDENT PROFILES** Rangmei Li (Beijing Normal University)
11:45	**EXAMINING ADAPTIVE EXPERTISE OF EMERGING AND EXPERIENCED INSTRUCTIONAL COACHES THROUGH NOTICING** Sze Looi Chin (Massey University)
July 20, Sunday	
10:30	**NAVIGATING STATISTICS AND PROBABILITY INSTRUCTION THROUGH MKT: A CONTENT ANALYSIS OF TEACHER REFLECTIONS** Joseph Ma. Steven Cabalo (Ateneo de Manila University)
10:45	**MANIFESTATIONS OF TEACHER AGENCY IN MATHEMATICS EDUCATION: PERSONAL, SOCIAL, AND CONTEXTUAL INSIGHTS FROM A GRADUATE COURSE ON TEACHING STATISTICS AND PROBABILITY** Nhoriel Toledo (Centro Escolar University, Ateneo de Manila University)
11:00	**LESSON PLANNING AS A REFLECTION OF PRE-SERVICE MATHEMATICS TEACHERS' MODELING TEACHING COMPETENCE** An Thi Tan Nguyen (Hue University)
11:30	**PERCEPTIONS AND PRACTICES OF FILIPINO JUNIOR HIGH SCHOOL MATHEMATICS TEACHERS ON PROBLEM-POSING** John Patrick Cultura (La Salle College Antipolo)
11:45	**TAIWANESE SECONDARY MATHEMATICS TEACHERS' PERSPECTIVES ON CALCULUS INSTRUCTION: INSIGHTS FROM THE CURRICULAR AND CONTEXTUAL FACTORS** Pin-Chen Guo (National Taiwan Normal University)
12:00	**CONCEPTIONS OF MATH TEACHING AND LEARNING IN CONVERSATIONS ABOUT CURRICULAR CHANGE** Nurdan Turan (New York University)
July 21, Monday	
10:30	**EVALUATION OF STUDENTS' COUNTEREXAMPLES BY PRE-SERVICE PRIMARY SCHOOL TEACHERS** Kayo Miura (Hiroshima University)
10:45	**SHORT-DURATION OBSERVATION INSTRUMENT FOR ELEMENTARY MATHEMATICS TEACHERS' COMPETENCE (SDOI-EMTC)** Ting-Chun Kuo (National Taipei University of Education)

11:00	**KNOWLEDGE NEEDED FOR TEACHING MATHEMATICS TO STUDENTS WITH LEARNING DISABILITIES** Roslinda Rosli (The National University of Malaysia)
11:15	**PRESERVICE MATHEMATICS TEACHERS' EMERGING PEDAGOGICAL CONTENT KNOWLEDGE IN A LESSON STUDY-MEDIATED PRACTICE TEACHING** Monalisa Sasing (University of the Philippines National Institute for Science and Mathematics Education Development)
11:40	**EXPLORING THE DEVELOPMENT OF PRIMARY MATHEMATICS PRE-SERVICE TEACHERS' TEACHING SELF-EFFICACY: A PRELIMINARY ANALYSIS** Hui Min Chia (University of Macau)
11:55	**A TEACHER'S DEVELOPMENT OF STUDENTS' METACOGNITION IN A MATHEMATICS CLASSROOM** Leng Low (Academy of Singapore Teachers, Ministry of Education)
12:10	**SHIFT IN TEACHER'S PERSPECTIVES IN MATHEMATICS BASED ON INTERVENTIONS ON INDIAN KNOWLEDGE SYSTEMS AND DISTINCTIONS BETWEEN EMPIRICAL AND FORMAL MATHEMATICS** Sanjeev Ranganathan (C3STREAM Land, SAIIER)

Topic Study Groups 5

The Essence of Mathematics Education in Learning and Cognition

Chair: Ban Heng Choy (Nanyang Technological University)
Co-Chair: Yuriko Yamamoto Baldin (Universidade Federal de São Carlos), Insook Chung (Saint Mary's College, Notre Dame)
Corrdinator: Jiyoung Lee (Gongju National University of Education)

July 19, Saturday	
10:30	**ANALYSIS OF THIRD-GRADE STUDENTS' STRATAGIES FOR SOLVING MULTI-DIGIT MULTIPLICATION PROBLEMS** Johyeon Chang (Korea National University of Education)
10:40	**INVESTIGATING THIRD GRADERS' PLACE VALUE CONCEPT AND MULTI-DIGIT ADDITION THINKING STRATEGIES** Hannah Mortensen (Saint Mary's College)
10:50	**TEACHERS SEE 1 NUMBER WHERE KIDS SEE 3 NUMBERINGS IN 507, WHO IS NUMBER-BLIND?** Allan Tarp (MATHeCADEMY.net)
11:00	**CHARACTERISTICS OF MATHEMATICAL NEGOTIATION THAT PROMOTE FRACTION CONCEPT FORMATION : AN ANALYSIS OF CLASS UTILIZING THE NUMBER LINE** Taketo Shimomura (Shimane University)
11:10	**"DIVIDING THE DENOMINATORS AND NUMERATORS SEPARATELY": SIXTH GRADERS' STRATEGIES OF THE MEASUREMENT FRACTION DIVISION** Jeongwon Kim (Bongmyeong Elementary School)
11:20	**HOW JAPANESE CHILDREN FOCUSING ON SPONTANEOUS PATTERNS CREATE PATTERNS?** Nanae Matsuo (Chiba University)
11:30	**A VISUALIZATION OF THE STRUCTURE OF KNOWLEDGE BASED ON THE CONNECTIONS OF MATHEMATICAL CONCEPTS** Ye-Jin Lee (Seoul National University of Education)

11:40	**CRITICAL THINKING IN MATHEMATICS EDUCATION: A SYSTEMATIC REVIEW** Ersi Cha (East China Normal University)
11:50	**IS THERE A CONNECTION BETWEEN REASONING OF MATHEMATICS AND SCIENCE: A DOCUMENT ANALYSIS OF TIMSS ASSESSMENT FRAMEWORK** Jin Mun (Seoul National University)
12:00	**CONTEXT-BASED LEARNING FOR ELEMENTARY STUDENTS: CONSTRUCTING REPRESENTATIVENESS IN ARITHMETIC MEAN USING AIC THEORY** Youngin Kim (Seoul National University Graduate School)

July 20, Sunday	
10:40	**A STUDY ON SPATIAL SKILLS TO ENHANCE MENTAL OPERATIONS IN ELEMENTARY SCHOOL STUDENTS** Satoshi Watanabe (Jissen Women's University)
10:50	**EARLY SPATIAL REASONING: THE CASE OF PATH FINDING AND CONSTRUCTION** Kazuya Kageyama (Hiroshima University)
11:00	**A DESIGN-BASED STUDY ON INTEGRATING CREATIVE AND CRITICAL THINKING INTO MATHEMATICS-GROUNDING ACTIVITIES: PROPORTIONAL REASONING** Bo-Ting Liu (National Taiwan Normal University)
11:10	**SURVEY ON THE FORMATION OF THE CONCEPT OF SPEED** Masahide Tsuda (Soka University)
11:20	**EXPLORING THE ABSTRACTION PROCESS DURING ALGEBRA PROBLEM-SOLVING IN A PROGRAMMING-ENHANCED ENVIRONMENT** Haoyi Wang (University of Pennsylvania)
11:30	**STRUCTURE SENSE AND FLEXIBILITY IN ALGEBRAIC PROBLEM-SOLVING: AN EXPLORATORY STUDY** Haoyi Wang (University of Pennsylvania)
11:40	**A COMPARATIVE STUDY OF INVERSE FUNCTIONS AMONG KOREAN AND CHINESE HIGH SCHOOL STUDENTS** Wenting Liu (Kaili University)
11:50	**GRADUATE STUDENTS IN EDUCATION' UNDERSTANDING OF ALGORITHMIC THINKING IN MATHEMATICS CLASSROOM THROUGH A COLORING BOOK WORKSHOP** Wiramon Kanlayarat (Khon Kaen University)
12:00	**EXAMINATION OF MATHEMATICAL DISCOURSE FOR INDIRECT PROOF OF THE CONVERSE OF THE INSCRIBED ANGLE THEOREM** Keiko Watanabe (Shiga university)
12:10	**ENGAGING THROUGH REASONING: A CULTURAL-HISTORICAL ACTIVITY THEORY (CHAT) ANALYSIS OF STUDENTS' ENGAGEMENT IN ACCOUNTABLE TALK IN A HYFLEX MATHEMATICS CLASSROOM** Edrian Peter Villanueva (De La Salle Univerisity)

July 21, Monday	
10:30	**THE COHERENT AND STRATEGIC USES OF PIAGET'S NOTION OF PERTURBATION IN MATHEMATICS (TEACHER) EDUCATION** Biyao Liang (The University of Hong Kong)
10:40	**USING ACE TEACHING CYCLE IN DEVELOPING A PRELIMINARY GENETIC DECOMPOSITION: A PROPOSED FRAMEWORK** Christian Abasta (Ateneo de Manila University)
10:50	**THE EFFECTS OF DIFFERENT INSTRUCTIONAL REPRESENTATIONS ON THE STUDENT ENGAGEMENT IN ELEMENTARY MATHEMATICS REMEDIAL TEACHING** Lan-Hsin Zhang (National Taipei University of Education)
11:00	**THE TEACHER AS A MEDIATOR IN A PROPORTIONAL REASONING TASK** Elaine Yu Ling Cai (Nanyang Technological University)
11:10	**ANALYZING INTERDEPENDENCY OF COGNITIVE ATTRIBUTES IN MATHEMATICS AND SCIENCE** Jihyun Hwang (Korea National University of Education)
11:20	**CHILDREN'S MATHEMATICAL NOTICING IN AN ONLINE PEER-GENERATED STRATEGY VIDEO** Ho-Chieh Lin (Qing Shui Elementary School)
11:30	**READING COMPREHENSION OF REAL-WORLD PROBLEMS: AN ERP STUDY** Hui-Yu Hsu (National Tsing Hua University)
11:40	**PHONE FREQUENCY DISTRIBUTIONS IN MATHEMATICAL LANGUAGE: A STATISTICAL ANALYSIS OF MANDARIN** Pu Yu (National Chengchi University)
11:50	**A NEUROCOGNITIVE PERSPECTIVE ON THE DEVELOPMENT OF FRACTIONAL UNDERSTANDING BASED ON APOS THEORY** Doyeon Ahn (Korea National University of Education)
12:00	**FALLACY OF COMPOSITION AND COIN TOSS SEQUENCES: COMPARING SEQUENCES WITH THE SAME HEADS-TO-TAILS RATIOS** Ippo Ishibashi (Okayama University)

Topic Study Groups 6

The Essence of Mathematics Education in the Use of Digital Technology

Chair: Mi Yeon Lee (Arizona State University)
Co-Chair: Oi-Lam Ng (The Chinese University of Hong Kong), Sheunghyun Yeo (Daegu National University of Education)
Corrdinator: Dongjo Shin (Pusan National University)

Topic Study Groups 6-1 AI Integration in Mathematics Education	
July 19, Saturday	
11:00	**LEARNING TEACHING MATHEMATICS WITH INSTRUCTIONAL TECHNOLOGY: A PRAXEOLOGICAL ANALYSIS OF A GHANAIAN MATHEMATICS TEACHER EDUCATION COURSE** Farouq Sessah Mensah (Stockholm University)

11:12	**EXPLORING PRESERVICE TEACHERS' INTEGRATION OF AI TOOL THROUGH AI-TPACK AND SAMR MODEL** Yujin Lee (Heungryong Elementary School)
11:24	**PRE-SERVICE TEACHERS' SELF-EFFICACY OF USING GENERATIVE ARTIFICIAL INTELLIGENCE IN MATHEMATICS INSTRUCTION** Norbert Noster (University of Würzburg)
11:36	**STUDENTS' PERCEPTIONS OF GENERATIVE ARTIFICIAL INTELLIGENCE AND UNDERSTANDING OF MATHEMATICAL PROOFS** Yujin Lee (Kangwon National University)
11:48	**SIMULATED PRACTICE-BASED TEACHING FOR PRESERVICE TEACHERS USING AN AI CHATBOT** Taekwon Son (Bongmyong Elementary School)
12:00	**HOW SECONDARY MATHEMATICS TEACHERS LEVERAGE AI-POWERED TOOLS TO ENHANCE TEACHING EFFICIENCY** Wing Kin Cheng (Hong Kong Metropolitan University)

July 20, Sunday	
10:30	**CRITICAL REFLECTIONS ON ALGORITHMIC LEARNING DESIGN AND HUMAN EXISTENCE THROUGH HEIDEGGERIAN ONTOLOGY** Jiyoung Kim (Seoul Dungyang Elementary School)
10:40	**EXPLORING THE LONG-TERM LEGACY OF COVID DISRUPTION TO MATHEMATICS TEACHING IN AUSTRALIAN PRIMARY SCHOOLS** Kathryn Holmes (Western Sydney University)
10:50	**KOREAN TEACHERS' PERCEPTION ON AI-EMPOWERED DIGITAL TEXTBOOKS IN MATHEMATICS** Jukyung Park (Seoul Oryun Elementary School)
11:00	**HARNESSING AI IN SINGAPORE PRIMARY MATHEMATICS CLASSROOM - REDEFINING THE ROLE OF A TEACHER** Suo Hui Chang (Academy of Singapore Teachers)
11:10	**ANALYSIS OF THE STRUCTURING FEATURES OF CLASSROOM PRACTICE IN A TRIGONOMETRIC RATIOS LESSON FOR ACUTE ANGLES USING GENERATIVE AI** Jaeeun Joo (Mudung Middle school)
11:20	**EXPLORING ACTOR-NETWORKS IN MATHEMATICS CLASSES USING GENERATIVE AI** Jungeun Yoon (Seoul National University)
11:30	**UTILIZATION OF CHEATING-RESISTANT QUESTIONS USING STACK IN THE COURSE OF LINEAR ALGEBRA** Kentaro Yoshitomi (Osaka Metropolitan University)
11:40	**STUDENT INTERACTIONS WITH CHATGPT: ANALYZING PROBLEM-POSING PROCESSES IN A NUMBER THEORY COURSE** Ah Ra Cho (Seoul National University)
11:50	**THE DEVELOPMENT OF PRESERVICE MATHEMATICS TEACHERS' SPATIAL REASONING: STRENGTHS AND CHALLENGES** Lizhen Chen (Western Washington University)

12:00	**DESIGNING AND REVISING DYNAMIC GEOMETRY TASKS BY PRE-SERVICE ELEMENTARY TEACHERS: THEORETICAL NETWORKING OF NOTICING AND TPACK** Nayoung Ku (Gyeonggi Science High School)

July 21, Monday	
10:30	**TOWARDS A DIGITAL MATHEMATICS TEACHING ANALYTIC FRAMEWORK** Erlina Ronda (University of the Philippines)
10:40	**ELECTRONIC FEEDBACK IN FORMATIVE ASSESSMENT: ENHANCING STUDENT UNDERSTANDING IN MATHEMATICS** Huan Chin (Universiti Sains Malaysia)
10:50	**PERCEIVED IMPACT OF AI-GENERATED FEEDBACK ON UNDERGRADUATE MATHEMATICS LEARNING IN GEOMETRIC PROOF** Lady Angela Rocena (University of the Philippines Integrated School)
11:00	**ANALYSIS OF MATHEMATICAL PERFORMANCE AND REASONING PROCESS OF GENERATIVE AI: FOCUSING ON THE COLLEGE SCHOLASTIC ABILITY TEST** Sejun Oh (Hongik University)
11:10	**SHIFT OF OUTPUT RESULTS WHEN INPUT TO THE GENERATIVE AI ARE CHANGED : A CASE STUDY OF THE 2024 NATIONAL STUDENT ACHIVEMENT SURVEY FOR GRADE 9** Ruka Kikuchi (Saitama university)
11:20	**CHATGPT AS PROBLEM POSER: CONTEXTUALIZED AI-ASSISTED PROBLEM GENERATION USING STRUCTURED CHAIN-OF-THOUGHT PROMPTING** Joseph Ma. Steven Cabalo (Ateneo de Manila University)
11:30	**POCKET CALCULATORS IN GRADE ONE TO PREDICT DIVISION TABLES** Allan Tarp (MATHeCADEMY.net)
11:40	**AN ANALYSIS OF MATHEMATICAL CAPABILITIES OF GENERATIVE ARTIFICIAL INTELLIGENCE: FOCUSING ON CHATGPT O4-MINI-HIGH, COPILOT, GEMINI 2.5 PRO, AND DEEPSEEK R1** Hong Kyung Yun (Seoul National University)
11:50	**NOTICING CLASSES OF PRESERVICE TEACHERS: RELATIONS TO TEACHING MOVES THROUGH AI CHATBOT SIMULATION** Taekwon Son (Bongmyeong Elementary School)
12:00	**EXPLORING PRESERVICE TEACHERS' UNDERSTANDING OF FRACTION MULTIPLICATION WITH A TECHNOLOGICAL TOOL** Ji Nam Hwang (Yongjeong Elementary School)

Topic Study Groups 6-2 **Theoretical and Practical Exploration with Digital Tools in Mathematics Education**	
July 19, Saturday	
11:00	**EQT-TECH: FRAMEWORK FOR INTEGRATING TECHNOLOGY TO ADVANCE MATH THINKING AND LEARNING** Jennifer Suh (George Mason University)

11:12	**EVOLVING TRENDS IN MATHEMATICS EDUCATION RESEARCH ON DIGITAL TECHNOLOGY: A DYNAMIC TOPIC MODELING ANALYSIS** Yeongjun Kim (Seoul National University)
11:24	**INVESTIGATING PRESERVICE TEACHERS' USE OF A DYNAMIC DIGITAL TOOL TO SOLVE A FRACTION MULTIPLICATION PROBLEM** Mi Yeon Lee (Arizona State University)
11:36	**USE OF GEOGEBRA IN ADDITION AND SUBTRACTION OF FRACTIONS WITH DIFFERENT DENOMINATORS IN MOZAMBIQUE** Fernando Macuacua Junior (Naruto University of Education)
11:48	**'GREEDY PIG' FOR A MATHEMATICAL INQUIRY WITH PROGRAMMING IN SECONDARY SCHOOL** Dai Kishimoto (Waseda university)
12:00	**STUDENTS' REASONING OF QUADRATIC GROWTH IN A PROGRAMMING-BASED ENVIRONMENT** Xuan Su (The University of Hong Kong)
12:12	**ENHANCING COMPUTATIONAL THINKING THROUGH MACHINE LEARNING IN DAILY WASTE SORTING PROBLEM SITUATIONS** Anucha Koyata (Khon Kaen University)

July 20, Sunday	
10:30	**META-ANALYSIS OF THE EFFECTS OF EDUCATIONAL ROBOTICS ON K-12 STUDENTS' MATHEMATICS LEARNING** Ziyi Xiao (East China Normal University)
10:40	**EXPLORING MATHEMATICAL NARRATIVES AND CITIZENS' COMMUNICATION IN KOREAN YOUTUBE SHORTS** Kyungwon Lee (Seoul National University)
10:50	**DESIGNING AUGMENTED PAPER-BASED MATHEMATICS LEARNING ENVIRONMENT FROM AN EMBODIED COGNITION PERSPECTIVE: APPLICATION TO THE TRANSLATION OF QUADRATIC FUNCTIONS** Hyowon Wang (Sookmyung Women's University)
11:00	**COLLABORATIVE PRACTICES IN VIRTUAL GROUP WORK ON DYNAMIC GEOMETRY TASKS** Younggon Bae (The University of Texas Rio Grande Valley)
11:10	**PROVING ACTS WITH DIGITAL TOOLS FOR INQUIRING INTO MATHEMATICAL PROPOSITIONS IN LEARNING GEOMETRY** Keiko Watanabe (Shiga university)
11:20	**CAPTURING COMPUTATIONAL THINKING IN EARLY MATHEMATICS EDUCATION: A CASE STUDY FROM A CHINESE KINDERGARTEN** Ying Zhang (University of Hong Kong)
11:30	**INTERACTIVE HIGH SCHOOL APPS FOR BRIDGING MATHEMATICAL LEARNING: PERSPECTIVES FROM A HIGH SCHOOL IN QUEZON CITY, PHILIPPINES** Maria Alva Aberin (Ateneo De Manila University)
11:40	**A PRELIMINARY STUDY ON 5- TO 6-YEAR-OLD CHILDREN'S SELECTION PREFERENCES AND SALIENT INFORMATION IN MATHEMATICAL SHORT VIDEOS.** Wan-Hua Deng (National Pingtung University)
11:50	**AN ANALYSIS ON THE DIGITAL ARTIFACT AFFORDANCE THROUGH ANT AND INSTRUMENTAL APPROACH** Jiyoung Kim (Seoul Dungyang Elementary School)

July 21, Monday	
10:30	**A STUDY ON THE CONSTRUCTION OF ARCUATE FIGURES IN WASANSHO** Jun Yamada (Tsushima High School)
10:40	**STUDENT VOICE WHILE USING ICT IN MATHEMATICS LESSONS: THE CASE OF INFECTION EPIDEMICS** Minoru Ohtani (Kanazawa University)
10:50	**CO-CONSTRUCTING UNDERSTANDING OF PROGRAMMING-BASED MATHEMATICS TEACHING PRACTICES IN A COMMUNITY OF INQUIRY** Huiyan Ye (The Chinese University of Hong Kong)
11:00	**EFFECTIVE USE OF SCRATCH FOR UNDERSTANDING REGULAR POLYGON IN NICARAGUAN PRIMARY EDUCATION** Isaias Alberto Granados Gonzalez (Naruto University of Education)
11:10	**FIRST-GRADE STUDENTS' IDEAS AROUND NEGATIVE NUMBERS IN MATHEMATICS-CODING INTEGRATED LESSONS** Boram Lee (Utah State University)
11:20	**THE EFFECTS ON STUDENTS' PROBLEM-SOLVING SKILLS OF ESPORTS-BASED MATHEMATICAL VIDEO LESSONS** Ronie Jake Mamala (Mariano Marcos State University, University of the Philippines - Diliman)
11:30	**A MID-CAREER MATHEMATICS TEACHER'S NOTICING OF STUDENTS' DATA INVESTIGATION PRACTICES WITH A DATA ANALYSIS AND VISUALIZATION TOOL** Takashi Kawakami (Utsunomiya University)
11:40	**USING AN AI CHATBOT TO DEVELOP PRESERVICE TEACHERS' RESPONSIVE TEACHING SKILLS** Sheunghyun Yeo (Daegu National University of Education)
11:50	**INVESTIGATING THE PEDAGOGICAL VALUE OF DIGITAL MANIPULATIVES BETWEEN US AND KOREA** Sheunghyun Yeo (Daegu National University of Education)

Topic Study Groups 7

The Essence of Mathematics Education in Affective and Emotional Aspects

Chair: Ting Ying Wang (National Taiwan Normal University)
Co-Chair: Ting Ying Wang (National Taiwan Normal University), Hiroyuki Ninomiya (Saitama University)
Corrdinator: Yunmin Kim (Chinju National University of Education)

July 19, Saturday	
10:30	**PROFILE ANALYSIS USING CLUSTERING TO EXPLORE STUDENTS' MATHEMATICS GRIT** Yuan-Horng Lin (National Taichung University of Education)
10:40	**EXPLORING THE RELATIONSHIP BETWEEN STUDENTS' PERCEPTIONS OF DOING WELL IN MATHEMATICS, MATHEMATICS CONFIDENCE, AND GROWTH MINDSETS** Anni E (The University of Melbourne)

10:50	**EFFECTS OF COLLABORATIVE CRITICAL MATHEMATICS IN A FLIPPED CLASSROOM ON STUDENTS' MATHEMATICAL DISPOSITION** Eric Loyd P. Hilario (University of the Philippines Diliman)
11:10	**MATHEMATICAL ANXIETY OF CHILEAN STUDENTS FROM A TECHNICAL-VOCATIONAL HIGH SCHOOL** Claudia Vargas (Universidad de Santiago de Chile)
11:20	**A DYNAMIC SYSTEMS APPROACH TO EMOTIONS IN MATHEMATICS LEARNING AND EDUCATION** Soohyun Baek (The University of Sydney)
11:30	**A STUDY ON THE EFFECTS OF MATHEMATICAL TOPICS AND ACTIVITY TYPES ON ELEMENTARY SCHOOL STUDENTS' MATHEMATICS ANXIETY** Chiao-En Wang (National Taipei University of Education)
11:50	**THE EFFECTIVENESS OF INQUIRY-BASED TEACHING INTEGRATED WITH COMPUTATIONAL THINKING ON MATHEMATICS ATTITUDES IN VOCATIONAL HIGH SCHOOLS** Lin-Chieh Tsai (Taipei Municipal Muzha Vocational High School)
12:00	**FOSTERING A GROWTH MINDSET IN ELEMENTARY MATHEMATICS INSTRUCTION** Giwoo Kwak (Daeheung Elementary School)

July 20, Sunday	
only proceedings	**THE TRENDS IN INSTRUCTIONAL APPROACH AND THEIR EFFECTS ON STUDENTS' MATHEMATICS ATTITUDES: AN ANALYSIS OF TIMSS DATA** Rae Yeong Kim (Korea Institute for Curriculum and Evaluation)
10:40	**IDENTIFYING APPRAISAL PROFILES AND THEIR RELATIONS WITH ACADEMIC EMOTIONS IN MATHEMATICS** Xin Chen (East China Normal University)
11:00	**AFFECTIVE COMPUTING AND NLP-DRIVEN METHODOLOGIES FOR MEASURING PROBLEM-SOLVING IN MATHEMATICS EDUCATION** I-Ping Wan (National Chengchi University)
11:10	**DEVELOPMENT OF TEACHING AND LEARNING MODEL USING META-AFFECT FOR COLLABORATIVE PROBLEM SOLVING IN MATHEMATICS** Bumi Kim (Wonkwang University)
11:20	**THE STUDY OF PRIMARY SCHOOL STUDENTS' OPINIONS IN MATHEMATICAL LITERACY-ORIENTATION QUESTIONS** Jing-Wen Chiu (Taoyuan Municipal Bade Elementary School)
11:40	**DATA LITERACY IN MATHEMATICS AND SCIENCE TEACHING: A NARRATIVE EXPLORATION OF CLASSROOM PRACTICES, TEACHER EFFICACY, AND ATTITUDE** Rachel Dorcas A. Lim (University of the Philippines)
11:50	**DEVELOPMENT OF AN ASSESSMENT TOOL FOR MEASURING STEM HABITS OF MIND** Hye-Young Byeon (Pusan National University)

July 21, Monday	
10:30	**A STUDY ON VALUE FORMATION AND TEACHER INFLUENCE OVER THREE YEARS OF JUNIOR HIGH SCHOOL IN JAPAN** Hiroshi Ishii (Hokkaido University of Education)
10:40	**VALUES ALIGNMENT IN TEACHERS' PERSPECTIVE: A STUDY IN CHINA** Haomin Fang (The University of Melbourne)
10:50	**INVESTIGATING VALUES OF TEACHERS IN COURSES FOR LOW-PERFORMING UPPER-SECONDARY STUDENTS** Justine Sakurai (The University of Melbourne)
11:10	**INTELLECTUAL HUMILITY AND MATHEMATICS PERFORMANCE** Frederick K. S. Leung (Beijing Normal University)
11:20	**A STUDY ON THE RELATIONSHIP BETWEEN MATHEMATICS ADVERSITY QUOTIENT AND MATHEMATICAL ACHIEVEMENT OF SECOND-GRADE JUNIOR HIGH SCHOOL STUDENTS —THE MEDIATING ROLE OF MATHEMATICAL ANXIETY** Yumei Xia (Beijing Normal University)
11:30	**USING HISTORICAL CONTEXTS IN NON-HISTORICAL MATHEMATICS COURSES.** Amanda Jo Meiners (Northwest Missouri State University)
11:50	**EMOTIONAL DIMENSIONS AND SOCIAL SOLIDARITY IN MATHEMATICS EDUCATION: EVERYONE ENJOYS GAMES - OR DO THEY?** Linda Bonne (Victoria University of Wellington)
12:00	**RETHINKING NEO-CONFUCIAN CONCEPTS OF LEARNING & WELLBEING** Emily Sum (The University of Melbourne)
12:10	**A STUDY ON CHILDREN'S PERCEPTIONS OF "GOOD EXPLANATION" IN ARITHMETIC** Shota Mastumoto (Shimane University)

Topic Study Groups 8

The Essence of Mathematics Education with Equity and Culture

Chair: Catherine P. Vistro-Yu (Ateneo de Manila University)
Co-Chair: Sri Adi Widodo (Universitas Sarjanawiyata Tamansiswa), Takuya Baba (Hiroshima University)
Corrdinator: Sungjae Moon (Chungbuk National University)

July 19, Saturday	
10:30	**FROM TRADITION TO TECHNOLOGY: RETHINKING MATHEMATICS EDUCATION THROUGH ETHNOMATHEMATICS IN INDONESIA** Nur Robiah Nofikusumawati Peni (Universitas Ahmad Dahlan)
10:45	**INDIGENOUS EPISTEMOLOGY AND ETHNOMATHEMATICS: INSIGHT FROM BHUJEL COMMUNITY** Jaya Bishnu Pradhan (Tribhuvan University)
11:00	**A STUDY ON MATHEMATICAL CURRICULUM IN AN INDIGENIOUS SCHOOL IN TAIWAN: FOCUSING ON THE PROCESS OF CURRICULUM DESIGN** Tai-Jung Lee (National Academy for Educational Research)

11:15	**MATHEMATICS ANXIETY ACROSS ETHNIC COMMUNITIES IN NEPAL** Krishna Chandra Paudel (Tribhuvan University)
11:30	**A STUDY ON MATHEMATICS CURRICULUM AT AN INDIGENIOUS SCHOOL IN TAIWAN: FOCUSING ON THE PRODUCTION OF MATHEMATICAL TASK** Ying-Hsuan Lee (National Academy for Educational Research)
11:45	**A PRACTICAL STUDY ON THE INDIGENOUS RITUAL CULTURE IN ELEMENTARY SCHOOL INQUIRY EDUCATION** Felisa (Hsueh-Yun) Yu (National Changhua University of Education)
12:00	**SURVEY AND STUDY OF "ETSUKI-SANGAKU," MATHEMATICAL TABLETS WITH ILLUSTRATIONS, IN FUKUI PREFECTURE, JAPAN, AND EMPIRICAL RESEARCH ON ITS UTILIZATION FOR MATHEMATICS EDUCATION** Hiroshi Kazama (University of FUKUI)

July 20, Sunday	
10:30	**SURVIVING MASTERY IN MATHEMATICS EDUCATION** Nikki Mann (Simon Fraser University)
10:45	**SECONDARY SCHOOL STUDENTS' LEVEL OF INVENTIVE THINKING IN MATHEMATICAL LEARNING** Roslinda Rosli (The National University of Malaysia)
11:00	**DIVERSE VALUES AND MATHEMATICAL THINKING FOSTERED THROUGH SOCIALLY OPEN-ENDED PROBLEMS: SIMPLIFICATION OF SOCIAL CONTEXTS** Yuichiro Hattori (Okayama University)
11:15	**DIGITAL MATHEMATICS STORYTELLING IN INDONESIA, VIETNAM, AND THE ASIAN DIASPORA: EQUITY, CULTURE, AND COMMUNITY THROUGH NARRATIVE IDENTITIES** Theodore Chao (California State University)
11:30	**CULTIVATING CRITICAL MATHEMATICS CONCIOUSNESS IN PRE-SERVICE TEACHERS THROUGH DIGITAL RESOURCES AND THE RWC MODEL** Artorn Nokkaew (Naresuan University)
11:45	**ETHICAL FOUNDATIONS FOR EXPLORING INCLUSIVE SCIENCE AND MATHEMATICS EDUCATION** Mitsuru Matsushima (Kagawa University)
12:00	**NEW POSSIBILITIES FOR PROBLEM FINDING THROUGH MATHEMATICAL MODEL AND CARING** Rintaro Ueda (Tokyo Gakugei University)

July 21, Monday	
10:30	**Small Group Discussion**
11:30	**TSG 8 Group Reporting**
12:00	**TSG 8 General Discussion**
12:15	**TSG 8 Wrap-up and Closing**

Topic Study Groups 9

The Essence of Mathematics Education in Undergraduate Level

Chair: Vilma Mesa (University of Michigan)
Co-Chair: Hans Stefan Siller (University of Würzburg), Tatsuya Mizoguchi (Tottori University)
Corrdinator: Sunyoung Han (Sungkyunkwan University)

July 19, Saturday	
11:30	**ON THE HURDLES OF PROBLEM SOLVING IN UNDERGRADUATE MATHEMATICS DISCOURSE** Igor' Kontorovich (The University of Hong Kong)
11:45	**THE ROLE OF COGNITIVE RESOURCES IN MATHEMATICS PROBLEM SOLVING** Xin Li (Suzhou University of Technology)
12:00	**AN ANALYSIS OF STUDENTS' BASIC MENTAL MODELS OF THE INTEGRAL: THE CASE OF PHYSICAL CHEMISTRY** Christian Heinz (Julius Maximilian University of Würzburg)

July 20, Sunday	
10:30	**A CASE STUDY ON STUDENTS' SELECTIVE NOTE-TAKING IN UNDERGRADUATE NUMBER THEORY LECTURE** Jeong-Won Noh (Busan National University of Education)
10:45	**ENHANCEING MATHEMATICS STUDENT TEACHERS' UNDERSTANDING IN ACTIVE LEARNING THROUGH ABSORB-DO-CONNECT LEARNING FRAMEWORK** Wanintorn Poonpaiboonpipat (Naresuan University)
11:00	**PROVING ACTIVITIES IN A GRAPH-THEORETICAL INQUIRY FOR PRE-SERVICE TEACHERS: A PRELIMINARY ANALYSIS** Minh Hong Doan Le (Hiroshima University)
11:15	**CONCEPT-BASED LEARNING IN GEOMETRY WITH GEOGEBRA IN PAPUA NEW GUINEA PRIMARY TEACHERS COLLEGES** Hanson Wambu (Naruto University of Education)
11:30	**PRESERVICE MATHEMATICS TEACHERS' CONFIDENCE TO TEACH STATISTICS** Hyung Won Kim (University of Texas Rio Grande Valley)
11:45	**PRESERVICE SECONDARY TEACHERS' PROOF ANALYSIS IN SPHERICAL GEOMETRY** Younggon Bae (The University of Texas Rio Grande Valley)
12:00	**GUIDED REINVENTION OF THE DEFINITIONS OF REDUCIBLE AND IRREDUCIBLE ELEMENTS** Younggon Bae (The University of Texas Rio Grande Valley)

July 21, Monday	
10:30	**TEACHING MATERIALS FOR DIFFERENTIAL EQUATIONS WITH APPLICATION TO STEM PROBLEMS** Satoru Takagi (Waseda University)
10:45	**A STUDY OF ACTIVITIES USING GEOGEBRA IN METHOD OF LAGRANGE MULTIPLIER** Manabu Yoshida (Yamato University)

11:00	**ROLE OF MATH DUAL ENROLLMENT ON STUDENT OUTCOMES** Jungmin Lee (University of Kentucky)
11:15	**DECISION-MAKING IN A DESIGN-BASED RESEARCH APPROACH TO COLLABORATIVE DEVELOPMENT OF INTERACTIVE TEXTBOOK QUESTIONS** Vilma Mesa (University of Michigan)

Topic Study Groups 10

The Essence of STE(A)M Education

Chair: Hyunyi Jung (Texas A&M University)
Co-Chair: Masitah Shahrill (Universiti Brunei Darussalam), Hyunkyoung Yoon (California State Polytechnic University, Pomona)
Corrdinator: Yujin Lee (Kangwon National University)

July 19, Saturday	
10:40	**IDENTIFYING ZAMBIAN GRADE 10 STUDENTS' 21ST CENTURY SKILLS IN CROSS-CURRICULAR LEARNING OF MATHEMATICS AND HOME ECONOMICS** Koki Nakashima (Yokohama National University)
11:00	**EXPLORING INQUIRY-BASED LEARNING IN SENIOR HIGH SCHOOL MATHEMATICS: A STUDY ON FINALIZED SRPS** Mitsuhiro Iwata (Senior High School at Otsuka, University of Tsukuba and Waseda University)
11:30	**A STUDY ON THE ASSESSMENT OF CREATIVE PROBLEM-SOLVING COMPETENCE USING FERMI ESTIMATION** Jungin Heo (Samjung High School)
11:50	**ENHANCING STE(A)M EDUCATION THROUGH PROJECT-BASED LEARNING: A FOCUS ON AGE-PERIOD-COHORT MODELING** Diana Jasmin S. Costales (University of Santo Tomas)

July 20, Sunday	
10:40	**EVALUATING PRACTICAL STEAM ACTIVITIES FOR STUDENT LEARNING AND ENGAGEMENT** Masitah Shahrill (Universiti Brunei Darussalam)
11:10	**EMPOWERING FEMALE MATHEMATICIANS FOR THE FUTURE OF STEAM IN BRUNEI DARUSSALAM** Masitah Shahrill (Universiti Brunei Darussalam)
11:30	**INTERDISCIPLINARY APPROACHES TO CONNECT MATHEMATICS AND SCIENCE: A CASE STUDY OF PRIMARY TEACHERS' ADAPTIVE EXPERTISE** Wanty Widjaja (Deakin University)

July 21, Monday	
10:40	**ENHANCING SELF-REGULATION AND MOTIVATION THROUGH STEM PROJECT-BASED LEARNING** Yujin Lee (Kangwon National University)
11:00	**FOSTERING AGENCY IN MATHEMATICAL MODELLING FOR CRITICAL CITIZENSHIP** Jimbo Juanito B. Villamor (Ateneo de Manila University and Surigao del Norte State University)
11:30	**INVESTIGATING TEACHER'S DESIGN OF MATHEMATICAL MODELLING TASKS TO FOSTER MATHEMATICAL LITERACY** Sakon Tangkawsakul (Kasetsart University)
11:50	**FROM STEM OVER STEAM TO STEEM BUILT ON ECONOMICS** Allan Tarp (MatheCademy.net)

Contents

Topic Study Groups

Topic Study Groups 1

PRESERVICE TEACHERS' BELIEFS AND VIEWS ON CURRICULUM AND TEACHER GUIDE COMPONENTS 5
Ji-Eun Lee (Oakland University)

DO DIGITAL CURRICULUM RESOURCES SUPPLEMENT THE EDUCATIVE FEATURES OF TEACHER GUIDEBOOKS? 10
MinYoung Oh (Graduate School of Korea National University of Education)

A GUIDING FRAMEWORK FOR DESIGNING GRAPHING TASKS FOR STUDENT LEARNING 15
Hwa Young Lee (Texas State University)

THE IMPACT OF TEACHING MATERIAL FORMATS ON TWO-STEP PROBLEM SOLVING: A COGNITIVE LOAD PERSPECTIVE 21
Yen-Ting Lai (National Changhua University of Education)

EQUITY IN THE KOREAN ELEMENTARY MATHEMATICS CURRICULUM 25
MinKyung Kim (Graduate School of Korea National University of Education)

A COMPARISON OF THE CATEGORIES OF NON TEXT ELEMENTS IN HIGH SCHOOL TEXTBOOKS - A CASE STUDY OF PEP A VERSION AND A-LEVEL VERSION 30
Xiaolin Yuan (East China Normal University)

A COMPARATIVE STUDY OF DERIVATIVE CONTENT IN HIGH SCHOOL MATHEMATICS TEXTBOOKS 36
Chunxia Qi (Beijing Normal University)

A COMPARATIVE STUDY ON THE PRAXEOLOGICAL ANALYSIS OF ELEMENTARY SCHOOL MATHEMATICS TEXTBOOKS IN KOREA AND FINLAND: FOCUSING ON SYMMETRY AND TRANSFORMATION 41
Jaehyeon Park (Seoul National University)

DE/COLONIALITY IN MATHEMATICS CURRICULUMS OF THE PHILIPPINES AND INDIA 46
Harita Raval (Stockholm University)

A STUDY OF PROOF TASKS IN SECONDARY SCHOOL MATHEMATICS TEXTBOOKS IN SINGAPORE 51
Weng Kin Ho (National Institute of Education)

A STUDY ON EXPLORING THE CONTEXTUAL FEATURES OF QUESTIONS IN MATHEMATICS TEXTBOOK 58
Yi-An Cho (National Academy for Educational Research)

ACTION RESEARCH OF IMPLEMENTING A PROBLEM-BASED LEARNING MODEL INTO THE CURRICULUM OF TEACHING MODELS FOR ELEMENTARY MATHEMATICS GIFTED STUDENTS 62
Yuan Yuan (National Taichung University of Education)

FROM CLASSROOM TO REAL LIFE: EMBRACING MATHEMATICAL MODELLING IN HONG KONG'S CURRICULUM—INSIGHTS FROM SINGAPORE 66
Ka Shing Chui (The Education University of Hong Kong)

EFFECTIVENESS OF THE TEACHING STRATEGY USING A CONFLICT MAP IN LESSONS ON MULTIPLICATION WITH DECIMAL FRACTIONS 74
Tadayuki Kishimoto (University of Toyama)

MATHEMATICAL MODELING IN HONG KONG PRIMARY MATHEMATICS EDUCATION: A CONTENT ANALYSIS OF SCHOOL CURRICULUM 79
Hange Yun (The Education University of Hong Kong)

AN ANALYSIS OF THE TYPES OF CONNECTIONS BETWEEN THE KOREAN MATHEMATICS COLLEGE SCHOLASTIC ABILITY TEST AND EBS-LINKED MATHEMATICS TEXTBOOKS 84
Minsik Woo (Seoul National University)

A METHOD OF TEXTBOOK ANALYSIS FROM A PRAXEOLOGICAL PERSPECTIVE: THE CASE OF STATISTICS 89
Seitaro Hanasaki (Graduate School of Humanities and Social Sciences in Hiroshima University)

PREALGEBRAIC CONCEPTS IN SOLVING NUMBER SENTENCES: A TEXTBOOK ANALYSIS 94
Maria Alva Aberin (Ateneo de Manila University)

THE HIGHER-ORDER THINKING SKILLS (HOTS) CONTENT ANALYSIS ON MATHEMATICS TEXTBOOKS OF SENIOR HIGH SCHOOL 99
Haili Liang (Beijing Normal University)

DEVELOPING AND VALIDATING IB-BASED MATHEMATICS ASSESSMENT TASKS: FOCUSING ON STUDENT THINKING AND RELIABILITY 104
Yoonjoo Cho (Seoul National University)

FROM COUNTABLE TO UNCOUNTABLE: DEVELOPING TEACHING MATERIALS FOR IRRATIONAL NUMBERS 110
Toshikazu Ikeda (Yokohama National University)

INSIGHTS FROM GERMAN SECONDARY STATISTICAL EDUCATION ON THE DEVELOPMENT OF RISK LITERACY 115
Che-Yu Hsu (National Central University)

CONTEXT IN STATISTICAL TASKS: INSIGHTS FROM LOWER SECONDARY MATHEMATICS TEXTBOOKS IN THAILAND 120
Dhanachat Anuniwat (Hiroshima University)

EXPLORING DIGITAL LITERACY IN THE 2022 REVISED KOREAN MATHEMATICS CURRICULUM AND TEXTBOOKS 126
Jungeun Yoon (Seoul National University)

A DECOLONIZED CURRICULUM AND CHILDREN'S BUNDLE-NUMBERS WITH UNITS MAY REACH THE UN DEVELOPMENT NUMERACY GOAL 132
Allan Tarp (MATHeCADEMY.net)

GRADE 10 CALCULUS: WHY AND HOW 137
Wei-Chang Shann (National Central University)

TRENDS AND PERSPECTIVE IN RESEARCH ON DIGITAL MATHEMATICS TEXTBOOKS: INSIGHTS FROM A SYSTEMATIC REVIEW 141
Hyosup Shim (Seoul National University)

SOME LANGUAGE ISSUES IN DEFINING A NEW MATHEMATICAL CONCEPT: THE CASE OF JAPANESE SECONDARY SCHOOL STUDENTS 146
Shogo Murata (Nippon Sport Science University)

DEVELOPMENT OF TEACHING MATERIALS ON THE LOCUS OF THE CIRCUMCENTER OF A REGULAR POLYGON 150
Yuki Osawa (Kaichi Mirai Junior and Senior High School)

GEOMETRIC CONSTRUCTION IN TAIWANESE AND JAPANESE JUNIOR HIGH SCHOOL TEXTBOOKS 154
Yi-Wen Su (University of Taipei)

Contents

Topic Study Groups 2

TEACHING AND LEARNING OF SOLID GEOMETRY IN PRIMARY EDUCATION IN NICARAGUA: A DIDACTIC PROPOSAL BASED ON THE IMPLEMENTATION OF THE VAN HIELE MODEL AND THE PROBLEM-SOLVING APPROACH 165
Douglas Jared Martínez Cruz (Naruto University of Education)

FOURTH-GRADE STUDENTS' JUSTIFICATIONS OF THE PROPERTIES OF EVEN AND ODD NUMBERS 170
Hae Song Oh (Graduate School of Korea National University of Education)

A STUDY ON MANIPULATIVE ACTIVITIES FOR THE UNDERSTANDING SUBSUMPTION RELATIONS AMONG OF FIGURES : THROUGH CLASSES ON QUADRILATERALS FOR SIXTH-GRADE ELEMENTARY SCHOOL STUDENTS 175
Daisuke Itagaki (Graduate Student at the Graduate School of Education, Shimane University)

USING TEACHING WITH VARIATION IN AFRICAN MATHS CLASSROOMS: A PARTICIPATORY CASE STUDY 180
Xumai Lin (University of Macau)

HOW TEACHERS' INSTRUCTION INFLUENCES STUDENTS' UNDERSTANDING IN THE AREA OF A CIRCLE 184
Yi-Lun Tsai (National Pingtung University)

IMPACT OF LESSONS EMPHASIZING A RELATIONAL UNDERSTANDING OF THE EQUAL SIGN: FOCUSED ON SAMENESS AND SUBSTITUTION MEANINGS 189
Juhyeon Kim (Korea National University of Education)

EXPLORING THE POTENTIAL OF CHATGPT IN MATHEMATICS EDUCATION: A STUDY ON EXPLANATIONS ON SUPERSCRIPT (-1) IN PEDAGOGY AND MATHEMATICS 194
Xiaoying Chen (China West Normal University)

NARRATIVES OF A HIGH SCHOOL TEACHER IN A MATHEMATICS CLASS 200
Mayumi Kawamura (Oita University)

EXPLORING THE CURRENT STATE OF GIFTED EDUCATION IN MATHEMATICS IN JAPAN: A CASE STUDY OF CLASSROOM PRACTICES IN A SUPER SCIENCE HIGH SCHOOL 204
Natsuki Uchikubo (University of Tsukuba)

PROPOSAL FOR WIDE-AREA DEVELOPMENT AND CLASSROOM PRACTICE USING ICT TEACHING MATERIALS THAT CAN BE CONSIDERED FROM DAILY LIFE 209
Satoru Sakanashi (Tokyo Metropolitan Minato Municipal Odaiba-Gakuen Koyo Junior High School)

EMBRACING STUDENTS' DIVERSITY IN MATHEMATICS THROUGH DIFFERENTITATED INSTRUCTION AND OPEN APPROACH: A FRAMEWORK DEVELOPMENT 213
Yoga Dwi Windy Kusuma Ningtyas (Hiroshima University)

TEACHING WHOLE-NUMBER MULTIPLICATION, COMBINING CHILDREN'S LEARNING TRAJECTORY OF PROPORTIONAL REASONING: A CLASSROOM-BASED STUDY 220
Hino Keiko (Utsunomiya University)

AN ANALYSIS OF STUDENT' FOCUS ON THE RELATIONSHIP BETWEEN ORIGINAL SIZE AND FRACTIONS: A CASE STUDY OF 2ND GRADE LESSONS ON "FRACTION" 226
Yuri Masutani (Shimane University)

THE USE OF MATHEMATICAL AND NATURAL LANGUAGE BY ELEMENTARY SCHOOL TEACHERS IN DIVISION LESSONS 231
Yun Hsia Pai (National Tsing Hua University)

CURRICULUM ALIGHMENT THROUGH MULTILITERACY IN MATHEMATICS EDUCATION: INSIGHTS FROM FINLAND 235
Eunji Kim (Tampere University)

ENHANCING MATHEMATICAL CREATIVITY USING THE CONCEPTS OF INCUBATION BREAK AND THE FIVE LEGS OF CREATIVITY 240
Rhett Anthony C. Latonio (Ateneo de Manila University)

MANAGING THE TRANSITIONS IN CONCRETE-PICTORIAL-ABSTRACT 244
Yew Hoong Leong (National Institute of Education, Nanyang Technological University)

GRADE 10 STUDENTS CONCEPTIONS OF PROOF - AN EXPLORATORY STUDY 248
Berinderjeet Kaur (National Institute of Education)

THE IMPACT OF PROBLEM POSING ON STUDENT'S MATHEMATICAL INQUIRY SKILLS : AN EMPIRICAL INVESTIGATION 252
Jung Sook Park (Jungwha High School)

PROBLEM-POSING PRACTICES: A CASE STUDY OF A FILIPINO SCIENCE HIGH SCHOOL MATHEMATICS TEACHER 257
Lester Hao (Ateneo de Manila University)

THE ROLE OF REFLECTION CARD IN FACILITATING APPROPRIATION PROCESSES IN MATHEMATICS EDUCATION 262
Shinya Nakao (Kansai University of Welfare Sciences)

USING STUDENT'S SELF-ASSESSMENT OF THEIR INTERPRETED PREPARATION FOR CLASS: A PILOT OF USING THE MASTU EVALUATION 266
Amanda Jo Meiners (Northwest Missouri State University)

ANALYSIS OF TYPES OF DEALING WITH THE CONTEXT OF TASK IN SIXTH GRADE STUDENTS' MATHEMATICAL MODELING 272
Nahee Kim (Seoul National University of Education)

DESIGNING MODELLING TASKS TO ENHANCE STATISTICAL THINKING: INTEGRATING STUDENTS' EXPERIENCES OF HITTING THE LITTLE TOE WITH SCIENTIFIC PROBLEMS 277
Daiki Fujiwara (Naruto University of Education)

A PROBABILITY MODELING APPROACH TO PROMOTE STUDENTS' CONNECTIONS ACROSS THREE PERSPECTIVES OF PROBABILITY 282
Minh Kiem Tran (Hue University of Education)

MATHEMATICS CLASSROOM MANAGEMENT IN EL SALVADOR ELEMENTARY SCHOOLS AND ITS RELATIONSHIP WITH STUDENTS' ATTITUDE AND ACADEMIC PERFORMANCE 287
Félix Abraham Guevara (Naruto University of Education)

RESULTS OF TEACHER QUESTION ANALYSIS IN A MATHEMATICS RESEARCH LESSON 292
Ganbaatar Tumurbaatar (Mongolian National University of Education)

TEACHERS INTERPRETING STUDENT WORK IN GEOMETRY: USING CURRICULAR TEXT TO SUPPORT INSTRUCTIONAL EXCHANGE 296
Soobin Jeon (University of Michigan)

A STUDY ON ASPECTS OF GRASPING QUANTITATIVE RELATIONS IN WORD PROBLEMS INVOLVING INVERSE CALCULATION : FOCUSING ON THE PROCESS OF DRAWING THE DIAGRAMS 301
Maoko Nakahata (Shimane University)

STREAMING IN SECONDARY MATHEMATICS: A VIEW FROM AUSTRALIA 306
Elena Prieto-Rodriguez (University of Newcastle)

Contents

Topic Study Groups 3

THE RELATIONSHIP BETWEEN MIDDLE SCHOOL STUDENTS' MATHEMATICS SELF-EFFICACY, MATHEMATICS ANXIETY, AND MATHEMATICAL LITERACY 315
Yi-Jhen Li (National University of Tainan)

AN EXPLORATORY STUDY OF MIDDLE SCHOOL STUDENTS' PERFORMANCE AND TYPICAL ERRORS IN MODELING TASKS 322
Qianhui Zhao (East China Normal University)

EXAMINING MASTERY OF COGNITIVE ATTRIBUTES IN MATHEMATICS USING SOUTH KOREAN NATIONAL ASSESSMENT OF EDUCATIONAL ACHIEVEMENT DATA 327
Jihyun Hwang (Korea National University of Education)

FROM ICT USE TO CREATIVE THINKING: THE MEDIATION OF ATTITUDES TOWARDS CREATIVE THINKING AMONG EAST ASIAN STUDENTS 331
Si Chen (Beijing Normal University)

FACTORS PREDICTING EAST ASIAN STUDENTS' PERCEIVED QUALITY OF MATHEMATICS INSTRUCTION: A MACHINE LEARNING APPROACH 337
Xu Wei (Beijing Normal University)

CREATIVITY IN MATH ASSESSMENTS 342
Yujin Lee (Kangwon National University)

A VALIDATION STUDY MEASURING INTERDISCIPLINARY MATHEMATICAL PROBLEM SOLVING ABILITY 348
Zhongtian Ji (Beijing Normal University)

THE EFFECTS OF TECHNOLOGICAL INNOVATION IN NATIONAL MATHEMATICS ASSESSMENT 353
Chang-Geun Song (Cheongju National University of Education)

STUDENTS' SELF-REFLECTION IN A GEOMETRY LESSON: PERSPECTIVES AND QUALITY 359
Satoshi Kusaka (Naruto University of Education)

COMMUNICATION SKILLS PERFORMANCE IN QUANTITATIVE RELATED LITERACY AMONG ELEMENTARY SCHOOL STUDENTS 364
Yun-Chi Tsou (Southern Illinois University Carbondale)

IN BUNDLENUMBER-MATH YOU JUST ASK THE BUNDLEBUNDLE BOARD 369
Allan Tarp (MATHeCADEMY.net)

A STUDY ON PERSPECTIVES FOR EVALUATING VISUAL REPRESENTATIONS IN MATHEMATICS LESSONS 376
Naoya Miwa (University of Tsukuba)

THE ROLE OF ORIGINAL PROBLEMS IN MATHEMATICAL PROBLEM POSING ASSESSMENTS 381
Ziyi Jin (Hiroshima University)

SCAFFOLDED INTERVIEW TASKS BUILD ON WRITTEN TASKS TO REVEAL ALGEBRAIC THINKING 386
Catherine Pearn (The University of Melbourne)

ENHANCING ADAPTIVE MATHEMATICS ASSESSMENT THROUGH GENERATIVE AI: INTEGRATING THE COGNITIVE DISTRACTOR GENERATION MODEL (CDGM) INTO THE GENERATIVE AI-BASED MULTI-STAGE ASSESSMENT (GAMSA) FRAMEWORK 391
Jin Min Chung (University of Iowa)

Topic Study Groups 4

EXPLORING LESSON STUDY TEAM COLLABORATION IN TEACHING POSITIVE AND NEGATIVE NUMBERS TO GRADE 7 STUDENTS 401
Noppasorn Boonsena (Khon Kaen University)

COMPARISON OF LESSON PLANNING PROCESSES BY PRE-SERVICE MATHEMATICS TEACHERS: FROM THE PERSPECTIVE OF DESIGN RESEARCH 407
Hayato Hanazono (Miyagi University of Education)

COLLABORATIVE RESEARCH LEARNING: COMMUNITIES OF PRACTICE AMONG PRE-SERVICE AND IN-SERVICE MATHEMATICS TEACHERS 412
Mailizar Mailizar (Syiah Kuala University)

TRANSFORMING LESSON REFLECTION MEETING USING 'LEARNER OBSERVATION SHEET' 417
Naoki Yachimoto (Hokkaido University of Education)

SUSTAINING LESSON STUDY AS A PROFESSIONAL DEVELOPMENT MODEL: FROM THE TEACHERS' PERSPECTIVE 423
Edna Callanta (University of the Philippines National Institute for Science and Mathematics Education Development)

TEACHERS' EXPERIENCE OF REFLECTIVE PRACTICE IN THE OPEN CLASS 427
Narumon Changsri (Khon Kaen University)

INVESTIGATING PRESERVICE TEACHERS' CURRICULAR NOTICING ABILITY THROUGH TEXTBOOK ANNOTATION ACTIVITIES 432
Chunxia Qi (Beijing Normal University)

MATHEMATICS TEACHERS' TEXTBOOK UNDERSTANDING: A MEDIATION EFFECT ANALYSIS 438
Qinqiong Zhang (Fujian Normal University)

SUPPORTING EARLY CAREER TEACHERS' NOTICING: A FACILITATOR'S INTENT IN LESSON STUDY 443
Hyomin Kang (University of Tsukuba)

VISUAL PERCEPTION OF EXPERT AND NOVICE MATHEMATICS TEACHERS ACROSS STUDENT PROFILES 448
Rangmei Li (Beijing Normal University)

EXAMINING ADAPTIVE EXPERTISE OF EMERGING AND EXPERIENCED INSTRUCTIONAL COACHES THROUGH NOTICING 453
Sze Looi Chin (Massey University)

AN ANALYSIS OF MATHEMATICS TEACHER'S SUPPORT IN HIGH SCHOOLERS' INQUIRY: FOCUSING ON THE MOMENT OF SUPERVISIONS IN SRP 460
Toshihiko Ogawa (Waseda University)

TOWARDS REFASHIONING TEACHERS' TASK DESIGN IN TRIGONOMETRY: A PROFESSIONAL DEVELOPMENT MODEL 465
Jake Garnace (Ateneo de Manila University and University of Northern Philippines)

REFLECTING IN-SERVICE ELEMENTARY TEACHERS' MATHEMATICAL KNOWLEDGE FROM TEACHER PROFESSIONAL EDUCATION PROGRAM 471
Zetra Hainul Putra (Universitas Riau)

Contents

THE DEVELOPMENT OF DIDACTIC PRAXEOLOGIES IN LESSON DESIGN THROUGH SOCIAL RESOURCE INTERACTIONS: EVIDENCE FROM TWO NOVICE MATHEMATICS TEACHERS 476
Xingfeng Huang (Shnaghai Normal University)

A STUDY ON INSTRUCTIONAL REPRESENTATIONS IN DIFFERENT TEACHING SCENARIOS: INTRODUCING TEXT MINING TECHNIQUES 482
Ling-Yi Huang (National Taipei University of Education)

THE ROLE OF PROFESSIONAL DEVELOPMENT IN ENHANCING MATHEMATICAL CREATIVITY: A CASE STUDY OF NOVICE TEACHERS 487
Yujin Lee (Kangwon National University)

INTERVENTIONS TO DEVELOP PRE-SERVICE MATHEMATICS TEACHERS' COMPETENCE IN DIFFERENTIATED INSTRUCTION: A SCOPING REVIEW 491
Iden Rainal Ihsan (National Taiwan Normal University)

NAVIGATING STATISTICS AND PROBABILITY INSTRUCTION THROUGH MKT: A CONTENT ANALYSIS OF TEACHER REFLECTIONS 497
Joseph Ma. Steven Cabalo (Ateneo de Manila University)

MANIFESTATIONS OF TEACHER AGENCY IN MATHEMATICS EDUCATION: PERSONAL, SOCIAL, AND CONTEXTUAL INSIGHTS FROM A GRADUATE COURSE ON TEACHING STATISTICS AND PROBABILITY 502
Nhoriel Toledo (Centro Escolar University, Ateneo de Manila University)

LESSON PLANNING AS A REFLECTION OF PRE-SERVICE MATHEMATICS TEACHERS' MODELING TEACHING COMPETENCE 506
An Thi Tan Nguyen (Hue University)

PERCEPTIONS AND PRACTICES OF FILIPINO JUNIOR HIGH SCHOOL MATHEMATICS TEACHERS ON PROBLEM-POSING 513
John Patrick Cultura (La Salle College Antipolo)

TAIWANESE SECONDARY MATHEMATICS TEACHERS' PERSPECTIVES ON CALCULUS INSTRUCTION: INSIGHTS FROM THE CURRICULAR AND CONTEXTUAL FACTORS 518
Pin-Chen Guo (National Taiwan Normal University)

CONCEPTIONS OF MATH TEACHING AND LEARNING IN CONVERSATIONS ABOUT CURRICULAR CHANGE 523
Nurdan Turan (New York University)

TEACHERS' INTENTION TO IMPLEMENT BLENDED LEARNING IN ALGEBRA: INSIGHTS FROM A PILOT STUDY 530
Nursaadah Jailani (The National University of Malaysia)

INNOVATION IN MATHEMATICS TEACHER EDUCATION THROUGH A STEAM-BASED GLOBAL PROFESSIONAL LEARNING COMMUNITY MODEL 535
Jungin Heo (Samjung High school)

DEVELOPMENT OF A TEACHER TRAINING MODEL FOR ENHANCING AI DIGITAL TEXTBOOK UTILIZATION COMPETENCY 540
Seo Hyun Ahn (University of Ajou)

"AM I BEING TOO UNHELPFUL?" MATHEMATICS TEACHER DECISION-MAKING & RESPONSIBILITY 545
Samantha A. Marshall (North Carolina State University)

ANALYZING STUDENT REFLECTIONS IN TEACHING PRACTICUM TO ENHANCE MATHEMATICS INSTRUCTION COURSES 551
Hideyo Makishita (Yamato University)

INVESTIGATING BEST PRACTICES OF HIGH SCHOOL MATH TEACHERS 555
Hyunkyoung Yoon (California Polytechnic Univsersity, Pomona)

EVALUATION OF STUDENTS' COUNTEREXAMPLES BY PRE-SERVICE PRIMARY SCHOOL TEACHERS 560
Kayo Miura (Hiroshima University)

SHORT-DURATION OBSERVATION INSTRUMENT FOR ELEMENTARY MATHEMATICS TEACHERS' COMPETENCE (SDOI-EMTC) 565
Ting-Chun Kuo (National Taipei University of Education)

KNOWLEDGE NEEDED FOR TEACHING MATHEMATICS TO STUDENTS WITH LEARNING DISABILITIES 570
Roslinda Rosli (The National University of Malaysia)

PRESERVICE MATHEMATICS TEACHERS' EMERGING PEDAGOGICAL CONTENT KNOWLEDGE IN A LESSON STUDY-MEDIATED PRACTICE TEACHING 574
Monalisa Sasing (University of the Philippines National Institute for Science and Mathematics Education Development)

EXPLORING THE DEVELOPMENT OF PRIMARY MATHEMATICS PRE-SERVICE TEACHERS' TEACHING SELF-EFFICACY: A PRELIMINARY ANALYSIS 580
Hui Min Chia (University of Macau)

A TEACHER'S DEVELOPMENT OF STUDENTS' METACOGNITION IN A MATHEMATICS CLASSROOM 585
Leng Low (Academy of Singapore Teachers, Ministry of Education)

SHIFT IN TEACHER'S PERSPECTIVES IN MATHEMATICS BASED ON INTERVENTIONS ON INDIAN KNOWLEDGE SYSTEMS AND DISTINCTIONS BETWEEN EMPIRICAL AND FORMAL MATHEMATICS 590
Sanjeev Ranganathan (C3STREAM Land, SAIIER)

Topic Study Groups 5

ANALYSIS OF THIRD-GRADE STUDENTS' STRATAGIES FOR SOLVING MULTI-DIGIT MULTIPLICATION PROBLEMS 601
Johyeon Chang (Korea National University of Education)

INVESTIGATING THIRD GRADERS' PLACE VALUE CONCEPT AND MULTI-DIGIT ADDITION THINKING STRATEGIES 606
Hannah Mortensen (Saint Mary's College)

TEACHERS SEE 1 NUMBER WHERE KIDS SEE 3 NUMBERINGS IN 507, WHO IS NUMBER-BLIND? 612
Allan Tarp (MATHeCADEMY.net)

CHARACTERISTICS OF MATHEMATICAL NEGOTIATION THAT PROMOTE FRACTION CONCEPT FORMATION : AN ANALYSIS OF CLASS UTILIZING THE NUMBER LINE 617
Taketo Shimomura (Shimane University)

"DIVIDING THE DENOMINATORS AND NUMERATORS SEPARATELY": SIXTH GRADERS' STRATEGIES OF THE MEASUREMENT FRACTION DIVISION 622
Jeongwon Kim (Bongmyeong Elementary School)

HOW JAPANESE CHILDREN FOCUSING ON SPONTANEOUS PATTERNS CREATE PATTERNS? 627
Nanae Matsuo (Chiba University)

Contents

A VISUALIZATION OF THE STRUCTURE OF KNOWLEDGE BASED ON THE CONNECTIONS OF MATHEMATICAL CONCEPTS 632
Ye-Jin Lee (Seoul National University of Education)

CRITICAL THINKING IN MATHEMATICS EDUCATION: A SYSTEMATIC REVIEW 638
Ersi Cha (East China Normal University)

IS THERE A CONNECTION BETWEEN REASONING OF MATHEMATICS AND SCIENCE: A DOCUMENT ANALYSIS OF TIMSS ASSESSMENT FRAMEWORK 643
Jin Mun (Seoul National University)

CONTEXT-BASED LEARNING FOR ELEMENTARY STUDENTS: CONSTRUCTING REPRESENTATIVENESS IN ARITHMETIC MEAN USING AIC THEORY 648
Youngin Kim (Seoul National University Graduate School)

A STUDY ON SPATIAL SKILLS TO ENHANCE MENTAL OPERATIONS IN ELEMENTARY SCHOOL STUDENTS 654
Satoshi Watanabe (Jissen Women's University)

EARLY SPATIAL REASONING: THE CASE OF PATH FINDING AND CONSTRUCTION 659
Kazuya Kageyama (Hiroshima University)

A DESIGN-BASED STUDY ON INTEGRATING CREATIVE AND CRITICAL THINKING INTO MATHEMATICS-GROUNDING ACTIVITIES: PROPORTIONAL REASONING 664
Bo-Ting Liu (National Taiwan Normal University)

SURVEY ON THE FORMATION OF THE CONCEPT OF SPEED 669
Masahide Tsuda (Soka University)

EXPLORING THE ABSTRACTION PROCESS DURING ALGEBRA PROBLEM-SOLVING IN A PROGRAMMING-ENHANCED ENVIRONMENT 674
Haoyi Wang (University of Pennsylvania)

STRUCTURE SENSE AND FLEXIBILITY IN ALGEBRAIC PROBLEM-SOLVING: AN EXPLORATORY STUDY 682
Haoyi Wang (University of Pennsylvania)

A COMPARATIVE STUDY OF INVERSE FUNCTIONS AMONG KOREAN AND CHINESE HIGH SCHOOL STUDENTS 687
Wenting Liu (Kaili University)

GRADUATE STUDENTS IN EDUCATION' UNDERSTANDING OF ALGORITHMIC THINKING IN MATHEMATICS CLASSROOM THROUGH A COLORING BOOK WORKSHOP 692
Wiramon Kanlayarat (Khon Kaen University)

EXAMINATION OF MATHEMATICAL DISCOURSE FOR INDIRECT PROOF OF THE CONVERSE OF THE INSCRIBED ANGLE THEOREM 698
Keiko Watanabe (Shiga university)

ENGAGING THROUGH REASONING: A CULTURAL-HISTORICAL ACTIVITY THEORY (CHAT) ANALYSIS OF STUDENTS' ENGAGEMENT IN ACCOUNTABLE TALK IN A HYFLEX MATHEMATICS CLASSROOM 703
Edrian Peter Villanueva (De La Salle Univerisity)

THE COHERENT AND STRATEGIC USES OF PIAGET'S NOTION OF PERTURBATION IN MATHEMATICS (TEACHER) EDUCATION 708
Biyao Liang (The University of Hong Kong)

USING ACE TEACHING CYCLE IN DEVELOPING A PRELIMINARY GENETIC DECOMPOSITION: A PROPOSED FRAMEWORK 713
Christian Abasta (Ateneo de Manila University)

THE EFFECTS OF DIFFERENT INSTRUCTIONAL REPRESENTATIONS ON THE STUDENT ENGAGEMENT IN ELEMENTARY MATHEMATICS REMEDIAL TEACHING 718
Lan-Hsin Zhang (National Taipei University of Education)

THE TEACHER AS A MEDIATOR IN A PROPORTIONAL REASONING TASK 723
Elaine Yu Ling Cai (Nanyang Technological University)

ANALYZING INTERDEPENDENCY OF COGNITIVE ATTRIBUTES IN MATHEMATICS AND SCIENCE 727
Jihyun Hwang (Korea National University of Education)

CHILDREN'S MATHEMATICAL NOTICING IN AN ONLINE PEER-GENERATED STRATEGY VIDEO 731
Ho-Chieh Lin (Qing Shui Elementary School)

READING COMPREHENSION OF REAL-WORLD PROBLEMS: AN ERP STUDY 736
Hui-Yu Hsu (National Tsing Hua University)

PHONE FREQUENCY DISTRIBUTIONS IN MATHEMATICAL LANGUAGE: A STATISTICAL ANALYSIS OF MANDARIN 742
Pu Yu (National Chengchi University)

A NEUROCOGNITIVE PERSPECTIVE ON THE DEVELOPMENT OF FRACTIONAL UNDERSTANDING BASED ON APOS THEORY 747
Doyeon Ahn (Korea National University of Education)

FALLACY OF COMPOSITION AND COIN TOSS SEQUENCES: COMPARING SEQUENCES WITH THE SAME HEADS-TO-TAILS RATIOS 751
Ippo Ishibashi (Okayama University)

Topic Study Groups 6

LEARNING TEACHING MATHEMATICS WITH INSTRUCTIONAL TECHNOLOGY: A PRAXEOLOGICAL ANALYSIS OF A GHANAIAN MATHEMATICS TEACHER EDUCATION COURSE 761
Farouq Sessah Mensah (Stockholm University)

EXPLORING PRESERVICE TEACHERS' INTEGRATION OF AI TOOL THROUGH AI-TPACK AND SAMR MODEL 766
Yujin Lee (Heungryong Elementary School)

PRE-SERVICE TEACHERS' SELF-EFFICACY OF USING GENERATIVE ARTIFICIAL INTELLIGENCE IN MATHEMATICS INSTRUCTION 772
Norbert Noster (University of Würzburg)

STUDENTS' PERCEPTIONS OF GENERATIVE ARTIFICIAL INTELLIGENCE AND UNDERSTANDING OF MATHEMATICAL PROOFS 777
Yujin Lee (Kangwon National University)

SIMULATED PRACTICE-BASED TEACHING FOR PRESERVICE TEACHERS USING AN AI CHATBOT 782
Taekwon Son (Bongmyong Elementary School)

HOW SECONDARY MATHEMATICS TEACHERS LEVERAGE AI-POWERED TOOLS TO ENHANCE TEACHING EFFICIENCY 786
Wing Kin Cheng (Hong Kong Metropolitan University)

Contents

EQT-TECH: FRAMEWORK FOR INTEGRATING TECHNOLOGY TO ADVANCE MATH THINKING AND LEARNING 791
Jennifer Suh (George Mason University)

EVOLVING TRENDS IN MATHEMATICS EDUCATION RESEARCH ON DIGITAL TECHNOLOGY: A DYNAMIC TOPIC MODELING ANALYSIS 795
Yeongjun Kim (Seoul National University)

INVESTIGATING PRESERVICE TEACHERS' USE OF A DYNAMIC DIGITAL TOOL TO SOLVE A FRACTION MULTIPLICATION PROBLEM 800
Mi Yeon Lee (Arizona State University)

USE OF GEOGEBRA IN ADDITION AND SUBTRACTION OF FRACTIONS WITH DIFFERENT DENOMINATORS IN MOZAMBIQUE 806
Fernando Macuacua Junior (Naruto University of Education)

'GREEDY PIG' FOR A MATHEMATICAL INQUIRY WITH PROGRAMMING IN SECONDARY SCHOOL 811
Dai Kishimoto (Waseda university)

STUDENTS' REASONING OF QUADRATIC GROWTH IN A PROGRAMMING-BASED ENVIRONMENT 816
Xuan Su (The University of Hong Kong)

ENHANCING COMPUTATIONAL THINKING THROUGH MACHINE LEARNING IN DAILY WASTE SORTING PROBLEM SITUATIONS 822
Anucha Koyata (Khon Kaen University)

CRITICAL REFLECTIONS ON ALGORITHMIC LEARNING DESIGN AND HUMAN EXISTENCE THROUGH HEIDEGGERIAN ONTOLOGY 828
Jiyoung Kim (Seoul Dungyang Elementary School)

EXPLORING THE LONG-TERM LEGACY OF COVID DISRUPTION TO MATHEMATICS TEACHING IN AUSTRALIAN PRIMARY SCHOOLS 834
Kathryn Holmes (Western Sydney University)

KOREAN TEACHERS' PERCEPTION ON AI-EMPOWERED DIGITAL TEXTBOOKS IN MATHEMATICS 838
Jukyung Park (Seoul Oryun Elementary School)

HARNESSING AI IN SINGAPORE PRIMARY MATHEMATICS CLASSROOM - REDEFINING THE ROLE OF A TEACHER 843
Suo Hui Chang (Academy of Singapore Teachers)

ANALYSIS OF THE STRUCTURING FEATURES OF CLASSROOM PRACTICE IN A TRIGONOMETRIC RATIOS LESSON FOR ACUTE ANGLES USING GENERATIVE AI 849
Jaeeun Joo (Mudung Middle school)

EXPLORING ACTOR-NETWORKS IN MATHEMATICS CLASSES USING GENERATIVE AI 853
Jungeun Yoon (Seoul National University)

UTILIZATION OF CHEATING-RESISTANT QUESTIONS USING STACK IN THE COURSE OF LINEAR ALGEBRA 858
Kentaro Yoshitomi (Osaka Metropolitan University)

STUDENT INTERACTIONS WITH CHATGPT: ANALYZING PROBLEM-POSING PROCESSES IN A NUMBER THEORY COURSE 863
Ah Ra Cho (Seoul National University)

THE DEVELOPMENT OF PRESERVICE MATHEMATICS TEACHERS' SPATIAL REASONING: STRENGTHS AND CHALLENGES 868
Lizhen Chen (Western Washington University)

DESIGNING AND REVISING DYNAMIC GEOMETRY TASKS BY PRE-SERVICE ELEMENTARY TEACHERS: THEORETICAL NETWORKING OF NOTICING AND TPACK 874
Nayoung Ku (Gyeonggi Science High School)

META-ANALYSIS OF THE EFFECTS OF EDUCATIONAL ROBOTICS ON K-12 STUDENTS' MATHEMATICS LEARNING 879
Ziyi Xiao (East China Normal University)

EXPLORING MATHEMATICAL NARRATIVES AND CITIZENS' COMMUNICATION IN KOREAN YOUTUBE SHORTS 886
Kyungwon Lee (Seoul National University)

DESIGNING AUGMENTED PAPER-BASED MATHEMATICS LEARNING ENVIRONMENT FROM AN EMBODIED COGNITION PERSPECTIVE: APPLICATION TO THE TRANSLATION OF QUADRATIC FUNCTIONS 891
Hyowon Wang (Sookmyung Women's University)

COLLABORATIVE PRACTICES IN VIRTUAL GROUP WORK ON DYNAMIC GEOMETRY TASKS 897
Younggon Bae (The University of Texas Rio Grande Valley)

PROVING ACTS WITH DIGITAL TOOLS FOR INQUIRING INTO MATHEMATICAL PROPOSITIONS IN LEARNING GEOMETRY 903
Keiko Watanabe (Shiga university)

CAPTURING COMPUTATIONAL THINKING IN EARLY MATHEMATICS EDUCATION: A CASE STUDY FROM A CHINESE KINDERGARTEN 907
Ying Zhang (University of Hong Kong)

INTERACTIVE HIGH SCHOOL APPS FOR BRIDGING MATHEMATICAL LEARNING: PERSPECTIVES FROM A HIGH SCHOOL IN QUEZON CITY, PHILIPPINES 912
Maria Alva Aberin (Ateneo De Manila University)

A PRELIMINARY STUDY ON 5- TO 6-YEAR-OLD CHILDREN'S SELECTION PREFERENCES AND SALIENT INFORMATION IN MATHEMATICAL SHORT VIDEOS. 918
Wan-Hua Deng (National Pingtung University Department of Early Childhood Education)

AN ANALYSIS ON THE DIGITAL ARTIFACT AFFORDANCE THROUGH ANT AND INSTRUMENTAL APPROACH 926
Jiyoung Kim (Seoul Dungyang Elementary School)

TOWARDS A DIGITAL MATHEMATICS TEACHING ANALYTIC FRAMEWORK 934
Erlina Ronda (University of the Philippines)

ELECTRONIC FEEDBACK IN FORMATIVE ASSESSMENT: ENHANCING STUDENT UNDERSTANDING IN MATHEMATICS 938
Huan Chin (Universiti Sains Malaysia)

PERCEIVED IMPACT OF AI-GENERATED FEEDBACK ON UNDERGRADUATE MATHEMATICS LEARNING IN GEOMETRIC PROOF 943
Lady Angela Rocena (University of the Philippines Integrated School)

ANALYSIS OF MATHEMATICAL PERFORMANCE AND REASONING PROCESS OF GENERATIVE AI: FOCUSING ON THE COLLEGE SCHOLASTIC ABILITY TEST 948
Sejun Oh (Hongik University)

Contents

SHIFT OF OUTPUT RESULTS WHEN INPUT TO THE GENERATIVE AI ARE CHANGED : A CASE STUDY OF THE 2024 NATIONAL STUDENT ACHIVEMENT SURVEY FOR GRADE 9 953
Ruka Kikuchi (Saitama university)

CHATGPT AS PROBLEM POSER: CONTEXTUALIZED AI-ASSISTED PROBLEM GENERATION USING STRUCTURED CHAIN-OF-THOUGHT PROMPTING 958
Joseph Ma. Steven Cabalo (Ateneo de Manila University)

POCKET CALCULATORS IN GRADE ONE TO PREDICT DIVISION TABLES 963
Allan Tarp (MATHeCADEMY.net)

AN ANALYSIS OF MATHEMATICAL CAPABILITIES OF GENERATIVE ARTIFICIAL INTELLIGENCE: FOCUSING ON CHATGPT O4-MINI-HIGH, COPILOT, GEMINI 2.5 PRO, AND DEEPSEEK R1 968
Hong Kyung Yun (Seoul National University)

NOTICING CLASSES OF PRESERVICE TEACHERS: RELATIONS TO TEACHING MOVES THROUGH AI CHATBOT SIMULATION 973
Taekwon Son (Bongmyeong Elementary School)

EXPLORING PRESERVICE TEACHERS' UNDERSTANDING OF FRACTION MULTIPLICATION WITH A TECHNOLOGICAL TOOL 978
Ji Nam Hwang (Yongjeong Elementary School)

A STUDY ON THE CONSTRUCTION OF ARCUATE FIGURES IN WASANSHO 984
Jun Yamada (Tsushima High School)

STUDENT VOICE WHILE USING ICT IN MATHEMATICS LESSONS: THE CASE OF INFECTION EPIDEMICS 990
Minoru Ohtani (Kanazawa University)

CO-CONSTRUCTING UNDERSTANDING OF PROGRAMMING-BASED MATHEMATICS TEACHING PRACTICES IN A COMMUNITY OF INQUIRY 995
Huiyan Ye (The Chinese University of Hong Kong)

EFFECTIVE USE OF SCRATCH FOR UNDERSTANDING REGULAR POLYGON IN NICARAGUAN PRIMARY EDUCATION 1000
Isaias Alberto Granados Gonzalez (Naruto University of Education)

FIRST-GRADE STUDENTS' IDEAS AROUND NEGATIVE NUMBERS IN MATHEMATICS-CODING INTEGRATED LESSONS 1005
Boram Lee (Utah State University)

THE EFFECTS ON STUDENTS' PROBLEM-SOLVING SKILLS OF ESPORTS-BASED MATHEMATICAL VIDEO LESSONS 1011
Ronie Jake Mamala (Mariano Marcos State University, University of the Philippines - Diliman)

A MID-CAREER MATHEMATICS TEACHER'S NOTICING OF STUDENTS' DATA INVESTIGATION PRACTICES WITH A DATA ANALYSIS AND VISUALIZATION TOOL 1015
Takashi Kawakami (Utsunomiya University)

USING AN AI CHATBOT TO DEVELOP PRESERVICE TEACHERS' RESPONSIVE TEACHING SKILLS 1021
Sheunghyun Yeo (Daegu National University of Education)

INVESTIGATING THE PEDAGOGICAL VALUE OF DIGITAL MANIPULATIVES BETWEEN US AND KOREA 1026
Sheunghyun Yeo (Daegu National University of Education)

Topic Study Groups 7

PROFILE ANALYSIS USING CLUSTERING TO EXPLORE STUDENTS' MATHEMATICS GRIT 1037
Yuan-Horng Lin (National Taichung University of Education)

EXPLORING THE RELATIONSHIP BETWEEN STUDENTS' PERCEPTIONS OF DOING WELL IN MATHEMATICS, MATHEMATICS CONFIDENCE, AND GROWTH MINDSETS 1042
Anni E (The University of Melbourne)

EFFECTS OF COLLABORATIVE CRITICAL MATHEMATICS IN A FLIPPED CLASSROOM ON STUDENTS' MATHEMATICAL DISPOSITION 1047
Eric Loyd P. Hilario (University of the Philippines Diliman)

MATHEMATICAL ANXIETY OF CHILEAN STUDENTS FROM A TECHNICAL-VOCATIONAL HIGH SCHOOL 1052
Claudia Vargas (Universidad de Santiago de Chile)

A DYNAMIC SYSTEMS APPROACH TO EMOTIONS IN MATHEMATICS LEARNING AND EDUCATION 1056
Soohyun Baek (The University of Sydney)

A STUDY ON THE EFFECTS OF MATHEMATICAL TOPICS AND ACTIVITY TYPES ON ELEMENTARY SCHOOL STUDENTS' MATHEMATICS ANXIETY 1060
Chiao-En Wang (National Taipei University of Education)

THE EFFECTIVENESS OF INQUIRY-BASED TEACHING INTEGRATED WITH COMPUTATIONAL THINKING ON MATHEMATICS ATTITUDES IN VOCATIONAL HIGH SCHOOLS 1064
Lin-Chieh Tsai (Taipei Municipal Muzha Vocational High School)

FOSTERING A GROWTH MINDSET IN ELEMENTARY MATHEMATICS INSTRUCTION 1069
Giwoo Kwak (Daeheung Elementary School)

THE TRENDS IN INSTRUCTIONAL APPROACH AND THEIR EFFECTS ON STUDENTS' MATHEMATICS ATTITUDES: AN ANALYSIS OF TIMSS DATA 1076
Rae Yeong Kim (Korea Institute for Curriculum and Evaluation)

IDENTIFYING APPRAISAL PROFILES AND THEIR RELATIONS WITH ACADEMIC EMOTIONS IN MATHEMATICS 1081
Xin Chen (East China Normal University)

AFFECTIVE COMPUTING AND NLP-DRIVEN METHODOLOGIES FOR MEASURING PROBLEM-SOLVING IN MATHEMATICS EDUCATION 1086
I-Ping Wan (National Chengchi University)

DEVELOPMENT OF TEACHING AND LEARNING MODEL USING META-AFFECT FOR COLLABORATIVE PROBLEM SOLVING IN MATHEMATICS 1091
Bumi Kim (Wonkwang University)

THE STUDY OF PRIMARY SCHOOL STUDENTS' OPINIONS IN MATHEMATICAL LITERACY-ORIENTATION QUESTIONS 1095
Jing-Wen Chiu (Taoyuan Municipal Bade Elementary School)

DATA LITERACY IN MATHEMATICS AND SCIENCE TEACHING: A NARRATIVE EXPLORATION OF CLASSROOM PRACTICES, TEACHER EFFICACY, AND ATTITUDE 1100
Rachel Dorcas A. Lim (University of the Philippines)

DEVELOPMENT OF AN ASSESSMENT TOOL FOR MEASURING STEM HABITS OF MIND 1105
Hye-Young Byeon (Pusan National University)

Contents

A STUDY ON VALUE FORMATION AND TEACHER INFLUENCE OVER THREE YEARS OF JUNIOR HIGH SCHOOL IN JAPAN 1112
Hiroshi Ishii (Hokkaido University of Education)

VALUES ALIGNMENT IN TEACHERS' PERSPECTIVE: A STUDY IN CHINA 1118
Haomin Fang (The University of Melbourne)

INVESTIGATING VALUES OF TEACHERS IN COURSES FOR LOW-PERFORMING UPPER-SECONDARY STUDENTS 1123
Justine Sakurai (The University of Melbourne)

INTELLECTUAL HUMILITY AND MATHEMATICS PERFORMANCE 1129
Frederick K. S. Leung (Beijing Normal University)

A STUDY ON THE RELATIONSHIP BETWEEN MATHEMATICS ADVERSITY QUOTIENT AND MATHEMATICAL ACHIEVEMENT OF SECOND-GRADE JUNIOR HIGH SCHOOL STUDENTS—THE MEDIATING ROLE OF MATHEMATICAL ANXIETY 1134
Yumei Xia (Beijing Normal University)

USING HISTORICAL CONTEXTS IN NON-HISTORICAL MATHEMATICS COURSES. 1139
Amanda Jo Meiners (Northwest Missouri State University)

EMOTIONAL DIMENSIONS AND SOCIAL SOLIDARITY IN MATHEMATICS EDUCATION: EVERYONE ENJOYS GAMES - OR DO THEY? 1145
Linda Bonne (Victoria University of Wellington)

RETHINKING NEO-CONFUCIAN CONCEPTS OF LEARNING & WELLBEING 1149
Emily Sum (The University of Melbourne)

A STUDY ON CHILDREN'S PERCEPTIONS OF "GOOD EXPLANATION" IN ARITHMETIC 1154
Shota Mastumoto (Shimane University)

Topic Study Groups 8

FROM TRADITION TO TECHNOLOGY: RETHINKING MATHEMATICS EDUCATION THROUGH ETHNOMATHEMATICS IN INDONESIA 1165
Nur Robiah Nofikusumawati Peni (Universitas Ahmad Dahlan)

INDIGENOUS EPISTEMOLOGY AND ETHNOMATHEMATICS: INSIGHT FROM BHUJEL COMMUNITY 1169
Jaya Bishnu Pradhan (Tribhuvan University)

A STUDY ON MATHEMATICAL CURRICULUM IN AN INDIGENIOUS SCHOOL IN TAIWAN: FOCUSING ON THE PROCESS OF CURRICULUM DESIGN 1174
Tai-Jung Lee (National Academy for Educational Research)

MATHEMATICS ANXIETY ACROSS ETHNIC COMMUNITIES IN NEPAL 1179
Krishna Chandra Paudel (Tribhuvan University)

A STUDY ON MATHEMATICS CURRICULUM AT AN INDIGENIOUS SCHOOL IN TAIWAN: FOCUSING ON THE PRODUCTION OF MATHEMATICAL TASK 1183
Ying-Hsuan Lee (National Academy for Educational Research)

A PRACTICAL STUDY ON THE INDIGENOUS RITUAL CULTURE IN ELEMENTARY SCHOOL INQUIRY EDUCATION 1189
Felisa(Hsueh-Yun) Yu (National Changhua University of Education)

SURVEY AND STUDY OF "ETSUKI-SANGAKU," MATHEMATICAL TABLETS WITH ILLUSTRATIONS, IN FUKUI PREFECTURE, JAPAN, AND EMPIRICAL RESEARCH ON ITS UTILIZATION FOR MATHEMATICS EDUCATION 1193
Hiroshi Kazama (University of FUKUI)

SURVIVING MASTERY IN MATHEMATICS EDUCATION 1200
Nikki Mann (Simon Fraser University)

SECONDARY SCHOOL STUDENTS' LEVEL OF INVENTIVE THINKING IN MATHEMATICAL LEARNING 1205
Roslinda Rosli (The National University of Malaysia)

DIVERSE VALUES AND MATHEMATICAL THINKING FOSTERED THROUGH SOCIALLY OPEN-ENDED PROBLEMS: SIMPLIFICATION OF SOCIAL CONTEXTS 1209
Yuichiro Hattori (Okayama University)

DIGITAL MATHEMATICS STORYTELLING IN INDONESIA, VIETNAM, AND THE ASIAN DIASPORA: EQUITY, CULTURE, AND COMMUNITY THROUGH NARRATIVE IDENTITIES 1214
Theodore Chao (California State University)

CULTIVATING CRITICAL MATHEMATICS CONCIOUSNESS IN PRE-SERVICE TEACHERS THROUGH DIGITAL RESOURCES AND THE RWC MODEL 1218
Artorn Nokkaew (Naresuan University)

ETHICAL FOUNDATIONS FOR EXPLORING INCLUSIVE SCIENCE AND MATHEMATICS EDUCATION 1223
Mitsuru Matsushima (Kagawa University)

NEW POSSIBILITIES FOR PROBLEM FINDING THROUGH MATHEMATICAL MODEL AND CARING 1228
Rintaro Ueda (Tokyo Gakugei University)

Topic Study Groups 9

ON THE HURDLES OF PROBLEM SOLVING IN UNDERGRADUATE MATHEMATICS DISCOURSE 1239
Igor' Kontorovich (The University of Hong Kong)

THE ROLE OF COGNITIVE RESOURCES IN MATHEMATICS PROBLEM SOLVING 1243
Xin Li (Suzhou University of Technology)

AN ANALYSIS OF STUDENTS' BASIC MENTAL MODELS OF THE INTEGRAL: THE CASE OF PHYSICAL CHEMISTRY 1248
Christian Heinz (Julius Maximilian University of Würzburg)

A CASE STUDY ON STUDENTS' SELECTIVE NOTE-TAKING IN UNDERGRADUATE NUMBER THEORY LECTURE 1254
Jeong-Won Noh (Busan National University of Education)

ENHANCEING MATHEMATICS STUDENT TEACHERS' UNDERSTANDING IN ACTIVE LEARNING THROUGH ABSORB-DO-CONNECT LEARNING FRAMEWORK 1258
Wanintorn Poonpaiboonpipat (Naresuan University)

PROVING ACTIVITIES IN A GRAPH-THEORETICAL INQUIRY FOR PRE-SERVICE TEACHERS: A PRELIMINARY ANALYSIS 1263
Minh Hong Doan Le (Hiroshima University)

CONCEPT-BASED LEARNING IN GEOMETRY WITH GEOGEBRA IN PAPUA NEW GUINEA PRIMARY TEACHERS COLLEGES 1268
Hanson Wambu (Naruto University of Education)

Contents

PRESERVICE MATHEMATICS TEACHERS' CONFIDENCE TO TEACH STATISTICS 1273
Hyung Won Kim (University of Texas Rio Grande Valley)

PRESERVICE SECONDARY TEACHERS' PROOF ANALYSIS IN SPHERICAL GEOMETRY 1278
Younggon Bae (The University of Texas Rio Grande Valley)

GUIDED REINVENTION OF THE DEFINITIONS OF REDUCIBLE AND IRREDUCIBLE ELEMENTS 1283
Younggon Bae (The University of Texas Rio Grande Valley)

TEACHING MATERIALS FOR DIFFERENTIAL EQUATIONS WITH APPLICATION TO STEM PROBLEMS 1290
Satoru Takagi (Waseda University)

A STUDY OF ACTIVITIES USING GEOGEBRA IN METHOD OF LAGRANGE MULTIPLIER 1295
Manabu Yoshida (Yamato University)

ROLE OF MATH DUAL ENROLLMENT ON STUDENT OUTCOMES 1299
Jungmin Lee (University of Kentucky)

DECISION-MAKING IN A DESIGN-BASED RESEARCH APPROACH TO COLLABORATIVE DEVELOPMENT OF INTERACTIVE TEXTBOOK QUESTIONS 1304
Vilma Mesa (University of Michigan)

Topic Study Groups 10

IDENTIFYING ZAMBIAN GRADE 10 STUDENTS' 21ST CENTURY SKILLS IN CROSS-CURRICULAR LEARNING OF MATHEMATICS AND HOME ECONOMICS 1315
Koki Nakashima (Yokohama National University)

EXPLORING INQUIRY-BASED LEARNING IN SENIOR HIGH SCHOOL MATHEMATICS: A STUDY ON FINALIZED SRPS 1320
Mitsuhiro Iwata (Senior High School at Otsuka, University of Tsukuba and Waseda University)

A STUDY ON THE ASSESSMENT OF CREATIVE PROBLEM-SOLVING COMPETENCE USING FERMI ESTIMATION 1325
Jungin Heo (Samjung High School)

ENHANCING STE(A)M EDUCATION THROUGH PROJECT-BASED LEARNING: A FOCUS ON AGE-PERIOD-COHORT MODELING 1329
Diana Jasmin S. Costales (University of Santo Tomas)

EVALUATING PRACTICAL STEAM ACTIVITIES FOR STUDENT LEARNING AND ENGAGEMENT 1336
Masitah Shahrill (Universiti Brunei Darussalam)

EMPOWERING FEMALE MATHEMATICIANS FOR THE FUTURE OF STEAM IN BRUNEI DARUSSALAM 1341
Masitah Shahrill (Universiti Brunei Darussalam)

INTERDISCIPLINARY APPROACHES TO CONNECT MATHEMATICS AND SCIENCE: A CASE STUDY OF PRIMARY TEACHERS' ADAPTIVE EXPERTISE 1346
Wanty Widjaja (Deakin University)

ENHANCING SELF-REGULATION AND MOTIVATION THROUGH STEM PROJECT-BASED LEARNING 1352
Yujin Lee (Kangwon National University)

FOSTERING AGENCY IN MATHEMATICAL MODELLING FOR CRITICAL CITIZENSHIP 1356
Jimbo Juanito B. Villamor (Ateneo de Manila University and Surigao del Norte State University)

INVESTIGATING TEACHER'S DESIGN OF MATHEMATICAL MODELLING TASKS TO FOSTER MATHEMATICAL LITERACY 1362
Sakon Tangkawsakul (Kasetsart University)

FROM STEM OVER STEAM TO STEEM BUILT ON ECONOMICS 1367
Allan Tarp (MatheCademy.net)

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

KOREA 2025

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 1:

The Essence of Mathematics Education in Curriculum and Materials

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 1
(Day 2)

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

TSG1-01

PRESERVICE TEACHERS' BELIEFS AND VIEWS ON CURRICULUM AND TEACHER GUIDE COMPONENTS

Ji-Eun Lee
Oakland University, USA, lee2345@oakland.edu

Sangmee Kim
Chuncheon National University of Education, South Korea, sangmee@cnue.ac.kr

This study investigated how preservice teachers (PSTs) perceive school curricula and assess the necessity and flexibility of teacher guide elements. Data were gathered from 71 U.S. PSTs through a two-part questionnaire and analyzed using descriptive statistics and inductive content analysis. The preliminary findings revealed some conflicting beliefs about learning progressions and the use of open-source curricula. PSTs considered teaching practices, such as teacher questions and anticipated student thinking in the teacher guide, to be less necessary and more adaptable, while standard-related items and math content were viewed as more essential and less flexible. Additional insights and recommendations for future research are provided.

Keywords: curriculum materials, productive beliefs, unproductive beliefs, pre-service teachers, teacher guide

Purpose of the Study

Teachers' beliefs and understanding of mathematics standards influence their instructional decisions as they interact with curriculum materials (Remillard, 2000; Taylor, 2016). Although research has explored curriculum materials and the knowledge required for their effective use (Cai et al., 2020), limited attention has been given to how preservice teachers (PSTs) in initial teacher education programs perceive school curricula and teacher guides early in their training when envisioning learning opportunities to support students' mathematical development. As an effort to capture PSTs' initial beliefs and perceptions, this study probes the following research questions:

RQ1: What are the levels of agreement among PSTs regarding statements that depict productive and unproductive beliefs about the school curriculum?

RQ2: How do PSTs perceive the necessity and flexibility of the elements presented in teachers' guides?

Methods

Participants and Context

Seventy-one preservice teachers (PSTs) participated in this study. These PSTs were enrolled in a required elementary mathematics methods course at a Midwestern university in the United States and were working toward their initial elementary teaching certifications. Among them, 51 were undergraduate PSTs, while 20 were graduate PSTs. The undergraduate PSTs were enrolled in the second

course of a three-course mathematics methods sequence, with most graduating from high school within the past two years. In contrast, the graduate PSTs were enrolled in the first course of a two-course mathematics methods sequence, having previously earned undergraduate degrees in other fields before transitioning to a teaching career.

Tasks and Data Collection

A two-part written questionnaire was utilized in this study. Part 1 included 10 statements representing both productive and unproductive beliefs that impact the implementation of an effective curriculum, as outlined in *Principles to Actions* (NCTM, 2014, p. 72). PSTs rated their agreement with each statement using a 4-point Likert scale, with options ranging from "strongly disagree" to "strongly agree." In Part 2, excerpts from teacher guides for a curriculum, including the unit introduction and a lesson within the unit, were provided. Twenty elements were identified and labeled in the teacher guides (e.g., relevant standards, ideas for extension, etc.). PSTs were asked to evaluate each element on two aspects using a 4-point Likert scale: (a) the perceived level of necessity/importance of the element in teacher guides (ranging from "very unnecessary" to "very necessary") and (b) the perceived level of flexibility for teachers to adapt or modify the element (ranging from "very inflexible" to "very flexible").

Data Analysis

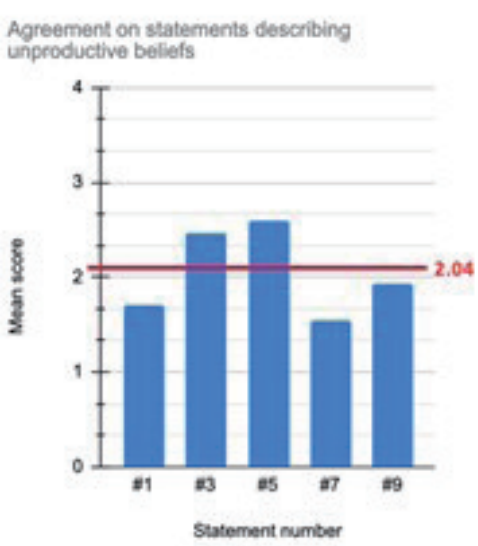
The data from the Likert scales in Parts 1 and 2 were analyzed by calculating the mean scores for each individual items and the overall grand means. The grand means provided an overview of the general trends across all items in each part of the questionnaire. Additionally, individual item means were compared to the grand means to identify any deviations or outliers. The rationales provided in Part 1 were examined through inductive content analysis to uncover patterns or themes in the PSTs' justifications for their ratings.

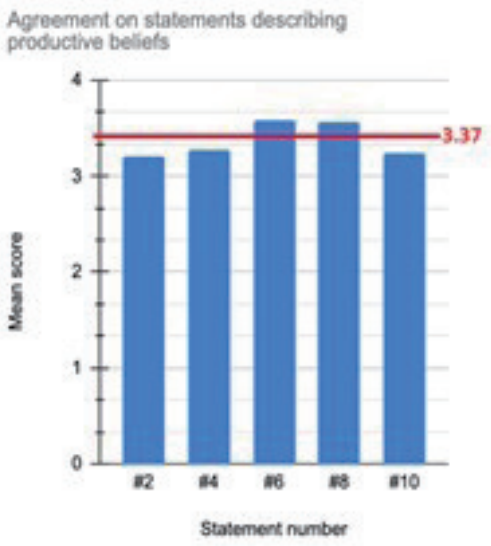
Findings

PSTs' Agreement on Productive/Unproductive Beliefs

Figure 1 summarizes the findings from Part 1. Overall, PSTs demonstrated lower levels of agreement with statements reflecting unproductive beliefs (grand mean = 2.04) and higher levels of agreement with those reflecting productive beliefs (grand mean = 3.37). To gain deeper insights, we analyzed PSTs' rationales for their ratings, focusing on items that deviated from the grand means. For the unproductive beliefs, statements #3 and #5 had mean scores higher than the grand mean. In response to statement #3, PSTs who selected "agree" or "strongly agree" primarily emphasized the importance of teachers' knowledge of the curriculum. However, their rationales rarely addressed whether knowledge limited to a specific grade-level curriculum is sufficient. Regarding statement #5, PSTs who agreed or strongly agreed with this unproductive belief highlighted the importance of staying on track, being organized, ensuring quality and continuity, and upholding the principle of equity (or equality). For the productive beliefs, statements #2, #4, and #10 had mean scores below the grand mean. For #2, PSTs who disagreed or strongly disagreed questioned the comprehensiveness of the standards, noting that some essential skills or content might not be included. For statement #4, PSTs raised concerns about feasibility when disagreeing with this productive belief. Their perspectives included comments such as, "I don't think I need to be able to do calculus to teach counting" and "If I fully understand my grade level and the lower and the one higher, that would be sufficient, but knowing all the curriculum across multiple grade levels is unnecessary." For question

#10, the PSTs' disagreement was attributed to their uncertainty regarding the quality of open-source materials.





#1. The content and sequence of topics in a textbook always define the curriculum. Everything included in the textbook is important and should always be addressed , and what is not in the book is not important.
#3. Knowing the mathematics curriculum for a particular grade level or course is sufficient to effectively teach the content to students.
#5. Implementation of a pacing guide ensures that teachers address all the required topics and guarantees continuity so that all students are studying the same topics on the same days.
#7. Mathematics is a static, unchanging field.
#9. The availability of open-source mathematics curricula means that every teacher should design his or her own curriculum and textbook.

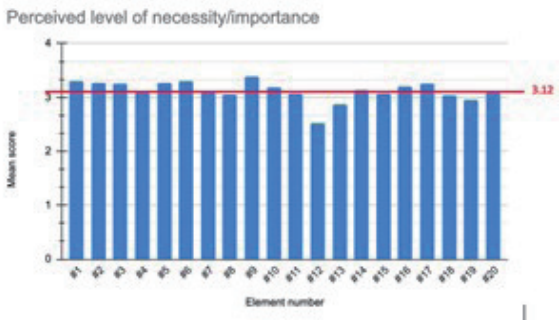
#2. Standards should drive decisions about which topics to address and which to omit in the curriculum. How a textbook is used depends on its quality that consistently support implementation of the Mathematics Teaching Practices.
#4. Mathematics teachers need to have a clear understanding of the curriculum within and across grade levels to effectively teach a particular grade level or course in the sequence.
#6. Curriculum maps and pacing guides attempt to ensure coverage of content but do not guarantee that students learn the mathematics. Adequate time to provide for meaningful learning, differentiation, and interventions must be provided for students to develop deep understanding of the content.
#8. Mathematics is a dynamic field that is ever changing. Emphases in the curriculum are evolving, and it is important to embrace and adapt to appropriate changes.
#10. Open-source curricula are resources to be examined collaboratively and used to support the established learning progressions of a coherent and effective mathematics program.

Note. Strongly Disagree = 1; Disagree = 2; Agree = 3; Strongly Agree = 4. The red lines indicate grand means.

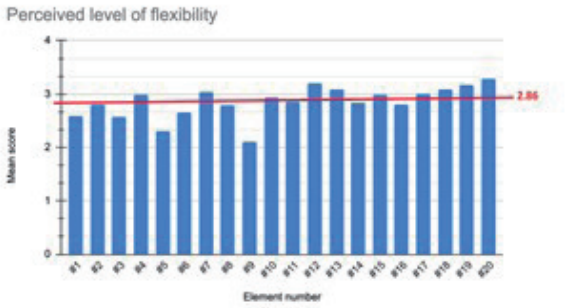
Figure 1. PSTs' agreement on productive and unproductive beliefs

PSTs' Perceived Level of Necessity and Flexibility on Elements in Teachers Guides

Figure 2 summarizes the findings from Part 2. Due to space limitations, we highlight a few key observations. PSTs indicated that several elements related to official standards (e.g., #1, #5, #6, #9) are essential to know and offer limited flexibility. Two elements received relatively low scores for necessity/importance: #12 (List of teacher questions) and #13 (Anticipated student thinking). However, these same elements were rated as having relatively high flexibility. While guidelines on using and presenting models/visuals (#16) were considered necessary, guidance/examples for what and how to record on the board (#18) were deemed less important to include in the teacher guide.



Note. Very unnecessary = 1, Unnecessary = 2, Necessary = 3, very necessary = 4. The red lines indicate grand means.



Note. Very inflexible = 1, Unnecessary = 2, Necessary = 3, very necessary = 4. The red lines indicate grand means.

Elements: 1. Unit overview, 2. Unit Planner, 3. Mathematical background, 4. Teaching tips, 5. Skills across the grade levels, 6. Assessment, 7. Differentiation, 8. Lesson summary, 9. Relevant standards, 10. Materials needed, 11. Step-by-step sequences to follow, 12. List of teacher questions, 13. Anticipated student thinking, 14. List of needed materials for the activity, 15. Tasks/problems, 16. How to use and present models/visuals to students, 17. Tasks for differentiation, 18. Examples of what and how to record on the board, 19. Activities/tasks for extension, 20. Homework

Figure 2. PSTs' perceived level of necessity and flexibility on elements in teacher guides

Discussion and Implications

Overall, PSTs tended to agree with productive belief statements and disagree with unproductive ones. They also acknowledged the importance of elements in teacher guides, though with some flexibility in their application. However, a few items deviated from this trend and warrant further attention, as they may require more focused support as PSTs continue in their teacher education. These include the need to understand learning progression, the use of pacing guides and open-source curricula, and the assumptions about teaching practices like teacher questioning and anticipating student thinking. Follow-up studies will investigate three areas: (a) whether these beliefs and perspectives change over time as PSTs gain more field experience, (b) whether these patterns vary based on PSTs' background and experiences, and (c) whether PSTs from different cultures and regions exhibit different belief and view patterns.

References

Cai J., Morris A., Hohensee C., Hwang S., Robison V., Cirillo M., Kramer S.L., Hiebert J., & Bakker A. (2020). Addressing the problem of always starting over: Identifying, valuing, and sharing professional knowledge for teaching. *Journal for Research in Mathematics Education, 51*(2), 130–139.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematics success for all.*

Remillard J. T. (2000). Can curriculum materials support teachers' learning? Two fourth-grade teachers' use of a new mathematics text. *The Elementary School Journal, 100*(4), 331–350.

Taylor M. W. (2016). From effective curricula toward effective curriculum use. *Journal for Research in Mathematics Education, 47*(5), 440–453.

TSG1-02

DO DIGITAL CURRICULUM RESOURCES SUPPLEMENT THE EDUCATIVE FEATURES OF TEACHER GUIDEBOOKS?

JeongSuk Pang
Korea National University of Education, Korea, jeongsuk@knue.ac.kr

MinYoung Oh
Graduate School of KNUE, Korea, omy8529@gmail.com

With the advent of the digital age, curriculum resources have shifted from exclusively print-based materials to digital formats. Previous studies examining how printed curriculum resources, such as teacher guidebooks, support teacher learning have identified both strengths and weaknesses in their educative features. We anticipate that digital curriculum resources can enhance features. This paper analyzes how digital curriculum resources address the weaknesses identified in teacher guidebooks' educative features, providing specific examples. Based on this analysis, it further discusses ongoing and emerging issues related to the educative design and use of curriculum resources.

Keywords: educative features, teacher learning, teacher guidebooks, digital curriculum resources, mathematics

Introduction

Curriculum resources, which provide specific information on what and how to teach, support both student learning and teacher learning (Stein et al., 2007). Various elements in curriculum resources (e.g., mathematical content knowledge, methods for addressing errors or misconceptions, assessment methods, etc.) are considered *educative* because they aid teacher learning in lesson design and implementation (Davis & Krajcik, 2005). Previous research has used terms such as *educative elements* and *educative features* interchangeably to refer to these elements. In this paper, we define the "educative features" as those elements of curriculum resources that support teacher learning.

With the advent of the digital age, curriculum resources have expanded from print-based to digital formats. According to a study examining the educative features of printed curriculum resources, such as teacher guidebooks (Pang et al., 2023), these guidebooks have various educative features but also exhibit certain strengths and weaknesses. Because the structure and content of digital curriculum resources can be easily modified to suit user needs, and they can offer immediate feedback through interactive components (Bates, 2017), the interaction between teachers and digital curriculum resources supports teachers' professional learning (Gueudet & Pepin, 2020). Based on this background, we posit that digital curriculum resources could expand or supplement the educative features of teacher guidebooks. Accordingly, our research question is as follows: How do digital mathematics curriculum resources address the identified weaknesses of the educative features in elementary school mathematics teacher guidebooks?

Theoretical Background

According to Pang et al. (2023), teacher guidebooks for elementary mathematics in Grades 3 and 4 in Korea exhibit both strengths and weaknesses in their educative features. For instance, regarding mathematics content knowledge for teaching, teacher guidebooks offer detailed and systematic explanations of what teachers should teach, in what order, and the mathematical knowledge they need to understand before the lesson. Similarly, regarding teacher knowledge of disciplinary discourse in mathematics, teacher guidebooks present examples of discourse, teachers' questioning, and discussion focal points, thereby helping teachers understand the overall flow of mathematical discourse. However, because the guidebooks do not address the decision-making process involved in conducting mathematical discourse, it can be difficult for teachers to handle unexpected situations during actual discussions. Finally, regarding teacher knowledge of manipulative materials and technology use in mathematics, teacher guidebooks provide specific methods for incorporating manipulatives and technology into mathematics lessons, but these are limited to particular mathematical topics or units.

Digital curriculum resources differ from printed ones in several ways. First, they integrate various media—text, images, videos, and interactive interfaces—making them highly engaging. Beyond reading, the modes of interaction include viewing, manipulating, creating, and responding (Remillard, 2016). Consequently, teacher learning spaces can be tailored to meet individual needs and connected to teacher communities (Pepin et al., 2017). Although many studies advocate for the potential of digital curriculum resources to enhance teacher learning, relatively few explore how these resources should be designed and used. According to Pepin (2020), teachers face challenges in evaluating and utilizing the quality of digital curriculum resources due to their sheer abundance. This paper examines how digital curriculum resources support teacher learning and aims to lay a foundation for further discussion on their educative design and use.

Methods

The analysis focused on mathematics curriculum resources provided by digital platforms operated by eight publishers of elementary school mathematics textbooks in Korea. The first step in the analysis was to carefully review these digital curriculum resources to distinguish educative elements from teaching convenience functions. An example of a convenience functions is the provision of templates for creating quizzes, surveys, games, word clouds, enabling teachers to develop new digital resources easily. In contrast, resources shared by colleagues were classified as educative elements, as they serve as supplementary or alternative activities that can replace or augment textbook-based instruction.

Next, the educative elements identified in the digital curriculum resources were categorized into the seven dimensions of Teacher Learning Opportunities in Korean Mathematics Curriculum Materials (Pang et al., 2023) and other relevant categories. To avoid overlapping codes, elements were further subdivided where needed. For example, videos explaining mathematical concepts or principles were labeled as "video (concept explanation)" and coded under mathematics content knowledge for teaching, whereas videos showing teachers conducting remote lessons were labeled as "video (overall lesson)" and coded under teacher knowledge of disciplinary discourse in mathematics. Videos designed to boost interest or motivation were labeled as "video (motivation)" and coded under teacher knowledge of mathematics community. Finally, we distinguished between *supplementation* types, in which digital curriculum resources address weaknesses in the educative features of teacher guidebooks, and *maintenance* types, in which such supplementation does not occur, thereby pre-

serving these weaknesses in the digital resources. Throughout data collection and analysis, we took measures to minimize researcher bias.

Results

The analysis showed that, in most cases, weaknesses in the educative features of the teacher guidebooks were supplemented by digital curriculum resources. Due to space limitations, we provide one specific example each for the supplementation and maintenance types.

An example of supplementation appeared in Dimension 6 (teacher knowledge of manipulative materials and technology use in mathematics). The teacher guidebooks provided information on using concrete manipulatives or technology but only for a limited range of mathematical topics or units. In contrast, the digital curriculum resources provided a wide variety of digital manipulatives spanning multiple mathematics content strands. For example, in the geometry domain, a digital manipulative (Figure 1, left) dynamically visualizes the transformation of plane figures. With a single click, it automatically executes the transformation and records both the pre- and post-transformed shapes. This feature helps students grasp the direction and degree of rotation of a figure, which can be otherwise difficult to conceptualize, and facilitates easy comparison between the initial and final shapes. Specific teaching strategies for using this digital tool are provided in the lesson plan and worksheet at the bottom of Figure 1.

In the operations domain, another digital manipulative (Figure 1, right) visualizes fraction operations using a length model. It automatically demonstrates the process of manipulating the model with a click and displays the correct answer. However, when subtracting whole numbers and then an additional 3/5 from the remaining 1 and 1/5, the critical step of converting the mixed number 1 and 1/5 into the improper fraction 6/5 is omitted on-screen, even though the final answer is correct. Consequently, teachers must provide additional explanations to connect the visual model with the formal algorithm. Such considerations, however, are not reflected in the lesson plan or worksheet at the bottom of Figure 1.

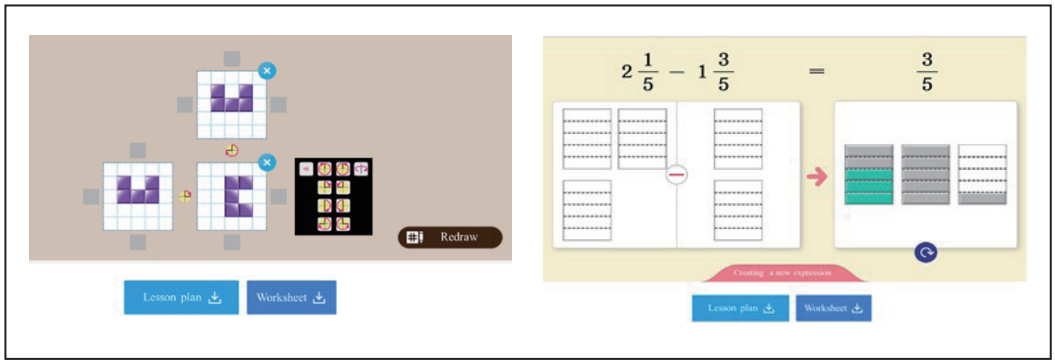


Figure 1. Digital manipulatives provided on a digital curriculum platform

An example of maintenance was identified in Dimension 3 (teacher knowledge of disciplinary discourse in mathematics). Teacher guidebooks help teachers understand the overall flow and focus of mathematical discussions by providing examples of discourse. However, these examples assume an

ideal classroom scenario and do not address student responses containing errors, misconceptions, or non-standard solution strategies. Digital curriculum resources, such as lesson PPTs and remote teaching videos, assist teachers in grasping the flow of discourse, key questions, and methods of explanation. Yet, these digital resources are typically structured around a one-way lecture format and also assume an ideal classroom context, making it challenging for teachers to respond effectively when faced with student misconceptions or unconventional solution strategies.

Implications

Some weaknesses in the educative features of teacher guidebooks were supplemented by digital curriculum resources, but certain aspects remain unaddressed. While a diverse array of digital mathematics manipulatives is now available, the effective utilization of these tools has emerged as a new challenge. As teachers encounter difficulties in using digital curriculum resources (Pepin, 2020), it is crucial that these resources not only supply the necessary tools but also provide detailed guidance on their effective application and relevant considerations. Furthermore, to prevent teachers from being confined to a passive role of merely clicking through content (Van Steenbrugge & Remillard, 2020), curriculum resources should be designed to accommodate a range of teaching scenarios rather than assuming an ideal classroom situation. Although research on digital curriculum resources is gaining momentum internationally (Bikner-Ahsbahs & Johnson, 2024), further investigation into their educative design and utilization is needed.

Limitations

Several limitations should be noted when interpreting this study's findings. First, although we examined the educative features of digital curriculum resources currently used in Korea, it remains unclear how far these results generalize to other settings. Further research across a wider range of contexts is needed to validate the broader applicability of our conclusions. Second, while our analysis suggests that digital curriculum resources may supplement weaknesses in teacher guidebooks' educative features, further investigation is required to determine whether and how these resources meaningfully support teacher learning in real classroom practice. Third, this study does not offer a specific design framework or set of design principles for developing educative digital curriculum resources. Instead, it highlights the pedagogical potential of such resources and underscores the need for continued inquiry in this area.

References

Bates, M. S. (2017). Leveraging digital tools to build educative curricula for teachers: Two promising approaches. *ZDM Mathematics Education, 49*(5), 675-686.

Bikner-Ahsbahs, A., & Johnson, H. L. (2024). Advancing mathematics education on digital resources: A reciprocity between theory, methodology, and design. In B. Pepin, G. Gueudet, & J. Choppin (Eds.), *Handbook of digital resources in mathematics education* (pp. 43-67). Springer.

Davis, E. A., & Krajcik, J. S. (2005) Designing educative curriculum materials to promote teacher learning. *Educational Researcher, 34*(3), 3-14.

Gueudet, G., & Pepin, B. (2020). Digital curriculum resources in/for mathematics teaching learning: A documentational approach perspective. In S. Llinares & O. Chapman (Eds.), *International handbook of mathematics teacher education Vol. 2* (pp. 139-161). Brill.

Pang, J., Oh, M., & Park, Y. (2023). An analysis of the educative features of mathematics teacher guidebooks for grades 3 and 4 (in Korean with English abstract). *The Mathematical Education, 62*(4), 531-549.

Pepin, B. (2020). Quality of (digital) resources for curriculum innovation. In A. Donevska-Todorova, E. Faggiano, J. Trgalova, Z. Lavicza, R. Weinhandl, A. Clark-Wilson, & H. Weigand (Eds.), *Proceedings of the 10th ERME topic conference on mathematics education at the digital age* (pp. 19-26). Johannes Kepler University.

Pepin, B., Choppin, J., Ruthven, K., & Sinclair, N. (2017). Digital curriculum resources in mathematics education: Foundations for change. *ZDM Mathematics Education, 49*(5), 645-661.

Remillard, J. T. (2016). Keeping an eye on the teacher in the digital curriculum race. In M. Bates, & Z. Usiskin (Eds.), *Research on mathematics textbooks and teachers' resources: Advances and issues* (pp. 69-88). Springer.

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 319-369). Information Age.

Van Steenbrugge, H., & Remillard, J. (2020). Digital curriculum resources' connectivity: An attempt to conceptualization. In A. Donevska-Todorova, E. Faggiano, J. Trgalova, Z. Lavicza, R. Weinhandl, A. Clark-Wilson, & H. Weigand (Eds.), *Proceedings of the 10th ERME topic conference on mathematics education at the digital age* (pp. 491-498). Johannes Kepler University.

TSG1-03

A GUIDING FRAMEWORK FOR DESIGNING GRAPHING TASKS FOR STUDENT LEARNING

Hwa Young Lee
Texas State University, U.S., hylee@txstate.edu

Brandi Rygaard Gaspard
Texas State University, U.S., brr102@txstate.edu

Stephanie Tarigan
Texas State University, U.S., sit11@txstate.edu

Lucinda Ford
Texas State University, U.S., llf48@txstate.edu

Graph literacy is critical for students to succeed in STEM coursework and careers and to make sense of and critically respond to information portrayed in graphical representations in everyday life. Studies that examined whether US textbooks are providing students with productive opportunities for developing graph literacy found several issues. In response to these issues, we present a guiding framework that we developed and used to design graphing tasks to examine and support middle grade students' graph literacy, explicate the framework via a task, and make a call for collaborative research with researchers in East Asia.

Keywords: graph literacy, graphing tasks, task design, design-based research

Introduction

Graph literacy is critical for students to succeed in STEM coursework and careers (Costa, 2020). It is also important for students to make sense of and critically respond to information portrayed in graphical representations, with the rapid development of technological devices and media (e.g., Kwon et al., 2021). However, plenty of studies have shown that U.S. students continue to experience difficulties with graphical representations (e.g., Clement, 1989; Lai et al., 2016; Yavuz, 2010). Relatedly, some studies have examined whether U.S. textbooks are providing students with opportunities that could support productive graphing activity in relation to STEM. For example, Roth et al. (1999) studied different types of graphs displayed in high school biology textbooks and scientific ecology journals. Paoletti et al. (2020) examined different types of graphs in mathematics, science, and engineering undergraduate textbooks and practitioner journals. Collectively, these studies documented the discrepancy between the frequency and type of graphical representations students encounter in their math and science textbooks versus those in scientific journal articles. From their analysis of middle school mathematics textbooks, Lee and Guajardo (2023) suggested that more opportunities for students are needed to engage in creating coordinate systems and graphs. Building on these findings, we have designed graphing tasks to support middle school students' graph liter-

acy. We present a framework we developed and used to design graphing tasks to examine middle grade students' graph literacy, explicate the framework and design process via a sample task, and make a call for collaborative research with researchers in East Asia.

Task Design Theoretical Underpinnings

We define a graphical representation as a spatial depiction of quantities (Thompson, 2011) to mathematize some phenomenon. Here, a graphical representation consists of three layers: reference frames (RFs), a coordinate system (CS; e.g., Cartesian plane), and a graph. RFs are mental structures used to gauge the relative extents of various attributes in the phenomenon being depicted (Levinson, 2003; Lee, 2017; Joshua et al., 2015). Thinking within RFs entails attending to and establishing reference points, directionality, and having an idea of what and how to measure the quantities being depicted (Joshua et al., 2015; Lee et al., 2019). CSs are the geometric embodiment of the RFs (e.g., axes) that allow an individual to systematically express and coordinate RFs. Finally, a graph is a collection of points depicted upon the underlying CS. The ways of thinking about a graph fundamentally depends on the RFs and CSs upon which they are created. Extant graph literacy research has predominantly focused on the graph component and often overlooked underlying RFs and/or CSs (Lee et al., 2020). In our task design, we focused on all three layers.

The tasks were first sorted into Reading vs. Writing problems, i.e., whether the student was asked to interpret a given graph or create a graph (Figure 1). Within Reading problems, we distinguished between Reading From, Reading a Trend, and Reading Beyond type problems (Curcio, 1989; Friel et al. 2001). We developed an analogous distinction for Writing problems, namely, Plotting Onto, Drawing a Relationship/Trend, and Predicting Beyond. Within each of these problem types, we also distinguished between whether the task involved a spatial, quantitative, or neither type of coordinate system (Lee et al., 2020). A spatial CS is used to quantitatively organize a space in which a phenomenon is situated. Constructing a spatial CS involves an individual organizing a space by (mentally) overlaying a CS onto some physical or imagined space being represented where objects within that space are tagged with coordinates. A quantitative CS is used to coordinate sets of quantities by constructing a geometrical representation of the product of measure spaces (imagine a speed-time graph). Constructing a quantitative CS involves an individual abstracting quantities from the space in which a phenomenon occurs and projecting them onto a new space, different from the space in which the quantities were originally conceived.

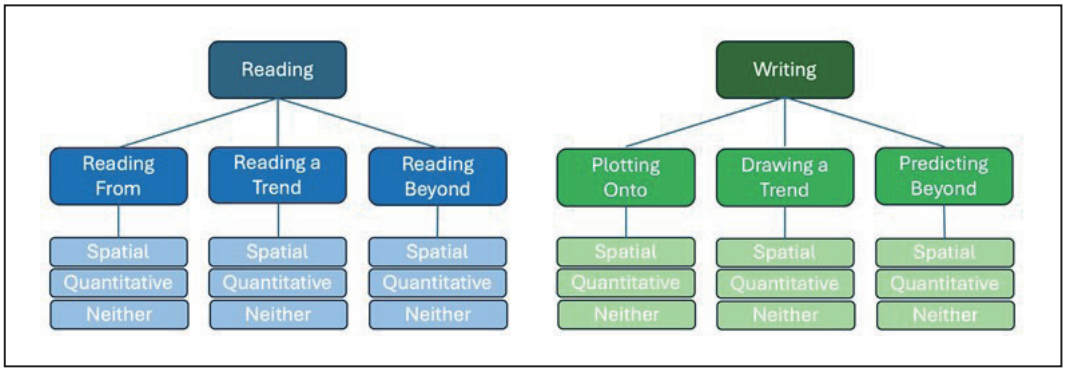


Figure 1. Task design problem type framework

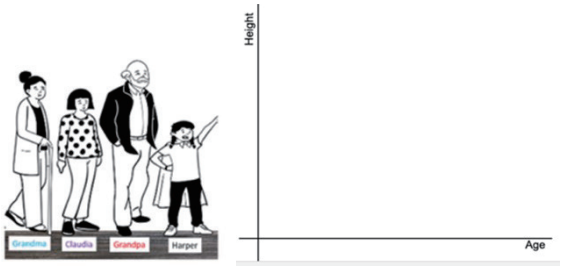
Methods

Our work comes from two projects, both examining and supporting middle grades (grades 5–8 in the U.S.) students' developing graph understandings. Both involve design-based studies (Cobb et al., 2003; DBRC, 2003) consisting of cyclic iterations of task design, testing, analyzing, and reflecting. The tasks were implemented in clinical interviews (CIs) and teaching experiments (TEs) (Steffe & Thompson, 2000). Our graphing problems drew from contexts that build on students' knowledge of and experiences with real-life phenomena. To further enhance the accessibility of these problems, we illustrated/modeled the phenomena via physical manipulatives or dynamic geometry software (e.g., GeoGebra, Desmos). In this report, we explicate our task design process via the Family Frenzy Task. Rather than discussing how the task was implemented in each project, we will illustrate the progression of the task and its design across both projects.

Findings

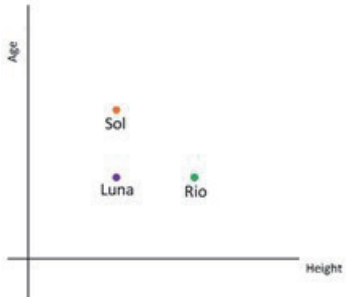
We present each sub-problem in the Family Frenzy Task sequence in Table 1. The Family Frenzy Task was modified from Swan's (1985) original "Bus Stop Queue" task, in which students were given a picture of seven people at a bus stop along with a scatterplot with age and height axes and seven corresponding points. Students had to match each person to their appropriate scatterplot point. One original goal of Swan's task was to perturb students who interpreted graphs as literal pictures, as in this case height was labeled on the horizontal axis.

Table 1. Family frenzy task sequence overview

1. The Sun Family: Write, Plotting Onto, Quantitative The picture below shows the Sun Family. Please use the representation below their picture to show the height and age of the family members. 	**1'. The Johnson Family: Sun Family Revised** The picture below shows the Johnson Family of Grandma and her grandchildren. Please use the representation below their picture to show the height and age of the family members.

2. The Moon Family:
Read, Reading From, Quantitative

The heights and ages of the Moon Family are represented below.

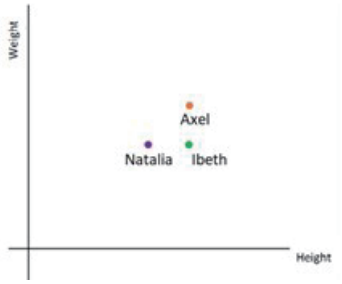


a. Can you rank the heights and ages of the family members?

b. If Rio is 5ft tall and 10 years old, how tall is Luna? How old is Sol?

2'. The Gonzales Family:
Moon Family Revised

The heights and ages of the Gonzales Family are represented below.



a. Can you rank the heights and ages of the family members?

b. If Ibeth is 5ft tall and 150lbs, estimate Natalia and Axel's height and weight.

3. Variation 1:
Star, Sun, & Bright Family in Desmos

https://teacher.desmos.com/activitybuilder/custom/6557d73a809673ed40429d3c

4. Variation 2:
DeVante's Growth: Write, Drawing a Trend

Let's say DeVante in the Johnson Family grew older over the years and he is now 60 years old. We know that as he started to grow, his weight and height both increased during the first fifteen years he grew. During the next 15 years, both his weight and height stayed the same. Finally, during the next 15 years, his weight decreased and his height decreased. Create a graph to explain how DeVante's weight and height changed together throughout the 60 years of his life.

Items 1 and 2 were were later revised to Items 1' and 2' after testing, analyzing, and reflecting. Note, we intentionally placed interpret tasks (Items 2, 2') to come later in all sequences, to examine students' own ideas without being influenced by pre-made graphs. Item 1 was designed as a Write, Plotting Onto, Quantitative type, although we anticipated students may view it as Spatial. Different from Swan's original task, we presented students with the Sun Family picture and a blank plane. We switched the axes back to their natural orientation with height vertically; we wanted to see what spatial insights students may draw from to organize height in this way before giving them the rest of the task, where axes were switched. However, we noticed that students associated linearity to height and age in the Sun Family (the older, the taller). Thus, in Item 1', we switched to the Johnson Family to mitigate that association. We also made the cartoon more realistic and culturally diverse. We designed Item 2 as a Read, Reading From, Quantitative type, although we anticipated students may view it as Spatial. We switched the axes so that height was now on the horizontal axis, consistent with Swan's original task. We noticed that students associated linearity to height and age in the Moon Family (the older, the taller). Therefore, in Item 2' we switched to height and weight to explore whether that association might be mitigated. We also added a Reading a Trend follow-up problem. Item 3 was a variation of Item 1 in a dynamic geometry software, in which students could drag points around and use various tools to support their problem solving. Item 4 was an extension problem of Item 1', asking students to draw a trend by starting with creating a CS on their own.

Discussion

From these tasks, we observed a variety of student strategies, such as using spatial transfer, indexing, ordering, coordinating number lines, etc. (see Zolt et al., 2024 for more details). We have seen several affordances in using our framework and tasks like the Family Frenzy Task. First, they gave us insights into various students' graphing strategies and resources students are capable of drawing from. We find this to be crucial because when teachers can better anticipate students' strategies and resources, they are better supported to orchestrate mathematical discussions that encourage connections and understanding (Smith & Stein, 2018). Second, using the framework allowed us to balance the distribution of graphing activities across reading/writing, and CS/graph type, affording more opportunities for students to engage with and reflect upon a variety of graphing tasks. Third, the variety of problems in the task sequence allows multiple entry points and various levels of cognitive demand, which can be useful for teachers and be supportive of student learning (Smith & Stein, 2018). We have seen several limitations as well. For instance, the process is time consuming; the overview presented in Table 1 is a culmination of a year's worth of designing, testing, analyzing, reflecting, and revising cycles with 27 students. We call for collaboration with other researchers, especially those in East Asia, to examine students' strategies across 18 types of problems (Figure 1) so that we as a field can build stronger documentation of and supportive curricular materials for students' developing graph understandings.

References

Clement, J. (1989). The concept of variation and misconceptions in Cartesian graphing. *Focus on Learning Problems in Mathematics, 11*(1–2), 77–87.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in education research. *Educational Researcher*, *32*(1), 9–13.

Costa, S. A. (2020). Graphical literacy, graphicacy, and STEM subjects. In S. A. Costa, M. Danesi, D. Martinovic (Eds.), *Mathematics (Education) in the Information Age*, Mathematics in Mind, (pp. 65–72). Springer. https://doi.org/10.1007/978-3-030-59177-9_5

Curcio, F. R. (1989). Developing graph comprehension: Elementary and middle school activities. Reston, VA: National Council of Teachers of Mathematics.

Design-Based Research Collective (DBCR). (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher*, *32*(1), 5–8.

Friel, S. N., Curcio, F. R., & Bright, G. W. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. *Journal for Research in Mathematics Education*, *32*(2), 124–158. https://doi.org/10.2307/749671

Joshua, S., Musgrave, S., Hatfield, N., & Thompson, P. W. (2015). Conceptualizing and reasoning with frames of reference. In T. Fukawa-Connelly, N. Infante, E., K. Keene, & M. Zandieh (Eds.), *Proceedings of the 18th Meeting of the MAA Special Interest Group on Research in Undergraduate Mathematics Education* (pp. 31–44). Pittsburgh, PA: RUME.

Kwon, O. N., Han, C., Lee, C., Lee, K., Kim, K., Jo, G., & Yoon, G. (2021). Graphs in the COVID-19 news: a mathematics audit of newspapers in Korea. *Educational Studies in Mathematics*, 1–18.

Lai, K., Cabrera, J., Vitale, J. M., Madhok, J., Tinker, R., & Linn, M. C. (2016). Measuring graph comprehension, critique, and construction in science. *Journal of Science Education and Technology*, *25*(4), 665–681.

Lee, H. Y. (2017). Students' construction of spatial coordinate systems (Unpublished doctoral dissertation). University of Georgia, Athens, GA.

Lee, H. Y., & Guajardo, L. R. (2023). A content analysis of tasks involving two-dimensional Cartesian graphs in grade 6 – 8 U.S. textbooks. *Investigations in Mathematics Learning*, *15* (3), 222–240.

Lee, H. Y., Moore, K. C., & Tasova, H. I. (2019). Reasoning Within Quantitative Frames of Reference: The Case of Lydia. *The Journal of Mathematical Behavior, 53,* 81–95.

Lee, H. Y., Hardison, H., & Paoletti, T. (2020) Foregrounding the background: Two uses of coordinate systems. *For the Learning of Mathematics*, 40(*1*), 32–37.

Levinson, S. C. (2003). *Space in language and cognition: Explorations in cognitive diversity* (Vol. 5). United Kingdom: Cambridge University Press.

Paoletti, T., Lee, H. Y., Rahman, Z., Vishnubhotla, M., & Basu, D. (2020) Comparing graphical representations in mathematics, science, and engineering textbooks and practitioner journals, *International Journal of Mathematical Education in Science and Technology*, https://doi.org/10.1080/0020739X.2020.1847336

Roth, W. M., Bowen, G. M., & McGinn, M. K. (1999). Differences in graph-related practices between high school biology textbooks and scientific ecology journals. *Journal of research in science teaching*, 36 (9), 977–1019.

Smith, M., & Stein, M. K. (2018). *5 Practices for orchestrating productive mathematics discussion (2nd edition)*. National Council of Teachers of Mathematics.

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In R. A. Lesh & A. E. Kelly (Eds.), *Handbook of research design in mathematics and science education* (pp. 267–307). Hillside, NJ: Erlbaum.

Swan, M. (1985). The language of functions and graphs: An examination module for secondary schools. Testing strategic skills. https://www.mathshell.com/materials.php?item=lfg&series=tss

Thompson, P. W. (2011). Quantitative reasoning and mathematical modeling. In S. Chamberlin, L. L. Hatfield, & S. Belbase (Eds.), *New perspectives and directions for collaborative research in mathematics education: Papers from a planning conference for WISDOM^e* (pp. 33–57). Laramie, WY: University of Wyoming.

Yavuz, I. (2010). What does a graphical representation mean for students at the beginning of function teaching? *International Journal of Mathematical Education in Science and Technology, 41*(4), 467–485.

Zolt, H., Gaspard, B. R., Lee, H.Y., Paoletti, T., Hardison, H. L., Ford, L., Tarigan, S., & Bui, M.(2024). Modeling students' strategies when creating a graph: A focus on reference frames and coordinate systems. In Kosko, K. W., Caniglia, J., Courtney, S. A., Zolfaghari, M., & Morris, G. A., (Eds.) *Proceedings of the forty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Kent State University (pp. 695-701).

Acknowledgement

This work was made possible by funding from the National Science Foundation, Grants #2239316 and #2200778. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

TSG1-04

THE IMPACT OF TEACHING MATERIAL FORMATS ON TWO-STEP PROBLEM SOLVING: A COGNITIVE LOAD PERSPECTIVE

Yen-Ting Lai
National Changhua University of Education, Taiwan, leo840509@yahoo.com.tw

Yi-Ting Zhuang
Dapeng Elementary School, Taiwan, dpes10901@dpes.tc.edu.tw

Erh-Tsung Chin
National Changhua University of Education, Taiwan, abegracechin@hotmail.com

To achieve immediate academic success, parents and educational programs often focus on teaching formulas, keywords, and require extensive practice through assessments. However, in a high-pressure environment, students' learning effectiveness varies, and interest in learning tends to be low. For example, in application questions, keywords like "total," "less," "more," "long," and "short" lead students to solve problems with simple operations, neglecting a deeper understanding. This results in students becoming "human calculators," which detracts from the true essence of meaningful learning in mathematics. This study, based on cognitive load theory, examines the learning effectiveness of different teaching material formats in 100 second-grade students across four classes. Comparing caricature books and traditional textbooks, the research looks at learning effectiveness, interest, and cognitive load. Results show significant differences in learning effectiveness, interest, and cognitive load between the two groups, with varying perceptions of difficulty and complexity.

Keywords: two step problem solving, cognitive load, learning interest

Introduction

Mathematics education serves as a foundation for students' cognitive development, with instructional materials playing a pivotal role in shaping learning outcomes. Among various pedagogical approaches, the integration of visual and textual elements has attracted increasing attention for its potential to enhance student engagement and problem-solving skills. This study compares two types of instructional materials—comic-based and traditional text-based—in the context of teaching two-step word problems to second-grade students.

Prior research has highlighted the difficulties students face in solving word problems, particularly those involving multiple steps (Lin, 2000). These difficulties often stem from limited understanding of problem structures and are further intensified by the cognitive load associated with processing complex textual information (Sweller, 2010). Visual representations have been shown to reduce extraneous cognitive load, thereby facilitating access to and engagement with abstract mathematical concepts (Mayer, 2008).

This study aims to explore the effects of comic-based materials on second-grade students' ability to solve two-step word problems, as well as their cognitive load and learning interest. Specifically,

it seeks to answer the following research questions: (a) How do comic-based instructional materials compare to traditional text-based materials in terms of students' performance on two-step word problems? (b) Does the use of comic-based materials reduce students' cognitive load compared to traditional materials? (c) How do comic-based materials influence students' interest in learning mathematics?

Theoretical Background

Two-Step Word Problems

Two-step word problems require students to perform two consecutive arithmetic operations. Their difficulty is influenced by both numerical reasoning and semantic comprehension (Lin, 2000). Research suggests that well-structured instructional materials that explicitly support these relational understandings can enhance students' problem-solving performance.

Cognitive Load Theory

Cognitive load theory identifies three types of cognitive load: intrinsic, extraneous, and germane (Sweller, 2010). Intrinsic load is determined by the inherent complexity of the content, whereas extraneous load is affected by how information is presented. Mayer (2008) emphasized that well-designed multimedia materials can reduce extraneous load and thereby improve learning outcomes.

Visual Instructional Materials and Learning Interest

Learning interest is categorized into situational and individual types (Hidi & Renninger, 2006). Materials that stimulate situational interest can enhance motivation and engagement. Wang and Chen (2015) found that visuals combined with text are more attention-grabbing than text alone. This integration reduces cognitive load and improves comprehension, suggesting that comic-based materials may better support young learners than traditional formats.

Method

Participants

The study participants consisted of 100 second-grade students (ages 7-8) from a primary school in Taiwan, randomly assigned to two groups: a Comic-based Material Group (50 students) and a Traditional Material Group (50 students), drawn from four classes.

Research Process

This study employs a standard experimental design comprising five stages: (1) Pre-test and Grouping: Through random assignment, 100 second-grade students are allocated into two groups—Comic-based Material Group (50 students) and Traditional Material Group (50 students)—ensuring comparable learning backgrounds at baseline. (2) Material Learning: Students read assigned materials independently. Both groups had similar prior exposure to word problems. Minimal guidance was provided to reduce bias, and researchers supervised all sessions to ensure consistency. (3) Testing and Data Collection: Students completed a two-step problem-solving test, a cognitive load questionnaire, and a learning interest scale to capture learning outcomes and subjective experience. (4) Data Analysis: SPSS was used to conduct descriptive statistics, one-way ANOVA, post-hoc analysis, and correlation tests to examine the effects of material format on performance, cognitive load, and interest. (5) Result Interpretation and Discussion: The findings were interpreted to evalu-

ate the instructional impact of each material type, informing recommendations for teaching design.

Research Tools

This study uses the following tools to collect data: (a) Comic-based and Traditional Materials: Both materials present identical content but differ in format. The comic-based material integrates sequential panels with images and embedded dialogue resembling comics, while the traditional version presents the same examples and explanations using only continuous prose without visual aids. (b) Test Paper: Contains two-step application problems to assess students' problem-solving performance. (c) Cognitive Load Questionnaire: Uses a five-point scale to assess students' perceived cognitive load. (d) Learning Interest Scale: Assesses students' interest in learning mathematics. Figure 1 shows examples of the comic-based and traditional materials used in the study, highlighting key differences in visual and textual presentation.

Figure 1. Sample pages from (a) comic-based and (b) traditional instructional materials.

Data Analysis

Data were analyzed using SPSS to examine the effects of comic-based and traditional materials on students' problem-solving performance, cognitive load, and learning interest. The analysis included: (a) Descriptive statistics – mean and standard deviation were calculated to observe group trends across variables; (b) One-way ANOVA – used to identify significant differences in test scores, cognitive load, and learning interest between groups; (c) Post-hoc tests – conducted when ANOVA revealed significance, to further specify between-group differences; and (d) Correlation analysis – used to explore relationships among performance, cognitive load, and learning interest.

Results

This study examined the impact of comic-based and traditional instructional materials on students' performance, cognitive load, and interest in learning mathematics.

Test Scores and Learning Performance

The first research question asked whether the type of instructional material affects students' performance in two-step word problems. The results showed that students using comic-based materials scored significantly higher than those using traditional materials ($p < .01$), suggesting that comics enhance problem-solving performance. Comic-based materials, through visual aids, simplify

the understanding of quantity relationships, reducing confusion and supporting real-world problem-solving. These findings align with Mayer's (2008) multimedia learning theory and Wang & Chen (2015), who also found that visual aids improve learning outcomes.

Cognitive Load Comparison

The second research question investigated whether material type affects perceived cognitive load. Students using comic-based materials reported significantly lower extraneous load than those using traditional materials ($p < .05$). This suggests that the integration of visuals and text made concepts easier to grasp and reduced language processing. In contrast, the text-heavy format increased load, especially for students with lower language proficiency. These findings support Sweller's (2010) view that reducing extraneous load improves learning.

Learning Interest Impact

The third research question asked whether the type of material affects students' interest in learning mathematics. The comic-based group scored higher on the learning interest scale ($p < .05$), suggesting that comics increase learning motivation. Comic-based materials stimulate situational interest through vivid visuals and engaging dialogues, while traditional materials may struggle to maintain students' attention, especially those with lower interest. These results confirm that comic-based materials foster a more engaging and motivating learning environment.

Conclusion and Discussion

Overall, the results of this study show that comic-based instructional materials can effectively enhance students' problem-solving performance, reduce cognitive load, and increase learning interest. While supporting prior findings, this study uniquely highlights how image-text integration benefits young learners' independent problem solving. It provides early evidence that visual scaffolds can enhance learning without direct instruction, offering insight for curriculum design at the lower primary level. Future studies could further explore the long-term effects of comic-based materials and extend the research to other subject areas. Future studies could also explore how visual-text integration affects learners with different reading proficiency levels.

References

Hidi, S., & Renninger, K. A. (2006). The four-phase model of interest development. *Educational Psychologist, 41*(2), 111-127. https://doi.org/10.1207/s15326985ep4102_4

Lin, J. (2000). Challenges in solving word problems: The case of the two-step word problem. *Educational Research, 22*(3), 251-267. https://doi.org/10.1080/0031383002001791

Mayer, R. E. (2008). *Multimedia learning* (2nd ed.). Cambridge University Press.

Sweller, J. (2010). Cognitive load theory. *Psychology of Learning and Motivation, 55*, 37-76. https://doi.org/10.1016/B978-0-12-380877-8.00002-8

TSG1-05

EQUITY IN THE KOREAN ELEMENTARY MATHEMATICS CURRICULUM

MinKyung Kim
Graduate school of KNUE, Korea, cocoa@knue.ac.kr

JeongSuk Pang
Korea National University of Education, Korea, jeongsuk@knue.ac.kr

This study examines the extent to which equitable mathematics teaching practices, as defined by Bartell et al. (2017), into the Korean elementary mathematics curriculum through qualitative research methods and content analysis. The findings indicate a strong focus on attending to students' mathematical thinking and establishing classroom norms for participation. However, practices such as explicitly addressing race and culture, and recognizing multiple forms of discourse and language as valuable resources were less evident. Given the increasing migrant populations in Korea, future curriculum revisions should incorporate these practices to foster greater inclusivity. This study aims to provide in-depth insights into the curricula, policies, and educational practices not only in Korea but also in other countries, with the goal of promoting educational equity and embracing all students within diverse socio-cultural contexts.

Keywords: curriculum, teaching practices, mathematics, equity

Background to the Study

Equity has emerged as a fundamental and timely issue in contemporary mathematics education, driven by efforts to address the marginalization of students based on factors such as socio-economic status, ethnicity, and gender. In mathematics education, equity refers to the intentional effort to expand learning opportunities for all students, particularly those who have historically been marginalized in mathematics due to these factors (Ruef & Ahearn, 2022). Consequently, educators and policymakers are increasingly prioritizing equity to create more inclusive learning environments. However, according to Kim (2023), research on equity in mathematics education within Korea remains limited, with only two domestic publications on this topic published domestically between 2019 and 2023.

Looking ahead, revisions in mathematics curriculum policy can promote equity and shape research in mathematics education (Vithal et al. 2024). As Cerna et al. (2021) and the OECD (2024) observe, the curriculum is a critical instrument for fostering inclusion and equity by embedding principles of rights and social justice into educational systems. However, rather than serving marginalized students, mathematics education policies and reforms frequently reinforce the interests of the economically, technologically, and socially dominant groups (Berry, 2018)

Mathematics curriculum plays a pivotal role in shaping teachers' classroom instructional practices (Osborne, 2021; Remillard et al., 2010). Omitting explicit attention to equity in educational policies and teaching methods can lead to standards failing to fulfill their objectives for students (Bartell et al., 2017). In this context, examining equitable mathematics teaching practices (EMTPs) within the mathematics curriculum is of critical importance. Therefore, this study analyzes the extent to which equity is reflected in Korean elementary mathematics curriculum.

Research Questions

How are EMTPs integrated into the Korean elementary mathematics curriculum? Specifically, to what extent are EMTPs incorporated into each section of the curriculum?

Methods

This study examines how EMTPs are incorporated into the most recently revised Korean elementary mathematics curriculum, using qualitative research methods. EMTP, as defined by Bartell et al. (2017), consists of nine key practices:

> "drawing on students' funds of knowledge, establishing classroom norms for participation, positioning students as capable, monitoring how students position each other, attending explicitly to race and culture, recognizing multiple forms of discourse and language as a resource, pressing for academic success, attending to students' mathematical thinking, and supporting development of a sociopolitical disposition" (Bartell et al., 2017, pp. 11-12)

The curriculum is organized into three main sections: Characteristics & Objectives, Content Structure & Achievement Standards, and Teaching, Learning, & Assessment. Among the various sections, the "Characteristics" section, which consists of three paragraphs, was analyzed at the sentence level for the presence of EMTP. The remaining sections were examined based on individual items.

We conducted a content analysis to identify and tally occurrences of each of the nine EMTPs. Each instance was counted separately, even when multiple EMTPs appeared in the same sentence of item. For example, the item "Recognizing students themselves as the agents of their mathematics learning and encouraging them to actively engage in mathematics learning" (Korean Ministry of Education, 2022, p. 45) was coded both as "Establishing classroom norms for participation" (Practice 2) and "Positioning students as capable" (Practice 3). Because section lengths and content vary, raw EMTP frequencies alone cannot support direct quantitative comparisons across sections.

Therefore, in this study, the frequency of each EMTP identified in each main section was represented as a percentage graph (Figure 1). This visual analysis aims to assess the extent to which EMTPs are incorporated into the curriculum and to highlight areas for potential improvement.

Findings and Conclusions

Figure 1 illustrates that the majority of EMTPs—except for Practices 5 and 6—are consistently observed and well-integrated within the Korean elementary mathematics curriculum. Unlike Practice 1, which appears in relatively uniform proportion across sections, the other practices vary in frequency depending on the characteristics of each section. Despite these variations, Practice 8 was the most frequently reflected practice overall.

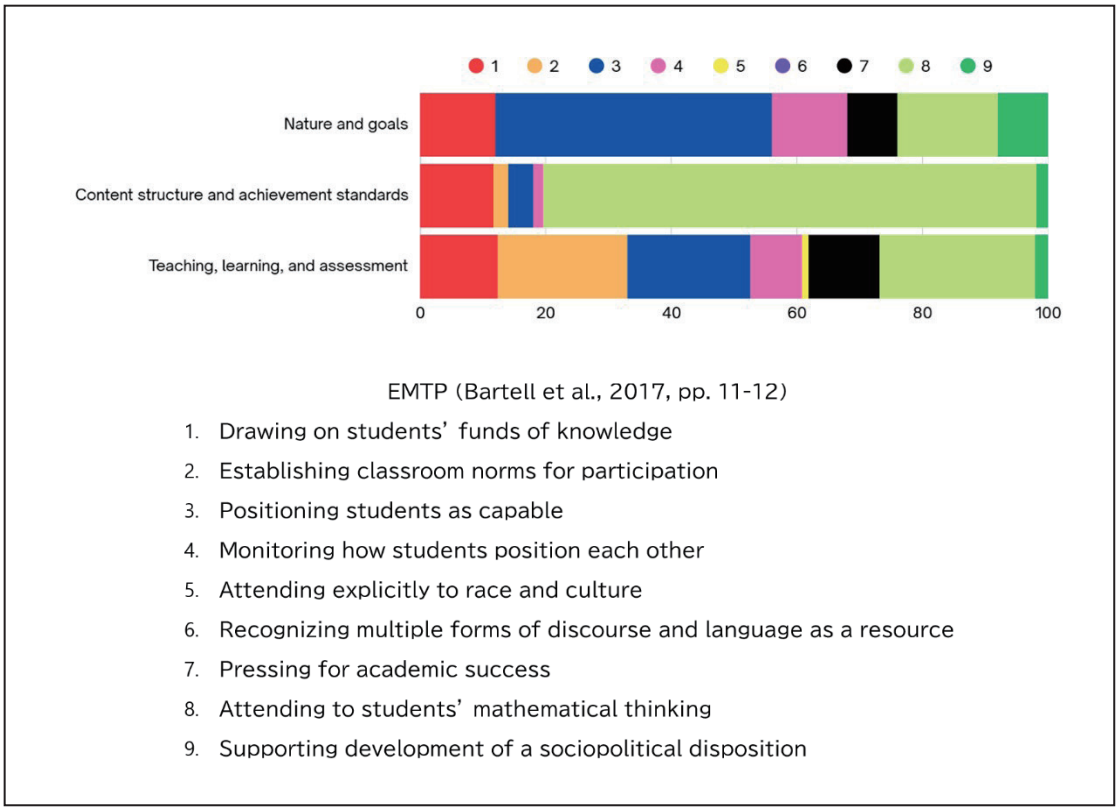


Figure 1. Distribution of EMTPs in the Korean elementary mathematics curriculum

Drawing on Students' Funds of Knowledge (Practice 1)

A comparative analysis revealed that most practices—except for two—were consistently observed. Notably, Practice 1 was evenly distributed across all sections, aligning with the objectives of the mathematics curriculum revision (Lee et al., 2022), which emphasizes deep learning connected to students' lives. By integrating mathematics with other subjects and real-life contexts, students can expand their knowledge and relate lessons to their own experiences and interests.

Attending to Students' Mathematical Thinking (Practice 8)

The remaining practices exhibited significant variations in proportion across different sections of the curriculum. Notably, "Attending to students' mathematical thinking" (Practice 8) was the most frequently observed practice overall. This emphasis on core mathematical ideas is intended to foster reflective and predictive learning, promoting a deeper understanding of transferable content (Lee et al., 2022). Consequently, the curriculum places a strong emphasis on developing students' mathematical thinking.

Attending Explicitly to Race and Culture (Practice 5) and Recognizing Multiple Forms of Discourse and Language as a Resource (Practice 6)

Among the nine practices, Practice 5 played a minimal role in the teaching, learning, and assessment areas, while Practice 6 was rarely observed throughout the curriculum. This may be attributed to the relatively low relevance these Practices 5 and 6 to the nature of educational disparities in Korea. Park & Baik (2019) identified key factors contributing to educational inequality in Korea, including socio-economic background, participation in private education, and parents' educational expectations and support. Additionally, Korea remains one of Asia's most ethnically homogeneous developed countries (Cunningham, 2021), with Korean as the sole official language.

However, since 2020, the number of long-term foreign residents in Korea has been increasing (Korean Ministry of Justice, 2024), indicating that the gradual incorporation of Practices 5 and 6 into the curriculum may become necessary in the future. Consequently, when revising their educational curricula, other East Asian countries with a high degree of ethnic homogeneity and a dominant official language, could consider Korea's findings on Practices 5 and 6 as a reference.

The analysis reveals that, while the emphasis on EMTPs varies, the Korean curriculum prioritizes students' mathematical thinking as a means of promoting equitable education. This focus reflects an intention to assess students' mathematical comprehension and design instruction accordingly. However, it is difficult to conclude that the curriculum fully addresses social changes and diversity, highlighting the need for revisions to enhance educational equity.

This study seeks to provide global insights into the effective integration of EMTPs by analyzing their presence in Korea's national-level curriculum documents. Furthermore, by comparing curricula across different countries, this research aims to identify diverse implementations of EMTPs and encourage further studies on their adaptation to various educational contexts.

References

Bartell, T., Wager, A., Edwards, A., Battey, D., Foote, M., & Spencer, J. (2017). Toward a framework for research linking equitable teaching with the standards for mathematical practice. *Journal for Research in Mathematics Education, 48*(1), 7–21. https://doi.org/10.5951/jresematheduc.48.1.0007

Berry, R. Q. (2018). Disrupting policies and reforms in mathematics education to address the needs of marginalized learners. In T. Bartell (Ed.), *Toward equity and social justice in mathematics education* (pp. 3–20). Springer. https://doi.org/10.1007/978-3-319-92907-1_1

Cerna, L., Mezzanotte, C., Rutigliano, A., Brussino, O., Santiago, P., Borgonovi, F., & Guthrie, C. (2021). *Promoting inclusive education for diverse societies: A conceptual framework* (OECD Education Working Papers, No. 260). OECD. https://doi.org/10.1787/94ab68c6-en

Cunningham, E. (2021, November 11). South Korea's population is barely growing. It should accept more refugees. *International Law and Policy Brief*. https://studentbriefs.law.gwu.edu/ilpb/2021/11/11/south-koreas-population-is-barely-growing-it-should-accept-more-refugees/

Kim, R. (2023). Equity in school mathematics education: A review of the literature [in Korean with English abstract]. *Communications of Mathematical Education, 37*(3), 369–392. https://doi.org/10.7468/jksmee.2023.37.3.369

Korean Ministry of Education. (2022). *Mathematics curriculum.*

Korean Ministry of Justice. (n.d.). *Introduction to the Ministry of Justice*. https://www.moj.go.kr/moj/2412/subview.do

Lee, K. H., Kim, S., Kim, Y., Kang, E., Kim, N., Choi, I., Joo, S., Cho, S., Lim, M., Kim, E., Na, G., Byun, H., Jeong, J., Choi, J., Kim, D., Yang, J., Tak, B., Lee, S., Jeon, J., ... Kim, J.. (2022). *Research on the development of the draft for the 2022 revised mathematics curriculum* (Research Report 11-B552111-000033-01) [in Korean]. Korea Foundation for the Advancement of Science and Creativity.

OECD. (2024). *An evolution of mathematics curriculum: Where it was, where it stands and where it is going.* https://doi.org/10.1787/0ffd89d0-en.

Osborne, M. C. (2021). Teacher instructional practices and student mathematics achievement. *Journal of Educational Research and Practice, 11*(1), 345-358. https://doi.org/10.5590/JERAP.2021.11.1.25.

Park, J. H., & Baik, J. M. (2019). The systematic analysis of the empirical studies measuring the educational gap [in Korean with English abstract]. *Korean Education Inquiry, 37*(1), 213–238. https://doi.org/10.22327/kei.2019.37.1.213.

Remillard, J. T., Herbel-Eisenmann, B. A., & Lloyd, G. M. (Eds.). (2010). *Mathematics teachers at work: Connecting curriculum materials and classroom instruction*. Routledge.

Ruef, J. L., Willingham, C. J., & Ahearn, M. R. (2022). Math and equity in the time of COVID: Teaching challenges and successes. *International Electronic Journal of Mathematics Education*, *17*(2), em0681. https://doi.org/10.29333/iejme/11818

Vithal, R., Brodie, K., & Subbaye, R. (2024). Equity in mathematics education. *ZDM Mathematics Education, 56*(1), 153–164. https://doi.org/10.1007/s11858-023-01504-4

TSG1-06

A COMPARISON OF THE CATEGORIES OF NON-TEXT ELEMENTS IN HIGH SCHOOL TEXTBOOKS – A CASE STUDY OF PEP A VERSION AND A-LEVEL VERSION

Xiaolin Yuan
East China Normal University, China, 51265500097@stu.ecnu.edu.cn

Han Qiu
East China Normal University, China, 51265500130@stu.ecnu.edu.cn

The research focuses on the categories of non-text elements, and makes a comparative study on the non-text elements of Chinese and English high school mathematics textbooks. The research finds: in terms of category distribution, the category distribution of non-text elements in Chinese and British textbooks is relatively consistent, mainly concentrated in notes and mathematical concepts classes.

Keywords: Non-text elements, Category, Textbook analysis

Introduction

As an important part of mathematics textbooks, non-text elements are not only the carrier of information, but also the tool of thinking. In addition, scholars have systematically studied the quality of illustrations in textbooks, and most of the existing domestic literature is qualitative research, lacking of systematic research for the categories of non-text elements. Besides, there are few comparative studies on non-text elements in mathematics textbooks between China and Britain. Therefore, the research takes Chinese and British mathematics textbooks as the research object, and analyzes their categories and functions. Due to space constraints, this paper mainly solves the following problems from the category dimension: What are the categories of non-text elements in different versions of mathematics textbooks in China and Britain? What is the overall distribution of it? What are the similarities and differences in categories?

Research Design

Non-text Element Classification

According to the characteristics of mathematics, non-text elements can be divided into life, pure mathematics, portraits and experimental diagrams. Some researchers also divide it into geometric figures, charts, pictures, composite figures and other categories (Sun, 2018). Because the classification of the latter is more prominent in the geometry of mathematics, Weizhong Zhang and Zhizhi Hu further explained it (Weizhong Zhang & Hu Zhizhi, 2019). Based on the characteristics of textbooks in the two countries, the study divides mathematics into mathematical concepts and data charts bases on life, mathematics, combination, portraits and other categories. The portraits are changed to the history of mathematics, and ICT and notes are added. Finally, combined with the

classification of Lingzhu Li (Lingzhu Li & Qiuyu Chen, 2024), non-text elements are divided into eight categories, namely, life, mathematical concept, data graph table, history of mathematics, ICT, combination, note and other classes.

Research Object

The research selected the Chinese people's education A version textbook(referred to as PEP A version) and the British A-level mathematics textbook(referred to as A-level version) as the research object . Because the use of illustrations in different countries is not the same, considering the comparability of illustration categories in two textbooks, combined with the comparison between the ' Ordinary High School Mathematics Curriculum Standards ' and A-level version, the Chinese and British high school mathematics curriculum content is corresponding. The researchers removed the collection, common logical terms, complex numbers, solid geometry preliminary, solid geometry part, and probability statistics part in PEP A version to ensure the correspondence between Chinese and English knowledge points.

Research Process

In the process, the researchers divided the categories of non-text elements. If a non-text element appears in multiple categories, it is divided according to the most prominent category. If a non-text element appears in multiple categories with the same status, it is regarded as a combination class. Due to a total of 3347 of non-text elements in 11 textbooks, it was finally decided to extract 20 % of them to calculate the consistency as the reliability of non-text elements. Firstly, the two researchers independently coded the non-text elements, discussed the inconsistent coding, and revised the coding standard. Then, the researchers extracted the non-text elements in two version textbooks in proportion, and each volume adopted the method of stratified sampling. Then, the two researchers independently coded the categories and functions of 20 % non-textual elements. Finally, the consistency test was performed on the non-text elements of the categories encoded by the two researchers. It is calculated that Kappa category = 0.81, which indicates that the category has a relatively high consistency.

Analysis of Research Results

Overall Analysis

Table1. The distribution of textbook categories in PEP A version

	Life class	Mathematics Concepts class	Data Graph Table class	History of Mathematics class	ICT class	Combination class	Notes class	Other class	total	Average density
Compulsory 1	16	135	57	6	18	7	159	0	398	1.49
Compulsory 2	11	81	0	0	0	13	55	0	160	0.58
Optional Compulsory 1	13	185	1	3	2	13	125	0	342	2.25
Optional Compulsory 2	18	45	21	8	3	6	85	0	186	1.22
Optional Compulsory 3	7	6	13	3	0	2	41	0	72	0.64

There are 1158 non-text elements in PEP A version (as shown in Table 1). The average density of the 5 textbooks is 1.207 (average density = non-text elements/total number of pages). The order of single volume density is: Compulsory 2 (0.58) < Optional Compulsory 3 (0.64) < Optional Compulsory 2 (1.22) < Compulsory 1 (1.49) < Optional Compulsory 1(2.25). 8 categories of non-text elements in order of number are: history of mathematics class (20) <ICT class (23) <life class (65) <combination class (41) <data graph table class (92) <mathematical concepts class (452) <notes class (465).

Table2. Distribution of textbook categories in A-level version

	Life class	Mathematics Concepts class	Data Graph Table class	History of Mathematics class	ICT class	Combination class	Notes class	Other class	total	Average density
Core1	5	94	0	3	0	6	320	0	428	2.18
Core2	16	259	11	0	4	33	495	1	819	3.49
Core3	10	162	3	1	18	27	374	1	596	2.95
Core4	8	98	3	0	1	10	322	5	227	1.34

There are 2,189 non-text elements in A-level version (as shown in Table 2). The average density of the 4 textbooks is 2.73.The density of the single book is ranked: Core4(1.34)< Core1(2.18)< Core3(2.95) <Core2(3.49).From the perspective of categories, the 8 types of non-text elements are: history of mathematics class (4)< other class (7)<data graph table class (17)<ICT class (23)<life class (37)<combination class (75) <mathematical concepts class(593) <notes class (1434).

A Comparison of the Categories of the Two Editions of Textbooks

Table3. The difference analysis of non-text element categories in the two editions of textbooks

category	version (%)		Pearson chi-square	Asymptotically significant (two-sided)
	PEP A version	A-level version		
Life class	5.61	1.68	0.229	0.018
Mathematics Concepts class	39.03	27.07		
Data Graph Table class	7.94	0.77		
History of Mathematics class	1.72	0.18		
ICT class	1.98	1.05		
Combination class	3.54	3.42		
Notes class	40.15	65.47		
Other class	0	0.32		

As shown in Table 3, there is no significant test for the categories of non-text elements in the two editions of textbooks (P=0.229> 0.05) after the chi-square test. This shows that the distribution of illustrations in two versions is not much different, and the overall distribution is more consistent. Although two versions tend to be consistent in the overall distribution of categories, there are significant differences in type preference and forms of expression. Specifically, PEP A version is dominated by elements of mathematical concepts and notes class: mathematical concepts class are presented intuitively based on geometric graphics (as shown in Figure 1) to strengthen the visual expression of abstract concepts; The notes class uses a variety of forms such as question mark boxes and bubble notes (as shown in Figure 2), aim to arouse students' in-depth thinking through questions. On the contrary, in A-level version, the proportion of notes class is significantly higher than other types, but its form of expression is relatively simple. It mainly notes key steps or theorems through circle icons, supplemented by arrows to establish logical associations (as shown in Figure 3), in order to highlight the core knowledge points and their internal connections.

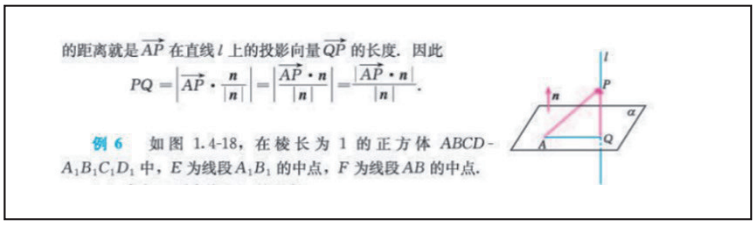


Figure 1. One mathematical concepts class of PEP A version

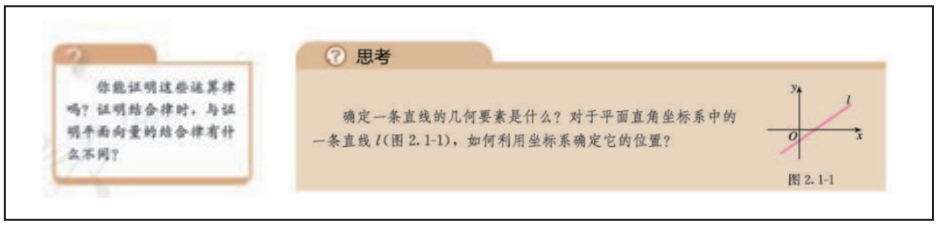


Figure 2. One notes class of PEP A version

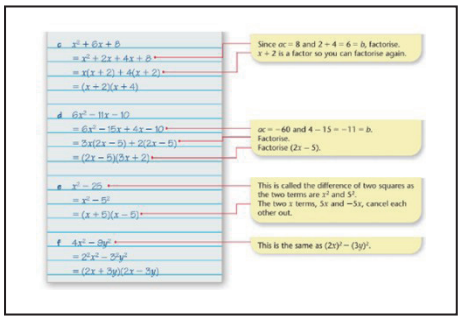


Figure 3. One notes class of A-level version

Revelation

Through comparative analysis, the main conclusions are as follows: the non-text elements in two textbooks are divided into eight categories, namely, life, mathematical concepts, data charts, mathematical history, ICT, combination, notes and other classes. In terms of category distribution, the non-text elements in two textbooks show high similarity, but the presentation methods are different. It is worth noting that the A-level version shows distinctive features in the use of non-text elements: its notes class effectively highlight the key steps and important conclusions of problem solving through many circle chart annotations, process arrows, etc. (as shown in Figure 3). This design can guide students to focus on the core knowledge nodes with the help of visual markers, and clearly present the progressive process of mathematical reasoning with the help of graphical language. Therefore, PEP A version also appropriately introduces such interactive notes in the writing process, especially in the derivation of complex theorems and problem solving, to reduce students' cognitive load and realize the "dialogue" between students and textbooks.

References

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational studies in mathematics, 52*(3), 215-241.

Gunzel, M., & Binterova, H. (2016). Evaluation of Nonverbal Elements in Mathematics Textbooks. *Universal Journal of Educational Research*, *4*(1), 122-130.

Hunter, B., Crismore, A., & Pearson, P. D. (1987). Visual displays in basal readers and social studies textbooks. In *The Psychology of Illustration: Volume 2: Instructional Issues* (pp. 116-135). Springer.

Kim, R. Y. (2012). The quality of non-textual elements in mathematics textbooks: an exploratory comparison between South Korea and the United States. *ZDM*, *44*, 175-187.

Levin, J. R. (1981). On functions of pictures in prose. *Neuropsychological and cognitive processes in reading*, *203*, 28.

Baozhen Li, Guoyi Chen, & Jiawei Gong. (2021). A comparative study of the problem situation of ' mathematical modeling ' in the new high school mathematics textbook. *Mathematical communication* (10), 4-10 + 29.

Lingzhu Li, & Qiuyu Chen. (2024). Research on the characteristics and functions of non-text elements in junior high school mathematics textbooks-Taking Zhejiang Education Edition as an example. *Journal of Mathematics Education*, *33* (01), 8-15 + 43.

Zhuochen Li, & Xiaoqin Wang. (2022). Comparative study of mathematical culture in the front page of Chinese and French junior high school mathematics textbooks. *Journal of Mathematics Education, 31* (2), 26-34.

Ming Liu. (2012). Analysis and coping strategies of ' negative effects ' of mathematical illustrations. *Teaching and management (primary school edition)* (4), 4647.

Qianwen Liu, Yuqing Chen, Yue Wu, & Xiaoqin Wang. (2024). A comparative study of illustrations in Chinese and French junior high school mathematics textbooks. *Journal of Mathematics Education, 33* (05), 18-26.

Qingkuo Sun. (2018) The classification, function and application strategy of illustrations in high school mathematics textbooks in China. *Curriculum teaching research.*

Xiaoli Wang. (2018). A comparative study of rational number examples in Chinese and American junior high school mathematics textbooks, *Henan University*.

Yiru Wang, & Xueli Liu. (2021). Study on the characteristics of illustrations in junior high school mathematics textbooks published by People ‘s Education Press. *Journal of Inner Mongolia Normal University (Education Science Edition)*.

Zhang, & Hu. (2019).Analysis of non-linguistic elements in mathematics textbooks-Taking the eighth grade mathematics textbook of Zhejiang Education Edition as an example. *Journal of Zhejiang Normal University (Natural Science Edition)*, 3.

TSG1-07

A COMPARATIVE STUDY OF DERIVATIVE CONTENTS IN HIGH SCHOOL MATHEMATICS TEXTBOOKS

Gengxin Li
Beijing Normal University, China, hughlee526@163.com

Chunxia Qi
Beijing Normal University, China, qichxia@126.com

This study conducts a comparative analysis of derivative contents in high school mathematics textbooks from China, Britain, America, and Australia. Through qualitative and quantitative methods, it explores differences in the compilation style of knowledge content, cognitive level distribution, integration of information technology and mathematical modeling activities. The research results demonstrate the characteristics of different textbook editions and reveal the educational orientations of various countries. Chinese textbooks emphasize real-life cases and problem-solving, while foreign ones focus on spiral knowledge structures, systematic summaries and advanced IT applications. The study concludes that diverse textbook approaches reflect different educational orientations, offering insights into enhancing students' understanding and mastery of derivative concepts.

Keywords: curriculum and materials, derivative contents, textbook comparison

Introduction

Mathematics education plays a pivotal role in nurturing students' logical thinking, problem-solving skills, abstract reasoning abilities and so on. With the release of updated curriculum standards, high school mathematics textbooks, as primary learning resources, have undergone significant revisions in China, with derivatives emerging as a crucial topic due to its foundational role in calculus and its wide range of practical applications. However, students often encounter cognitive barriers when learning derivatives, attributed to its abstraction and complexity. Research indicates that students often fail to grasp the essence of derivatives, focusing instead on rote computation (Bressoud, 2022; Huang, 2022). This study seeks to analyze the presentation of derivative contents in textbooks from four countries, identifying strengths and weaknesses to inform pedagogical practices and explore the essence of mathematics education.

Methods

This study employs a comparative research design, focusing on seven editions of high school mathematics textbooks from China (PEP A and PEP B published by People's Education Press in 2020, BNU Edition published by Beijing Normal University Press in 2020, JE Edition published by Jiangsu Phoenix Education Press in 2021), the UK (Advanced Level Pure Mathematics 1 published by Cambridge University Press in 2018), the USA (AP Calculus Early Transcendentals published by Cengage Learning in 2020), and Australia (VCE Mathematical Methods published by Cam-

bridge University Press in 2006). The textbooks from China and Australia are at the high school level, whereas British A-Level and American AP are university preparatory materials. However, their teaching content and objectives in the content of derivatives are largely consistent and comparable.

The research questions revolve around the following aspects: the compilation style, cognitive level distribution, the integration of information technology and mathematical modeling activities. Their significance lies in ensuring that textbooks effectively facilitate students' conceptual understanding, cognitive development, technological engagement, and real-world application of derivatives. Data collection involves comprehensively organizing and scientifically analyzing the content related to derivatives in the seven versions of textbooks, followed by quantitative and qualitative analyses to summarize the characteristics of each version.

Results

Compilation Style of Knowledge Content

The various editions of textbooks differ in the content framework of chapters, as well as in the arrangement of various column modules within the main text, demonstrating diversified compilation styles. The way derivative content is structured and presented in textbooks significantly impacts students' learning experiences and understandings of the essence of mathematics. Fan & Zhu (2007) demonstrate that differences in textbook organization across countries affect students' learning pathways and problem-solving strategies.

Chinese textbooks embark from real-life cases, adopting a learning model of "situations and problems, research and analysis, learning new knowledge, and practice for consolidation." This learning approach is reinforced through clear column settings, enabling students to comprehend the significance of new knowledge while cultivating mathematical thinking and core competencies in the process of problem-solving.

Foreign textbooks emphasize the spiral structure of knowledge. For example, the British AL edition clarifies the previous chapters related to the current content, while the Australian MM edition outlines learning objectives in the preface, acquainting students with the key and difficult points of the study in advance. Foreign versions all systematically summarize important core knowledge points at the end of each chapter.

Cognitive Level Distribution in Derivative Content

Bloom categorized the cognitive domain into six hierarchical levels based on educational objectives: remembering, understanding, applying, analyzing, synthesizing, and evaluating (Bloom, 1956). Comparatively, China's curriculum standards delineate knowledge acquisition into four progressive levels: knowing, comprehending, mastering, and applying. These two classification systems demonstrate significant correspondence (Zhong & Cui, 2018). To facilitate analytical clarity, this study consolidates cognitive levels of knowledge into three comprehensive tiers: knowing, understanding and applying.

Table 1. Cognitive level distribution in derivative content across textbook editions

	PEP A	PEP B	BNU	JE	British AL	American AP	Australian MM
Knowing	9	8	4	9	3	7	7
Understanding	7	8	11	8	11	20	9
Applying	5	5	5	4	3	9	3
Total	21	21	20	21	17	36	19

All four Chinese textbook editions each contain 20-21 knowledge points, demonstrating comparable quantities. This configuration strictly aligns with the national curriculum standards' requirements for students' knowledge competencies. In contrast, the American AP edition contains the highest number of knowledge points (36), given its nature as preparatory materials for collegiate-level calculus courses. The British AL edition presents the fewest knowledge elements, primarily due to its pedagogical approach that foregoes introductory exposition and proceeds directly to formal definitions when addressing derivative concepts.

Integration of Information Technology

With the rapid development of artificial intelligence, the cultivation of information literacy has become increasingly crucial. Consequently, there has been a deep integration of mathematics education with information technology, utilizing information technology as an auxiliary tool in the process of learning mathematics to stimulate students' interest in it. Due to the complexity of derivatives, the use of information technology in mathematics education can make these abstract concepts more comprehensible and accessible (Tall, 1997). The textbooks studied demonstrate varying degrees of IT integration.

Chinese textbooks employ GeoGebra for graphing and exploring function properties, with the BNU Edition providing detailed instructions on software operation. However, the use of these tools is presented as optional recommendations rather than mandatory activities.

In contrast, foreign textbooks, including the British AL Edition, American AP Edition, and Australian MM Edition, tend to embrace IT more comprehensively. They design numerous examples and exercises requiring technological solutions, embedding IT learning within training activities to genuinely achieve the goal of enhancing students' IT capabilities. These textbooks also encourage students to explore through interactive software and online resources.

Mathematical Modeling Activities

Mathematical modeling activities connect derivative concepts to real-world applications, enhancing students' ability to use derivatives in practical situations, which are crucial in cultivating students' mathematical literacy, critical thinking, innovative spirit, and problem-solving abilities (Blum & Leiß , 2007; Schoenfeld, 2014; Cevikbas & Schukajlow, 2022). The textbooks studied offer varying levels of opportunities for such activities. The PEP A Edition presents comparative explorations of the speed of Newton's tangent method and the bisection method for finding approximate solutions to equations through "Inquiry & Discovery" sections, while PEP B Edition and the JE Edition incorporate these as post-chapter exercises, allowing students to experience the optimization effects

of numerical computation processes. A mathematical modeling activity in PEP Edition B focuses on describing the relationship between body weight and pulse rate, emphasizing the application of functional modules.

In contrast, the American AP Edition textbook contains the highest number of mathematical modeling activities, including investigations such as analyzing the motion trajectory of a pilot's parachute landing and exploring why the optimal solution for beverage can design is uncommonly seen in real life. Such modeling activities are rooted in real-life situations, thus posing greater challenges, so typically provide students with extensive guidance and support to help them successfully tackle these complex tasks.

Discussions

Due to limitations in class hours, the chapter on "Derivatives and Their Applications" cannot cover all aspects such as mathematical culture, information technology, and mathematical modeling. Therefore, selections and omissions are necessary, which reflect respective orientations.

The PEP A edition emphasizes student initiative in learning, yet features monotonous and repetitive exercise types. The PEP B edition distinguishes itself with interdisciplinary contexts like marginal costs in finance and biological population dynamics, helping students appreciate mathematics in daily life, though its application of IT is relatively basic. The BNU edition offers clear hierarchy and logical flow, supplements calculus history, and provides abundant cases of IT applications and modeling activities. The JE Edition stands out with its visual presentation, emphasizing graphical-numerical integration and geometric intuition development.

The British AL edition is the most difficulty, constructing knowledge networks through connecting new and prior concepts. The American AP Edition incorporates substantial advanced mathematics content with extensive marginal annotations. The Australian MM Edition features the most extensive page count with meticulous explanations, though accompanied by considerable repetitive practice.

Conclusions

This study reveals that the essence of mathematics education in curriculum and materials is reflected in the diverse compilation styles, use of information technology, and promotion of modeling and inquiry activities in textbooks. Each country's textbooks exhibit unique strengths and approaches to teaching derivatives. To enhance students' understanding and mastery of derivative concepts, educators should draw from the strengths of these diverse textbooks.

References

Bloom, B. S. (Ed.). (1956). Taxonomy of educational objectives: *The classification of educational goals. Handbook I: Cognitive domain.* Longmans, Green.

Blum, W. & Leiß, D. (2007). How do students and teachers deal with mathematical modeling problems? *The International Journal on Mathematics Education*, 39(5), 777–787.

Bressoud, D., Ghedamsi, I., Martinez-Luaces, V., & Törner, G. (2016). *Teaching and learning of calculus: A literature review. International Journal of Mathematical Education in Science and Technology*, 47(2), 285–299.

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2022). A systematic literature review of mathematical modeling education in higher education. *International Journal of STEM Education*, *9*(1), 1–24.

Fan, L. & Zhu, Y. (2007). From convergence to divergence: the development of mathematical problem solving in research, curriculum, and classroom practice in Singapore. *ZDM Mathematics Education*, 39(5-6), 491-501.

Huang, S. (2022). *The research on construction of senior high school student's derivative cognitive structure.* Zhangzhou: Minnan Normal University.

Schoenfeld, A. H. (2014). Mathematical modeling, critical thinking, and the arts. *Educational Studies in Mathematics*, 86(2), 149–164.

Tall, D. (1997). Functions and calculus. In A. J. Bishop (Ed.), *International Handbook of Mathematics Education* (pp. 289–325). Springer.

Zhong, Q., & Cui, Y. (2018). Reforming China's curriculum: From knowledge transmission to competency development. *Journal of Curriculum Studies*, 50(3), 291–308.

TSG1-08

A COMPARATIVE STUDY ON THE PRAXEOLOGICAL ANALYSIS OF ELEMENTARY SCHOOL MATHEMATICS TEXTBOOKS IN KOREA AND FINLAND: FOCUSING ON SYMMETRY AND TRANSFORMATION

Jaehyeon Park
Seoul National University, Korea, jaehynp1@snu.ac.kr

Seunghwan Jeon
Seoul National University, Korea, jsh095@snu.ac.kr

Kyungwon Lee
Seoul National University, Korea, kyungwon.lee.snu@gmail.com

Oh Nam Kwon
Seoul National University, Korea, onkwon@snu.ac.kr

This study analyzes the structure of tasks covered in the symmetry and transformation units of Korean and Finnish elementary school mathematics textbooks through a praxeological analysis. The concept of praxeology, central to the Anthropological Theory of the Didactic (ATD), provides a perspective for analyzing human activity and institutional practices. This study applies praxeological analysis to mathematics textbooks to provide insights into the structure of tasks. The analysis of the symmetry and transformation tasks in Korean and Finnish elementary school mathematics textbooks identified eight praxeologies. Differences in the sequence of these praxeologies were observed, which can be attributed to whether certain sub-concepts are introduced in the curricula of each country.

Keywords: praxeological analysis, Anthropological Theory of Didactic (ATD), mathematics textbooks

Introduction

Symmetry and transformation are fundamental concepts in geometric learning, serving as essential foundations for students' spatial awareness and understanding of geometry. The teaching and learning of these concepts can be considered significant in the curriculum of any country, yet it has often been addressed within specific educational contexts. Takeuchi and Shinno (2020) analyzed the similarities and differences in praxeologies found in Japanese and English middle school textbooks, providing insights into the descriptions of symmetry and transformation in the textbooks of these two countries. Building on such research cases, this study conducts a comparative analysis of the praxeologies in the symmetry and transformation units of elementary school mathematics textbooks in Korea and Finland. The study not only aims to enhance the understanding of the content domains related to symmetry and transformation taught in each country but also seeks to provide implications for better textbook descriptions through praxeological analysis-based textbook comparison. Specifically, the research addresses the following question: *What praxeologies and their sequences are presented in Korean and Finnish elementary school mathematics textbooks?*

Theoretical Background

The Anthropological Theory of the Didactic (ATD), developed by Chevallard, is a theoretical framework that explains the nature and meaning of mathematical knowledge in school mathematics and the methods of didactics from an anthropological perspective. This perspective considers school mathematics and didactics as part of human activity and culture, shaped by the evolution and transmission of institutions over time. Central to ATD is the concept of praxeology, which serves as the fundamental unit for analyzing human activity and institutional practices (Bosch & Gascón, 2006). Praxeology consists of two main components: the *Praxis Block*, which includes the *Type of Task* (specific problems of a certain type) and *Technique* (methods to address these tasks), and the Logos Block, which includes *Technology* (justifications for the techniques) and *Theory* (a deeper framework to explain or refine the technology) (Chevallard & Sensevy, 2014). Bosch and Gascón (2014) further classified praxeologies into three levels: point praxeology, focusing on a single type of task; local praxeology, encompassing multiple types of tasks sharing a common technological discourse; and regional praxeology, integrating all point and local praxeologies under a shared theoretical framework. Through these components and levels, ATD provides a comprehensive lens to analyze the processes of teaching, learning, and the organization of mathematical knowledge. Praxeological analysis facilitates a structural understanding of mathematical tasks, identifies institutional constraints that determine which tasks or techniques are included or excluded, and provides insights into the cultural context of mathematics teaching and learning. This study aims to analyze the praxeologies of problems presented in Finnish and Korean elementary school mathematics textbooks to offer new insights and knowledge regarding the teaching of symmetry and transformations.

Methods

This study aims to enhance the understanding of teaching symmetry and transformation by conducting a praxeological analysis of the units involving symmetry and congruence in elementary school mathematics textbooks from Korea and Finland. In the Korean elementary school mathematics curriculum, the concept of transformations is introduced in Grade 3, while symmetry-related content is covered in Grade 5. The Finnish elementary school mathematics curriculum integrates the concept of transformations within the context of symmetry-related content. The elementary school mathematics curricula of the two countries are comparable, as they share similar approaches to introducing symmetry and transformations. In Korean mathematics textbooks (KMTs) (Kim et al., 2021a, 2021b), the symmetry and transformation units appear in the first semester of Grade 4 and the second semester of Grade 5, whereas in Finnish mathematics textbooks (FMTs) (Ilmavirta et al., 2012a, 2012b), they are introduced in the first semester of Grade 3 and the first semester of Grade 5. For the KMTs, the I-Scream Publisher was selected considering its market share, while for the FMTs, the Laskutaito Publisher was chosen based on accessibility. The first and second authors analyzed the tasks presented in the textbooks using the framework of types of tasks, techniques, technologies, and theories established by Takeuchi and Shinno (2020) in their analysis of Japanese and English middle school mathematics textbooks. When the tasks could not be coded using the foundational framework, open coding was employed to identify and analyze new types of tasks, techniques, and technologies. Theories were coded under the single theory of isometry. The list of types of tasks, techniques, and technologies was refined through this process. All authors reviewed the analysis to ensure its validity and derived the praxeologies. We identified the sequences of praxeologies based on individual textbooks and analyzed both the common praxeologies and those that differ.

Findings

The results of the analysis of the praxeologies of KMTs and FMTs are summarized in Table 1. The distribution of the number of problems according to the type of task is presented. In the case of KMTs, the tasks are ordered by frequency as T3, T1, and T4, whereas in Finland, the tasks are ordered as T6, T1, and T3. A notable finding is that a task type with a low proportion in KMTs (T6, 11.7%) is observed with a significantly higher proportion in FMTs (T6, 27.66%). Additionally, tasks related to T4 and T8 are found exclusively in KMTs. These findings suggest the need to analyze the semester-by-semester sequence of praxeologies. Furthermore, the results demonstrate that the components of praxeology (*Type of Task*, *Technique*) differ from those identified by Takeuchi and Shinno (2020).

Table 1. Praxeological analysis of KMTs and FMTs

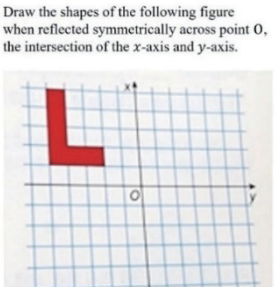
Type of Task(T)	Technique(τ)	Problems		Technology(θ)		Theory(Θ)
		KMTs (n=94)	FMTs (n=47)	KMTs	FMTs	
T1. Identify symmetric shapes	τ1: epistemic τ2: physical	14.5 (15.43%)	10 (21.27%)	θ_1	θ_1	Isometry
T2. Draw symmetric shapes	τ2: physical τ3: manipulative	5.3 (5.64%)	6 (12.76%)	θ_2	θ_2	
T3. Identify symmetry axes and corresponding points/lines/angles/ centers of symmetry	τ1: epistemic τ2: physical	24.3 (25.85%)	9 (19.14%)	θ_1	θ_1	
T4. Explore properties of symmetric shapes	τ1: epistemic τ3: manipulative τ4: symbolic	11.9 (12.66%)	–	θ_4	–	
T5. Identify transformed shapes	τ1: epistemic τ3: manipulative	11 (11.7%)	6 (12.76%)	θ_3	θ_3	
T6. Draw transformed shapes	τ3: manipulative τ2: physical τ1: epistemic	11 (11.7%)	13 (27.66%)	θ_3	θ_2	
T7. Explain transformations matching one shape to another	τ3: manipulative τ1: epistemic	13 (13.83%)	3 (6.38%)	θ_3	θ_3	
T8. Explore properties of given transformations	τ4: symbolic	3 (3.19%)	–	θ_4	–	

θ_1: one shape is symmetrical; θ_2: A is symmetrical to B in relation to C; θ_3: geometric relationships between the before-and-after shapes that are transformation; θ_4: symmetry conserves lengths and angles

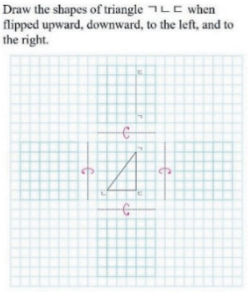
The praxeological sequences observed in KMT and FMT are presented in Table 2. The common praxeologies shared by KMTs and FMTs include [T1/τ/θ_1/Θ], [T2/τ/θ_2/Θ], [T3/τ/θ_1/Θ], [T5/τ/θ_3/Θ], and [T7/τ/θ_3/Θ], while the differing praxeologies include [T4/τ/θ_4/Θ], [T6/τ/θ_2/Θ], [T6/τ/θ_3/Θ], and [T8/τ/θ_4/Θ]. The order of semesters for common praxeologies in KMT and FMT differs, as shown in Table 2. For example, while [T1/τ/θ_1/Θ], [T2/τ/θ_2/Θ], and [T3/τ/θ_1/Θ] are covered in the first semester of FMTs focusing on symmetry and transformation, these praxeologies are covered in the second semester of KMTs. Similarly, [T5/τ/θ_3/Θ], and [T7/τ/θ_3/Θ] are addressed in the first semester of KMTs, focusing on symmetry and transformation; those praxeologies are covered in the second semester of FMTs. Despite being distinct praxeologies, [T6/τ/θ_2/Θ] and [T6/τ/θ_3/Θ] differ in the order of semesters in which they are taught. These differences highlight the structural characteristics of the tasks addressed in each country and suggest the need for an institutional interpretation of the content covered in the respective national curricula.

Table 2. Sequences of praxeologies in KMTs and FMTs

Semester	KMTs	Semester	FMTs
1st Semester Grade 4	[T5/τ/θ_3/Θ], [T6/τ/θ_3/Θ], [T7/τ/θ_3/Θ], [T8/τ/θ_4/Θ]	1st Semester Grade 3	[T1/τ/θ_1/Θ], [T2/τ/θ_2/Θ], [T3/τ/θ_1/Θ]
2nd Semester Grade 5	[T1/τ/θ_1/Θ], [T2/τ/θ_2/Θ], [T3/τ/θ_1/Θ], [T4/τ/θ_4/Θ]	1st Semester Grade 5	[T5/τ/θ_3/Θ], [T6/τ/θ_2/Θ], [T7/τ/θ_3/Θ]



Example of [T6/τ/θ_2/Θ] in a FMT



Example of [T6/τ/θ_3/Θ] in a KMT

Figure 1. Examples of problems in FMT and KMT

The tasks of [T6/τ/θ_2/Θ] in FMT and [T6/τ/θ_3/Θ] in KMT are illustrated in Figure 1. The task in KMT is presented on a grid, whereas the task in FMT includes both a grid and a coordinate plane. Since the coordinate plane serves as a reference for the object to be drawn, the technology utilized in the two tasks is considered different. This suggests that the approach to transformations varies based on the introduction of coordinates in the curricula of the two countries.

Conclusion

This study analyzed the structure of tasks through the results of the praxeological analysis of Korean and Finnish elementary school mathematics textbooks. This analysis in this study provides further insights into the structure of symmetry and transformation, building upon the findings of Takeuchi and Shinno (2020). Furthermore, by analyzing the sequence of praxeologies, this study implies a methodological potential for capturing the detailed concepts embedded within the tasks.

References

Bosch, M., & Gascón, J. (2006). Twenty-five years of the didactic transposition. *ICMI Bulletin, 58*, 51–65.

Bosch, M., & Gascón, J. (2014). Introduction to the anthropological theory of the didactic (ATD). In A. Bikner-Ahsbahs & S. Prediger (Eds.), *Networking of theories as a research practice in mathematics education* (pp. 67–83). Springer.

Chevallard, Y., & Sensevy, G. (2014). Anthropological approaches in mathematics education: French perspectives. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 38–43). Springer.

Ilmavirta, R., Koivisto, M., Salonen, M., Sintonen, A., & Uus-Leponiemi, T. (2012a). *Laskutaito in English 3A*. WSOY Oppimateriaalit Oy.

Ilmavirta, R., Koivisto, M., Salonen, M., Sintonen, A., & Uus-Leponiemi, T. (2012b). *Laskutaito in English 5A*. WSOY Oppimateriaalit Oy.

Kim, S., Kang, E., Kang, Y., Ko, C., Kim, B., Kim, Y., Noh, S., Park, Y., Park, H., An, H. , Lee, D., Jeong, S., Jeong, Y., Jeong, J., & Hwang, Y. (2021a). *Elementary school mathematics 4-1*. I-Scream Media.

Kim, S., Kang, E., Kang, Y., Ko, C., Kim, B., Kim, Y., Noh, S., Park, Y., Park, H., An, H. , Lee, D., Jeong, S., Jeong, Y., Jeong, J., & Hwang, Y. (2021b). *Elementary school mathematics 5-2*. I-Scream Media.

Takeuchi, H., & Shinno, Y. (2020). Comparing the lower secondary textbooks of Japan and England: A praxeological analysis of symmetry and transformations in geometry. *International Journal of Science and Mathematics Education, 18*, 791–810.

TSG1-09

DE/COLONIALITY IN MATHEMATICS CURRICULUMS OF THE PHILIPPINES AND INDIA

Ivy Estrella
Stockholm University, Sweden, ivy.angel.estrella@su.se

Harita Raval
Stockholm University, Sweden, harita.raval@su.se

This paper aims to examine how coloniality, that is the control of economy, authority, gender and sexuality, as well as of subjectivities and knowledge, is (re)produced in the primary and lower secondary mathematics curriculum in the Philippines and India. Using decolonial perspective, we examined the mathematics curriculums focusing on curriculum goals, standards, contents, resources, pedagogy, and language. Using a decolonial lens, we aim to bring forward and make visible the power relations in the specificities of mathematics curriculums in both countries by critiquing the Eurocentric epistemology which positions people in hierarchical structures and influences the mathematics education of the Philippines and India.

Keywords: mathematics curriculum, coloniality, decoloniality

Introduction

The education systems in India and the Philippines, similar to former colonial countries, are built and based on its former colonial administrations. This is extended in the present in the form of coloniality, which refers to colonial situations without the presence of a colonial administration in which cultural, political, intersubjective, epistemic, and economic relations are imposed upon people and positions them as subaltern to a dominant 'colonial' culture (Grosfoguel, 2011; Maldonado-Torres, 2013). In mathematics education, coloniality permeates mathematics curriculum by determining who is at the center or who controls mathematics education; which knowledge and thinking are privileged inside mathematics classrooms; and what type of people are being created through mathematics education. This can be determined through mathematics curriculum goals, standards, contents, resources, pedagogy, and language. In this sense, in the contexts of the Philippines and India, mathematics curriculum, positions learners in specific places in power relations. From this, we aim to question how coloniality is (re)produced in the primary and lower secondary mathematics curriculums in the Philippines and India.

Colonization of Education Systems in the Philippines and India

India and the Philippines share a similar pattern of colonial history as both countries were under different colonial administrations during the time of colonial expansion. Colonization in both countries also extends to the present through coloniality - the control of economy, authority, gender and sexuality, as well as of subjectivities and knowledge, without the presence of colonial administrations (Mignolo, 2011).

During the time of colonial expansion, the Philippines was under three colonial administrations namely, Spain, United States, and Japan. During this time, colonialism was used to occupy Philippine territories and to subjugate the Filipino people through Christianity and civilizing missions under the narrative of modernization. Spanish colonization lasted for more than 300 years in which primary education system was established with literacy schools providing systematic literacy education in Spanish (Gonzalez, 2003). During the Spanish colonization, the main goal was to disseminate Christianity to the Filipino people (Errington, 2007). With this, primary to tertiary level of education focused on the teaching of Christian doctrines and conversion of Filipinos to catholicism (Nathan & Ishii, 2022). The education system include Spanish as the language of intruction, teaching of subject areas such as arithmetic, handcrafts, reading and writing. Mathematics and Science were not the main focus of education during this time (Nathan & Ishii, 2022). Following more than 300 years of Spanish colonization, Spain surrendered to the United States, that occupied the Philippines for more than 40 years after the Philippine- American war (Tupas, 2022; Tupas & Lorente, 2014). Using a similar logic to the one used by Spain, race became a tool to justify the cultural assimilation and oppression of Filipinos. Education as an ideological tool was used with the imposition of English as the language of the public school system starting 1901, and its general intention of 'Americanization' of the Filipino people (Tupas, 2022; Pariona, 2018). United States focused on mass education of Filipino people and education emphasized English grammar, writing, reading and arithmetic (Nathan & Ishii, 2022). Japanese occupation, on the other hand, lasted only for three years in which education policies mandated by the Japanese administration ended after the war. The guidelines during this period is the adoptation of Nihongo, introduction of primary and vocational education, and promotion of the love of work (Nathan & Ishii, 2022). With this, the education system of the Philippines is built by and through its colonial administrations. In the present, mathematics education is based on Western epistemology of knowledge in which English is still the main language of instruction, particulary in mathematics and sciences.

India's education system has been shaped by multiple colonial influences, extending beyond the British colonial administration to include precolonial knowledge hierarchies. While British colonialism institutionalized a Eurocentric model of education, privileging Western epistemologies, older structures of caste-based knowledge production remained intact. Colonial education policies restructured Indian schooling systems, prioritizing English as the medium of instruction and promoting mathematics and science as subjects for modernization while simultaneously erasing Indigenous knowledge systems (Manoharan, 2020).The British colonial administration's introduction of formal education sought to create a class of intermediaries—Indians who could serve as administrators while upholding British economic and political interests (Chakravorty, 2019). The Macaulay Minute of 1835 formalized this through the promotion of English-medium education, which continues to dominate Indian classrooms today. This period also reinforced the exclusionary nature of education, as it was primarily accessible to upper-caste elites, with marginalized communities, particularly Dalits and Adivasis, remaining excluded from meaningful participation. Even after independence, the colonial structure of education persisted, with national policies largely mirroring the hierarchical frameworks of colonial schooling. The post-independence period saw efforts to expand access to education, but mathematics and science education remained deeply embedded in a Eurocentric paradigm. The dominance of colonial languages (English in elite institutions and Hindi in state curricula) and the continued marginalization of Indigenous and vernacular knowledge further entrenched coloniality within mathematics education. The British colonial administration in India formalized a Western-style education system, replacing Indigenous knowledge systems with Eurocentric curricula. Macaulay's Minute of 1835 laid the foundation for English-medium education, aiming to create a class of intermediaries proficient in Western knowledge but disconnected

from their own intellectual traditions. Mathematics education was restructured to fit British administrative needs, emphasizing computational skills over problem-solving approaches rooted in Indigenous traditions. This epistemological shift marginalized local mathematical systems, such as those used by artisans, traders, and astronomers, reinforcing the dominance of colonial knowledge. Even post-independence, the Indian education system largely retained this structure, privileging Western mathematical models while excluding historically marginalized communities, particularly Dalits and Adivasis, from equitable access to learning. The persistence of colonial frameworks, coupled with pre-existing caste hierarchies, continues to shape the power dynamics within mathematics education in India.

Mathematics Curriculum: De/Coloniality in Primary and Lower Secondary Level

Mathematics knowledges and languages were used as effective tools of colonization during the colonial era through the transformations and adoptations of mathematical practices imposed by colonial administrations. In the present, mathematics education continues to function as a tool of coloniality. That is, even without the presence of the former colonial administrations, mathematics education still have a significant influence in the current education systems of the Philippines and India. In this paper, we examined the current mathematics curriculums in the primary and lower secondary levels in the Philippines and India. Using a decolonial lens, we aim to bring forward and make visible the power relations in the specificities of mathematics curriculums in both countries. With this, we aim to critique Eurocentric epistemology, which dominates the mathematics curriculums in both countries, in which race is used for hierarchical structures and ideologies in the education system; and to understand the ingrained expectations and outcomes that give meaning and shape social interactions (Diem & Young, 2023). Through this lens, the mathematics curriculums were examined in terms of wider issues of power relations with focus on interrogating: (a) who is at the center or who who holds the control of knowledge and power of decisions in the classrooms and in the society; (b) who has access to institutional power, to (mathematical) knowledge, and who produces knowledge; (c) how is human agency operative, negotiated, contradictory, and constrained, and (d) who is considered human and not human, whose story is told and from whose point of view. From this, we examined the mathematics curriculum with regards to curriculum goals, standards, contents, resources, pedagogy, and language.

The current mathematics curriculum of the Philippines is called the Matatag Curriculum which aims to prepare Filipino learners for the life in the 21st century in which the main goal is for learners to be mathematically proficient and critical problem solvers (Department of Education, 2023). The focus on 21st century skills pertains to the mathematics curriculum as encompassing the development of information, media, and technology skills; learning and innovation skills; communication skills; and life and career skills. With this, the curriculum follows eurocentric epistemology in which the content, assessment, and standards are based on Western knowledge which disregards indigenous knowledge systems, which persists outside the school system. In addition, the main language of instruction is English, which is a colonial language, despite the country's diverse languages and cultures. With this, the mathematics curriculum of the Philippines do not acknowledge indigenous knowledges, languages, and cultures and puts western knowledge, culture, and language in a superior position.

Mathematics education in India, like in the Philippines, remains shaped by colonial histories and enduring epistemic hierarchies. While the British colonial administration institutionalized a Western-style curriculum, the pre-existing Brahminical order also influenced the production and access to mathematical knowledge. This dual legacy has resulted in a system where Eurocentric mathe-

matics dominates formal education, while Indigenous mathematical traditions—especially those developed outside upper-caste circles—remain largely absent (Manoharan, 2020). Unlike in the Philippines, where English was introduced as part of the colonial schooling system, India already had deeply entrenched hierarchies of knowledge production, where access to education was historically restricted by caste. This means that decolonizing mathematics education in India requires not just a critique of Eurocentrism but also a dismantling of caste-based knowledge gatekeeping (Grosfoguel, 2011). Epistemic Coloniality and the Persistence of Western Mathematical Standards in India's National Curriculum Framework (NCF) aligns closely with globalized mathematical standards that are rooted in Eurocentric epistemologies. The emphasis on abstract, universalized mathematical concepts over localized and context-driven learning reflects the continued dominance of Western mathematical traditions. While India has a rich history of mathematical advancements, these contributions are rarely acknowledged in contemporary curricula. Instead, the education system prioritizes Western methodologies, assessment patterns, and problem-solving techniques, reinforcing the perception that 'real' mathematics is European in origin (Freter, 2024). The role of language in perpetuating coloniality is also significant as English remains the dominant language of higher education in mathematics, creating a divide where students from English-speaking backgrounds—often from upper-caste and urban elite families—are at an advantage. Meanwhile, regional-language instruction is often treated as a second-tier education, with translated content failing to integrate Indigenous mathematical reasoning.

Mathematics curriculum positions learners in specific places in power relations by regulating what happens inside mathematics classrooms. The goals, standards, content, resources, pedagogy, and language determine who is at the center of mathematics education, which knowledges are privileged in the classroom, who is heard and allowed to speak, and the type of learners being (re)produced in mathematics classrooms. The Need for a 'Debrahminized' Mathematics Curriculum as Manoharan (2020) argues, decolonization in India is incomplete without debrahminization. While postcolonial critiques of education often focus on Western epistemologies, they fail to acknowledge that Brahminical knowledge structures also played a role in shaping what counts as 'legitimate' mathematics. In the Philippines, decolonization could be achieved by recognizing indigenous knowledge systems which are ignored and not included in education systems. Based on our investigation, both curriculums do not recognize the local cultures, languages, and knowledges of the learners, thereby centering a Eurocentric paradigm of knowledge in mathematics classrooms.

References

Chakravorty, S. (2019). *The truth about us*. Hachette UK.

Errington, J. (2007). *Linguistics in a colonial world: A story of language, meaning, and power*. John Wiley & Sons.

Gonzalez, A. (2003). Language planning in multilingual countries: The case of the Philippines 1 1.

Grosfoguel, R. (2011). Decolonizing post-colonial studies and paradigms of political-economy: Transmodernity, decolonial thinking, and global coloniality.*Transmodernity: journal of peripheral cultural production of the luso-hispanic world*, *1*(1).

Maldonado-Torres, N. (2013). On the Coloniality of Being: Contributions to the development of a concept. In *Globalization and the decolonial option* (pp. 94-124). Routledge.

Mignolo, W. (2013). Introduction: Coloniality of power and de-colonial thinking. In *Globalization and the decolonial option* (pp. 1-21). Routledge.

Natan Jr, A., & Ishii, H. (2022). Comparative Study of Mathematics Education in the Philippines and Japan. 北海道教育大学紀要. 教育科学編, *72*(2), 173-187.

Pariona, A. (2018, July 24). What languages are spoken in the philippines? World Atlas. Retrieved from https://www.worldatlas.com/articles /what-language-do-they-speak-in-the-philippines. html

Tupas, R. (2022). Language Policy. In *Philippine English* (pp. 257-267). Routledge.

Tupas, R., & Lorente, B. P. (2014). A 'new'politics of language in the Philippines: Bilingual education and the new challenge of the mother tongues. In Language, Education and Nation-building (pp. 165-180). Palgrave Macmillan, London.

Young, M. D., & Diem, S. (Eds.). (2023). Handbook of critical education research: Qualitative, quantitative, and emerging approaches.

TSG1-10

A STUDY OF PROOF TASKS IN SECONDARY SCHOOL MATHEMATICS TEXTBOOKS IN SINGAPORE

Thanabalasingam Navinesh
Raffles Girls' School, Singapore, navinesh.t@rafflesgirlssch.edu.sg

Weng Kin Ho
National Institute of Education, Singapore, wengkin.ho@nie.edu.sg

Berinderjeet Kaur
National Institute of Education, Singapore, berinderjeet.kaur@nie.edu.sg

This paper examines the support provided in curriculum materials, namely textbooks, for the teaching and learning of proof in Singapore secondary schools. Drawing on a purposive Reasoning and Proving Analytic Framework (RPAF) practice tasks involving proof in a series of textbooks used for Additional Mathematics were examined. There were 1135 practice tasks in the 2 books of the series. 256 (22.6%) of the tasks were proof practice tasks. The highest number of proof practice tasks (45.7%) were for the strand: Geometry and Trigonometry. In addition, the most frequent attributes of proving amongst the tasks were for Dimension 1- demonstration, Dimension 2 – verification and Dimension 3 – deductive reasoning. The findings lend support to past claims that proof is not a key focus of mathematics instruction in secondary schools. In addition, there appears to be a narrow focus on type of proof practice tasks suggesting that proof tasks have fixed structure or purpose.

Keywords: proof, proving, school mathematics textbooks, secondary school, Singapore

Introduction

Mathematical proof and the aspects of proof are the very foundations of mathematics. The arguments, justifications and reasoning typically used in a proof are ubiquitous in all of mathematics. However, it is surprising then that the concept and process of proving have not been ascribed sufficient importance in the secondary mathematics curricula worldwide until recently (Ball et al, 2003; Healy & Hoyles, 1998). There lies a belief that proof should not be covered in an in-depth manner in the secondary mathematics curriculum as it is beyond students' access and teachers' expertise.

The Study

The research reported in this paper is guided by the research question: What support does the school mathematics curriculum in Singapore provide for the teaching and learning of proof in secondary schools. In Singapore textbooks manifest the intended curriculum. As such, the quantity and type of proof tasks in the curriculum materials, mainly textbooks used in grades 9 and 10 were examined.

Theoretical Framework

There have been numerous analyses of mathematics textbooks regarding the support for reasoning and proof (Bieda et al., 2014; Cai & Cirillo, 2014; Davis, 2012; Fujita & Jones, 2014; Otten, et al., 2014; Stylianides, 2008, 2009; Thompson et al., 2012) in many countries. One of the most comprehensive and robust frameworks that covers many aspects of proof for textbook analysis is the analytic framework by Stylianides (2009). It focuses on aspects of proof in general suited for the study of tasks in textbooks. It is 2 dimensional. Dimension (1) comprises the components and subcomponents of reasoning-and-proving (e.g., plausible and definite pattern, conjecture, generic example, demonstration, empirical argument and rationale). Dimension (2) picks on the purposes of pattern, conjecture, and proof (conjecture precursor and non-precursor, proof precursor and non-precursor, explanation, verification, falsification, and generation of new knowledge).

For the purpose of this study, drawing on past relevant studies, the framework was further enriched to include as part of "providing a proof", *visual proof*. In addition, Dimension (3) was created to amplify the three important reasoning types (deductive, inductive, and abductive) (Peirce, 1992). The framework used in this study is shown in Figure 1.

Dimension	**Process of Reasoning and Proving**			
	I. Making Mathematical Generalisations		II. Providing Support to Mathematical Claims	
	(a) Identifying a Pattern	(b) Making a Conjecture	(c) Providing a Proof	(d) Providing a Non-Proof Argument
Dimension 1 Components and Subcomponents of Reasoning and Proving	1.Plausible Pattern 2.Definite Pattern	3. Conjecture	4. Generic Example 5. Visual Proof 6. Demonstration	7. Empirical Argument 8. Rationale
Dimension 2 Purposes of Pattern, Conjecture, and Proof	1. Conjecture Precursor 2. Conjecture Non-Precursor	3. Proof Precursor 4. Proof Non-Precursor	5. Explanation 6. Verification 7. Falsification 8. Generation of New Knowledge	
Dimension 3 Reasoning Type	1. Deductive Reasoning 2. Inductive Reasoning 3. Abductive Reasoning			

Figure 1. The Reasoning and Proving Analytic Framework (RPAF)

Method

The Textbooks

The textbooks (Additional Mathematics (AM)) examined were the ones most commonly available to teachers and students in secondary schools. In these books, the focus were practice tasks for student work in the textbooks, as teachers mostly draw on these tasks for their student work. Hence, the unit of analysis in this study is the practice task, known as a *task* (similar to that in Stylianides' (2009)).

Coding the Tasks

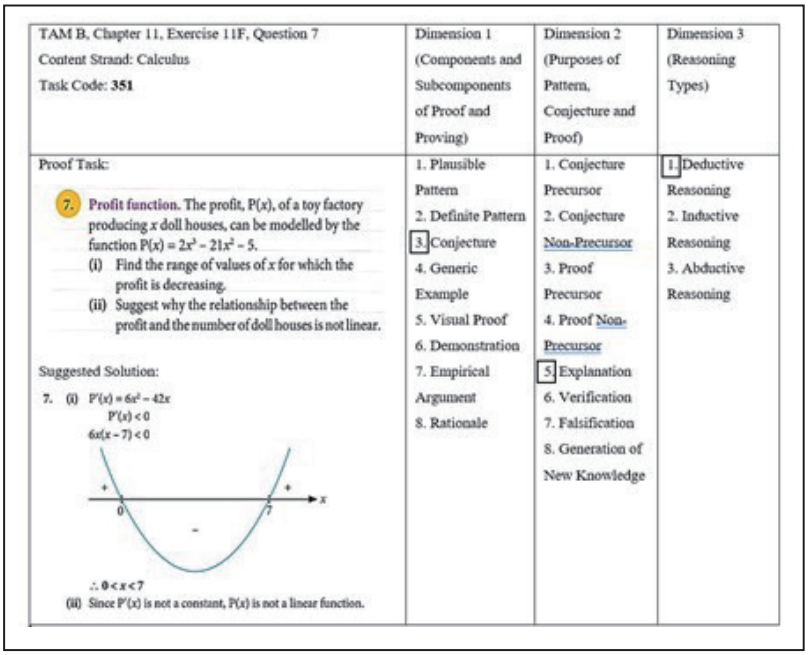
Two coders were involved in coding the tasks. Both were experienced mathematics teachers with deep mathematical and pedagogical knowledge for teaching of mathematics at the secondary school level. There were 2 phases involved in coding the tasks.

The first phase involved identifying the practice tasks as proof type and non-proof type. In the context of the study a proof type task must be solvable using one of the following: direct proof, proof by mathematical induction, proof by contraposition, proof by contradiction, proof by construction, proof by exhaustion, nonconstructive proof, and visual proof. Although other proof types, such as probabilistic and combinatorial, exist they are not part of the school mathematics curriculum. The coders scanned the textbooks and collected practice tasks that required proving, deducing, verifying, showing, justifying, explaining with reasons, and generalizing a statement. These tasks also had to offer opportunities for reasoning and proving according to at least one of the four major categories (identifying a pattern, making a conjecture, providing a proof, and providing a non-proof argument) in the RPAF framework.

The Cohen's Kappa statistic (McHugh, 2012) was used to establish the inter-rater reliability of the two coders involved in coding of the practice tasks as proof type and non-proof type. The inter-rater reliability was found to be 0.9974 (near perfect).

In the second phase the tasks were coded for the three dimensions in the RPAF. To understand the intention of the textbook authors in developing these tasks, three resources (curricular document, teachers' resource, and textbook) were drawn on to decipher the expected pre-requisite knowledge required and recommended procedures at each stage of content coverage. Student knowledge based on this content sequencing was considered when the coders worked through the proofs of the tasks. As such, the coding was aligned with what the curriculum intended, and this provided a systematic way of analyzing the content.

Next the coders coded the practice task's affordance according to the 3 Dimensions of the RPAF. A three-digit code was assigned to every task by the coders independently. The first digit depicts Dimension 1, the second digit depicts Dimension 2, and the third digit depicts Dimension 3. There were instances where a task involved more than one aspect in a dimension. In such cases the predominant objective of the question led us to identify the aspect for coding.



TAM B, Chapter 11, Exercise 11F, Question 7 Content Strand: Calculus Task Code: **351**	Dimension 1 (Components and Subcomponents of Proof and Proving)	Dimension 2 (Purposes of Pattern, Conjecture and Proof)	Dimension 3 (Reasoning Types)
Proof Task: **7.** **Profit function.** The profit, P(x), of a toy factory producing x doll houses, can be modelled by the function $P(x) = 2x^3 - 21x^2 - 5$. (i) Find the range of values of x for which the profit is decreasing. (ii) Suggest why the relationship between the profit and the number of doll houses is not linear. Suggested Solution: 7. (i) $P'(x) = 6x^2 - 42x$ $P'(x) < 0$ $6x(x - 7) < 0$ + 0 − 7 + $\therefore 0 < x < 7$ (ii) Since $P'(x)$ is not a constant, $P(x)$ is not a linear function.	1. Plausible Pattern 2. Definite Pattern 3. Conjecture 4. Generic Example 5. Visual Proof 6. Demonstration 7. Empirical Argument 8. Rationale	1. Conjecture Precursor 2. Conjecture Non-Precursor 3. Proof Precursor 4. Proof Non-Precursor 5. Explanation 6. Verification 7. Falsification 8. Generation of New Knowledge	1. Deductive Reasoning 2. Inductive Reasoning 3. Abductive Reasoning

Figure 2. Example of a task coded as 351

Figure 2 shows an example of a task that was coded using the RPAF. This task requires students to consider the claim "The relationship between profit and the number of doll houses" is not linear. Before this claim is proven or disproven, one can at most regard it as a conjecture; that is, one requires some form of reasoning or explanation to either support or refute this claim. So, for Dimension 1, it is coded as "3" ("Conjecture"). The action word in (ii) of the Task invites for a suggestion to be given by the student. This amounts to asking the student to explain or supply some logical arguments to justify the conjecture is true. Therefore Dimension 2 is coded as "5" ("Explanation"). The overall emphasis of this task is to give a sequence of logical arguments to support the claim that the relationship between the two variables is non-linear. Essentially these arguments are an articulation of the "falling-rising" nature of the quadratic graph that would be sufficient to refute any possible linearity. Hence, this task is calls for deductive reasoning and Dimension 3 is coded as "1".

Data, Findings and Conclusion

In this section, we will only share the key findings due to the constrain of the page limit of the paper. The key findings are:

1. There were a total of 1135 practice tasks in the books. Only 256 (22.6%) of them were proof practice tasks.
2. The 256 proof practice tasks were spread across the strands, Algebra, Geometry and Trigonometry and Calculus, with 45.7% for Geometry and Trigonometry.
3. The most frequent attributes of proving amongst the tasks were for Dimension 1- demonstration, Dimension 2 – verification and Dimension 3 – deductive reasoning.

The data and findings show that:

1. There is inadequate emphasis on proof tasks in the curriculum materials that support the learning of secondary school mathematics in Singapore schools. Bergwall (2021) proposed a threshold of at least 25% of student work involving proof and proving.
2. Distribution of proof tasks across content strands was not comparable, with almost half of them in Geometry and Trigonometry. This lends to students' perception that proof and proving is predominantly in this strand of mathematics. This is similar to the study conducted by Fujita et al., (2009).
3. The type of proof practice tasks in the curriculum materials makes apparent the limited exposure of students to proof and proving. Several attributes such as, 'Empirical Argument' and 'Rationale' in Dimension 1, as well as 'Generation of New Knowledge,' 'Conjecture Precursor,' 'Conjecture Non-Precursor,' 'Proof Precursor' in Dimension 2 and total absence of abductive reasoning amongst the proof practice tasks merits attention of curriculum developers. As noted by Magnani (2011), abductive reasoning skills facilitate creativity and conjecturing whilst working with mathematical knowledge.

References

Ball, D. L., Hoyles, C., Jahnke, H. N., & Movshovitz-Hadar, N. (2003). The teaching of proof. *arXiv preprint math/0305021*.

Bergwall, A. (2021). *Proof-related reasoning in upper secondary mathematics textbooks: Characteristics, comparisons, and conceptualizations*. Malardalen University (Sweden).

Bieda, K. N., Ji, X., Drwencke, J., & Picard, A. (2014). Reasoning-and-proving opportunities in elementary mathematics textbooks. *International Journal of Educational Research, 64*, 71-80.

Cai, J., & Cirillo, M. (2014). What do we know about reasoning and proving? Opportunities and missing opportunities from curriculum analyses. *International Journal of Educational Research, 64*, 132-140.

Davis, J. D. (2012). An examination of reasoning and proof opportunities in three differently organized secondary mathematics textbook units. *Mathematics Education Research Journal, 24*(4), 467-491.

Fujita, T., & Jones, K. (2014). Reasoning-and-proving in geometry in school mathematics textbooks in Japan. *International Journal of Educational Research, 64*, 81-91.

Fujita, T., Jones, K., & Kunimune, S. (2009). The design of textbooks and their influence on students' understanding of 'proof'in lower secondary school.

Healy, L., & Hoyles, C. (1998). *Justifying and proving in school mathematics: Technical report on the nationwide survey*: Institute of Education, University of London.

Magnani, L. (2011). *Abduction, reason and science: Processes of discovery and explanation*: Springer Science & Business Media.

McHugh, M. L. (2012). Interrater reliability: the kappa statistic. *Biochemia medica*, *22*(3), 276-282.

Otten, S., Gilbertson, N. J., Males, L. M., & Clark, D. L. (2014). The mathematical nature of reasoning-and-proving opportunities in geometry textbooks. *Mathematical Thinking and Learning, 16*(1), 51-79.

Peirce, C. S. (1992). *Reasoning and the logic of things: The Cambridge conferences lectures of 1898*: Harvard University Press.

Stylianides, G. J. (2008). An analytic framework of reasoning-and-proving. *For the Learning of Mathematics, 28*(1), 9-16.

Stylianides, G. J. (2009). Reasoning-and-proving in school mathematics textbooks. *Mathematical Thinking and Learning, 11*(4), 258-288.

Thompson, D. R., Senk, S. L., & Johnson, G. J. (2012). Opportunities to learn reasoning and proof in high school mathematics textbooks. *Journal for Research in Mathematics Education, 43*(3), 253-295.

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 1
(Day 3)

TSG1-11

A STUDY ON EXPLORING THE CONTEXTUAL FEATURES OF QUESTIONS IN MATHEMATICS TEXTBOOK

Yi-An, Cho
National Academy for Educational Research, Taiwan, scottiess0625@mail.naer.edu.tw

Sheng-Yuan Wang
Taipei Municipal Yangming High School, Taiwan, wjames0717@gmail.com

This study uses content analysis to categorize textbook questions based on their contextual focus, revealing three key findings. First, across all analyzed textbook versions, over 92% of questions are rooted in mathematical contexts, emphasizing a strong focus on mathematical concepts and problem-solving. Second, in all of textbook versions, personal-context questions ranked second in frequency. Third, questions related to social, occupational, and scientific contexts are notably scarce in all versions, with most accounting for less than 1.5% of the total. Additionally, five versions of textbook contain no occupational-context questions. Given these findings, the study suggests that textbook design should incorporate more questions related to personal, occupational, social, and scientific contexts. Increasing the diversity of problem contexts would better support students in applying mathematics to real-world and professional situations, enhancing their problem-solving skills and practical mathematical understanding.

Keywords: Mathematics textbook, textbook design, problem-solving

Introduction

Mathematics education worldwide aims to equip students with the ability to apply mathematical concepts, methods, and problem-solving strategies in real-life situations. It emphasizes mathematics as a practical tool rather than just an abstract discipline, bridging theoretical knowledge with practical applications. Taiwan's 2018 implementation of 12-Year Basic Education focuses on enhancing students' mathematical competence, problem-solving skills, and ability. This curriculum shifts from rote learning to fostering a deep and flexible understanding of mathematical principles, and to ensuring that students can apply mathematics in daily life.

Textbooks play a key role in curriculum reform by reflecting educational goals. Taiwan's new curriculum moves beyond repetitive practice and rote learning, advocating for varied problem types across different contexts that promote higher-order thinking (Gu, Huang, & Marton, 2004). Enriching textbook content with varied problem contexts fosters higher-order thinking and deeper understanding (Van den Heuvel-Panhuizen, 1996). This diversity enhances students' academic achievement by broadening learning opportunities. Analyzing contexts of questions in textbook helps ensure whether they support curriculum goals. Identifying gaps in contextual representation can further improve the curriculum, ensuring students are well-prepared for real-world mathematical challenges. In Taiwan, high school offers two types of mathematics courses: General High School Mathematics and Vocational School Mathematics, with the latter further divided into mathematics

for engineer, mathematics for business, and mathematics for art and design. In this article, we will examine the distribution and differences of problem contexts in mathematics textbooks for engineer.

Literature Review

Students' mathematical learning should not depend on repetitive practice of similar problems (Gu, Huang, & Marton, 2004). Instead, textbooks should present a variety of problems in different contexts to enrich content and foster a deeper understanding (Van den Heuvel-Panhuizen, 1996). Introducing problems in diverse contexts encourages students to develop flexible problem-solving strategies, stimulating cognitive engagement and inspiring creative approaches. A strong positive correlation exists between exposure to varied mathematical contexts and academic achievement. Diverse problem situations help students connect concepts to real-world scenarios, enhancing their ability to transfer knowledge across domains. This approach is crucial for fostering mathematical thinking and adaptive problem-solving skills.

The PISA 2022 assessment framework classifies problem contexts into four categories: Personal, Occupational, Social, and Scientific. Currently, mathematical contexts fall under scientific contexts, but mathematics and science are fundamentally distinct. While mathematics is often applied in scientific fields, it operates with its own principles, problem-solving methods, and abstract reasoning that do not always align with scientific methodologies.

This project advocates for the separation of mathematical contexts from scientific ones to better highlight the unique cognitive skills and problem-solving strategies essential to mathematics. Recognizing mathematics as an independent context allows for a clearer understanding of its real-world applications beyond science, including finance, technology, engineering, and daily decision-making. This reclassification would also refine assessments of students' mathematical literacy, ensuring their mathematical reasoning is evaluated separately from scientific inquiry. Ultimately, this distinction would lead to more precise and meaningful evaluations of students' problem-solving abilities in both domains.

Textbooks play a crucial role in fostering diverse problem-solving strategies by integrating varied contexts into the curriculum. They should move beyond rote memorization, emphasizing the practical application of mathematics. As part of curriculum reform, textbooks must highlight problem-solving as a core objective, ensuring students develop mathematical reasoning for real-world challenges. Critically analyzing textbook contexts is essential to enhance students' problem-solving abilities. By broadening problem situations, textbooks better support curriculum goals, equipping students with the adaptability and creativity needed to navigate an evolving world.

Methodology

This study, based on the PISA 2022 Mathematics Framework, analyzes question types in mathematics textbooks for engineer using content analysis. Following Wimmer and Dominick's (2003) systematic approach and based on the fact that mathematics is a required subject for all first-year students, researchers selected all 10 first-year textbook versions in Taiwan. The study defines questions as the fundamental unit of analysis (Stein, Remillard, & Smith, 2007), categorizing them into personal, social, occupational, scientific, and mathematical contexts. A detailed comparison and analysis of these textbooks provide insights into their structure and emphasis on mathematics education.

To ensure reliability and validity, researchers engaged two experienced mathematics teachers for data extraction and continuous comparison, refining data sampling. Reliability was assessed through inter-rater reliability, ensuring consistency in data classification. This approach enhances the precision of data selection and analysis accuracy (Wimmer & Dominick, 2012), strengthening the study's credibility in evaluating the role of textbooks in mathematical problem-solving.

Results

This study analyzes mathematics textbooks for engineer, which are published by seven different publishers. It examines the differences in context-based questions among publishers, providing insights into the characteristics of textbook question design and serving as a reference for teachers and students in textbook selection.

As shown in Table 1, the version-E contains the highest number of questions (1,758), followed by the version-G (1,608), while the version-D version has the fewest (1,237). A chi-square test was conducted to examine differences in the distribution of question contexts among textbooks from different publishers. The chi-square value was 109.979 ($p<.01$), indicating significant differences in question distribution across publishers and contexts. In all versions, "mathematical" context questions were the most prevalent, suggesting that the design of textbook questions generally lacks real-life contextualization. This absence makes it difficult for students to connect textbook problems with real-world situations.

Table 1. The frequency of different contexts in mathematics textbooks from different versions

The versions of textbooks	Contexts						X^2	p-value
	Personal	Occupational	Social	Mathematical	Scientific	Sum		
A	56 3.8%	0 0.0%	1 0.1%	1,415 95.7%	7 0.5%	1,479 100%	109.979	.000
B	75 5.2%	0 0.0%	20 1.4%	1,334 92.7%	10 0.7%	1,439 100%		
C	76 4.8%	1 0.1%	4 0.3%	1,501 94.3%	9 0.6%	1,591 100%		
D	47 3.8%	0 0.0%	0 0.0%	1,187 96.0%	3 0.2%	1,237 100%		
E	117 6.66%	0 0.0%	0 0.0%	1,627 92.5%	14 0.8%	1,758 100%		
F	88 6.0%	1 0.1%	10 0.7%	1,355 92.9%	5 0.3%	1,459 100%		
G	74 4.6%	0 0.0%	0 0.0%	1,519 94.5%	15 0.9%	1,608 100%		
sum	533 5.0%	2 0.0%	35 0.3%	9,938 94.0%	63 0.6%	10,571 100%		

An analysis of all versions of mathematics textbooks for engineer found that over 92% of questions were rooted in a mathematical context, emphasizing problem-solving and mathematical applications. In all versions, personal-context questions ranked second, while social, occupational, and scientific-context questions were scarce, each accounting for less than 1.5% of the total. Additionally, five versions contained no occupational-context questions. These findings suggest that textbooks should incorporate more diverse contextual questions, helping students apply mathematics to real-world scenarios. Increasing vocational-context questions is particularly crucial to better prepare students for solving problems in their professional fields.

Conclusions and Suggestion

The research findings show that mathematic textbooks for engineer heavily emphasize mathematical-context questions, with significantly fewer in other contextual dimensions. This bias results in a lack of real-life or professional connections, making it difficult for students to apply mathematics in practical situations. This trend may stem from the nature of mathematics as a discipline, where textbooks are structured around problem-based learning, using diagrams, tables, and symbols to explain concepts. As a result, questions focus on direct mathematical knowledge rather than integrating real-world or vocational contexts, limiting students' ability to connect mathematics to their professional fields.

Acknowledgement

This study was funded by the Taiwanese Ministry of Science and Technology under grant number NSTC 113-2410-H-656-005.

References

Gu, L., Huang, R., & Marton, F. (2004). Teaching with Variation: An effective way of mathematics teaching in China. In L. Fan, N. Y. Wong, J. Cai, & S. Li (Eds.), How Chinese learn mathematics: Perspectives from insiders (pp. 309-345). Singapore: World Scientific.

Ministry of Education. (2018). *Curriculum guidelines of 12-year basic education: Language Arts-Mandarin*. Taipei, Taiwan: Author.

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. Lester Jr. (Ed.), Second handbook of research on mathematics teaching and learning (pp. 319-369). Greenwich, CT: Information Age.

Van del Heuvel-Paanhuizen, M. (1996). Assessment and realistic mathematics. Utrecht, The Netherlands: Freudenthal Institute, University of Utrecht.

Wimmer, R. D. & Dominick, J. R. (2012). Mass media research: An introduction. (9th ed). Belont: Wadsworth Publishing.

TSG1-12

ACTION RESEARCH OF IMPLEMENTING A PROBLEM-BASED LEARNING MODEL INTO THE CURRICULUM OF TEACHING MODELS FOR ELEMENTARY MATHEMATICS GIFTED STUDENTS

Yuan Yuan
National Taichung University of Education, Taiwan, yuanyuan@mail.ntcu.edu.tw

The teaching methods of teachers are influenced by their learning experience. Therefore, the teaching methods received by pre-service teachers in the pre-service education professional courses also have a profound impact on their future teaching methods. I am teaching a university course titled Models of Gifted Education in Elementary School Mathematics, which aims to cultivate students with the professional ability of elementary school mathematics gifted curriculum design. However, I found that only emphasizing multiple curriculum design concepts and examples is not enough to enable students to develop accordingly the ability to design the mathematics gifted curriculum program, so I think about seeking a core teaching concept, which is the characteristic of the curriculum based on development. This project explores the feasibility of integrating problem-based learning (PBL) teaching strategy as a curriculum design model using action research and analyzes its impact on students' curriculum design ability. The research found that the gradual presentation of the three teaching cases can help students understand PBL's gifted mathematics curriculum model; the actual curriculum design review and practice can help students gain practical experience. After this course, the students significantly improved their PBL teaching method, designed courses for gifted students in mathematics, taught confidence in gifted students, and showed interest in teaching them mathematics. Students still need to be strengthened and improved in terms of learning sheet design and group teaching guidance for exploring with questions.

Keywords: mathematics gifted, project-based learning, action research, teaching model

Research Background and Purposes

In Taiwan, elementary schools have established gifted classes for students with advanced abilities. These classes or courses often extend and deepen the curriculum based on student's learning abilities and performance. However, in current practice, each teacher usually determines the curriculum content independently without standardized curriculum guidelines. As a result, teachers may introduce challenging problems or concepts, sometimes covering foundational content from junior or senior high school mathematics. This approach can lead to gifted math education focusing excessively on test-taking skills, often neglecting the development of mathematical creativity and practical application. Gifted students may become overly focused on solving speed and competition scores, overlooking exploring the nature of mathematics and improving problem-solving skills.

Problem-Based Learning (PBL) was first introduced by Barrows (1996), who used real-world case problems to stimulate students' learning and as a tool to develop problem-solving skills. From the educational philosophy of PBL, learning occurs through solving problems, with the importance placed not on the answers but on the problems themselves (Barrows & Tamblyn, 1980). According to this view, problems are the best teachers, while answers are merely hypotheses awaiting verification, requiring continuous inquiry and evidence. PBL, as a teaching and learning model centered on real-world case problems, has the potential to significantly enhance students' problem-solving

skills, particularly in gifted students. Teachers can design a curriculum based on the PBL approach to stimulate student learning and guide them in solving problems. Students, in turn, engage in peer collaboration and self-directed learning to cultivate critical thinking and problem-solving skills.

Gifted students often possess greater motivation, responsiveness, and strategic abilities than other learners. The real-world problems offered by PBL, embedded in a rich context of knowledge and social relevance, are highly suitable for gifted students (Sternberg et al., 1996; Gallagher, 1997). Therefore, the effectiveness of applying PBL to teaching gifted students has gained researcher recognition. PBL enhances problem-solving skills and fosters a deep interest in mathematics among students. This aspect of PBL is particularly essential in teaching gifted math students, as it can inspire them to explore the subject further and develop a lifelong passion for mathematics.

While numerous studies have explored PBL applications in higher education, its application in teacher training programs remains limited (for example, Erdogan & Senemoglu, 2017; Martin, 2022); moreover, its application primarily targets educational subjects related to practical teaching issues rather than the entirety of teacher training courses (Chang & Lin, 2016). Research shows that PBL in teacher training can help trainee teachers construct professional knowledge in education, promote the dialectic and integration of educational theory and practice, establish habits of professional collaboration, enhance critical and reflective abilities, improve problem-solving skills and techniques, and increase teaching confidence and efficacy (cited in Bai, 2021). However, these studies mainly focus on enhancing pre-service teachers' educational skills and competencies, with fewer efforts to assist them in designing PBL instructional cases focused on subject content learning. Therefore, this study integrates PBL (Problem-Based Learning) into a course titled " Models of Gifted Education in Elementary School Mathematics " and explores, through an action research process, the impact of PBL-integrated curriculum design on students' ability to develop gifted mathematics curricula.

Research Method

This study aims to train students in designing teaching plans that integrate Problem-Based Learning (PBL) into gifted mathematics education at the elementary school level. The research setting is the Department of Mathematics Education at a national university. The participants are 29 students enrolled in the "Models of Gifted Mathematics Education" course during the second semester of the 111th academic year.

This action research follows three iterative cycles, and three PBL-integrated gifted mathematics instructional cases were used as examples: The Longest Prime Train, Numbers in Playing Cards, and Garbage Trucks & Eulerian Paths. Each cycle consists of four steps before proceeding to the next cycle.

1. Developing PBL-integrated gifted mathematics teaching plans
2. Developing and implementing problem-solving strategies
3. Implementing PBL and identifying potential issues
4. Collecting data and evaluating the effectiveness of the strategies

Data collected during each action research cycle—including teacher journals, worksheets, student assignments, end-of-term feedback and student interviews—were analyzed to refine and improve the instructional approach. This study adopts thematic analysis as the primary qualitative data analysis method.

Research Results

This study develops an action plan by presenting three PBL-integrated mathematics-gifted instructional cases, gradually developing teaching content to cultivate pre-service teachers' ability to design mathematics-gifted courses with PBL integration. The findings from the three action processes are as follows:

"The Longest Prime Train" as a PBL-Integrated Mathematics Gifted Instructional Case

In the first action process, the students taking the course assumed the role of mathematics-gifted students. After presenting the main problem, they collaborated in groups to find the longest prime train and further discussed why 13 was the longest. Since this problem was unfamiliar and required different problem-solving approaches, students recognized the importance of group discussions and engaging actively in problem-solving during class.

> *"Every question the teacher posed was carefully designed, perfectly suited for strategic questioning with gifted students. It encouraged us to explore, experiment, and discover while considering a problem, ultimately achieving our learning goals."* (End-of-term feedback from student 110120)

> *"The teacher provided many interesting and meaningful mathematics gifted activities. They enhanced our learning motivation, and the need for critical thinking and exploration in the course also improved our positive learning attitude and concentration."* (End-of-term feedback from student 110124)

"Numbers in Playing Cards" Focuses on the Design of Worksheets, but Students Use Algebraic Methods Instead

The "Numbers in Playing Cards" activity aimed to have students record their findings using a pre-designed worksheet. However, these university students quickly solved the problem using algebraic methods rather than following the intended process of recording their observations. While the magic trick had novelty, requiring university students to document their step-by-step thinking proved challenging. After watching a live teaching video, they realized that elementary school students could not immediately determine the problem-solving direction and needed step-by-step guidance to reach an understanding.

Due to their lack of experience designing worksheets, the second assignment failed to meet expectations and required revision.

> *"Once again, we did not meet the teacher's standard and had almost to redo everything. Lesson design is a big challenge, haha!"* (Individual interview)

"Garbage Trucks and Eulerian Paths" Best Stimulates Problem-Based Design and Real-Life Applications but Is Too Easy for the Students

The third cycle focused on the problem of how a garbage truck could complete its route in one go. Among the three lesson plans, this best illustrated the application of mathematics in daily life and encouraged students to extend their problem-solving thinking to real-world decision-making. However, since the central mathematical concept involved Eulerian paths—something the students were already familiar with—group discussions did not yield much engagement.

Instead, students mainly observed and discussed teaching videos of elementary school gifted students. They found that elementary students could identify examples of Eulerian and non-Eulerian paths when guided but required more time to deduce the necessary conditions for a successful Eu-

lerian path, unlike the university students who immediately grasped it. This highlighted elementary gifted students' diverse, unique, and divergent problem-solving approaches, emphasizing the need for careful guidance to direct their thinking effectively.

Due to limited experience in guiding student group discussions, some students reflected:

> *"When using group teaching for gifted students, how should we assign tasks? How should we guide students?"* (End-of-term feedback)

Among the three cases, "The Longest Prime Train" was the most suitable for a complete trial teaching session with university students. For "Numbers in Playing Cards" and "Garbage Trucks and Eulerian Paths," since the mathematical content was relatively simple, future lessons could focus on designing worksheets and guiding questions rather than teaching procedures.

> *"Perhaps students should first practice designing worksheets and posing questions, followed by discussion and better examples. This might help them master this aspect of lesson design."* (Reflection)

Conclusion

This action research aimed to explore whether PBL-integrated mathematics gifted instruction could serve as the primary teaching model for this course and whether students could enhance their curriculum design and teaching abilities. After completing the course, students reported increased teaching confidence and curriculum design proficiency. This study confirmed that a PBL-integrated mathematics-gifted teaching model could guide pre-service teachers in curriculum design. However, students still need to be strengthened and improved in learning sheet design and group teaching guidance for exploring problems.

Reference

Bai, H. J. (2021). A study on the teaching practice of problem-based learning (PBL) in the teacher education course "Educational Trends and Practices." Retrieved from http://dspace.lib.pu.edu.tw/handle/123456789/22762. (In Chinese)

Barrows, H. S. (1996). Problem-based learning in medicine and beyond: A brief overview. *New Directions for Teaching and Learning, 69*, 3-12.

Barrows, H. S. & Tamlyn, R. (1980). *Problem-based learning: An approach to medical education.* New York: Springer.

Chen, C. Y. (2017). A preliminary study on the application of problem-based learning (PBL) in pre-service teacher education courses. *Taiwan Educational Review Monthly, 6*(10), 70-77. (In Chinese)

Erdogan, T., & Senemoglu, N. (2017). PBL in teacher education: its effects on achievement and self-regulation. *Higher Education Research & Development, 36*(6), 1152-1165.

Gallagher, S. A. (1997). Problem-based learning: Where did it come from, what does it do, and where is it going? *Journal for the Education of the Gifted, 20*(4), 332-362.

Martin, D. A. (2022). The Impact of Problem-based Learning on Pre-service Teachers' Mathematics Pedagogical Content Knowledge. *Australian Journal of Teacher Education, 47*(4).

Sternberg, R. J., Ferrari, M., Clinkenbeard, P., & Grigorenko, E. L. (1996). Identification, instruction, and assessment of gifted children: A construct validation of a triarchic model. *Gifted Child Quarterly*, *40*(3), 129-137.

TSG1-13

FROM CLASSROOM TO REAL LIFE: EMBRACING MATHEMATICAL MODELLING IN HONG KONG'S CURRICULUM—INSIGHTS FROM SINGAPORE

Ka Shing Chui
The Education University of Hong Kong, Hong Kong SAR, China, ksdanielchui@gmail.com

This paper explores the integration of mathematical modelling in the curricula of Hong Kong and Singapore, aiming to identify effective strategies for enhancing problem-solving skills among students. While both education systems excel in mathematics education, Singapore has gained international recognition for its innovative approaches to embedding mathematical modelling within its curriculum. This study examines the curricular designs employed in Singapore, highlighting their potential applicability to Hong Kong's educational context. Through comparative analysis, this study investigate how these approaches can foster critical thinking, creativity, and real-world problem-solving skills in students. The findings suggest that adopting Singapore's 14-years practices could significantly enhance Hong Kong's mathematics curriculum, ultimately preparing students for a rapidly evolving global landscape. This research contributes to the discourse on education reform, offering insights that can inform policy decisions and curriculum development in Hong Kong and beyond.

Keywords: Mathematical modelling, Curriculum development, Education reform, Comparative analysis

Introduction

Mathematical education plays a vital role in developing students' mathematical skills, which encompass essential components like critical analysis, mathematical reasoning, mathematical modelling, problem-solving, communication, and the effective use of mathematical tools and resources. Research by Nhung and Thang (2021) indicates that mathematical modelling is widely acknowledged as a crucial competency in several countries, including the United States, Germany, France, England, China, Russia, and Singapore. Modeste et al. (2023) point out that Singapore has undergone significant curriculum reform emphasizing mathematical modelling. The 2003 update to the Singapore mathematics curriculum integrated mathematical modelling to highlight its importance in mathematics education and to tackle the challenges of the 21st century (Balakrishnan et al., 2011).

In point 71, "Continue to enhance STEAM education," from the Chief Executive's 2023 Policy Address, there is a significant focus on advancing STEAM education in Hong Kong SAR. This includes particular initiatives aimed at incorporating mathematical modelling into both primary and secondary education:

> "Implementation of a pilot scheme on mathematical modelling at the primary level within the 2023/24 school year, with the goal of completing all tryout lessons by the 2024/25 school year."
>
> "Provision of professional development training on mathematical modelling for at least one Mathematics teacher in each publicly-funded secondary school by the end of the 2025/26 school year (EDB)."

Hong Kong has recently initiated pilot programs to emphasize modelling at the primary level, starting in 2023. Due to their similar contexts and cultural backgrounds, Hong Kong could benefit from Singapore's experience in integrating mathematical modelling into both primary and secondary education. In international assessments such as PISA and TIMSS, Singapore consistently outperforms Hong Kong (see table 1). PISA places a stronger focus on modelling skills (Rosyidi et al., 2021; Liu et al., 2024), which Singapore has incorporated into its primary curriculum since 2007. This study will focus on investigating how Singpare embedded mathematical modelling in the curriculum to provide insights that can assist governments, particularly in the education sector, looking to implement mathematical modelling in both primary and secondary schools.

Table 1. Mathematics performance in PISA results for Singapore and hong kong

	2009	2012	2015	2018	2022
Singapore	562 (2^{nd})	573 (2^{nd})	564 (1^{st})	569 (2^{nd})	575 (1^{st})
Hong Kong	555 (3^{rd})	561 (3^{rd})	548 (2^{nd})	531 (4^{th})	540 (4^{th})

Note. The numbers represent the scores, while the positions in brackets indicate their ranking in the PISA assessment.

The Evolution of Mathematical Modelling in Singapore Curriculum Framework

Mathematics education in Singapore has been revised to align with global reform efforts. Since the early 1990s, the Singapore Mathematics Curriculum Framework (SMCF) has focused on mathematical problem-solving as a central goal. The framework emphasizes that cultivating students into skilled mathematical problem-solvers relies on five interconnected components: skills, concepts, attitudes, metacognition, and processes (MOE, 2001). In 2007, the Ministry of Education (MOE) incorporated additional mathematical procedures such as logic, communication, making connections, practical applications, and modelling into mathematics education (MOE, 2007). This intentional integration of new mathematical strategies is seen as a significant step forward in improving the relevance of problem-solving. While applications and modelling were previously emphasized mainly in the context of differential equations at the pre-university level, the revised curriculum now mandates their integration across all educational levels. Additionally, it stresses that "students should learn to utilize various data representations and to choose and apply suitable mathematical methods and tools for solving real-world issues" (MOE, 2007, p.14). These changes signify a significant shift, as fostering these mathematical processes will require different teaching methods and assessment strategies compared to traditional practices (Chan, 2009). Within the framework, mathematical modelling is acknowledged as a key competency essential to the mathematical problem-solving process. The SMCF explicitly identifies mathematical modelling as a vital element of the wider range of mathematical skills and abilities that students should acquire and demonstrate as part of their mathematical proficiency. By emphasizing modelling as a core competency, the framework seeks to promote the holistic development of students' mathematical capabilities.

The SMCF defines mathematical modelling as "the process of formulating and refining a mathematical model to represent and solve real-world problems". Applications and modelling enable students to link the mathematical concepts they have learned to real-life situations, enhancing their understanding of fundamental mathematical ideas and techniques while developing their mathematical skills. Students should be provided with opportunities to apply problem-solving and reasoning abilities to tackle a range of problems, including open-ended and real-world challenges. Through mathematical modelling, students gain skills in managing ambiguity, making connections, selecting

and applying relevant mathematical concepts and techniques, recognizing assumptions, evaluating solutions to real-world problems, and making informed decisions based on available or gathered data (MOE, 2013, p.15-16). The mathematical procedures refer to "the process skills involved in acquiring and applying mathematical knowledge. This encompasses reasoning, communication and connections, applications and modelling, as well as thinking skills and heuristics that are crucial in mathematics and other areas. These thinking skills and heuristics are vital for solving mathematical problems" (MOE, 2013, p.15, 17). Table 2 presents a comparative assessment of Singapore's mathematics curriculum in relation to mathematical modelling. The incorporation of mathematical modelling into the primary curriculum started in 2007. However, the emphasis has now moved towards creating mathematical expressions and evaluating their contextual relevance at the primary level, while mathematical modelling is primarily designated for the secondary level in the 2021 curriculum.

Table 2. Comparative analysis of singapore's mathematics curriculum regarding mathematical modelling

	Modelling process	Modelling competency	Modelling tasks
2007	A linear problem solving process.	Use diverse data representations and select appropriate mathematical methods to solve real-world problems.	-
2013	A cycle connecting real world with mathematics.	Manage ambiguity, make connections, apply relevant mathematical concepts, identify assumptions, reflect on solutions, and make informed decisions based on data.	Carry out a mathematical modelling project.
2021	-	-	Students are exposed to mathematical modelling at the secondary level.

Discussion and Conclusion

There are several commonalities between the mathematics curricula of Singapore and Hong Kong, from which Hong Kong can draw lessons for embedding mathematical modelling. The emphasis on conceptual understanding is a key aspect shared by both curricula, as they aim to foster a deep comprehension of mathematical concepts. Hong Kong has the opportunity to enhance its focus on this area within mathematical modelling, ensuring that students grasp the underlying principles rather than merely acquiring procedural skills. Additionally, both educational systems prioritize problem-solving as a fundamental component of mathematics education. Singapore's successful integration of real world problems into mathematical modelling serves as a valuable example for Hong Kong, which could adopt similar practices to make mathematics more relevant and engaging for students. Furthermore, both curricula are characterized by their structured frameworks, featuring clear learning objectives. Hong Kong could refine its curriculum framework by incorporating specific guidelines and examples for embedding mathematical modelling, thereby promoting consistent implementation across schools. This approach would not only strengthen students' mathematical skills but also enhance their ability to apply these skills in practical contexts.

The in-depth discussion in this study regarding the integration of mathematical modelling in mathematics education in Singapore paves the way for several future research:

Longitudinal Studies on Modelling Competency Development. Additional research is needed to track the development of students' mathematical modelling competencies over time, from primary to secondary education, while identifying key progression patterns and influencing factors. It is important to explore the long-term effects of the focus on modelling in the curriculum on students' problem-solving skills and overall mathematical proficiency.

Cross-Cultural Comparisons. Comparative studies between Singapore and other educational systems that also emphasize mathematical modelling could help uncover best practices and cultural influences on the implementation and evaluation of modelling competencies. This research should investigate how the conceptualization and assessment of modelling competencies may differ across various educational contexts and curricula.

The study concentrates on Singapore's practices, which may be influenced by biases like cultural factors and the lack of examination of other successful mathematical modelling implementations in primary and secondary education. Future research could explore how Singapore will advance students' modelling competencies after 2021, especially since the integration of mathematical modelling has been removed from the primary curriculum.

The study significantly enhances the understanding of how to effectively integrate mathematical modelling into mathematics education, especially regarding Singapore's curriculum reforms and students' modelling competencies. The insights gained can guide both research and practical applications in this crucial aspect of mathematics education.

References

Ambarita, S. M., Asri, L., Agustina, A., Octavianty, D., & Zulkardi. (2018). Mathematical modeling skills on solving PISA problems. *Journal of Physics: Conference Series*, 1097, 012115. https://doi.org/10.1088/1742-6596/1097/1/012115

Balakrishnan, G., Yen, Y. P., & Goh, L. E. (2011). Mathematical modelling in the secondary school mathematics curriculum. In B. Kaur & J. Dindyal (Eds.), *Mathematical applications and modelling: Yearbook 2010 – Association of Mathematics Educators* (pp. 247-257). Singapore: World Scientific.

Blomhøj, M. (1993). Modellerings betydning for tilegnelsen af matematiske begreber [The significance of modelling for the acquisition of mathematical concepts]. *Nordisk matematikkdidaktikk*, 1, 18–39.

Blomhøj, M., & Jensen, T. H. (2003). Developing mathematical modelling competence: Conceptual clarification and educational planning. *Teaching Mathematics and its Applications, 22(3)*.

Blomhøj, M., & Jensen, H. T. (2007). What's all the fuss about competences? Experiences with using a competence perspective on mathematics education to develop the teaching of mathematical modelling. In W. Blum (Ed.), *Modelling and applications in mathematics education: The 14th ICMI-study* (pp. 45-56). Springer-Verlag.

Borromeo Ferri, R. & Blum, W. (2012). Barriers and Motivations of Primary Teachers for Implementing Modelling in Mathematics Lesson. *In Proceeding of 8th Congress of European Research in Mathematics Education*. Retrieved from http://www.cerme8.metu.edu.tr/wgpapers/wg6_papers.html, by 20 Jun 2024.

Chan, C. M. E. (2008). Using model-eliciting activities for primary mathematics classrooms. *The Mathematics Educator, 11(1/2)*, 47-66.

Chan, C. M. E. (2009). Mathematical modelling as problem solving for children in the Singapore mathematics classrooms. *Journal of Science and Mathematics Education in Southeast Asia, 32(1)*, 36-61.

Chan, C. M. E. (2011). Primary 6 Students' Attitudes towards Mathematical Problem-Solving in a Problem-Based Learning Setting. *The Mathematics Educator, 13(1)*, 15-31.

Chan, C. C. M., Ng, K. E. D., Widjaja, W., & Seto, C. (2012). Assessment of primary 5 students' mathematical modelling competencies. *Journal of Science and Mathematics Education in Southeast Asia, 35(2)*, 146-178.

Cheah, U. H. (2008). *Introducing Mathematical Modelling to Secondary School Teachers: A Case Study. The Mathematics Educator, 11(1/2)*, 21-32.

Diehl, W. J., Grobe, T. A., Lopez, J., & Cabral, O. (1999). *Successful project-based learning: Pedagogy, performance, and implementation*. ERIC Document Reproduction Service No. ED434137.

Doerr, H. M., & English, L. D. (2003). A modelling perspective on students' mathematical reasoning about data. *Journal for Research in Mathematics Education, 34*, 110-136.

English, L. D. (2006). Mathematical Modeling in the Primary School: Children's Construction of a Consumer Guide. *Educational Studies in Mathematics, 63(3)*, 331-355. DOI: 10.1007/s10649-005-9013-1

English, L. D. (2010). Young Children's early modelling with Data. *Mathematics Education Research Journal, 22 (2)*, 24-47.

English, L. D. (2012). Data Modelling with First-Grade Students. *Educational Studies in Mathematics*. Springer. DOI 10.1007/s10649-011-9377-3.

Greer, B., Verschaffel, L., & Mukhopadhyay, S. (2007). *Modelling for life: Mathematics and children's experience*. In W. Blum, P. L.

Galbraith, P. L. (2018). *Mathematical Modelling: Guidebook for teachers and teams*. Australian Council for Educational Research (ACER).

Gregersen, P., & Jensen, T. H. (1998). *Problemløsning og modellering i en almendannende matematikundervisning* [Problem solving and modelling in general mathematics teaching] (Technical Report 353). IMFUFA, Roskilde University, Denmark. Available to order from IMFUFA@ruc.dk.

Haines, C.R., & Crouch, R.M. (2007). Mathematical modelling and applications: Ability and competence frameworks. In W. Blum, P. Galbraith, M. Niss, & H-W. Henn (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 417–424). New York: Springer.

Hjalmarson, M., & Diefes-Dux, H. (2008). Teacher as designer: A framework for teacher analysis of mathematical model-eliciting activities. Interdisciplinary *Journal of Problem-based Learning, 2(1)*, 58-78.

Hong Kong Government. (2023). *The Chief Executive's 2023 Policy Address: A Vibrant Economy for a Caring Community*. Retrieved from https://www.policyaddress.gov.hk/2023/en/policy.html

Højgaard, T. (2009). Communication: The essential difference between mathematical modeling and problem solving. In *ICTMA 13* (pp. 255-264).

Jones, B. F., Rasmussen, C. M., & Moffitt, M. C. (1997). *Real-life problem solving: A collaborative approach to interdisciplinary learning*. ERIC Document Reproduction Service No. ED407630.

Krajcik, J., Blumenfeld, P., Marx, R., & Soloway, E. (1994). *A collaborative model for helping middle grade science teachers learn project-based instruction*. ERIC Document Reproduction Service No. ED369038.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning: *A project of the National Council of Teachers of Mathematics* (pp. 763-803). Charlotte, NC: Information Age Publishing.

Lesh, R., & Doerr, H. (2003). Foundations of a models and modeling perspective in mathematics teaching and learning. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching* (pp. 3-34). Mahwah, NJ: Lawrence Erlbaum Associates.

Liu, X., Hansen, K. Y., Valcke, M., & De Neve, J. (2024). A Decade of PISA: Student Perceived Instructional Quality and Mathematics Achievement across European Countries. *ZDM: Mathematics Education, 56(5)*, 859-891. Retrieved from ERIC database. (ERIC Number: EJ1443626)

Maaß, K. (2006). Classification of modelling tasks. Mathematics meets reality. In H. Burkhardt (Ed.), Improving educational design worldwide: *ISDDE Annual Conference 2006*. Oxford.

Marx, R. W., Blumenfeld, P., Krajcik, J., & Soloway, E. (1994). Learning with technology: Middle school students' use of computers as cognitive tools. *Journal of Research in Science Teaching, 31(10)*, 1053-1076.

Mehraein, S., & Rafiepour Gatabi, A. (2014). Gender and mathematical modelling competency: primary students' performance and their attitude. *Procedia - Social and Behavioral Sciences, 128*, 198-203.

Modeste, S., Giménez, J., Lupiáñez Gómez, J. L., Carvalho e Silva, J., & Nguyen, T. N. (2023). Coherence and relevance relating to mathematics and other disciplines. In Y. Shimizu & R. Vithal (Eds.), *Mathematics curriculum reforms around the world* (pp. 151-172). Springer International Publishing. https://doi.org/10.1007/978-3-031-13548-4_11

MOE. (2001). *Primary mathematics syllabus - Curriculum Planning and Development Division*, Ministry of Education Singapore.

MOE. (2007). *Primary mathematics syllabus - Curriculum Planning and Development Division*, Ministry of Education Singapore.

MOE. (2013). *Mathematics syllabus primary one to six - Curriculum Planning and Development Division*, Ministry of Education Singapore.

MOE (2021). *Mathematics syllabus primary one to six - Curriculum Planning and Development Division*. Ministry of Education Singapore.

Moursund, D. (1999). *Project-based learning using information technology*. Retrieved from ERIC database. (ED433677)

Mousoulides, N., Sriraman, B., & Christou, C. (2007). *From problem solving to modelling: The emergence of models and modelling perspectives*. Nordic Studies in Mathematics Education, 12(1), 23-47.

Ming Eric, C. C. (2012). Using Model-Eliciting Activities for primary Mathematics Classrooms. *The Mathematics Educator (Special Issue for ICME12)*, pp. 140-158.

NG K. E. D. (2010). Initial Experiences of Primary School Teachers with Mathematical Modelling. In Association of Mathematics Educators (Ed.), *Mathematical Applications and Modelling Yearbook 2010* (pp. 129). World Scientific.

Ng, K. E. D. (2018). *Towards a professional development framework for mathematical modelling: The case of Singapore teachers*. ZDM, 50, 287–300. https://doi.org/10.1007/s11858-018-0910-z

Nhung, N. T. P., & Thang, N. C. (2021). *Building a process of practising the skill to teach mathematical modelling for students of primary education*. Vinh University Journal of Science, Vol. 50 - No. 2B/2021, pp. 69-81.

Niss, M. (1989). Aims and scope of applications and modelling in mathematics curricula. In W. Blum et al. (Eds.), *Applications and modelling in learning and teaching mathematics* (pp. 22–31). Horwood.

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction to modelling and applications in mathematics education. In W. Blum, P. L. Galbraith, H. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: 14th ICMI study* (pp. 3-32). Springer.

Niss, M., Blum, W. (2020). *The Learning and Teaching of Mathematical Modelling*. United Kingdom: Taylor & Francis.

Niss, M. & Højgaard, T. (Eds.) (2011). *Competencies and Mathematical Learning: Ideas and Inspiration for the Development of Mathematical Teaching and Learning in Denmark*. Roskilde University: IMFUFA. English translation of Danish original (2002).

OECD. (2010). PISA 2009 Results: *What Students Know and Can Do – Student Performance in Reading, Mathematics and Science (Volume I)*. OECD Publishing. http://dx.doi.org/10.1787/9789264091450-en

OECD. (2014). *PISA 2012 results in focus: What 15-year-olds know and what they can do with what they know*. Retrieved from https://www.oecd.org/content/dam/oecd/en/about/programmes/edu/pisa/publications/national-reports/pisa-2012/pisa-2012-in-focus/pisa-2012-results-overview.pdf

OECD. (2015). *Assessment and analytical framework: Mathematics, reading, science, problem solving and financial literacy* (pp. 67-108). OECD Publishing.

OECD. (2016). PISA 2015 Results (Volume I): Excellence and Equity in Education. OECD Publishing. http://dx.doi.org/10.1787/9789264266490-en

OECD. (2019). *PISA 2018 Results (Volume I): What Students Know and Can Do*. OECD Publishing. https://doi.org/10.1787/5f07c754-en

OECD. (2023). *PISA 2022 Results (Volume I): The State of Learning and Equity in Education*. PISA. OECD Publishing. https://doi.org/10.1787/53f23881-en

Rosyidi, A. H., Fadhilah, L. F., & Hartono, S. (2021). Students' mathematical modeling on PISA quantity problems of formulation category: Explicit model vs implicit model. *International Joint Conference on Science and Engineering 2021* (IJCSE 2021). Atlantis Press International B.V.

Seto, C., Dawn, N. K. E., Magdalene, T. M., Eric, C. C. M., & Widjaja, W. (2012). Mathematical Modelling for Singapore Primary Classrooms: From a Teacher's Lens. In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), *Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia)*. Singapore: MERGA.

Tan, L. S., & Ang, K. C. (2016). A school-based professional development programme for teachers of mathematical modelling in Singapore. *Journal of Mathematics Teacher Education, 19*, 399–432. https://doi.org/10.1007/s10857-015-9305-z

Tan, O. S. (2003). *Problem-based learning innovation: Using problems to power learning in the 21st century*. Singapore: Thomson Learning.

Thomas, J. W., Mergendoller, J. R., & Michaelson, A. (1999). *Project-based learning: A handbook for middle and high school teachers*. Novato, CA: The Buck Institute for Education.

Thomas, J. W. (2000). *A review of research on project-based learning*. Retrieved from http://www.bie.org/research/study/review_of_project_based_learning_2000

Tomas H. J. (2009). *Competencies, Skills and Assessment*. The Danish School of Education, Aarhus University, Denmark.

Trinter, C. P., Moon, T. R., & Brighton, C. M. (2015). Characteristics of Students' Mathematical Promise When Engaging With Problem-Based Learning Units in Primary Classrooms. *Journal of Advanced Academics, 26(1)*, 24-58. doi:10.1177/1932202X14562394

Verschaffel, L., Greer, B. & De Corte, E. (2002). Everyday knowledge and mathematical modeling of school word problems. In K. Gravemeijer, R., Lehrer, B., Oers, B., van and L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 257–276). Dordrecht, Netherlands: Springer.

Verschaffel, L. (2002). Taking the modelling perspective seriously at the elementary school level: promises and pitfalls. In A. Cockburn & E. Nardi (Eds.), *Proceedings of 26th Annual Meeting of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 64– 80). Norwich: University of East Anglia.

Waykar, S. R. (2020). Mathematical Modelling: A study of Corruption in the society. *International Journal of Scientific & Engineering Research, Volume 4*, Issue 7, July-2013. Retrieved from https://www.researchgate.net/publication/338609190

Wei, Y., Zhang, Q., & Guo, J. (2022). Can Mathematical Modelling Be Taught and Learned in Primary Mathematics Classrooms: A Systematic Review of Empirical Studies. *Education Sciences, 12(12)*, 923. https://doi.org/10.3390/educsci12120923

TSG1-14

EFFECTIVENESS OF THE TEACHING STRATEGY USING A CONFLICT MAP IN LESSONS ON MULTIPLICATION WITH DECIMAL FRACTIONS

Tadayuki Kishimoto
University of Toyama, School of Education, Japan, kisimoto@edu.u-toyama.ac.jp

This study aims to examine the effectiveness of using a conflict map as a teaching strategy to support teachers in lessons in multiplication with decimal fractions. The study utilized qualitative methods and included three stages: a pretest, a teaching program, and a posttest. Based on the findings, the following conclusions can be drawn: (1) Developing a conflict map proved advantageous for lesson planning, as it provided a structured approach to addressing preconceptions, organizing teaching materials, and facilitating students' responses. (2) Students who experienced a cognitive conflict performed more effectively than those who did not. Awareness of a cognitive conflict enhanced students' understanding by prompting them to reflect their preconceptions. (3) Students in the Division group, who initially selected division as the operation, achieved a higher correct answer rates than those in the Multiplication group. The Division group demonstrated greater awareness of the need to reassess their reasoning. In contrast, many students in the Multiplication group relied on intuition rather than clear reasoning, resulting.

Keywords: multiplication, decimal fraction, conceptual change, conflict map

Introduction

All students meet various numbers and basic operations such as multiplication and division at the elementary school level. To support their comprehension, teachers must present these operations in a way that deepens students' mathematical understanding. Several researchers have proposed models to students' support in their conceptual understanding. Tsai (2000) introduced a conceptual change strategy called "conflict maps." Many educators have adopted conflict maps to address students' misconceptions and refine teaching strategies (Bawaneh *et al*., 2010; Oh, 2010).

This study evaluates the effectiveness of conflict maps as a teaching strategy for lessons in multiplication with decimal fractions. This approach could provide teachers with a valuable tool for enhancing students' conceptual understanding.

Method

The study employed qualitative methods to investigate the effectiveness of a conflict map teaching strategy for the teaching program. 33 fifth-grade students from an elementary school participated in this study. The investigation comprised of two stages: (1) The teaching program was designed based on the conflict map framework, which aimed to notice students' misconceptions and enhance their correct conceptual understanding of multiplication and division. (2) The posttest required students to identify the correct operation for word problems involving multiplication. Additionally, students also compared the sizes of the multiplicand and the resulting product.

Framework

Conflict Map

Tsai (2000) developed the conflict map as a teaching strategy to support meaningful learning in mathematics. Students must solve two conflicts during conceptual change: (1) between new perceptions and their informal conceptions (Conflict 1) and (2) between their informal conceptions and scientific concepts (Conflict 2). Conflict 1 can be solved through discrepant events, where students confront unexpected or contradictory situations. Conflict 2 is solved by using critical events, explanations, representations (P2, P3, P4,...) and conceptions (C1, C2, C3,...) that clarify the scientific concepts.

Teaching Program

The teaching program was planned as five stages based on the conflict map shown in Figure 1.

C1': Students' informal conceptions stage: Students solved word problems using previous informal mathematical knowledge: (a) A ribbon costs 90 yen per meter. How much does 3 m of ribbon cost? (90 × 3), (b) A ribbon costs 90 yen per meter. How much does 2.6 m of ribbon cost? (90 × 2.6), (c) A ribbon costs 270 yen for 3 m. How much does 1 m of ribbon cost ?(270 ÷ 3)

DE(P1): Discrepant events stage: Students encountered an unfamiliar word problem: (d) A ribbon costs 90 yen per meter. How much does '0.6 m of ribbon' cost? Students were asked to choose between multiplication and division and answer their reasonable statements.

C1: Scientific conceptions stage: The teacher explained the meaning of multiplication using the following example: 'A × p' refers to finding the amount corresponding to the rate p, considering A as a unit.

CE: Critical events or explanation stage: Students represented the situations from word problems on a number line diagram.

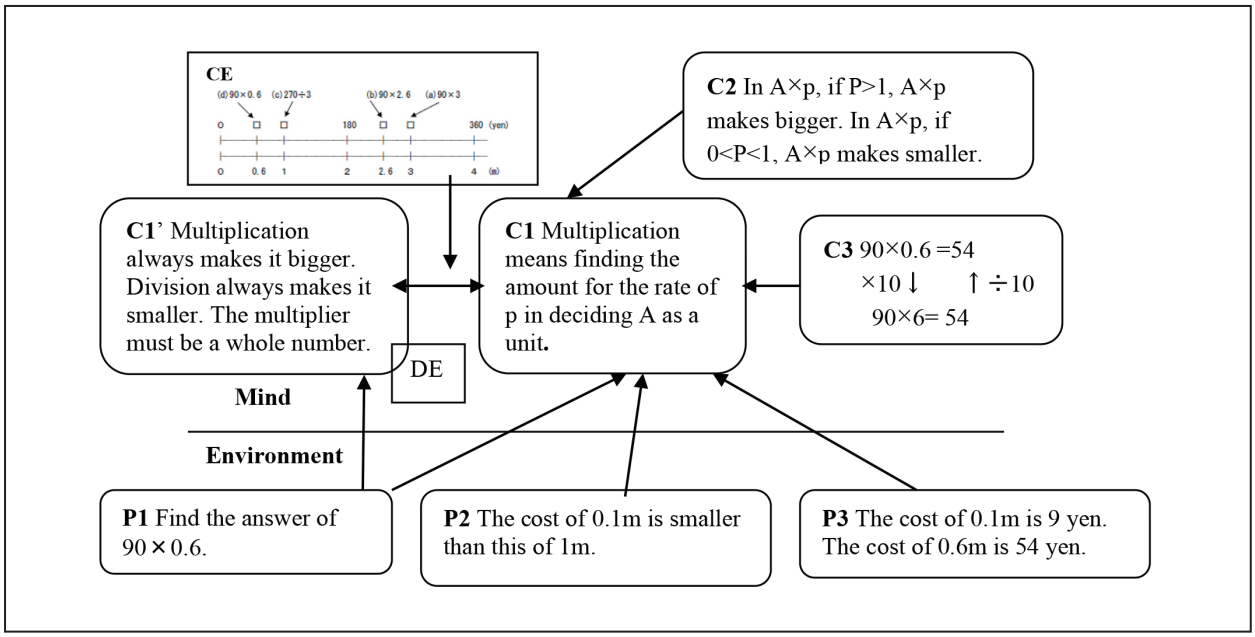


Figure 1. Conflict map of multiplication with decimal fractions

P1 and P2 stage: Students found correct mathematical statements about multiplication such as scientific conceptions. For example, "the answer for (90 × 1) is 90. As 0.6 m is shorter than 1 m, the cost for 0.6 m must be less than 90 yen."

Posttest

Students were tested on operation selection problems presented in Figure 2. These word problems also included addition, subtraction, multiplication, and division to help students easily decide the correct operation.

(1) You buy beans that cost 580 yen per kilogram. How much does 2.4 kg of beans cost? (580 × 2.4)
(2) A store uses 7.6 L of sauce, with each liter weighing 1.2 kg in a month. How many kilograms of sauce did they use over the month? (1.2 × 7.6)
(3) There is some cooking oil that costs 600 yen per liter. How much does 0.3 L of this cooking oil cost? (600 × 0.3)
(4) There is an iron pole that is 1 meter long and weighs 1.2 kg. How many kilograms do 0.8 meters long weigh? (1.2 × 0.8)

Figure 2. Deciding operation problems

Analysis

(C1') Students' Informal Conception

For the word problem (90×0.6), among them who chose division as an error answer, 11 students could provide explanations, as outlined in Table 1. 9. Students wrote, "Division makes smaller". One student explained the calculation procedure: "The cost of 1 m is 90 yen. I want to find the cost of 0.6 m. The length of 1 m is 0.1 m × 10. I want to find 6 of the 10." One student asserted, "Because the right operation must be division."

Table 1. Reasons for division group (Error group)

Reasons for deciding on operation	Number of students
Statement (Division makes smaller)	9
Calculation procedure	1
Assertion	1

Table 2 presents the reasoning of 22 students who selected multiplication as a correct answer. 9 students could correctly state that "Multiplication makes it smaller". 8 students only asserted that "Because the right operation must be multiplication." 4 students explained the calculation procedure: "The cost of 1 m is 90 yen. If one divides 90 by 10, one finds 9 as the answer. One multiplies 9 by 6 and finds 54." One student explained the calculation in reverse: "One multiplies 0.6 by 90. 0.6 × 90 makes 54."

Table 2. Reasons for multiplication group (Correct group)

Reasons for deciding on operation	Number of students
Statement (Multiplication makes smaller)	9
Assertion	8
Calculation procedure	4
Calculation backwards	1

(DE) Discrepant Event

We conjectured about their cognitive conflict aware based on their answer and statement as follows. Table 3 shows the classification results. 7 students in the Division group(error group) and 13 students in the Multiplication group (correct group) were classified as aware of a conflict of C1'.

Table 3. Cognitive conflict aware

	Aware of C1'	Not aware of C1'
Division group (Error group)	7	4
Multiplication group (Correct group)	13	9

(CE) Critical Event or Explanation

On the critical event explanation phase in this lesson, they in the Multiplication group had a lower percentage of correct answers for the number line diagram problem than them in the Division group. The score of them in the Multiplication group was very low in the 90 × 0.6 problem.

(C1) Students' Scientific Conception

Table 4 presents the percentage of students aware of a scientific conceptions. In the Division group, students who recognized a cognitive conflict scored higher on average than those who did not. Similarly, in the Multiplication group, students who were aware of a cognitive conflict outperformed those who were not.

Table 4. Results of deciding operation in word problems

	580 × 2.4	1.2 × 7.6	600 × 0.3	1.2 × 0.8
Correct group				
Division / Conflict aware	100.0%(7)	85.7%(6)	57.1%(4)	71.4%(5)
Division / No conflict aware	100.0%(4)	75.0%(3)	75.0%(3)	50.0%(2)
Error group				
Multiplication / Conflict aware	76.9%(10)	76.9%(10)	61.5%(8)	76.9%(10)
Multiplication / No conflict aware	66.7%(6)	44.4%(4)	55.6%(5)	77.8%(7)

Discussion

In the 90 × 0.6 problem, 11 students selected division as a correct operation, while 22 did multiplication. Among the Division group, 9 students explained their reasoning by stating the misconception such as "Division makes smaller." In the Multiplication group, 9 students claimed the misconception such as "Multiplication makes smaller." 13 students in the Multiplication group were unable to provide a clear explanation for their choice.

7 students in the Division group demonstrated awareness of a cognitive conflict, representing 63.6% of the group. In the Multiplication group, 13 students (59.1%) were aware of a cognitive conflict. Overall, the number of students recognizing cognitive conflict was slightly higher in both groups.

In solving word problems in post-test, students in the Multiplication group achieved lower scores than those in the Division group. Regarding understanding right mathematical concepts, the correct answer rate for solving word problems was lower for multiplication problems involving pure decimals than for those with mixed decimals. Students who recognized a cognitive conflict scored higher than those who did not, regardless of group.

References

Bawaneh, A.K.A., Zain, A.N.M., and Ghazali, M. (2010). The effectiveness of conflict maps and the V-Shape teaching method in science conceptual change among eighth-grade students in Jordan. *International Education Studies*, 3(1), 96-108.

Oh, J.Y., Lee, H., and Lee, S.S. (2017). Using the lakatosian conflict map for conceptual change of pre-service elementary Teachers about the Seasons. *Research in Science & Technological Education*, 35(1), 17-41.

Tsai, C.-C. (2000). Enhancing science instruction: The use of 'conflict maps'. *International Journal of science education*, 22(3), 285-302.

TSG1-15

MATHEMATICAL MODELING IN HONG KONG PRIMARY MATHEMATICS EDUCATION: A CONTENT ANALYSIS OF SCHOOL CURRICULUM

Hange Yun
The Education University of Hong Kong, Hong Kong SAR, China, s1154358@s.eduhk.hk

Qiaoping ZHANG
The Education University of Hong Kong, Hong Kong SAR, China, zqiaoping@eduhk.hk

While mathematical modeling has been traditionally emphasized at the secondary level, scholars advocate its early introduction in primary education. This study explores how modeling can be integrated into Hong Kong's primary mathematics curriculum by analyzing aims, content, structure, and assessment. Findings show that although modeling is not explicitly stated, its principles are embedded in curriculum objectives, STEM elements, and e-learning practices. Challenges remain due to the lack of explicit goals, task guidelines, and examples. The study recommends providing structured frameworks and flexible content organization: inquiry-based STEM activities and project-based learning foster students' modeling awareness and competencies.

Keywords: Primary mathematics education, Mathematical modeling, Curriculum guidelines, Content analysis

Introduction

Mathematical modeling is widely recognized for its capacity to develop students' mathematical application skills, critical thinking, and interdisciplinary problem-solving (Asempapa, 2015; Suh et al., 2021). It bridges school mathematics and real-world contexts by engaging students in iterative processes involving assumptions, approximations, and multiple representations (Stohlmann & Albarracín, 2016). With the global rise of STEM education, modeling has been formally integrated into national curricula in countries such as Singapore and Australia to foster 21st-century competencies (Choy & Dindyal, 2024; Geiger, 2015).

While traditionally emphasized at the secondary level, recent studies argue for introducing modeling in primary education, where students possess emerging mathematical skills and cognitive flexibility (English, 2014; Jung & Brand, 2021). Early exposure to modeling can promote reasoning and real-world problem-solving from a young age (Carpenter & Romberg, 2004). However, challenges remain, including a lack of teacher training, resources, and understanding of young learners' modeling capabilities (English et al., 2016).

In Hong Kong, modeling has gained policy attention, notably through the Chief Executive's 2023 Policy Address and professional development initiatives (Curriculum Development Council, 2017; Lo et al., 2022). However, how mathematical modeling can be effectively integrated into the primary mathematics curriculum remains underexplored. This study investigates the implicit and explicit presence of modeling within Hong Kong's curriculum framework, aiming to inform curriculum revisions, textbook development, and classroom practices.

Method

This study adopts a content analysis approach to examine the 2017 edition of the "Mathematics Education Key Learning Area Curriculum Guide (P1–S6)," the latest version issued by the Hong Kong Education Bureau (Curriculum Development Council, 2017). Using keyword searches for "model," "modeling," and "mathematical modeling," both quantitative and qualitative analyses were conducted. Quantitative analysis assessed keyword frequency to determine explicit references to modeling, while qualitative analysis explored implicit expressions of modeling concepts embedded in curriculum aims, content, organization, and assessment components.

Research Findings

Search Results

The term "model" appears 73 times across the curriculum guide, primarily in teaching practices, STEM tasks, and technology-assisted learning. Of these, 43 instances are in specific teaching examples—mostly within upper primary levels (P.4–P.6)—involving measurement and geometry, such as using models for volume estimation or trajectory drawing. Nine mentions appear in the content section, referring to geometric and data models that support analysis and reasoning. Models are also used in diagrams, images, and software tools to enhance spatial visualization.

"Modeling" appears 6 times in STEM-related contexts and 10 times across e-learning and other subject areas, but rarely in primary-level content. "Mathematical modeling" appears only 4 times, mainly within the STEM education section, where its role in solving real-world problems is emphasized.

Modeling Concepts in Curriculum Objectives

The Hong Kong mathematics curriculum aims to develop critical thinking, problem-solving, logical expression, and positive attitudes toward mathematics. While not explicitly stated, its goals implicitly support various forms of mathematical expression, deepen conceptual understanding, enhance logical analysis, and lay the groundwork for students to master the process from problem identification to model construction.

The curriculum updates emphasize mathematical modeling through four key areas: First, STEM education fosters real-world problem-solving through interdisciplinary modeling activities. Second, e-learning tools help optimize the visualization and refinement of mathematical concepts and models. Third, the Cross-Curriculum Language (LaC) approach strengthens modeling thinking through mathematical expression and logical reasoning. Fourth, developing general capabilities, such as IT skills, teamwork, critical thinking, and creativity, prepares students for complex modeling tasks. These updates provide systematic support for modeling in a multidisciplinary context.

Modeling Concepts in Curriculum Content

The primary mathematics curriculum covers five core areas: numbers, algebra, measurement, geometry and space, and data processing. In the lower grades (1st to 3rd), the focus is on hands-on activities, where students learn to select appropriate data recording, organizing, and analysis tools through tasks like measuring length, weight, capacity, and creating bar and pictorial graphs. These tasks, grounded in real-life contexts, guide students to extract data and establish basic mathematical relationships. For example, tasks related to "currency" and "time" involve basic operations and data organization, helping students grasp the application of mathematics in daily life and laying the foundation for future modeling skills.

More complex tasks are introduced in the upper grades (4th to six[th]). For example, the "Rubber Band Powered Car" STEM task encourages students to extract key variables from real-world phenomena, build mathematical relationships for rate calculations, and optimize models to explore energy transfer and power characteristics, engaging students in the process of mathematical abstraction and model refinement. The "Cross Sections of 3-D Shapes" task uses geometric models to help students explore the planar properties of three-dimensional shapes, allowing them to understand spatial relationships visually through dynamic tools. Additionally, functions in the statistics unit, such as categorization, graph creation, and trend analysis, provide students with practical descriptive and predictive modeling opportunities.

Modeling Concepts in Curriculum Organization

The Hong Kong mathematics curriculum promotes a diversified instructional approach—including direct instruction, inquiry-based learning, and co-constructivist methods—to support students' understanding of mathematical concepts while cultivating core competencies and positive learning attitudes. A student-centered approach, supported by technology, enables multiple pathways for engaging with mathematical modeling. Inquiry-based learning introduces modeling through group work and hands-on tasks, while co-constructivist learning enhances communication and collaboration, especially in interdisciplinary and project-based settings.

Life-wide Learning (LWL) expands modeling beyond the classroom through activities such as statistical surveys, thematic studies, and math games. The curriculum also encourages students to explore the complexity of social issues in cultural and community contexts, guiding them to distinguish between data-driven problems and those shaped by value judgments (English, 2013). The relevance of mathematics to real-world contexts significantly enhances students' motivation and engagement (Carlson, 2021).

Modeling in Curriculum Assessment

The Hong Kong curriculum outlines five assessment modes—classroom discussions, assignments, projects, inquiry-based tasks, and tests—providing multidimensional support for modeling instruction. Discussions assess reasoning and communication, while real-life tasks (e.g., tracking water usage) evaluate students' data processing, model construction, and problem-solving abilities. Inquiry tasks and hands-on activities offer timely feedback to support model refinement. Although formal tests rarely include complex modeling, simplified application problems help assess foundational modeling skills.

While mathematical modeling is not explicitly listed as core content, its principles are embedded across curriculum objectives, structure, and assessment, especially in STEM and e-learning components. The frequent appearance of "model" in upper primary content, particularly in measurement, geometry, and data tasks, suggests implicit support. However, the lack of defined modeling competencies and learning goals limits systematic teacher guidance and student engagement. Textbooks and digital platforms often treat modeling as a supplementary tool rather than a fully integrated instructional process, reducing its interdisciplinary potential.

Discussion

Although the Hong Kong primary mathematics curriculum integrates modeling tools and highlights its relevance to STEM education, mathematical modeling remains unsystematic in classroom practice. According to PISA 2022, about 27% of Hong Kong secondary students achieved Level 5 or 6

in modeling tasks, surpassing the international average (OECD, 2024). This indicates strong student performance despite limited exposure at the primary level. Introducing structured modeling instruction earlier could enhance awareness and build a stronger foundation for STEM learning (Stillman, Brown, & Galbraith, 2010).

Clear instructional frameworks are essential. The Australian Curriculum defines stage-based achievement standards and emphasizes key modeling skills, including problem formulation and interpretation (Australian Curriculum, 2021). Germany's assessment framework further supports classroom implementation (Jahnke & Hefendehl-Hebeker, 2019).

To improve alignment, the curriculum should offer case examples and adaptable resources. Flexible strategies—such as inquiry-based STEM tasks, digital exploration, and project-based learning—can promote engagement and deepen modeling understanding (Gilbert & Suh, 2021). Clear task themes and difficulty gradients are also necessary. Teachers should adopt a "modeling perspective" to link curricular goals with students' lived experiences (Suh et al., 2021). Context-rich tasks related to school or community issues can develop mathematical thinking, interdisciplinary skills, and social responsibility (English, 2021).

References

Asempapa, R. S. (2015). Mathematical modeling: Essential for elementary and middle school students. *Journal of Mathematics Education, 8*(1), 16-29.

Australian Curriculum, Assessment and Reporting Authority. (2022). *HASS F-6 glossary.* https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-1_year-2_year-3_year-4_year-5_year-6?view=quick&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0

Carpenter, & Romberg. (2004). *Powerful practices in mathematics and science.* Madison, WI: National Center for Improving Student Learning and Achievement in Mathematics and Science.

Choy, B. H., & Dindyal, J. (2024). The Singapore mathematics curriculum: Influences and confluences. In *Research handbook on curriculum and education* (pp. 534-549). Edward Elgar Publishing.

Curriculum Development Council. (2017). *Mathematics education curriculum guide (Primary 1 to Secondary 6).* Hong Kong: Hong Kong Printing Press. https://www.edb.gov.hk/attachment/en/curriculum-development/kla/ma/curr/ME_KLACG_eng_2017_12_08.pdf

English, L. D. (2021). Mathematical and interdisciplinary modeling in optimizing young children's learning. In J. M. Suh, M. H. Wickstrom, & L. D. English (Eds.), *Exploring mathematical modeling with young learners: Early Mathematics Learning and Development* (pp. 3-23). Springer. https://doi.org/10.1007/978-3-030-63900-6_1

English, L. D. (2014). Promoting statistical literacy through data modelling in the early school years. In E. Chernoff & B. Sriraman (Eds.), *Probabilistic thinking: Presenting plural perspectives* (pp. 441–458). Dordrecht, The Netherlands: Springer.

English, L. D., Arleback, J. B., & Mousoulides, N. (2016). Reflections on progress in mathematical modelling research. In A. Gutierrez, G. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education* (pp. 383–413). Rotterdam, The Netherlands: Sense Publishers.

Geiger, V. (2015). Mathematical modelling in Australia. In N. H. Lee & K. E. D. Ng (Eds.), *Mathematical modelling: From theory to practice* (pp. 73–82). World Scientific.

Gilbert, A., & Suh, J. M. (2021). Convergent nature of modeling principles across the STEM fields: A case study of preservice teacher engagement. In J. M. Suh, M. H. Wickstrom, & L. D. English (Eds.), *Exploring mathematical modeling with young learners: Early Mathematics Learning and Development* (pp. 321-342). Springer. https://doi.org/10.1007/978-3-030-63900-6_15

Jahnke, H. N., & Hefendehl-Hebeker, L. (2019). *Traditions in German-speaking mathematics education research.* Springer Nature.

Jung, H., Brand, S. (2021). Synthesizing Research of Mathematical Modeling in Early Grades. In: Suh, J.M., Wickstrom, M.H., English, L.D. (eds) Exploring Mathematical Modeling with Young Learners. Early Mathematics Learning and Development. Springer, Cham. https://doi.org/10.1007/978-3-030-63900-6_2

Lo, C. K., Huang, X., & Cheung, K. L. (2022). Toward a design framework for mathematical modeling activities: An analysis of official exemplars in Hong Kong mathematics education. *Sustainability, 14*(15), 9757. https://doi.org/10.3390/su14159757

OECD. (2024, October 5). PISA 2022 database. https://www.oecd.org/en/data/datasets/pisa-2022-database.html

Stohlmann, M. S., & Albarracín, L. (2016). What is known about elementary grades mathematical modelling. *Education Research International*, *2016*, 1–9. https://doi.org/10.1155/2016/5240683

Suh, J. M., Matson, K., Birkhead, S., Green, S., Rossbach, M., Seshaiyer, P., & Jamieson, S. (2021). Elementary teachers' enactment of the core practices in problem formulation through situational contexts in mathematical modeling. In J. M. Suh, M. H. Wickstrom, & L. D. English (Eds.), *Exploring mathematical modeling with young learners: Early Mathematics Learning and Development* (pp. 123-144). Springer. https://doi.org/10.1007/978-3-030-63900-6_6

TSG1-16

AN ANALYSIS OF THE TYPES OF CONNECTIONS BETWEEN THE KOREAN MATHEMATICS COLLEGE SCHOLASTIC ABILITY TEST AND EBS-LINKED MATHEMATICS TEXTBOOKS

Minsik Woo
Seoul National University, Korea, abcv2145@snu.ac.kr

Kyungwon Lee
Seoul National University, Korea, kyungwon.lee.snu@gmail.com

Oh Nam Kwon
Seoul National University, Korea, onkwon@snu.ac.kr

This study explores the types of connections between the Korean mathematics CSAT and EBS-linked mathematics textbooks. In Korea, EBS publishes EBS-linked textbooks as part of public education, and KICE announces the linkage rate and linked problems between the Korean CSAT and the EBS-linked textbooks. This study analyzes the connections in 86 problems presented in the Korean mathematics CSAT and EBS-linked mathematics textbooks over the past four years. The analysis of external connection, which focuses on the format of the problems, reveals that the problems that are not connected accounted for a larger proportion. The analysis of internal connection, based on the skills required in the problems and the main principles of problem-solving, shows that problems reflecting both the skills and the main principles had the highest proportion. These findings can serve as foundational data for developing linked mathematics problems and for analyzing the methods used to establish connections between mathematics problems.

Keywords: Korean CSAT, EBS, Textbooks, Mathematics problems, Connections

Introduction

This study aims to explore the types of connections between the Korean mathematics CSAT (College Scholastic Ability Test) and EBS (Educational Broadcasting System) mathematics textbooks. The Korean CSAT, a standardized examination taken by Korean students for university entrance, is widely regarded as a representative assessment of mathematical proficiency in Korea. As a public system, EBS develops EBS-linked textbooks serving as auxiliary educational materials rather than officially certified school textbooks. The KICE (Korea Institute for Curriculum and Evaluation), which administers the Korean CSAT, annually discloses the linkage rate, categories of connections, and linked problems between the Korean CSAT and EBS-linked textbooks (e.g., KICE, 2024). However, there is limited information on the analytical relationships between the linked problems in the Korean mathematics CSAT and EBS-linked mathematics textbooks. In this study, we aim to conduct a systematic analysis of the types of connections between linked mathematics problems in the Korean mathematics CSAT and EBS-linked mathematics textbooks to deepen the understanding of the structural patterns and the functional aspects of problem-solving in the connections between linked mathematical problems. The research question is as follows: *What are the types of connections between the Korean CSAT mathematics problems and the EBS-linked mathematics problems?*

Theoretical Perspective

The term *curriculum* can be broadly used to encompass curriculum documents, textbooks, teaching, and assessment. The Korean CSAT is an assessment that students who have completed the curriculum in Korea must take for university entrance. EBS textbooks are supplementary educational materials that support the official curriculum by facilitating its flexible implementation and achievement, functioning alongside officially certified textbooks. Both the Korean CSAT and EBS textbooks can be considered part of the category of the curriculum. The connection of the curriculum can be found in the three fundamental principles of organizing curriculum content proposed in Tyler's (1949) seminal work: *continuity*, *sequence*, and *integration*. *Continuity* refers to the vertical emphasis on significant elements of the curriculum, ensuring that concepts, knowledge, and skills are repeatedly addressed throughout the curriculum. *Sequence* involves designing the curriculum to build on the previous content, allowing for more comprehensive and in-depth development of knowledge and skills. *Integration* refers to the horizontal relationships within the curriculum, aiming to establish organic connections among knowledge, and skills within the content. Tyler (1949) emphasized the connectedness of the curriculum by pointing out the importance of recurring opportunities for learning content within the curriculum. These three principles are applicable to exploring the connections between curriculum resources, such as the Korean mathematics CSAT and EBS-linked mathematics textbooks, serving as a framework for analyzing the types of connections present.

Methods

This study analyzes the items linked between the Korean mathematics CSAT and EBS-linked mathematics textbooks over the past four years. The Korean mathematics CSAT is administered based on the 2015 revised curriculum. The common subjects include *Mathematics I*, and *Mathematics II*, with a total of 22 problems being presented. For elective subjects, there are *Probability and Statistics*, *Calculus*, and *Geometry*, with 8 problems being asked from one selected subject. In other words, the Korean mathematics C SAT consists of 30 problems across three types. The EBS-linked mathematics textbooks are divided into CSAT Special Lecture (CSL) and CSAT Completion (CC). CSL publishes five textbooks per subject annually, while CC publishes three textbooks for each selected subject, depending on subject selection (See Table 1).

Table 1. Types of korean mathematics CSAT and EBS-linked mathematics textbooks

Category	Type
Korean Mathematics CSAT	Selection <*Probability and Statistics*> including *Mathematics I*, *Mathematics II*, *Probability and Statistics*
	Selection <*Calculus*> including *Mathematics I*, *Mathematics II*, *Calculus*
	Selection <*Geometry*> including *Mathematics I*, *Mathematics II*, *Geometry*
EBS-Linked Mathematics Textbooks	CSL five mathematics textbooks *Mathematics I*, *Mathematics II*, *Probability and Statistics, Calculus*, *Geometry*
	CC three mathematics textbooks Selection <*Probability and Statistics*>, Selection <*Calculus*>, Selection <*Geometry*>

Considering the accessibility of EBS-linked mathematics textbooks at the current time, this study analyzes the type of connections between the Korean mathematics CSAT and EBS-linked mathematics textbooks over the past four years. Information regarding EBS-linked CSAT mathematics problems is provided on the EBS website, categorized by problem numbers in CSAT mathematics, CSL mathematics, and CC mathematics. The years for the Korean mathematics CSAT and EBS-linked mathematics textbooks are based on the students' university entrance year.

The number of EBS-linked mathematics problems in the CSAT is 45 annually, but excluding the overlapped problems in the common subjects, the total number of problems amounts to 86 problems (21 in 2022, 21 in 2023, 21 in 2024, and 23 in 2025) (EBS, 2021, 2022, 2023, 2024). This study investigates the types of connections between the 86 EBS-linked CSAT mathematics problems and the 86 problems linked to CSL and CC. We differentiated between external and internal connections. External connection refers to the format of the problem description. Since task format can affect students' mathematical problem solving (e.g., Zhang et al., 2022), the external connection focusing on the format is analyzed by comparing the problem descriptions of EBS-linked mathematics problems in CSL and CC with those of CSAT-linked mathematics problems. Internal connection pertains to how the skills required in the problem and fundamental principles of mathematical problem solving are interrelated. The differentiated internal connection is categorized based on skills and the main principles of problem-solving. This categorization was conducted through open coding (Strauss & Corbin, 1998) with inter-researcher examination and consensus.

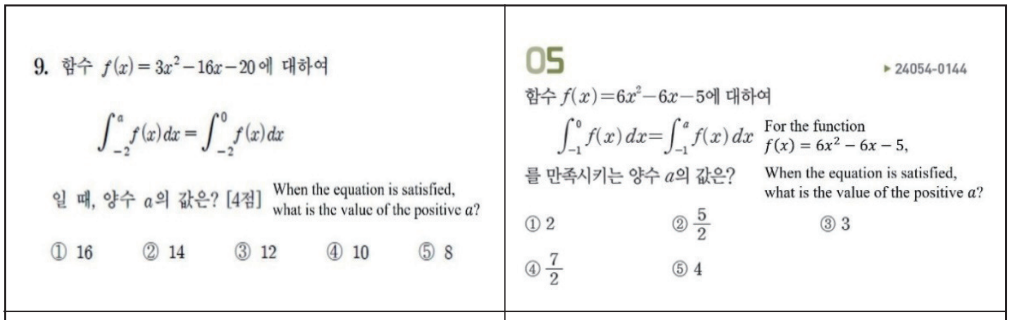
Findings

The findings classify the types of connections between the Korean mathematics CSAT and EBS-linked mathematics textbooks into external connections and internal connections. Among the 86 problems analyzed, 41 problems (47.7%) demonstrated external connections related to the formal format, while 45 problems (52.3%) did not. Although the number of Korean mathematics CSAT in the format of the EBS-linked mathematics textbooks was smaller than that of those that were not, it is not considered an insignificant proportion. The number of problems demonstrating external connections between *Mathematics I* and *Mathematics II* showed significant variation, whereas the variation was relatively minor in *Probability and Statistics*, *Calculus*, and *Geometry*.

The types of internal connections between the Korean mathematics CSAT and EBS-linked mathematics textbooks for the 86 problems are categorized into three types (See Table 2). First, the skills required for problems and the main principles of problem solving are connected (n = 62, 72.1%). This type contains the largest number of problems, and it is evident that connected problems are consistently included across years. Second, the skills required in the problems are connected (n = 16, 18.6%). Although this type was not included in the 2024 CSAT, the number of such problems is the highest in the 2022 CSAT, followed by the 2025 CSAT and the 2023 CSAT. Third, the main principles of problem-solving are connected (n = 8, 9.3%). Problems requiring skills and the main principles of problem solving are mostly related to *Mathematics II*, including the topics of limits of functions, differentiation, and integration.

Table 2. Type of internal connection between the Korean mathematics CSAT and EBS-linked mathematics textbooks problems and distribution of the annual problems

Type	2022 CSAT	2023 CSAT	2024 CSAT	2025 CSAT	Category Total (n = 86)
Skills required in problems and the main principle of problem solving	10	16	20	16	62(72.1%)
Skills required for problems	8	3	·	5	16(18.6%)
The main principle for problem solving	3	2	1	2	8(9.3%)
Total	21	21	21	23	86(100%)



An Example of the 2025 CSAT Problem	An Example of the 2025 CL Problem

Figure 1. An example of an internal connection between the skills and the main principle for problem solving

The example of integration problems for the skills required in the problems and the main principles of problem-solving, which constitute the largest proportion, is shown in Figure 1. The 2025 CSAT problem involves a problem with the definition and properties of definite integrals. By calculating that the result of $\int_0^a f(x)dx$ equals 0, the value of the positive constant a. can be determined. Similarly, the 2025 CL problem also requires finding the value of the positive constant a, where calculating the result of $\int_0^a f(x)dx$ as 0 leads to the value of a. It can be observed that both problems share the same result form (a positive constant a) and are based on the same main principle of problem-solving (the definition and properties of definite integrals).

Conclusion

This study illuminates the types of connections between the Korean mathematics CSAT problems and EBS-linked mathematics textbook problems. The external and internal connections of mathematics problems can help students engage with the problems in a more familiar way. By offering problems with different formats or designing them to require a variety of skills and principles, students can further enhance their problem-solving abilities and apply them in an integrative manner. This study implies fundamental insights for designing such problems or analyzing the connections between mathematics problems to provide these learning opportunities.

References

EBS. (2021). An analysis of linkage details for the 2022 College Scholastic Ability Test. Retrieved from https://www.ebsi.co.kr/ebs/ent/enta/retrieveEntAnlyStrdDataVw.ebs?bbsCd=B114&datNo=127071&cookieGradeVal=high3

EBS. (2022). An analysis of linkage details for the 2023 College Scholastic Ability Test. Retrieved from https://www.ebsi.co.kr/ebs/ent/enta/retrieveEntAnlyStrdDataVw.ebs?bbsCd=B114&datNo=127075&cookieGradeVal=high3

EBS. (2023). An analysis of linkage details for the 2024 College Scholastic Ability Test. Retrieved from https://www.ebsi.co.kr/ebs/ent/enta/retrieveEntAnlyStrdDataVw.ebs?bbsCd=B114&datNo=127082&cookieGradeVal=high3

EBS. (2024). An analysis of linkage details for the 2025 College Scholastic Ability Test. Retrieved from https://www.ebsi.co.kr/ebs/ent/enta/retrieveEntAnlyStrdDataVw.ebs?bbsCd=B114&datNo=127086&cookieGradeVal=high3

KICE. (2024). *Guidelines for studying for the 2025 College Scholastic Ability Test*. CSAT CAT 2024-2-1.

Strauss, A., & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (2nd ed.). Thousand Oaks, CA: Sage.

Tyler, R. W. (1949). *Basic principles of curriculum and instruction*. Chicago: University of Chicago Press.

Zhang, L., Cai, J., Song, N., Zhang, H., Chen, T., Zhang, Z., & Guo, F. (2022). Mathematical problem posing of elementary school students: The impact of task format and its relationship to problem solving. *ZDM Mathematics Education, 54*(3), 497-512.

TSG1-17

A METHOD OF TEXTBOOK ANALYSIS FROM A PRAXEOLOGICAL PERSPECTIVE: THE CASE OF STATISTICS

Seitaro Hanasaki
Graduate School of Humanities and Social Sciences in Hiroshima University, Japan, d255099@hiroshima-u.ac.jp

Yusuke Shinno
Hiroshima University, Japan, shinno@hiroshima-u.ac.jp

Mathematics textbooks are considered as one of the primary curricular resources. The aim of this study is to propose and illustrate a method of textbook analysis from a praxeological perspective. Based on the Anthropological Theory of the Didactic, we draw some theoretical concepts, didactic transposition, praxeology, and didactic paradigm, which underpin the proposed method. To describe both the mathematical and didactic aspects of textbooks, we explore epistemological/didactic models, that provide descriptions of the mathematical/didactic knowledge in textbooks used in the particular institutional context. In this paper, we illustrate this method by referring to the Grade 9 chapter "sample survey" taken from a textbook used in Japanese lower secondary schools. We also discuss the possibilities and limitations of the proposed method.

Keywords: textbook analysis, praxeology, statistics

Introduction

Research on textbooks is a growing field of research in mathematics education (Fan et al., 2013; Fan et al., 2017). Textbooks can be recognised as a "potentially implemented curriculum" that is an intermediate document produced in-between "intended" and "implemented" curriculum (Fan et al., 2013; Valverde et al., 2002). In addition, textbooks are developed under the influence of local and institutional contexts. For example, in some countries, textbooks must be approved by the government based on the national curriculum, but in other countries, textbooks can be published without such approval. Thus, textbooks can be considered cultural products in a given region and in a given time. To better understand the organisation of textbooks from both a mathematical and educational perspective, in this paper, we aim to offer a theoretical approach to analyse mathematics textbooks used in Japanese middle schools.

Theoretical Framework

We adopt the Anthropological Theory of the Didactic (ATD; Chevallard & Bosch, 2020) as the theoretical basis of the proposed method of textbook analysis. The ATD is an epistemological research programme in which researchers work on the "institutional relativity of knowledge" (Gascón, 2024; Gascón & Nicolás, 2019a). To examine the multi-faceted aspects of textbooks, we draw some basic concepts of the ATD: *Didactic transposition*, *praxeology*, and *didactic paradigm*.

The theory of the didactic transposition conceptualises that mathematical knowledge is produced, consumed, and diffused in institutions (Bosch & Gascón, 2006). For example, "scholarly knowledge", "knowledge to be taught", "taught knowledge", and "learned knowledge" exist in different institutions: such as mathematicians' communities, educational systems, classrooms, and communities of study. The transposition from "scholarly knowledge" to "knowledge to be taught" is called the *external didactic transposition* (EDT), and other transpositions between educational systems, classrooms, and communities of study are called the *internal didactic transposition* (Bosch & Winsløw, 2020). Knowledge to be taught is a product of EDT, which involves complex processes in which various "intermediate texts" (e.g., scholarly documents, educational documents, earlier textbooks). Although textbooks are considered empirical sources of knowledge to be taught (Takeuchi & Shinno, 2020), these intermediate texts can be sources for studying EDT.

Praxeology is a model of human knowledge from the "procedural" aspect (*praxis*) and the "declarative or propositional" aspect (*logos*) (Gascón, 2024, p.1322). The praxis block consists of *types of tasks* (T) and *techniques* (τ), describing a set of problems and methods to address them, and the logos block consists of *technologies* (θ) and *theories* (Θ) explaining and justifying the praxis block. There are two basic kinds of praxeologies: Mathematical praxeologies (MP) describes what to teach and didactic praxeologies (DP) explains how to teach and learn. In the praxeological analysis, praxis elements often appear explicitly, but the logos elements are rather implicit. To better characterise the logos parts, we draw an epistemological model (EM) and a didactic model (DM). EM and DM constitute the assumptions of an institutional didactic paradigm (Gascón & Nicolás, 2022). The didactic paradigm consists of didactic ends (to explain the purpose of the study), means (to state what is done for the study), and EMs (to provides a description of what is to be studied) (Gascón & Nicolás, 2019a, 2022). We consider DMs a model for providing us with didactic descriptions in textbooks, including didactic ends and means. In this paper, we try to describe the mathematical and didactic aspects of mathematics textbooks in terms of the EM and the DM.

Praxeology has been used for analysing textbooks (e.g., Takeuchi & Shinno, 2020; Wijayanti & Winsløw, 2017). However, most of the previous studies focus on MP, while DP remains outside their scope. To better understand the organisation of textbooks, in this paper, we consider both MP and DP. We then illustrate this approach using the chapter "sample survey" as a case study.

Table1. Praxeological elements (descriptions adapted from Takeuchi & Shinno, 2020)

	Types of tasks (T)	**Technique (τ)**	**Technology (θ)**	**Theory (Θ)**
MP	Given tasks or problems (MT)	How to solve MT (Mτ)	Ways to explain and justify Mτ (Mθ)	Mathematical ideas, notions, and assumptions to explain and justify Mθ (MΘ)
DP	Teaching purposes (DT)	How to achieve DT (Dτ)	Ways to explain and justify Dτ (Dθ)	Didactic ideas, notions, and assumptions to explain and justify Dθ (DΘ)

An Illustrative Case from a Japanese Textbook

The Object of the Analysis

In Japan, textbooks must be approved by the Ministry of Education based on the national curriculum. Teachers have obligations to use textbooks in their teaching. This means that textbooks play a prominent role in teaching and learning. We selected examples from the statistics domain in lower secondary mathematics to illustrate the textbook analysis method. Statistical education is a growing field of research and practice around the world (e.g., Braham & Ben-Zvi, 2017). This is also the case with the Japanese recent curriculum (MEXT, 2018a, 2018b). We focus on the chapter "sample survey" in the Grade 9 textbook (Fujii & Majima, 2021, 2022).

The Way of the Analysis

According to Table 1, we first identify the praxis block using textbooks. We then draw EM to identify the logos elements of MP. For drawing EM, we refer to some literature used for pre-service mathematics teacher education in Japan. We adopt the basic structure of statistical reasoning (BSSR; Kageyama, 2007) to explore EM (see Figure 1), because BSSR explains the entire structure and process of statistical reasoning, and the relationships between statistics and probability. As this model has been more or less cited and used in research and practice in Japan, it is possible to interpret it as one of "intermediate texts" affecting EDT to statistical knowledge to be taught in Japan.

Regarding DM in textbooks, we employ the investigative cycle (PPDAC; Wild & Pfannkuch, 1999) (see Figure 2). PPDAC describes the process for solving statistical problems. As PPDAC is introduced and described in the latest teaching guide of the Course of Study (MEXT, 2018a, 2018b), we consider this one of the "intermediate texts" affecting EDT as well. To better characterise the DP elements, we also refer to Teacher's Guidebooks (TG), which are supplementary materials of published textbooks.

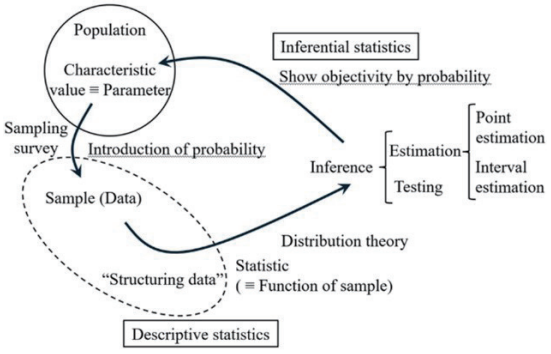


Figure 1. BSSR model (Kageyama, 2007, p.80) (translated by the authors)

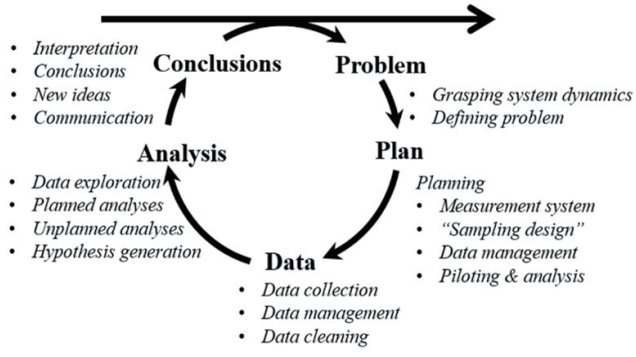


Figure 2. PPDAC cycle (Wild & Pfannkuch, 1999, p.226)

Table 2 summarises the results of the analysis. From the perspective of EM, the importance of random sampling can be interpreted by MΘ (relationship between statistical reasoning and probability). From the perspective of DM, teaching approaches are described Dθ that explains understanding

problems and planning the survey. Moreover, Dθ is described by DΘ that is the process of statistical problem solving. Thus, the logos block of MP and DP were described using the frameworks (BSSR, PPDAC) that provide some terminologies of MP and DP.

Table 2. Result of the praxeological analysis

T	**τ**	**θ**	**Θ**
MT1: What is the population and samples of the specific survey?	Mτ1: Identify the entire group of interest and the actual subjects examined	Mθ: Relationship between the samples and population	MΘ: Relationship between statistical reasoning and probability
MT2: What is the method for selecting samples in the population?	Mτ2: Select samples with random method		
DT: Enable students to understand the need for random sampling	Dτ: Have students make sure the sampling assumptions (purposes) and think about the selection methods	Dθ: Understanding problems and planning the survey	DΘ: The process of statistical problem solving

Discussion and Conclusion

In this paper, we proposed a method for mathematics textbooks in terms of MP and DP in a given institution (in particular, statistical knowledge to be taught in the Japanese textbooks). Although the praxis blocks are often visible in textbooks, the logos parts are rather implicit. To better explain the logos elements, we considered and explored "intermediate texts" produced in the process of EDT used to identify possible MP and DP in a given institution.

However, further research is needed to take into account the relationships between textbooks and classroom practices: How do teachers and students use textbooks in the class, and how does the class proceed with the textbooks? Within ATD, there is the potential for considering these issues. The first one is to analyse textbooks with the theory of *didactic moments* (Chevallard & Bosch, 2019). Didactic moments provide descriptions of generations and proceedings of MP and DP in study processes with textbooks and can be used for textbook analysis (Tonnesen, 2024). The second one is to construct the *reference epistemological/didactic model* (REM/RDM; Bosch & Gascón, 2006; Gascón & Nicolás, 2019a, 2019b). REM and RDM provide objective explanations based on the didactic transposition process and all institutions, including "taught knowledge" and "learned knowledge", while EM and DM describe the specific institution. One of the approaches to developing EM/DM is to consider the nature of "scholarly (statistical) knowledge" and the diversity and specificity of internal didactic transposition in the international educational context.

References

Bosch, M., & Gascón, J. (2006). Twenty-five years of the didactic transposition. *ICMI bulletin*, *58*, 51-65.

Braham, H. M., & Ben-Zvi, D. (2017). Students' emergent articulations of statistical models and modeling in making informal statistical inferences. *Statistics education research journal*, *16*(2), 116-143.

Chevallard, Y., & Bosch, M. (2019). A short (and somewhat subjective) glossary of the ATD. In M. Bosch, Y. Chevallard, F. J. García, & J. Monaghan (Eds.), *Working with the anthropological theory of the didactic in mathematics education: A comprehensive casebook* (pp.xviii-xxxvii). Routledge.

Chevallard, Y., & Bosch, M. (2020). Anthropological theory of the didactic (ATD). In S. Lerman (Eds.), *Encyclopedia of mathematics education* (pp.53-61). Springer.

Fan, L., Trouche, L., Qi, C., Rezat, S., & Visnovska, J. (2017). Topic study group no. 38: Research on resources (Textbooks, Learning Materials etc.). In G. Kaiser (Eds.), *Proceedings of the 13th international congress on mathematical education* (pp.561-564). Springer.

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: development status and directions. *ZDM mathematics education*, *45*, 633-646.

Fujii, T., & Majima, H (Eds.). (2021). 新しい数学*3*　教師用指導書　指導編（朱書） *[New mathematics 3: Teacher's guidebook, the instructional parts (vermilion annotation)]*. Tokyo Shoseki. (In Japanese)

Fujii, T., & Majima, H (Eds.). (2022). 新しい数学 *3 [New mathematics 3]*. Tokyo Shoseki. (In Japanese)

Gascón, J. (2024). Contributions of the anthropological theory of the didactic to the epistemological programme of research in mathematics education. *ZDM mathematics education*, *56*, 1319-1330.

Gascón, J., & Nicolás, P. (2019a). Research ends and teaching ends in the anthropological theory of the didactic. *For the learning of mathematics*, *39*(2), 42-47.

Gascón, J., & Nicolás, P. (2019b). What kind of results can be rationally justified in didactics?. In M. Bosch, Y. Chevallard, F. J. García, & J. Monaghan (Eds.), *Working with the anthropological theory of the didactic in mathematics education* (pp.3-11). Routledge.

Gascón, J., & Nicolás, P. (2022). Paradigm crisis in the step from tertiary to secondary mathematics education. *International journal of mathematical education in science and technology*, *53*(5), 1153-1169.

Kageyama, S. (2007). 教育統計に関する内容 [The content of educational statistics]. In R. Ikehata, S. Kageyama, & T. Shimomura (Eds.), これだけは知っておきたい教員のための数学Ⅱ―解析・統計・コンピューター *[Essential knowledge of mathematics II for teachers: Analysis, statistics and computer]* (pp.53-113). Baifukan. (In Japanese)

Ministry of Education, Culture, Sports, Science and Technology [MEXT]. (2018a). *The guide of course of study for elementary school: Mathematics*. Nichibun. (In Japanese)

Ministry of Education, Culture, Sports, Science and Technology [MEXT]. (2018b). *The guide of course of study for junior high school: Mathematics*. Nichibun. (In Japanese)

Takeuchi, H., & Shinno, Y. (2020). Comparing the lower secondary textbooks of Japan and England: a praxeological analysis of symmetry and transformations in geometry. *International journal of science and mathematics education*, *18*, 791-810.

Tonnesen, P. B. (2024). A comparative study of didactic moments in a first chapter on algebra in Danish and Japanese middle school textbooks. *Hiroshima journal of mathematics education*, *17*, 1-24.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the Book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Kluwer.

Wijayanti, D., & Winsløw, C. (2017). Mathematical practice in textbooks analysis: Praxeological reference models, the case of proportion. *Journal of research in mathematics education*, *6*(3), 307-330.

Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International statistical review*, *67*(3), 223-265.

TSG1-18

PREALGEBRAIC CONCEPTS IN SOLVING NUMBER SENTENCES: A TEXTBOOK ANALYSIS

Reisid May Sumbilon
Villanueva National High School, Philippines, reisidmay.sumbilon@deped.gov.ph

Maria Alva Aberin
Ateneo de Manila University, Philippines, maberin@ateneo.edu

The shift from arithmetic to algebra is widely recognized as a complex cognitive transition, often referred to as the "didactic cut" (Filloy & Rojano, 1989). Pillay, Wilss, and Boulton-Lewis (1998) proposed a sequential developmental model that bridges arithmetic skills and algebraic reasoning, highlighting the significance of a pre-algebraic stage. This study examined the presence and prevalence of pre-algebraic concepts in number sentence solutions within two widely used elementary mathematics textbook series in the Philippines. A content analysis of two textbooks (Textbook A and Textbook B) revealed that equivalence was the most frequently occurring pre-algebraic concept, whereas the use of inverses was notably scarce. Moreover, significant gaps were identified in how these concepts were presented within number sentence solutions, potentially leading to misconceptions and mathematical inaccuracies in student learning. These findings underscore the need for a curriculum review to enhance the integration of pre-algebraic concepts.

Keywords: pre-algebraic concepts, equivalence, textbook analysis

Introduction

Arithmetic is a branch of mathematics that deals with the study of numbers and the properties of operations on them. It is a branch of mathematics that helps in understanding things that happen on a day-to-day basis. In addition to this, the understanding of arithmetic is foundational to the development of symbolic algebra and that making sense of arithmetic is critical to students' development in algebra and the other areas of mathematics and statistics (Alton-Lee, 2003).

In the Philippines' K-12 curriculum, arithmetic is introduced to students from kindergarten to Grade 6 within the strand of the Number and Number Sense. This curriculum segment encompasses fundamental concepts such as numbers, properties, operations, estimation, and their practical applications. Research has underscored the significance of nurturing students' number sense, as it plays a crucial role in promoting algebraic thinking (Kaput, 2002). The development of algebraic thinking can be facilitated when number sense activities incorporate efficient strategies, capacity to think flexibly with numbers and operations, and a propensity for making informed mathematical judgments.

One activity aimed at cultivating number sense involves working with number sentences. Exposure to diverse types of number sentences can provide opportunities to develop number sense, one aspect of which is the knowledge of and facility with operations, which is the second component introduced in the Number Sense Framework (Mcintosh et al. ,1992). It includes the effect of oper-

ations as well as their properties and relationships between numbers. Pillay et. al (1998) set up a three-stage model of sequential development of knowledge for understanding algebraic concepts with pre-algebra as the second stage linking arithmetic and algebra. The pre-algebra concepts consist of recognition of the unknown or variable in equations (or number sentences), concatenation, and equivalence.

This study focused on analyzing the number sentences found in elementary textbooks. Research has shown the role textbooks play in the teaching and learning of Mathematics is important since textbooks are used to aid students with their learning process. Maruyama and Kurosaki's (2022) study on the impact of newly developed mathematics textbooks for second-grade students found an average improvement of 0.49 standard deviations in mathematics test scores over the course of one year. The findings highlight the effectiveness of well-structured, high-quality textbooks in enhancing student learning outcomes.

Moreover, textbooks make up the main tool teachers use in their teaching (McDuffie & Mather, 2007). These learning resources help teachers in deciding the content that must be taught, the pace and the timing of the teaching, recommending projects for the students, and deciding either directly or indirectly what is to be assessed (Gene et al, 2018). Furthermore, Valverde et al., (2002) considered textbooks and other organized resource materials as the 'potentially implemented' curriculum because these act as a bridge between intention and implementation. Exploring a study that examines the presence of pre-algebraic concepts in textbook discussions is significant. It can be instrumental in fostering the early development of algebraic thinking in mathematics education.

Statement of the Problem

This study aimed to determine whether the two series of Grade 1 to Grade 6 mathematics textbooks, labeled Textbook A and Textbook B, provide opportunities where pre-algebraic concepts. It focused on the discussion given in the textbooks dealing with number sentences. Specifically, this study intended to answer the question: How are the following pre-algebraic concepts presented and discussed in the solutions of number sentences of the textbook series? (a) properties of operations; (b) use of inverse; (c) equivalence; (d) use of unknown or variable and, (e) use of concatenation.

Methodology

This study analyzed two textbook series covering Grade 1 to Grade 6, totaling 12 textbooks from reputable publishing houses. The selection criteria for these series included their commercial availability, production by well-known publishing houses, and alignment with the K-12 curriculum.

In the textbook discussion section, the researcher checked how number sentences are answered and the methods used in presenting the textbook discussion. These are the pre-algebraic concepts investigated: (1) properties of operations, (2) use of inverses (additive inverse and multiplicative inverse), (3) equivalence, (4) use of concatenation, and (5) use of unknown or variable.

Results and Discussion

Based on the data gathered, the following findings were obtained. Textbook A had 634 textbook discussions, while Textbook B had 669 textbook discussions. A discussion in the textbook includes the solution of a number sentence. It must be noted that more than one pre-algebraic concept can be observed in one textbook discussion. Across both textbook series, students encounter pre-algebraic concepts in fewer than 25% of the number sentence solution discussions.

Table 1. Frequency of textbook discussion that has at least one pre-algebraic concept and no concept in both series of textbooks

	Textbook A	Textbook B
No Pre-algebraic Concept	492 (77.60%)	534 (79.82%)
At least One Pre-Algebraic Concept	142 (22.40%)	135 (20.18%)
Total	634	669

The presence of the following pre-algebraic concepts was examined in the textbook discussion section of the textbook: (a) properties of operations, (b) use of inverses, (c) equivalence, (d) use of concatenation, and (e) use of unknown or variable.

Table 2. Frequency and percentage of pre-algebraic concepts in both series of textbooks

Concepts	Textbook A	Textbook B
Property of Operations	28 (19.71%)	57 (42.22%)
Use of Inverse	13 (9.15%)	6 (4.44%)
Equivalence	128 (90.14%)	88 (65.19%)
Use of unknown and variable	62 (43.66%)	33 (24.44%)
Use of concatenation	19 (13.38%)	10 (7.41%)

The following results represent the total number of discussions with the occurrence of each concept in relation to the total number of discussions that has a pre-algebraic concept. In Textbook A, it is observed that the highest percentage is 90.14% for the Concept 3, which is equivalence. Equivalence emerges as the most prevalent pre-algebraic concept, likely due to its fundamental role in understanding different number representations and facilitating comparisons between expressions.

There are different themes found related to the presence of pre-algebraic concepts. Discussions at the lower grade levels often rely on pictures to aid in conceptual visualization. In contrast, narrative explanations were used to enhance student comprehension of the problems at the upper grade levels. Additionally, only a limited number of examples include narrative explanations detailing their solutions. It is observed that most of the discussions in the textbook do not provide alternative strategies in solving number sentences. The textbook series make use of the initial letter of the missing quantity and mostly use N as a variable to represent a number. The emphasis on utilizing place value for facilitating simpler calculations is also observed. Finally, it has also been noted that certain discussions contain misleading solutions, and some exercises are mathematically incorrect.

Implications to Teaching Practice

The findings of the analysis of the textbook have important implications for instruction in the classroom, especially concerning the development of students' initial algebraic thinking. The low level of integration of the important pre-algebraic ideas—properties of operations, use of inverses, equivalence, concatenation, and variables, each of which occurred in fewer than 20% of the items analyzed—suggests that teachers need to take an active role in augmenting textbook content. Instruction does not have to be limited to that taught in textbooks; instead, teachers are taught to use other methods and materials that directly construct these fundamental concepts. For instance, the underrepresentation of inverse operations makes it necessary for teachers to present this concept by using concrete activities and problem-solving experiences that construct students' preparation for formal equation solving. Similarly, while equivalence was the most discussed pre-algebraic idea, its depiction remained under 25%, indicating that more focus is required during teaching. Teachers can facilitate the understanding of this idea by using visual models such as balance scales and number sentences, to enhance students' understanding of equality.

Teachers should likewise critically examine their textbooks' contents and make decisions based on the information to address any conceptual lapses that will impede students' learning. The developmental change observed in the use of visual aids—from concrete pictures in lower grade levels to increasingly abstract representations in higher levels—also has instructional planning implications. Teachers need to make sure that the visual supports they employ are developmentally appropriate, starting with concrete images and progressing to more abstract representations as students move along. In general, these findings highlight the importance of ongoing professional development to assist teachers in providing effective early algebra teaching. Providing teachers with a greater understanding of pre-algebraic ideas and the instructional materials with which to tackle them can facilitate a smoother progression from arithmetic to algebra, thus enhancing the mathematical competency of students.

Conclusion

This study examined the extent to which primary textbooks in the Philippines integrate pre-algebraic concepts in solution explanations, aiming to assess their role in supporting the transition from arithmetic to algebra. Findings reveal that most key pre-algebraic concepts—such as properties of operations, inverses, concatenation, and variables—are underrepresented, with only equivalence appearing slightly more frequently, yet still below 25%. Differences between Textbooks A and B suggest that students' exposure to these concepts may vary depending on the resource used. The limited emphasis on inverse operations, despite their importance for later algebraic learning, points to a need for improved textbook content. Additionally, the study notes a shift in the use of visual aids, with simpler illustrations in lower grades and more complex visuals in higher grades, reflecting an adaptation to students' developing cognitive abilities.

References

Alton-Lee, A., New Zealand, & Ministry of Education. (2003). Quality teaching for diverse students in schooling: best evidence synthesis. *Wellington, New Zealand: Ministry of Education.*

Filloy, E., & Rojano, T. (1989). Solving equations: The transition from arithmetic to algebra. *For the Learning of Mathematics, 9*(2), 19-25.

Gene, K., Zacharos, K., Lavidas, K., & Koustourakis, G. (2018). An Analysis of School Mathematics Textbooks in Terms of Their Pedagogical Orientation. *Open Journal for Educational Research*, *2*, 1–18. https://doi.org/10.32591/coas.ojer.0201.01001g

Maruyama, T. & Kurosaki, T. (2022). Developing Textbooks to Improve Math Learning in Primary Education: Empirical Evidence from El Salvador. *Economic Development and Cultural Change. 72(2).* https://doi.org/10.1086/721768

McDuffie, A. M. R., & Mather, M. (2006). Reification of instructional materials as part of the process of developing problem-based practices in mathematics education. *Teachers and Teaching*, *12*(4), 435–459. https://doi.org/10.1080/1345060060064428

Mcintosh, A., Reys, B., & Reys, R. (1992). A proposed framework for examining basic number sense. For the Learning of Mathematics, 12, 2–8.

Pillay, H., Wilss, L., & Boulton-Lewis, G. (1998). Sequential development of algebra knowledge: A cognitive analysis. *Mathematics Education Research Journal*, *10*(2), 87–102. https://doi.org/10.1007/BF03217344

Valverde, G., Bianchi, L. J., Wolfe, R., Schmidt, W. H., & Houang, R. T. (2002). *According to the Book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Springer Netherlands. https://doi.org/10.1007/978-94-007-0844-0

TSG1-19

THE HIGHER-ORDER THINKING SKILLS (HOTS) CONTENT ANALYSIS ON MATHEMATICS TEXTBOOKS OF SENIOR HIGH SCHOOL

Haili Liang
Beijing Normal University, China, hely_l@163.com

Chunxia Qi
Beijing Normal University, China, qichxia@126.com

Guihua Huang
Beijing Normal University, China, 592802835@qq.com

Higher-order thinking skills (HOTS) are skills demanded in the 21st century. This research aimed to analyse the HOTS content in the Mathematics Textbook of Senior High School. Based on a three-dimensional framework (i.e., Cognitive Level, Mathematical Ability, Problem Situation) adapted from Bai et al. (2023), the HOTS content analysis of the textbook was conducted. The source of the data used in this study was two senior high school mathematics textbooks, Compulsory Version I & II, released by People's Education Press in 2019. In addition, the study examined the HOTS content in relation to various content areas. When contrasted with the curriculum standard in China, the results found that there is an alignment regarding the HOTS content between the mathematics curriculum standards and textbooks. Last but not least, contributions and suggestions for future studies were made.

Keywords: higher-order thinking skills, mathematics textbook, high school, China

Introduction

This Higher-order thinking skills (HOTS), which are "a non-algorithmic, complex mode of thinking that often generates multiple solutions", are essential for tackling the difficulties and challenges of the 21st century (Miri et al., 2007). In the field of mathematics, HOTS enhancement has also become one of the priorities of mathematics education (Budiman & Jailani, 2014). Students with higher HOTS in mathematics tend to be more proficient in problem-solving strategies, more confident in learning mathematics, and more skillful in addressing non-routine problems (Butkowski, 1994). In the case of China, the emphasis on thinking skills is also a major agenda in the Mathematics Curriculum Standards for General High Schools (MOE, 2020), which highlights that students must improve cognitive skills to analyse facts, organize ideas, make generalizations, build conclusions, evaluate arguments and solve problems creatively.

The textbook serves as a prominent learning medium for both teachers and students, making it a valuable resource for cultivating students' HOTS in class (Pratama & Retnawati, 2018). The selection of teaching materials and tactics by teachers is significantly influenced by the choice of textbooks utilized in the classroom, hence, as indicated by several studies (e.g., Singh et al., 2020; Törnroos., 2005), there is a significant correlation between the choice of textbooks and students' academic performance. According to Pratama & Retnawati (2018), it could be argued that textbooks

with more HOTS content are more likely to effectively develop and transmit HOTS to students. Therefore, it is essential to conduct a content analysis of HOTS in mathematics textbooks to find out whether the HOTS requirements in curriculum standards and textbooks are aligned, and to assist teachers in selecting and utilizing the textbook content to cultivate students' HOTS. In response to this research trend, the purpose of the present study was to investigate how the HOTS content is distributed in senior high school mathematics textbooks in China.

Theoretical Framework

Based on Bloom's Taxonomy of Educational Objectives, Bai et al. (2023) developed a model for evaluating the HOTS of mathematics (as shown in Figure 1). According to Fig. 1, the horizontal direction represents the increasing complexity of cognitive processes in the last three levels of the Revised Bloom's Taxonomy. Under *Analyse*, students are required to determine, organize, and assign attributes. Under *Evaluate*, students are required to critique or examine a particular mathematical problem. Under *Create*, students are required to generate, plan, or create a piece of work, such as an idea, a new solution, or a product. The vertical direction, on the other hand, illustrates the dimension of mathematical ability that students need to show when solving mathematical problems requiring HOTS, and it includes *Abstraction* and *Generalization*, *Reasoning* and *Proof*, and *Problem-Solving*. The direction of the inside of the paper represents the third dimension of the Problem Situation, which was categorized into two types: intramathematical situations and realistic situations. The present study adopted Bai et al.'s (2023) framework and considered that the HOTS content should include subskills of the last three levels of the revised Bloom's Taxonomy of Educational Objectives (Mitana et al., 2018), in each cognitive level, the HOTS content was coded according to two other dimensions, i.e., the *Mathematical Ability* and the *Problem Situation*.

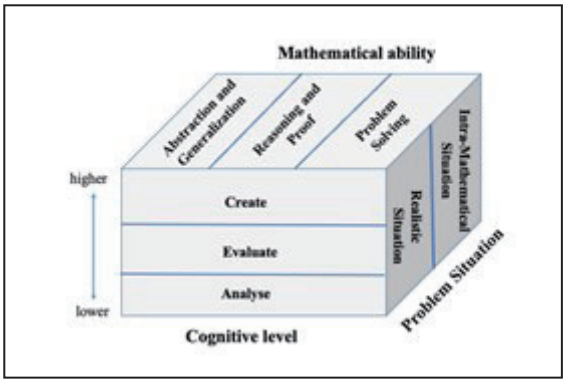


Figure 1. The three-dimensional framework for measuring HOTS in mathematics (Bai et al., 2023).

Methods

Textbook Selection

The textbooks we selected in the current study were the Compulsory Version I & II published in 2019 by People's Education Press, considering that they reflect the advanced rationale of new national mathematics curriculum reform, and most of the subject content inside these textbooks is tested on the high school entrance exam in China. In our analysis, we examined the HOTS opportunities that were directly provided by these two 10th-grade textbooks rather than the implementation of textbooks in the classroom.

Unit of Analysis

Both the narratives and exercises of the mathematics textbooks would be coded. The number of each HOTS category in textbooks was counted separately. We met the situation that one exercise contained several tasks, then we counted the number of its separate tasks (Zhang & Qi, 2019). For example, if one exercise comprised tasks (1), (2), and (3), this exercise counted as 3. The unit of analysis for narratives was one sentence or more sentences that included at least one mathematical term (e.g., Davis et al., 2014). For those narratives that included several sentences but only contained one purpose or single theme, we viewed this situation as an analysis unit and marked one code only.

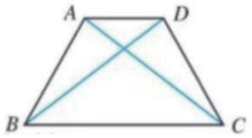
Data Analysis

Two mathematics education doctoral candidates independently coded all 2567 textbook items utilizing the HOTS coding framework (see Table 1). If an item does not belong to HOTS, it will be coded as C0. The coding example is shown in Table 2. The intercoder reliability coefficient was measured, and Cohen's Kappa was 0.81 (>0.7), demonstrating good reliability. After completing this separate coding, the two researchers reviewed the messages with conflicting codes and established agreement on all of them before conducting further analysis.

Table 1. Coding scheme of mathematics textbooks' HOTS content analysis.

		Cognitive Level		
		Analyse（C1）	Evaluate（C2）	Create（C3）
Mathematical ability	Abstraction & Generalization（A1）			
	Reasoning & Proof（A2）			
	Problem-Solving（A3）			
Problem Situation	Intra-mathematical（B1）			
	Realistic（B2）			

Table 2. Coding examples of mathematics textbooks' HOTS content analysis.

Item Examples	Codes
Example 1: Please prove: As shown in the figure, the necessary and sufficient condition for trapezoid ABCD to be an isosceles trapezoid is that AC = BD : 	A2B1C0
Example 2: Summer is the peak period for electricity consumption, especially at night. To ensure that residents can use electricity for air conditioning, the power department has to implement power cuts for enterprises and institutions. However, after midnight, there is a surplus of electricity. As a result, daily electricity usage exhibits periodic fluctuations. In order to ensure electricity supply for residents, the power department proposed the idea of "shifting peak to valley," which involves raising electricity prices during the peak period at night and lowering prices during the low-demand period in the early morning. This encourages businesses and institutions to use electricity during the low-demand period. Please investigate the daily electricity usage in your area and formulate an electricity pricing plan based on the "shifting peak to valley" concept.	A3B2C3

Findings and Discussions

The result of this study shows that the HOTS content took 50.95% across the two textbooks. Specifically, in the *Cognitive Level* dimension, the *Analyse* level encompassed the most amount of the HOTS content (78.98%), followed by the Evaluate level (17.66%), while the *Create* level had the least amount of the content (3.36%); in the *Mathematical Ability* dimension, the HOTS content related with *Problem-Solving* emerged most frequently (84.10%), followed by *Reasoning and Proof* (12.54%), while *Abstract & Generalization* exhibited the least frequency (3.36%); in the *Problem Situation* dimension, the inclusion of the HOTS content was predominantly set in Intra-mathematical situation (65.29%), with 34.71% set in *Realistic* situation. It is noteworthy to mention that, the HOTS content that at the *Create* level was all presented in a realistic situation.

Furthermore, the HOTS content in the textbooks varied across content areas, with 15.67% of *Preparatory Knowledge*, 36.68% of *Functions*, 26.45% of *Geometry & Algebra*, and 18.2% of *Probability & Statistics*, respectively. Table 3 shows the comparison between the two textbooks with Mathematics Curriculum Standards for General High Schools (MOE, 2017) regarding the HOTS content. As can be seen from the standardized results in Table 3, Porter's alignment indices were 0.877>0.7, indicating that mathematics textbooks aligned with the standards' HOTS content across different areas. Also notable is the fact that the proportion of the HOTS content in the textbooks exceeds the standards in the area of *Function*.

The present paper is the first study to analyse the HOTS content in mathematics textbooks of senior high schools in China. The result revealed Chinese students' learning opportunities of HOTS to

some extent and helped examine the development of the alignment between textbooks and curriculum standards based on the HOTS content analysis. Further studies on the alignment among curriculum standards, textbooks, classroom instruction, assessments, and professional development for teachers regarding HOTS should be conducted.

Table 3. Comparison of the HOTS content between curriculum standards and textbooks.

	HOTS Content			
	Preparatory Knowledge	Functions	Geometry & Algebra	Probability & Statistics
Curriculum Standards	0.0556	0.0556	0.1667	0.1389
Textbooks	0.0347	0.2022	0.1348	0.0927
Alignment indices	0.877			

References

Bai, Y., Liang, H., Qi, C., & Zuo, S. (2023). An Assessment of Eighth Graders' Mathematics Higher Order Thinking Skills in the Chinese Context. *Canadian Journal of Science, Mathematics and Technology Education*, 1-18.

Budiman, A., & Jailani, J. (2014). Developing an assessment instrument of higher order thinking skill (HOTS) in mathematics for junior high school grade VIII semester 1. *Jurnal Riset Pendidikan Matematika, 1*(2), 139-151.

Butkowski, J. (1994). *Improving Student Higher-Order Thinking Skills in Mathematics* [Master thesis, Saint Xavier University-IRI]. Chicago, Illinois, USA.

Davis, J. D., Smith, D. O., Roy, A. R., & Bilgic, Y. K. (2014). Reasoning-and-proving in algebra: The case of two reform-oriented US textbooks. *International Journal of Educational Research, 64*,92–106.

Ministry of Education of the People's Republic of China. (2020). *Mathematics Curriculum Standards for Senior Secondary Schools (2017 version 2020 revised)*. People's Education Press.

Miri, B., David, B. C., & Uri, Z. (2007). Purposely teaching for the promotion of higher-order thinking skills: A case of critical thinking. *Research in science education, 37*, 353-369.

Mitana, J. M. V., Muwagga, A. M., & Ssempala, C. (2018). Assessment of Higher Order Thinking Skills: A Case of Uganda Primary Leaving Examinations. *African Educational Research Journal, 6*(4), 240-249.

Pratama, G. S., & Retnawati, H. (2018, September). Urgency of higher order thinking skills (HOTS) content analysis in mathematics textbook. In *Journal of Physics: Conference Series* (Vol. 1097, No. 1, p. 012147). IOP Publishing.

Singh, P., Yusoff, N. M., & Hoon, T. S. (2020). Content Analysis of Primary School Mathematics Textbooks and Its Relationship with Pupils Achievement. *Asian Journal of University Education, 16*(2), 15-25.

Törnroos, J. (2005). Mathematics textbooks, opportunity to learn and student achievement. *Studies in educational evaluation, 31*(4), 315-327.

Zhang, D., & Qi, C. (2019). Reasoning and proof in eighth-grade mathematics textbooks in China. *International Journal of Educational Research, 98*, 77-90.

TSG1-20

DEVELOPING AND VALIDATING IB-BASED MATHEMATICS ASSESSMENT TASKS: FOCUSING ON STUDENT THINKING AND RELIABILITY

Yoonjoo Cho
Seoul National University, Korea, yoonjoo.cho@snu.ac.kr

Oh Nam Kwon
Seoul National University, Korea, onkwon@snu.ac.kr

The IB curriculum is based on a criterion-referenced approach, employing extended-response assessments to evaluate learning and cognitive competencies. This study developed mathematics assessment tasks aligned with both the IB curriculum and the 2022 revised Korean curriculum and administered them to secondary school students in Korea. The results indicate that the assessment approach of the IB curriculum effectively highlights diverse problem-solving strategies and variations in conceptual understanding. A structured rater training process was implemented, resulting in an ICC of 0.75 or higher across all tasks, thereby demonstrating the reliability of the assessment. This study explores both the validity and challenges of implementing assessment methods from the IB curriculum within the Korean curriculum and offers suggestions for improving the assessment system in Korea.

Keywords: IB Curriculum, Extended-response assessment, Reliability, Assessment Task Development

Introduction

The International Baccalaureate (IB) curriculum is implemented in over 5,500 schools across 159 countries. Recently, its adoption has expanded in Korea, with approximately 60 schools expected to implement the IB curriculum by 2027 (Lim, 2023). The IB curriculum employs a criterion-referenced assessment approach, utilizing extended-response assessments. Although the 2022 revised Korean curriculum emphasizes an achievement-based system aligned with criterion-referenced assessment, it continues to incorporate norm-referenced methods, such as the five-level school grading system. The norm-referenced assessment system in Korea has predominantly relied on multiple-choice assessments, which, while advantageous for ensuring reliability, are limited in assessing students' creative problem-solving competencies and higher-order thinking processes. This study develops IB-based mathematics assessment tasks in an extended-response format and adapts them to the Korean context. By administering these tasks to secondary school students and analyzing their responses, this study aims to explore how IB-based assessments can contribute to promoting competency-based learning within the Korean education system. The research question is as follows: *How can IB-based extended-response assessments be applied in Korea to measure the depth of mathematical thinking while ensuring reliability?*

Related Literature: IB Curriculum

The IB curriculum has recently gained attention in Korea as an alternative assessment system, offering distinctive features compared to traditional Korean assessment methods. First, the IB curriculum adopts a criterion-referenced assessment system, enabling students to achieve the highest possible score if they meet predefined standards. Unlike Korean multiple-choice-based assessments, which emphasize rote memorization and standardized problem-solving, the assessment of the IB curriculum promotes critical and creative thinking rather than the mere identification of correct answers (Song et al., 2022). Second, the IB curriculum employs a multi-layered rating system, where student responses undergo multiple rounds of grading by examiners at different levels, from general to principal examiners. This structured process mitigates individual bias and ensures consistency through continuous monitoring (Kim & Ahn, 2020). Third, the IB curriculum emphasizes coherence between learning objectives, curriculum, and assessment. The IBO guide clearly outlines assessment criteria and achievement levels, providing teachers with consistent guidelines for designing and conducting assessments (Oh et al., 2021). These features position the IB curriculum as a model for fostering critical and creative thinking.

Methods

This study developed assessment tasks based on the mathematical competencies outlined in the 2022 revised Korean curriculum (problem-solving, reasoning, communication, connections, and information processing) and the assessment criteria of the IB curriculum (knowing and understanding, investigating patterns, communicating, and applying mathematics in real-life contexts). The tasks were developed in an extended-response format to encourage students' logical reasoning and structured problem-solving. To ensure alignment with both curricula, the study adapted tasks from the IB MYP eAssessment and the IB DP external assessments to the Korean context. A structured rubric, based on the IB markscheme, was established to ensure evaluation consistency. An example of the assessment tasks is as follows. The study involved 164 students (119 students from middle schools and 45 students from high schools) across four middle schools and two high schools. After excluding non-responses, a random sample of 80 middle school and 38 high school responses was analyzed.

Table 1. An example of assessment tasks for first-year middle school students

Task 4-3	Answer the following questions. (1) Write one sentence that includes a positive value and one that includes a negative value from real life. (2) Underline the positive and negative values in your sentence, and represent them using the + and - symbols.	
Marking Criteria	• Any reasonable answer that expresses the meaning of a positive number • Identifying a positive number and representing it using the + symbol • Any reasonable answer that expresses the meaning of a negative number • Identifying a negative number and representing it using the - symbol	A1 A1 A1 A1
	Note: If a response includes both the negative sign (-) and the word "loss" together, such as "-5 million loss", do not award the mark A1.	

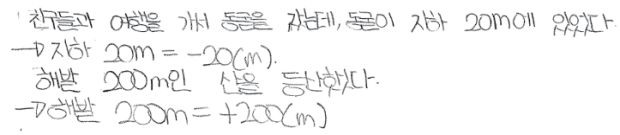
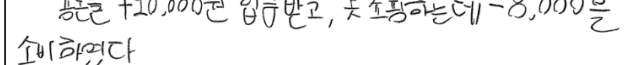
To ensure the reliability, a structured rater training process was implemented. A team of six experienced teachers (three for middle school and three for high school) participated in the rating process. A pre-rating workshop was conducted to ensure that raters clearly understood the study objectives, the structure of the assessment task, and the rubric. Raters individually scored three sample responses, followed by a group discussion to standardize scoring practices. After the final rubrics were established, all responses were rated. Each response was independently rated by three raters, and Intraclass Correlation Coefficient (ICC) analysis was conducted to verify reliability.

Findings

Implementation of the Assessment Tasks

The findings demonstrate that the IB-based assessment tasks enabled a more detailed evaluation of conceptual understanding and problem-solving processes. Student responses revealed various aspects of learning and cognitive competencies, including logical reasoning and real-life application. As presented in Table 1, students exhibited varying levels of understanding regarding positive and negative numbers. As shown in the student response in Table 2, a student correctly connected mathematical concepts to real-life contexts by representing elevation above sea level as a positive number and depth below ground as a negative number. In contrast, other students displayed misunderstandings by writing expressions such as "spent -$6", confusing the negative sign as both a numerical value and an operation, resulting in an improper mathematical representation. These variations in thinking and approaches are difficult to assess through simple calculations or multiple-choice questions. Nevertheless, the IB-based extended-response assessment offers a more in-depth evaluation of reasoning processes and problem-solving approaches, highlighting its potential advantages in assessing mathematical understanding.

Table 2. Examples of correct and incorrect answers to Task 4-3

Correct Answer	Incorrect Answer
 The cave is located 20 meters below ground. → Underground 20m = -20 (m) I hiked a mountain with an elevation of 200 meters above sea level. → Elevation 200m = +200 (m)	 I received +$10 as an allowance and spent -$6 on clothes shopping.

Reliability in the Assessment Tasks

Ensuring reliability in extended-response assessments is more challenging than in multiple-choice assessments, as they are influenced by raters' expertise and their interpretation of the rubric. Because of these challenges, multiple-choice assessments have remained the dominant method in Korean secondary schools. However, given the advantages of extended-response assessments in capturing diverse problem-solving approaches, the Korean metropolitan and provincial offices of education are increasingly promoting the use of extended-response assessments, recommending

that they constitute at least 20% to 40% of total assessments. Moreover, continuous teacher training programs have been conducted to enhance fairness in evaluation. To verify the reliability of the assessment tasks, ICC analysis was conducted using the ICC function in the psych package of R 4.4.1. The analysis was performed under the assumption that both subjects and raters were randomly selected. As ICC is widely regarded as the most suitable measure for generalizing results in such cases, it was used to assess inter-rater reliability. As presented in Table 3, the ICC analysis confirmed that all tasks achieved an ICC value of 0.75 or higher, with most tasks exceeding 0.85, indicating a very high level of reliability. An ICC value of 0.75 or above is generally regarded as an indicator of good reliability, indicating that the assessment tasks developed in this study can serve as a reliable evaluation method. To ensure reliability, three raters participated in a structured rater training process, including extensive pre-rating workshops. A detailed rubric was also designed to minimize subjectivity and enhance rating consistency. These results highlight the importance of well-defined assessment criteria and structured rater training in ensuring fairness and consistency in extended-response assessments.

Table 3. Intraclass correlation coefficient (ICC) analysis for all tasks

Middle School			High School		
Task	Score	ICC	Task	Score	ICC
1	2	.95	1-1	3	.98
2-1	4	.99	1-2	3	.93
2-2	4	.98	2-1	5	.91
3-1	1	.90	2-2	3	.88
3-2	2	.94	3-1	3	.87
3-3	5	.92	3-2	7	.95
4-1	2	.77	4-1	5	.96
4-2	4	.85	4-2	7	.95
4-3	4	.89			
5-1	4	.86			
5-2	3	.95			
5-3	4	.89			
5-4	1	.90			

Conclusion

The IB curriculum ensures assessment reliability through structured rater training and well-defined assessment criteria. This approach indicates that IB-based extended-response assessments, when implemented within the Korean curriculum, can maintain reliability and effectively evaluate students' problem-solving strategies and conceptual understanding. However, the complexity of rating extended-response assessments may increase teachers' workload. Successful implementation of the IB-based assessment approach requires continuous teacher training and a national rater training system. Future research can prioritize longitudinal studies to examine the long-term impact of IB-based assessments on student achievement and cognitive development.

References

International Baccalaureate Organization (2019a). *Mathematics: analysis and approaches guide.* IBO.

International Baccalaureate Organization (2019b). *Mathematics: applications and interpretation guide.* IBO.

International Baccalaureate Organization (2020). *Middle Years Programme Mathematics guide.* IBO.

Kim, S. H., & Ahn, S. I. (2020). Ways to Settle Process-focused Assessment Through Analysis of the IB MYP and DP Mathematical Assessment System. *Journal of Educational Research in Mathematics, 30*(3), 445-463.

Lim, Y. N. (2023). *The current status and challenges of introducing the International Baccalaureate (IB) in Korea.* Korea Educational Development Institute, Education Policy Network, Educational Issues Report.

Ministry of Education. (2022). *Mathematics curriculum.* Proclamation of the Ministry of Education # 2022-33 [Annex 8]. Authors.

Oh, K. H., Lee, C. S., Lee, K. W., & Kwon, O. N. (2021). A Study on Coherence in the Structure of IB DP Mathematics Curriculum Documents. *Communications of Mathematical Education, 35*(1), 75-96.

Song, J. W., Kang, H. S., Kwon, O. N., Kwon, O. H., Kim, H. J., So, Y. S., Lee, H. J., Kim, S. Y., Kim, E. S., & Han, C. R. (2022). *Research on improving extended-response-based assessment and university entrance exams in the transition to a future-oriented education system.* Daegu Metropolitan Office of Education.

Topic Study Groups 1
(Day 4)

TSG1-21

FROM COUNTABLE TO UNCOUNTABLE: DEVELOPING TEACHING MATERIALS FOR IRRATIONAL NUMBERS

Toshikazu Ikeda
Yokohama National University, Japan, toshi@ynu.ac.jp

Noriaki Inaguma
Shizuoka University Faculty of Education Attached Shimada Junior High School, Japan, inaguma.noriaki@shizuoka.ac.jp

Makoto Suzuki
Tokoha University, Japan, m-suzuki@sz.tokoha-u.ac.jp

This study aimed to explore how the Euclidean Algorithm, through concrete, geometric, and symbolic operations, can help students understand the uncountable nature of irrational numbers. The key research question is: How can a multi-faceted, exploratory approach using concrete, geometric, and symbolic methods be integrated into teaching materials to support students in engaging with the transition from the countable to the uncountable world? The study found that the following process is effective in deepening students' exploratory activities. Specifically, the material exploring the common unit between 1 and √2 prompted students to inquire—while revising their reasoning repeatedly—whether it is possible to tile a rectangle with squares, thereby raising the issue of irrational numbers. Key student insights included predicting the feasibility of tiling for rational versus irrational numbers, realizing that proving the impossibility of tiling for irrationals requires demonstrating that the operation can repeat indefinitely, and understanding that this repetition depends on the existence of similar rectangles.

Keywords: Euclidean Algorithm, irrational number, common unit, tiling, concrete operation

Background of the Study

Experiencing activities through which mathematics evolves and develops is a fundamental principle in mathematics education (Freudenthal, 1973, 1991). This principle is particularly relevant when extending the concept of numbers. Starting with counting quantities and progressing to lengths, decimals, and fractions introduces students to the world of countable numbers. From there, a curriculum should guide students to encounter arbitrary lengths, ultimately leading to the exploration of uncountable numbers. In the Japanese curriculum, Euclid's algorithm is typically introduced in Grade 11 as part of number theory (MEXT, 2018). However, this study explores the possibility of introducing it earlier, in Grade 9, to provide students with a more direct experience of the uncountable nature of irrational numbers. While considerable attention has been given to the early stages of students' understanding of irrational numbers, including the exploration of geometric representations and the concept of incommensurability (Fischbein, Jehiam, & Cohen, 1995; Sirotic & Zazkis, 2007; Voskoglou & Kosyvas, 2013; Womack, 2021; Rottoli & Riva, 2021), limited research has focused on how students engage in a multifaceted, exploratory approach to understanding irra-

tional numbers, especially with a focus on concrete operation. The key research question is: How can a multi-faceted, exploratory approach using concrete, geometric, and symbolic methods be integrated into teaching materials to help students engage with the transition from the countable to the uncountable world?

Intended Curriculum Sequences: From Countable to Uncountable

When considering mathematics as a dynamic activity, it is essential to think about curriculum sequences that guide the extension of numbers. This involves not just abstract concepts but also their practical application in a structured and progressive way, leading students from counting discrete quantities to understanding fractions, decimals, and ultimately irrational numbers. The act of counting depends on how we define "1," which, in turn, affects how quantities are calculated. Once students master counting, they begin to explore whether this method can be applied to other contexts, such as measuring length. By treating objects like pencils as "1," they can express length in terms of these units, extending counting to measurement. Challenges arise when fractional units are involved. While counting objects works smoothly, measuring length becomes more complicated with fractional parts. This challenge led to the development of fractions and decimals. With these, addition and subtraction become possible by finding a common unit, allowing consistent counting ("1, 2, 3...") and arithmetic. This idea extends to ratios, such as the lengths of the sides of a rectangle, which can be expressed in fractions. However, when lengths cannot be expressed as fractions or decimals, it becomes impossible to find a common unit for counting. In such cases, a new representation, such as $\sqrt{2}$, is introduced. This challenges the assumption that all ratios can be expressed as integers by defining a suitable "1." Without a common unit, arithmetic operations such as addition or subtraction cannot be performed. Additionally, since it is impossible to find a unit square that tiles a rectangle whose length or width is an irrational number, it becomes impossible to consider the area in the definition where area is counted as the number of unit squares. This exploration of countability versus uncountability forms the basis for understanding irrational numbers. The goal of this research is to develop teaching materials that enable students to experience the concept of uncountability firsthand, helping them grasp the distinction between countable and uncountable sets.

Focus on Developing Teaching Materials by Interconnecting Three Worlds

To enable the curriculum sequence described earlier, it is essential to begin with concrete operations and gradually refine them through geometric operations, eventually progressing to symbolic operations. At the same time, it is also crucial to encourage students to reverse the process when necessary, reducing symbolic operations back to geometric or concrete operations. Through these activities, students will develop a tangible understanding of both the countable and uncountable worlds. This paper focuses on the development of two teaching materials to facilitate this sequence for 9th-grade junior high school students. The first is the introduction of Euclid's algorithm, and the second is a teaching material that explores whether it is possible to tile a rectangle with a ratio of 1 to $\sqrt{2}$ using squares, in the context of square roots. The experimental teaching was conducted with 9th-grade students by one of the authors in 2024. In the experimental lesson, while the teacher presented the questions to be considered, the ideas and insights were left to the students, with the lesson unfolding in response to their thoughts. By focusing on the key ideas that the students brought out, the central issue became whether the students could reach the conclusion that, when the ratio of rectangle is an irrational number, there is no square that can tile the rectangle.

Material to introduce Euclidean Algorithm

For the introduction of Euclid's algorithm, we propose a teaching material in which students learn how to tile a rectangle with the largest possible squares using a compass, without using a ruler. In the experimental teaching, the teacher first presented a narrow rectangle, as shown in Figure 1 (left), and asked the students to predict the ratio of the length to the width. The teacher encouraged them to think about how they could check the ratio without using a ruler. The key idea was to focus students' attention on how many squares, each with a side equal to the shorter length of the rectangle, could fit along the length. This activity introduces the foundational concept of Euclidean Algorithm. Students would then check whether three squares or four squares fit along the length. In this case, the ratio was found to be 1:3 (see the center rectangle in Fig. 1). Next, the teacher guided students to apply the same reasoning to new rectangles that could not be solved with just one operation, encouraging them to apply the previous method in a similar way. The students performed the concrete operation twice and, using the compass, derived the solution as 3:4, as shown in Figure 1 (right). The teacher then asked students to reflect on the two methods and identify the essence of the process. They confirmed that the essential step was to repeatedly divide the longer side of the rectangle by the shorter side, using squares with sides equal to the shorter length of the rectangle.

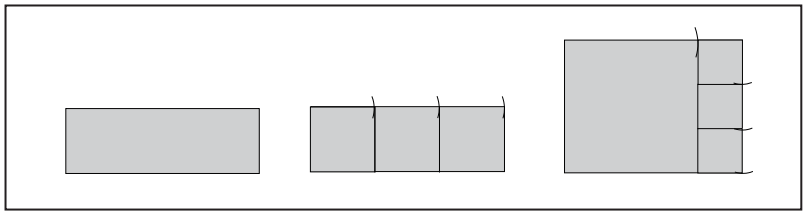


Figure 1. Operational method of euclidean algorithm: from simple to slightly extended cases material to inquire the common unit between 1 and the square root of 2

Next, the teacher asked the students whether it is possible to tile any rectangle with squares. Some students predicted that if the ratio of the rectangle is a rational number, tiling would be possible, and if the ratio is an irrational number, tiling might be impossible. However, as some students disagreed with this prediction, it became an issue for students to clarify. After confirming that it is possible to tile rectangles with ratios expressed as fractions (i.e., by finding a common unit that makes both lengths countable in terms of a single integer ratio), the teacher directed the students to consider the case of a rectangle with a ratio of 1 to $\sqrt{2}$. Using the operational method of Euclidean Algorithm—starting with one square, then two squares, and finally the remaining rectangle—students observed that the original rectangle and the final remaining rectangle share similar properties, as shown in Fig. 2 (left). They concluded that if the similarity can be proven, and the iteration continues indefinitely, it is impossible to find a square that will tile the rectangle. Then, the students checked the ratio using a numerical method, as shown in Fig. 2 (second from the left). However, the calculation of $3 - 2\sqrt{2} : \sqrt{2} - 1$ showed that the ratio of the remaining rectangle is not $1 : \sqrt{2}$, but $1 : 1 + \sqrt{2}$. Additionally, another student demonstrated, using a geometrical method, that the angles of the two right triangles formed by the diagonals of the rectangle are different. Therefore, the two rectangles are not similar, as shown in Fig.2 (second from the right). The students need to find another pair of similar rectangles to show that the iteration will continue indefinitely. After discussing in a group, some students found out two rectangles which seem to be similar as shown Fig.2 (left). By calculating of $\sqrt{2} - 1 : 2 - \sqrt{2}$ showed that the ratio of the remaining rectangle is $1 : \sqrt{2}$. By navigating

through three different types of operations —concrete, geometric, and symbolic— students were able to conclude that there is no square that can tile the rectangle. In other words, it is impossible to find a common unit that allows both lengths to be counted in terms of a single integer ratio, thus entering the realm of the uncountable. After the numerical approach, the teacher introduced a geometrical proof as an alternative method. Finally, the teacher explained that, since it is impossible to find a unit square that can tile this rectangle, the traditional definition of area—typically understood in elementary school mathematics, which counts the number of unit squares—needs to be expanded.

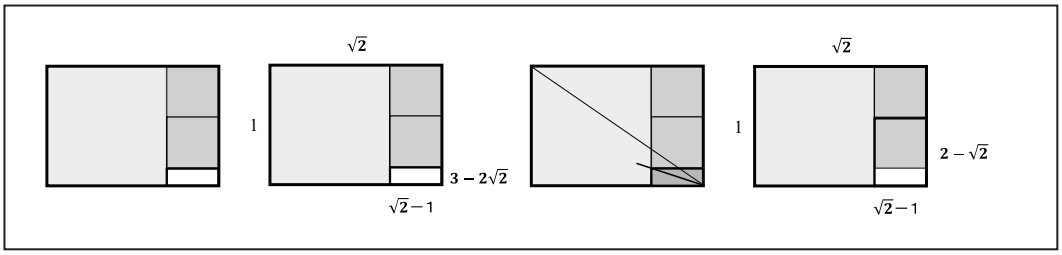


Figure 2. Students' trial to prove the similarity of two rectangles

Conclusion: Implications for Teaching Materials Development

It can be concluded that using concrete operations with a compass (instead of a ruler) helped students derive the Euclidean Algorithm by dividing a rectangle into squares with side lengths equal to the shorter side. This concrete operational approach allowed for more direct engagement with the division process. The exploration of the common unit between 1 and √2 prompted students to question whether it is possible to tile any rectangle with squares, a question directly addressed through the Euclidean Algorithm. Key intuitive insights from students were central to the lesson's development, including: predicting that tiling is possible for rational numbers but not for irrational ones; realizing that demonstrating the impossibility of tiling for irrational numbers requires showing that the operation of removing squares can be repeated indefinitely; understanding that this proof depends on the existence of similar rectangles; and investigating which rectangles can be similar. Further development of these initial student ideas, particularly regarding similar rectangles, could be explored, such as considering the similarity between the discovered rectangle and one formed by removing a square from the original. Expanding the definition of area for students remains a challenging task and should be explored further in future work.

The analysis and discussion on how instructional design can help students understand the impossibility of commensurability, as well as the cognitive and emotional responses to the uncountable nature of irrational numbers, will be presented in detail during the session.

References

Fischbein, E., Jehiam, R. & Cohen, D. (1995), The concept of irrational numbers in high-school students and prospective teachers, *Educational Studies in Mathematics*, 29, 29-44.

Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht: D. Reidel Publishing Company.

Freudenthal, H. (1991). *Revisiting Mathematics Education*, Kluwer Academic Publishers.

Ministry of Education, Culture, Sports, Science and Technology (2018). Course of Study for High Schools: Mathematics. *Gakkō Tosho Publishing*.

Rottoli, E., & Riva, G. (2021). ON AN “OLD” CONCEPT OF QUANTITY. *Academia Letters*, Article 3472. https://doi.org/10.20935/AL3472

Sirotic, N., & Zazkis, R. (2007). Irrational numbers: The gap between formal and intuitive knowledge. *Educational Studies in Mathematics*, 65(1), 49–76. https://doi.org/10.1007/s10649-006-9041-5

Voskoglou, M. Gr. (2013). An application of the APOS/ACE approach in teaching the irrational numbers, *Journal of Mathematical Sciences and Mathematics Education*, 8(1), 30-47

Womack, D. (2021). Mathematics without boundaries! *Academia Letters*, Article 305. https://doi.org/10.20935/AL305

TSG1-22

INSIGHTS FROM GERMAN SECONDARY STATISTICAL EDUCATION ON THE DEVELOPMENT OF RISK LITERACY

Che-Yu Hsu
Research Center for Science and Technology for Learning and Office of Institutional Research National Central University, Taoyuan, Taiwan, cheyuhsu37@gmail.com

Wei-Chang Shann
Center of Teacher Education and Dept. of Math, National Central University, Taiwan, shann@math.ncu.edu.tw

Yu-Chi Kang
Taipei Municipal Nan-Gang High School, Taiwan, n075@mail.nksh.tp.edu.tw

This study analyzes 120 tasks from the statistical education content of the Lambacher Schweizer Mathematik (LSM) textbook to examine its integration of statistical cognition and risk literacy. Results indicate broad alignment with Lei and Yang's (2012) framework of statistical cognition and graph comprehension. Additionally, applying Spiegelhalter's risk literacy framework reveals that the textbook effectively embeds risk-based thinking through coherent task design and content structure, demonstrating its potential to support students' critical engagement with uncertainty.

Keywords: statistical education, risk literacy, textbook

Introduction

Statistics, emerging in the 17th century, has become essential across disciplines and now underpins Data Science in the Big Data era. Though a later branch of mathematics, it is central to 21st-century education, as reflected in PISA's 2022 inclusion of "Conditional Decision-Making" to address modeling and uncertainty (Gal, 2003). To promote statistical literacy, Ben-Zvi and Garfield (2011) emphasize integrating statistics into primary and lower secondary education. Textbooks play a key role in shaping curricula and learning outcomes (Pingel, 2010; Chou & Zheng, 2008; Chambliss & Calfee, 1999; Fan, Zhu, & Miao, 2013; Yang & Chen, 2011). Responding to PISA, Germany revised its core curricula after 2003, with improved mathematics performance observed post-2009 (Chan, 2012).

This study analyzes the Lambacher Schweizer Mathematik (LSM) textbook—widely used in Germany and at the Taipei European School—to explore its role in fostering uncertainty and risk literacy. Specifically, it examines the statistical units in LSM 6 and LSM 7 (2003 Bavarian edition) for grades six and seven, excluding LSM 12 (high school level). The analysis focuses on the "Statistics" sections of the main textbooks, including examples (Beispiel), exercises (Aufgaben), topics (Thema), and review activities (Aufgaben zum Üben und Wiederholen), while excluding narrative texts, supplements, and workbooks. This focused scope allows for consistent comparison of core statistical content relevant to early secondary education.

Research Instruments

This study adopts the Statistical Cognition Analysis Framework and the Graph Comprehension Analysis Framework, developed by Lei and Yang (2012), to analyze textbook tasks. These frameworks are based on Garfield et al.'s (2003) classification of statistical cognition and Friel et al.'s (2001) concept of graph sense, and were translated and synthesized by Lei and Yang.

The classification procedure consists of three steps: 1. All tasks are first analyzed using the Statistical Cognition Analysis Framework. 2. Tasks involving statistical graphs are further classified using the Graph Comprehension Analysis Framework. 3. A single task may fall into multiple categories depending on its cognitive features. Inter-rater reliability was calculated to ensure coding consistency. The agreement coefficients were 0.95 for statistical cognition analysis and 0.93 for graph comprehension analysis, both exceeding the 0.80 threshold, indicating high reliability in this study.

Results of the Statistical Cognition and Graph Comprehension Framework

An analysis of 120 tasks in the LSM textbook revealed comprehensive coverage of statistical knowledge, reasoning, and thinking. Most tasks integrated multiple cognitive dimensions, with a focus on statistical knowledge—such as graph creation, calculations (mean, median, percentiles), and data interpretation—often set in real-world contexts. Reasoning tasks required inference, tool selection, and prediction of central tendency or dispersion. Thinking tasks emphasized critique, justification, and applying methods to authentic data, fostering statistical literacy and a questioning mindset.

Among 75 graph-related tasks, most aligned with "G2: Explaining the Phenomena Represented by Graphs." Nearly one-fourth targeted higher-order skills like "G5: Choosing Appropriate Graphs" and "G6: Avoiding Subjective Interpretation," highlighting the textbook's emphasis on critical graph evaluation and effective statistical communication.

Example of the Analysis Framework

This section is drawn from a set of tasks in the LSM textbook, illustrating the integration of statistical cognition and graph comprehension. Please refer to the following task: Despite the economic downturn, energy consumption has not decreased in recent years. In 2003, Germany converted 489.1 million tons of anthracite coal equivalent (Standard Coal Equivalent, SKE) into electricity, an increase of 400,000 tons compared to 2002. (Coal equivalent refers to standard coal.) Nevertheless, the most important energy source remained mineral oil. In 2003, it accounted for 36.4% of primary energy consumption, compared to 37.3% in the previous year. Meanwhile, the importance of natural gas continued to rise; in 2003, it accounted for 22.5% of primary energy consumption (equivalent to 110 million tons of SKE).

a. In the context of electricity generation, how is primary energy consumption defined?

b. Calculate the percentage of total primary energy consumption in 2003 attributed to each type of energy. Present your results using an appropriate chart.

c. By what percentage did total primary energy consumption increase from 2002 to 2003?

d. The text mentions that the share of mineral oil in primary energy consumption decreased, and this is also visible in the chart. However, the two statements seem contradictory. Explain why this apparent contradiction arises.

e. (Hint: absolute vs. relative percentage difference)

f. What was the percentage share of natural gas in primary energy consumption in 2002? What is that equivalent to in tons of SKE?

g. Assuming the rate of change in energy consumption remains constant, estimate how long it will take for the share of natural gas to equal that of mineral oil.

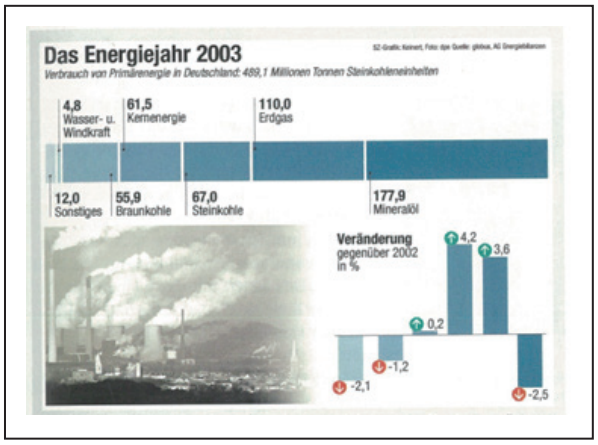


Figure 1. Primary energy consumption

Source: Klett Verlag. (2014). Lambacher Schweizer Mathematik 7

This study systematically analyzes each subtask based on statistical cognition and graph comprehension levels. Subtask a, being a textual definition, is excluded. Subtask b involves percentage calculation and graph construction, classified as L7, L9 (statistical cognition) and G5 (graph comprehension). Subtask c requires extracting values and computing percentages, categorized as L9 and G2. Subtask d includes data collection, graph interpretation, statistical calculation, and critical evaluation, classified as L9, T2, T4, and G6. Subtask e mirrors c. Subtask f focuses on interpreting data, calculating statistics, and identifying trends, thus classified as L9, R3, and G4. These classifications reflect the cognitive demands of each task.

This example provides a valuable opportunity to apply Spiegelhalter's framework for risk and uncertainty communication(Table 1). The diagram itself integrates both quantitative and visual elements, supporting or challenging students' statistical literacy depending on how the data is framed. Consistent with Spiegelhalter's recommendations, the use of proportional bars offers a cognitively accessible "gist" that supports quick comparison, particularly for audiences with varied levels of numeracy.

Table 1. Key dimensions in risk literacy education and assessment

Aspect	Description	Source
Core Definition	Risk literacy involves not only numerical understanding but also critical thinking, evaluation of assumptions, and communication of uncertainty. Guiding questions include: "Why did you choose this method?" or "How would you explain the uncertainty in the result?"	pp. 33–34 (Uncertainty taxonomy)
Types of Uncertainty	• Aleatory Uncertainty: Stemming from inherent randomness, inevitable variation. • Epistemic Uncertainty: Arising from limitations in model knowledge and parameters. • Ontological Uncertainty: Related to unknowns and limitations in the overall modeling process.	pp. 33–34 (Three-level uncertainty)
Use of Graphics and Language	Graphics (e.g., diagrams, bullet charts) should aid understanding and avoid misleading representations. Language should be accompanied by quantitative explanations; vague terms like "high risk" should be avoided unless clearly defined.	pp. 36–40; pp. 42–43 (Graphics & Words)

Note. Adapted from Spiegelhalter, D. (2017).

The diagram illustrates epistemic and ontological uncertainty, key elements of Spiegelhalter's taxonomy. While it shows energy consumption changes (e.g., +3.6% mineral oil, −2.1% coal), it omits underlying causes like economic or environmental factors—reflecting epistemic uncertainty from incomplete knowledge. Its reliance on year-to-year data suggests a linear view, ignoring systemic variability, thus illustrating ontological uncertainty. Instructionally, this design enables open-ended questions central to risk literacy, such as "Why did mineral oil use increase?" or "What uncertainties influence future trends?" These prompts foster critical thinking about data, sources, and assumptions, aligning with Spiegelhalter's framework and embedding real-world uncertainty reasoning into classroom practice.

Discussion and Conclusion

The LSM textbook exemplifies the integration of statistical education and risk literacy to foster data-driven competencies. It addresses key dimensions of statistical knowledge, reasoning, and thinking. Drawing on Spiegelhalter's (2017) taxonomy, the textbook distinguishes among aleatory, epistemic, and ontological uncertainties, and encourages critical evaluation through open-ended tasks. It promotes graph interpretation and the application of statistical reasoning in real-world contexts, such as energy consumption. Embedded within Germany's educational reforms, the LSM model offers valuable insights into cultivating learners who can reason under uncertainty and make informed decisions in complex, risk-laden situations.

References

Adams, P. (2006). Exploring social constructivism: Theories and practicalities. *Education*, 34(3), 243-257.

Aven, T. (2024). Risk literacy: Foundational issues and its connection to risk science. *Risk Analysis*, 44(5), 1011-1020.

Ben-Zvi, D., & Garfield, J. (Eds.). (2011). *The challenge of developing statistical literacy, reasoning, and thinking.* Dordrecht, the Netherlands: Kluwer Academic Publishers.

Chambliss, M. J., & Calfee, R. C. (1999). *Textbooks for learning*. London, England: Blackwell.

David Chan. (2012). German Secondary Education in the 21st Century: Its Reform and Achievements. *Bulletin of Educational Resources and Research*, 54, 253-282.

Yang, D. C. & Chen, R. H. (2011). A Study of the Methods for Developing 7th-Graders' Mathematical Ability in Selected Algebra Textbooks from Taiwan, U.S.A. and Singapore. *Chinese Journal of Science Education*, 19(1), 39-67. https://doi.org/10.6173/CJSE.2011.1901.03

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: development status and directions. *ZDM—The International Journal on Mathematics Education*, 45(5), 633-646.

Friel S. N., Curcio, F. R., & Bright, G. W. (2001). Making sense of graphs: Critical factors influencing comprehension and instructional implications. *Journal for Research in Mathematics Education, 32*, 124-158.

Gal, I. (2003). Teaching for statistical literacy and services of statistics agencies. *The American Statistician*, *57*, 80-84.

Garfield, J., delMas, R., & Chance, B. (2003). *The web-based ARTIST: Assessment resource too LSM for improving statistical thinking project.* Paper presented at AERA annual meeting. Retrieved from http://www.amstat.org/publications/jse/v10n3/garfield.html

Lei, K. H., & Yang, K. L. (2012). Analysis of Statistical Content in Junior High School Mathematics Textbooks Based on Statistical Cognition and Graph Comprehension. *Journal of Textbook Research, 5*(2), 31-72. https://doi.org/10.6481/JTR.201208.0031

Chou, P. I., & Cheng, M. C. (2008). Methodologies on Textbook Research. *Curriculum & Instruction Quarterly, 11*(1), 193-222. https://doi.org/10.6384/CIQ.200801.0193

Pingel, F. (2010). *UNESCO guidebook on textbook research and textbook revision* (2nd revised and updated edition). Paris: UNESCO.

Spiegelhalter, D. (2017). Risk and uncertainty communication. *Annual Review of Statistics and Its Application*, 4(1), 31-60.

Till, C. (2014). Fostering risk literacy in elementary school. *International Electronic Journal of Mathematics Education*, 9(2), 83-96.

Tversky, A., & Kahneman, D. (1973). Availability: A heuristic for judging frequency and probability. *Cognitive psychology*, 5(2), 207-232.

Zoi Nikiforidou. (2017).'It is riskier': preschoolers' reasoning of risky situations, *European Early Childhood Education Research Journal*, (25)4, 612-623.

TSG1-23

CONTEXT IN STATISTICAL TASKS: INSIGHTS FROM LOWER SECONDARY MATHEMATICS TEXTBOOKS IN THAILAND

Dhanachat Anuniwat
Hiroshima University, Japan, d245626@hiroshima-u.ac.jp

Supanida Silaba
Sriayudhya School, Thailand, supanida.s@sriayudhya.ac.th

This study examined the inclusion of context in statistical tasks in lower secondary mathematics textbooks in Thailand and the types of contexts represented. A qualitative content analysis was conducted on 82 statistical tasks extracted from widely used national mathematics textbooks for grades 7 to 9. These tasks were analyzed to identify the types of contexts included–societal, personal, occupational, and scientific–using a deductive content analysis approach. The results revealed that 89.02% of the statistical tasks contained context, while occupational and scientific contexts were less frequently represented. Given the importance of diverse contexts for developing students' statistical reasoning and literacy, we recommend incorporating a broader range of contexts in these tasks.

Keywords: context, secondary mathematics, statistical tasks, textbook analysis

Introduction

Statistical literacy and reasoning are crucial for students, empowering them to make informed decisions in both personal and professional contexts (Gal, 2004; Bargagliotti et al., 2020; Hauge, 2022). In today's data-driven world, the teaching and learning of statistics have become increasingly important in school education, prompting mathematics curricula worldwide to place greater emphasis on this discipline (Bargagliotti et al., 2020). Mathematics curricula vary across jurisdictions and can be categorized into three levels: intended, implemented, and attained (Inprasitha, 2019). The intended curriculum consists of official documents and mathematics textbooks that guide classroom instruction based on these materials, while the attained curriculum represents what students actually learn (Inprasitha, 2019). Textbooks primarily serve as resources for teachers to design lesson plans, worksheets, and other instructional materials (Baba, 2010; Fan et al., 2013). Thus, textbooks provide significant learning opportunities (Choppin et al., 2021) and bridge the gap between what is outlined in the curriculum materials and what can be implemented in classroom practice (Baba, 2010). Textbooks play a vital role in shaping how statistics is taught and learned (Jones & Jacobbe, 2014). Since teachers often rely on textbooks to plan their lessons, the tasks within them significantly influence classroom instruction (Fan & Kaeley, 2000).

These statistical tasks can be analyzed from two perspectives: content (e.g., Jones & Jacobbe, 2014) and context (e.g., Fukuda, 2017, 2020). This study focused on context because statistics inherently requires context (Rao, 1975), which is crucial for developing students' statistical reasoning and literacy (Wild & Pfannkuch, 1999; Burrill & Biehler, 2011). Unlike mathematics education, statistics necessitates a distinct way of thinking, as it is inherently tied to data—data that are not simply numbers but numbers embedded in context. While contextual elements can sometimes obscure

mathematical structures, they provide meaning to statistics (Cobb & Moore, 1997; Fukuda, 2020). Therefore, understanding the role of context in statistics education and its integration into statistical tasks is essential. Although some studies have examined the context in statistics education (e.g., Pfannkuch, 2011; Ben-Zvi & Aridor-Berger, 2016), few have analyzed statistical tasks in textbooks (e.g., Fukuda, 2017, 2020), particularly in the context of Thailand.

As a result, little is known about the characteristics of statistical tasks presented in Thai mathematics textbooks. This study addressed this knowledge gap by examining statistical tasks in lower secondary mathematics textbooks in Thailand, with a particular emphasis on context. Lower secondary education is the focus because it marks the formal introduction of statistics, whereas elementary education primarily centers on data representation (for complications, see Ministry of Education, 2017). Accordingly, this study seeks to answer two research questions: RQ1) To what extent do lower secondary mathematics textbooks in Thailand incorporate context into their statistical tasks? RQ2): What types of context are included?

Literature Review

Numerous scholars have acknowledged the criticality of context in teaching and learning statistics (e.g., Watson, 2006; Pfannkuch, 2011; Ben-Zvi & Aridor-Berger, 2016). As noted above, statistics involves more than the manipulation of numbers; it requires an understanding of their contextual meanings (Cobb & Moore, 1997; Burrill & Biehler, 2011). Wild and Pfannkuch (1999) endorsed that context clarifies tasks and supports the planning of data collection at the initial stage of a statistical investigation. Furthermore, the presence of context enables students to transition between real-world situations and statistical methods, thereby enhancing their ability to engage with statistical reasoning and literacy.

As mentioned previously, this study examined the context of statistical tasks. To guide the analysis, we adopted the framework proposed by Fukuda (2017), which examines the presence and absence of context in statistical tasks and the type of context incorporated, as categorized by the Organization for Economic Co-operation and Development [OECD] (2023). Fukuda (2017, pp. 153–154) structured his framework into three components: (a) whether context is included; (b) whether the included context is realistic; and (c) whether the context is indispensable for answering the statistical tasks. (I) If context is not included (a = "no"), then components (b) and (c) are not applicable, and the tasks is classified as (no, –, –). (II) If context is included (a = "yes"), the tasks may fall into one of three categories: (i) (yes, no, no): context is present but unrealistic and not used to answer the tasks; (ii) (yes, yes, no): context is present and realistic but not used to answer the tasks; (iii) (yes, yes, yes): context is present, realistic, and essential for answering the tasks. Building on this framework, when context is present, it is further classified into four types: personal, occupational, societal, and scientific (OECD, 2023, pp. 40–41).

Methodology

This study employed qualitative content analysis (Mayring, 2022), a method well-suited for examining textual data–specifically, statistical tasks–where language functions as a medium of communication. Statistical tasks were extracted from the Statistics Unit of three widely-used national textbooks for lower secondary education (Grades 7–9), published by the Institute for the Promotion of Teaching Science and Technology [IPST] (2022, 2024a, 2024b).

A deductive content analysis approach was used to analyze 82 tasks drawn from the exercise sections of the textbooks: 36 from Grade 7, 32 from Grade 8, and 14 from Grade 9. The analysis began with an individual, detailed review of each task to determine the presence and nature of context, following Fukuda's (2017) framework. When context was identified, it was further categorized according to the OECD's (2023) classification system. The analysis was cross-validated using one-to-one comparisons, with 94% of the tasks showing consistent coding. Discrepancies were resolved through discussions with an external expert specializing in mathematics education. Table 1 provides examples of the statistical tasks analyzed, with parentheses indicating the type of context. The results are presented in the following sections.

Table 1. Examples of statistical tasks. sources: IPST (2022, 2024a, 2024b)

(I) no, – , –	(II) yes, –,–		
	(i) yes, no, no	(ii) yes, yes, no	(iii) yes, yes, yes
(Computational Task) Find the mode of each of the following data sets: 5, 7, 4, 8, 7, 11, 7, 4, 10, and 8.	(Bottles Task) How many bottles of drinking water were sold at the [...] shop on Loy Krathong Day 2017?	(Dice Task) From an experiment of rolling two dice together 100 times, the sum of the numbers rolled was recorded. The results are shown in the table below: [table excluded] Find the mean, standard deviation, and mode of the sums of the points rolled.	(Olympic Task) From the 2016 Olympic Games held in Brazil, the recorded times of female swimmers in the 100-meter freestyle event are shown in the box plot below. [Box plots for the final round, semi-final Group 1, and semi-final Group 2 are excluded]. Based on the box plot, how can you summarize the swimming times of the three groups of athletes?
(None)	(Occupational)	(Personal)	(Societal)

Results and Discussions

An analysis of 82 statistical tasks from Thai lower secondary mathematics textbooks revealed that 89.02% (73 tasks) included context (see Table 2). Among these, the largest proportion (36.58%) featured realistic context that were *not* used to answer the tasks, followed closely by those with realistic contexts that were *used* in solving the tasks (34.15%). This high percentage of context inclusion suggests that textbook developers acknowledge the importance of context in connecting abstract statistical concepts to real-world applications.

Table 2. Context in statistical tasks by grade level

(a, b, c)/Grade	Grade 7 (n=36)	Grade 8 (n=32)	Grade 9 (n=14)	Grade 7 to 9 (n=82)
(I) no, – , –	0% (0)	21.88% (7)	14.29% (2)	10.98% (9)
(II) yes, –,–	100% (36)	78.12% (25)	85.71% (12)	89.02% (73)
(i) yes, no, no	41.67% (15)	0% (0)	0% (0)	18.29% (15)
(ii) yes, yes, no	30.55% (11)	46.87% (15)	28.57% (4)	36.58% (30)
(iii) yes, yes, yes	27.78% (10)	31.25% (10)	57.14% (8)	34.15% (28)

The context embedded in the statistical tasks presented in Table 1 can be discussed from two key perspectives. First, context that fosters statistical reasoning: Tasks in which the context is realistic and indispensable for solving the problem–classified as "yes, yes, yes" and comprising 34.15% of the total–encourage students to view statistics as relevant to real-life situations. For example, the Olympic task situates data within a societal context, prompting students not only to compute statistical measures such as the mean and standard deviation but also to interpret what these values reveal about athletic performance. Second, context fosters statistical literacy: Realistic contexts support students in moving beyond routine calculations toward interpreting, evaluating, and making decisions based on data (see Gal, 2004; Watson, 2006). Tasks involving personal contexts, such as the dice task, help students connect statistics to everyday experiences. Occupational contexts, exemplified by the bottles task, demonstrate the relevance of statistics to business scenarios, while societal contexts, such as the Olympic task, underscore the role of statistics in public discourse. This multifaceted contextual exposure contributes to the development of statistical reasoning and statistical literacy, which are essential for informed citizenship (Wild & Pfannkuch, 1999; Gal, 2004; Watson, 2006).

Table 3. Types of contexts in statistical tasks

Context/Grade	Grade 7 (n=36)	Grade 8 (n=32)	Grade 9 (n=14)	Grade 7 to 9 (n=82)
Personal	19.45% (7)	56% (14)	16.67% (2)	31.51% (23)
Occupational	11.1% (4)	20% (5)	8.33% (1)	13.70% (10)
Societal	47.22% (17)	16% (4)	58.33% (7)	38.36% (28)
Scientific	22.22% (8)	8% (2)	16.67% (2)	16.43% (12)

Although most tasks included context (see Table 3), the predominance of societal (38.36%) and personal (31.51%) contexts, with less emphasis on occupational (13.70%) and scientific (16.43%) contexts, indicates a need for greater contextual diversification. The dominance of societal and personal contexts in textbooks can be interpreted as reflecting their close relation to daily lives. However, expanding the variety of contexts could further enhance students' statistical reasoning and literacy. Incorporating a broader range of occupational and scientific contexts, as observed in countries such as New Zealand and Japan (Fukuda, 2020), would provide students with richer opportunities to engage with authentic data contexts. Building on these findings, future revisions of Thai mathematics textbooks might consider incorporating a broader range of contextual scenarios to further enrich students' development of statistical reasoning and literacy.

Conclusion

This study reveals the contexts and types of statistical tasks in lower secondary mathematics textbooks in Thailand. Before contextualizing this study, its limitations must be acknowledged. Since this study focused on context in statistical tasks, it did not analyze the statistical content. The discussion is theoretical and limited in its empirical data support. Future studies should examine this content to better understand the characteristics of statistical tasks in textbooks.

References

Baba, T. (2010). Analysis of the structure of mathematics curriculum in Zambia. *Africa Educational Research Journal, 1*, 41–51. https://doi.org/10.50919/africaeducation.1.0_41 (in Japanese)

Bargagliotti, A., Franklin, C., Arnold, P., Gould, R., Johnson, S., Perez, L., & Spangler, D. A. (2020). *Pre-K–12 guidelines for assessment and instruction in statistics education II (GAISE II): A framework for statistics and data science education*. American Statistical Association.

Ben-Zvi, D., & Aridor-Berger, K. (2016). Children's wonder: How to wander between data and context. In D. Ben-Zvi & K. Makar (Eds.), *The teaching and learning of statistics: International perspectives* (pp. 25–36). Springer. https://doi.org/10.1007/978-3-319-23470-0_3

Burrill, G., & Biehler, R. (2011). Fundamental statistical ideas in the school curriculum and in training teachers. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics—Challenges for teaching and teacher education: The 18th ICMI study* (pp. 57–69). Springer. https://doi.org/10.1007/978-94-007-1131-0_10

Choppin, J., Davis, J., McDuffie, A. R., & Drake, C. (2021). Influence of features of curriculum materials on the planned curriculum. *ZDM – Mathematics Education, 53*(6), 1249–1263. https://doi.org/10.1007/s11858-021-01305-7

Cobb, G. W., & Moore, D. S. (1997). Mathematics, statistics, and teaching. *The American Mathematical Monthly, 104*(9), 801–823. https://doi.org/10.1007/s11858-013-0539-x

Fan, L., & Kaeley, G. S. (2000). The influence of textbooks on teaching strategies: An empirical study. *Mid-Western Educational Researcher, 13*(4), 2. https://doi.org/10.1007/s11858-013-0539-x

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: Development status and directions. *ZDM – Mathematics Education, 45*(5), 633–646. https://doi.org/10.1007/s11858-013-0539-x

Fukuda, H. (2017). The characteristic of statistics education in Japan by comparison with New Zealand: From the viewpoint of contexts and social values. *Journal of JASME: Research in Mathematics Education, 23*(2), 151–158. https://doi.org/10.24529/jasme.23.2_151. (in Japanese)

Fukuda, H. (2020). *Research towards a principle for the statistics curriculum in Japan from the perspective of context* [Doctoral Dissertation, Hiroshima University]. Hiroshima University. https://hiroshima.repo.nii.ac.jp/records/2003218

Gal, I. (2004). Statistical literacy: Meanings, components, responsibilities. In J. B. Garfield & D. Ben-Zvi (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 47–78). Kluwer. https://doi.org/10.1007/1-4020-2278-6_3

Hauge, K. H. (2022). A tool for reflecting on questionable numbers in society. *Studies in Philosophy and Education, 41*(5), 511–528. https://doi.org/10.1007/s11217-022-09836-6

Inprasitha, M. (2019). A new model of mathematics curriculum and instruction system in Thailand. In C. Vistro-Yu & T. Toh (Eds.), *School mathematics curricula: Asian perspectives and glimpses of reform* (pp. 51–77). Springer. https://doi.org/10.1007/978-981-13-6312-2_4

Institute for the Promotion of Teaching Science and Technology [IPST]. (2022). *Basic mathematics, Grade 7, Volume 2*. Suksapan Press. (in Thai)

Institute for the Promotion of Teaching Science and Technology [IPST]. (2024a). *Basic mathematics, Grade 8, Volume 2*. Suksapan Press. (in Thai)

Institute for the Promotion of Teaching Science and Technology [IPST]. (2024b). *Basic mathematics, Grade 9, Volume 1*. Suksapan Press. (in Thai)

Jones, D. L., & Jacobbe, T. (2014). An analysis of the statistical content in textbooks for prospective elementary teachers. *Journal of Statistics Education, 22*(3). https://doi.org/10.1080/10691898.2014.11889709

Ministry of Education. (2017). *Core indicators and core learning content: Mathematics: Revised edition 2017 of the Basic Education Core Curriculum BE 2551* (AD 2008). Office of the Basic Education Commission, Ministry of Education, Thailand. (in Thai)

Mayring, P. (2022). *Qualitative content analysis: A step-by-step guide*. SAGE Publications.

Organization for Economic Co-operation and Development [OECD]. (2023). *PISA 2022 assessment and analytical framework*. OECD Publishing. https://doi.org/10.1787/dfe0bf9c-en

Pfannkuch, M. (2011). The role of context in developing informal statistical inferential reasoning: A classroom study. *Mathematical Thinking and Learning, 13*(1–2), 27–46. https://doi.org/10.1080/10986065.2011.538302

Rao, C. R. (1975). Teaching of statistics at the secondary level: An interdisciplinary approach. *International Journal of Mathematical Education in Science and Technology, 6*(2), 151–162. https://doi.org/10.1080/0020739750060203

Watson, J. M. (2006). *Statistical literacy at school: Growth and goals*. Routledge.

Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review, 67*(3), 223–248. https://doi.org/10.1111/j.1751-5823.1999.tb00442.x

Acknowledgment

This study was supported by JST SPRING (Grant Number: JPMJSP2132). The views expressed are solely those of the authors and do not necessarily reflect those of the funding agency.

TSG1-24

EXPLORING DIGITAL LITERACY IN THE 2022 REVISED KOREAN MATHEMATICS CURRICULUM AND TEXTBOOKS

Jungeun Yoon
Seoul National University, Korea, yoonhoho1004@snu.ac.kr

Oh Nam Kwon
Seoul National University, Korea, onkwon@snu.ac.kr

Digital literacy should be integrated as a meaningful component of each subject rather than merely being suggested as a competency. In this study, we explored digital literacy in the 2022 Revised Korean Mathematics Curriculum and Textbooks based on a digital literacy framework. In the 2022 Revised Mathematics Curriculum, the proportion of D2 and D3 decreased, whereas the proportion of D1 increased with higher school levels. Also, D4 was rarely present in the mathematics curriculum as a whole. In the 2022 Revised Mathematics Textbooks, digital literacy elements were predominantly focused on D1 and D2 across all school levels. The 'Use of AI (D1-3)' was mainly reflected as a material for textbook, such as chatbots and AI learning assistants, rather than being used directly as a tool. 'Data collection and storage (D2-1)' and 'Information analysis and representation (D2-2)' have been more prominently incorporated in the curriculum and textbooks of elementary and middle schools compared to those of high schools in terms of content organization. Based on findings, this study discussed supplementary measures for each digital literacy elements and proposed digital teaching and learning strategies, including establishing digital mathematics education guidelines, expanding programming-related digital content creation activities, structuring digital literacy elements hierarchically, and strengthening digital ethics education.

Keywords: digital literacy, 2022 Revised Korean Mathematics Curriculum, 2022 Revised Korean Mathematics Textbook, mathematics education, digital teaching & learning

Introduction

As the need to revise education to reflect digital transformation has been raised, educational changes to foster students' digital and AI literacy have been attempted in Korea. Ministry of Education established an AI-based education policy and foundation through the 'Comprehensive Plan for Fostering Digital Competencies' in 2022, and sought ways to foster digital literacy from the perspective of future education (Joint of Related Ministries, 2022). In the 2022 Revised Korean Curriculum, 'digital literacy' was designated as a basic literacy to respond to the uncertainty caused by rapid digital transformation (MOE, 2022).

Many studies interpret digital literacy at the level of competency and propose components or frameworks of digital literacy (Ferrari, 2012). However, digital literacy needs to be operated and organized as meaningful component of each subject, rather than merely being suggested as a competency (Kim & Park, 2023). This study explored the degree and aspect of digital literacy in the 2022 Revised Korean Mathematics Curriculum and Textbooks to provide insights to Korean mathematics

education. To this end, we explored the reflection of digital literacy in the mathematics curriculum and textbooks for 3rd and 4th grade elementary school students, 1st grade middle school students, and 1st grade high school students for which the 2022 revised textbook was released.

Methods

Scope of Analysis

For analysis, the scope of this study was outlined in Table 1, which examined how digital literacy was reflected in the 2022 Revised Korean Mathematics Curriculum and Textbooks. Currently, the 2022 Revised Korean Mathematics Curriculum is released to all elementary, middle, and high schools, but the 2022 Revised Korean Mathematics Textbooks are only available for certain grade levels from some publishers. Accordingly, the mathematics curriculum and textbooks for the 3rd and 4th grades of elementary school, the 1st year of middle school, and the 1st year of high school to which the 2022 Revised Mathematics Textbooks will be applied in 2025 were targeted for analysis.

Table 1. Scope of analysis

Category	Scope of analysis
Curriculum	? Nature, goals, teaching and learning methods, and evaluation of 2022 Revised Korean Mathematics Curriculum (3rd and 4th grades of elementary school, 1st year of middle school, 1st year of high school) ? Content system and achievement standards of 2022 Revised Korean Mathematics Curriculum (3rd and 4th grades of elementary school, 1st year of middle school, 1st year of high school)
Textbook	? Examples, problems, exploring activities of 2022 Revised Korean Elementary School 3-1, 3-2, 4-1, 4-2 Textbooks ? Examples, problems, exploring activities of 2022 Revised Korean 1st year Middle School Textbooks ? Examples, problems, exploring activities of 2022 Revised Korean 1st year High School <Common Mathematics 1>, <Common Mathematics 2> Textbooks

Analytical Framework

Since this study analyzed Korean mathematics curriculum and textbooks, it was judged appropriate to use digital framework of domestic curriculum as an analytical framework. Since a domestic digital literacy framework for mathematics has not yet been developed, the digital framework by Kim et al. (2023) was adopted due to its cross-curricular applicability and widespread use as an analytical framework in various subjects. This framework presented four areas and detailed elements for each: Use of digital devices and software, Use and creation of digital information, Digital communication and problem solving, and Digital ethics and information protection. Based on Table 2, digital literacy in the elementary and secondary mathematics curriculum (nature, goals, achievement standards, achievement standard explanations, teaching and learning methods, and evaluation) and three textbooks with highest market share and accessible revised files for 3rd and 4th grade of elementary schools, 1st grade of middle school, and 1st grade of high school were analyzed.

Table 2. Digital literacy framework (Kim et al., 2023)

Category	Details	Description of details
Use of digital devices and software	Use of digital devices	Understand and utilize the basic principles and functions necessary to operate digital devices.
	Use of Software	Understand the basic principles and functions of software and utilize software in various tasks.
	Use of AI	Tools equipped with AI technology are used in various problem-solving processes.
Use and creation of digital information	Data collection and storage	Consider the purpose of use, collect data, evaluate information from a critical perspective, and manage necessary information efficiently.
	Information analysis and presentation	Analyze, synthesize, and visualize information to effectively convey it.
	Digital content creation	Creates various types of content acquired through digital media.
Digital communication and problem solving	Digital communication	Critically analyze and share information, participate in decision-making, and collaborate in a digital environment.
	Digital problem solving	Design a solution to a problem and implement it using digital tools.
Digital ethics and Information protection	Digital ethics	As a mature citizen of the digital society, be considerate of others and practice etiquette and ethics.
	Digital information protection	Practice measures to protect your own and others' information.

Results

In this study, we analyzed 2022 Revised Korean Mathematics Curriculum and Textbooks for 3rd and 4th grade elementary school students, 1st year middle school students, and 1st year high school students. Distribution of digital literacy is presented as Table 3. In terms of curriculum, the proportion of D2 and D3 tended to decrease as the school grade level increased, while the proportion of D1 tended to increase. Meanwhile, the distribution of digital literacy in the revised textbooks showed a high proportion of D1 and D2 in elementary, middle, and high schools. In contrast to the situation where digital ethics is emphasized worldwide, the D4 element was hardly reflected in the elementary, middle, and high school curricula and textbooks in Korea.

Table 3. Distribution of digital literacy in the 2022 Revised Korean Mathematics Curriculum and Textbooks [value (%)]

Category	Details	3rd & 4th grades of elementary school				1st grade of middle school				1st grade of high school			
		curriculum		textbooks (A,B,C)		curriculum		textbooks (A,B,C)		curriculum		textbooks (A,B,C)	
Use of digital devices and software **(D1)**	Use of digital devices (D1-1)	2 (18.2)	3 (27.3)	7.16 (13.2)	14.8 (26.3)	1.45 (11.2)	7.73 (59.5)	8 (13.7)	26 (44.5)	1 (25)	3 (75)	3 (9.5)	27.5 (87.3)
	Use of Software (D1-2)	1 (9.1)		6.66 (12.2)		6.28 (48.3)		18 (30.8)		2 (50)		23.5 (74.6)	
	Use of AI (D1-3)	0 (0)		1 (0.9)		0		0 (0)		0		1 (3.2)	
Use and creation of digital information **(D2)**	Data collection and storage (D2-1)	1.66 (15.1)	3.5 (31.8)	13.65 (25.1)	40.2 (73.7)	0.78 (6)	3.56 (27.4)	8.5 (14.5)	26.5 (45.3)	0.33 (8.33)	0.66 (16.6)	0 (0)	4 (12.7)
	Information analysis and presentation (D2-2)	1.33 (12.1)		19.16 (35.2)		2.78 (21.4)		14 (24)		0.33 (8.33)		0 (0)	
	Digital content creation (D2-3)	0.5 (4.6)		7.33 (13.4)		0 (0)		4 (6.8)		0 (0)		4 (12.7)	
Digital Communication and problem solving **(D3)**	Digital communication (D3-1)	0.66 (6)	2.5 (22.7)	0 (0)	0 (0)	1 (7.7)	1.7 (13.1)	4 (6.8)	6 (10.2)	0 (0)	0.33 (8.3)	0 (0)	0 (0)
	Digital problem solving (D3-2)	1.83 (16.7)		0 (0)		0.7 (5.4)		2 (3.4)		0.33 (8.33)		0 (0)	
Digital ethics and information protection **(D4)**	Digital ethics (D4-1)	2 (18.2)	2 (18.2)	0 (0)	0 (0)	0 (0)	0(0)	0 (0)	0(0)	0(0)	0 (0)	0 (0)	0 (0)
	Digital information protection (D4-2)	0 (0)		0 (0)		0 (0)		0(9)		0 (0)			
Total		11(100)		55(100)		13(100)		58.5(100)		4(100)		31.5(100)	

Notably, the representation of 'Use of AI (D1-3)' was limited. AI was not explicitly included in the mathematics curriculum, but it was indirectly reflected in textbooks as materials such as 'AI robot', 'chatbot', and 'AI learning assistant'. This is in line with the recent recognition of the role of generative AI in collaborating with learners in mathematics education (Yoon & Kwon, 2024).

Also, digital content creation (D2-3) appeared in conjunction with mathematics and competencies such as problem solving, communication, information processing, and connection in textbooks. The activity of drawing polygonal pictures using points, lines, and planes (first year middle school) and the activity of drawing shapes using block coding (first year high school) contained digital content creation (see Figure 1).

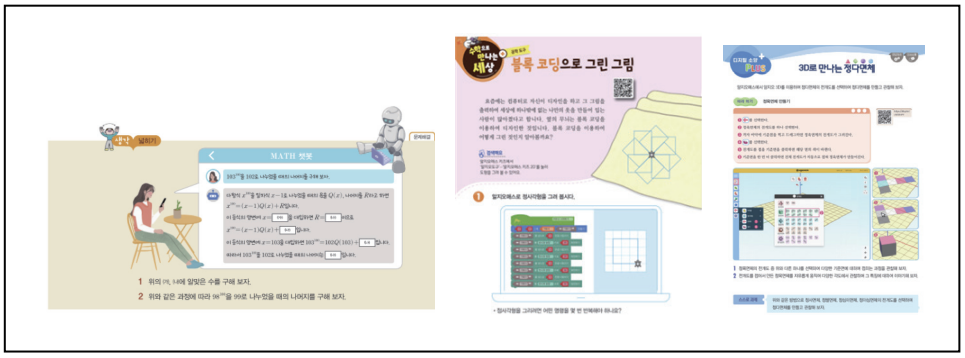


Figure 1. Tasks in mathematics textbooks that reflect 'Use of AI(D1-3)' (left), 'Digital content creation(D2-3)' (middle & right)

'Digital ethics and information protection (D4)' was minimally addressed in the mathematics curriculum and textbooks. Usually, use of technology has accounted highest in Korean curriculum, while digital ethics has shown highest rates in UK or Canadian curriculum (Park et al., 2021). The purpose of digital literacy education of UK or Canada is to foster digital citizens and includes the attitude and citizenship contained in digital literacy framework. These findings indicate that future curriculum revisions should prioritize the integration of AI and ethical considerations to align with international standards and enhance students' digital literacy.

Conclusion & Discussion

The results presented how the sub-elements of digital literacy are reflected in 2022 Revised Korean Curriculum and Textbooks at elementary, middle, and high schools. In order to foster digital literacy in mathematics education, it is necessary to balance sub-categories of digital literacy in curriculum and textbooks. First, it is necessary to support students' use of AI in Korea, referring to the AI-based learning platform SLS(Student Learning Space) provided by the Singapore Ministry of Education (https://www.learning.moe.edu.sg/). Second, the hierarchy of four areas of digital literacy needs to be considered. Digital literacy has a pyramidal structure that 'use and creation of digital information' and 'digital communication and problem solving' based on 'use of digital devices and software' and 'digital ethics and information protection' (Kim et al., 2023, p. 268). However, this hierarchy was not revealed in the 2022 Revised Mathematics Curriculum and Textbooks and each element of digital literacy was reflected independently. The hierarchy of the sub-elements of digital literacy should be considered when revising the curriculum and textbooks in the future. Third, digital ethics education needs to be implemented in each subject because digital attitude and safety are essential parts of digital literacy. Ethical considerations such as digital responsibility, data privacy, and plagiarism awareness should be explicitly included in future curriculum revisions.

This study contributes to identifying the current state of digital literacy integration in 2022 Revised Korean Mathematics Curricula and Textbooks, while also proposing ways to enhance its implementation. To enhance digital well-being in mathematics education, future curriculum revisions should reinforce AI learning, consider hierarchy of digital literacy, and include ethics education.

References

Ferrari, A. (2012). *Digital Competence in Practice: An Analysis of Frameworks.* Publications Office of the European Union.

Joint of Related Ministries. (2022). *Announcement of a comprehensive plan for science, mathematics, information, and convergence education (2025-2029) to foster future talent for a cutting-edge science and technology society.* Joint of Related Ministries press release. 2022.8.22.

Kim, J., Kang, W., Kim, Y., & 23 others. (2022). *Research on the development of the 2022 revised information department curriculum draft (final draft)* (A23010015). Korea Foundation for Science and Creativity.

Kim, M. & Park, S. (2023). AI education curriculum: Focusing on the middle school curriculum of AI education leading schools. *The Journal of Curriculum Studies, 41*(4), 29-54.

Ministry of Education. (2022). *Mathematics curriculum.* Proclamation of the Ministry of Education # 2022-33 [Annex 8]. Authors.

Park, H., Jin, M., Park, J., & Lim, K. (2021). The Analysis of National and International Curricula for Digital Literacy Education. *Journal of Education & Culture, 27*(5), 75-101.

Yoon, J. & Kwon, O. N. (2024). Systematic literature review on AI-based mathematics teaching and learning: Focusing on the role of AI and teachers. *The Mathematics Education, 63*(3), 573-591.

TSG1-25

A DECOLONIZED CURRICULUM AND CHILDREN'S BUNDLE-NUMBERS WITH UNITS MAY REACH THE UN DEVELOPMENT NUMERACY GOAL

Allan Tarp
MATHeCADEMY.net, Denmark, Allan.Tarp@gmail.com

*The fourth United Nations Development Goal wants all youth and most adults to achieve numeracy. In 'essence-based' mathematics using one-dimensional line-numbers without units, numeracy is at the beginning and calculus is at the end. But the opposite is the case within the alternative 'existence-based' paradigm built on the numbers children bring to school, two-dimensional bundle-numbers with units, where 2 3s is short for 2 bundles with 3s per bundle. Here calculus is needed in grade one for next-to addition of 2 3s and 4 5s as 8s since they add by their areas. Here, adding is preceded by counting leading to a recount-formula, T = (T/B)*B, with T and B for Total and Bundle, used to change units in STEM, and here recounting between icon-numbers and tens leads to a division and a multiplication table as well as to solving equation. Recounting physical units leads to per-numbers becoming fractions with like units and trigonometry when recounting the sides in a bundle-stack. So, changing from an essence-based to an existence-based curriculum may allow reaching the UN Development Goal and may provide new ways for students to interact with curriculum and materials.*

Keywords: early childhood mathematics, numeracy, special education, curriculum, arithmetic

Background, the 4th United Nations Sustainable Development Goal

The fourth of the 17 UN Sustainable Development Goals defines quality education as 'ensuring inclusive and equitable quality education and promoting lifelong learning opportunities for all.' Here the subgoal 4.6 wants to "By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy". However, different definitions of 'numerate' seem to exist. The English Oxford Dictionary defines it as being "competent in the basic principles of mathematics, esp. arithmetic". In contrast, the American Merriam-Webster dictionary defines it as "having the ability to understand and work with numbers."

The difference in the definitions is interesting. The English uses the passive term 'competent' where the American uses the active term 'work'. The word 'competent' is a predicate, a non-action word, I cannot 'competent' something, I can only be judged as competent by someone who is competent. In contrast, 'work' is an action word, a verb, since with my body and mind I can work on something.

Also, there is a difference between the words 'mathematics' and 'numbers.' Again, mathematics is a non-action word, I cannot 'mathematics' or even 'math' a thing. In contrast, 'number' is both a verb and a noun since I can number different degrees of Many to produce a number for later calculations.

Finally, Many exists in the outside world where humans see and name it differently. In contrast, mathematics does not do so, it is an institutionalized essence that is socially constructed as inside abstractions from outside examples, or as inside examples from inside abstractions.

To understand these differences, we now consult the three grand theories, philosophy and sociology and psychology. Philosophy may be able to illuminate the different nature of predicates and verbs. Sociology may be able to illuminate the different inter-human power effects coming from using predicates instead of verbs. And psychology may be able to illuminate the different learning results coming from listening to predicates or practicing verbs.

Grand Theory Looks at Mathematics Education

Within philosophy, Existentialism holds that 'existence precedes essence' (Sartre, 2007) so that in a judging is-sentence, an existing subject is being colonized by its predicates. 'Many' thus should be seen ontologically in itself, instead of epistemologically, how some may perceive and verbalize it.

Within psychology, Piaget (1969) sees learning as adapting to outside existence, whereas a Vygotsky (1986) sees learning as adapting to inside institutionalized socially constructed essence.

Within sociology, a structure-agent debate discusses if institutions should be obeyed or negotiated between peers. Here, a Weberian viewpoint (1930) asks if SET is a rationalization gone too far by leaving Many de-enchanted and leaving learners in an 'iron cage'. As to the goal, Bauman (1990) suggests that, by institutionalizing mastery of Math as the means to reach mastery of Many, 'essence-math' has created a 'goal displacement' making the means a goal instead. Then, with the end goal, mastery of Many or existence-math, essence-math may be a means, but are there other means also?

Essence-math sees Many as an example of 1D linear cardinality always able to absorb one more by being built on the assumption that '1+1 = 2'. In contrast, humans see Many as a union of 2D stacks coming from numbering singles, bundles, and bundles of bundles, e.g., T = 345 = 3*BB + 4*B + 5*1. Essence-math sees mastery of math as its goal. A difference could see mastery of Many as the goal to be reached directly by using the bundle-numbers with units that children bring to school and that allows learning core math automatically (Tarp, 2018, 2024). Essence-math could be more meaningful by de-modelling it (Tarp, 2020) built on the French poststructuralist version of Existentialism where deconstruction is used to replace predicates with verbs (Derrida, 1991). As to differences, Difference Research (Tarp, 2018) searching for differences making a difference may design micro curricula to test with Design Research and building on the observation that when asked "How many years next time?" a 3-year-old child will say "That is not 4, that is two 2s" if seeing the 4 fingers held 2 by 2.

Counting Many with Bundles, Children Deserve a Bundle-Number Curriculum

A01. Bundle-counting with units and snap-cubes or a ten-by-ten BundleBundleBoard, two 3s is two bundles with 3s per bundle. The per-number 3s exists in space, the counting-number two in time. The Algebra Square reunites unlike and like counting- & per-numbers (table 1). Polynomials use bundle-counting with units. 43 = 4*B+3*1 = 4B3 tens, and 543 = 5BB4B3 tens. Bundle-numbers falsify '1+1 = 2' with two V-signs where one 1s + one 1s = one 2s and two 1s + one 2s = one 4s, and not three 3s.

A02. Flexible Bundle-counting in space. Space-count five and ten fingers in 2s, 3s, 4s and 5s. 5 = 1B3 = 3B-1 = 2B1 = 1BB0B1 2s, and Ten = 2BB0B2 = 1BBB0BB1BB0 2s. And T = 38 = 3B8 = 2B18 = 4B-2. T = 35+46 =3B5+4B6 = 7B11 = 8B1. T = 6*28 = 6*2B8 = 12B48 = 16B8 = 168. And T = 4507 = 4BBB 5BB 0B 7, T = 4*B^3 + 5*B^2 + 0*B + 7*1. Place value and carrying is unneeded.

A03. Add and subtract 1digit numbers counted in half-bundles. T = 6+7 = ½B1 + ½B2 = 1B3 = 13. T = 4+7 = ½B-1 + ½B2 = 1B1 = 11. T = 3+4 = ½B-2 + ½B-1 = 1B-3 = 7. And T = 8-6 = ½B3 - ½B1 = 3-1 = 2. T = 6-4 = ½B1 - ½B-1 = 1 - -1 = 2 (so, - - = +). T = 6-8 = ½B1 - ½B3 = 1-3 = -2

A04. Time-counting fingers in ½B, "1,2,3,4,5,6" no!, "0B1, 0B2, 0B3, 0B4, 0B5 or ½B0, ½B1". Time-count from 88 to 100: "8B8, 8B9, 8Bten or 9B0, …, 9B9, 9Bten or tenB0 or 1BB0B0".

A05. Digits are icons. 4 strokes as a 4-icon: ❙❙❙❙ → 𝍸 → 4. And 5 as a 5-icon: ❙❙❙❙❙ → 𝍸❙ →5.

A06. Operations are icons also. Push-away and -back to lift to stack, (division & multiplication), 6/2 means 'from 6 push-away 2s to lift', so 6 = 3x2 = (6/2)x2, T = (T/B)xB (the recount-formula). Pull-away and -back (minus and plus) to get decimals, fractions and negatives. 7 = 3B1 = 3½B = 4B-1 2s.

A07. Recounting between icon and tens. "? 5s gives 40": u*5 = 40 = (40/5)*5, so u = 40/5, i.e., "To Opposite Side with Opposite Sign". 6 7s = ? 8s and 6 7s = ? tens leads to division and multiplication tables where 6 7s = 6*7 = (B-4)*(B-3) = From BB, pull-away 3B & 4B and pull-back the 4*3 pulled-away twice = 3B12 = 4B2 = 42. So (B-4)*(B-3) = BB - 3B - 4B + 4*3. Here, minus * minus is +.

A08. Recounting physical units gives per-numbers as 2$/5kg. 20kg = (20/5)*5kg = (20/5)*2$ = 8$. Meter = (meter/second)*second = speed*second. Fractions with like units: 2$/5$ = 2/5. Trigonometry in a stack: height = (height/base)*base = tan(Angle)*base. Pi = n*tan(180/n) for n high enough.

A09. Bundle-bundles are squares. 3 3s= 1BB 3s= 1BB2B1 2s= 1BB-2B1 4s. So, 1BB2B1 = next BB.

A10. Squaring stacks. T = 6 4s = 1BB where B=√(6*4). Guess 1: '(6-1) (4+1)s' or '5 5s', since ½(6-4) = 1. The empty 1-corner needs two 't 4s' stacks, and t*4 = ½ gives t = 1/8. Guess 2: '4.9 4.9s'.

A11. Solving quadratics. A (u+3) square has two squares and stacks: (u+3)^2 = u^2 + 3^2 + 2*3*u = u^2 + 6*u + 9. If u^2 + 6*u + 8 = 0, all disappears but 1, so, (u + 3)^2 = 1, so u = -4 or u = -2.

A12. Adding next-to and on-top or reversed. 2 3s + 4 5s = ? 8s. Here integral calculus adds areas, and recounting change units. 2 3s + ? 5s = 3 8s. Here ? = (T2-T1)/5 = ΔT/5 roots differential calculus.

A13. Adding per-numbers & fractions. 2kg at 3$/kg+ 4kg at 5$/kg=(2+4)kg at (3*2+5*4)/(2+4) $/kg. And, 2 apples with ½ red + 3 apples with 2/3 red = (2+4) apples of which (2*½+3*2/3)/(2+3) red. Integral calculus adds piecewise or locally constant per-numbers.

A14. The Algebra square. The 'Algebra Square' reunites unlike and like unit- & per-numbers

Table 1. The algebra square shows the ways to reunite unlike and like unit- and per-numbers

Calculations unite/*split Totals in*	Unlike	Like
Unit-numbers m, s, kg, $	T = a + n $T - n = a$	T = a * n $T/n = a$
Per-numbers m/s, $/100$ = %	T = ∫ f dx $dT/dx = f$	T = a^b $\sqrt[b]{T}=a$ $\quad log_a(T) = b$

A15. Fact, fiction & fake models. Fact 'since-then' models quantify and predict predictable quantities by using factual numbers and formulas. Typically modeling the past and the present, they need to be checked for truth and units. Fiction 'if-then' models quantify and predict unpredictable quantities by using assumed numbers and formulas. Typically modeling the future, they need to be supplied with scenarios building on alternative assumptions. Fake 'what-then' models quantify and predict unpredictable qualities by using fake numbers and formulas or by adding without units (Tarp, 2001).

Conclusion, Recommendation and Discussion with a Reviewer

"By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy". We may reach this UN Development Goal by replacing an essence-based curriculum with an existence-based curriculum answering the question 'how many?' by counting and recounting totals before adding them. Digits then occur as icons with as many strokes as they represent, thus becoming units when counting totals existing in time and space with 2D bundle-numbers that are flexible by allowing both overloads and underloads, which makes place values and carrying unneeded. Bundling bundles also lead to squares and square roots; and to power as the first of the operations. The operations are icons also, but with different meanings and opposite order. Division now means counting, iconized by a broom to push-away bundles. Multiplication is iconized by a lift uniting the bundles in a stack that a subtraction rope pulls-away to find the unbundled, seen as decimals, fractions, or negative numbers on top of the stack. Combined, bundling and stacking create a recount-formula with a per-number that changes units and used all over STEM. Thus, both proportionality and trigonometry occur in year one. Once counted and recounted, totals may add on-top, or next-to by areas as with integral calculus, also used to add per-numbers. An existence-based curriculum will finally allow a communicative turn within the number-language as within the word-language in the 1970s (Widdowson, 1978). Using children's own flexible bundle-number with units thus represents a paradigm shift (Kuhn, 1962) that opens new areas for research and innovation; as well as self-organized pre-service and in-service teacher training asking the subject on a BundleBundleBoard instead of the instructor as exemplified on the MATHeCADEMY.net website. **Reviewer**: *Students will find it easier to understand the relationship between units and quantities. The calculation process becomes more flexible, without relying too heavily on traditional algorithms. But I suggest adding case studies and experimental data to make the argument more convincing.* **Answer**: Grade one class A has an essence-math curriculum with unit-free 1D line-numbers and the traditional order from addition to power where only few hear about calculus. And class B has an existence-math curriculum with 2D Bundle-numbers with units and the opposite order. With counting preceding adding, they will meet addition in part 12 and meet the core of math directly, calculus and linearity. What will happen if two students exchange classes? How ethical is it to test essence-math against existence-math in an ordinary school? Or in special education where the students will return far ahead to normal education. Ethical testing is for teacher education and home education. And a composer should be allowed to compose and leave it to others to play and evaluate the music.

References

Bauman, Z. (1990). *Thinking sociologically.* Blackwell.

Derrida, J. (1991). *A Derrida reader: between the blinds.* P. Kamuf (ed). Columbia Uni. Press.

Kuhn T.S. (1962). *The structure of scientific revolutions.* University of Chicago Press.

Piaget, J. (1969). *Science of education of the psychology of the child.* Viking Compass.

Sartre, J.P. (2007). *Existentialism is a humanism.* Yale University Press.

Tarp, A. (2001). Fact, fiction, fiddle - three types of models. in J. F. Matos, W. Blum, K. Houston & S. P. Carreira (Eds.). Modelling and mathematics education: ICTMA 9: Applications in Science and Technology. *Proc. 9th Int. Conf. on the Teaching of Mathematical Modelling and Applications* (pp. 62-71). Horwood Publishing.

Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: From inside-inside to outside-inside understanding. *Ho Chi Minh City Univ. of Education Journal of Science 17*(3), 453-466.

Tarp, A. (2024). *Many before math, math decolonized by the child's own bundle bundle-numbers with units.* https://youtu.be/uV_SW5JPWGs.

Vygotsky, L. (1986). *Thought and language.* MIT press.

Weber, M. (1930). *The protestant ethic and the spirit of capitalism.* Unwin Hyman.

Widdowson, H. G. (1978). *Teaching language as communication.* Oxford University Press.

TSG1-26

GRADE 10 CALCULUS: WHY AND HOW

Wei-Chang Shann
National Central University, Taiwan, shann@math.ncu.edu.tw

The rationale of high school calculus is reviewed. Accordingly, the author explains why the calculus curricula should start at the first year of high school, namely grade 10 for most East Asia countries. But how can we do that? The author proposes an approach which is a combination of the insight of local straightness by David Tall and the technique of iterated synthetic division by Jiushao Qin. This approach, a polynomial approach, implements the idea of treating calculus as algebra by Joseph-Louis Lagrange. One feature of the proposed curriculum is that students manipulate exact algebra only and no concept of limit is required. Instead, the necessity and motivation of the notion of limits is developed during the study and leads naturally to the standard approach of calculus, namely the limit approach. With the fundamental experiences of polynomial calculus at grade 10, students can move on, hopefully more smoothly, to the calculus of elementary transcendental functions by the standard approach at grades 11 and 12. At the end of this short presentation, pilot teaching experiments are reported.

Keywords: high school, calculus, polynomial approach, locally straight

Background and Motivation

Calculus made its way to high school in the first half of the 20th century. This movement is considered a legacy of the German mathematician Felix Klein (1849–1925) (Isoda, 2019). Klein started to propose the idea of making the studies of *function* the hub for high school mathematics curriculum in late 19th century. He argued that function can be the central topic that links arithmetic, geometry, algebra and analysis. By doing so, the subjects of mathematics can be made an integrated one as it should be, instead of being a composition of separated courses on each branch of mathematics (Klein, 2016). As the mathematical meaning of *function* was invented by Gottfried Wilhelm Leibniz (1646–1716) for the expression of concepts in calculus, it was a natural consequence that calculus was considered a topic in the integrated high school math curriculum. Klein's idea invoked discussions among mathematicians from West Europe, North America and Japan. As a result, the International Commission on Mathematical Instruction (ICMI) was established in 1908 at the 4th International Congress of Mathematicians (ICM) in Rome, with Klein the first elected chairperson. It is well known that, since 1969, ICMI organized the International Congress on Mathematical Education (ICME).

However, before 1950s, Klein's agenda did not prevail in most of the world outside of the German speaking countries due to the interruption of two disastrous world wars. In late 1950s, as the world was recovering from the wars and Soviet's launch of the first artificial satellite shocked United States, mathematics education was under discussion again. This time, the stake holders were no longer the mathematicians alone. People think of mathematics education in terms of national needs, they may or may not understand Klein's ideology. With or without Klein, calculus entered quickly into high school curricula either as a topic in the required integrated courses or as an elective or advanced placement course.

Why Should Calculus Start at Grade 10?

There are at least two perspectives on the rationale of encompassing calculus in the high school mathematics curricula. One is mathematically intrinsic: to link together different topics and hopefully to make sense of some learning topics. For instance, without calculus, there makes little sense to write the radicals $\sqrt[n]{x}$ in the exponential form $x^{1/n}$, there are no reasons to solve polynomial equations and inequalities, and there is no need of the notation $f(x)$. The other one is extrinsic: to support personal developments in the presumption of necessities for industry, economy, or even national defense.

Calculus is already in the high school curricula for most countries. Over the years, even the entry level college textbooks employ calculus. Notably the textbooks meant for freshman on the subjects of physics, chemistry, biology, statistics and economy, they all employ concepts and notations of calculus freely (Kao, 2009). For this reason, high school calculus becomes more and more popular and there is no way of turning back.

For most of us in East Asia, the introduction of calculus into high school in 1960s was done in a hurry.[1)] It is understandable that the rationale for doing so was mostly pragmatic and the intrinsic motivation was not well received. As a result, most high school calculus materials are not well integrated with the rest of topics. Curricula designers paid little attention to the ideal of making calculus a hub that integrates branches of the mathematical concepts and objects. A typical approach of high school calculus teaching is to trim down the college calculus. In particular, most high school calculus are taught by the standard limit approach, only *without the limit*, or with a certain kind of *limit in disguise*.

The author stands with Klein. He believes the integration is good for both pedagogic and pragmatic reasons of mathematics education. Under the assumption that the materials in grade 10 are mostly polynomial functions and equations, namely line equations and circle equations, this article proposes a teaching plan, the *polynomial approach*, that integrates calculus with grade 10 mathematics. It offers advantages over the traditional limit approach in the sense that differentiation and antiderivatives are developed along the learning of polynomial functions, and the complete process can be done with exact algebra, no limit is involved. The techniques for dealing with limits are postponed to grade 11 when students are introduced to further calculus on transcendental functions such as trigonometric and logarithmic functions. The algebraic techniques will serve as a convincing reason for the introduction of limits. If there is an appropriate physics curriculum for grade 10, this teaching plan can associate with rectilinear motion, free fall and free projectile.

How to Teach Grade 10 Calculus: A Polynomial Approach

The proposed approach of teaching grade 10 calculus focuses on polynomial functions. Students have the opportunity to acquire all key concepts and typical applications of calculus within the scope of polynomials. If time allows, the scope can be extended to rational functions and radicals of polynomials, but they are not necessary for grade 10.

Conceptually, there are two key points which come from the West: local straightness by David Tall (1941–2024), and derivatives as Taylor series coefficients by Joseph-Louis Lagrange (1736–1813). Technically, there is a Chinese antique: the iterated synthetic polynomial division by Jiushao Qin (1202–1261).

1) Americans gave up the intention of integrating calculus with high school mathematics. They put college calculus directly into high school as Advanced Placement courses. Today, their high school mathematics are still mostly offered by separated courses for individual subjects. They have their historical and social contexts for being so.

Tall (1985, 2012) argues that how students can really understand calculus is by the recognition that all (smooth) curves, if looked close enough, are locally straight lines. This statement is certainly not generally true (ignoring the premise of being smooth), but it is true enough for the graphs of polynomial functions and simple polynomial equations like circles and conic sections. At the outset of graphical calculators, Tall realized its great potential for the assistance of calculus teaching and learning. Now, with apps like Desmos on browsers and smart phones, we are at even better position. A carefully engaged explorational activity can firmly convince learners that all polynomial curves are locally straight.

The next step is to embody the local straightness algebraically. Lagrange tried to build the foundation of calculus on Taylor series (Grabiner, 1990). He attempted to define the *n*th derivative of a function at a point by its *n*th Taylor coefficient multiplied by *n*!. Now we know his idea could not work because there are non-analytical functions. However, it works for polynomial functions, and Lagrange developed the concept of limit in order to make his model work. His successors, including Augustin-Louis Cauchy (1789–1857), finally devised the ε-δ, or the error-difference, argumentation, which became the standard approach of today. The failure of Lagrange can be revived to be a perfect plan for the introduction of calculus on polynomials to grade 10 students.

For a polynomial function $f(x)$ of degree *n*, we can always change its reference point from 0 to *c*, namely it can be converted to the Taylor polynomial centered at *c*:

$$f(x) = a_0 + a_1(x-c) + a_2(x-c)^2 + \text{L} \ + a_n(x-c)^n$$

It is a standard knowledge that $a_0 = f(c)$ and a_n is simply the original leading coefficient of *f*. The local straight line we have just explored is actually the line $y = a_1(x-c) + a_0$. We now call this line the tangent line of *f* at $(c, f(c))$, and the slope of the tangent is denoted by $f'(c)$, which notation was invented by Lagrange. Here comes the crucial technique: the iterated synthetic polynomial division, which can proceed easily for the coefficients a_0, a_1, a_2, ⋯, a_n.

After a few practicing exercises with concrete numbers, students are led to realize that $f'(c)$ is in fact a formula for the slope of tangent for any number *c*. In this sense, the formula $f'(c)$ is a function in the variable *c*. Why don't we change the name of variable to our beloved *x*? And the derivative function $f'(x)$ is born. The geometrical meaning of the signs of $f'(x)$ and the concept of critical points can be understood. We have the first type of applications: the optimization problem for polynomial models. The linearity of derivatives, the product rule and the quotient rule of derivatives, can be justified algebraically. A special kind of chain rule for $[f(x)]^m$ can be studied by mathematical induction.

If we divide $f(x)$ by $x-c$, the quotient $q(x)$ is a polynomial of one less degree. It is perfectly all right to substitute *x* for *c* and obtain $a_1 = q(c)$. However, when we attempt to rewrite $q(x)$ in the quotient form $\dfrac{f(x)-f(c)}{x-c}$, we simply cannot substitute *x* for *c*! An intention to resolve this awkward situation is the motivation for the notion of limit. It is to be continued at grade 11.

We omit antiderivatives, definite integrals and their applications due to the limitation of space.

Conclusion

The author started to write high school calculus lecture notes and make teaching videos in the MOOCs style since 2015. He personally taught experimental courses to grade 10 students in the academic years of 2018 and 2019. The first year was done with 15 students in Zhongli Senior High School and the second year with 36 students in Dayuan International Senior High School. Since then, the author organized an online course of high school calculus with 103 videos (7–15 minutes each) completed with lecture notes (in pdf format), homework assignments, quiz and examination question banks. Eight teachers and 416 students from four schools in Taipei were involved.

We conclude that calculus for polynomial functions with elementary applications—optimizations, rectilinear motions, and vloumes of revolutional solids—can be learned successfully by more than 70% of grade 10 students. The curriculua that includes exponential and logarithmic functions, and the inclusion of subtle applications and further techniques of integration, can be learned by more than 60% of grade 11 students (Shann, 2025). One notable exception is *related rates*. We finally dropped related rates from the online course. Two interesting findings are worthy to report. First, even when students found a formula like $f'(c) = 2c + 1$, they still cannot perceive $f'(c)$ as a function in c; this is a chance to learn *function* for real. Secondly, when we write an equation like $2x^2 - 3x + 1 = a_0 + a_1(x-1) + a_2(x-1)^2$, many students thought they were supposed to solve the equation for x; this is where polynomials must be seen as mathematical objects. These are the examples of learning opportunities provided by grade 10 calculus.

References

Grabiner (1983). Who gave you the epsilon? Cauchy and the origins of rigorous calculus. *Amer. Math Monthly*, *90*, 185–194.

Grabiner (1990). *The calculus as algebra: J.-L. Lagrange, 1736–1813*. Garland.

Isoda, M. (2019). The road of the German book *Praktische Analysis* into Japanese secondary school mathematics textbooks (1943-1944): An influence of the Felix Klein movement on the Far East. In H.-G. Weigand et al. (Eds.), *The legacy of Felix Klein* (pp. 117-129). ICME-13 Monographs.

Kao, C.-C. (2009). *A discussion on the compulsory courses in the first semester of freshman year and the calculus course of high school.* Unpublished master thesis. National Central University. [in Chinese] Retrieved from https://shann.idv.tw/articles/gaocj.pdf

Klein, F. (2016). *Elementary mathematics from a higher standpoint* (Vols. 1, 2 translated by G. Schubring; Vol. 3 translated by M. Menghini & A. Baccaglini-Frank). Berlin: Springer.

Shann, W.-C. (2025). *Grade 10 Calculus*. Unpublished webpages. [in Chinese] Retrieved from https://shann.idv.tw/edu/curr.html#calc10

Tall, D. (1985). Understanding the calculus. *University of Warwick institutional repository.* Retrieved from https://www.researchgate.net/publication/37143298_Understanding_the_calculus

Tall, D. (2012). A sensible approach to the calculus. *El Cálculo y su Enseñanza*. Retrieved from https://www.semanticscholar.org/paper/A-Sensible-approach-to-the-Calculus-Tall/dec6867f6c814776abc1aa929dd654278b0ae0d3

TSG1-27

TRENDS AND PERSPECTIVE IN RESEARCH ON DIGITAL MATHEMATICS TEXTBOOKS: INSIGHTS FROM A SYSTEMATIC REVIEW

Kyungwon Lee
Seoul National University, Korea, kyungwon.lee.snu@gmail.com

Hyosup Shim
Seoul National University, Korea, hyosupshim@snu.ac.kr

Oh Nam Kwon
Seoul National University, Korea, onkwon@snu.ac.kr

Digital resources are increasingly recognized as more than mere tools—they serve as critical educational assets that open new possibilities for teaching and learning. This study aims to analyze the existing research on digital mathematics textbooks (DMTs). Adopting the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines, a systematic literature review was conducted. The findings reveal that the number of studies on DMTs has gradually increased. European countries, especially Germany, have led the research on digital resources. Additionally, studies focusing on high school and university are the most prevalent. Quantitative research methods were conducted relatively more frequently, with a strong focus on studies examining interactions between students and DMTs. This study contributes to a broader understanding of global interest in DMTs, highlighting regional and school-level, research methods, sample sizes, and participants in this evolving field.

Keywords: digital mathematics textbooks, digital resources, systematic literature review

Introduction

With the continuous advancement of engineering technologies, the distribution of digital devices for educational purposes is steadily increasing. The use of digital devices in classrooms can take two primary forms: utilizing software for educational purposes while focusing on printed textbooks as the main resource, or implementing printed textbooks directly on digital devices. By embedding printed textbooks into digital devices, it becomes possible to integrate various types of software into the textbooks, enabling their functionality in digital environments. This potential distinguishes digital textbooks from printed ones, highlighting the importance of developing appropriate resources (e.g., Pepin et al., 2015). The study of digital mathematics textbooks (DMTs), which represent digitally implemented mathematics textbooks, is a relatively recent field of research due to its close connection with technological advancements. Consequently, the amount of information available in this area remains limited. This study aims to provide foundational knowledge about DMTs and offer information essential for designing empirical research by conducting a systematic review of existing literature on DMTs. The research question is as follows: *How are the studies on digital mathematics textbooks distributed, and how can they be characterized by the country, appearance over time, research methods, and focus?*

Theoretical Framework

The didactical tetrahedron model, proposed by Rezat and Sträßer (2012), extends the traditional didactical triangle—which describes the interactions between the teacher, student, and mathematics—by incorporating artifacts as a fourth vertex. This model highlights the significant role of instructional tools, such as textbooks and digital technologies, in structuring the teaching and learning environment. It emphasizes how these artifacts mediate teacher-student interactions and contribute to mathematics learning. This model has been widely applied in various studies, including research on teachers' use of instructional resources (Hjelmborg et al., 2020), students' engagement with textbooks (Rezat, 2013), and the analysis of curriculum resources (Rezat, 2024). In this study, we systematically review research on DMTs. Within the framework of the socio-didactical tetrahedron, DMTs are categorized as artifacts, meaning they interact with teachers, students, and mathematical content, thereby shaping the didactical environment. This emphasizes the importance of investigating how DMTs function in mathematics education and analyzing the existing research in this domain. Building on this theoretical framework, this study systematically reviews existing research on DMTs in mathematics education, providing a comprehensive overview of the current literature and identifying key areas for future research and discussion.

Methods

To structure this study and ensure the quality of the systematic literature review, we followed the Preferred Reporting Items for Systematic reviews and Meta-Analysis (PRISMA) guidelines (Page et al., 2021). Our article selection process consisted of three stages: (1) identification, (2) screening, and (3) inclusion. In the identification process, we obtained search results for "digital mathematics textbooks" and "e-mathematics textbooks" from the Web of Science database. A total of 868 articles were identified during this process. In the screening process, we determined whether the articles were published in journals rather than book chapters or conference proceedings. Additionally, we assessed whether the articles were related to mathematics education. Following these screening procedures, 819 articles were excluded. In the inclusion process, 48 articles related to DMTs published in mathematics education journals were selected. 48 articles were reviewed by the authors and were classified and analyzed based on the number of publications per year, the countries of affiliation of the first authors, the focus of the research, and the school levels targeted by the studies.

Results

The distribution of the number of studies on 48 DMTs by year is illustrated in Figure 1. Prior to 2016, only 1 to 2 studies were published annually. However, the number of studies began to increase starting in 2016, with an average of 5 studies reported annually between 2016 and 2024. It is also noteworthy that a significant concentration of studies has been observed since 2020. This trend may be attributed to the increased attention to online mathematics teaching and learning following the COVID-19 pandemic, which likely spurred further research on DMTs. Specifically, 9 studies were reported in 2022, and 10 studies in 2024. This distribution highlights that research on DMTs has become a recent focal point and demonstrates that it is an increasingly recognized area of study.

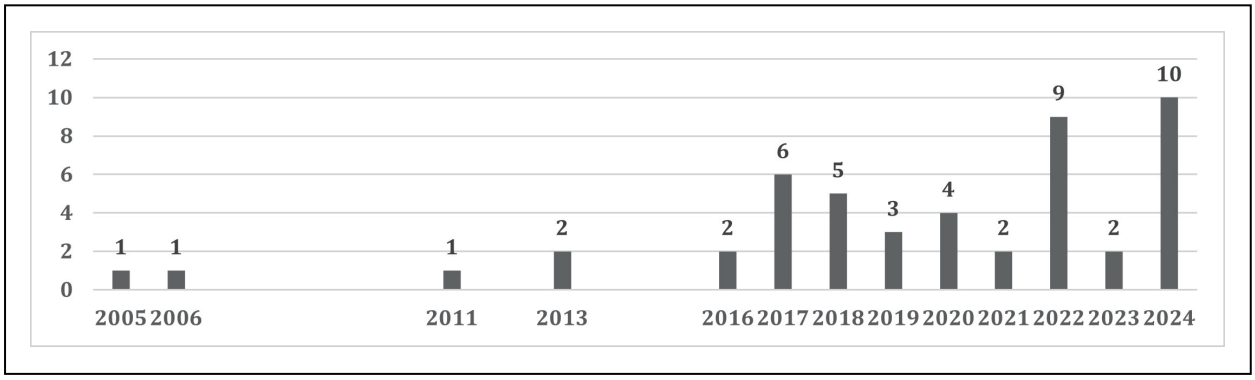


Figure 1. The distribution of the number of studies from 2005 to 2024

The geographical distribution of research on DMTs is presented in the diagram of Figure 2. Europe, the continent with the highest number of studies on DMTs, has conducted research across 11 countries (n=21, 43.8%). Among the European countries, Germany (n=6) has the highest number of studies, making it the leading country in this field. In European countries except Germany, between one and three studies were reported. Asia follows Europe as the region with the next highest number of studies on DMTs (n=16, 33.3%) across 11 countries. Among Asian countries, Türkiye (n=4) emerged as the leading contributor, reporting the highest number of studies. In other Asian countries, only 1 or 2 studies were reported. The United States (n=6) ranks as the second-highest contributor to research on digital textbooks, following Germany. These findings provide insights into the varying levels of interest in digital mathematics textbook research across different countries.

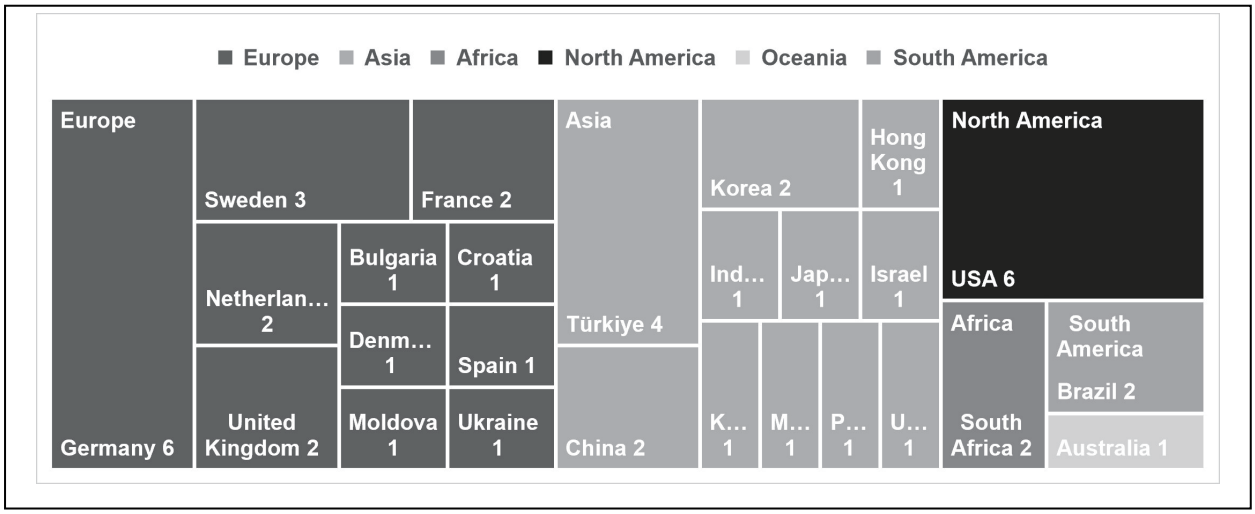


Figure 2. The geographical distribution of the studies

Of the 48 studies reviewed, 35 (72.9%) were empirical studies that involved obtaining data from instructors or students, significantly outnumbering the 13 non-empirical studies (27.1%). The non-empirical studies primarily focused on analyzing the features, structure, or tasks of DMTs or comparing printed mathematics textbooks with DMTs.

When 35 empirical studies were categorized by school level, the distribution was as follows: University level (n=13, 37%), high school level (n=8, 22.9%), elementary school level (n=8, 22.9%), middle school level (n=4, 11.4%), middle and high school levels (n=1, 2.9%), and elementary and middle school levels (n=1, 2.9%). Empirical studies on DMTs have predominantly been conducted at the elementary, high school, and university levels. This trend may be attributed to the limited availability of resources related to previously developed DMTs at lower school levels, as well as systemic factors such as the distribution and implementation of digital devices within school systems.

The research methods of the 35 empirical studies were classified as follows. Quantitative studies accounted for 17 papers (48.6%), while qualitative studies comprised 16 papers (45.7%). The sample sizes in quantitative studies ranged from a minimum of 22 to a maximum of 1,218. Specifically, four studies had sample sizes between 1–50, five studies between 50–100, seven studies between 100–500, 1 study between 500–1,000, and 1 study with a sample size exceeding 1,000. In one study, the sample size was not recorded; however, the number of textbook records analyzed was reported, totaling 7,126. Quantitative studies employed statistical techniques such as correlation analysis and factor analysis. Qualitative studies utilized methods such as classroom observations, interviews, task design, and textbook use records. Mixed-method studies, which applied both quantitative and qualitative methods, were reported in two papers (5.7%). The predominance of quantitative studies can be attributed to the use of statistical techniques aimed at providing information on the methods and effects of DMTs use, and comparison between DMTs and printed mathematics textbooks.

35 empirical studies were classified based on the subject of DMTs as follows: 21 studies (60%) focused on the interaction between students and DMTs, nine studies (25.7%) focused on the interaction between teachers and DMTs, and 5 studies (14.3%) focused on the interaction among teachers, students, and DMTs. The majority of studies emphasized how students use DMTs in classroom settings. Given that the majority of the 35 empirical studies were conducted at the high school and university levels, it can also be inferred that the focus lies on the practical use of digital devices by students who can independently operate them.

Conclusion

This study systematically analyzed research reports in DMTs, identifying the major developments and gaps in the field. The results indicate that DMTs have emerged as a prominent research topic alongside the digitalization of education following the COVID-19 pandemic. The research has been particularly active in Europe, with Germany leading the way. The most frequently studied educational level of 35 empirical studies was elementary, high school and university. Among the 35 empirical studies, quantitative research accounts for a relatively larger proportion, with the most common sample size ranging between 100 and 500. Studies focusing on interactions between students and digital mathematics textbooks (DMTs) represent the largest share. This analysis provides fundamental insights into the emerging research field of DMTs. Additionally, these findings suggest that DMTs are garnering significant attention and hold the potential to enhance learning outcomes and the efficiency of teaching and learning processes.

References

Hjelmborg, M. D., Larsen, D. M., Jensen, M. E., Gents, S. D., Christensen, M. S., Junge, L., & Hansen, R. (2020). From teachers' predispositions to different socio-didactical situations. *IARTEM E-Journal, 12*(1), 1-18. https://doi.org/10.21344/iartem.v12i1.728

Page, M. J., Moher, D., Bossuyt, P. M., Boutron, I., Hoffmann, T. C., Mulrow, C. D., ... & McKenzie, J. E. (2021). PRISMA 2020 explanation and elaboration: Updated guidance and exemplars for reporting systematic reviews. *BJM*.

Pepin, B., Gueudet, G., Yerushalmy, M., Trouche, L., & Chazan, D. I. (2015). E-textbooks in/for teaching and learning mathematics: A potentially transformative educational technology. In *Handbook of International Research in Mathematics Education* (pp. 636-661). Routledge.

Rezat, S. (2013). The textbook-in-use: students' utilization schemes of mathematics textbooks related to self-regulated practicing. *ZDM Mathematics Education, 45*, 659–670. https://doi.org/10.1007/s11858-013-0529-z

Rezat, S. (2024). Research on curriculum resources in mathematics education: a survey of the field. *ZDM Mathematics Education, 56*, 223–237. https://doi.org/10.1007/s11858-024-01559-x

Rezat, S., & Sträßer, R. (2012). From the didactical triangle to the socio-didactical tetrahedron: artifacts as fundamental constituents of the didactical situation. *ZDM Mathematics Education, 44*, 641–651. https://doi.org/10.1007/s11858-012-0448-4

TSG1-28

SOME LANGUAGE ISSUES IN DEFINING A NEW MATHEMATICAL CONCEPT: THE CASE OF JAPANESE SECONDARY SCHOOL STUDENTS

Shogo Murata
Nippon Sport Science University, Japan, smurata@nittai.ac.jp

This study focuses on how students construct and refine mathematical definitions. Definition plays a crucial role in introducing or extending mathematical concepts, such as when extending exponents to negative or rational numbers. However, definitions are often presented authoritatively by teachers and textbooks, which can hinder students' understanding of their significance. This study treats defining as a process rather than definition as a product, aiming to promote students' defining activities through task design. The study also highlights the impact of language issues on these activities. For instance, the terms "exterior angle" and "round off" in Japanese affect students' conceptual understanding, as language differences may influence their defining process. The findings suggest that the terminology used in mathematics can create challenges for students and calls for further research on the role of language in mathematical definitions from an international perspective.

Keywords: mathematical definition and defining, extending a concept, task design, labels of a concept

Introduction

In mathematics learning, definitions play a crucial role in introducing a new mathematical concept or extending a familiar concept to a new domain (Borasi, 1992). For example, when we extend an exponent in an exponentiation of natural numbers to negative numbers or rational numbers, we need to consider the definition of the exponent from the perspective of the exponential law. These mathematical activities are essential for authentic learning. Indeed, the importance of constructing and refining definitions has been emphasized (Freudenthal, 1973). However, teachers and textbooks often provide definitions with authority in mathematics classes. It has been found that students do not understand the significance and role of definitions. To address this issue, this study focuses on defining as a process rather than definition as a result. In recent years, several frameworks have been proposed to capture the process of defining by students (Zandieh & Rasmussen, 2010; Kobiela & Lehrer, 2015), and various tasks have been designed to foster students' defining in primary and secondary school mathematics (e.g. Komatsu, et al., 2024).

This study is based on the methodology of topic-specific design research (Gravemeijer & Prediger, 2019). Design research has two goals: to improve teaching and learning in the classroom and to generate theory about learning by students. In this study, I focused on secondary students' defining activities in the classroom as a topic and have designed and implemented several types of tasks to promote students' defining activities. While the designed tasks are effective as an intervention role for students learning, some difficulties related to Japanese language are detected through this design research process. This type of difficulty seems to be important and interesting, because previous studies related to mathematical definition and defining have not focused on it. Given the above, the research question explored in this paper is: What are language issues in the process of defining a new mathematical concept by Japanese secondary school students?

Task Design and Implementation

Defining in Mathematical Inquiry

Defining is characterized as the activity of examining examples and properties of a mathematical concept, constructing a tentative definition, and refining the definition based on the requirements for mathematical definitions (Zandieh & Rasmussen, 2010; Kobiela & Lehrer, 2015). We construct definitions to achieve specific purposes such as proving a conjecture, extending a concept, or organizing a system. If defining does not have these purposes, the activity will have no direction (Borasi, 1992). Therefore, defining should be positioned in a mathematical inquiry. This paper focuses on defining in extending a mathematical concept.

Task Design for Defining in Extending a Mathematical Concept

Based on the above framework, the learning goal in task design was set, as students can recognize the need for definitions while extending a familiar concept to an unfamiliar domain and can engage in activities to refine the tentatively constructed definitions. To achieve this goal, I designed two tasks to promote defining in extending a mathematical concept (see Figure 1 & 2).

Q1-1: Is the sum of the exterior angles 360° even in the case of a concave quadrilateral like the following figure?

Q1-2: When we extentd a polygon to a concave one, how should we define an exterior angle of it?

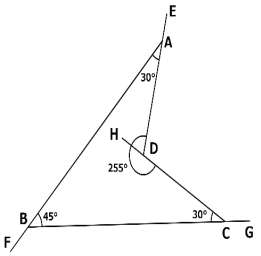


Figure 1. The exterior angle task

Q2-1: Let y be the number obtained by rounding off the first decimal place of the value of x ($x \geqq 0$). Draw the graph the relationship between x and y in the xy-plane.

Q2-2: When we extentd a round function to negative numbers, how should we define it?

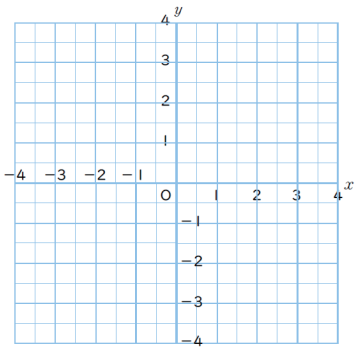


Figure 2. The round function task

The exterior angle task (Figure 1) is intended to get students to consider the definition of an exterior angle of a concave polygon in the context of the sum of the exterior angles of a quadrilateral. If the exterior angle at point D is ∠ADC, then the sum of the exterior angles is 150 + 135 + 150 + 105 =

540 (°). On the other hand, if the exterior angle at point D is ∠ADH, and it can be negative values, then the sum of the exterior angles is 150 + 135 + 150 + (−75) = 360 (°) .

The round function task (Figure 2) is intended to get students to consider the definition of a round function in $x < 0$ in the context of drawing the graph of a round function. If the graph of the function becomes successively ●—○, then -1.5 is rounded off to -1. If the graph of the function becomes a point symmetry at the origin, then -1.5 is rounded off to -2.

Task implementation and the results

Using the above tasks, the classroom interventions wert implemented at a public lower secondary school and an affiliated lower secondary school. The teachers were regular mathematics teachers at these schools. The lessons were transcribed from video and audio recordings.

Both lessons were basically carried out as intended. Most students were able to participate in defining the problematized mathematical concepts. Some students agreed with one definition of the concept, and the other students agreed with the other definition of the concept. In the case of the exterior angle task, the students who claimed that the sum of the exterior angles of a concave quadrilateral is 540° considered the definition of the exterior angle from the perspective of the label, "exterior".

Yui:	The exterior angle is ∠ADC because this angle is placed outside of the quadrilateral.
Masa:	Yes, an exterior angle means an angle of the exterior part.

Similarly, in the case of the round function task, the students who claimed that -1.5 is rounded off to -2 considered the definition of the exterior angle from the perspective of the label, "round off".

Kohei:	Round off (shi-sha-go-nyu, in Japanese) means that if the number of digits to be rounded off is 4 or less, it is rounded down, and if it is 5 or more, it is rounded up.
Miki:	I have the same opinion. If -1.5 is rounded off to -1, then it is no longer round off.

In Japanese, the operation of rounding off is called shi-sha-go-nyu (四捨五入 in Chinese characters). The term "shi-sha (四捨)" literally means "to discard the number four", and it means to round down any number that is less than or equal to four. On the other hand, the term "go-nyu (五入)" literally means "to include the number five", and it means to round up any number that is more than or equal to five. As a result, the students were able to consider two types of definitions of the extended mathematical concept, but compared to the exterior angle task, the students tended to focus more on the labels of the concepts they were defining in the round function task.

Discussion and Conclusion

In both tasks, there was a certain number of students who thought that when considering the definition of a new mathematical concept, they had to completely follow the meaning of the label. From a mathematical view, this tendency is not desirable, because even after extension, in order to preserve the theorem that the sum of the exterior angles of a polygon is 360°, or to preserve the rule that the value exactly midpoint is rounded up to the positive direction in the case of rounding, we need abandon the label of the concept. The difficulty related to labels in defining a new mathematical concept could be due to the influence of language difference. As I showed, while the students worked on the round function task, they have considered the consistency between the operation of

round off and the label of "round off (shi-sha-go-nyu) ". This result suggests that the difference in labels caused by language difference will lead to differences in the students' defining activities.

Related to the exterior angle task, in Japanese, an exterior angle is called by gai-kaku (外角 in Chinese characters). The term "gai (外)" literally means "exterior" and "kaku (角)" literally means "angle". In this case, the system of term construction, "exterior angle" and "gai-kaku" is not so different between English and Japanese. However, related to the round function task, the system of term construction, "round off" and "shi-sha-go-nyu" is quite different between English and Japanese. This paper pointed out that the difficulties Japanese students encounter in learning definitions may be due to the Japanese language system. However, this does not mean that English-speaking learners can easily learn the definitions of a new mathematical concept. Furthermore, other languages such as Chinese or Korean, may present new kinds of difficulties due to the language differences in the labels of a concept. In any case, it is important to recognize that differences in language systems can affect the learning of definitions of a new mathematical concept.

It has been pointed out that the language differences between Japanese and English may cause difficulties in learning mathematics (Shinno et al., 2019), but this point has not yet been fully addressed in research on mathematical definitions and defining. The findings in this paper are based on a qualitative analysis of the activities of Japanese students in two classes. Therefore, it is necessary to increase the amount of data and conduct quantitative analysis in order to enhance the validity of the findings. Furthermore, it is also necessary to conduct a comparative analysis of defining activities between Japanese-speaking and English-speaking students, or to reveal other type of difficulties arising from language systems other than Japanese.

References

Borasi, R. (1992). *Learning mathematics through inquiry.* Heinemann.

Freudenthal, H. (1973*). Mathematics as an educational task.* D. Reidel Publishing Company.

Gravemeijer, K., & Prediger, S. (2019). Topic-specific design research: An introduction. In G. Kaiser, & N. Presmeg (Eds.), *Compendium for Early Career Researchers in Mathematics Education, ICME-13 Monographs* (pp. 33–57). Springer.

Kobiela, M., & Lehrer, R. (2015). The codevelopment of mathematical concepts and the practice of defining. *Journal for Research in Mathematics Education*, 46(4), 423–454.

Komatsu, K., Murata, S., Stylianides, A. J., Stylianides, G. J. (2024). Introducing students to the role of assumptions in mathematical activity. *Cognition and Instruction*, 42(2), 327–357.

Shinno, Y., Miyakawa, T., Mizoguchi, T., Hamanaka, H., & Kunimune, S. (2019). Some linguistic issues on the teaching of mathematical proof, U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp.318-325), Utrecht, Netherlands: CERME.

Zandieh, M., & Rasmussen, C. (2010). Defining as a mathematical activity: A framework for characterizing progress from informal to more formal ways of reasoning. *Journal of Mathematical Behavior*, 29(2), 57–75.

Acknowledgment

This research is supported by JSPS KAKENHI (JP23K12778).

TSG1-29

DEVELOPMENT OF TEACHING MATERIALS ON THE LOCUS OF THE CIRCUMCENTER OF A REGULAR POLYGON

Yuki Osawa
Kaichi Mirai Junior and Senior High School, Japan, yosawa23@outlook.com

This study examines a teaching material on the locus of the circumcenter of a regular polygon for active learning. We have developed a simple teaching material for this purpose. We gave two actual classes to private Japanese high school students using this teaching material. We showed that this teaching material helps students understand the value of vectors, and it is effective for active learning. Moreover, we concluded that this teaching material is effective in increasing students' motivation for generalization and helping students understand the essence of the problems in it through generalization.

Keywords: generalization, active learning, locus of the circumcenter, value of vector

Aim and Methodology of the Study

Active learning in mathematics class has been studied in recent years. For example, Hetmanenko examined the role of active learning in mathematics and its impact on stimulating students' interest in the subject (2024). In addition, there are some studies on the loci of five centers of a triangle (e.g. Komoda, Nishimura, & Takeuchi, 2018). The aim of this study is to develop an effective teaching material on the locus of the circumcenter of a regular polygon for active learning and generalization. In this paper, we will focus on applying vectors to find the locus. First, we will study our new teaching material, and then we will give two classes to high school students (15-16 years old) using it to investigate the value of the teaching material.

Teaching Material

We have created the following problem as a teaching material.

Problem: Given a line l and a point A not on l, find the locus of the circumcenter of a regular n-gon with the segment AP as one of its sides, where the point P moves along the line l.

When we conduct classes, we start with the case where $n = 3$, and then gradually move on to $n = 4, 5$, and the general case of .

Analysis of the Teaching Material

Let the coordinates of P be $(t, 0)$, and A be $(0, a)$,, where $a > 0$. Moreover, let G and G' be the circumcenters of the regular n-gons with AP as one of their sides as shown in Figure 1. Since G and G' are symmetric with respect to AP, the locus of G and that of G' are symmetric with respect to the y-axis, so we can obtain the locus of G' by reflecting that of G across the y-axis. Therefore, we will study the locus of G first.

Let O be the origin, and M be the midpoint of AP. Then, we obtain $\overrightarrow{\mathrm{AM}} = \frac{1}{2}\binom{t}{-a}$. Since the point G is the intersection of perpendicular bisectors of all the sides, $\overrightarrow{\mathrm{MG}}$ is vertical to $\overrightarrow{\mathrm{AM}}$. Hence, $\overrightarrow{\mathrm{MG}}$ can be expressed as $\overrightarrow{\mathrm{MG}} = x\binom{a}{t}$, where x is a positive real number (see Figure 2). Thus, we can express $\overrightarrow{\mathrm{OG}}$ as follows.

$$\overrightarrow{\mathrm{OG}} = \overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{AM}} + \overrightarrow{\mathrm{MG}} = \binom{0}{a} + \frac{1}{2}\binom{t}{-a} + x\binom{a}{t} = \binom{x}{\frac{1}{2}}\, a + \binom{\frac{1}{2}}{x}\, t$$

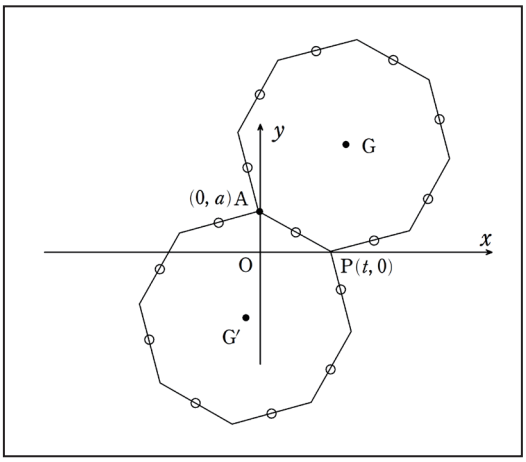


Figure 1.

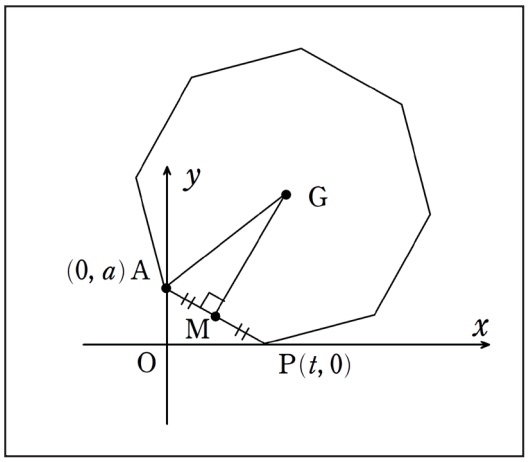


Figure 2.

Here, the angle between the vector $\binom{\frac{1}{2}}{x}$ and the x-axis is the same as $\angle \mathrm{GAM}$, since. $|\overrightarrow{\mathrm{AM}}| : |\overrightarrow{\mathrm{MG}}| = \left|\frac{1}{2}\binom{t}{-a}\right| : \left|x\binom{a}{t}\right| = \frac{1}{2} : x$.Therefore, the locus of G is a line that forms an angle with the x-axis equal to half of an interior angle of the regular n-gon and passes through the point $\left(xa, \frac{1}{2}a\right)$.. The loci of G and G' are shown in Figure 3.

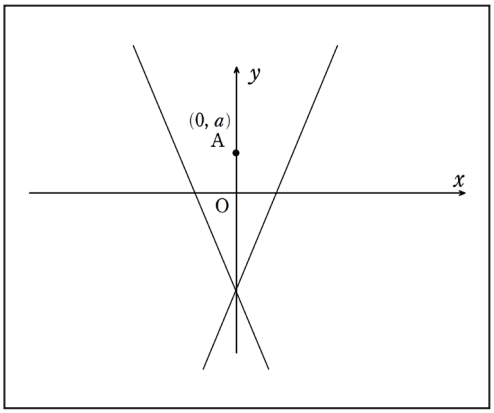


Figure 3.

Actual Classes to High School Students

Outline of the Classes

We gave two classes (50 minutes each) to 10th grade students (15-16 years old) at a private Japanese high school using this teaching material. The number of the students was 24 (one student was absent in each class). They had learned coordinate geometry before, and they had just finished learning plane vectors at school. We gave three types of handouts to the students, and they wrote their ideas on them.

Actual Classes

In the actual classes, we focused on the locus of G (not the locus of G'). In the first class, we started from the case where $n = 3$ in the problem. We had the students think of the problem in groups, and we sometimes gave them hints. We also used dynamic mathematics software GeoGebra to see how the equilateral triangle moves and predict the locus of G. In the second class, we asked the students about the case where $n = 4$, 5, and the general case of n. We told the students that they could use GeoGebra only for checking their answers (but not for making predictions).

Results of Questionnaire and Discussion

Case Where $n = 3$ (equilateral triangle)

According to the questionnaire results, about 96% of the students understood the meaning of the problem, and about 70% of all repeated trials and improvements, although no one could solve it. Therefore, we can say that the problem is difficult but manageable. Moreover, for a question "What kind of questions would you like to explore on the problem?", about 70% of the students formulated questions about the case of another figure such as a general triangle or a square. In other words, the motivation for solving the advanced problem was generated by the students themselves. From this result, we conclude that this teaching material engages students' interest in generalization, and it is effective for improving their perspectives of generalization.

We had the students use GeoGebra to see how to move the equilateral triangle and predict the locus of G after they repeated trials and improvements. It seems difficult for many students to predict the answer correctly without any dynamic software like GeoGebra. In fact, about 65% of the students did not predict that the locus of G is a line. However, they can easily predict the correct answer if they use dynamic software. It helps students to investigate $\overrightarrow{OG}$.

Case Where $n = 4$ (square)

According to the questionnaire results and writings on the handouts, about 39% of the students could prove that the locus is a line, and about 61% of all could express $\overrightarrow{OG}$ in terms of its components, whereas nobody could prove it, and only about 22% of all could express $\overrightarrow{OG}$ in terms of its components in the case where $n = 3$. This means that students understood how to approach the problem through the case where $n = 3$. We can say that students become able to consider more advanced problems through this teaching material.

Case Where $n = 5$ and general n (regular pentagon and n-gon)

According to the questionnaire results, only about 13% of the students could prove that the locus of G is a line that forms an angle with the x-axis equal to 54 in the case where $n = 5$, although they

learned the cases where $n = 3$ and 4. We believe that this is because expressing $\overrightarrow{MG}$ without any unknown quantities is hard in the case where , which is different from the cases where $n = 5$ and 4. Through solving the problem in the case where , students can find that the locus of G is a line that forms an angle with the x-axis equal to half the interior angle of the regular n-gon, and they can understand it is the essence of this problem. After that, they can find the answer for the general case of n.

Throughout This Teaching Material

As we saw above, there are two important roles of generalization in this teaching material. One is to increase students' motivation for generalization, and the other one is to have students understand the essence of the problems through generalization. Watarai proposed that there are two important kinds of generalization, which are intensive generalization and extensive generalization (2015). In this teaching material, thinking of the case where $n = 4$, 5 and the general case of after the case where $n = 3$ corresponds to extensive generalization, and understanding the essence of the problem in the case of $n = 5$ or the general case of n corresponds to intensive generalization. In addition, about 87% of the students repeated trials and improvements through solving the problems, about 83% enjoyed thinking of the problems, and about 91% became interested in the problems, although they were difficult for about 52%, and a little difficult for about 48%, according to the questionnaire results. This result shows that this teaching material is effective in promoting active learning. Moreover, about 87% of all understood the value of vectors through solving these problems. Actually, solving these problems with vectors is simpler and easier than that with coordinate geometry or elementary geometry. Therefore, this teaching material is effective for students who already have learned plane vector if we focus on understanding a value of vectors and on generalization.

Conclusion

We have developed a teaching material on the locus of the circumcenter of a regular polygon. Students can understand the value of vectors with the teaching material. It is effective in promoting active learning, because the problems are so simple that students can repeat trials and improvements, students will become interested in the problems, and they will enjoy thinking of the problems. Moreover, this teaching material is effective in increasing students' motivation for generalization and helping students understand the essence of the problems through generalization.

References

Hetmanenko, L. (2024). The role of interactive learning in mathematics education: fostering student engagement and interest. *Multidisciplinary Science Journal*, 6:2024ss0733.

Komoda, K., Nishimura, K., & Takeuchi, N. (2018). Investigation of the Loci of 5 Centers of Triangles: Perceiving the Connections Between Discipline Education and Discipline. *Journal of Japan Society of Mathematical Education. Volume C,* 3-10.

Watarai, R. (2015). Teaching to promote progressive thinking focused on generalization in mathematics education. *Epsilon / Journal of Mathematics Education Society in Aichi University of Education*, Vol. 57 179-184.

TSG1-30

GEOMETRIC CONSTRUCTION IN TAIWANESE AND JAPANESE JUNIOR HIGH SCHOOL TEXTBOOKS

Takeshi Miyakawa
Waseda University, Japan, tmiyakawa@waseda.jp

Yi-Wen Su
University of Taipei, Taiwan, yiwen@utaipei.edu.tw

Geometric construction has played a crucial role in the historical development of mathematics. However, traditional constructions using only a straightedge and compass are sometimes perceived as outdated today. This paper aims to deepen the understanding of the geometry curriculum—specifically, the role of geometric construction in lower secondary education—through an international comparative analysis of textbooks from Taiwan and Japan. Although Taiwan and Japan share similar educational systems and have historically influenced each other, a praxeological analysis of their textbooks reveals distinct approaches to geometric construction. In Taiwanese textbooks, a straightedge-and-compass construction is consistently used to introduce geometric properties. In contrast, Japanese textbooks place less emphasis on geometric construction, and it does not play a significant role in structuring or introducing the geometric system.

Keywords: international comparative study, textbooks analyses, geometry education, praxeology

Introduction

Geometric construction has held a prominent position in Euclidean geometry since antiquity, contributing to novel perspectives in mathematics. From the perspective of geometry teaching, solving geometric construction problems can serve as a motivation for the development of geometric theorems and properties (Chevallard, 2002). Consequently, it is considered a bridge between the practical and theoretical aspects of geometry (Mariotti, 2002; Fujita et al., 2010).

With the advent of dynamic geometry software (DGS), geometric construction has become more accessible than ever before, leading to numerous studies on its use in classrooms (Sinclair & Robutti, 2013). The use of tools, including straightedges, compasses, and DGS, is regarded as a critical component of geometry learning (Kuzniak, 2015). However, traditional constructions using only a straightedge and compass is sometimes perceived as outdated in today's educational contexts.

This paper examines the role of geometric construction in the geometry education in Taiwan and Japan, where straightedge-and-compass constructions remain part of the mathematics curriculum. Through a comparative analysis of textbooks, we investigate the following questions: *How is geometric construction organized within the junior high school curricula of Taiwan and Japan? What roles does geometric construction play, and how is it connected to other key concepts in geometry?*

Theoretical and Methodological Perspectives

This study adopts the *institutional perspective* proposed within the Anthropological Theory of the Didactic (ATD; Chevallard, 2019). From this perspective, the nature of mathematical knowledge is considered to vary depending on the *institution* (Chevallard & Bosch, 2019, p. xxxi). The institutions under consideration in this study are junior high schools in Taiwan and Japan.

To investigate the mathematical knowledge in an institution, ATD provides an analytical tool called *praxeology* (Chevallard, 2019; Takeuchi & Shinno, 2020). Praxeology is a model of human activity, encompassing both practical and theoretical aspects. It is composed of two components: (1) *praxis block* including the *types of tasks* and the *techniques* used to accomplish those tasks, describing the practical aspects; (2) *logos block* including the theoretical elements that support and justify the praxis. Specifically, it consists of *technology*, which explains and justifies the techniques, and *theory*, which further underpins the technology.

In this study, praxeology is employed to model the mathematical knowledge and practices associated with geometric construction. We identified the following praxeological elements in the textbooks: (1) praxis block: tasks involving the construction or drawing of geometric figures, along with the techniques and instruments used to complete these tasks; (2) logos block: the mathematical properties or justifications supporting these techniques, identified as technology. Theory will be discussed after identifying other elements, as it represents a system that integrates these theoretical components.

This study examines major textbook series: the *Keirinkan* series for Japan (Okamoto et al., 2021) and the *Kang-Hsuan* series for Taiwan (Hung, 2023). The analysis focuses on chapters dedicated to geometric construction, specifically the plane geometry chapter in the Grade 7 textbook for Japan and two chapters in the Grade 8 textbook for Taiwan. A broader overview of how geometry is structured within the curricula of Taiwan and Japan is available in Miyakawa & Su (2025).

Results

The results of the praxeological analyses are presented in Tables 1 and 2. While a variety of types of tasks can be observed in these tables, the number of types of tasks is not directly comparable. Symmetric figures and circles, which appear in Japanese Grade 7 textbooks, are covered in Taiwanese textbooks in Grades 7 and 9, respectively, and thus are not included in Table 1.

The types of tasks fall into two genres: construction and drawing. Construction tasks require the use of a straightedge and compass, whereas drawing tasks, particularly in Japanese textbooks, permit the use of additional instruments. The scope of geometric construction tasks is more limited in Japan than in Taiwan. In Japanese textbooks, straightedge-and-compass constructions are restricted to basic constructions such as perpendicular bisectors and angle bisectors. In contrast, Taiwanese textbooks also include the construction of angles and triangles related to triangle congruence.

Regarding techniques, due to space limitations, we were unable to explicitly list them in Tables 1 and 2. In Taiwanese textbooks, all construction tasks are performed exclusively with a straightedge and compass, with the exception of angle bisector construction, where folding is also introduced as a technique. For tasks involving instruments other than these, the specific instruments used are indicated in square brackets following the types of tasks, particularly for drawing tasks in Japanese textbooks. For instance, techniques involve set squares, protractors, and grid paper. This practice continues in higher grades in Japanese textbooks. In Grade 8, parallel lines are drawn using a set

square, and triangle congruence properties such as SAS and ASA are introduced through the drawing of congruent triangles using a protractor. Additionally, in both countries, DGS is not utilized at all.

Regarding technology, two kinds of technologies can be identified: some for justifying construction techniques and others for justifying their results. In Tables 1 and 2, technologies marked with an asterisk indicate that the construction technique is based on previously established constructions or geometric properties, classifying them as application tasks. In some cases, the justification for a technique relies on the instrument itself. For instance, a compass is justified as a tool for transferring distances, and a set square is justified as a tool for drawing perpendicular lines.

In Taiwanese textbooks, some construction results are validated through measurement or superposition using concrete objects. In the case of constructing a given angle (displacement), the congruency of triangles (e.g., SSS) is employed, even though this property has not yet been formally introduced at the time of teaching angle construction. Consequently, the result is verified by measuring the angles with a protractor. This construction technique is later used as a method for other constructions, such as constructing a triangle with SAS. Similarly, superposition is employed to verify whether constructed triangles are congruent to given triangles in tasks with SSS, SAS, and ASA.

In Japanese textbooks, the distinction between the justification for a technique and the justification for the result of a construction is not always clearly made. For instance, the rhombus is used both to determine the technique for constructing a perpendicular bisector (by recognizing that all four sides are equal) and to justify that the constructed line is indeed a perpendicular bisector. At this stage, no additional justification is provided (later in Grade 8, it is proven using congruent triangles).

Regarding theory, the extent to which a systematic geometric framework is established varies. In Japan, geometric system with mathematical proofs is introduced in Grades 8 and 9 (Miyakawa, 2017). However, in Grade 7, which serves as an introductory phase, the underlying geometric system is not explicitly structured. Some geometric properties are introduced, but most (e.g., translation, reflection) are not subsequently used to prove other properties in later grades. Straightedge-and-compass construction does not play a foundational role in constructing a geometric system. When triangle congruence is introduced in Grade 8, straightedge-and-compass construction is not emphasized. Instead, students continue using protractors and set squares for tasks such as drawing parallel lines and congruent triangles.

In contrast, in Taiwan, geometric construction serves as a means of introducing and justifying new geometric properties, such as SSS and SAS. The construction of equal angles and equal line segments using a straightedge and compass is introduced at an early stage, forming a foundation of geometry. These constructions are later used to develop triangle congruencies and to construct parallel lines. The consistency in types of tasks and techniques, where straightedge-and-compass construction is consistently required, contributes to construct a coherent geometric system.

Table 1. Elements of praxeology in taiwanese grade 8 textbook

Chapter	Section	Types of tasks	Technologies
2-3 Basic properties of triangles	2. Straightedge and compass construction	Construct a line segment	Instrumental
		Construct a given angle	Measurement
		Construct a perpendicular bisector	Rhombus
		Construct an internally dividing point	*perp. bis.
		Construct an angle bisector [folding]	Kite
		Construct a particular angle (45°, 30°, 15°, 75°)	*angle bis., perp. bis.
		Construct a perpendicular line through a point	Iso. triangle, kite
		Construct an equilateral triangle	Equilateral triangle
	3. Congruency of triangles (21)	Construct triangles with SSS, SSA, ASA	Superposition
		Construct triangles with RHS, AAS	RHS, ASA
2-4 Parallel lines and quadrilateral	1. Parallel lines	Construct a parallel line	Corresponding angle
	2. Parallelogram	Construct parallelograms	Parallelogram

Note. Instruments in brackets are used in addition to a straightedge and/or compass.

Table 2. Elements of praxeology in japanese grade 7 textbook

Chapter	Section	Types of tasks	Technologies
5 Plane figures	1. Straight lines and figures	Draw a perpendicular line [set square]	Instrumental
		Draw a parallel line [set square]	Instrumental
		Draw a triangle [protractor]	Instrumental
	2. Motion and geometric construction	Draw a translated figure [set square]	Translation
		Draw a rotated figure [protractor]	Rotation
		Draw a reflected figure [on the grid paper]	Reflection
		Construct a perpendicular bisector	Rhombus
		Construct a midpoint	*perp. bis.
		Construct an angle bisector	Rhombus
		Construct a perpendicular line	Rhombus
		Construct a particular angle (90°, 45°, 30°, 15°, 75°)	*perp. line, angle bis., equilateral triangle
	3. Circles and sectors	Construct a tangent line	*perp. line
		Draw a sector of a circle [protractor]	Sector of a circle
		Draw a straight line	Instrumental
		Draw a reflected figure	*perp. line
		Construct an equidistant point	*perp. bis.
		Construct a shortest point to a line	*perp. line
		Construct a median	*perp. bis.

Discussion and Conclusion

The comparative analysis in this paper highlights distinct approaches to geometric construction in lower secondary geometry education. In Taiwanese textbooks, straightedge-and-compass construction is consistently employed to introduce geometric properties, implicitly supporting the development of a geometric system. In contrast, in Japan, geometric construction is more limited and does not play a significant role in constructing or introducing the geometric system. However, even in Taiwan, the emphasis on geometric construction has gradually diminished compared to previous curricula. Additionally, DGS is not integrated into the geometry curricula of either country. This situation raises broader questions about the role of geometry teaching beyond geometric construction.

References

Chevallard, Y. (2002). Organiser l'étude : 3. Ecologie et régulation. In J.-L. Dorier, M. Artaud, M. Artigue, R. Berthelot, & R. Floris (Eds.), *Actes de la 11e Ecole d'Eté de Didactique des Mathématiques* (pp. 41-56). La Pensée Sauvage.

Fujita, T., Jones, K., & Kunimune, S. (2010). Students' geometrical constructions and proving activities: A case of cognitive unity? In M. F. Pinto & T. F. Kawasaki (Eds.), *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education (PME34)* (Vol. 3, pp. pp. 9-16). PME.

Hung, Y.-C. (Ed.) (2023). *Junior High School Mathematics Textbook Vol. 4*. Kang-Hsuan Educational Publishing Group. (In Chinese)

Kuzniak, A. (2015). Understanding the Nature of the Geometric Work Through Its Development and Its Transformations. In S. J. Cho (Ed.), *Selected Regular Lectures from the 12th International Congress on Mathematical Education* (pp. 1-15). Springer International Publishing. https://doi.org/10.1007/978-3-319-17187-6_1

Mariotti, M. A. (2002). Justifying and Proving in the Cabri Environment. *International Journal of Computers for Mathematical Learning*, *6*(3), 257-281. https://doi.org/10.1023/A:1013357611987

Miyakawa, T. (2017). Comparative analysis on the nature of proof to be taught in geometry: the cases of French and Japanese lower secondary schools. *Educational Studies in Mathematics*, *94*(1), 37-54. https://doi.org/10.1007/s10649-016-9711-x

Miyakawa, T. & Su, Y.-W. (2025). Geometry to be taught in Taiwanese and Japanese junior high schools. *Pre-proceedings of CERME14*.

Okatmoto et al. (2021). *Mathematics for the future 1*. Keirinkan. (in Japanese)

Sinclair, N., & Robutti, O. (2013). Technology and the Role of Proof: The Case of Dynamic Geometry. In M. A. Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Third International Handbook of Mathematics Education* (pp. 571-596). Springer New York. https://doi.org/10.1007/978-1-4614-4684-2_19

Takeuchi, H., & Shinno, Y. (2020). Comparing the Lower Secondary Textbooks of Japan and England: a Praxeological Analysis of Symmetry and Transformations in Geometry. *International Journal of Science and Mathematics Education, 18*(4), 791-810. https://doi.org/10.1007/s10763-019-09982-3

MEMO

MEMO

KOREA 2025

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 2:

The Essence of Mathematics Education in Classroom Practice

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 2
(Day 2)

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

TSG2-1-1

TEACHING AND LEARNING OF SOLID GEOMETRY IN PRIMARY EDUCATION IN NICARAGUA: A DIDACTIC PROPOSAL BASED ON THE IMPLEMENTATION OF THE VAN HIELE MODEL AND THE PROBLEM-SOLVING APPROACH

Douglas Jared Martínez Cruz
Naruto University of Education, Japan, martinezcdouglas@gmail.com

The objective of this research is to analyze the effectiveness of a didactic proposal based on Van Hiele's model in combination with the Problem-Solving Approach in the learning of solid geometry in fifth and sixth grade students in Nicaragua. For this purpose, 8 classes were designed, in the experimental group consisting of 20 5th and 26 6th grade students the Problem-Solving Approach methodology was used and in the control group consisting of 24 5th and 17 6th grade students the usual methodology of the classroom teacher was used. When analyzing the pre- and post-test results, it was evident that the didactic proposal had significant impacts on the three levels of reasoning in the fifth and sixth grades of the experimental group. However, a comparison between the control and experimental groups revealed a significant statistical difference only at level 3 among fifth graders and at levels 1 and 2 among sixth graders.

Keywords: nicaragua, solid geometry, primary education, problem-solving approach, van hiele model

Introduction

In Nicaragua, students' low academic performance is evident in both national and international assessments, with geometry test scores for fifth and sixth graders at 49.7% and 24.0% correct answers, respectively (JICA, 2022). The ERCE 2019 study further highlights the challenge, showing that only 34.6% of third grade students and 3.1% of sixth grade students achieved or exceeded the Minimum Proficiency Level (MPL), which is significantly lower than the regional averages of 52.3% and 17.4%, respectively (UNESCO, 2022). These results emphasize the importance of strengthening teaching practices, as effective methods and well-prepared teachers significantly impact student outcomes. In this context, the research explores the application of the Van Hiele Model (VHM) combined with the Problem-Solving Approach (PSA) to enhance solid geometry knowledge among primary school students in Nicaragua.

Literature Review

According to Jaime (1993) and Guillén (1997), the Van Hiele Model (VHM) consists of two dimensions: a descriptive aspect that categorizes geometric reasoning into five levels (Visualization, Analysis, Informal Deduction, Deduction, and Rigor) and an instructional aspect that organizes learning into five phases (Information, Directed Orientation, Explicitation, Free Orientation, and Integration) to enhance reasoning development. The characteristics are synthesized in studies by Usiskin (1982) and Jaime and Gutiérrez (1990). Research, such as Jaime (1993) and Vargas and Gamboa (2013), emphasizes focusing on the first three levels during primary education, as levels 4 and 5 are not achievable at this stage.

This research contextualizes VHM by replacing its learning phases with the eight steps of the Problem-Solving Approach (PSA), used in mathematics teaching in Nicaragua. PSA fosters problem-solving skills in students through clearly defined steps, as outlined by MINED (2019). By combining VHM levels to structure content and PSA steps to guide methodology, this study introduces an innovative approach aimed at improving solid geometry knowledge and aligning teaching practices with local educational strategies.

Methodology and Research Question

The research followed a quasi-experimental design, based on a pretest-intervention-posttest sequence. The sample consists of 44 fifth grade students, 24 in the control group (CG) and 20 in the experimental group (EG). For sixth grade, the sample includes 43 students, 17 in CG and 26 in EG. The control and experimental groups are in two different schools in Managua within the same urban area, both offering primary education in the morning shift. The experimental school was selected because of its teaching experience in the application of the PSA, while the other school was chosen because of its proximity. Both have one group per grade.

The eight solid geometry contents taught are shown in Table 1. Prior to the intervention, a pretest was administered to all groups. During the intervention, the lessons of the experimental group followed the PSA methodology (Initiation → Central problem of the class → Individual student resolution → Presentation of ideas on the blackboard → Explanation of ideas presented → Establishment of conclusions → Exercise → Culmination), while the lessons of the control group followed the usual classroom teacher methodology. After the intervention, a post-test was conducted to evaluate the results. The pretest and posttest contained 10 items each, some of which could be answered at different levels of reasoning. For example, item 2a could only be solved at level 3 (L3) and item 2b at L1, L2 and L3. Responses were scored as completely incorrect (0), partially correct (1) or correct (2). To ensure comparability, the total scores for each level were standardized to a scale of 0 to 1.

Table 1. Solid geometry contents

Contents	Fifth grade	Sixth grade
Identifying solids geometry	L1	L1
Modelling of solids geometric with commercially available material	L1	L1
Net of prisms and pyramids	L2	L2
Elements of solids geometry	L2	L2
Characteristics of solids geometry	L2	L2
Classification of solids geometry	L3	L3
Definition of prism and pyramid	L3	L3
Lateral area and total area of cubes and rectangular prisms	L2	-
Volume of cubes and rectangular prisms	-	L3

Note. Considering the class achievement indicators and the specific characteristics of Van Hiele's levels, each content was carefully aligned with a corresponding level of geometric reasoning.

To assess the significance of differences between groups, an independent samples t-test was performed. A paired t-test was used to examine improvements in performance between pretest and posttest. The study aims to answer the following research question: How does the learning of solid geometry improve in Nicaraguan fifth and sixth grade students?

Results

The paired samples t-test revealed significant differences between pretest and posttest results in both groups. In grade 5, the experimental group showed remarkable improvements in all levels of reasoning (L1, L2 and L3), demonstrating the effectiveness of the PSA intervention. In contrast, the control group only showed significant progress at the L1 and L2 levels, probably due to the limitations of the traditional methodology, which affected their performance in L3. In sixth grade, both groups improved significantly at all levels, but the experimental group exhibited more pronounced advancements, further confirming the efficacy of PSA in fostering the acquisition of Van Hiele levels in geometry.

The t-test for independent samples, used to analyze the differences (d=post-test-pre-test), showed that fifth grade students in the experimental group obtained significantly better results only at L3. For sixth-grade students, statistical analyses identified significant progress in levels L1 and L2 within the experimental group. While both groups showed improvements in their results, the outcomes are more notable in the experimental groups, as illustrated in Figure 1.

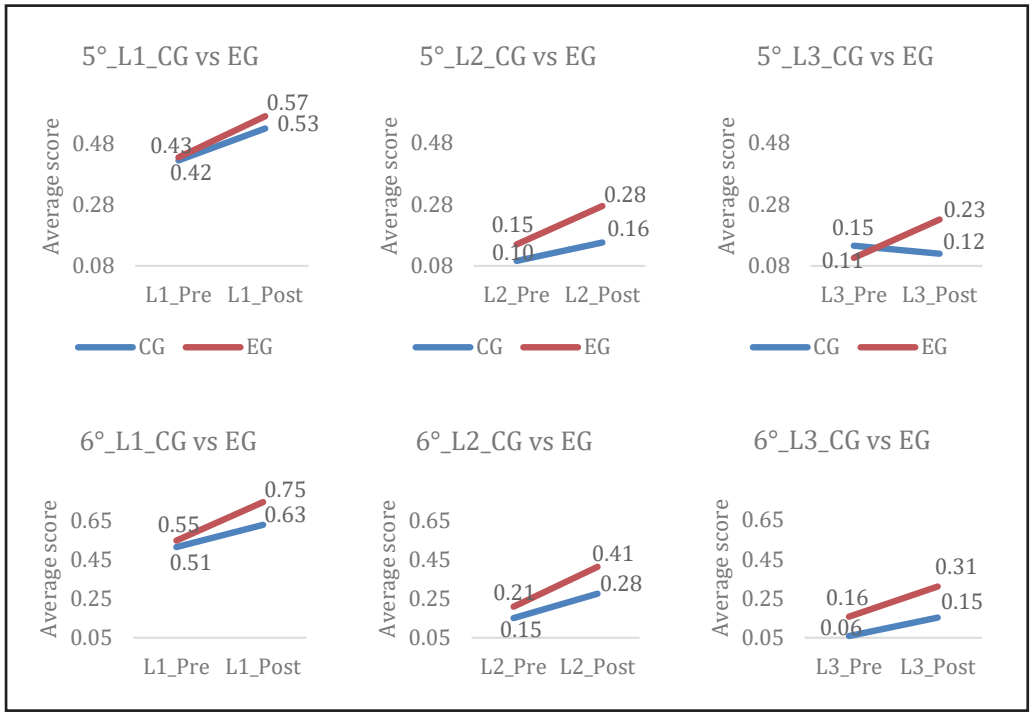
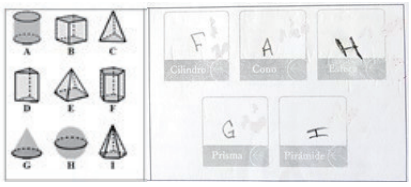
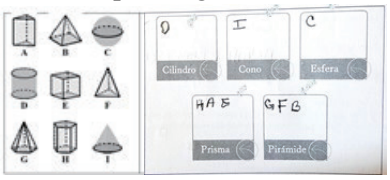
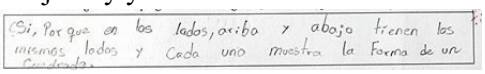
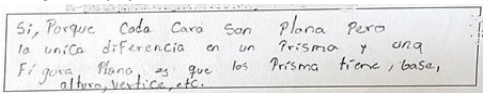


Figure 1. Comparison of pre- and post-test results between EG and CG

Table 2 shows how students improved their learning in geometry of solids. For example, in item 2a, the student in the pretest failed to recognize geometric solids, classifying his answer at level L3 as incorrect. In the post test, however, the student classifies them correctly, reaching level L3. In item 2b, they were asked to justify why the figure is a prism. In the pretest, the student's vocabulary was limited to everyday terms such as 'top' and 'bottom', corresponding to L1 level. In the posttest, an advance is evidenced with the use of geometric vocabulary, arguing by means of elements of the geometric solid such as 'base' and 'height'. Although he does not manage to provide a complete definition of the solid to reach L3 level, the use of elements allows him to classify his response at L2 level. In addition, improvements in the graphical representation of cubes and related problem solving are identified.

Table 2. Sample student responses

Geometric skills	Pretest	Post test
Recognize geometric solids	2a. Write the letter of each solid in the corresponding. 	2a. Write the letter of each solid in the corresponding. 
Argue geometric propositions/definitions	2b. Is figure D a prism? YES or NO, justify your answer.  *Yes, because the sides, top and bottom have the same sides, and each one shows the shape of a square.*	2b. Is figure E a prism? YES or NO, justify your answer.  *Yes, because each face is flat, but the only difference between a prism and a plane figure is that prisms have base, height, vertex, etc*

Conclusions

The intervention based on the Van Hiele model significantly improved the geometric reasoning of fifth and sixth graders, especially in the experimental groups. Fifth graders excelled at level 3 of the Van Hiele model, while sixth graders showed significant progress at levels 1 and 2. The impact of the intervention was more pronounced in the experimental groups than in the control groups.

In addition, students made progress in solid geometry by improving their ability to recognize, classify and describe geometric solids, albeit without precise definitions. Their graphical representations became more precise, and they showed significant improvements in understanding and calculating the volume of cubes and prisms.

References

Guillén S, G. (1997). *El modelo de Van Hiele aplicado a la geometría de los sólidos: Observación de procesos de aprendizaje* [Van Hiele's model applied to the geometry of solids: Observation of learning processes]. https://dialnet.unirioja.es/servlet/tesis?codigo=9178

Jaime, A. (1993). *Aportaciones a la interpretación y aplicación del modelo de Van Hiele: La enseñanza de las isometrías del plano. La evaluación del nivel de razonamient*o [Contributions to the interpretation and application of Van Hiele's model: The teaching of plane isometries. The evaluation of the level of reasoning]. Doctoral thesis, University of Valencia. https://www.uv.es/gutierre/archivos1/textospdf/Jai93.pdf

Jaime, A., & Gutiérrez, A. (1990). *Una propuesta de fundamentación para la enseñanza de la geometría: El modelo de Van Hiele*. [A proposal for a foundation for teaching geometry: Van Hiele's model]. https://www.uv.es/angel.gutierrez/archivos1/textospdf/JaiGut90.pdf

JICA (2022). *Recopilación de datos para el estudio sobre el plan de estudios de matemáticas, los libros de texto y las guías para docentes de educación primaria* [Data collection for the study on the mathematics curriculum, textbooks and teacher's guides for elementary school teachers]. https://openjicareport.jica.go.jp/pdf/12369963.pdf

Ministry of Education. (2019). *Guía de Matemática: Quinto grad*o [Math Guide: Fifth Grade]. https://www.mined.gob.ni/biblioteca/product/guia-de-matamatica

UNESCO (2022). *Estudio Regional Comparativo y Explicativo (ERCE 2019): Resumen nacional de resultados, Nicaragua* [Regional Comparative and Explanatory Study (ERCE 2019): National summary of results, Nicaragua]. UNESDOC Digital Library: https://unesdoc.unesco.org/ark:/48223/pf0000382940?posInSet=28&queryId=N-EXPLORE-2e3e4925-4b04-4c4e-b3eb-e86e35a054a2

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry*. https://files.eric.ed.gov/fulltext/ED220288.pdf

Vargas, G., & Gamboa, R. (2013). The Van Hiele model and the teaching of the geometry. *Uniciencia, 27*(1), 74-94. https://www.redalyc.org/pdf/4759/475947762005.pdf

TSG2-1-2

FOURTH-GRADE STUDENTS' JUSTIFICATIONS OF THE PROPERTIES OF EVEN AND ODD NUMBERS

HaeSong Oh
Graduate School of KNUE, Korea, gothd05@naver.com

JeongSuk Pang
Korea National University of Education, Korea, jeongsuk@knue.ac.kr

Experiencing the justification of generalized mathematical claims is crucial for developing mathematical understanding. Although the concept of even and odd numbers is familiar to students, the Korean curriculum does not explicitly cover activities for justifying their properties. This study implemented a reconstructed lesson focused on exploring the properties of even and odd numbers and analyzed how fourth-grade students in Korea justify these properties. Two criteria were established for analysis: the definitions students employed and the types of justification they provided. The analysis revealed that students frequently used the Pairing definition to justify the properties of even and odd numbers; however, the use of multiplicative or division-based definitions increased over time. Moreover, students progressed from justifying with specific numerical examples to using generalized expressions through mathematical discussions. This study demonstrates that structured mathematical discussions facilitate the transition to generalized justifications and offers a comparative case for international research on the justification of even and odd number properties.

Keywords: even and odd numbers, even-odd properties, justification, generalization

Background to the Study

Justification —demonstrating the validity of a mathematical conjecture —is fundamental to students' mathematical reasoning and concept construction (Staples et al., 2012). Carpenter et al. (2003) categorized justification into three levels: appeal to authority, justification by example, and generalizable arguments. In the context of early algebra, Blanton et al. (2022) underscored the importance of engaging students in constructing and justifying mathematical claims. They classified students' justifications of the properties of even and odd numbers into three types: empirical justification (pattern recognition through examples), representation-based justification (use of visual models), and algebraic justification (use of algebraic expressions).

Kim and Pang (2023) analyzed mathematics textbooks from Finland, Singapore, Japan, and Korea. Their findings indicate that these countries introduce the concept of even and odd numbers at different grade levels: Finland in Grade 1, Singapore in Grade 2, and Japan in Grade 5. Notably, Finnish textbooks explicitly address the properties of even and odd numbers. In Korea, curriculum changes have affected both the timing of instruction on even and odd numbers and the methods used to define them. Prior to the 2009 revision, fifth-grade students learned about even and odd numbers through the concepts of multiples and factors. Since the 2009 revision, however, the "pairing" definition has been introduced to first-grade students and has remained in use in the 2022 revision. Despite these changes, there is no explicit instruction on alternative definitions or on the properties

of even and odd numbers beyond Grade 1. Kim and Pang (2023) underscored the need for explicit instruction on these properties in Korean mathematics education.

This study aims to provide fourth-grade students, who were introduced to even and odd numbers only in first grade, with an opportunity to explore the properties of even and odd numbers and to analyze their justification processes. The research question is as follows: How do fourth-grade students justify the properties of even and odd numbers?

Method

This study employed a qualitative case study approach to analyze in depth how fourth-grade students justify the properties of even and odd numbers. The participants comprised 22 fourth-grade students in Korea who had previously learned division with remainders but had not been introduced to multiples and factors. Since these students were exposed only to the "pairing" definition in first grade, they were expected to utilize alternative definitions in their justifications.

The lesson was designed within the fourth-grade "Finding Patterns" unit, reconstructing it to focus on addition patterns related to the properties of even and odd numbers. The lesson was conducted over two 40-minutes sessions. In the first session, students explored cases where the sum is odd, and in the second session, they examined cases where the sum is even. Students worked in pairs, forming 11 groups. The lesson consisted of four main activities. The first activity, "exploring definitions of even and odd numbers," began with a 10-minute review of various definitions, given that students use definitions when justifying the properties of even and odd numbers. The second activity, "exploring even and odd sums in an addition table," was designed with reference to Finnish textbooks and the study by Lin and Tsai (2016). In this task, students examined an addition table and marked the results with different colors depending on whether the sum was even or odd. This visual representation enabled students to intuitively recognize patterns and employ inductive reasoning to infer the properties of even and odd numbers.

The third activity, "justifying properties with a partner," involved students working in pairs to record their justifications on activity sheets. They were encouraged to explore and articulate multiple justification methods rather than relying on a single approach. The fourth activity, "whole-class discussion," was adapted from Isler et al. (2013), who observed that students tend to use generalized expressions when prompted to consider the broader applicability of their findings. In this activity, each group presented the justifications they had developed. During these presentations, other students had the opportunity to ask questions and offer additional perspectives. The teacher facilitated the discussion by connecting students' reasoning and guiding them toward generalizing the properties of even and odd numbers.

Data were gathered via lesson recordings and the collection of students' worksheets. We applied two analytical criteria to examine students' justification processes in their written work and then calculated the frequency of each category. First, we classified students' definitions of even and odd numbers into four categories—*Pairing, Multiples of Two, Division by Two*, and *Other*—because these definitions vary depending on how they are introduced in the curriculum. The Other category comprised responses that recognized patterns (e.g., alternating parity) or simply labeled a number as even or odd without reference to a formal definition. Second, drawing on the frameworks of Blanton et al. (2022) and Carpenter et al. (2003), we coded students' justifications as either using specific numerical cases or using generalized expressions. Additionally, we performed an episodic analysis of whole-class discussions to identify moments when students' justification types shifted or developed through mathematical discourse.

Results: Students' Justifications of the Properties of Even and Odd Numbers

The activity sheets from 11 groups were analyzed based on the established criteria. First, students' justifications were examined according to the definitions of even and odd numbers they applied. The frequencies of each definition used in the first and second sessions are presented in Table 1.

Table 1. Frequency of definitions of even and odd numbers applied by students

	Pairing	Multiples of two	Division by two	Other	No response	Total
First session	10	1	1	5	1	18
Second session	13	2	4	6	0	25

In the first session, students predominantly relied on the Pairing method to justify even-odd properties. Although Pairing remained the most frequent strategy in the second session, there was a slight increase in the use of other definitions. Next, activity sheets were analyzed by justification type (see Table 2). In both sessions, justifications using specific numerical cases were most common. However, the second session saw a rise in mixed justifications that combined specific examples with generalized expressions. Among students who relied on the Pairing definition, most continued using numerical examples, while a few shifted toward generalized expressions in the second session.

Table 2. Frequency of students' justification types

	Using specific numerical cases	Using generalized expressions	No response	Total
First session	14	3	1	18
Second session	19	6	0	25

When comparing the use of generalized expressions across sessions, the frequency increased from three to six. Most justifications involving generalized expressions consisted of verbal explanations, such as: "When two odd numbers are grouped in pairs, each leaves one remaining, and the remaining ones combine to form an even number." Additionally, one group of students who used Multiples of Two expressed their generalization using algebraic notation.

Meanwhile, certain students who initially explained even-odd properties using specific numerical examples on the activity sheet gradually transitioned to using generalized expressions during the whole-class discussion. The following episode illustrates this progression.

Student A: (Stands in front of the whiteboard and begins presenting.) So, um… when you add two odd numbers, like… each leaves one… And then… those remaining come together and make two, and, uh… then it just… yeah, *it* happens. [omitted dialogue]

Teacher: Any other questions?

Student B: I'm not sure if I understood correctly, but… are you saying that when you group odd numbers in pairs, they always leave something behind? And then those leftovers add up to make an even number?

Student A: ……

Teacher: Why don't you come up and point to the diagram while explaining to your classmates? Let's listen carefully to what he says.

Student B: So… if you take 3 and group it in twos, there's one left. And if you do the same for 7, there's also one left. And then if you put those two remaining together, you get 2, which makes an even number.

Student A: Yeah, that's it!

Student B expanded on Student A's explanation by introducing a generalized expression. Through this exchange, students collectively identified the key mathematical idea related to even-odd properties.

Conclusions and Implications

Based on the analysis of students' justifications, this study draws the following conclusions. First, mathematical discussions play a crucial role in fostering generalization when students justify even-odd properties. In this study, students first engaged in justification activities with a partner and later shared their reasoning in a whole-class discussion. The analysis of their activity sheets revealed a gradual increase in the use of generalized expressions. Additionally, some students who initially relied on specific numerical examples on their activity sheets began using generalized expressions during the discussion. This finding aligns with previous research (Isler et al., 2013; Lin & Tsai, 2016), which suggests that mathematical discourse helps students transition from reasoning with specific numbers to making generalizations.

Second, this study illustrates how fourth-grade students in Korea justify even-odd properties, thereby contributing to comparative research in mathematics education. In Korea, parity is introduced implicitly rather than taught explicitly, whereas curricula in some countries, such as Finland, address these properties directly. Prior studies (e.g., Blanton et al., 2022; Isler et al., 2013; Lin & Tsai, 2016) have similarly examined how students' reasoning about parity. By comparing our findings with international research, we can gain insight into how differences in instructional timing and methods influence students' justification processes. Future research might also investigate how students' preferred definitions of parity shape their choice of justification strategies.

References

Blanton, M., Gardiner, A. M., Ristroph, I., Stephens, A., Knuth, E., & Stroud, R. (2022). Progressions in young learners' understandings of parity arguments. *Mathematical Thinking and Learning, 26*(1), 90-121.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically*. Heinemann.

Forsback, M., Kalliola, A., Tikkanen, A., & Waneus, M. (2015). *Star Maths 1B*. Otava.

Isler, I., Stephens, A., Gardiner, A. M., Knuth, E., & Blanton, M. (2013). Third graders' generalizations about even numbers and odd numbers: The impact of an early algebra intervention. In M. Martinez & A. Superfine (Eds.), *Proceedings of the 35th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 140–143). PME-NA.

Kim, L., & Pang, J. (2023). Reflections on the instruction of even and odd numbers in elementary mathematics textbooks (in Korean). *The Mathematical Education, 62*(4), 551-567.

Lin, P. J., & Tsai, W. H. (2016). Enhancing students' mathematical conjecturing and justification in third-grade classrooms: The sum of even/odd numbers. *Journal of Mathematics Education, 9*(1), 1-15.

Staples, M. E., Bartlo, J., & Thanheiser, E. (2012). Justification as a teaching and learning practice: Its (potential) multifacted role in middle grades mathematics classrooms. *Journal of Mathematical Behavior, 31(*4), 447–462.

TSG2-1-3

A STUDY ON MANIPULATIVE ACTIVITIES FOR THE UNDERSTANDING SUBSUMPTION RELATIONS AMONG OF FIGURES: THROUGH CLASSES ON QUADRILATERALS FOR SIXTH-GRADE ELEMENTARY SCHOOL STUDENTS

Daisuke Itagaki
Graduate Student at the Graduate School of Education, Shimane University, Japan,
itaita1006@gmail.com

The second-year middle school students in Japan do not fully understand the subsumption relations of parallelograms. To improve this situation, this study examined manipulative activities in elementary schools that contribute to the understanding of subsumption relations of parallelograms. Therefore, students whose perceptions were transformed by manipulative activities in the experimental class were extracted and discussed by classifying their posttest descriptions. In conclusion, as an operational activity that contributes to the correct understanding of the subsumption relations, it was pointed out that "the difference between two figures should be captured from the conditions, and the transformation of the figures should be used as material to determine which condition is more restrictive."

Keywords: manipulative activities, subsumption relation of figures, GeoGebra

Introduction

In Japan, students learn the subsumption relations of parallelograms in the second grade of junior high school. However, three surveys conducted in the past 40 years have shown that children's understanding is not sufficient (Koseki et al., 1984 ; Kunimune, 2000 ; Kondo et al., 2024). Regarding this situation, it has been pointed out that it is more important to provide instruction in elementary school that is intended to connect to junior high school than to provide instruction in junior high school itself (Enokido, 1987 ; Matsuo, 2000). In addition, a recent study by Fujita et al. (2019) is confirmed as one that captures actual classroom situations. In their study, they conducted an experimental class with British junior high school students and pointed out that the group parallelogram image was transformed by children's discussions with each other. In this study, we discuss learning instruction at the elementary school level that contributes to the understanding of the subsumption relations of parallelograms in middle school within a class that incorporates such interactions among children.

Theoretical Background

Matsuo (1997) lists the factors for moving from a state in which two figures are regarded as the same kind and can be explained to a state in which two figures are regarded as an inclusive relation and can be explained. The three factors are: (1) to relate the differences, (2) to have a clear picture of the relationship between the general and specific, and (3) to accurately grasp the meaning of the definition. In this study, among the three factors, we will focus on (2) in particular. This is aimed at transforming the concept of figure by working on learners' images through manipulative activities such as drawing and transforming figures. However, Pratt & Davison (2003) argue that the use of

interactive white boards (IWBs) may be inadequate for children's shape concept formation depending on how they are used, and Fujita (2012) notes that dynamic shape manipulation activities such as GeoGebra and IWM should be used with caution. It has also been pointed out that the reason behind the difficulty of such figure deformation operations is that they emphasize the opposing relationship of one figure to another (Yokoyama, 1976).

Research Question

Therefore, the following items were set as research questions in this study :

〔*R.Q.*〕 *What are the characteristics of manipulative activities that for understanding the correct inclusive relationship between two figures?*

Method

Survey design

In this study, we planned and conducted a three-hour experimental class with Matsuo (1997) as the theoretical perspective, and extracted the transformation of figure recognition from the data obtained in the class. The survey was conducted on 21 students in the sixth grade of a public elementary school in Japan. The data collected from the study consisted of worksheets written by the children, post-tests, and transcripts created from VTR recordings and audio recordings.

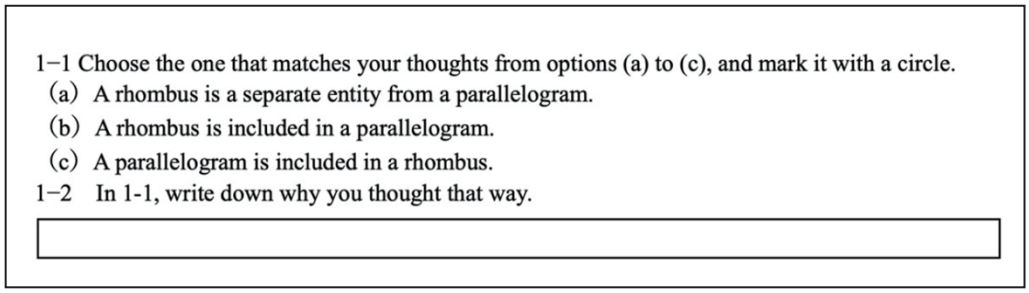

1−1 Choose the one that matches your thoughts from options (a) to (c), and mark it with a circle.
(a) A rhombus is a separate entity from a parallelogram.
(b) A rhombus is included in a parallelogram.
(c) A parallelogram is included in a rhombus.
1−2 In 1-1, write down why you thought that way.

Figure 1. Part of the post-test (questions 2-1 and 2-2 for rectangle-parallelogram intersections)

Out of the three hours, the main analysis was conducted in the second and third hours. The goal of the second and third periods was to reach a state in which the two figures could be considered and explained as subsumption relations. In the second and third periods, which are the main analysis periods, two measures were taken to achieve the goal. First, By presenting examples of familiar organisms, the students understand that conditions added to a superordinate concept are subordinate concepts, which can be used to compare figures (Figure 2).

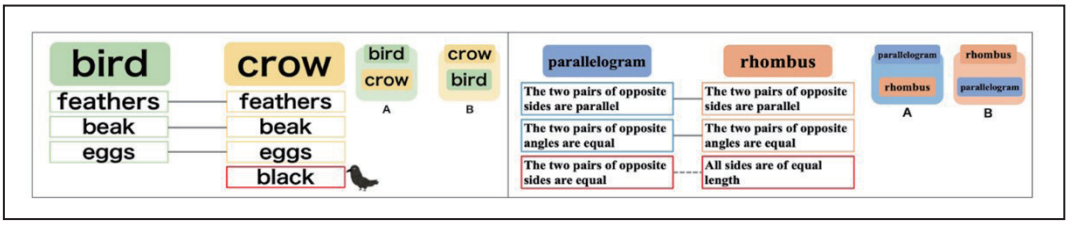


Figure 2. Comparison of two figures (right) with an example of a familiar creature (left)

Second, as a manipulative activity, the students used GeoGebra to deform figures (Figure 3).

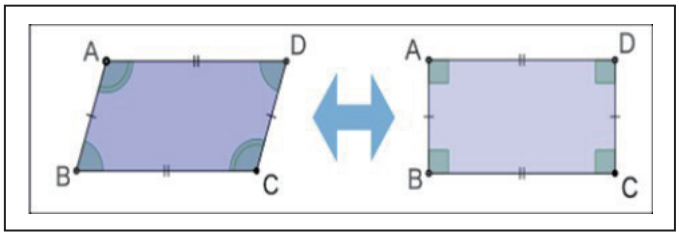


Figure 3. Manipulative activity by transforming a figure (at time 3)

Data Analysis

The analytical procedure in this study is as follows. First, we select children who changed their opinion from B(the opposite of the upper and lower concepts)to A(Correct concept) after the manipulative activity. Second, we classify the extracted children based on their responses and descriptions in the posttest. Third, we discuss the characteristics of the manipulative activity that for the understanding of correct subsumption relations among figures.

Result

There were 10 cases in which the children changed their opinions from B to A after the manipulative activity, and the extracted children were classified as follows according to their understanding subsumption relations among figures based on their responses and descriptions in the post-test.

Table 1. Classification of extracted children

I Select (c) for post-test (superordinate concept and subordinate concept are reversed)		
the second period	Iy	Because sometimes a parallelogram becomes a rhombus if all its lengths change.
	Om	As a result, if the lengths are not equal, they cannot be a rhombus.
the third period	Nk	When the angle and length of a parallelogram change, it becomes a rectangle.
	Hr	If the shape of a parallelogram changes, it becomes a rectangle.
II Select (b) for post-test (superordinate concept and subordinate concept are correct)		
(1) Description based on deformation of figures		
the second period	Ks	(no description)
	Mr	If the angle of a parallelogram changes, it's a rhombus.
the third period	Kk	Because a parallelogram can be turned into a rectangle.
	Ks	(no description)
	Mr	If the angle changes, it's a parallelogram.
(2) Description based on number of conditions		
the third period	Km	Rectangles have more requirements to be parallelograms.

The results of the post-test showed that in case (I), statements such as, "When ~ is made, it becomes a rhombus (rectangle)" were identified. This statement was reminiscent of the view that figures change into different figures (Yokoyama, 1976). On the other hand, in case (1) of case (II), although the correct choice was selected, the rationale was not described (Ks), or the explanation of the rationale for the subsumption relations was not sufficient (Mr, Kk). However, in case (II) (2), only Km, whose description was based on the number of conditions, was taken to have reached an understanding of correct subsumption relations among figures. Therefore, from this point on, we will focus on how the manipulative activity contributed to Km's understanding: Km, at the beginning of the third period, after the similarities and differences between rectangles and parallelograms were identified as a whole, the following statement was made in the question "Is the correct relationship A or B and why?" in the worksheet.

Comparing "two pairs facing each other ..." and "four pairs facing each other ...," I thought that "two pairs facing each other ..." had more restrictive conditions.

Km compares the property of a parallelogram, in which two sets of opposite corners are equal, with the property of a rectangle, in which four corners are equal, and incorrectly judges that the property of a parallelogram is more demanding, and chooses B (the opposite of the upper and lower concepts). Later, through manipulative activities, a change in perception toward A (correct concepts) was observed, and the following statement was made in the question "Is the correct relationship A or B and why?" in the worksheet.

I thought the condition that all the corners must be exactly 90 degrees to be a rectangle (compared to two sets of opposite corners that are equal) was too strict.

From this statement, it can be taken that Km compared the property of two sets of opposite corners being equal and the property of four corners being equal again from the deformation operation of the figure,and decided again that the property of rectangle is more restrictive condition.

Discussion and Conclusion

From the results of the survey, it is considered that the correct understanding of the subsumption relations may not be achieved if the deformation operation of the figure itself is used as the basis for determining the upper and lower concepts, as in (I) and (II) (1). However, as in Km in (II)(2), if the difference between the two figures is considered in terms of conditions, and the deformation operation of the figure is used as a material to determine which condition is more restrictive, it is considered that it may contribute to the correct understanding of the subsumption relations. Therefore, the manipulative activity that contributes to the understanding of the correct inclusive relation is "to take the differences between the two figures in terms of conditions, and then to perform deformation operations on the figures as materials for judging which condition is more restrictive.

References

Enokido, A. (1987). Developmental study of figure cognition, Edited by Kiyoshi Koseki, *instruction in geometric proof* , 90-125,Meiji books. (in Japanese)

Fujita, T. (2012). Learners' level of understanding of the inclusion relations of quadrilaterals and prototype phenomenon, *The Journal of Mathematical Behavior*, 31,60-72.

Fujita, T., Doney, J., & Wegerif, R. (2019). Students' collaborative decision-making processes in defining and classifying quadrilaterals: a semiotic/dialogic approach, *Educational Studies in Mathematics*,101,341-356.

Koseki, K., Ieda, Y., & Kunimune, S. (1984). Developmental Studies of Figure Cognition : Formation process of the concept of "parallelogram". *Journal of Japan Society of Mathematical Education*, 66（R4142）, 3-23. (in Japanese)

Kunimune. S. (2000). Change in understanding of the argumentation of the figure. *Journal of Japan Society of Mathematical Education*,82(3),66-76. (in Japanese)

Kondo. U., Kumakura. H., Fujita. T., Miyawaki. S., Kunimune. S. (2024). Middle school students' understanding of parallelogram concepts : Results of three surveys with the same problem and its change over time. *Journal of Japan Society of Mathematical Education*, 106, (7), 2-10. (in Japanese)

Matsuo. N. (1997). Examination of factors that promote understanding of the inclusion relationship of the concept of figure : Through a lesson on parallelograms in the second grade of junior high school. *Proceedings of the 30th Annual Meeting of Japan Society of Mathematics Education*, 397-402. (in Japanese)

Matsuo. N. (2000). Improving the Teaching of Figures in Arithmetic and Mathematics, *Toyokan Press*. (in Japanese)

Pratt, D., & Davison, I. (2003). Interactive Whiteboards and the Construction of Definitions for the Kite, *Proceedings of the 27th PME conference*, 4, 31-38

Yokoyama, S. (1976). Investigating the Problems of Inclusion Relations in Elementary School. *Journal of Japan Society of Mathematical Education*, 58, (10), 19-26. (in Japanese)

Acknowledgements

We would like to thank the teachers and students at public elementary schools in Shimane Prefecture for their cooperation in this study. I am also sincerely grateful to Mr. Takehito Shimomura for his guidance.

TSG2-1-4

USING TEACHING WITH VARIATION IN AFRICAN MATHS CLASSROOMS: A PARTICIPATORY CASE STUDY

Xumai Lin
University of Macau, China, yc47174@um.edu.mo

Lianghuo Fan
University of Macau, China, lhfan@um.edu.mo

Teaching with variation is a widely-used method in Chinese maths classrooms which helps design of teaching. Many case studies have discussed its function as "instruction" for discerning critical aspects of concepts and "scaffolding" for forming concepts and solving problems. In this paper, we describe how instruction and scaffolding were designed and applied in an African educational context. We illustrate this by focusing on the formation of concept and exploration of property section in four consecutive lessons of transformation in geometry. The analysis of teaching design implies variation from shallow and wide perspectives help form property. And the number and variety variation should be more as expected and the content or examples should be carefully interpreted in intercultural situation when exploring narrow and deep object like properties. Feedback from students showed that teacher's approach impressed some of students in the conceptual, procedural and emotional aspects. Results concluded from an insider perspective will help teachers' development in terms of teaching design.

Keywords: Teaching with variation, Intercultural, Mathematics, Transformation

Teaching with Variation

Teaching with variation has been a tradition in Chinese maths classrooms for a long time (Bao, Huang, Yi & Gu, 2003a; Sun, Huang, Lin & Zhang, 2006). Due to the outstanding performance in international assessments like PISA (OECD, 2010, 2014), there has been an increasing number of research that shed lights on this unique teaching design method in China (Fan & Zhu, 2004; Huang & Zhang, 2019; Kullberg, Runesson, & Marton, 2017; Wong, Lam, & Chan, 2012). Many researchers argued that it was developed by the theory of variation derived from phenomenology (Marton & Booth, 1997; Bao et al, 2003a; Marton & Tsui, 2004; Huang et al, 2015; Huang & Zhang, 2019). Generally speaking, learning happens when discerning the variation and in-variance (Marton & Booth, 1997; Marton, 2015). Additionally, Huang & Zhang (2019) pointed out that the theory of variation illustrated the value of teaching with variation in China from the perspective of learning space, which resulted in the outstanding performance of Chinese students in academic (Fan & Zhu, 2004). Some research considered the teaching with variation as various ways of constructing scaffolding (Bruner, 1985) assisting students' learning (Bao et al, 2003c; Huang et al, 2004; Seidel, Rimmele & Prenzel, 2005). The scaffolding theory can be supported by Vygotsky's theory: Zone of proximal development (ZPD) (Fani & Ghaemi, 2011), which means that students can acquire knowledge steadily following a series of scaffolding (Vygotsky, 1978; Bao et al, 2003c; Huang et al, 2004). Scaffolding in theory is similar to "Pu Dian" (Procedural variation) in Chinese context, as they have the same function -- bridging learned knowledge with intended knowledge--promoting

the development of ZPD. However, Procedural variation pay more attention on the hierarchy and process (Bao et al, 2003c; Gu & Yang, 2007).

Theoretical Framework

Gu (1981) analyzed this method of teaching systematically and expanded it from the conceptual variation to procedural variation (Gu, 1981; Bao et al, 2003a). Conceptual variation means that teachers use various examples, including counterexamples, remaining features of the intended concept but changing the context or instances, to help students figure out the core of the concept from various perspectives (Bao et al, 2003b, 2003c; Huang, Miller & Tzur, 2015).

Table 1. Conceptual variation framework

Type of conceptual variation	Conceptual variation strategy	Intended feature of knowledge to be discerned	Ways of representation
A.Conceptual (Gu et al., 2017)	a.Change context	I key word II the set of the concept III boundary of the concept	1. Concrete examples in real-life 2. Animation/Image 3. Symbol 4. Word problem 5. One-question-multi-problems (Gu et al., 2017) 6. One-question-multi-solutions (Gu et al., 2017) 7. Body language
	b.Change condition		
	c.Change method		
B.Non-conceptual (Gu et al., 2017)	d.Counterexample		
	e.Change instance		

Procedural variation means that teachers use a number of various variations to support the students' learning process step by step, either formulate concepts or solve problems (Bao et al, 2003b, 2003c; Huang et al, 2015). Comparing to the Conceptual variation, the Procedural variation is dynamic, focusing on the whole process and changes in teachers' teaching and students' learning. As a result, the procedural variation is divided into 3 types according to the change of difficulty (cognitive load): bridge, scaffolding and convergence (Gu et al., 2017) .

Research Context

The teacher involved in this study came from a high-performing state-owned secondary school of modern city in China and had ten-year experience in teaching. The mother tongue of the teacher is Chinese, but the lessons were taught in English. The Tanzanian school was also an outstanding public school in the busiest city. English was the teaching language in this school, but Kiswahili was their native language. Lessons were consecutive and took place in the class of 56 students in Form 2. In Tanzania, students need to experience 7 years in primary school and 6 years in secondary school. Form 2 is their second year in secondary school. The teacher gave five lessons of the topic "(Geometric) transformation". Before this topic, students had learned definition and theorems of congruent triangles and similar figures. In terms of research, the advantage of choosing this topic is that "Transformation" as a mathematics topic includes the features of teaching with variation: variation and in-variance (Nie, 2004).

Instruments

The first author acted as teacher and researcher in this study. The insider perspective of the first author will increase the exploratory sense of coding. In another word, the first author can be regarded as an instrument in the analytic process. Lessons were compared horizontally and vertically according to the conceptual variation and procedural variation framework. The used ppt files, recorded videos, written dairy and recorded reflection were materials to analyze lesson design. Students' learning was evaluated by worksheets, which consisted of three parts: examples and practices and self-evaluation. Therefore, the results of worksheets served as evidence. A clustered sample was made to see how it changed different students from the first lesson to the last lesson.

(Preliminary) Results

The topic of the first lesson was "translation". The main teaching goal for this lesson is the formation of the definition and property of translation. The analysis for this lesson focuses on these two parts. At the phase of concept formation, there were 4 variations. First, the teacher showed three types of congruent transformation (translation, reflection and rotation) by animation and asked students: "How did they change?". Students could describe these transformations precisely. Secondly, the teacher invited two students to stand in front of the classroom and move following instructions: "Move forward", "Move 2 steps" and "Move forward 2 steps". The activity aimed to let students experience translation and discern the differences among three instructions. Thirdly, teacher asked students to discern the dialogue between arguments of two birds according to the animation. At last, the teacher asked students to think about translation in daily life and try to conclude the definition of translation in mathematics using students' own words. Scaffolding can be noticed in the process of concept formation. In variation 3, the argument of two birds on who went further can be regarded as a scaffolding, which arose students' interests to think about the core mathematical meaning of "the distance translated". Additionally, it could help students reflect the meaning of translation as well. In variation 4, concrete examples in daily life introduced by students were a small steps to help themselves reach the abstract definition of translation tin mathematics. When concluding the concept of translation, one student said: "Translation is which move all the points on the plane". After the teacher asked students to recall all the activities they experienced, another student answer: "The translation is which moves an object or a figure in the same distance, in the even directions".

Table 2. Variation in the first lesson

	Type of variation	Ways of representation	Intended feature of knowledge to be discerned
Translation Variation 1	A	① ②	move
Translation Variation 2	C	⑦	Direction of a translation
Translation Variation 3	C	② ⑤	Same direction Same distance
Translation Variation 4	C	①	Same direction Same distance

In conclusion, teaching with variation can help students form concept effectively. But given the nature of this study and the limit of the length of this article, further studies with large scales and using different research methods are needed.

References

Fan, L., & Zhu, Y. (2004). How have Chinese students performed in mathematics? A perspective from large-scale international comparisons. In L. Fan, N. Y. Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 3–26). World Scientific.

Fan, L., Miao, Z. & I.A.C., Mok. (2015). How Chinese teachers teach mathematics and pursue professional development: Perspectives from contemporary international research. In L. Fan, J. Cai, & S. Li (Eds.). *How Chinese teach mathematics: Perspectives from insiders* (pp. 43–70). World Scientific.

Gu, L., HUNAG, R., & Marton, F. (2004). Teaching with variation: A Chinese way of promoting effective mathematics learning. In *How Chinese learn mathematics: Perspectives from insiders* (pp. 309-347).

Gu, F., Huang, R., & Gu, L. (2017). Theory and development of teaching through variation in mathematics in China. In R. Huang & Y. Li (Eds.), *Teaching and learning mathematics through variation. Mathematics Teaching and Learning*. SensePublishers, Rotterdam.

Gu, L. (2021). From "Qingpu Experience" to Investigating Chinese Students' Mathematical Competencies. In: Xu, B., Zhu, Y., Lu, X. (Eds) *Beyond Shanghai and PISA*. *Research in Mathematics Education. Sprin*ger, Cham.

Huang, R., Leung, F.K.S. (2017). Teaching Geometrical Concepts through Variation. In R. Huang & Y. Li (Eds.) *Teaching and Learning Mathematics through Variation. Mathematics Teaching and Learning*. SensePublishers, Rotterdam.

Kullberg, A., Runesson Kempe, U., & Marton, F. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? *ZDM – Mathematics Education, 49*(4), 559-569.

TSG2-1-5

HOW TEACHERS' INSTRUCTION INFLUENCES STUDENTS' UNDERSTANDING IN THE AREA OF A CIRCLE

Yi-Lun Tsai
National Pingtung University, Taiwan, e0976606969@gmail.com

Tsu-Nan Lee
National Pingtung University, Taiwan, tsunanl@mail.nptu.edu.tw

Geometry is a topic to discuss the relationship between geometric shapes and their properties through reasoning. This study aimed at discussing students' learning problems in the circle topic and investigated 90 grade 6 students after learning the circle topic. This study developed a test of area of a circle (TAC) with 5 items, and 5 items were related to three cognitive competences, including knowing, applying and reasoning. The results found that students performed the best in knowing items and the worst in a reasoning item. It was difficult to understand the relationship between the area of a circle formula and the circumference formula for low-achieving students. According to the results, this study suggests that using technology to combine hands-on manipulatives with visualization could promote students' reasoning and understanding in the area of a circle.

Keywords: primary students, area of a circle, reasoning, technology

Introduction

Geometry is a topic to understand the relationship between geometric shapes and their properties, and the relationship in geometry is related to reasoning (National Council of Teachers of Mathematics, 2000). In a topic of circle, primary students would understand why Pi (π) equals to 3.14 approximately, and the relationship between the circumference formula and the area of a circle formula through hands-on manipluatives or measurement in Taiwan (Ministry of Education, 2018). However, Hsu (2017) interviewed 45 elementary school teachers and tried to understand how these teachers employed mathematics textbooks to teach. The results found that elementary school teachers in Taiwan tended to do lectures, and taught students how to solve problems, but ignored developing students' thinking and connecting different concepts in teaching mathematics (Hsu, 2017). Hsu (2017) addressed that this type of instruction might not enhance students' thinking and reasoning, and the results in some international assessments revealed similar patterns (Mullis et al., 2020; von Davier et al., 2024). Doing lectures is the main instruction to teach mathematics in Taiwan, and this instruction depends on whether teachers could explain a mathematical concept clearly. In TIMSS 2024 (von Davier et al., 2024), 60% of grade 4 students in Taiwan reported that they thought their teachers explained mathematical concepts clearly, but 60% was lower than average (69%). It shows that teachers' instruction influences students' understanding. Therefore, students in learning the topic of circle might focus on how to use formulas to calculate or solve problems and perform weakly in understanding. According to these discussions, this study was an initial study to investigate students' performance in the topic of circle after teaching, and the research questions are as follows: 1. did students' scores have differences between different cognitive competences? 2. What were low-achieving students' learning problems?

Method

Participants and Design

One hundred and two grade 6 students were from four different primary schools in Kaohsiung, Taiwan, but twelve students were excluded from this study because they could not understand the difference between the circumference and the area of a circle. So, ninety students participated in this study. Sixty-five and thirty-five students were identified as high- and low-achieving students by their homeroom teachers, and all schools adopted the same version of textbook, Kang Hsuan, to teach. The topic of area of a circle is arranged in the first semester.

The test of area of a circle (TAC) consisted of 6 items, and only 5 items were analyzed. One item without analyzing was to identify whether students could distinguish the difference between circumference and the area of a circle. The other 5 items were related to students' procedural knowledge, applying the area of a circle formula, and understanding the area of a circle formula. Two items of procedural knowledge were to ask students to calculate area of a circle, and two items of applying the area of a circle formula were to provide situations to solve problems. For example, a piece of colored paper is a square shape, and each side is 10 cm. If you would like to cut a circle from this colored paper, how could you cut the largest circle from this colored paper? What is the approximate area of a circle? This last item was about a concept of area of a circle formula, and asked students that "where did the area of a circle formula come from?" The five items could be identified as different cognitive competences: items of procedural knowledge were related to knowing, items of applying the area of a circle formula were related to applying, and the item of understanding the area of a circle formula was related to reasoning. Table 1 shows the structure of five items to reveal the content validity in the TAC. This study adopted Kuder-Richardson coefficient (KR20) to show the reliability in the TAC, and the KR20 value was .62. The total score in the TAC was 10 points, and each item was 2 points. If students had a calculation error, students would lose 1 point.

Table 1. The content validity in the TAC

Learning Target \ Cognitive Competences	Knowing	Applying	Reasoning
• Using the area of a circle formula to calculate	2		
• Using the area of a circle formula to solve problems		2	
• Understanding the area of a circle formula through dissecting the circle			1

Procedure

According to mathematics curriculum (Ministry of Education, 2018), students in this topic would use a centimeter grid board to measure area of irregular shapes and circles and then understand the area of a circle formula through dissecting the circle. Teachers dissect the circle into many sectors and reconstructing sectors to form a parallelogram, and it would help students understand the area of a circle formula from a circumference formula. Next, students use the area of a circle formula to

calculate the area of a circle and a sector. Finally, students use the area of a circle formula to calculate the area of compound shapes which consist of many sectors through students' daily experiences. After learning the topic of the area of the circle, all students did the TAC within 30 minutes. Teachers' instructions relied on homeroom teachers' self-reports and online observations to confirm how they taught.

Data Analysis

As the research question was mentioned, this study adopted a one-way repeated measured ANOVA to answer the research question 1. There was an independent variable, three cognitive competences. One dependent variable was average scores in three cognitive competences. To answer the research question 2, this study adopted descriptive statistics to reveal problems for low-achieving students.

Results

Research Question 1 (RQ 1)

According to RQ 1, this study adopted the one-way repeated measured ANOVA, and the descriptive statistics are shown in Table 2. However, the value of Mauchly's sphericity test was significant, Mauchly's = .87, $p = .002$, and the degree of freedom was adjusted. The result showed that students' scores had a significant difference between three cognitive competences, F (1.77, 157.39) = 54.45, $p > .001$, $\eta^2 = .38$. The post hoc analysis showed that students' scores in knowing were higher than in applying and reasoning, and students' scores in applying were also higher than in reasoning. It revealed that students' scores in knowing performed the best, and students' scores in reasoning were the worst in the TAC.

Table 2. The descriptive statistics in the TAC

Knowing		Applying		Reasoning	
M*	SD	M*	SD	M	SD
1.74	.53	1.59	.52	1.06	.63

Note. The average mean from 2 items.

Research Question 2 (RQ 2)

As RQ 2 was mentioned, this study aimed to understand students' learning problems, especially for low-achieving students. The descriptive statistics of the TAC are shown in Table 3. This study found that high-achieving students got nearly full scores in knowing items, and it meant that these students could calculate circles' area through the area of a circle formula. Although low-achieving students could calculate circles' area through the area of a circle formula, they might make some calculation errors, especially in decimal multiplication and division. Meanwhile, high-achieving students could imagine cutting a compound figure into some sectors and using a square to cut the largest circle, but some of low-achieving students could not imagine how to cut a compound figure in applying items. It revealed that high-achieving students had an imagination to show how to form a circle or sectors, but low-achieving students might not have enough relevant experiences to show the imagination to solve problems in applying items. In a reasoning item, a circle was divided into

several sectors, and these sectors were rearranged to a rectangle: the rectangle's width was the radius of the sector, and its length was half the circumference of the circle. This study found that some high-achieving students and few low-achieving students could understand these relationships to infer from the circle to the rearranged rectangle. Although the textbook introduced the relationship between the area of a circle formula and a circumference formula, the result showed that just some students could understand the relationship.

Table 3. The descriptive statistics between high- and low-achieving students in the TAC

	Knowing		Applying		Reasoning	
	M*	SD	M*	SD	M	SD
High	1.95	.20	1.79	.37	1.24	.58
Low	1.40	.69	1.27	.56	.77	.60

Note. The average mean from 2 items.

Discussion and Conclusion

According to research questions and results, this study indicated two points to address, including teachers' instructions and what students are to be supported. To promote students' reasoning and understanding, it would be related to how students construct geometric concepts (Duval, 1998; Fyfe et al., 2014). Both Duval (1998) and Fyfe et al. (2014) point out that hands-on manipulatives and visualization could support students' understanding and reasoning. However, students in this study only experienced visualization, such as teachers' demonstrations, e-book's animation, and these students had no chance to experience relevant measurement or hands-on manipulatives. It could be explained why students in this study performed better in knowing and applying, but worse in reasoning. However, Taiwanese teachers faced much pressure in teaching mathematics: teachers should use less time to teach more mathematics topics in comparison with other countries (Mullis et al., 2020). Teachers should adopt effective instructions to teach mathematics. As hands-on manipulatives and visualization were mentioned previously, Liao and Chen (2021) and Sarama and Clements (2009) found that using technology could support students to learn mathematics, and technology could combine the function of hands-on manipulatives with visualization. Hands-on manipulatives with visualization in technology calls dynamic visualization. Therefore, teachers' instruction could provide technology to support students to learn the topic of circle, and the function of technology reveals dynamic visualization.

References

Duval, R. (1998). Geometry form a cognitive point a view. In C. Mammana & V. Villani (Eds.), *Perspectives on the Teaching of Geometry for the 21st Century* (pp. 37-52). Kluwer Academic Publishers.

Fyfe, E. R., McNeil, N. M., Son, J. Y., & Goldstone, R. L. (2014). Concreteness fading in mathematics and science instruction: a systematic review. *Educational Psychology Review*, *26*(1), 9-25. https://doi.org/10.1007/s10648-014-9249-3

Hsu, W.-M. (2017). Investigation of elementary mathematics textbooks use in classrooms in Taiwan. *Journal of Textbook Research*, *10*(2), 99-132. https://doi.org/10.6481/jtr.201708_10(2).04

Liao, Y.-K. C., & Chen, Y.-H. (2021). Integrating technology in mathematics instruction on grade school academic achievement in Taiwan: a meta-analysis. *Bulletin of Educational Psychology*, *52*(4), 781-805. https://doi.org/10.6251/bep.202106_52(4).0003

Ministry of Education. (2018). *Curriculum guidelines for the 12-year basic education elementary school, junior high school, and upper secondary school: the domain of mathematics*. Ministry of Education.

Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). *TIMSS 2019 International Results in Mathematics and Science*. TIMSS & PIRLS International Study Center, Boston College. https://timssandpirls.bc.edu/timss2019/international-results/

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.

Sarama, J., & Clements, D. H. (2009). "Concrete" computer manipulatives in mathematics education. *Child Development Perspectives*, *3*(3), 145-150. https://doi.org/10.1111/j.1750-8606.2009.00095.x

von Davier, M., Kennedy, A., Reynolds, K., Fishbein, B., Khorramdel, L., Aldrich, C., . . . Yin, L. (2024). *TIMSS 2023 International Results in Mathematics and Science*. Boston College, TIMSS & PIRLS International Study Center. https://doi.org/10.6017/lse.tpisc.tr2103.pcbxxxx

TSG2-1-6

IMPACT OF LESSONS EMPHASIZING A RELATIONAL UNDERSTANDING OF THE EQUAL SIGN: FOCUSED ON SAMENESS AND SUBSTITUTION MEANINGS

JeongSuk Pang
Korea National University of Education, Korea, jeongsuk@knue.ac.kr

Juhyeon Kim
Korea National University of Education, Korea, enfdl1380@daum.net

Recent interest in the relational understanding of the equal sign emphasizes both "sameness" and "substitution." Building on previous research, this study developed an instructional approach focused on these meanings to enhance third-grade students' relational understanding of the equal sign. The lessons were designed around these meanings and implemented in the classroom. This study then analyzed the impact of the lessons on students' understanding of the equal sign. Results showed that students recognized both interpretations and applied relational strategies to solve equations. Pre- and post-test analyses revealed significant improvements in understanding and strategy use. Implications for enhancing relational understanding of the equal sign are discussed.

Keywords: equal sign, sameness, substitution, relational understanding, instructional approach

Background to the Study

The equal sign is a fundamental mathematical symbol that denotes equivalence, and understanding its meaning lays the groundwork for comprehending equation transformations in later algebra learning (Alibali et al., 2007). Studies related to the equal sign have primarily analyzed instructional cases that focus on the "sameness" meaning of the equal sign (McNeil et al., 2015; Stephens et al., 2022). In contrast, recent studies suggest that a relational understanding of the equal sign encompasses not only "sameness" but also "substitution," where one expression can be replaced by an equivalent expression (Jones et al., 2012; Simsek et al., 2019). Moreover, some studies have shown that teaching both meanings can enhance students' relational understanding of the equal sign (Donovan et al., 2022).

Previous research has commonly categorized students' understanding of the equal sign as either operational or relational (Rittle-Johnson et al., 2011). An operational view interprets the equal sign as a prompt to carry out a calculation or as a placeholder for the answer. In contrast, a relational view treats the equal sign as denoting equivalence between the two sides of an equation. More recently, scholars elaborated on the relational perspective by distinguishing two complementary meanings**:** *sameness*, which stresses that both sides share the same value, and *substitution*, which emphasizes that one expression may be replaced by an equivalent one (Jones et al., 2012; Lee & Pang, 2023; Simsek et al., 2019).

However, few studies have implemented lessons that emphasize both the sameness and substitution meanings of the equal sign in actual classrooms and analyzed their impact on students' understanding. Therefore, this study developed an instructional approach to teach a relational understanding

of the equal sign. The approach was implemented in classroom lessons, and students' relational understanding of the equal sign was analyzed. Specifically, the instructional approach incorporates the following components. First, it is essential to teach the substitution meaning alongside the sameness meaning to foster a deeper understanding of the equal sign (Jones & Pratt, 2012). Second, students should engage in tasks that allow them to explore relational strategies and grasp the structure of equations (Donovan et al., 2022; Lee & Pang, 2023). Third, opportunities should be given for students to apply relational strategies in solving various types of equation tasks, thereby extending their understanding of the equal sign (Ki & Chong, 2008). Based on this research background, the guiding research question is: How does a lesson emphasizing relational understanding—grounded in both the sameness and substitution meanings of the equal sign—impact students' understanding?

Methods

The participants of this study were a third-grade class (12 males, 10 females) from a school in Sejong City, Korea, all of whom exhibited average academic achievement. The research design involved administering a pre-test, adapted from Lee (2024) to suit third-grade students, followed by three lessons implementing the previously described instructional approach. In the first lesson, students focused on understanding the meanings of sameness and substitution of the equal sign. In the second lesson, they explored relational strategies by paying attention to the structure of equations. In the third lesson, students applied these relational strategies to solve various types of equations. A post-test, equivalent to the pre-test, was administered after the lessons to assess changes in students' understanding of the equal sign.

Data were collected through recorded lesson videos, transcripts, and student activity sheets. These materials were used to extract lesson episodes that focused on students' understanding of the equal sign, the structure of equations, and the application of relational strategies. The students' understanding was analyzed qualitatively. Additionally, the post-test was divided into sections addressing the meaning of the equal sign, equation structure, and solving equations. The test consisted of multiple-choice questions (scored 1 point for a correct answer and 0 points for incorrect ones) and open-ended questions, with responses coded into categories (operational, relational-sameness, relational-substitution, and incorrect explanations). The frequencies of each category were then analyzed.

Results

Due to space constraints, this paper focuses on analyzing students' use of relational strategies during lessons, whereas the pre- and post-test analysis assesses their understanding of the equal sign through open-ended questions. For a more detailed examination of lesson design and instructional episodes, see Pang & Kim (2024).

The use of relational strategies revealed during lessons

In this study, students effectively employed relational strategies to evaluate whether a given equation was true or false, based on their understanding of the equal sign (see Episode 1).

Episode 1. Using relational strategies to determine whether 28+34=30+32 is true or false

Teacher: How can we compare both sides of the equation 28+34=30+32 to determine whether it is true or false? Please explain.

Student A: (Annotates 34 as 2 and 32) If you subtract 2 from 34 and give it to 28, it becomes 30, so both sides are the same.

Student B: (Partitions 34 into 2 and 32 beneath it) If you split 34 into parts of 32 and 2, it [the left side] becomes 28+2+32, which is the same as 30+32.

Student C: To make 28 into 30 you need 2, and to make 34 into 32 you subtract 2, so you can say they are equal.

In this way, the students who participated in this study, such as Student A and Student B, focused on the structure of the equation and employed a decomposition strategy, dividing the numbers and substituting the parts into the equation. Additionally, Student C applied a compensation strategy by considering the relationships between the addends to demonstrate that both sides are equal.

Students' understanding of the meaning of the equal sign in pre- and post-tests

Table 1 summarizes students' written responses to the question "What does the equal sign (=) mean?" in the pre- and post-tests. Since multiple responses were permitted, students could offer more than one interpretation of the equal sign.

Table 1. Students' pre- and post-test understanding of the meaning of the equal sign

Category	Operational meaning (add, answer)	Relational meaning (sameness)	Relational meaning (substitution)	Other
Pre-test	15 (68.2%)	10 (45.5%)	0 (0%)	3(13.6%)
Post-test	0 (0%)	22 (100%)	3 (13.6%)	0 (0%)

In the pre-test, 15 out of 22 students (68.2%) described the equal sign using an operational meaning, such as "add" or "the answer" while 10 students (45.5%) offered a relational meaning focused on sameness. In contrast, no students referred to the substitution meaning, and three students (13.6%) gave responses that did not fit into the predefined categories. However, in the post-test, no students provided an operational interpretation of the equal sign, and all students described it using a relational meaning based on sameness. Among them, three students (13.6%) explicitly articulated the substitution meaning, stating that one side of the equation can be replaced with the other.

Items requiring students to explain why given equations were true showed a clear shift from the pre-test to the post-test. In the pre-test, students typically justified each equation by calculating its two sides separately. In the post-test, however, they not only performed calculations but also employed relational strategies—attending to the structural relationship between both sides—to justify equality (see Table 1).

Table 2. Examples of students' flexible strategy use across post-test items

No	Item	Student Response
1	28+67=29+66	67 is 1 more than 66, and 29 is 1 less than 28, so it is the same.
2	67+84=69+82	I split 69 into 67 and 2, and because 82 + 2 is 84, both sides are equal.
3	38+44=40+42	38 + 44 is 82 and 40 + 42 is also 82 too, so they are same.
4	66+48=79+35	The answer is 114, so they are the same.
5	54+9+19=63+19	I broke 63 into 54 and 9, so it's the same when you add it like that.

Discussion

The study analyzed the impact of lessons emphasizing the sameness and substitution meanings of the equal sign on students' understanding. Based on the results, the following implications are derived.

First, this study confirmed that lessons emphasizing both the sameness and substitution meanings of the equal sign produced a positive change in students' understanding and increased their use of relational strategies. Donovan et al. (2022) noted that changes in the use of relational strategies during equation solving were difficult to observe in online learning environments, even when lessons focused on these meanings. In contrast, our findings indicate that face-to-face lessons had a positive impact on both students' understanding of the equal sign and their application of relational strategies. Therefore, it is essential to further explore teaching methods based on the sameness and substitution meanings of the equal sign, not only in Korea but also in other international contexts.

Second, the study is significant in that it presents specific teaching and learning strategies along with real classroom examples for lessons that emphasize both the sameness and substitution meanings of the equal sign. However, because the study was conducted with only a single class, future research should expand to include multiple grades and classes.

References

Alibali, M. W., Knuth, E. J., Hattikudur, S., McNeil, N. M., & Stephens, A. C. (2007). A longitudinal examination of middle school students' understanding of the equal sign and equivalent equations. *Mathematical Thinking and Learning, 9*(3), 221-247.

Donovan, A. M. et al. (2022). Is a substitute the same? Learning from lessons centering different relational conceptions of the equal sign. *ZDM-Mathematics Education, 54*(6), 1199-1213.

Jones, I., Inglis, M., Gilmore, C., & Dowens, M. (2012). Substitution and sameness: Two components of a relational conception of the equals sign. *Journal of Experimental Child Psychology, 113*(1), 166-176.

Jones, I., & Pratt, D. (2012). A substituting meaning for the equals sign in arithmetic notating tasks. *Journal for Research in Mathematics Education, 43*(1), 2-33.

Ki, J., & Chong, Y. (2008). The analysis of elementary school students' understanding of the concept of equality sign in contexts and the effects of its teaching methods (in Korean). *School Mathematics, 10*(4), 537-555.

Lee, Y., & Pang, J. (2023). An analysis of elementary school students' understanding of equation structure and solution strategies by their understanding of the equal sign (in Korean). *School Mathematics, 25*(2), 193-212.

Lee, Y. (2024). Analysis of students' understanding of equal sign through equal sign introduction lessons emphasizing their relational understanding (in Korean). *Educational of Primary School Mathematics, 27*(1), 39-55.

Matthews, P., Rittle-Johnson, B., McEldoon, K., & Taylor, R. (2012). Measure for measure: What combining diverse measures reveals about children's understanding of the equal sign as an indicator of mathematical equality. *Journal for Research in Mathematics Education, 43*(3), 316-350.

McNeil, N. M., Fyfe, E. R., & Dunwiddie, A. E. (2015). Arithmetic practice can be modified to promote understanding of mathematical equivalence. *Journal of Educational Psychology, 107*(2), 423.

Pang, J., & Kim, J. (2024). An Analysis of Third Grade lessons emphasizing a Relational Understanding of the Equal sign Based on Sameness and Substitution (in Korean). *School Mathematics*, *26*(4), 469–487.

Rittle-Johnson, B., Matthews, P. G., Taylor, R. S., & McEldoon, K. L. (2011). Assessing knowledge of mathematical equivalence: A construct-modeling approach. *Journal of Educational Psychology,* 103, 85-104.

Simsek, E., Xenidou-Dervou, I., Karadeniz, I., & Jones, I. (2019). The conception of substitution of the equals sign plays a unique role in students' algebra performance. *Journal of Numerical Cognition, 5*(1), 24-37.

Stephens, A., Sung, Y., Strachota, S., Torres, R. V., Morton, K., Gardiner, A. M., Blanton, M., & Stroud, R. (2022). The role of balance scales in supporting productive thinking about equations among diverse learners. *Mathematical Thinking and Learning, 24*(1), 1-18.

TSG2-2-1

EXPLORING THE POTENTIAL OF CHATGPT IN MATHEMATICS EDUCATION: A STUDY ON EXPLANATIONS ON SUPERSCRIPT (-1) IN PEDAGOGY AND MATHEMATICS

Xiaoying Chen
China West Normal University, China, cherrylaw39@163.com

Bomi Shin
Chonnam National University, Korea, bomi0210@chonnam.ac.kr

Teacher explanations serve as a critical bridge between the abstract concepts of mathematics and the concrete understanding of students. Generative Artificial Intelligence, represented by ChatGPT, has strong abilities in understanding dialogues and generating inspiring content, providing the possibility for simulating teacher explanations. In mathematics the same symbol, superscript (-1), is used to indicate an inverse of a function and a reciprocal of a rational number. This paper analyzed ChatGPT's explanations on this notation situated in mathematics and pedagogy. The results shows that ChatGPT's explanations of the superscript (-1) symbol vary significantly between pedagogical and mathematical situations, and lacking visual aids and comprehensive integration. The findings indicate that ChatGPT's potential in mathematics education is limited by its lack of conceptual depth in teaching strategies.

Keywords: ChatGPT in mathematics education, teacher explanations, inverse function, reciprocal, scrip writing

Introduction

Empirical evidence highlights the crucial role of teacher explanations in facilitating students' knowledge acquisition and understanding in mathematics. Clear and effective explanations help clarify ambiguities, build on existing knowledge, and enable students to apply concepts in various contexts (Brown & Edmunds, 2019). However, teachers must provide timely, professional, and accurate explanations to meet diverse student needs (Khashaba, 2020). The emergence of generative AI tools like ChatGPT offers new possibilities for enhancing classroom conversations. This study explores ChatGPT's potential in explaining math concepts by analyzing the characteristics of its mathematical and pedagogical explanations.

Literature Review

Teacher Explanations & ChatGPT in Education

Effective teacher explanations in mathematics classrooms have key characteristics: using examples and analogies to enhance comprehension, emphasizing and structuring explanations to highlight critical aspects, being interactive and responsive to students' needs, and providing metacognitive support. However, ChatGPT, based on Large Language Models, can generate human-like discourses and texts (Kasneci et al., 2023). In education, it's used for creating teaching materials, designing

activities, and providing feedback. Studies analyzed ChatGPT generated artifacts using theoretical frameworks, including one for mathematical concept understanding in this study.

Framework for Understanding the Inverse Function Symbol $\square^{-1}$

This study specifically focuses on students' curiosity regarding the superscripts (-1). Teachers' explanation hinge on their comprehension of the concept within the realms of mathematics and pedagogy, which in turn shapes how they direct students' attention. In this study, teachers' understanding of inverse functions symbol reflects their advanced mathematical thinking (AMK) (Wasserman, 2017; Tall, 2013), exemplifying inverse function as a *procept* in AMK, which is usually interpreted as an algebraic process, a geometric reflection, or a process reversal, but struggle to connect them. This connection is evident from the perspective of group theory, where superscript (-1) denotes an inverse element in a group structure. This study adopts the framework of understanding the inverse functions symbol (Table 1), viewing functions as objects (object-oriented function), or process-oriented function.

Table 1. Categories of understanding inverse function symbol.

Categories	Descriptions
Process-oriented function	A1. Generated by exchanging the domain and range of the given function. A2. Generated by exchanging x and y in the expression of the given function. A3. The image of inverse function is symmetrical to the image of the given function about line x = y.
Object-oriented function	B1. The function satisfied $f \circ f^{-1} = I = f^{-1} \circ f$ with the given one-to-one function f for the identity function I. B2. The inverse element of function composition for operation ○. B3. $\square^{-1}$ is the symbol of inverse element for binary operation.

This study focuses on exploring the potential of ChatGPT in explaining math concepts. Specifically, we need to solve: (a) How ChatGPTs explained the superscript (-1) in pedagogical situations? (b) How ChatGPTs explained the superscript (-1) in mathematical situations?

Methods

This study analyzes ChatGPT-4.0's ability to generate mathematical explanations through Zazkis and Zazkis' (2014) scripting task framework. Six identical prompt iterations under fixed parameters produced response variants for two inverse function questions (pedagogical/mathematical contexts), revealing the model's homogenization tendency through algorithmic determinism. Content analysis using Table 1's framework showed three patterns: (1) consistent procedural explanations ("undoing functions"), (2) limited integration of polysemous concepts (reciprocal vs. inverse), and (3) scarce visual representations. While demonstrating basic explanatory stability, ChatGPT exhibited critical pedagogical limitations in addressing conceptual ambiguities and multi-modal communication. The findings highlight generative AI's current constraints in mathematical concept development despite procedural reliability, underscoring the necessity of human educators for contextual adaptation and conceptual bridging in classroom discourse.

Results

Explanations on Superscript (-1) by ChatGPT in Pedagogical Situation

In pedagogical situation (Q1), 4 explanations (out of 6) provided by ChatGPT were categorized as A1, as illustrated in the following examples:

> The "-1" refers to the process of **undo**ing the function. Specifically, if f(x) maps an input to an output y, the inverse function $f^{-1}(y)$ maps y **back to** x. It's like reversing the operation of f. (3#)

> The inverse function essentially "**undo**es" what the original function does. For example, if f(x) takes x to y, then $f^{-1}(x)$ will **take** y **back** to x. (4#)

> For the reciprocal, ..., the "-1" refers to the operation of finding the reciprocal of a number. It's part of the notation for exponents in arithmetic. ..., It's shorthand notation in mathematics to represent the function that **reverse** the action of . ... The "-1" indicates that we are **undo**ing the function f, not dividing anything. (5#)

Despite employing diverse terminology such as "undo" "reverse" and "take back" to convey the concept of inverse function, these explanations failed to establish a connection with the meaning of both inverse functions and reciprocals. Notably, 5# highlighted a critical issue, which regarded the reciprocal as an exponent operation in arithmetic, even as the division specifically. It lacked the understanding of the reciprocals in the same way as the relationship between an inverse function and its original function. However, the explanations from 2# and 6# seemed to demonstrate characteristics of B2, shown as the following:

> If you apply the function to a value and then apply its inverse function , you will get back the original value. So, $f^{-1}(f(x)) = x$. (2#)

> If $f(x) = 2x + 3$, then $f^{-1}(x) = \frac{x-3}{2}$. When you apply f^{-1} to the result of f, you get back to the original input: $f^{-1}(f(x)) = x$. (6#)

Even with the use of concrete examples and composition of functions, they did not find the inverse element or exhibit a deeper understanding of the superscript (-1), which could recognize them as the object-oriented understanding. In fact, all explanations provided by ChatGPT interpreted the superscript (-1) as a homonym, that is, refers to the same symbol with different meanings depending on the context (Zazkis & Kontorovich, 2016).

Explanations on Superscript (-1) by ChatGPT in Mathematical Situation

In mathematical situation (Q2), the explanations provided by ChatGPT quite diverse. Firstly, only one instance (2#) exhibited as A1, explicitly indicating a tendency to interpret the superscript (-1) as a homonym, shown as "the concept of an inverse function f^{-1} also extends to the idea of homeomorphisms in topology", "the key takeaway is that while both symbols share the same notation, their meanings are context-dependent". Moreover, 5 cases demonstrated a deep grasp of group theory and abstract algebra, with their explanations being particularly well-articulated. They directly pointed out the identity elements in group theory and discussed inverse functions and reciprocals in a higher field. For example,

> The inverse of a function can be viewed as a type of operation that satisfies the condition $f^{-1}(f(x)) = x$ for all in the domain of , and similarly $f(f^{-1})(x)) = x$ for all in the domain of f^{-1}. This is akin to the identity element in a group, where an element combined with its inverse results in the identity (or "no change"). This concept of inverse in group theory and other algebraic structures is a more generalized and abstract idea than the reciprocal. (3#)

> Both function inversion and multiplicative reciprocals can be viewed through the lens of group theory. The set of bi-jective functions on a set, under composition, forms a group, and the inverse function f^{-1} is the group inverse. Similarly, the nonzero real numbers under multiplication form a group, where a^{-1} is the group inverse. (5#)

Consequently, their interpretations of the superscript (-1) leaned more towards polysemy rather than homonymy, though the specific elaboration varied. For instance,

> The commonality is that in both contexts, -1 denotes an "inverse" in some sense. however, the type of inverse **depends on the mathematical structure**: in scalar algebra, it's about multiplication; in functions, it's about composition. (1#)

> Thus, while both the inverse function and the reciprocal involve a form of "reversal", they operate in different contexts. The confusion arises from a superficial resemblance in notation, but the **underlying_mathematical ideas are quite distinct**. (3#)

> These two uses of the -1 notation are fundamentally related to inverse operations, but they **apply to different structures**. (5#)

They all agreed on the similarity in the sense of group theory, but the polysemy in different mathematical structures and knowledge domains was no longer attributed to context. In mathematical and pedagogical situations, the explanations generated by ChatGPT exhibited a striking polarization, diverging significantly from human explanations. Human with Object-oriented understanding, conversely, have the ability of viewing the structure of mathematics to explain students' ideas mathematically and analyze students' learning obstacles pedagogically (Shin, 2020; Mellone et al., 2015).

Discussion & Implications

This result reveals ChatGPT's contextual disparities in explaining (-1): Pedagogically, it emphasizes inverse functions' "undo" nature without connecting to reciprocals; mathematically, it recognizes inverse structures but omits graphical symmetry (A3) and inverse element notation (B3). These gaps likely stem from ChatGPT's neglect of visual representations like function graphs and multiple mathematical registers, critical for conceptual understanding (Huntley et al., 2000). The findings demonstrate ChatGPT's limited capacity to integrate pedagogical visualization with formal mathematical conventions when teaching inverse operations.

This study reveals ChatGPT's dual role in mathematics education: while capable of generating procedural explanations (e.g., "undoing operations"), it lacks pedagogical depth for conceptual teaching. Key limitations include insufficient integration of visual representations, polysemous concept differentiation, and contextual teaching strategies. Although useful knowledge recall and brainstorming multiple approaches, ChatGPT's outputs require critical educator evaluation to ensure disciplinary rigor and prevent oversimplification. Effective integration demands teacher expertise in pedagogical content knowledge to transform fragmented AI suggestions into structured learning frameworks, i.e., while generating formulaic explanations, it cannot autonomously design scaffold activities meeting diverse needs. Pre-service teachers should use ChatGPT as a supplementary brainstorming tool while maintaining central control over teaching design. Future development should enhance AI's capacity for object-oriented conceptual integration and cross-platform instructional alignment. These findings underscore the necessity for human educators to lead pedagogical adaptations, cultivating adaptive expertise that synergizes AI-generated content with pedagogical knowledge.

References

Brown, G., & Edmunds, S. (2019). *On explaining*. In O. Hargie (Ed.), *Handbook of communication skills* (4th ed.). New York: Routledge.

Mellone, M., Jakobsen, A., & Ribeiro, C. M. (2015). Mathematics educator transformation(s) by reflecting on students' non-standard reasoning. In K. Krainer & N. Vondrova (Eds.), *Proceedings of CERME 9* (pp. 2874-2880). Prague: ERME.

Huntley, M. A., Rasmussen, C., Villarubi, R., Sangtong, J., & Fey, J. (2000). Effects of standards-based mathematics education: A study of the core-plus mathematics project algebra and functions strand. *Journal for Research in Mathematics Education,* (31), 328–361.

Kasneci, E., Seßler, K., Küchemann, S. et al. (2023). ChatGPT for good? On opportunities and challenges of large language models for education. *Learning and Individual Differences, 103*, 102-274.

Khashaba, A. (2020). Evaluation of the effectiveness of online peer-based formative assessments (PeerWise) to enhance student learning in physiology: A systematic review using prisma guidelines. *International Journal of Research in Education and Science, 6*(4), 613-628.

Shin, B. (2020). An analysis on prospective teachers' HCK: Focused on understandings of inverse function symbol. *Journal of the Korean School Mathematics Society, 23*(1), 67-88. [in Korean]

Tall, D. (2013). *How humans learn to think mathematically: Exploring the three worlds of mathematics.* NY: Cambridge University Press.

Wasserman, N. H. (2017). Making sense of abstract algebra: Exploring secondary teachers' understandings of inverse functions in relation to its group structure. *Mathematical Thinking and Learning, 19*(3), 181–201.

Zazkis, R., & Kontorovich, I. (2016). A curious case of superscript (−1): Prospective secondary mathematics teachers explain. *Journal of Mathematical Behavior, 43*, 98-110.

Zazkis, R., & Zazkis, D. (2014). Script writing in the mathematics classroom: Imaginary conversations on the structure of numbers. *Research in Mathematics Education, 16*(1), 54–70.

Appendix

You are playing the role of an undergraduate student majoring in Mathematics Education. You are proficient in primary and secondary school mathematics content and have studied undergraduate-level mathematical knowledge, including Analysis, Geometry, Topology, and so on. You have also learned relevant knowledge of Pedagogy and completed courses such as Fundamentals of Education and Educational Psychology. You are currently taking courses related to Mathematics Education.

Next, you need to answer two questions. You will be provided with a dialogue between a teacher and a student in a high school mathematics class, as follows:

T: So today we will continue our exploration of how to find an inverse function for a given function. Consider for example f(x)=2x+5. Yes?

S: So you said yesterday that f^{-1} stands for an inverse function. But we learned that this power (-1) means 1 over, that is, $5^{-1} = \frac{1}{5}$, right?

T: Right.

S: So, is this the same symbol, or what?

Q1: Please carefully read the dialogue and, based on your professional knowledge, extend this imaginary interaction in a form of a dialogue between a teacher and a student (or students).

Q2: The way you understand the idea yourself could be different from the way you explain it to a student. If this is the case, please indicate how you could clarify the issue for yourself, or for a "mathematically mature" colleague.

TSG2-2-2

NARRATIVES OF A HIGH SCHOOL TEACHER IN A MATHEMATICS CLASS

Mayumi Kawamura
Oita University, Japan, skmv606vmks@gmail.com

The study poses the research question: How do high school teachers interact between mathematizing (focusing on mathematics) and subjectifying (emphasizing people doing mathematics) in classes? To obtain answers, the study conducted a qualitative analysis on the class of a mathematics teacher in a Japanese high school using the perspective of commognitive theory. The result clearly indicated that the mathematics teacher communicated the effectiveness that he encouraged students to adopt by demonstrating consistency between words and actions.

Keywords: commognitive theory, mathematizing, subjectifying

Introduction

The teaching and learning process consists of interactions between the narratives about mathematics and people doing mathematics (Heyd-Metzuyanim & Sfard, 2012; Sfard, 2008). In commognitive theory, narratives about mathematics are conceptualized as *mathematizing*, and narratives about people doing mathematics are conceptualized as *subjectifying*. The latter is also called *identifying*, because the product of subjectifying activities is the formation of identity. Previous scholars have reported the effects of mathematizing and subjectifying on student learning (cf. Heyd-Metzuyanim & Cooper, 2023). However, the mainstream research on *teachers* mainly focuses on identities formed by subjectifying and changes in identities (cf. Nachlieli & Heyd-Metzuyanim, 2022). The way that teachers teach will likely influence the way students that form their identities; thus, elucidating how teachers utilize the interaction between mathematizing and subjectifying during teaching is important. Against this background, the study poses the following research question: How do high school teachers interact between mathematizing and subjectifying in classes?

Mathematizing and Subjectifying in the Commognitive Theory

Commognitive theory clearly states the basic idea that thinking is a form of communication and emphasizes the fact that interpersonal communication and individual cognition are two facets of the same phenomenon (Sfard, 2008). This theory assumes that the mathematical teaching and learning processes are a result of the interaction between mathematizing and subjectifying (Heyd-Metzuyanim & Sfard, 2012). As such, capturing mathematical teaching and learning from the perspectives of mathematics and communication is possible. As examples, "$2 \times 3 = 6$" and "an equilateral triangle has three sides of equal length" represent mathematizing, while "I don't understand" and "she is good at math" represent subjectifying. Heyd-Metzuyanim and Sfard (2012) classified subjectifying utterances into three levels according to the generality of the utterances. The first level pertains to utterances related to specific performances (e.g., using past tense such as "I forgot how to calculate fractions"), while the second denotes speech spoken in the present tense that can be observed as a routine performance (e.g., using present tense such as "It boggles my mind"). The last one is utter-

ances related to the properties or membership of the person (e.g., using "be" or "have" verb such as "My brain is so slow" and "She has a mathematical mind"). We can identify the parson's properties, that is, identity in subjectifying from the third level of utterances (Heyd-Metzuyanim & Sfard, 2012). In this analytical framework, utterances such as "I want you to try it" that do not include the be or have verbs are classified as the second level and cannot identify the properties of the person. This is because, out of the three levels of subjectification, the second and third levels can be differentiated by whether an action or be/have verbs are used. Therefore, the study refers to this classification as level "2.5" in order to use it as a point of analysis. This is a level where we can identify the properties of the person, even though the utterances do not use the be/have verbs.

Method and Subject

The teacher was Kei who had 22 years of experience and had been taught tenth grade students (aged 15–16 years) at a public high school in Japan. The class (n = 30) was the top out of three classes classified according to proficiency level out of 80 students. The content of the class was proof of the equations. The scene analyzed was the part of the class in which the students were asked to prove "If a / b = c / d, then prove that (a + c) / (b + d) = (a – c) / (b – d)".

This study employs qualitative analysis to capture the relationship between subjectifying and mathematizing conducted by the teacher. Specifically, data, which were derived from the recording of the class from the back of the class and interview for teacher, were first transcribed into Japanese then translated into English. First, I explained the activities of mathematizing, that began with an outline of the methods of proof used. I then extracted the utterances of the teacher and analyzed identity in subjectifying narratives using the abovementioned levels of subjectifying. Next, I ascertained the teachers' identity in subjectifying through utterances in class and interviews. Lastly, the study examined the interaction between mathematizing and subjectifying, and I explained the significance of setting the level "2.5".

Transcript Data and Methods of Proof

For brevity, the study only included a numbered selection of the utterances of teacher Kei and an outline of the method of proof covered in the class while following the flow of the class.

1Kei (after writing the problem on the blackboard): Of course, if you can lead and prove it properly without any special effort, that's the best. I think it is better to think hard and solve it yourself than to be taught by someone else and do it in a good way. (After reading the problem) Talk to your classmates for a bit. (omission … the students are each working on proving equality).

2Kei: There are lots of different ways to do things, so I want you to come up with lots of different ideas. I want you to think, "Oh, that is how you do it!" It is good to go through a process of trial and error like that. How did you do that? (omission … teacher Kei pointed to a student and, while interacting with the student's comments, wrote the first proof on the blackboard; (i) is an outline).

(i) Since a / b = c / d, ad = bc, (left side) – (right side) = 2(–ad + bc) / (b + d).

3Kei: Well, I know I've said this a lot, but it is more valuable to try and fail and try again on your own, rather than just having someone teach you or reading a textbook and thinking "Oh, I see!" and then just doing it in a very unrefined way. Has anyone tried substituting? For example, you could do a =. (omission …While interacting with the students' comments, teacher Kei wrote down the second method on the blackboard; (ii) is an outline).

(ii) Since a / b = c / d, a = bc / d, (left side) = (bc / d + c) / (b + d) = c / d (right side) = (bc / d–c) / (b–d) = c / d

4Kei: I have said this many times, but if you came up with this yourself, it is more valuable than if you were taught by someone else. You can do it by substitution. However, there is a typical way to solve this kind of problem, so I am going to add some more letters. (omission …The teacher explained and wrote the proof on the blackboard; (iii) is an outline).

(iii) Since a / b = c / d = k, a = bk, c = dk, (left side) = (b + d)k / (b + d) = k (right side) = (b – d)k / (b–d) = k

5Kei: When you compare them all, this one feels the clearest. That is because you do not have to include fractions within fractions. As you try out various things, you can find out which one is best for you. Probably, you can find out the good points of this one by comparing it with the others, rather than doing this one (pointing at (iii) on the blackboard) from the start.

Analysis

Among the listed mathematical objects, (i) is proof that uses the transformed condition ad = bc, while (ii) is a proof that transforms the condition to a = bc / d and substitutes a for the left and right sides. Alternatively, (iii) is proof that uses the proportional expression of the condition, which is a typical example of a proof given in the textbook. Therefore, the study infers that the development gradually demonstrates a way of thinking that approaches the typical solution.

The researcher also observes subjectifying utterances in addition to mathematizing ones. Before and during the cognition process of the students, the teacher conveyed the actions that he wanted the students to take using subjectifying utterances at level 2.5: “I think ...” (1Kei), “I want ...” (2Kei), and “It is good ...” (2Kei). In response to the subjectifying utterances of the teacher, the students who had worked on the proof of the formula took up the methods of proving (i) and (ii). After proving (i) and (ii), teacher Kei made subjectifying utterances at level 2.5, “It is more valuable ...” (3Kei and 4Kei), and valued the actions thought by the students. Finally, the teacher illustrated the proof of (iii). Teacher Kei used the second level subjectifying utterance “you can find …” (5Kei) to explain to the students that, even if the third method was the best way to prove the point, they could independently discover by trying out the other methods and comparing them.

During the interviews, the teacher made comments such as “I want students to discover how to think for themselves, even if it’s just a little” and “If students can solve the problems on their own, I want to let them try on their own”. From these comments, we can determine that the teacher has identity in subjectifying that places importance on students trying and making mistakes on their own, rather than on knowing smart ways of solving problems.

Discussion and Recommendation

Teacher Kei made similar comments multiple times during the interview. Therefore, we can interpret the teacher’s utterances at level 2.5 in the class as an expression of teacher Kei’s identity in subjectifying which places importance on students’ trying and making mistakes.

Many mathematics teachers use the methods of displaying alternative solutions and encouraging students to think of alternative solutions. Therefore, if the class were interpreted through mathematizing alone, then it would appear as a class that gradually exhibited the idea of approaching a typ-

ical solution. However, if it were interpreted to include the interaction between mathematizing and subjectifying, then it becomes a class that demonstrates the effectiveness of an attitude of arriving at a solution by maximizing knowledge and skills instead of only imitating a method that they were taught. The reason is that the teacher told the students the type of action they were expected to take after presenting the problem then explained and valued the methods of proof using such actions. Afterward, the teacher stated that they could identify the best method by comparing the results of the implementation of the various methods. If the method of proof in (iii), which is the reverse of (ii), were first shown to the entire class, then the students would have been taught a simple method of proof, and the value of formulating alternative solutions would have been halved. Lastly, the students would have perceived the argument of the teacher about his attitude as lacking conviction.

The study proposed a perspective on level 2.5 of subjectifying utterances that were not classified in Heyd-Metzuyanim and Sfard (2012). In addition to third level, it was also shown that level 2.5 subjectifying utterances can be used to identify the properties of the person. This makes it possible to identify the properties of the person from subjectifying utterances other than be/have verbs. This is through to be a useful tool for analyzing the meaning of actions in classes. In actual classes, many teachers do not use subjectifying utterances such as "I am …", so being able to analyze them as the teacher's identity in subjectifying is useful. Using this new level to analyze a Japanese high school mathematics class, the study clearly demonstrated that the teacher was communicating the effectiveness of the attitude that he encouraged the students to adopt by exhibiting consistency between words and actions. When teachers make subjectifying utterances in classes, it is suggested that the student's conviction and implementation of the action is encouraged by the consistency between the mathematical activities and the utterances. This case study can be viewed as a first step in elucidating how teachers interact with mathematizing and subjectifying in the classroom.

References

Heyd-Metzuyanim, E., & Cooper, J. (2023). When the problem seems answerable yet the solution is unavailable: affective reactions around an impasse in mathematical discourse. *International Journal of Research in Undergraduate Mathematics Education*, 9(3), 605-631.

Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom: On learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research*, 51-52, 128-145.

Nachlieli, T., & Heyd-Metzuyanim, E. (2022). Commognitive conflicts as a learning mechanism towards explorative pedagogical discourse. *Journal of Mathematics Teacher Education*, 25(3), 347-369.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge university press.

Acknowledgement

This work was supported by JSPS KAKENHI Grant Number 23K18944.

TSG2-2-3

EXPLORING THE CURRENT STATE OF GIFTED EDUCATION IN MATHEMATICS IN JAPAN: A CASE STUDY OF CLASSROOM PRACTICES IN A SUPER SCIENCE HIGH SCHOOL

Natsuki Uchikubo
Graduate School of Comprehensive Human Sciences, University of Tsukuba, Japan,
s2430306@u.tsukuba.ac.jp

This study aims to explore the current state and features of gifted education in mathematics in Japan with a particular focus on daily classroom practices in a Super Science High School (SSH). The survey was conducted at a high school designated as SSH and included observations of daily mathematics classes and semi-structured interviews with four students and with their mathematics teacher. As indicated by previous research, it was revealed that egalitarianism was influential in Japan's educational policy as well as educational practices in relation to gifted education in mathematics. Nevertheless, the teacher in a SSH recognized specific characteristics of mathematically gifted students and designed his daily classroom practices reflecting fundamental aspects of gifted education in mathematics. While Japan lacks a formal educational system for mathematically gifted students, the SSH programs as well as the surveyed teacher provide insight into thinking of daily classroom practices where mathematically gifted students demonstrate their creativity.

Keywords: mathematical giftedness, gifted education, creativity, Super Science Highschool

Introduction

Given that there are individuals with exceptional potential in a specific academic domain and they are in need of special educational support, gifted education has increasingly become a global concern (e.g., Dimitriadis, 2017). In mathematics education, *acceleration, enrichment and deepening* are widely recognized as core pedagogical models for addressing the needs of mathematically gifted students (Leikin, 2011). However, implementation and stance on gifted education vary across countries, which reflects the cultural, political and economic features of the country (Frantz & McClarty, 2016). Among countries strongly emphasizing on egalitarianism, there is sometimes no formal educational system which explicitly focuses on gifted education and Japan is not an exception.

Nevertheless, this does not imply that educational needs of intellectually gifted students are entirely neglected in Japan. Since 2002, the Ministry of Education, Culture, Sports, Science and Technology (MEXT) has implemented the Super Science High School (SSH) program to cultivate globally competitive talent in science and technology. The program supports high schools providing advanced mathematics and science education, and students in SSHs makes significant contributions at the global level including participation in the International Science Olympiad and publication of research findings in international academic journals (Council for Science and Technology, 2021). Moreover, a recent report by the Central Council for Education, which advocates for personalized learning, leads to a growing attention on gifted education as well as heightened expectations for the SSH program (Advisory Council on Educational Instruction and Support for Gifted Students, 2022).

The growing interest in gifted education is a relatively recent trend in Japan. Therefore, there has been little discussion about educational practices in SSHs from the perspective of nurturing mathematical giftedness, and even less about daily mathematics classes where mathematically gifted students spend most of their time.

This study aims to contribute to better understanding of the current state and features of gifted education in Japan. First, the features of the SSH program as nurturing giftedness are examined by exploring existing literature and documentation following recent structural changes in the SSH framework. Then, a case-study approach is adopted to obtain further in-depth information on how mathematics teachers in SSHs design their classroom practices.

Brief Overview of the Super Science High School

The SSH program was established in 2002 in response to the growing needs to cultivate globally competitive human capital in science and technology. According to the second report of the expert council on the SSH program (Council for Science and Technology, 2021), each high school designated as SSH may design their school curriculum to which the National Curriculum Standard may not apply "when it is especially necessary for performing research to contribute to the improvement of high school curricula" (Ministry of Justice, 2018).

In 2022 academic year, a new category of the SSH designation, certified schools (*Nintei-ko*), was introduced. As of the academic year 2024–2025, 15 high schools (out of 225 SSHs) with extensive experience in the SSH program are designated as certified schools. These schools serve as nationwide educational models for cultivating experts in science and technology. Leveraging their accumulated expertise and strengths, they are expected to not only implement but also disseminate their innovative educational practices. Furthermore, they are eligible to apply for the special exception to the standard for curricula when "it is necessary to provide education in the high school by organizing special curricula harnessing the characteristics of the relevant high school or relevant region in order to provide more effective education" (Ministry of Justice, 2018).

However, as of April 2024, no certified schools have applied for such special exception concerning with mathematics and science curriculum (MEXT, 2024). This indicates the significant influence of standardized national curriculum on public education in Japan that SSHs basically design their daily academic classes based on the National Curriculum Standard. Additionally, unlike other SSHs, certified schools do not receive financial support from Japan Science and Technology Agency (JST) (Council for Science and Technology, 2021), reflecting Japan's cultural feature of egalitarianism in educational policy.

Under such circumstances, SSHs play a critical role in realizing the potential of gifted students through their progressive initiatives such as independent research activities (*Kadai-Kenkyu*) and original advanced teaching materials. It is reported that the SSH program has enhanced students' understanding of the nature of science and increased the number of students to pursue STEM careers (Science and Technology Policy Bureau, 2023).

Method

In October 2023, the survey was conducted at one of the most academically advanced high schools in Japan. The school had been designated as SSH since 2002 at that time and later, was designated as certified school in 2024. The survey includes observations of three consecutive mathematics classes and semi-structured interviews with four students and with their mathematics teacher.

To explore how classroom practices in mathematics are designed at that school, this study focuses on the interview with the mathematics teacher. During the interview, the teacher was asked about the learning objectives of the observed classes and students' characteristics he considered when organizing the classroom practices as well as how they differ from those at a previous school. Collected data was transcribed and analyzed thematically and categorized under common theme. Transcripts of observed classes and interviews with four students served as supplementary references.

Results

First, the teacher prefaced by mentioning that no acceleration approach had been adopted, and mathematics curriculum was designed based on the National Curriculum Standard at that school, which confirmed the influence of standardized national curriculum on public education in Japan.

Nevertheless, the teacher's responses included some essences to design the learning environment for mathematically gifted students. The teacher's reflection on his classroom practices was categorized into two themes: perspectives on his teaching materials and perspectives on his teaching approach. Three distinct features were identified for perspectives on teaching materials, and two were identified for perspectives on teaching approach, as describing below.

Perspectives on teaching materials

It was revealed that the teacher was designing teaching materials which (i) are ill-structured, (ii) provide a deeper exploration of mathematical topics and (iii) ensure accessibility for students of varying mathematical abilities.

The teacher described ill-structured problems as "vague questions", referring to problems that are not formally defined and lack clearly specified givens. He also explained that "the existence of a solution is evident, yet articulating a clear explanation can be challenging, even for me. But since the solvability is unquestionable, variety of ideas might be generated by students". In this context, ill-structured problems permit a variety of interpretations and solution paths, allowing mathematically gifted students to explore the problems creatively (Silver, 1997).

"Deeper exploration" referred to "expanding what we can do using the specific mathematical knowledge in the textbook" according to the teacher. Through the observed classes, students indeed engaged in exploring "underlying rules for regular curricular topics" (Leikin, 2011, p.178). This approach corresponds to "deepening", one of the three primary approaches in gifted education, alongside acceleration and enrichment (Leikin, 2011).

These two features align with the key elements highlighted in existent research of gifted education in mathematics (e.g. Sheffield, 2009).

Lastly, according to the teacher, problems in observed classes were designed to

> (…) begin with accessible problems even for students with difficulty in mathematics. (…) The problems become increasingly demanding, with the final problem being so challenging that no one might be able to solve it.

The teacher ensured accessibility for students of varying mathematical abilities, reflecting a feature of such a country where students basically spend in heterogeneous classroom under the culture of egalitarianism.

Perspectives on Teaching Approach

As his teaching approach, it was revealed that the teacher (i) respect students' idea as well as their opinion and (ii) provide instructional support to ensure students complete their solutions with full logical reasoning, emphasizing the importance to fully develop and articulate solutions.

It is said that mathematically gifted students "are not afraid to try unusual methods that do not guarantee success" (Sheffield, 2009, p.89) and sometimes challenge the teacher's explanations by proposing alternative solutions. The teacher recognized such characteristics and demonstrated a positive reception to them (e.g. "I want them to be free to exploring any ideas. It's doesn't matter even if someone says that there is no need using given ideas").

His emphasis on the importance to fully develop and articulate solutions also reflects characteristics of mathematically gifted students: curtailing the process of mathematical reasoning and getting bored with routine tasks (e.g. Krutetskii, 1976). During the observed classes, the teacher often supported students to explain their ideas to their classmates through successive questioning.

Concluding Remark

As Frantz & McClarty (2016) suggests, egalitarianism is very influential in Japan's educational policy as well as educational practices in relation to gifted education in mathematics. Despite that such egalitarian system is often seen as a challenge to implementing gifted education, the teacher at the surveyed high school recognized specific characteristics of mathematically gifted students and his daily classroom practices included some fundamental aspects of gifted education in mathematics.

As a certified school of the SSH program, mathematics teachers in surveyed high school implement a continuous improvement cycle by presenting their teaching materials to other schools, incorporating the received feedback, and refining these materials accordingly. Such cycle, as well as the open-minded teacher, seemed to enable the provision of an educational environment where mathematically gifted students demonstrate their creativity during daily mathematics classes.

References

Advisory Council on Educational Instruction and Support for Gifted Students. (2022). Summary report on the advisory council on educational instruction and support for gifted students: As part of the comprehensive initiative to enhance both personalized and collaborative learning that fosters mutual recognition of diversity. https://www.mext.go.jp/content/20220928-mxt_kyoiku02_000016594_01.pdf (February 10, 2025)

Council for Science and Technology. (2021). The 2nd report of the advisory council on the future direction of the Super Science High School (SSH) Program. https://www.mext.go.jp/content/20211020-mxt_kiban03-000018518_8.pdf (February 2, 2025)

Dimitriadis, C., & Georgeson, J. (2017). Provision for mathematically able children in primary schools: a review of practice five years after England dropped the gifted and talented initiative. *Educational Review*, *70*(3), 358–380.

Frantz, R. S., & McClarty, K. L. (2016). Gifted education's reflection of country-specific cultural, political, and economic features. *Gifted and Talented International, 31*(1), 46–58.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren*. The University of Chicago.

Leikin, R. (2011). The education of mathematically gifted students: Some complexities and questions. *The Mathematics Enthusiast, 8*(1), 167–188.

MEXT. (2024). About the Special Exception to Curricula. https://curriculumdb.mext.go.jp/bc/tr/ (February 3, 2025)

Ministry of Justice. (2018). Regulation for enforcement of the school education act: Order of the ministry of education No. 35 of 2018. *Japanese law translation.* https://www.japaneselawtranslation.go.jp/ja/laws/view/4575#je_ch7sc1at7 (February 9, 2025)

Science and Technology Policy Bureau. (2023). Report on the evaluation of the Super Science High School (SSH) program outcomes. https://www.mext.go.jp/content/20230518-mxt_kiban01-000029519_1.pdf (February 10, 2025)

Sheffield, L. (2009). Developing mathematical creativity–Questions may be the answer. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 87-100). Sense.

TSG2-2-4

PROPOSAL FOR WIDE-AREA DEVELOPMENT AND CLASSROOM PRACTICE USING ICT TEACHING MATERIALS THAT CAN BE CONSIDERED FROM DAILY LIFE - CLASSES ON DIAGRAM OF PRIMARY FUNCTIONS TO CREATE AND THINK ABOUT DIAGRAMS WIDE-AREA LESSON DESIGN USING TABLET PC

Satoru Sakanashi
Tokyo Metropolitan Minato Municipal Odaiba-Gakuen Koyo Junior High School, Japan,
saksat1015@gmail.com

In this research, we have been investigating the possibility of using ICT equipment in the classroom for diagrams, which are treated in textbooks as a teaching material for the development of primary functions in junior high school mathematics, and have been working on the long-standing theme of cultivating students' thinking skills by "reading the assignments, creating graphs that match the tasks, and using them as diagrams to consider the problems."

In this research presentation, we aim to propose a lesson design that uses an ICT environment to simultaneously practice classes in multiple regions and schools, and to derive from graphs through the relationship between train travel, operating time, and stop stations through collaborative learning, and to pursue the development and possibility of the problem of diagrams using the ICT environment.

Keywords: Information Utilization Skills, Lesson Design, Tablet Computers, Collaborative Learning

Introduction

The theme of this research has been to incorporate "creative videos shot with digital cameras" into the classroom and to devise ways to handle teaching materials, which can be done according to the current state of the ICT environment.

As shown in the National Academic Achievement Survey Report (pp. 29-31, 57-69) published by the National Institute for Educational Policy Research in 2024, we have set up and proceeded with this research with the goal of creating a place to cultivate the ability to use the information given by the task by teaching in the form of functions, which is one of the areas of weakness in mathematics for junior high school students, using clip videos in class and having students solve problems through collaborative work and exchanging opinions.

In this research, we focused on the diagram of the primary function, which is a unit that is often used in daily life but is difficult for students to understand, and devised the use of ICT teaching materials in the classroom so that students can work on the tasks with interest and interest. I would like to consider lesson design using ICT and find the development and possibilities of classroom practice.

Details of the Research

(1) About this Research

In this research, I considered the following three points about primary functional learning, which more than 80% of students at the school where I work said they were not good at it.

i) To enable students to be motivated to work on various situations in the lesson through the study of diagram tasks.

ii) To revitalize the class by exchanging students' remarks and ideas within the group and in the class.

iii) In the lesson, students will be able to develop classes that enrich their mathematical expressiveness by making graphs, examining the passage of time and movement, and understanding the expressions expressed by graphs.

In addition, in the researching class, the "diagram" is positioned as the subject of the research

class in the following three points.

i) To be able to know the meaning of the word "diamond disturbance" broadcast by TV news programs in daily life by creating diagrams.

ii) Use ICT devices such as OHC and computers to stimulate the exchange of opinions during presentations and exchanges among students.

iii) Conduct a class practice to "visualize" the possibility of using ICT equipment for students who needs support.

In conducting the researching class, the class is usually divided into classes, but the class was held in a class format in which all students participated, considering the environment in which

ICT equipment is used, the size of the number of students, and the characteristics of the teaching materials.

As a teaching material to be used in this research, I focused on the "Keikyu Line" and "Hankyu Kyoto Line" that are used by students on a daily basis, although they are famous for their overcrowded schedules.

In addition, when creating the teaching materials, we asked the station staff to listen to the actual operation situation and reflected the content in the teaching materials.

(2) Teaching Method and Content

The class was conducted with the following three points in mind.

i) Divide students into 3 or 4 members to make into small groups.

ii) Give each group an assignment card cut into an elongated shape, as shown in Figure 1

iii) Read the assignment written on the card and make a diagram on the worksheet as shown in Figure 2 to solve the problem.

The students carefully read the text of the mission card in Figure 1 below, created a diagram (Figure 3) and examined it, derived the relationship between the run, time, and stop of the train from the graph through group collaborative learning, exchanged the remarks and suggestions of their peers,

and set the task to find the use and possibility of using ICT equipment as a summary of the primary function of the development of the diagram task.

What I Learned from the Classroom Practice.

In the research classes that have been held in the past, the students were able to work on the assignment with enthusiasm, and when working on the mission card in the group, they were willing to exchange opinions within the group, and the representative was able to exchange opinions and share mathematical ideas and expressions by presenting their own materials using tablet devices and presenting their thoughts.

Consideration

Based on the above practical results, as shown in the results in terms of students' mathematical thinking skills, interest and interest in teaching materials, and the integration of life and subject learning, students answered "I learned that mathematics is used in my daily life" or "I am not good at it, but I am interested in learning today." As a result, it was a research result that was expected to improve greatly through the class.

Therefore, we believe that the use of teaching materials for railways in the two major metropolitan areas of Tokyo and Osaka has made it possible to diversify class practice, and that it is possible to hold long-distance simultaneous events using Zoom and Teams at the same time using Zoom and Teams remotely, and to share the actual state of students' thinking and the current situation among the schools that conduct the classes.

Agenda for the Future

The following points are raised as teaching materials that can be used for school collaboration and wide-area collaboration.

Currently, the area of the created assignment is limited, so it is necessary to create a teaching material that is of interest to children and students according to the characteristics of the region.

- Since it is a living teaching material that Japan is proud of and is known around the world, we are trying to collaborate of junior high school, but it is difficult to achieve cooperation between schools and school types due to local communities and school events.
- Handling of mutual use of personal information when sharing data
- About the characteristics of the environment and application used for communication, the communication status, and how to deal with problems including communication errors,

If we can share the above three issues, collaborate with teaching materials across regions, and implement class practice, we will be able to develop classes using the ICT environment.

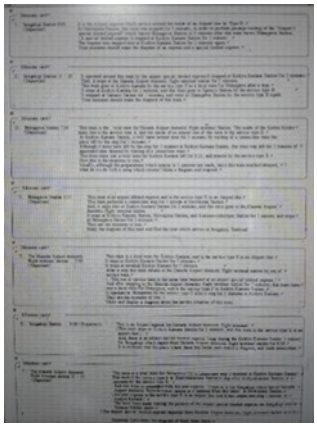


Figure 1. Mission card (working card)

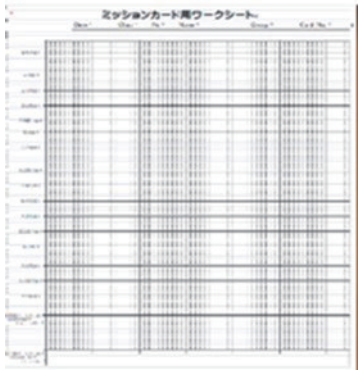


Figure 2. Diagrams' Graph sheet Original: written in Japanese

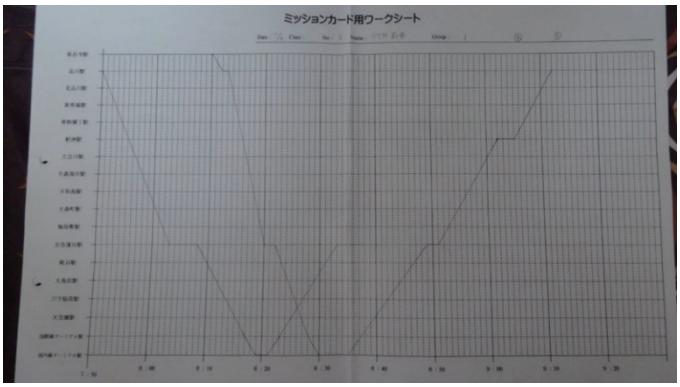


Figure 3. Student's work sheet

References

National Institute for Educational Policy Research2024 : *National Academic Achievement Survey Report* (pp. 29-31, 57-69)

TSG2-2-5

EMBRACING STUDENTS' DIVERSITY IN MATHEMATICS THROUGH DIFFERENTITATED INSTRUCTION AND OPEN APPROACH: A FRAMEWORK DEVELOPMENT

Yoga Dwi Windy Kusuma Ningtyas
Hiroshima University, Japan, d233565@hiroshima-u.ac.jp

Supporting student diversity and respecting varied mathematical ideas call for an appropriate instructional approach. Differentiated instruction seems promising for addressing these needs, but it is challenging to strengthen students' mathematical ideas. This study presents a framework that focuses on the integration of differentiated instruction (DI) and open approach (OA) in practice to support students' diversity and their mathematical ideas. The findings show that by integrating the key principles of DI and OA, supporting student growth, encouraging collaboration, and promoting equity, teachers can design mathematical activities that not only embrace students' diverse identities and mathematical learning experiences but also provide meaningful open-ended tasks.

Keywords: differentiated instruction, open-ended approach, design research, diversity in mathematics, mathematical ideas.

Introduction

Differentiated instruction (DI) has developed as a means of cultivating student diversity (Guberina, 2023). Students increasingly demonstrate a range of backgrounds such as interests, abilities, motivation, and learning styles (Suprayogi et al., 2017; Guberina, 2023). DI is considered a teaching approach used to proactively recognize and address the diverse characteristics of students by valuing their strengths and adapting teaching methods to support their individual needs and limitations (Tomlinson, 2014; Tomlinson & Imbeau, 2023). In Indonesia context, DI has been mandated in the national education curriculum (MoECRT, 2022) as it aligns with their education goal, which emphasizes on student-centered learning, diversity of students' need and character development, and creativity (Ruhimat & Darmawan, 2020; MoECRT, 2022; Zidan & Qamariah, 2023).

Theoretical Framework

In implementing DI in mathematics classrooms, teachers are required to see student differences (Aliyeva, 2021; Hackenberg et al., 2021; Guberina, 2023) that are used as the basis for lesson planning and tasks (assessment); however, at the same time, there needs to be greater attention to addressing and enriching students' mathematical ideas. Students will have different ideas in their development, bringing a wide range of experiences to their learning. The extensive notion of diversity in mathematics classrooms has turned from cultural diversity (social, ethnic, gender, language, etc.) to students' ideas and reasoning (Seah & Andersson, 2015). Students can present and discuss various ideas in mathematics classrooms, which are then refined and enhanced mathematically using an open approach (OA). It involves contributions from peers and carefully guided feedback from the teacher to foster a deeper understanding and improve the sophistication of their solutions (Becker & Shimada, 1997; Nohda, 2000).

The implementation of DI varies widely across different settings. Teachers may group students based on their skill levels or interests, assign different sets of exercises, offer additional support and feedback to struggling readers, provide enrichment materials for advanced learners, adjust expectations regarding learning objectives, modify allocated time for tasks, or offer individualized encouragement (Tomlinson & McTighe, 2006; Fox & Hoffman, 2011; Tomlinson, 2014; Tomlinson & Imbeau, 2023). Additionally, teachers may allow certain students to work independently instead of in groups or select different entry points in the experiential learning cycle (Heacox, 2012; Spencer-Waterman, 2014). This signifies the possible adaptation of DI and OA in mathematics classroom practices.

In the Indonesian context, differentiation must be reframed in terms of a social value agenda focused on embracing diverse mathematical ideas within a differentiated space. In response to strengthening students' mathematical ideas with an understanding of social matters, teachers may consider using an open approach (OA). This study introduces a framework that lies in how teachers perceive differentiated instruction so that its impact can be fully optimized and, at the same time, how the diversity in students' backgrounds and mathematical ideas through OA can strengthen their mathematical learning experiences. This raises the question of how differentiated instruction can be integrated with OA approach to elevate students' mathematical ideas.

Methods

This study utilized educational design research (DR), focusing on addressing the educational challenges faced in teaching mathematics to students with diverse needs by applying relevant theoretical insights (Plomp, 2013). The EDR process followed three phases (Gravemeijer & Cobb, 2006; Plomp, 2013): (1) initial phase: A literature review was conducted to conceptualize a framework for embracing student diversity in mathematics classroom practice through adopting DI and OA; (2) prototyping: initial design criteria were formulated and iteratively refined; and (3) The framework was empirically assessed for effectiveness through feedback from target group and classroom implementation.

This study focuses on the initial phase, specifically the literature review. A systematic literature review (SLR) was conducted using the Preferred Reporting Items for Systematic Reviews and Meta-Analyzes (PRISMA) procedures to ensure a transparent and reproducible process. The search keywords included students' backgrounds, diversity in mathematics, differentiated instruction, OA approach, and students' mathematical ideas. The findings from the literatures were utilized in developing a framework. A final total of 12 peer-reviewed articles were generated from Scopus and Google Scholar databases. The data were subsequently analyzed using qualitative methods, emphasizing the analysis of meaning, context involved the learning practices. This research explores the possible integration of DI and OA in embracing students' diversity in mathematics classroom settings and examining their principles.

Results and Discussion

The integration of differentiated instruction (DI) and open approach (OA) is underpinned by theoretical foundations that underscore the importance of addressing the diversity in students' backgrounds and their mathematical ideas through openness value in learning mathematics.

Table 1 presents the principles navigating the adaptation of DI and OA. DI is fundamentally centered on adapting teaching methods to meet the students' needs, considering their readiness, inter-

ests, and learning profiles. DI is grounded in constructivist theories emphasizing students' active construction of knowledge. (Tomlinson & McTighe, 2006; Tomlinson, 2014; Tomlinson & Imbeau, 2023) DI allows teachers to implement varied strategies to meet students' needs (Spencer-Waterman, 2014). Meanwhile, OA prioritizes the democratization of knowledge in response to possible mathematical ideas through open-ended problems and student discussions (Becker & Shimada, 1997; Nohda, 2000; Inprasitha, 2006). In a mathematics classroom employing the OA, students present and discuss various ideas, which are then refined and enhanced mathematically. This process involves contributions from peers and carefully guided feedback from the teacher to foster a deeper understanding and improve the sophistication of solutions (Nohda, 2000).

DI may risk compromising the depth of students' mathematical reasoning through ability-based grouping (Tomlinson & Imbeau, 2023); however, this limitation can be mitigated by integrating OA, which emphasizes deep thinking through open-ended tasks (Becker & Shimada, 1997; Nohda, 2000; Raethong & Thinwiangthong, 2019). OA, while fostering rich problem-solving and discussion, often overlooks diverse learning needs – something DI can address by providing tailored support and strategies to mathematical understanding (Heacox, 2012; Spencer-Waterman, 2014).

Table 1. The principles of DI and OA integration

Principles	DI	OA
Respects for diverse ideas	accommodates students' with varying abilities, learning styles, and interests (Tomlinson & McTighe, 2006; Tomlinson, 2014; Aliyeva, 2021; Hackenberg et al., 2021; Guberina, 2023; Tomlinson & Imbeau, 2023)	open-ended tasks cater to a wide range of abilities, allowing students to tackle real-world problems, express their unique ideas discussions (Becker & Shimada, 1997; Nohda, 2000; Raethong & Thinwiangthong, 2019).
Encourages students' growth and progress	students can learn at their own pace, fostering academic growth regardless of their starting point (Tomlinson & McTighe, 2006; Tomlinson, 2014; Tomlinson & Imbeau, 2023)	tasks that encourage reasoning and justification push students to engage with problems at their own level and progress, leading to more meaningful learning (Becker & Shimada, 1997; Nohda, 2000; Raethong & Thinwiangthong, 2019)
Supports collaborative learning	flexible grouping or multiple teaching strategies develop students' understanding of their ideas (Heacox, 2012; Spencer-Waterman, 2014)	peer discussion or class discussions (*neriage*) work as students critically evaluate and improve each other's ideas (Becker & Shimada, 1997; Nohda, 2000; Inprasitha, 2006)
Promotes equity in education	ensures that all students have access to meaningful and challenging learning experiences (Tomlinson & McTighe, 2006; Fox & Hoffman, 2011; Tomlinson, 2014; Tomlinson & Imbeau, 2023)	all students, regardless of skill level, to contribute and find value in their ideas (Becker & Shimada, 1997; Nohda, 2000; Inprasitha, 2006)

Figure 2 depicts the adaptation of DI and OA in practice in this study. Navigating the adaptation of DI differentiation, such as what students learn (content), how students learn (process), and how students demonstrate (product) and openness value of OA. When differentiating content, teachers strategically select what is to be taught and provide open-ended problems. This appreciates the diverse perspective of students' readiness, profile, and interest through the use of tasks. During the differentiation process, teachers empower students to question and critically analyze information through varied types of design activities, such as discussion/grouping, or even individual tasks. This appreciates the diverse perspectives of students' ideas. In differentiating the product, teachers encourage students' diverse findings through presentations, demonstrations, or writing reports.

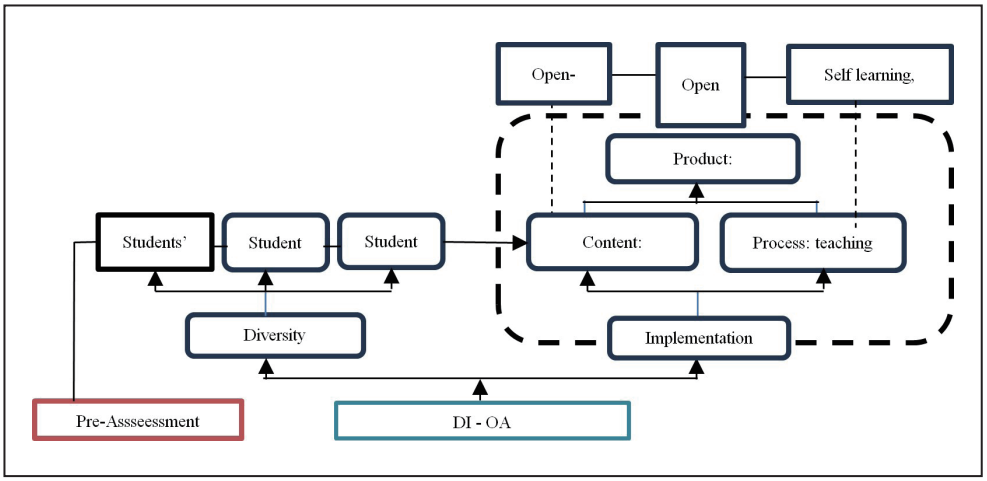


Figure 1. A framework of DI and OA in practice

Conclusion

Although empirical research specifically investigating the intersectionality in the practice of DI and OA remains limited, existing studies have demonstrated that bringing DI and OA together offers a promising framework that leverages the strengths of both approaches to address integration for supporting mathematical learning experiences among diverse students with regard to their mathematical ideas. By incorporating key principles of DI and OA, such as respecting diverse ideas, encouraging students' growth, supporting collaboration, and promoting equity, teachers can cultivate mathematical activities that affirm students' diverse identities and lived social experiences while offering meaningful open-ended real tasks. However, navigating this intersection presents challenges and considerations, including theoretical inconsistencies and a limited research base.

References

Aliyeva, A. (2021). Differentiated instruction, perceptions and practices. *Inclusive Education in a Post-Soviet Context: A Case of Kazakhstan*, 67–97.

Becker, J. P., & Shimada, S. (1997). *The Open-Ended Approach: A New Proposal for Teaching Mathematics.* ERIC.

Fox, J., & Hoffman, W. (2011). *The differentiated instruction book of lists* (Vol. 6). John Wiley & Sons.

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In *Educational design research* (pp. 29–63). Routledge.

Guberina, T. (2023). Cultivating inclusive learning environments: Incorporating diversity through culturally responsive pedagogy. *Social Science Chronicle*, *2*, 1–14.

Hackenberg, A. J., Creager, M., & Eker, A. (2021). Teaching practices for differentiating mathematics instruction for middle school students. *Mathematical Thinking and Learning*, *23*(2), 95–124.

Heacox, D. (2012). *Differentiating instruction in the regular classroom: How to reach and teach all learners (Updated anniversary edition)*. Free Spirit Publishing.

Inprasitha, M. (2006). Open-ended Approach and Teacher Education. *Progress Report of the APEC Project:" A Collaborative Study on Innovations for Teaching and Learning Mathematics in Different Cultures among the APEC Member Economies*, 99.

Nohda, N. (2000). *Teaching by Open-Approach Method in Japanese Mathematics Classroom.*

Plomp, T. (2013). Educational design research: An introduction. *Educational Design Research*, 11–50.

Raethong, K., & Thinwiangthong, S. (2019). Levels of Discourse in Mathematics Classroom using Lesson Study and Open Approach. *Journal of Physics: Conference Series*, *1340*(1), 012087.

Ruhimat, T., & Darmawan, D. (2020). Development of group-based differentiated learning (gbdl) models. *Adv. Sci. Technol. Eng. Syst. J*, *5*, 52–62.

Seah, W. T., & Andersson, A. (2015). Valuing diversity in mathematics pedagogy through the volitional nature and alignment of values. *Diversity in Mathematics Education: Towards Inclusive Practices*, 167–183.

Spencer-Waterman, S. (2014). *Handbook on differentiated instruction for middle & high schools*. Routledge.

Suprayogi, M. N., Valcke, M., & Godwin, R. (2017). Teachers and their implementation of differentiated instruction in the classroom. *Teaching and Teacher Education*, *67*, 291–301.

Tomlinson, C. A. (2014). *The differentiated classroom: Responding to the needs of all learners*. Ascd.

Tomlinson, C. A., & Imbeau, M. B. (2023). *Leading and managing a differentiated classroom*. Ascd.

Tomlinson, C. A., & McTighe, J. (2006). *Integrating differentiated instruction & understanding by design: Connecting content and kids*. Ascd.

Zidan, M. R., & Qamariah, Z. (2023). A Literature study on the implementation of merdeka curriculum. *Jurnal Riset Rumpun Ilmu Bahasa*, *2*(2), 153–167.

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 2
(Day 3)

TSG2-1-7

TEACHING WHOLE-NUMBER MULTIPLICATION, COMBINING CHILDREN'S LEARNING TRAJECTORY OF PROPORTIONAL REASONING: A CLASSROOM-BASED STUDY

Hino Keiko
Utsunomiya University, Japan, khino@cc.utsunomiya-u.ac.jp

Ichikawa Hiraku
Miyagi University of Education, Japan, hiraku-i@staff.miyakyo-u.ac.jp

Kato Hisae
Hyogo University of Teacher Education, Japan, katohi@hyogo-u.ac.jp

Narisawa Yukari
Elementary School Attached to Yamagata University, Japan, yukari3922@gmail.com

Kudo Suguru
Sendai City Board of Education, Japan, suguru.k.2636@gmail.com

This study explored how whole-number multiplication can be taught to facilitate children's proportional reasoning. To this end, the authors designed lessons that were included in the chapter on multiplication incorporating proportional reasoning problems based on three principles and the learning trajectory of using scalar ratios 1–5. The designed lessons were conducted in one classroom (33 children). The results showed that the children were finding and using one-to-many relations to deal with covarying two types of quantities with many-to-many correspondence. Some children interpreted and used the "one" found as the number of times the corresponding "many" was repeated. The process involved the interpretation of the multiplier as a ratio, showing an important connection between the learning trajectory of transitioning from scalar ratio 4 to 5 and the learning of multiplication.

Keywords: whole-number multiplication, proportional reasoning, learning trajectory, composed unit

Introduction

Proportional reasoning refers to "detecting, expressing, analyzing, explaining, and providing evidence in support of assertions about, proportional relationships" (Lamon, 2005, p. 4). Research on proportional reasoning has a long history, given its educational importance (e.g., Dooren et al., 2010; Harel & Confrey, 1994; Lamon, 2005/2020, 2007; Lobato et al., 2010). Most research targets middle school levels, and scholars have argued whether it is adequately fostered through schooling. In this study, we approach the issue by focusing on the formation of a foundation for proportional reasoning from the lower grades of elementary school. We conjectured on the learning trajectory regarding the uses of scalar ratios and functional rates, examined children's thinking, and discussed the possibility of intervention (e.g., Hino et al., 2021, 2025). Considering the significance of math-

ematical content in the development of proportional reasoning (Lamon, 2005/2020), we examine how the concept of whole-number multiplication connects the learning trajectory of proportional reasoning in young children. We formulate the following research question: *In what way do second-grade children reason to solve proportional reasoning problems, and what can we learn from their learning trajectory to teach whole-number multiplication?*

Framework of the Study

This study focuses on the learning trajectory of scalar ratios (SR). SR are ratios (scalars) in a relation between two types of quantities that covariate proportionally such that when one quantity increases *n* times, the other quantity also increases *n* times as much. We set SR1–5 for the progression of uses of scalar ratios by taking the perspective of previous research on proportional reasoning (e.g., Resnick & Singer, 1993; Vergnaud, 1994). Table 1 shows SR4 and SR5 among SR1–5.

Table 1. Conjectured SR4 and SR5 on the use of scalar ratios (Hino et al., 2021)

Use of SR	Description	Example
4: Relating two quantities	Use the change that accompanies the two quantities to solve problems. Mainly count and use additive relations for problem-solving.	A child writes 2 + 2 + 2 + 2 + 2 + 2 and 6 + 6 + 6 + 6 + 6 + 6
5: Mapping between two quantities using multiplicative relations	Use the change that accompanies the two quantities to solve problems. Focus on multiplicative relations between two quantities and actively and consciously use these relations.	A child writes 2 x 6 = 12 and 6 x 6 = 36 by using multiplication

Note. Examples come from anticipated solutions to a problem: One person is served two cookies and six candies. If 12 cookies were used to serve people, how many candies were used?

We looked at children's learning trajectory, aiming at promoting their use of SR. One important dimension was the way the children coordinated two types of quantities using "a composed unit" (Lobato et al., 2010, p. 19), which is a unit formed by combining two different types of quantities. At the beginning levels of proportional reasoning, "equivalent ratios can be created by iterating and/or partitioning a composed unit" (Lobato et al., 2010, p. 36). In a study on the progress of children aged 5–9 years in the ability to handle composed units, Vanluydt et al. (2020) identified large individual differences in their progression to solve proportional reasoning problems from one-to-many correspondence problems to many-to-many correspondence problems (e.g., the problem in Table 1 is a many-to-many correspondence problem). We designed lessons on whole-number multiplication by incorporating the use of composed units and iterations to produce equivalent ratios.

Method

In Japan, whole-number multiplication is introduced in the second grade (MEXT, 2018). The aim of this mathematical content includes helping children understand the meaning of multiplication, construct a multiplication table, and find and use the properties of multiplication, such as distributive property. As for the meaning of multiplication, "(unit quantity) × (number of units)" is introduced; the meaning of "how many times as large as" (倍or BAI in Japanese) is also taught.

We developed three design principles by considering the aim above as well as the learning trajectory on the uses of SR4 and SR5. The first principle is to incorporate problem situations that prompt children to be aware of two types of varying quantities in multiplication. In multiplication, "one" is not given much attention because it is shown in the problem statement, and children tend to view multiplication as a relation between three numbers (Askew, 2018). We developed problems on the relation between four numbers. Second, in proportional reasoning, covariation is treated by constructing many-to-many composed units, but this time, "one" is actively used as a preliminary step. In other words, although dealing with many-to-many correspondence problems, we devised numerical values and situations that are easy for children to grasp as one-to-many correspondence problems. Third, we reinforced the meaning of multiplication, "(unit quantity) × (number of units)," to teach children that a unit quantity constitutes a whole and that several parts can be added to make a whole. This principle was expected to lead to an understanding of the distributive property. We incorporated these principles in the design of a chapter on multiplication (31 lessons, summarized in Table 2). The lessons were held at a university-affiliated elementary school located in the northern region of Japan. Ms. Narusawa (the fourth author) conducted the lessons as the homeroom teacher.

Table 2. Feature and proportional reasoning problem in three phases of the design of lessons

Phase	Feature	Example of problems
Introduction to whole-number multiplication, Lessons 1–5	BAI is introduced using a measurement situation with arbitrary units. Multiplication is then introduced by emphasizing the functional relation between the multiplier and the product with the multiplicand constant.	The soprano recorder is four times the size of the teacher's fist, twice the size of Minnie's fist, and six times the size of the child's fist. A tenor recorder is eight times as large as the teacher's fist. (1) How many times is the tenor as many as Minnie's fist? (2) How many times is it as many as the child's fist?
Construction of multiplication table, Lessons 6–20	Each times-table (2 times, 5 times…) is constructed by emphasizing the distributive property.	How many pieces of sushi did I eat? (You can tell by the number of plates, since every plate has two pieces.)
Application of whole-number multiplication, Lessons 21–31	The lessons set up numbers and situations where "one" is easy to find. Reasoning is promoted by forming one-to-many composed units, and by iterating them.	A machine gives out 5 yen when 10 aluminum cans are inserted. (1) How many yens come out when 40 cans are put in? (2) Seven 5-yen coins came out. How many cans did you put in?

Summary of Results and Discussion

In the Many-to-many Correspondence Problems, the Children Found and Used "one" in Their Own Way.

In the recorder problem (1), children derived 2 + 2 = 4 by using the fact that two soprano recorders are equivalent to one tenor recorder in size. Some children found a one-to-many composed unit (two teacher's fists, one Minnie's fist) from the relation "four teacher's fists are equal to two Minnie's fists." They divided the teacher's eight fists into two fists and replaced the two fists with one

Minnie's fist (Fig. 1). However, the many-to-many composed unit (two teacher's fists, three child's fists) was difficult to find from the relation "four teacher's fists are equal to six child's fists" in (2). More children used 6 + 6 from the relation of two soprano recorders for one tenor recorder; some children derived the difference between 6 and 4.

The aluminum can problem is a many-to-many correspondence problem, with 10 cans costing 5 yen each. By showing that one 5-yen coin comes out, the problem called the children's attention to the number of 5-yen coins. By looking at the number of coins, they can use "one." For example, in (2), a child derived 40 + 30 = 70 by using the result of (1). To find the number of cans equivalent to three more coins, the child stated, "Since there are 10 cans per *coin*, there are 30 cans for 15 yen (three coins)." Another child devised a unique rule that when "0" is deleted from "10" cans, "1" is left, which equals the number of 5-yen coins.

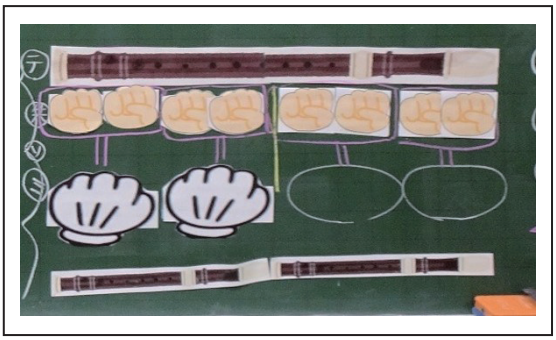


Figure 1. Composed units in problem (1)

Some Children Interpreted and Used the "one" Found as the Number of Times the Corresponding "many" is Repeated.

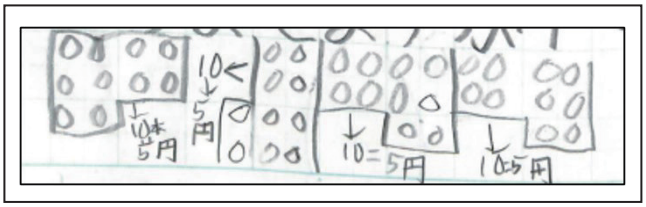


Figure 2. Composed units by a child

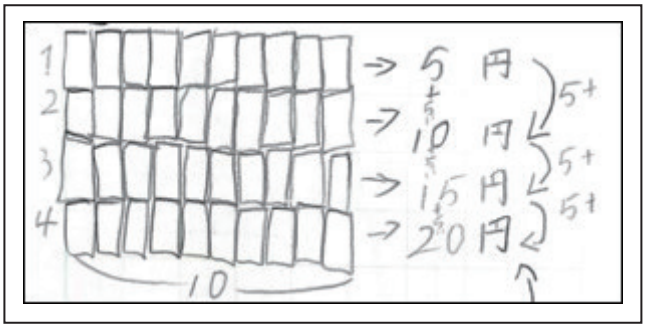


Figure 3. Record of 1, 2, …

The predominant strategy in the aluminum can problem (1) was to draw a picture of 10 cans and one 5-yen coin as a composed unit and then iterate them one by one to obtain 20 (Fig. 2). Meanwhile, some children wrote the 10 pieces as "1," "2," ... in their written work (Fig. 3). Interestingly, they wrote 4×5 instead of 5×4. We interpreted that the correspondence between the 10 cans and 5-yen coin by those children was solid, enabling them to reconstruct the whole (40 cans) by its parts (10 cans). This aspect may relate to the issue of whether the composite units (in this case, 10 cans) created by the children are *iterable* (Norton et al., 2015; Steffe, 1994).

The results showed that second graders can perform double iteration and proceed with the reasoning in SR4 and that they can create and utilize a one-to-many composed unit by finding "one" in the many-to-many relation. Furthermore, as implied by a child's problem solving in Figure 3, going beyond SR4 may require inference of the total amount by re-interpreting the "one" in the created one-to-many composed unit (e.g., 1 coin, 10 cans) as the number of times the corresponding "many" (10 cans) is repeated. Some children were able to make this inference as we found the record of "1," "2," … in the notebooks of three children to solve the aluminum can problem. This process involved making sense of the multiplier as a ratio between unit quantity and total quantity. The results demonstrated an important connection between the transition from SR4 to SR5 and multiplication, which can be highlighted in the teaching of multiplication.

References

Askew, M. (2018). Multiplicative reasoning: Teaching primary pupils in ways that focus on functional relations. *The Curriculum Journal, 29*(3), 406-423. https://doi.org/10.1080/09585176.2018.1433545

Dooren, W. V., Bock, D. D., & Verschaffel, L. (2010). From addition to multiplication … and back: The development of students' additive and multiplicative reasoning skills. *Cognition and Instruction, 28*(3), 360-381. https://doi.org/10.1080/07370008.2010.488306

Harel, G., & Confrey, J. (Eds.). (1994). *The development of multiplicative reasoning in the learning of mathematics*. State University of New York Press.

Hino, K., Kato, H., & Ichikawa, H. (2021). Learning trajectory of proportional reasoning in early grades of elementary school. *Proceeding of the Ninth Spring Research Conference of Japan Society of Mathematical Education*, 71-78. (in Japanese)

Hino, K., Ueno, Y., Tajima, T., Sakurai, A., Akizawa, K., Ichikawa, H., & Kato, H. (2025). A study of proportional reasoning in second and third-grade elementary school students. *Journal of Japan Society of Mathematical Education, 107*(2), 2-14. (in Japanese).

Lamon, S. J. (2005). *Teaching fractions and ratios for understanding (2nd ed)*. Lawrence Erlbaum Associates.

Lamon, S. J. (2020). *Teaching fractions and ratios for understanding (4th ed)*. Routledge.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629-668). Information Age Publishing.

Lobato, J., Ellis, A. B., Charles, R. I., & Zbiek, R. M. (2010). *Developing essential understanding of ratios, proportions, and proportional reasoning for teaching mathematics in grades 6–8*. Reston: National Council of Teachers of Mathematics.

MEXT (2018). *The course of study for elementary school mathematics*. Nihon Bunkyo Shuppan. (in Japanese).

Norton, A., Boyce, S., Phillips, N., Anwyll, T., Ulrich, C. & Wilkins, J. L. M. (2015). A written instrument for assessing students' units coordination structures. *IEJME-Mathematics Education, 10*(2), 111-136.

Resnick, L. B., & Singer, J. A. (1993). Protoquantitative origins of ratio reasoning. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational Numbers: An Integration of Research* (pp. 107- 130). Lawrence Erlbaum Associates.

Steffe, L. (1994). Children's multiplying schemes. In G. Harel & F. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 3-39). State University of New York Press.

Vanluydt, E., Degrande, T., Verschaffel, L., & Van Dooren, W. (2020). Early stages of proportional reasoning: A cross-sectional study with 5- to 9-year-olds. *European Journal of Psychology of Education, 35*, 529-547. https://doi.org/10.1007/s10212-019-00434-8

Vergnaud, G. (1994). Multiplicative conceptual field: What and why? In G. Harel & F. Confrey (Eds.), *The Development of Multiplicative Reasoning in the Learning of Mathematics* (pp. 41-59). State University of New York Press.

Acknowledgement

This study was supported by a Grant-in-Aid for Scientific Research (KAKENHI) from the Japan Society for the Promotion of Science (JSPS), Grant No. 24K00411, No. 21K02593.

TSG2-1-8

AN ANALYSIS OF STUDENT' FOCUS ON THE RELATIONSHIP BETWEEN ORIGINAL SIZE AND FRACTIONS : A CASE STUDY OF 2ND GRADE LESSONS ON "FRACTION"

Yuri Masutani
Graduate Student at the Graduate School of Education, Shimane University, Japan,
e249303@matsu.shimane-u.ac.jp

Taketo Shimomura
Graduate School of Education, Shimane University, JAPAN, tshimomura@edu.shimane-u.ac.jp

This paper aims to grasp in detail the aspects of second-grade elementary school student who focus on the relationship between "original size" and fraction size while studying fractions and to show the characteristics of the process of recognizing such a relationship in a case study. Therefore, we designed and conducted a survey targeting second graders, and analyzed the aspects observed in the survey from the theoretical perspective of unitizing and norming in Lamon. Consequently, we point out the importance of unitizing, in which the "original size" is defined as the unit, and norming, in which the size of the assumed fraction is interpreted again with the operation of extracting it, in order to grasp the relationship between the "original size" and the fraction.

Keywords: original size, fraction, relationship, second-grade student, unitizing and norming

Introduction

Understanding fractions is considered important for success in higher mathematics and even science (Lamon, 2007) and for dramatically refining the quality of mathematical content (Lamon, 2020). However, it has been pointed out in previous studies that understanding such fractions is highly difficult for children (e.g. Ishida, 1985). In understanding such fractions, learners are expected to comprehend that a numerical symbol can represent different quantities depending on what the unit or the whole unit is (Lamon, 1994). In other words, learning fractions requires consideration not only of fractions represented by numerical symbols but also of the entire unit of magnitude of the fraction. In Japan, however, from the second grade when students encounter fractions for the first time, they have little awareness of the "original size[1)]," and that they have difficulty in understanding fractions in relation to the "original size" (Nunokawa & Kasai, 2020). Consequently, students must be able to appropriately grasp the relationship between "original size" and the magnitude of fractions, starting in the second grade when only fractions with a numerator of 1 are the subject of discussion. However, there are few studies on fraction learning in the second grade, and to the best of our knowledge, no studies have focused on how children grasp the relationship between "original size" and the magnitude of fractions. Therefore, this study aims to capture in detail the aspects of children's attention to the relationship between "original size" and the magnitude of fractions in the study of fractions in the second grade and to show the characteristics of the process of recognizing such relationship through a case study.

Theoretical Background

In this section, we review Lamon's concepts of unitizing and norming, which are the theoretical perspectives of this study. Unitizing is the concept of taking a given quantity and putting it together in one's mind or reconstructing it into a coherent whole in order to suit one's purpose (Lamon, 2007). Referring to Freudenthal's (1983)[2] example, Lamon (1994) described norming as the concept of assuming a fixed unit or standard and reinterpreting a situation based on it. She stated that the concepts of unitizing and norming play an important role in the development of the concept of fractions (Lamon, 1994), and also proposed a better way of learning fractions based on these concepts (Lamon, 2020). In addition, it has been pointed out that using unitizing and norming as a framework for analysis makes it possible to identify the object of study as 1 and to observe changes in the standard (Ichikawa, 2004). In light of this perspective, it is thought that it is not limited to understanding how children perceive fractions as units, but also to pursuing how they interpret the relationship between the "original size" and fractions based on this understanding. Therefore, this paper adopts the theoretical framework of unitizing and norming to analyze the characteristics required for children to grasp the relationship between "original size" and the of fractions.

Research Question

How do second grade students grasp the relationship between "original size" and the magnitude of a given fraction? What are the characteristics of unitizing and norming that help them to understand the relationship between them?

Methodology

Survey Design

The survey was conducted on November 22, 2022, in the second grade of a public elementary school in Shimane Prefecture, Japan. The survey comprised a pretest, a posttest, and a one-hour class that focused on identifying the "original size". The data collected in the study included video recordings from fixed and 360° cameras, audio recordings from IC recorders, still image recordings from digital cameras, worksheets written by children in class, and pretest and posttest data. The worksheet shown in Figure 1 was used in this lesson, which focused on identifying the "original size." In Figure 1, the 1/8 piece in (4) was assumed to be 1/4, and the "original size" was asked in that case. The goal here was to encourage children to find the "original size" based on their understanding of the size of 1/4, and to grasp the relationship between the original size and the size of the given fraction through such an activity. In addition, the teaching tool shown in Figure 2 was created to help children identify the "original size" of the fractions.

---

1) The "original size" in this paper refers to the English-language math textbook shown below, and is equal to 1 for the magnitude of the fraction.
Shin Hitotsumatsu et al. (2020).Study with Your Friends Mathematics 1-6th Grade for Elementary School. GAKKO TOSHO CO.,LTD.

2) Freudenthal gives Norming's explanation with an example of reconceptualizing the solar system based on the definition of the earth as the size of a pin head (about 1 mm in diameter). For more details, see the following.
Freudenthal, H. (1983). Didactical Phenomenology of Mathematical Structures. D.Reidel Publishing Company.

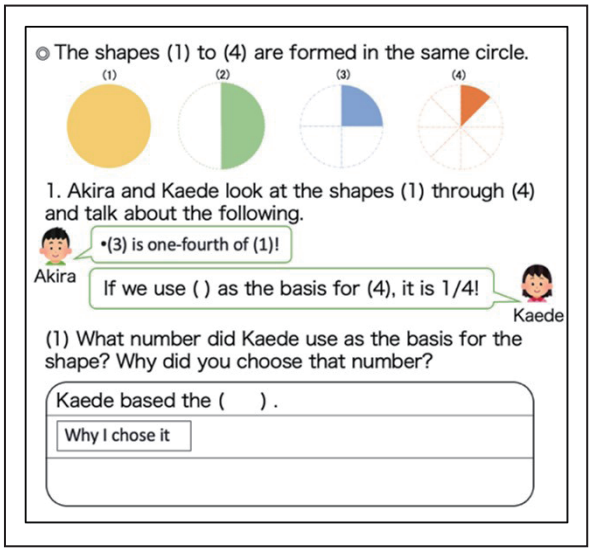


Figure 1. The worksheets used in class

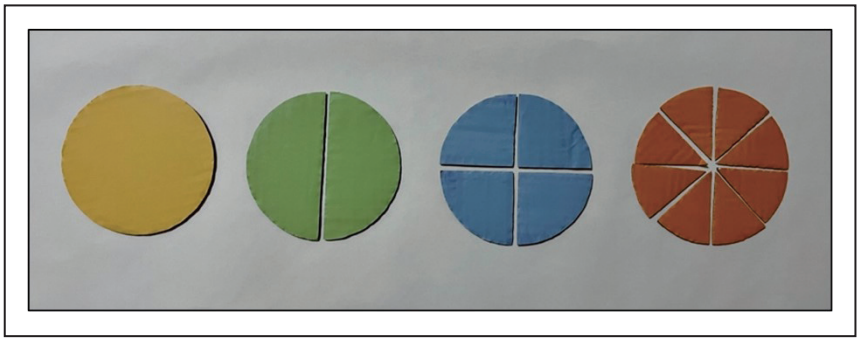


Figure 2. Teaching tools used by children in the classroom

Data Analysis

The procedure of analysis in this paper is as follows. First, the transcript was divided into utterances, referring to the video and audio of a one-hour class. The utterances were considered to be a series of one participant's spoken language, and the pauses in the utterance, changes in its content, and the speaker were considered as breaks in the utterance. Next, we set the target of analysis as Student A, who was observed to focus on the relationship between the "original size" and the size of the given fraction, by referring to the transcript data and the video of the child manipulating the teaching tool. The analysis was conducted using Lamon's unitizing and norming as the theoretical framework. Specifically, utterances that were judged to be unitizing were labeled with **[U]**, and those that were judged to be norming were labeled with **[N]**, respectively.

Result

We analyzed Student A's identification of the "original size" from the theoretical perspective of unitizing and norming in Lamon. In the pretest, when Student A was asked about the "original size" of the 1/8 piece when it was assumed to be 1/2 the size of the 1/8 piece, he assumed the entire circle to be the original size; subsequently, he was unable to find the appropriate "original size." Although

Student A had such readiness, the following statement was made during the phase in which the pairs explained their ideas to each other in response to the question shown in Fig. 1. The symbols used in the speech analysis are "{…}" to describe the manipulation of teaching tools and actions, and "**[…]**" as labels for unitizing and norming.

Okay? On top of this one of shape (2), I put (4) like this {Arrange four 1/8 pieces on a semicircle.} **[U]**, and then I put them over here, and they fit perfectly**[N]**, and then I took out three of them, and then I got a quarter, and then a quarter of this one **[U, N]**.

From the dialogue, it was confirmed that Student A not only found the appropriate "original size" but also focused again on the size of the given fraction. In addition, by referring to the video, it was confirmed that Student A stacked four 1/8 pieces on top of one 1/2 piece, saying, "On top of this one of shape (2), I put (4) like this {Arrange four 1/8 pieces on a semicircle.}." In this phase, the four 1/8 pieces were placed without any space between them; thus, we judged that she was unitized as a single unit and labeled it as **[U]**. In the following part of the sentence, "and then I put them over here, and they fit perfectly," the participants seemed to confirm that the sizes of one 1/2 piece and four 1/8 pieces were the same. In this case, the four 1/8 pieces were interpreted as being the same size as the semicircle, leading to the children reinterpreting the semicircle's size. This was labeled as **[N]**. Next, when stating "and then I took out three of them, and then I got a quarter, and then a quarter of this one," it was explained that 1/8 of a piece was equivalent to one-quarter of a semicircle, accompanied by the operation of removing three 1/8 pieces. In this case, after unitizing the original size, which was found to be the unit of measurement, norming was performed with the operation of removing three 1/8 pieces to interpret that one 1/8 piece corresponded to one-quarter of the size of a semicircle; therefore, the score was set to **[U・N]**. Based on the above, Student A, who struggled to find the appropriate "original size" in the pretest, is considered to have come to grasp the "original size" and the size of the fraction in the phase of identifying the "original size" while creating the size of the assumed fraction again from the identified "original size" and confirming that it is 1/4. In the process, it is suggested that the "original size" and the size of the fraction might have been captured. In addition, it is suggested that the norming, which is accompanied by unitizing, in which the "original size" is defined as the unit and then extracted, might play an important role in reinterpreting the assumed fractional size.

Concluding Remarks

We planned and conducted a lesson for second-grade elementary school students that emphasized the identification of the "original size," and analyzed and discussed the aspects of the lesson. Consequently, we pointed out the importance of unitizing the "original size" as the unit and then performing norming, which involves the operation of extracting the unit and interpreting the size of the assumed fraction again, when capturing the relationship between the "original size" and the fraction. However, the scope of this paper is limited, and new suggestions may be obtained by increasing the number of surveys. In the future, it is necessary to conduct more precise analysis by increasing the number of surveys.

References

Ichikawa, H. (2004). A Study on the Aspects of Understanding the Meaning of Fractional Multiplication Using a Number Line: From the Viewpoint of Unitizing and Norming. *Joetsu journal of mathematics education*, 19, 73-84. (in Japanese)

Ishida, T. (1985). *Why is it so difficult to understand the meaning of fractions and decimals?*. mathematics education, 327, Meiji Books Publishing Co, 21-27. (in Japanese)

Lamon, S, J. (1994). Ratio and Proportion: Cognitive Foundations in Unitizing and Norming. *The Development of Multiplicative Reasoning in the Learning of Mathematics*. Published by State University of New York Press, Albany, 89-120.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In Lester, F. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629-668), Charlotte, NC: Information Age Publishing.

Lamon, S. J. (2020). *Teaching Fractions and Ratios for Understanding: Essential Content 83 and Instructional Strategies for Teachers.* Routledge.

Nunokawa, K., & Kasai, M. (2020). An Examination of Arithmetic Lessons from a Discourse Perspective: The Case of Fractions in 2nd Grade Elementary School. *Joetsu journal of mathematics education,* 35, 29-44. (in Japanese)

Acknowledgements

This work was supported by JSPS KAKENHI (grant number: JP23K02341). I would like to thank Ms. Shimomura for her support in this study. I would also like to thank the teachers and students at public elementary schools in Shimane Prefecture for their cooperation in this study.

TSG2-1-9

THE USE OF MATHEMATICAL AND NATURAL LANGUAGE BY ELEMENTARY SCHOOL TEACHERS IN DIVISION LESSONS

Yun Hsia Pai
National Tsing Hua University, R.O.C., yunhsia@mx.nthu.edu.tw

This study aims to investigate teachers with different backgrounds utilize mathematical and natural language in their decimal division lessons, and when and how they utilize mathematical and natural language during instruction? The participants include two teachers: one with expertise in teaching Chinese reading comprehension, and the other specializing in mathematics education. Six lessons were observed for each teacher. Classroom discourse data was collected through audio recordings, and a discourse analysis approach was employed to analyze the timing and context of the teachers' use of mathematical and natural language. The study found that the two teachers predominantly used mathematical language when explaining and clarifying, restating/rephrasing, and summarizing, whereas they tended to use natural language more frequently when evaluating and giving task instructions. The teacher with expertise in mathematics education exhibited a higher frequency of mathematical language use when clarifying, whereas the teacher with expertise in reading comprehension demonstrated a higher frequency when explaining.

Keywords: mathematical language, natural language, elementary school teacher

Backgound

Mathematical language and natural language are deeply interconnected, complementing and reinforcing each other. Due to its unique and specialized nature, mathematical language requires specific background knowledge and reading strategies that go beyond those used for general texts. Understanding mathematical texts involves interpreting mathematical symbols, notations, and the underlying concepts they represent.

Recent research has explored the relationship between mathematical and natural language, highlighting their differences while advocating for connections to facilitate student comprehension. They argue that students need to grasp the similarities and differences between these language forms, including their deep structures, to effectively learn and communicate mathematical ideas (Ilany & Margolin, 2010; Roy et al., 2015). Additionally, natural language plays a vital role in solving mathematical word problems, as students must interpret and analyze problem statements to determine the appropriate solutions (Dries et al., 2017, Pai, 2022). However, research on how teachers utilize natural and mathematical language in math classroom remains limited.

In elementary schools in Taiwan, where mathematics is often taught by generalist teachers who may not have specialized training in mathematics education. This study explores how teachers with different backgrounds utilize mathematical and natural language in the mathematics classroom. Specifically, it compares the language use of a teacher with expertise in mathematics education to that of a teacher specializing in Chinese reading comprehension. The study investigates whether differences exist in their use of mathematical and natural language, and examines the contexts in which each type of language is employed.

Purpose

This study aims to investigate the following questions:

- How do teachers with different backgrounds (mathematics education vs. Chinese reading comprehension) utilize mathematical and natural language in their division lessons?
- When and how do they employ mathematical and natural language during instruction?

By exploring these questions, this study seeks to shed light on the complex interplay between mathematical and natural language in elementary classrooms and provide valuable insights for teacher professional development.

Method

Participants

Participants of this study were two teachers from the same school in Taiwan. Teacher A, who has expertise in teaching Chinese reading comprehension, and Teacher B, who specializes in mathematics education. Both teachers have had over 10 years of teaching experience.

Data collection and analysis

During the unit on decimal division, six lessons were observed for each teacher, with each lesson lasting 40 minutes. Classroom discourse data was collected through audio recordings, and a discourse analysis approach was employed to analyze the timing and context of the teachers' use of mathematical and natural language.

Framework of Discourse Analysis

All classroom dialogues between the teachers and students will be transcribed for discourse analysis. an analytic induction approach was employed to categorize the types of instructional behaviors observed in the classroom. teacher behaviors were classified into ten categories: initiating strategies, giving instructions for tasks, presenting problems, concept building, restating/rephrasing, evaluating, explaining, summarizing, clarifying, classroom management.

Classroom management, such as asking students to sit down or requesting them to speak louder, are not relevant to the analysis of mathematical and natural language and will therefore only be coded but not analyzed further.

Results

This study is interested in the timing and context of how fifth-grade teachers with different backgrounds utilize both natural and mathematical language. Through the analysis of classroom discourse, the following sections will illustrate how these two teachers, with distinct backgrounds, use natural and mathematical language at different times.

Across the six lessons, Teacher A's most frequent instructional behavior was explaining, followed by clarifying and then evaluating, along with giving instructions for tasks. In both explaining and evaluating, Teacher A used mathematical language slightly more often than natural language. However, for task instructions, Teacher A predominantly used natural language to explain the content of tasks. When clarifying mathematical concepts and student's questions, or summarizing, Teacher A showed a clear preference for using mathematical language.

Table 1. Teacher a's use of mathematical and natural language

Teaching behavior	Natural language	Mathematical language	Total
Initiating strategies	6	0	6
Giving instructions for tasks	40	27	67
Presenting problems	1	5	6
Concept building	3	8	11
Restating/Rephrasing	13	38	51
Evaluating	51	44	95
Explaining	82	97	179
Summarizing	2	22	24
Clarifying	36	68	104

In Teacher B's six lessons, clarifying was the most frequent instructional behavior, followed by giving instructions for tasks, explaining, evaluating, and restating or rephrasing students' contributions. When clarifying, explaining mathematical concepts, restating or rephrasing student contributions, Teacher B showed a clear preference for using mathematical language. Similar to Teacher A, Teacher B predominantly used natural language when giving task instructions, suggesting that both two teachers tend to use natural language when explaining the content of task. However, when evaluating students' mathematical answers or task outcomes, Teacher B tended to use natural language, mirroring the pattern observed with Teacher A.

Table 2. Teacher b's use of mathematical and natural language

Teaching behavior	Natural language	Mathematical language	Total
Initiating strategies	13	10	25
Giving instructions for tasks	98	27	126
Presenting problems	1	2	3
Concept building	5	13	18
Restating/Rephrasing	39	63	102
Evaluating	67	46	116
Explaining	23	96	122
Summarizing	14	30	44
Clarifying	74	188	269

Discussion and Conclusion

In the mathematics teaching of both teachers, explaining, clarifying, evaluating, summarizing, and restating/rephrasing were the most common dialogue types. The top three types of two teachers in terms of frequency were explaining, clarifying, and evaluating. However, Teacher A focused more on explaining, while Teacher B emphasized clarifying. This difference may be attributed to Teacher B's background in mathematics education, which may have led to a greater emphasis on clarifying student's mathematical misunderstanding. Teacher A, on the other hand, tended to focus more on explaining. Regardless, both teachers preferred to use mathematical language when clarifying or explaining.

When evaluating students' mathematical answers or task outcomes, both teachers tended to use natural language. because they often utilized more mathematical vocabulary when assessing students' responses to enhance their mathematical understanding.

Future research could investigate the occurrence of mathematical and natural language in teacher-student mathematical dialogues for students of different abilities.

References

Dries, A., Kimmig, A., Davis, J. Belle, V., & De Raedt, L. (2017). Solving Probability Problems in Natural Language. *Proceedings of the Twenty-Sixth International Joint Conference on Artificial Intelligence*, 3987.

Ilany, B. S. & Margolin, B. (2010). Language and Mathematics: Bridging between Natural Language and Mathematical Language in Solving Problems in Mathematics. *Creative Education, 1*(3), 138-148.

Roy, S., Vieira, T., Roth, D. (2015). Reasoning about Quantities in Natural Language. *Transactions of the Association for Computational Linguistics,* 3(2), 1-13.

Pai, Y. H. (2022). *The discourse of mathematical language and natural language in fifth-grade mathematics classrooms.* 45th PME (Conference of the International Group for the Psychology of Mathematics Education) 2022 conference. held in Spain, from July 12 to23, 2022.

TSG2-1-10

CURRICULUM ALIGHMENT THROUGH MULTILITERACY IN MATHEMATICS EDUCATION: INSIGHTS FROM FINLAND

Eunji Kim
Tampere University, Finland, eunji.kim@tuni.fi

Jorma Joutsenlahti
Tampere University, Finland, jorma.joutsenlahti@tuni.fi

Päivi Perkkilä
University of Turku, Finland, pperkkila@gmail.com

This study investigated the implementation of multiliteracy strategies in Finnish primary mathematics education and examined how these strategies influenced instructional quality. Drawing on classroom observations and a multiliteracy-aligned observation protocol, data were collected from five teachers across diverse school contexts. The findings revealed that multiliteracy practices—such as the use of multimodal representations, inclusive dialogue, and adaptive teaching materials—enhanced students' active participation, expression, and engagement in collaborative learning. Teachers frequently employed natural language to assess understanding, provided motivational feedback, and customized instructional resources to support learners' needs. Nevertheless, challenges persisted in addressing learning difficulties, supporting linguistically and culturally diverse classrooms, and consistently fostering creative and critical thinking. These findings offer practical implications for teacher education programs aiming to strengthen curriculum alignment and improve the quality of mathematics instruction through multiliteracy-informed pedagogy.

Keywords: multiliteracy, mathematics education, instructional quality, curriculum alignment, teacher practices

Introduction

The Finnish National Core Curriculum for Basic Education positions multiliteracy as one of seven transversal competencies, shaping learning objectives and instructional approaches across all subjects, including mathematics. Multiliteracy emphasizes the ability to interpret and produce a range of communicative forms—verbal, visual, symbolic, and embodied—encouraging students to become active, reflective learners. The curriculum outlines grade-level targets and provides guidance on learning environments, teaching strategies, and assessment practices (Finnish National Agency for Education, 2016). This competency-based framework promotes 21st-century skills such as collaboration, communication, and critical thinking.

While many East Asian countries follow centralized curricular mandates, the Finnish system grants considerable autonomy to teachers and local education providers in interpreting and implementing the national curriculum (Krzywacki, Pehkonen, & Laine, 2016; Sahlberg & Walker, 2021). This flexibility fosters pedagogical innovation but also presents challenges in achieving coherent curriculum alignment and consistent instructional quality. Despite the curriculum's strong emphasis on multiliteracy, empirical research on how teachers enact these strategies in mathematics classrooms remains limited.

This study addresses this gap by examining how Finnish primary school teachers implement multiliteracy strategies in their mathematics teaching. Drawing on classroom observations and qualitative data, the study explores how teachers use multimodal tools and communicative practices to promote conceptual understanding, foster engagement, and interpret students' mathematical thinking within an inclusive pedagogical framework.

Theoretical Background

Multiliteracy in mathematics education involves the use of diverse semiotic resources—such as speech, writing, diagrams, gestures, and symbolic representations—to support own meaning-making and conceptual development. These practices are grounded in constructivist and sociocultural learning theories, which emphasize individual knowledge construction (Piaget, 1952) and socially mediated learning (Vygotsky, 1978). Within this theoretical framework, multiliteracy becomes integral to teaching practices that support learners in expressing, negotiating, and refining their mathematical thinking.

The Finnish curriculum defines multiliteracy as the ability to interpret, produce, and critically evaluate various texts, enabling learners to navigate cultural communication, develop personal identities, and engage in ethical and aesthetic reflection (Finnish National Agency for Education, 2016, p. 23). It extends beyond language to include a broad repertoire of communicative strategies across traditional and digital learning environments. Learners are encouraged to manage their own learning using sensory, linguistic, and material resources in emotionally safe and responsive environments. Teachers facilitate this process through meaningful feedback and scaffolding.

Despite its pedagogical importance, multiliteracy implementation poses practical challenges. Teachers may struggle to interpret students' multimodal outputs, apply strategies in real time, and maintain curriculum coherence. Alignment across curricular levels is essential to address these challenges (Teig et al., 2024). The Finnish curriculum operates on four levels: national goals (Level 1), local adaptations (Level 2), teacher planning (Level 3), and classroom enactment (Level 4). Alignment across these levels directly influences instructional quality, as illustrated in Figure 1.

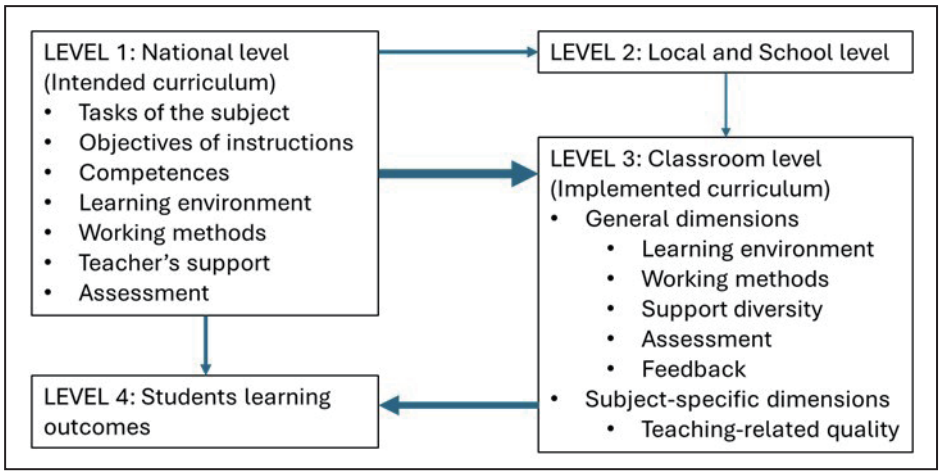


Figure 1. The flow of curriculum implementation from Level 1 to Level 4

This study investigates how teachers' instructional quality in mathematics aligns with the Finnish national curriculum through the lens of multiliteracy. *The guiding research question is: How does teachers' instructional quality in mathematics align with the Finnish National Core Curriculum, particularly in the implementation of multiliteracy strategies?*

Methods

Participants

This study recruited five female teachers from four municipalities in Finland using purposive sampling (Creswell & Creswell, 2018). The participants, aged 26 to over 55, included four primary school teachers and one special education teacher, all holding master's degrees and possessing 5 to 22 years of teaching experience. They taught Grades 2 to 4 in classes ranging from 6 to 20 students. All participants had prior exposure to multiliteracy practices through university coursework, professional development, or peer collaboration.

Classroom Observation

Classroom observations employed a custom-designed protocol to assess the implementation of multiliteracy strategies in mathematics instruction. The protocol addressed six dimensions: learning environment, working methods, support for diversity, assessment, feedback, and mathematics-specific instructional quality. These dimensions were informed by research from Luoto, Klette, & Blikstad-Balas (2022) and Schlesinger et al. (2018) and operationalized using observable indicators aligned with multiliteracy principles, as shown in Table 1.

Table 1. Dimensions and items of the classroom observation protocol: multiliteracy-aligned framework

Dimension		Multiliteracy-Aligned Interpretation
Learning environment	3	Critical use of materials
Working methods	3	Information or communication technology
Support diversity	5	Joy of learning and knowledge; Dealing with heterogeneity
Assessment	2	Using multiple representations for assessment; Correctness dealing with errors
Feedback	1	Motivational feedback
Teaching-related quality	4	Using multiple representations for teaching; Appropriate mathematical examples; Relevance of mathematics for students

Note. Teaching-related quality dimensions were adapted from Schlesinger et al. (2018).

Each item was rated on a four-point scale (*1 = does not apply at all to 4 = fully applies*), with non-observable items marked as "n/o." The protocol incorporated measures such as the use of multiple representations and multimodal assessment strategies. Qualitative field notes were collected concurrently to capture classroom discourse, engagement, and multimodal instructional practices.

Data Collection and Analysis

Observations were conducted between September and December 2024 over multiple sessions during regular mathematics lessons. Data were analyzed thematically using ATLAS.ti software (Cohen, Manion, & Morrison, 2002), with double coding to ensure reliability. Coding discrepancies were resolved through discussion. Member checking was conducted by sharing preliminary findings with participants to confirm the accuracy of interpretations.

The study received ethical approval from the appropriate institutional review boards. All participants provided informed consent and were assured of voluntary participation and confidentiality. Pseudonyms were used, and all data were stored securely in accordance with ethical guidelines.

Results

The findings indicate that integrating multiliteracy strategies significantly enhances students' active participation, multimodal expression, and engagement in mathematics learning. Teachers facilitated inclusive classroom dialogue by encouraging students to articulate their reasoning through multiple representational forms—spoken language, drawings, manipulatives, and symbolic notation. Visual aids and physical materials were frequently used to support conceptual development.

Motivational feedback and formative assessment were widely practiced, and many teachers demonstrated efforts to modify or create lesson materials beyond textbook use. Teachers also fostered emotionally supportive learning environments, with long-term teacher–student relationships contributing to a deeper understanding of learners' needs and capabilities.

Nevertheless, several challenges were identified. Some teachers found it challenges to address learning difficulties in real time or to differentiate instruction effectively for linguistically and culturally diverse learners. The pedagogical use of digital tools was inconsistent, and not all teachers regularly encouraged critical or creative thinking. These variations point to the need for strengthened school-level collaboration and professional development tailored to multiliteracy pedagogy.

This study provides insight into how multiliteracy strategies are enacted in mathematics classrooms and how they contribute to aligning practice with curricular objectives. The findings offer practical implications for teacher education programs seeking to promote inclusive, multimodal, and communicative approaches to mathematics instruction.

References

Cohen, L., Manion, L., & Morrison, K. (2002). *Research Methods in Education* (5th ed.). Routledge. https://doi.org/10.4324/9780203224342

Creswell, J. W., & Creswell, J. D. (2018). *Research design: Qualitative, quantitative, and mixed methods approaches* (5th ed.). Sage.

Finnish National Agency for Education. (2016). *National core curriculum for basic education 2014* (Publication 2016:5). https://www.ellibslibrary.com/book/9789521362590

Krzywacki, H., Pehkonen, L., & Laine, A. (2016). Promoting mathematical thinking in Finnish mathematics education. In H. Niemi, A. Toom, & A. Kallioniemi (Eds.), *Miracle of education: The principles and practices of teaching and learning in Finnish schools* (pp. 115–130). Sense Publishers. https://doi.org/10.1007/978-94-6091-811-7_8

Luoto, J. M., Klette, K., & Blikstad-Balas, M. (2022). Patterns of instruction in Finnish and Norwegian lower secondary mathematics classrooms. *Research in Comparative and International Education*, *17*(3), 399–423. https://doi.org/10.1177/1745499922107784

Piaget, J. (1952). *The origins of intelligence in children.* (M. Cook, Trans.). WW Norton & Company. https://doi.org/10.1037/11494-000

Sahlberg, P., & Walker, T. D. (2021). *In teachers we trust: The Finnish way to world-class schools*. WW Norton & Company.

Schlesinger, L., Jentsch, A., Kaiser, G., König, J., & Blömeke, S. (2018). Subject-specific characteristics of instructional quality in mathematics education. *ZDM*, *50*(3), 475–490. https://doi.org/10.1007/s11858-018-0917-5

Teig, N., Nilsen, T., & Yang Hansen, K. (Eds.). (2024). *Effective and Equitable Teacher Practice in Mathematics and Science Education: A Nordic Perspective Across Time and Groups of Students* (Vol. 14). Springer Nature Switzerland. https://doi.org/10.1007/978-3-031-49580-9

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press, Cambridge.

Acknowledgment

This paper is part of the author's doctoral dissertation project, which is under full engagement with supervisors, Jorma Joutsenlahti (Associate Professor, Tampere University, Finland) and Päivi Perkkilä (Associate Professor, Turku University, Finland).

TSG2-1-11

ENHANCING MATHEMATICAL CREATIVITY USING THE CONCEPTS OF INCUBATION BREAK AND THE FIVE LEGS OF CREATIVITY

Rhett Anthony C. Latonio
Ateneo de Manila University, Philippines, rhett.latonio@student.ateneo.edu

Catherine P
Vistro-Yu, Ateneo de Manila University, Philippines, cvistro-yu@ateneo.edu

Creativity is widely considered to be a skill students need to acquire to prepare themselves for an unforeseeable future. This study explores the effects of incubation breaks and classroom practices and activities which promote the five legs of creativity as interventions in enhancing students' mathematical creativity in the classroom setting. Results showed that the combination of the implementation of incubation breaks and classroom practices and activities promoting the five legs of creativity seemed effective in improving students' originality, elaboration and mathematical creativity while using incubation breaks alone only seemed to significantly improve the students' originality. Students seemed to enjoy the incubation breaks and multiple solution tasks while teachers deemed classroom practices and activities promoting the five legs of creativity as time-consuming and difficult yet effective in enhancing students' mathematical creativity.

Keywords: incubation break, five legs of creativity, mathematical creativity, multiple solution tasks

Introduction

Rapid technological advancement and the multifacetedness and globalization of society have catapulted creativity to the list of essential 21st-century skills needed to prepare students for an unforeseeable future (Ahmad et al., 2023; OECD, 2024) as creative individuals are deemed capable of formulating multiple innovative solutions to complex, multi-dimensional problems (Shvay, 2019). Creativity in general and mathematical creativity in particular have been discussed in academic circles for more than a century but a consensus definition of creativity or mathematical creativity has yet to be determined. Based on Liljedahl & Sriraman's (2006) definition which associates mathematical creativity with divergent thinking, this study defines mathematical creativity at school level as the ability to generate insightful and novel solutions and answers to given mathematical tasks, extend said solutions and answers using different perspectives, and/or present these in different contexts. Rhodes (1961, in Pitta-Pantazi et al., 2018) classified different perspectives of creativity into process, product, person, and press, which this study utilized as components of mathematical creativity in the classroom setting. Process refers to the thought processes which leads an individual to creative outputs, product are activities and/or outputs which provide individuals with opportunities to think creatively, person refers to the individual who is practicing mathematical creativity, while press refers to the environment which promotes mathematical creativity. This study is part of a larger dissertation study aimed at holistically assessing the ecology of the mathematics classroom by identifying and evaluating the different components of mathematical creativity in the classroom setting in order to transform a regular mathematics classroom into a mathematically creative classroom; hence, the discussion of this study will be in the context of the larger study.

Statement of the Problem

The study presented here is focused on determining the effects of classroom activities and practices (referred to as interventions) designed to promote the components of process and the combination of person and press on students' mathematical creativity in the classroom setting.

The chosen interventions for the process component are Incubation Breaks (IB), based on Hadamard's Concept of Mathematical Invention (Hadamard, 1945, in Sriraman, 2004). IBs are activities designed to shift a problem solver's attention from the problem being solved to an unrelated matter, which is theorized to expedite the transition from the incubation to the illumination stage of creative thinking, and have been shown to increase the number of math strategies used by students (Shaw et al., 2022). For this study, IBs were implemented when students were asked to perform multiple solution tasks (MST) to provide students with opportunities to produce multiple solutions and/or answers before and after IBs.

The interventions for the component of person/press are classroom activities and practices which promote intuition, impartiality, iconoclasm, inquisitiveness and investment, five affective states as defined by the Chamberlin and Mann's (2021) Five Legs of Creativity Theory (5LC). This theory posits that enhancing these five affective states positively impact the individual's abilities to produce creative outputs. Said classroom activities include the use of MSTs while classroom practices involve the teacher promoting a positive classroom culture through encouraging guessing and inquisitiveness, urging students to think of alternative solutions and answers, discussing wrong answers, and promoting group work, among others.

Methodology

The larger study is a mixed-method, experimental, univariate study with multiple independent variables. For the study presented here, the independent variables are the implementation of the chosen interventions, namely, IB only and the combination of IBs and classroom activities and practices promoting 5LC.

This study was conducted with three regular (heterogenous) sections of grade five students (sections 5A, 5M and 5P) from a public school in Metro Cebu, Philippines with a 3-week intervention plan per section implemented over a span of eight weeks. Section 5A employed both incubation breaks and practices which promoted 5LC, section 5M only used incubation breaks, while section 5P served as the control group.

The subjects' mathematical creativity levels were measured before and after the intervention implementation using a researcher-designed Mathematical Creativity Test for Grade Five Students (MCT5) composed of three activities based on items from the mathematics creativity scale of Akgul & Kahveci (2016) and model-eliciting activities (CPALMS, 2023), with one item each for problem-posing, problem-solving, and reasoning. The problem-posing item entails the generation of significantly different problems based on the student's chosen arithmetical solution. The problem-solving item involves the illustration of different polygons with the same area as a given figure. The reasoning item presents a realistic situation which requires students to make and justify their decisions using item-based or subject-supplied factors. The MCT5 utilizes the indicators of fluency, flexibility, originality and elaboration in measuring students' mathematical creativity using the pre-test/post-test model. Quantitative data generated by the MCT5 was analyzed using the paired-sample T-test to determine if the differences between the pre-test and post-test results were significant while ANOVA was used to determine which intervention was more effective in enhanc-

ing students' mathematical creativity. Qualitative data gathered from student and teacher interviews were analyzed using thematic analysis.

Discussion

Initial findings of the study showed that students of section 5A, who underwent interventions which combined IB with classroom practices and activities promoting 5LC, significantly increased their scores in the originality and elaboration indicators, and showed a significant increase in their over-all mathematical creativity scores. Students from section 5M, who underwent interventions involving only IB, showed a significant increase only in originality, and had no significant increases in all other indicators nor their over-all mathematical creativity scores. Students from section 5P, the control section with no interventions, showed no significant increase in all four indicators and in their mathematical creativity scores. Please refer to the significance levels in Table 1 below.

Table 1. Significance test results (at 0.0.5 significance level) of mathematical creativity indicators' pre-test and post-test scores' differences (DATAtab, 2025)

SECTION	Section 5A (IB/5LC)		Section 5M (IB only)		Section 5P (Control)	
	t-score	p-value	t-score	p-value	t-score	p-value
FLUENCY	-1.29	0.206	0.34	0.732	-0.8	0.428
FLEXIBILITY	0.09	0.929	0.12	0.907	0.9	0.373
ORIGINALITY	-4.46	<0.001	-2.72	0.01	-1.86	0.071
ELABORATION	-4.72	<0.001	-1.64	0.11	-1.22	0.232
MATHEMATICAL CREATIVITY	-3.88	0.001	-1.54	0.134	-1.08	0.288

Thematic analyses of students' interviews showed that students found MSTs novel and enjoyable albeit difficult while teachers' interviews showed that teachers deemed the implementation of MSTs and selected classroom practices and activities promoting 5LC difficult and time-consuming due to the utilization of activities which are open-ended and/or have multiple solutions. However, teachers found said practices effective in enhancing students' mathematical creativity and learning since it improved student engagement and gave equal opportunity to all students to express and justify their ideas and decisions.

Conclusion

Results of this study indicated that the intervention which combined MST-implemented IB with classroom practices and activities which promote the 5LC showed significant increases in originality and elaboration and the over-all mathematical creativity levels of students while the intervention which only used IB implemented through MSTs showed only a significant increase in the students' originality, which suggest that the combination of IBs and 5LC-promoting activities is more effective in enhancing students' mathematical creativity. Recommended guidelines for IB implementation include using varied IBs after students have been given ample time to process and analyze the given tasks. Suggested classroom practices to promote 5LC include encouraging students to consider multiple perspectives and solutions, providing opportunities for students to discuss and justify

their answers, and allowing students to guess the answers. The results of this study seemingly point to the effectiveness of using a combination of Incubation Breaks and practices promoting the Five Legs of Creativity in enhancing students' mathematical creativity in the classroom setting.

References

Ahmad, S., Umirzakova, S., Mujtaba, G., Amin, M.S. & Whangbo, T. (2023). Education 5.0: Requirements, enabling technologies, and future directions. https://www.researchgate.net/publication/372785224 _ Education_50_Requirements_Enabling_Technologies_and_Future_Directions

Akgul, S. & Kahveci, N.G. (2016). A study on the development of a mathematics creativity scale. *Eurasian Journal of Educational Research*, 62, 57-76. http://dx.doi.org/10.14689/ejer.2016.62.5

Chamberlin, S.A. & Mann, E.L. (2021). *The relationship of affect and creativity in mathematics: How the five legs of creativity influence math talent*. Routledge.

CPALMS. (2023). *Which apartment?* https://www.cpalms.com/teaching-resources/resource/47459

Liljedahl, P. G., & Sriraman, B. (2006). Musing on mathematical creativity. *For the Learning of Mathematics, 26*(1), 17–19.

DATAtab Team. (2025). DATAtab: Online Statistics Calculator. DATAtab e.U. Graz, Austria. URL https://datatab.net

OECD (2024). *PISA 2022 Results (Volume III): Creative Minds, Creative Schools*, PISA, OECD Publishing, Paris, https://doi.org/10.1787/765ee8c2-en.

Pitta-Pantazi, D., Kattou, M., & Christou, C. (2018). Mathematical creativity: Product, person, process and press. In F. M. Singer (Ed.), *ICME-13 monographs. Mathematical Creativity and Mathematical Giftedness: Enhancing Creative Capacities in Mathematically Promising Students* (pp. 27–53). Springer.

Shvay, R. (2019). *A Didactic Application of the Theory of Creativity in School Education*. Prima Education 2019. https://journals.umcs.pl/pe/article/download/9567/7407

Sriraman, B. (2004). The characteristics of mathematical creativity. *The Mathematics Educator 2004*, Vol. 14, No. 1, 19-34. https://files.eric.ed.gov/fulltext/EJ848493.pdf TED (2007, Jan. 7). *Do schools kill creativity?* [video]. Youtube. https://www.youtube.com/watch?v=iG9CE55wbtY

Acknowledgement

This study is fully funded by the Department of Science and Technology – Science Education Institute (DOST-SEI) and supported by the Ateneo de Manila University and Sotero B. Cabahug FORUM for Literacy.

TSG2-1-12

MANAGING THE TRANSITIONS IN CONCRETE-PICTORIAL-ABSTRACT

Yew Hoong Leong
National Institute of Education, Nanyang Technological University, Singapore, yewhoong.leong@nie.edu.sg

Tin Lam Toh
National Institute of Education, Nanyang Technological University, Singapore, tinlam.toh@nie.edu.sg

Lu Pien Cheng
National Institute of Education, Nanyang Technological University, Singapore, lupien.cheng@nie.edu.sg

Concrete-Pictorial-Abstract is an instructional heuristic that is used widely. But discussion on instructional moves between the stages is not common. In this paper, we delve more deeply into the Pictorial-Abstract transition as carried out by a teacher to unpack the deliberations that go beyond merely "fading" from one mode to the other.

Keywords: Concrete-Pictorial-Abstract; Teaching of Algebra; Making connections

Introduction

Concrete-Pictorial-Abstract (CPA) is a well-known instructional heuristic that has its historical roots in Bruner's Enactive-Iconic-Symbolic (Leong et al., 2015). It remains widely employed today (e.g., Naroth & Luneta, 2015; Puwadi et al., 2019;) because it is seen as a helpful rule-of-thumb for teachers to plan and implement lesson content in a way that students would be eased into what they would consider 'abstract' mathematics. As the name suggests, the instructional sequence begins with the Concrete Stage where students access the underlying mathematics concepts/skills in forms that are deemed 'concrete' to them; these experiences with the concepts/skills are then translated into Pictorial as an intermediary stage before students work on these concepts/skills in Abstract – that is, in the symbolic forms that are norms of doing school mathematics.

While the overall progression from C to P to A seems reasonable, there are many questions surrounding the actual classroom implementation of CPA. One non-trivial challenge is the transition between stages. When the continuity of ideas – both mathematically and visually – across the modes are perceived as broken by the students, then CPA is of little efficacy: "Why bother to start from C then P and so on if I don't see how C and P are related? We might as well just start with P, or even A". On the other hand, if one takes significant instructional time to develop a particular mode, say C, not only is it unrealistic in terms of lesson time but it may demotivate students from 'moving on' to P when they become overly fixated with C. These considerations give a glimpse of the type of practical challenges that come with implementing CPA.

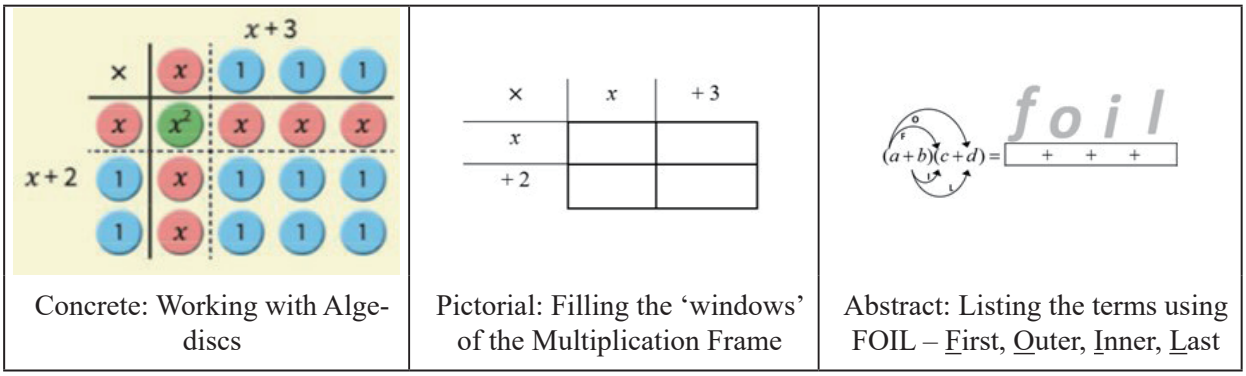
In this paper, we describe how a teacher attended to the challenge of inter-modal transition in CPA within the specific context of teaching expansion of algebraic expression in a Year 8 Singapore

classroom. From her experience, we reflect more broadly on the instructional practice of "fade-off" when planning inter-modal transition in CPA.

Method

The study reported here is part of a larger project known as HOME: Hands-On Minds-Engaged: A revisit of the Concrete-Pictorial-Abstract (CPA) Strategy. CPA has been ubiquitous in the Singapore curriculum – in official Ministry documents and Ministry-approved textbooks – for about 3 decades. The project seeks to revisit CPA by examining how teachers actually interpret and implement CPA in the classrooms, with a view of developing evidence-based embellishments to the theory. In this report, we focus on some aspects of implementation of Teacher Mong Tin (pseudonym).

Mong Tin was identified by her school leaders as one who was committed to use CPA as an instructional strategy in teaching. When asked to choose a topic to focus on for the project, she chose Quadratic Expansion and Factorisation as she felt the way she intended to develop the topic was amendable to the CPA heuristic. To capture her interpretation and implementation of CPA for the topic, we collected the following data: (a) the instructional materials she designed for the teaching of the topic. While most of the tasks in the instructional materials were taken from textbooks, she restructured them and weaved her design moves into the instructional materials. In this case, she undertook the process of re-designing without collaboration with other teachers; (b) fieldnotes of the lessons on the topic. We described critical events relating to CPA transitions as it happened in class; (c) interviews with her. We conducted four interviews with her to understand how she designed and used the instructional materials to realise CPA; we also asked her for her motivations behind the key instructional moves she made during the lessons. The analysis involved a drawing upon these data sources into a coherent narrative of how she made intermodal transitions. Figure 1 presents an overview of C, P, and A modes as it appeared in her instructional materials for Expansion. We emphasise that this paper reports how Teacher Mong Tin interprets CPA-use, including what is C, P, and A to her for this topic. It may be argued that Abstract *should* be more generalised (say, applicable also to product of trinomials) and not as narrowly captured by FOIL. But this discussion is beyond the scope of this paper. Readers may refer to Leong et al. (2015) for such a discussion.



Concrete: Working with Algediscs

Pictorial: Filling the 'windows' of the Multiplication Frame

Abstract: Listing the terms using FOIL – First, Outer, Inner, Last

Figure 1. Overview of the concrete, pictorial, and abstract representations as used by mong tin for the topic of expansion.

Due to space constraints, the focus for the rest of this paper is on the transition from Pictorial to Abstract, as this is the transition that was deemed more significant for Mong Tin.

Results

The careful transition from P to A was motivated by Mong Tin's concern that her students might not see the link between the modes easily: "I was a bit worried that they cannot see the continuity between the frame and this [pointing to the FOIL method]" (10.02, 2nd Interview).

The transition can be summarised as consisting of three moves: (1) First, she asked students to attempt some exercises to expand product of binomials using the Multiplication Frame. The instructional materials provided show the Multiplication Frame on the left panel and the right panel was dedicated to a re-writing of the terms obtained in the 'windows' in list form. At this point, the method of FOIL was not yet introduced formally. Figure 2 shows the arrangement of the instructional materials for one such exercise item at this stage of instruction. In other words, at this juncture, the Pictorial (Multiplication Frame) is foregrounded as a way of doing expansion and the Abstract (listing method) as an alternative way to write out the terms in the expansion.

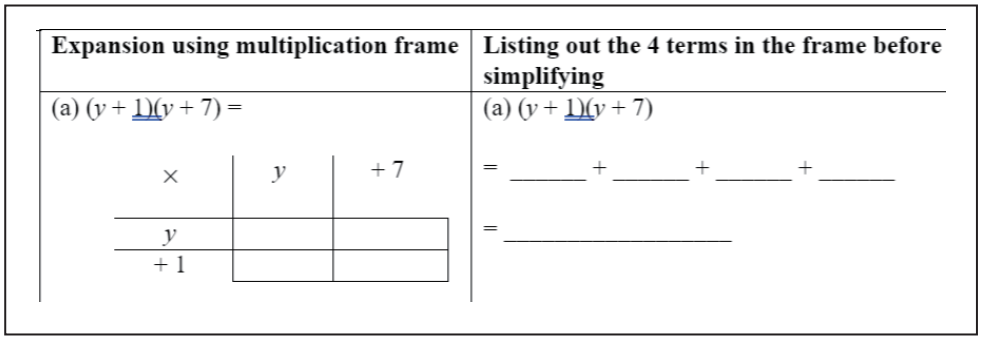


Figure 2. An extract of the instructional materials – Frame, then listing

(2) Thereafter, during classroom instruction, she placed the Frame and the Listing method side by side. As she filled up each 'window' of the Frame, she correspondingly drew each "rainbow" (her word) of the FOIL method. She also wrote the letters of the FOIL at each window and rainbow and also used different colours for each letter to strengthen the visual connection between both modes of representation. For example, when she filled in "x^2" (see Figure 1) in the top left window of the Frame, she used blue colour to write the letter "F" beside it and then move to the listing side to use the same blue colour to draw the rainbow to connect "x" (the First term) in the first binomial to the "x" (the First term) in the second binomial, labelling "F" on top of this rainbow. [It may appear that Mong Tin's meticulousness with the steps revealed her emphasis on procedure-following rather than conceptual understanding. We can make a case that she is committed to both but it will be beyond the scope of this paper]. (3) Next, the order is inverted: the Abstract is instead foregrounded first and the Pictorial is presented thereafter as a supporting "check". To Meng Tong, this "check" function of the Frame was important for a very specific reason: "I intend to do side by side so I can use frame or also by FOIL. So, they can see the comparison – most students when they do [Abstract] miss out the O and I. Using the frame, they can see what they miss out." (7.13, First Interview). In other words, the Pictorial Frame was not seen as an intermediate to be disposed of once students proceeded to Abstract; rather, she saw that Pictorial should be retained as a kind of fallback – when

students meet with difficulties or working in Abstract, they can then switch back to Pictorial to figure things out for themselves.

Discussion

Following Carbonneau et al. (2013), this continuity and transition between the CPA modes is commonly referred to as "fading". As elaborated in the previous section, Mong Tin carried out fading from P to A by putting the two modes side by side (fading presupposes a duration where both modes are present together). Moreover, just as fading means a process where less of one means more of another, the foregrounding of P and gradual introduction of A until the modes are inverted in prominence further brings out the feature of this metaphor. But beyond plainly fading, Mong Tin explicitly pointed out the visual links between the two modes so that both representations are presented as alternative ways of doing the same thing. And unlike fading which connotes that the movement is unidirectional – from P to A, Mong Tin wanted students to value P as fallback whenever there is a need to. There was more to the transition than what can be captured by the "fading" metaphor.

The case of Mong Tin's design as a way to manage intermodal transitions shows that CPA in practice is much more than "do C then P then A". The amount of careful deliberation – both in planning (as revealed in the interviews and the instructional materials) and in implementation – to bring about a smooth transition so that continuity of mathematical ideas would not pose hindrances to learning reminds us that multi-modal presentations in teaching mathematics is not a straightforward matter. It is generally understudied. And a moment's contemplation will reveal this challenge is not limited to those committed to the CPA heuristic. The learning of mathematics conceptually requires one to connect across multiple modes of representation – graphs, diagrams, symbols etc. As this study illustrates, managing the cognitive movements among these modes in teaching do require intentional and detailed instructional work.

References

Leong, Y.H., Ho, W.K., & Cheng, L.P. (2015). Concrete-Pictorial-Abstract: Surveying its origins and charting its future. *The Mathematics Educator, 16*(1), 1-19.

Carbonneau, K.J., Marley, S.C., & Selig, J.P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105*(2), 380-400.

Naroth, C., & Luneta, K. (2015) Implementing the Singapore Mathematics Curriculum in South Africa: Experiences of Foundation Phase Teachers. *African Journal of Research in Mathematics, Science and Technology Education, 19*(3), 267-277.

Purwadi, I.M.A; Sudiarta, I.G.P., & Suparta, I.N. (2019). The Effect of Concrete-Pictorial-Abstract strategy toward students' mathematical conceptual representation on fractions. *International Journal of Instruction, 12*(1), 1113-1126.

Acknowledgement

The research reported in this paper is funded by a grant from the Ministry of Education managed by the National Institute of Education under the project number OER23/21TTL. The study has cleared the University's Ethics Review Committee with the number IRB2022-330.

TSG2-2-6

GRADE 10 STUDENTS CONCEPTIONS OF PROOF – AN EXPLORATORY STUDY

Thanabalasingam Navinesh
Raffles Girls' School, Singapore, navinesh.t@rafflesgirlssch.edu.sg

Berinderjeet Kaur
National Institute of Education, Singapore, berinderjeet.kaur@nie.edu.sg

Weng Kin Ho
National Institute of Education, Singapore, wengkin.ho@nie.edu.sg

This paper examines grade 10 students' conceptions of proof in a secondary school in Singapore. Using a purposive survey comprising 9 mathematical items, the proofs written by 8 students were coded using Harel's proof schemes. The findings show that students did display a range of schemes, but three schemes were dominant. The three were ritual, non-referential symbolic and perceptual. In addition, several other deficits were present in proofs constructed by the students such as inability to articulate their arguments using correct mathematical language, over-reliance on inductive reasoning, and verifying a statement by using specific values or examples instead of establishing a general case.

Keywords: grade 10, students, proof schemes, secondary school, Singapore

Introduction

Students' conceptual knowledge (or conceptions) related to learning proofs encompasses their understanding of the fundamental principles and logical reasoning involved in constructing valid mathematical arguments (Basturk, 2010). It involves grasping abstract concepts, recognizing the logical structure of proofs, and comprehending the significance of proof techniques such as direct proof, proof by contradiction, and proof by induction. Conceptual knowledge also includes understanding the role of definitions, axioms, and theorems in establishing mathematical truth, as well as recognizing the connections between different mathematical concepts and how they can be applied in the context of proof construction (Basturk, 2010). Developing strong conceptual knowledge lays the foundation for students to effectively engage with and navigate the complexities of mathematical proofs.

This paper is situated within the context of classroom practice. Understanding students' proof schemes can inform pedagogical strategies for teaching proof. By identifying common misconceptions and types of reasoning students use, teachers can better design instruction that targets these specific challenges and supports the development of rigorous proof construction.

The Study

The study reported in this paper is part of a larger study on proof and proving in Singapore (Thanabalasingam, 2024). As part of an investigation related to "What shapes students' learning of mathematical proof in a secondary school" students' conceptions, experiences, approaches, perceptions and challenges were investigated. However, in this paper we will limit the scope to students' conceptions related to learning proof.

Theoretical Framework

In a landmark study, Harel and Sowder (1998) categorized how students approach problems related to proof as "*proof schemes*". It is a psychological framework, informed by historical, philosophical, and cultural analyses, for examining students' conceptions of proof. A few years later, backed by further empirical research, and taking into consideration historical and epistemological changes, Harel (2007) refined the proof schemes further (see Figure 1). Harel's (2007) framework categorizes proof schemes into three main types: External Conviction**,** Empirical**,** and Analytical**.** External Conviction relies on authority or ritual without understanding; Empirical involves perceptual or inductive evidence; Analytical includes transformational and axiomatic approaches, focusing on logical reasoning and internal consistency. We adopted the framework for our study of students' conceptions of proof.

External Conviction	*Empirical*	*Analytical*
○ Authoritative ○ Ritual ○ Non-referential symbolic	○ Inductive ○ Perceptual	○ Transformational ○ Axiomatic

Figure 1. Proof schemes (Harel, 2007)

Method

The Conceptions Survey

The conceptions survey questionnaire comprised nine modified past examination questions from the Elementary Mathematics (EM) and Additional Mathematics (AM) syllabuses from the Algebra and Geometry strands. The tasks were made more general, to provide students with opportunities to solve them in ways they deem fit. This would allow us to decipher their proof schemes from their solutions. Figure 2 shows two of the survey questions.

Subjects

A purposeful sample of eight grade 10 students (four males and four females) from a secondary school in Singapore participated in the study. They were doing both EM and AM. The students were keen to participate in the study and assess their readiness for their upcoming end of grade 10 national examination. Participants were selected to represent a diversity of performance levels in mathematics, based on prior results and teacher recommendations. Participation of the students in the study was governed by ethical requirements at the National Institute of Education in Singapore.

Data Collection and Analysis

Students were given one week to complete their take-home survey. The survey had to be completed in one-sitting within a duration of 150 minutes. This arrangement was based on mutual trust and a belief in students' integrity. The rationale for allowing a take-home survey was due to a tight curriculum schedule at school, as they were preparing for their end of grade 10 national examination. The researcher collected the completed surveys from the students a week after they were given out.

The solutions provided by the students were coded using Harel's (2007) proof schemes shown in Figure 1. The two coders were both experienced mathematics teachers with deep mathematical and pedagogical knowledge for teaching of mathematics at the secondary school level. The simple percent agreement between the 2 coders was at most near perfect (greater than 85%) for all solutions provided by the students.

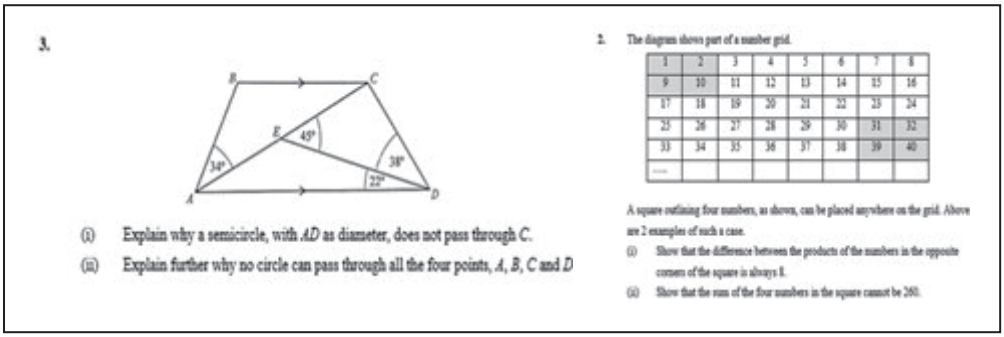


Figure 2. Sample conceptions survey questions

Findings and Conclusions

Due to limitation of a 4-page conference paper, we will only present the proof schemes used by the students for question 3. For part (i), students were asked to show that a circle with *AD* as the diameter cannot pass through *C.* The proof is equivalent to showing that angle *ACD* is not a right angle and students need to use the '*right-angled triangle in a semicircle property*' to prove the result. For part (ii), explaining why no circle can pass through all four points, *A*, *B*, *C* and *D*, is equivalent to transforming the problem to one that shows that quadrilateral *ABCD* is not cyclic. The proof involves using the '*opposite angles in a cyclic quadrilateral*' property by showing that the sum of the opposite angles in the quadrilateral does not add up to $180°$.

Amongst the solutions provided by the 8 students the proof schemes which occurred most regularly were the Ritual (5 out of 8 times), Perceptual (7 out of 8 times) and Transformational (7 out of 8 times) proof schemes. Evidence of the ritual proof scheme was seen in students who applied the Geometrical properties or theorems to explain a particular step incorrectly or without any real basis. For example, in proving part (ii) of the question, Student 3-1 stated that 'angles in the opposite segment do not add up to $180°$' in a ritualistic manner – using exactly the same words he was taught to – *without* acknowledging clearly that which segment of the cyclic quadrilateral he or she was referring to. Student 6-2 proved that both pairs of opposite angles do not have a sum of 180^0 – thinking that it would be safer to perform *twice* the ritual of checking that opposite angles are not supplementary confirm – when it was actually sufficient to show that just one pair does not add up to $180°$. In separate interviews with these students, it was affirmed that they were writing these justifications as the 'usual' geometrical reasons without actually understanding why.

Evidence of the perceptual proof scheme was seen in students who assumed some property without explicitly stating it or incoherence in their argument, as they assumed that it was needless to specify the implications of some statements. Student 1-1 had written, 'angle *DCE* is not equal to $90°$ because of right-angle in a semicircle rule'. He meant to state that since angle *DCE* was not equal to $90°$, by the right-angled triangle in a semicircle property, we cannot have a right-angled triangle *ACD* in a semicircle with diameter *AD*. Student 2-1 stated that 'properties in circle = $90°$ Δ in a semicircle. If *AD* is the diameter, *ACD* should be a right-angled triangle.' The statement›s incoherence and incorrect implication show that he had an incorrect perception of how the 'right-angled triangle in a semicircle property' should be applied. The transformational proof scheme was evident in those students who correctly transformed the problem into one that required showing the converse of the 'right-angled triangle in a semi-circle' property to show that angle *ACD* cannot be equal to $90°$ implies triangle *ACD* cannot be right-angled.

It is apparent from the findings, based on the students' solutions of the nine questions, that:

1. Students display especially the Ritual, and Perceptual proof schemes.

2. Students often struggle with articulating their arguments using correct mathematical language. They misuse symbols, connectors, or semantics, leading to incorrect or incoherent explanations.

3. Some students rely heavily on inductive reasoning, attempting to verify a statement by using specific values or examples instead of establishing a general case.

4. Students make assumptions or draw conclusions without properly defining variables, proving statements, or considering all possible cases.

5. Some students demonstrate the transformational proof scheme, where they successfully transform a problem into one that requires a different approach or proof technique.

6. Students do rely on memorized steps or formulas without fully understanding when and how to use them when proving, thereby adopting the ritual proof scheme.

The findings of this study have clear implications for classroom practice. Teachers should pay attention to helping students construct rigorous proofs using appropriate mathematical language. Explicit demonstration of different proof types, along with opportunities for students to critique and reason, can advance mathematical justification. Instruction that directly addresses students' common misconceptions on different proof schemes can support meaningful learning and align with curriculum goals. As such, proving should be placed at the core of mathematics instructions, akin to solving and evaluating, since proofs are foundational to mathematical understanding.

References

Basturk, S. (2010). First-year secondary school mathematics students' conceptions of mathematical proofs and proving. *Educational Studies*, *36*(3), 283-298.

Harel, G. (2007). Students' proof schemes revisited. In *Theorems in school* (pp. 65-78): Brill Sense.

Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. *American Mathematical Society, 7*, 234-283.

Thanabalasingam, N. (2024). Proof and proving in the Singapore secondary school mathematics curriculum. Unpublished PhD thesis. National Institute of Education, Singapore.

TSG2-2-7

THE IMPACT OF PROBLEM POSING ON STUDENT'S MATHEMATICAL INQUIRY SKILLS : AN EMPIRICAL INVESTIGATION

Jung Sook Park
Jungwha High School, Korea, pjs1924@sen.go.kr

This study investigates the relationship between mathematical problem posing and the development of students' questioning abilities and inquiry skills. Through a qualitative analysis of high school students' engagement with structured problem-posing activities, we examine how the practice of creating mathematical problems enhances conceptual understanding and mathematical thinking. The findings indicate that problem posing serves as both a pedagogical tool and a metacognitive strategy, facilitating deeper mathematical comprehension and fostering independent inquiry skills.

Keywords: problem posing

Introduction

Mathematical problem posing has emerged as a significant pedagogical approach that transcends traditional problem-solving activities in mathematics education. While problem solving has been central to mathematics education for decades, the emphasis on problem posing—the generation or reformulation of mathematical problems—has gained increasing attention for its potential to develop deeper mathematical understanding and inquiry skills (Cai & Hwang, 2020).

Recent research suggests that problem posing functions as both a means and an end in mathematics education (Silver, 1994). As a means, it provides opportunities for students to deepen their mathematical understanding and develop critical thinking skills. As an end, it cultivates students' ability to identify and formulate meaningful mathematical questions—a crucial component of mathematical inquiry (Cai & Leikin, 2020).

The relationship between problem posing and students' questioning abilities warrants particular attention. When students engage in problem posing, they must not only comprehend the mathematical concepts involved but also develop the capacity to generate meaningful questions about mathematical situations. This process necessitates deep consideration of mathematical relationships and multiple perspectives—essential components of inquiry-based learning.

The purpose of this study is to develop students' abilities to examine and think about problems through problem posing tasks. This purpose directly connects to the educational value of problem posing discussed above, and presents a practical educational approach for enhancing students' mathematical thinking and questioning abilities.

Theoretical Framework

Research in mathematics education conceptualizes problem posing as both a cognitive and metacognitive endeavor (Silver, 1994). When students engage in creating new mathematical tasks, they must:

1. Analyze Mathematical Situations and Relationships
   They evaluate contexts or prompts to detect patterns, constraints, and potential relationships that lend themselves to mathematical exploration.
2. Generate Novel Mathematical Questions
   By moving beyond standard problem-solving routines, students brainstorm original queries that may build upon or extend familiar concepts.
3. Evaluate Validity and Significance of Posed Problems
   Reflective thinking leads students to assess whether their generated tasks are coherent, solvable, and meaningful.
4. Refine Mathematical Questions through Feedback
   Problem posing is iterative, with students revising tasks based on peer or teacher input and self-reflection.

Development of Questioning Abilities

Questioning abilities, an essential component of inquiry-based learning, are cultivated when students learn to probe deeply into mathematical contexts (Cai & Leikin, 2020). In a problem-posing environment, they must:

- Identify mathematically significant features in a scenario
- Formulate questions that investigate these features
- Distinguish between superficial and substantive questions

Inquiry Skills and Mathematical Independence

Inquiry skills in mathematics involve the capacity to explore and justify mathematical claims independently (Singer, Ellerton, & Cai, 2015). By encouraging an active stance toward knowledge construction, problem posing positions students as "authors" of mathematics. This shift increases their engagement and fosters deeper comprehension of mathematical concepts.

Methodology

Participants and Setting

High school students (approximately ages 15–17), consisting of 90 participants, took part in this study. The problem-posing activities were integrated into the regular mathematics curriculum to investigate how such tasks could be implemented in authentic classroom contexts.

Problem-Posing Interventions

Five sequential sessions, each lasting 50 minutes, were introduced to develop problem-posing skills. After completing all five sessions, students were presented with eight images and asked to engage in problem posing to evaluate how much their skills had improved compared to their initial performance.

1. Fermi Problems
   Students tackled estimation problems that required logical thinking and real-world applications.
2. Video-Based Problem Posing
   Students watched short video clips, then formulated related mathematical problems.
3. Image-Based Problem Posing
   Static images served as prompts for creating geometry- or algebra-based questions.
4. What-If-Not Strategy
   Students systematically modified existing problem conditions, discovering how small changes can reshape entire problem structures.
5. Concept-Specific Problems (e.g., Geometric Sequences)
   Learners focused on a core mathematical concept to design more advanced-level problems.

Data Collection

The data collection is as follows:

- Student-Generated Problems: Collected to assess the breadth, depth, and creativity of posed tasks
- Observations and Field Notes: Documented student collaboration, reasoning, and iterative revisions
- Student Reflections: Provided qualitative insights into changes in attitudes, strategies, and perceived competence

Data Analysis

Data were analyzed thematically, focusing on indicators of cognitive and inquiry-based growth. Categories included conceptual understanding, questioning proficiency, and mathematical creativity.

Results and Discussion

Findings from the studies revealed notable outcomes in three main areas.

Transformation of Mathematical Perspective

Students reported a paradigm shift in how they approached mathematics. Instead of viewing problems as fixed puzzles to solve, they recognized that mathematics can be generated from real-world observations or theoretical manipulations (Silver, 2013). One student noted:

> *"While I've always been in the position of solving problems rather than creating them, this problem-posing activity gave me a new perspective on mathematical problems. Through the shift in thinking about 'how can I apply the concepts I know to create mathematical problems?' I was able to approach mathematics problems from various angles."*

This perspective fostered:

- A sense of agency in mathematical discourse
- Greater awareness of how mathematics applies to everyday phenomena

Deepening of Conceptual Understanding

The iterative nature of problem posing demanded a robust grasp of underlying concepts. One student noted:

> *"Compared to just solving problems, creating problems improved my ability to verify basic concepts and check for contradictions independently. I realized that concepts I sometimes used with uncertainty needed to be thoroughly mastered to create problems, which helped fill gaps in my understanding."*

By cross-verifying their own posed tasks, students identified conceptual gaps and improved their mastery of foundational ideas.

Development of Inquiry Skills

Students' questioning abilities grew substantially. They became adept at:

- Identifying key assumptions and variables within a scenario
- Evaluating the complexity and relevance of posed questions
- Moving beyond routine solutions to generate innovative queries

One student noted:

> *"I was surprised to find that we could create math problems from unexpected places. I felt a sense of achievement in discovering mathematics in everyday life and turning these discoveries into problems. Initially, I wondered 'how can these situations become math problems?' but it was fascinating to see how they could lead to problems even more challenging than those we usually solve."*

In particular, the What-If-Not strategy encouraged them to modify problem conditions methodically, fueling deeper exploration of functional relationships. This practice underscored how systematic variation of conditions can cultivate a more robust form of mathematical reasoning (Cai & Hwang, 2020).

Conclusion

Across both studies, problem posing emerged as a powerful pedagogical tool that advances students' questioning abilities and nurtures inquiry skills in mathematics. Students transitioned from passive recipients of problems to active co-creators of mathematical content, showcasing enhanced conceptual clarity and creative engagement.

Although initial hurdles—such as unfamiliarity with open-ended tasks—were present, consistent scaffolding and reflective practices helped students appreciate and leverage the value of problem posing. To maximize these benefits, future research and classroom practice should:

1. **Integrate Problem Posing Systematically**
   Embed activities at regular intervals across diverse mathematical topics.
2. **Offer Scaffolding and Support**
   Provide structured prompts and models for both teachers and students new to problem posing.

3. **Encourage Collaborative Exploration**
   Facilitate peer discussions where students refine and critique each other's posed problems.
4. **Promote Reflection**
   Foster metacognitive awareness through regular feedback and self-assessment cycles.

By aligning problem posing with broader curricular goals, educators can help develop a more inquiry-oriented mathematics classroom—one in which students continually hone their ability to ask thoughtful questions and pursue answers with creativity, rigor, and independence.

References

Cai, J., & Hwang, S. (2021). Teachers as redesigners of curriculum to teach mathematics through problem posing: Conceptualization and initial findings of a problem-posing project. *ZDM Mathematics Education, 53*, 1403–1416.

Cai, J., & Jiang, C. (2017). An analysis of problem-posing tasks in Chinese and U.S. elementary mathematics textbooks. *International Journal of Science and Mathematics Education, 15*(8), 1521–1540.

Cai, J., & Leikin, R. (2020). Affect in mathematical problem posing: Conceptualization, advances, and future directions for research. *Educational Studies in Mathematics, 105*(3), 287–301.

Cai, J., Hwang, S., Jiang, C., & Silber, S. (2015). Problem posing research in mathematics: Some answered and unanswered questions. In F.M. Singer, N. Ellerton, & J. Cai (Eds.), *Mathematical problem posing: From research to effective practice* (pp. 3–34). Springer.

Kontorovich, I., Koichu, B., Leikin, R., & Berman, A. (2012). An exploratory framework for handling the complexity of students' mathematical problem posing in small groups. *Journal of Mathematical Behavior, 31*(1), 149–161.

Leung, S. S., & Silver, E. A. (1997). The role of task format, mathematics knowledge, and creative thinking on the arithmetic problem posing of prospective elementary school teachers. *Mathematics Education Research Journal, 9*(1), 5–24.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19–28.

Silver, E. A. (2013). Problem-posing research in mathematics education: Looking back, looking around, and looking ahead. *Educational Studies in Mathematics, 83*, 157–162.

Singer, F. M., Ellerton, N., & Cai, J. (Eds.). (2015). *Mathematical problem posing: From research to effective practice*. New York: Springer.

TSG2-2-8

PROBLEM-POSING PRACTICES: A CASE STUDY OF A FILIPINO SCIENCE HIGH SCHOOL MATHEMATICS TEACHER

Christine Nicole Victorio
Ateneo de Manila University, National University Philippines, christine.victorio@student.ateneo.edu

Joseph Ma. Steven Cabalo
Department of Education Schools Division of Lipa City, Ateneo de Manila University, Philippines, joseph.cabalo@deped.gov.ph

Rodolfo Calimlim Jr.
University of Baguio High School, Philippines, rfcalimlimjr@e.ubaguio.edu

Mary Jane Castilla
University of Sto. Tomas, Philippines, macastilla@ust.edu.ph

John Patrick Cultura
La Salle College Antipolo, Philippines, patrick.cultura@lsca.edu.ph

Flordeliza Francisco
Far Eastern University, Philippines, ffrancisco@feu.edu.ph

Lester Hao
Ateneo de Manila University, Philippines, lhao@ateneo.edu

Vanessa Zubieta
South Luzon State University, Philippines, vzubieta@slsu.edu.ph

Despite the growing interest in research on problem-posing, it remains a relatively underexplored area in the Philippines. Through various teaching episodes, this paper examines the problem-posing practices of a Grade 7 mathematics teacher in a Philippine science high school. This paper discusses how Cai and Rott's problem-posing model and Vistro-Yu's innovation techniques are reflected in the teacher-led problem-posing activities observed in the teaching episodes. The findings show that it is possible not only for teachers to incorporate problem-posing into their everyday teaching but also to make sense of their problem-posing activities through several models.

Keywords: Teacher-led problem-posing, problem-posing model, innovation techniques

Introduction

Problem-posing consists of activities that enable teachers and students to create and express mathematical problems based on specific situations (Cai & Hwang, 2002). While problem-solving has

been articulated thoroughly in curriculum documents and professional development programs in the Philippines, problem-posing remains relatively underexplored here. This paper draws on Cai and Rott's (2023) problem-posing model and Vistro-Yu's (2009) innovation techniques to examine how teacher-led problem-posing occurs within a mathematics classroom. The case study is based on the teaching episodes from one of the six classroom observations conducted in a Grade 7 mathematics classroom at a science high school. Specifically, this study addresses two questions: (1) How does a teacher implement mathematics problem-posing in the classroom? (2) Which innovation techniques on mathematics problems were implemented in the classroom?

Conceptual Underpinnings

According to Cai and Rott (2023), there are four phases of problem-posing: orientation, connection, generation, and reflection. *Orientation* involves understanding the problem-posing task, which includes a specific situation, either mathematical or real-life, and a prompt that describes what the poser is being asked to do. In the *connection* phase, the poser draws possible connections and makes conjectures based on the task. During this phase, the problem poser explores elements involved in the situation, creating new problems by modifying the original scenario or using a *what-if-not* strategy. The *generation* phase focuses on making the posed problems visible to present relationships or specific facts in the form of questions. In this phase, the poser may articulate problems that they are curious about as conjectures. Finally, *reflection* involves assessing the problem-posing process; the problem-poser evaluates what has been accomplished throughout all the phases.

On the other hand, Vistro-Yu (2003) presented six innovation techniques for innovating *used* problems to develop *new* problems. *Replacement*, which does not require much time or thought, is an innovation technique characterized by posing the same problem but changing quantities, amounts, units, shapes, etc. In the *addition* technique, which requires some thought but not much time, the same problem is posed, but a new given, constraint, or obstacle is added. Taking the same given but changing the problem refers to *modification*, which is quite tricky. Meanwhile, *contextualizing a problem* for it to be more relevant could be a challenging innovation technique since there is a need to know what students could relate to in a particular time period. To *turn the problem around* or *reverse* it means that the poser takes the same problem and considers the end goal as the given and the given as the end goal; this is clearly a challenge because not all problems can be turned around to come up with a good, complex problem. Lastly, *reformulation* is quite tricky as well, since it entails posing the same problem in a different type (e.g., from a proving problem to a situational problem; see Butts, 1980). Adopting any innovation techniques to generate problems may sometimes not have solutions, but the key is to try and explore repeatedly until better problems are generated.

Methods

This qualitative study employed a case study research design to narrate how a teacher demonstrated mathematics problem-posing practices in the classroom and how the two mentioned models of mathematics problem posing were implemented. Case study design is an inquiry in which an investigator explores a case through detailed and in-depth data collection such as observations, interviews, reports, and documents (Creswell, 2007). Following this research design, data is gathered through classroom observations and interviews. The subject of this study is Teacher Rev, a mathematics teacher who has been teaching for about 20 years at a science high school in Valenzuela City, Philippines. This report is part of a larger study, and it only focuses on one of the six classroom observations, specifically the first day. Video recording of the first day of the classroom ob-

servation session was transcribed to produce a narrative detailing what transpired in Teacher Rev's mathematics classroom. The narrative was thematically analyzed using the lenses of Cai & Rott's (2023) problem-posing process model and Vistro-Yu's (2009) innovation techniques to map the problem-posing episodes present on that day.

Analysis and Discussion

As discussed, problem-posing episodes in mathematics classrooms can be observed in four phases: *orientation*, *connection*, *generation*, and *reflection* (Cai & Rott, 2023). Results from the first day of classroom observation reveal that the four phases were evident in Teacher Rev's teaching practices. The first phase, *orientation*, was commonly identified in the teaching episodes of Teacher Rev when he introduced a lesson or a task by recalling the previously discussed topic and harnessing students' prior knowledge. To illustrate, on the first day, he posed the question, "in which order do you wash the dishes at home?" to the students, and the responses gathered were used as the springboard for the discussion. Teacher Rev then mentioned, "so in statistics, we will be doing also that manner. So you have to organize data," and then he briefly explained the definition of statistics and asked the students if it would make sense if presentation of data comes before gathering of data. Students responded that it would not make sense, and then Teacher Rev posed a related question, asking the students how they organize data. The students' initial responses on washing dishes, which were used to discuss the definition of statistics, showcase how the second phase, *connection*, can be established subsequent to *orientation*. The third phase, *generation*, was also evident in the practices of Teacher Rev in multiple instances. For instance, he initially asked the students to narrate how they organize data, to which the students shared varied answers such as follows: they arrange it from the lowest value to the highest value, and separate data based on categories. Subsequent to this, Teacher Rev posed questions related to data organization which led to a guided activity and eventually followed by a similar independent task for the students. It was observed that he guided the students to recall their prior knowledge related to organizing (*orientation*), then used it to sort data and answer questions related to it (*connection*). Afterwards, he presented a different data set in which the students performed similar tasks, such as organizing data using back-to-back stem and leaf plot (*generation*). Meanwhile, the last phase, *reflection*, was only observed during the first day when he instructed his students to gather data from the student population and create their own set of questions and survey questionnaires to collect information.

Vistro-Yu's (2009) innovation techniques on mathematical problem-posing were used to identify the problem-posing techniques that were commonly present in the class of Teacher Rev during the first day classroom observation period. From the six techniques, only the *addition*, *modification*, *contextualization*, and *reformulation* were observed. Among these, the most frequent innovation technique observed in Teacher Rev's practices is *addition*. To recall, it is a way of generating new problems by adding a new given or constraint to the previously posed problem. This technique was apparent when Teacher Rev presented an example of a stem-and-leaf plot data table in which he initially asked the students to observe it. He used this as his base problem and then added another constraint or obstacle to pose a series of questions to the students. The next question he asked was to determine the lowest and highest data values as well as the frequency of each. This is categorized as *an addition* innovation technique, as it is essentially the same stem and leaf plot that was initially observed; however, a new constraint is added (i.e., identifying highest and lowest data values and frequencies). Meanwhile, the *modification* innovation technique was also observed when Teacher Rev gave a data set to the student and instructed them to determine which frequency distribution table is more appropriate to use. When the students answered that an ungrouped frequency distribu-

tion table is a better option, he then proceeded to pose the next activity: constructing an ungrouped frequency distribution table using the same data. This instance was identified as *a modification* since the previous given were used to generate new problems and tasks. Next, the *contextualization* innovation technique was also observed in Teacher Rev's practices. Initially, the base problem was generating frequency distribution tables and stem-and-leaf plots using random data; it was then contextualized when the students were asked to present and compare the mathematics test scores from two classes. Lastly, the *reformulation* innovation technique was also observed when Teacher Rev instructed the students to collect data from the student population, in which the students constructed their own survey questionnaire to gather information. From the reported teaching practices, it was evident that Teacher Rev posed problems to the students to assess their knowledge and to assist the students in finding the correct answers.

Conclusion

In this paper, we discussed what problem-posing means in the mathematics classroom by examining the teaching episodes, the viability of Cai and Rott's problem-posing model, and Vistro-Yu's innovation techniques that suggest ways to generate new problems and delve into problem exploration. Classroom observations presented a myriad of evidence on how problem-posing unfolds in mathematics classroom teaching. Such instances showcased the four phases of mathematical problem-posing and the evidence of mathematical problem-posing innovation techniques. The results described how problem-posing is possibly implemented in a mathematics classroom in the Philippines. Explicating the usefulness of problem-posing models and innovation techniques will help teachers better understand the problem-posing processes and thus, help them become better problem-posers. To promote problem-posing in mathematics classrooms, it is essential to equip teachers with more tools, time, and opportunities to explore these techniques. This support will encourage teachers to incorporate problem-posing into their classroom teaching. Establishing learning communities, such as partnerships between teachers and researchers, can further assist in discovering effective strategies for designing and implementing problem-posing activities in mathematics classes. Moreover, wider studies focusing on mathematical problem-posing in teaching can provide a narrative on how it can be modeled in Philippine classrooms.

References

Butts, T. (1980). Posing problems properly. In S. Krulik & R. E. Reys (Eds.), *Problem solving in school mathematics* (pp. 23–33). Reston, VA: National Council of Teachers of Mathematics.

Cai, J., & Hwang, S. (2002). Generalized and generative thinking in US and Chinese students' mathematical problem solving and problem posing. *The Journal of mathematical behavior*, *21*(4), 401–421.

Cai, J., & Rott, B. (2023). On understanding mathematical problem-posing processes. *ZDM–Mathematics Education*, *56*(1), 61–71.

Creswell, J. W. (2007). Qualitative inquiry and research design: Choosing among five approaches, 2nd ed. (pp. xvii, 395). Sage Publications, Inc.

Vistro-Yu, C. (2009). Using Innovation Techniques to Generate 'New' Problems. In *Mathematical Problem Solving* (pp. 185–207). https://doi.org/10.1142/9789814277228_0010

Acknowledgement

The authors would like to express their sincerest gratitude to the Philippine Council of Mathematics Teacher Educators (MATHTED), Inc. for making this collaborative endeavor possible.

TSG2-2-9

THE ROLE OF REFLECTION CARD IN FACILITATING APPROPRIATION PROCESSES IN MATHEMATICS EDUCATION

Shinya Nakao
Kansai University of Welfare Sciences, Japan, s-nakao@tamateyama.ac.jp

Masakazu Okazaki
Hiroshima University, Japan, masakazu-oka@hiroshima-u.ac.jp

This paper focuses on the appropriation process as a concept underlying the most proximal area of development in Vygotsky's theory, which is the foundation of the sociocultural approach, and on imitation as its central mechanism. This paper focuses on reflection through imitation and aims to clarify the role of reflection cards in facilitating the appropriation process. To achieve our objective, we first describe the stipulation of reflection in this paper and its significance. Next, we describe the theoretical background of the "LEAD card," a reflection card developed by Nakao (2001), and the "Reflection by description, imitation, and exclusivity. Then, the role of the reflection cards in facilitating the appropriation process was examined in a fourth-grade elementary school unit on "Multiplication of Decimals". The results suggest that repeatedly writing reflections on LEAD cards, a cultural tool, promotes the appropriation process.

Keywords: reflection, Vygotskyism, imitation, Appropriation

Introduction

The question of how to make use of assessment to enhance mathematical understanding is an ongoing issue in mathematics education. To make the most of assessment and improve learners' mathematical understanding, it is first necessary to visualize the learners' state of understanding. In this study, we focus on "reflection by description" to visualize learners' understanding and to make the most of evaluation.

Nakao (2021) has developed a reflection card named "LEAD Card ".The "LEAD Card" allows learners to write their reflections and visualize their understanding in concrete terms. On the other hand, one of the problems with written reflections is that they depend on whether the learner has writing skills. To solve this problem, it is possible to promote understanding by having the learners come into contact with the ideas of others and by having them refer to the board. In other words, the interaction with other people's writing advances one's own writing and learning. The sociocultural approach focuses on how an individual's cognitive development progresses within a social and cultural context. While Vygotskian theory is the foundation of the sociocultural approach, this paper focuses on "imitation" and "appropriation" as concepts underlying the most proximal areas of development. The purpose of this paper is to clarify the role of the reflection card in facilitating the appropriation process. The LEAD cards of Nakao (2021) are used for reflection.

First, we describe the definition of reflection and its significance in this paper. Next, I describe the theoretical background of the LEAD card and the descriptive reflection, imitation, and appropriation. Finally, the role of reflection cards in facilitating the appropriation process is examined in a

fourth-grade elementary school unit on “Multiplication of Decimals”.

Regulations and Significance of Reflection

Reflection” is an activity that leads to structuring knowledge, checking one’s own understanding, and becoming aware of one’s own transformation. In this paper, “reflection” is defined as “reflecting on one’s own thinking at the end of the class and grasping one’s current awareness. Here, the purpose of reflection is considered from two aspects.

The first is “to reflect on and confirm one’s learning. Reflection is the act of reflecting on one’s own thinking. By reflecting on one’s own thinking, one can reconfirm what one has learned and confirm one’s learning.

The second is the aspect of applying what they have learned to the next step. The structure of a unit of Japanese mathematics and arithmetic is not simply a collection of contents. In other words, the content of each hour of study is linked together to form a series of learning stories. For example, a unit has a systematic structure in which students first encounter a concept, then learn the meaning of the concept and how to handle it and finally confirm its effectiveness by applying the acquired concept and method. Rather than studying each lesson in isolation, the next lesson can be made even better by making use of “reflection”.

Reflection, Imitation and Appropriation Through Description

Learners can grow through imitation, but they cannot imitate everything, but only according to what they learn (Vygotsky, 2001).Appropriation is also a concept that intends to incorporate into one’s own thinking by imitating the ideas of others, a process that can be attributed to the personal reproducibility of historically formed human qualities, abilities, and modes of action (Leont’ev, 1981).Appropriation work requires the introduction of mediating tools, and in the sociocultural approach, mediation through “language” is of particular importance. Of language, especially with regard to written language, writing for Vygotsky was developmentally influential due to the characteristics that writing for him was planned and consciously done and based on an arbitrarily chosen symbol system (Sierpinska, 1998). With regard to written language, for example, one of the tools in the classroom is the board. One of the tools in the classroom, for example, is the writing board, in which the opinions of the learners are expressed through the written word. The transformation of one’s own ideas through contact with the board can be seen as an interaction with the writings of others.

In this paper, we will use the term “imitation” for situations in which the learner is actively trying to incorporate the ideas of others into his/her reflections, and “appropriation” for situations in which the learner describes the overall process of progress in his/her understanding.

The “ Lead Card”

The LEAD Cards were developed based on the requirements for effective reflective descriptive expressions identified by Ninomiya (2005): “core descriptions,” “concrete examples,” and “meta-knowledge descriptions. The LEAD Card consists of three parts: “Learning,” “Example,” and “ADvance.Each of these is an item to reflect on using words and expressions, and an item to describe specific examples using words, expressions, and charts. The “Learning” section is used to describe how to apply the learning and what else the student would like to know .

Investigation

The class to be studied is the fourth grade elementary school class "Decimal Multiplication," which covers the first through the fourth periods of dealing with decimal multiplication. In this paper, one student, F, is selected for analysis. The reason is that F's description of the LEAD card is likely to be an imitation of the ideas of others. In the first period, the relative view of numbers was confirmed through a review of multiplication of integers and addition of decimals, in the second period, students learned how to calculate 0.2×6, in the third period, they learned written operations using the properties of multiplication, and in the fourth period, they mastered multiplication of decimals.

From the first to the third period, F wrote on her LEAD card the ideas of others through imitation, rather than his own ideas that he had worked on in class. For example, Figure 1 shows a LEAD card written by F at the third hour. On the other hand, the ideas that were worked on during the class were not the ones written on the LEAD cards in the previous period. In other words, we can say that F did not make use of the reflection.

On the other hand, in the fourth period, F was solving the problem by using the ideas on his LEAD card, which he had obtained through imitation by the third period. During the introduction of the fourth period, the teacher commented on F's LEAD card, assigning values to it.In the fifth period, the ideas that had been written on the LEAD cards were also used in problem solving. The fourth period was a time of great growth for F. It was a time when he repeatedly recorded the ideas of others

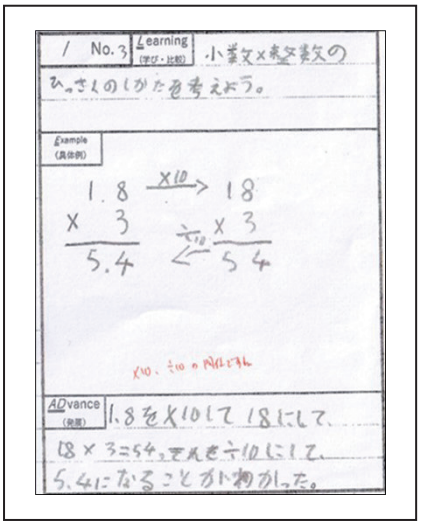


Figure 1. the LEAD Card

throughout the unit. This was probably due to the fact that he repeatedly recorded the ideas of others throughout the unit and the teacher assigned values to the ideas, which led to his awareness of "trying to use" them. In other words, the repeated writing of reflections on LEAD cards and the teacher's assigning values to the reflections are suggested to promote the appropriation process.

When human development is viewed from a sociocultural perspective, development is the process of developing cultural activities with the help of cultural tools, and learning is the process of qualitative changes in behavior that can occur when people participate in meaningful cultural activities

and receive advice to improve or appropriate their behavior (van Oers, 2010). The development of F observed in the present case is the result of the process of learning with the LEAD cards, a cultural tool, in which the process of imitation and utilization of others' ideas was repeated, and the process of possession was facilitated.

In this case, F was the subject of the study, and the process of ownership was promoted using LEAD cards. However, even if the process of ownership was not promoted, most of the children wrote on the LEAD cards by imitating the ideas of others. Since imitation is not something that everyone can do, it is expected that imitation of others' ideas will also lead to learning, but there is room to consider how to facilitate the process of ownership.

Conclusion

The purpose of this paper is to focus on reflection by imitation and to clarify the role of the reflection card in facilitating the appropriation process. The role of reflection cards in facilitating the appropriation process was examined in the unit "Multiplication of Decimals".

The results suggest that repeatedly writing reflections on LEAD cards, a cultural tool, promotes the appropriation process. In the future, it is necessary to examine how to promote the process of exclusive use by children who only imitate the ideas of others.

References

Nakao, S. (2021). Research on grasping mathematical thinking focusing on reflection: Through the development of reflection cards and the empirical understanding of thinking. *Journal of Japan Society of Mathematical Education Research Journal of Mathematical Education, 102(116)*, 17-29.

Sierpinska, A.(1998). Three Epistemologies, Three Views of Classroom Communication: Constructivism,Sociocultual Approaches, Interactionism. H. Steinbring, M. G. Bartolini-Bussi, & A. Sierpinska (eds.), *Language and Communication in the Mathematics Classroom*. NCTM.

Ninomiya, H. (2005). *A study on reflective descriptive expression activities in mathematics education.* Kazamashobo.

Van Oers, B. (2010). Emergent mathematical thinking in the context of play. *Educational Studies in Mathematics, vol. 74,* 23-37.

Vygotsky: Yoshimatsu Shibata (trans.) (2001). *New Translation: Thought and Language*. Shin-Dokusho-Sha.

TSG2-2-10

USING STUDENT'S SELF-ASSESSMENT OF THEIR INTERPRETED PREPARATION FOR CLASS: A PILOT OF USING THE MASTU EVALUATION.

Amanda Jo Meiners
Northwest Missouri State University, USA, ameiners@nwmissoui.edu

This is a pilot for the development of the MASTU self-assessment used by university students in their mathematics education focused content and mathematics context focused courses. This five question self-assessment was developed from the need for students to feel prepared for class and assessing if they had prepared in a manner that students deemed at an appropriate level. Such preparation was expected as the National Council of Teachers of Mathematics in the United State of America's eight Standard of Mathematics practice were used during course instill a Building Thinking Classrooms pedagogy. Basics of the assessment are included along with the initial results from an instructor's classes. Additionally, the results are discussed with next steps considered to promote student learning in a manner that promotes communication in the classroom during the more digital and semi-post CoVid-19 era.

Keywords: self-assessment; inquiry learning; university students

Background

Classroom Practices

The way we teach mathematics has shifted from teacher centered to that of more student centered (Sandra et al., 2014; Stein et al., 2008). However, these student-centered shifts have not seen the explicit progress that many students would expect in their feelings about learning mathematics (Brady et al. 2016; Grant & Sonnentag, 2010; Griggs et al., 2015). Mathematics is often still seen by students as a set of facts to learn and then regurgitate on the assessment (Boaler & Dweck, 2016). The National Council of Teachers of Mathematics (NCTM) in the United States of America has eight standards of mathematical practice (SMPs) they intend to be used in the classroom (pg. 8; Leinwand et al. 2014; 2024). These eight standards include such items as look for and *make use of structure*, *construct viable arguments and critique the reasoning of others*, and *make sense of problems and persevere in solving them*. Such items are significant in mathematics classrooms to promote student centered learning and design a learning environment that does so.

Building Thinking Classrooms

One such student-centered environment meets the eight SPMs is Building Thinking Classrooms pedagogy (Liljedahl, 2016; 2018; 2021). A brief overview includes using vertical non-permanent writing spaces, using visible random groupings, and tasks that build students thinking through thin sliced math tasks over mimicking what the teacher demonstrates. Within these concepts is the promotions of students leading and owning their learning of concepts while in an environment that allows for adjustments in the moments.

Although teachers and classrooms have made shifts to become more student-centered students are less inclined to know how to feel prepared and come to class ready with parental and

familiar stories often told of showing up for class. As such, the significance of this pilot study begins looking at what information a daily, short, self-assessment could offer educators as they prepare their student-centered classroom plans. Therefore, the research questions asked were

1) How much involvement are students having with a non-graded self-assessment?
2) What patterns are noticed about preparations patterns?
3) What other questions or rating may be opportunities to assist educators in their own course preparations?

Methods

Participants

The participants of this study those taking educational mathematics course work with a particular instructor as part of their degree seeking program. Approximately 20 to 30 students take part in the self-assessment before each day of class. The courses the metric was used in met 3 or 5 times a week, depending on the course hours. Students were asked to complete the assessment before coming to class. This could include completing the item anytime at least 23 hours in advance of the next class period.

Assessment

The self-assessment consisted of five main quantitative questions with a semi-Likert scale of three values. The five main questions included focus on materials, attention, sidekicks, timing and unassigned concepts. These five areas formed for each mathematics student what was their daily MASTU score. The use of the following general questions allowed students to self-assess their preparation for any course day on a scale 1-3. Where 1 was no, 2, yes 1 time, and 3, was two or more times.

- M-Materials: Did you read the materials for today's class meeting carefully?
- A-Attention: Did you come to class today with questions and thoughts ready to discuss the topic?
- S- Sidekick: Since we last met, did you talk with at least 1 or 2 classmates about the previously taught materials?
- T- Timing: Since our last class meeting, how much time have you spent reflecting on this course of study/recent class? (1) 20 uninterrupted minutes, (2) 40 uninterrupted minutes, (3) more than 60 uninterrupted minutes.
- U-Unassigned: Since our last meeting did you read any unassigned related to the course of study?

There was no grade connected to this assessment and students were guided to be honest with themselves. For example, discussions were held about whether students were submitted perfect scores of 15 each day, but were failing the course, that students' hours (sometimes called office

effort and performance were demonstrating. Further clarifying discussions were held about what uninterpreted time may mean. For example, if students worked on something for 60 minutes, but looked at or had their phone going off regularly, the uninterrupted time would be less than 20 likely and they should select 1.

Context and Data Collection

Students' courses typically met in the morning times with various assessments (variations of daily/weekly homework, quizzes, and tests) given throughout the course. In each course a summative exam was given about 1 time a month during the four-month long semester. In each course a final, cumulative exam was given. Throughout the course formative assessment was given at various depths of knowledge such as analyzing and creating tasks. In a three-hour course students could complete the assessment approximately 40 to 45 times, while a 5-hour course would complete the assessment approximately 60 to 75 times. The approximations allowed for students absent during school sanctioned events, illness, and a mental health day or two are allowed for in the course attendance policy.

Data Analysis

As this was a pilot study of the use of the MASTU assessment and a single instructor was using the assessment there was not a control group apart from this study. As such an analysis looked for trends in the overall involvement in the course and aligned with the research questions. Means and percentages were developed based on the number of entries that answered that scores divided by the total number entries for that course. These values were then multiplied by 100 to gain a percentage.

Results

Research Question 1: How much involvement are students having with a non-graded self- assessment?

The start of the answer to this research question can be noticed in Table 1.

Table 1. MASTU Scores by Course Type

Course Type	Number of students	Average Number of Self-Assessments	M- (3) (2) (1)	A- (3) (2) (1)	S- (3) (2) (1)	T- (3) (2) (1)	U- (3) (2) (1)
Mathematics Education Focused Content (MEFC)	110	25	15.7% 69.3% 14.9%	17% 66.3% 16.7%	20.8% 47.2% 32%	27% 60.4% 12.6%	10.7% 41% 48.3%
Mathematics Focused Content (MFC)	25	15	39.8% 49.4% 10.7%	39.5% 47.2% 13.3%	59.9% 19.2% 20.9%	40.1% 36.2% 23.7%	61.3% 26% 12.7%
Total	135						

Overall, it has been noticed that students who successfully complete the courses are engaged more consistently with the MASTU assessment. The MEFC has been completing the assessment for 4 semesters, and this is likely to account for the higher student numbers.

Research Question 2: What patterns are noticed about preparations patterns?

Patterns noticed aligned with the classroom shifts the instructor made during the courses. It was noticed over the course of the semester that students did start to, minimally, look at upcoming materials at least 1 time before the course would meet. Within the Attention section those in the MEFC course were coming with great questions that they were discussing in their building thinking classroom random groups. This also assisted in the improvement of reaching out to sidekicks as the semester progressed. What was interesting was the uninterrupted timing that students would spend on the course. Based on student feedback each semester, there was a high instance of "too much

time" or "work expected outside of class" kind of comments. This was interesting to the instructor as the systems in place allowed students to make multiple attempts on tasks before being assessed in a necessary grade way. Finally, in the unassigned category, it was curious to see the values adjust more as the semester progressed and it would be assumed that students were making decisions that aligned closer to university expectations rather than previous secondary expectations, because for many of these students this was their initial mathematics course at the university.

Research Question 3: What other questions or rating may be opportunities to assist educators in their own course preparations?

Based on these questions and answers, the instructor also considered adding in qualitative questions such as, "Is there anything else you would like to tell the instructor?" or a Likert scale that closer gave information about how students were viewing their time in tutoring sessions, attending student hours, or true working time with their sidekick. Furthermore, those in the MFC course may need more reminders and resources of using the MASTU over the MEFC. A possible reason for this may be their goal of also being an educator in the future and seeing more value in providing such information for a course instructor.

Discussion

Overall, this self-assessment provides an instructor with the opportunity to be even more flexible and informed decisions about class that day. This assessment allows the instructor to quickly discern if students feel they have come prepared for class. For example, if most students for a day are rating themselves as leaning toward scores that are in the lower range (under 10 points), implies students may not have come ready with previous practice and current readings completed, thus more thinly sliced questions tended to be better to use, rather than allowing larger jumps in the class, which aligns with Liljedahl (2021) thoughts on course questions for students.

References

Boaler, J., & Dweck, C. S. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching*. Jossey-Bass & Pfeiffer Imprints.

Brady, S. T., Reeves, S. L., Garcia, J., Purdie-Vaughns, V., Cook, J. E., Taborsky-Barba, S., Tomasetti, S., Davis, E. M., & Cohen, G. L. (2016). The psychology of the affirmed learner: Spontaneous self-affirmation in the face of stress. *Journal of Educational Psychology*, *108*(3), 353-373. https://doi.org/10.1037/edu0000091

Grant, A. M., & Sonnentag, S. (2010). Doing good buffers against feeling bad: Prosocial impact compensates for negative task and self-evaluations. *Organizational Behavior and Human Decision Processes*, *111*(1), 13-22. https://doi.org/10.1016/j.obhdp.2009.07.003

Griggs, M. S., Rimm-Kaufman, S. E., Merritt, E. G., & Patton, C. L. (2013). The Responsive Classroom approach and fifth grade students' math and science anxiety and self-efficacy. *School Psychology Quarterly*, *28*(4), 360-373. https://doi.org/10.1037/spq0000026

Leinwand, S., Brahier, D., Huinker, D., Berry, R., Dillion, F., Larson, M., Leiva, M., Martin, W., & Smith, M. (2014). *Principles to action: Ensuring mathematical success for all*. NCTM.

Leinwand, S., Brahier, D., Huinker, D., Berry, R., Dillion, F., Larson, M., Leiva, M., Martin, W., & Smith, M. (2024). *Principles to action: 10th Anniversary*. NCTM.

Liljedahl, P. (2016). Building thinking classrooms: Conditions for problem-solving. In *Posing and solving mathematical problems* (pp. 361-386). Springer.

Liljedahl, P. (2018). Building thinking classrooms. In *Teaching and Learning Secondary School Mathematics* (pp. 307-316). Springer.

Liljedahl, P. (2021). *Building thinking classrooms in mathematics, grades K-12: 14 teaching practices for enhancing learning*. Corwin Press.

Sandra, L. L., Marja-Liisa, H., Marina, K., & Timothy, J. W. (2014). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study. *Journal for research in Mathematics Education*, *45*(4), 406-418. https://doi.org/10.5951/jresematheduc.45.4.0406

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313-340.https://doi.org/10.1080/10986060802229675

Topic Study Groups 2
(Day 4)

TSG2-1-13

ANALYSIS OF TYPES OF DEALING WITH THE CONTEXT OF TASK IN SIXTH GRADE STUDENTS' MATHEMATICAL MODELING

Nahee Kim
Seoul National University of Education, Republic of Korea, henakim1107@gmail.com

Hyewon Chang
Seoul National University of Education, Republic of Korea, hwchang@snue.ac.kr

The primary goal of mathematics education is to help students develop into the integrating type in their approaches to deal with the context of task, as mathematical modeling requires integrating realistic and mathematical aspects to solve problems. For teachers to identify each student's type and provide appropriate scaffolding, it is essential to understand the characteristics and developmental factors of these types. Thus, this study analyzed the characteristics, patterns, and factors influencing changes in types of dealing with the context of task in sixth-grade students' mathematical modeling. The results revealed that group modeling activities resulted in a higher proportion of integrating type compared to individual activities. Furthermore, when students solved the main task again after the group activity, the integrating type has become the most common and the number of students in this category increased significantly compared to the first attempt. Based on the analysis, this study provides pedagogical implications for enhancing the implementation of mathematical modeling in education.

Keywords: mathematical modeling, context of task, integrating type, mathematics-bound type, reality-bound type, ambivalent type

Background of the Study

Mathematical Modeling

Although there are various definitions of mathematical modeling, there is a strong consensus that it involves transformations between real-world context and mathematics (Borromeo Ferri, 2014). Among different perspectives on modeling, the realistic perspective focuses on solving real-world problems, emphasizing that students should engage with authentic, realistic, and complex tasks (Abassian et al., 2019). Kaiser & Sriraman (2006) argued that authentic tasks allow students to go through the entire mathematical modeling cycle, rather than focusing on specific stages. Through participating in authentic tasks, students can develop the ability to solve real-world problems. Considering prior research, this study defines mathematical modeling as the entire process of constructing a mathematical model and solving problems that arise from real-life situations.

Four Ideal Types of Dealing with the Context of Realistic tasks

Busse (2001, 2005) highlighted the importance of situational contexts in tasks, stating that students may interpret these contexts differently and that individuals' approaches to dealing with the context can vary. Busse (2005) categorized the way people deal with the context of task into four ideal types. Ambivalent type people perceive real-world problems from both mathematical and realistic

perspectives but fail to integrate them into a coherent whole. People who are reality-bound type use non-mathematical concepts and methods but do not mathematize the problem or apply mathematical methods. Mathematics-bound type people rely solely on quantitative information presented in the task without considering personal or contextual knowledge and experiences. People who are integrating type utilize their personal knowledge and experiences to interpret the context and apply mathematical methods when they construct and verify mathematical models. Students' perceptions of task contexts can change or deepen throughout the problem-solving process (Busse, 2005). Additionally, while individuals may exhibit different types depending on the task, they tend to have a preferred type of dealing with the context of task (Busse, 2011). This preference can change and may develop into an integrating type over time (Chang, 2023; Vos & Roorda, 2017).

Since mathematical modeling involves interactions between reality and mathematics, students need to integrate both realistic and mathematical perspectives to solve the problem (Borromeo Ferri, 2018). Consequently, the goal of mathematics education in mathematical modeling is to help students develop into the integrating type. To enable teachers to identify each student's types and provide appropriate scaffolding, it is necessary to understand the characteristics and developmental factors of types. Therefore, this study aims to analyze the patterns and characteristics of types of dealing with the context exhibited by students in individual mathematical modeling activities, as well as the patterns and influencing factors of group-level types. Furthermore, it seeks to explore the impact of group mathematical modeling activities on individuals' types and the potential for their development into the integrating type through these experiences.

Method

Subjects

The participants of this study were twenty-one sixth-grade students (11-12 years old) from one school in Gyeonggi Province, South Korea, where one of researchers serves as the homeroom teacher. These students had prior experience participating in one mathematical modeling activity with their teacher.

Tasks

Students solve the main task independently following the modeling process outlined in their worksheets, without receiving assistance from the teacher or peers. One week later, students work on the isomorphic task in groups of three to four members. Each group was a heterogeneous group composed of students with different types as identified through their individual modeling activity results. Two weeks after the group work, they independently solved the main task again without additional explanations from the teacher. Table 1 shows mathematical modeling tasks given to students.

Table 1. Mathematical modeling tasks given to students

Task Type	Description
Main Task	Students estimate the amount of paper waste discarded in their class over a week, using a photo of a full paper recycling bin as a reference.
Isomorphic task	Using a school layout diagram and a photo of a paper recycling bin, students calculate the amount of paper waste generated at the entire school level and propose an appropriate size for a new recycling bin.

Data Collection and Analysis

The collected data include individual and group worksheets, video recordings of group activities, researcher's observation notes, and student interviews. To analyze student's types of dealing with the context, this study adopted Busse's framework (Busse, 2005). The presence of mathematical (M) and realistic (R) elements in students' responses was coded. Table 2 shows examples that were coded into M or R. A student or group was classified as mathematics-bound type if only mathematical factor was present. On the other hand, it was classified as reality-bound type if only realistic factor was present. If both M and R were present, the classification depended on their interaction. A student or group was categorized as integrating type when the two factors worked together to derive a conclusion. On the other hand, if the two factors functioned independently without influencing each other, the student or group was categorized as ambivalent type.

Table 2. Examples of realistic and mathematical factors

Factors	Examples
Realistic Factors (R)	- Types and sizes of discarded paper - Weekly class schedule and activities - Number of students and paper usage habits - Changes when compressing paper waste
Mathematical Factors (M)	- Volume calculation of the recycling bin or A4 paper - Ratio and proportion application: expressing the filled portion of the recycling bin as a proportion or adjusting for differences in class schedules or the number of students per classroom - Unit conversion and density considerations: using the thickness of a single sheet of paper to calculate the total amount of discarded paper or applying the density (weight) of paper to convert unit

Results

First Individual Students' Types of Dealing with the Context of Task

The results of the individual mathematical modeling activity revealed that among the twenty-one students, their types were distributed as mathematics-bound (10 students) > integrating (4 students) = ambivalent (4 students) > reality-bound (3 students). Students categorized as mathematics-bound used only mathematical methods while reality-bound students estimated the amount of paper waste through hand-on experience or observation. Integrating type students constructed models by incorporating realistic and mathematical elements. Students classified as ambivalent attempted to solve the problem mathematically but relied solely on their observations to draw conclusions.

Group-Level Types of Dealing with the Context of Task

The group-level analysis revealed that, among the six groups, four groups were categorized as integrating type, while one group belonged to the mathematics-bound type (Group 5) and another to the ambivalent type (Group 6). Groups categorized as integrating type successfully combined different approaches or cross-checked their methods, allowing them to synthesize both realistic and mathematical elements effectively. However, Group 5 focused excessively on mathematization while Group 6 relied on a single method, leading them to rely on approximation methods to estimate the required size of a new recycling bin.

Second Individual Students' Types of Dealing with the Context of the Task

A follow-up individual modeling activity conducted one month later revealed the distribution of types as integrating (11 students) > mathematics-bound (7 students) > reality-bound (2 students) > ambivalent (1 student). This distribution differed from the initial modeling activity. Among the twenty-one students, nine exhibited changes in their types. Seven students transitioned to integrating type and two students shifted from reality-bound or ambivalent to mathematics-bound type. According to student interviews, all students whose types changed attributed this shift to group activities, leading them to recognize the need to adjust volume estimations for the recycling bin or to be able to enhance their understanding of mathematical knowledge needed to solve the problem. One student(S3), who remained ambivalent, showed his consideration of both realistic and mathematical factors had expanded compared to his initial work.

Conclusions

The findings of this study lead to the following conclusions:

First, students exhibit diverse types when they deal with the context of the same task. The analysis of students' types confirmed the presence of all four types proposed by Busse (2005). However, unlike Busse's findings, the reality-bound type was the least common. Considering that most students attempted mathematization, it can be inferred that students tend to use mathematization as logical justification rather than relying solely on realistic elements when solving contextualized problems.

Secondly, group activities enhance the integration of realistic and mathematical elements. This is because students with different types engaged in discussions, leading them to consider both aspects comprehensively. Meanwhile, when students tried to minimize conflict by simply following one group member's approach or using arbitrary values, they tended to focus on either the realistic or the mathematical aspect, rather than integrating both. Therefore, teachers need to intervene during group activities by asking guiding questions or providing feedback that encourages students to consider both realistic and mathematical contexts in a balanced and productive way (Levenson, 2011).

Lastly, types of dealing with the context are dynamic and can evolve over time. Even when solving the same task, nine students exhibited changes in their types. Prior research (Busse, 2005; Vos & Roorda, 2017) has also suggested that these types can develop over time. This study further confirms that group activities significantly contribute to changes. Even for students whose types remained the same, their consideration of realistic factors expanded and tendency to apply mathematization increased. These highlight the positive impact of group activities on students' modeling competence.

References

Abassian, A., Safi, F., Bush, S. & Bostic, J. (2019). Five different perspectives on mathematical modeling in mathematics education. *Investigations in Mathematics Learning, 12(*1), 53–65.

Borromeo Ferri, R. (2014). Mathematical modeling-The teacher's responsibility. *Journal of Mathematics Education at Teachers College.*

Borromeo Ferri, R. (2018). *Learning how to teach mathematical modeling in school and teacher education* (pp. 121-133). Springer International Publishing

Busse, A. (2001). How do upper secondary school students respond to contextualised tasks? First results of an empirical study. In G. Törner et al. (Eds.), *Developments in mathematics education in German-speaking countries. Selected papers from the annual conference on didactics of mathematics, Ludwigsburg,* (pp. 19-28).

Busse, A. (2005). Individual ways of dealing with the context of realistic tasks—First steps towards a typology. *The International Journal on Mathematics Education, 37*(5), 354-360.

Busse, A. (2011). Upper secondary students' handling of real-world contexts. In G. Kaiser, W. Blum, R. Borromeo Ferri & G. Stillman (Eds.) *Trends in teaching and learning of mathematical modelling* (pp. 37-46). Springer.

Chang, H. (2023). An analysis of types of dealing with the context of task in mathematically gifted students' mathematical modelling. *Journal of Elementary Mathematics Education in Korea, 27*(4), 399-416.

Kaiser, G. & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *ZDM, 38*, 302-310.

Levenson, E. (2011). Exploring collective mathematical creativity in elementary school. *The Journal of Creative Behavior, 45*(3), 215-234.

Vos, P. & Roorda, G. (2017). Long-term development of how students interpret a model; Complementarity of contexts and mathematics. In G. A. Stillman, W. Blum, & G. Kaiser (Eds.), *Mathematical modelling and applications: Crossing and researching boundaries in mathematics education* (pp. 479-489). Springer.

TSG2-1-14

DESIGNING MODELLING TASKS TO ENHANCE STATISTICAL THINKING: INTEGRATING STUDENTS' EXPERIENCES OF HITTING THE LITTLE TOE WITH SCIENTIFIC PROBLEMS

Daiki Fujiwara
Naruto University of Education, Japan, dfujiwara@naruto-u.ac.jp

Satoshi Kusaka
Naruto University of Education, Japan, kusaka@ksm.world

Akio Matsuzaki
Saitama University, Japan, makio@mail.saitama-u.ac.jp

Shinichiro Matsumoto
Shizuoka University, Japan, matsugen@shizuoka.ac.jp

Toshikazu Ikeda
Yokohama National University, Japan, toshi@ynu.ac.jp

This study introduces the "Little Toe Gap" task, which connects real-world experiences with statistical concepts to explore whether students can generate scientific problems through statistical modelling. Two lesson studies were conducted in middle schools, differing in the level of freedom given to students in problem generation. Lesson I focused on statistical modelling and problem generation within a structured framework, while Lesson II emphasized planning the experiment and encouraged critical thinking with more freedom for problem generation. Students analyzed data using histograms, mean, median, mode, and relative frequency, drawing probabilistic conclusions. They also generated new scientific problems, demonstrating increased engagement and statistical reasoning. A key strength of the task is its ability to draw on students' informal knowledge, fostering statistical thinking.

Keywords: task design, statistical modelling, scientific problems, statistical thinking

Background

Task design is a key research area in mathematics education, explored from various perspectives, including mathematical modelling and statistics education (Watson & Ohtani, 2015; Kuntze et al., 2017; Barquero & Jessen, 2020). For teachers, designing tasks that align with students' real-world experiences while maintaining curriculum coherence is essential (Blum & Niss, 1991; Niss & Blum, 2020). Recently, statistics education has been reinterpreted from a modelling perspective (Pfannkuch et al., 2018), and research has emphasized task design that fosters both critical and statistical thinking through real-world connections (Kuntze et al., 2017). While studies highlight the need for embedded scientific problems (Gravemeijer et al., 2017), such tasks remain scarce. If the tasks for statistical modelling (Pfannkuch et al., 2018) can support problem generation, they may enhance statistical thinking (Garfield, 2011) and strengthen STEM/STEAM education.

Research Objective and Research Questions

The objective of this study is to design statistical modelling tasks that allow students to generate scientific problems and to clarify their effectiveness. To achieve this goal, the following two research questions (RQs) have been set: **RQ1:** What types of statistical modelling tasks are designed, and how are these tasks implemented to engage students with statistical contents in the classroom? **RQ2:** How do students generate scientific problems, and how does the level of freedom in problem generation influence their problem generation process?

Research Methodology

For RQ1, previous studies from outside math. informed the design of the statistical modelling tasks, which were implemented to engage students with statistical content and promote problem generation. For RQ2, two lessons were conducted, each incorporating the tasks with varying levels of freedom in problem generation. The lessons were recorded, and the resulting data and behaviors were analyzed to explore students' engagement with statistical activities and problem generation.

Results and Discussion

Task Development

Contextual knowledge is crucial for developing students' statistical thinking (Wild & Pfannkuch, 1999). A common example is accidentally hitting one's little toe, particularly among individuals who live indoors without shoes, such as in East Asia. In Japan, Kobayashi et al. (2007) investigated this by measuring the gap between a person's imagined and actual foot position, revealing a lateral discrepancy of about 10–15 mm. A television program later explained that people hit their toe because they walk about 1 cm further outward than they perceive. Building on this idea, we developed the 'Little Toe Gap' task to examine whether middle school students could generate scientific problems through statistical modelling. We hypothesized that students, reflecting on their experiences, could generate similar problems, such as 'Is there a difference in gap distance between the left and right foot?'. We conducted two lesson studies: In Lesson I, students are required to generate new problems in situations where the collected data is categorized limitedly. In Lesson II, students are required to generate new problems more freely, regardless of the data. Lesson I requires students to find conclusions about the new problems based on collected data, Whereas Lesson II doesn't. Unlike Kobayashi et al. (2007), we used a straight line on the floor as a reference and recorded gap distances as positive (outward) or negative (inward).

Overview of Lesson I and Collected Data

Lesson I was conducted over four sessions from January to February 2019 at a middle school (School A) in Japan, involving 108 grade 7 students across three classes. The entire class shared their experiences of hitting their little toe. The teacher then asked, 'Why does this happen?', introducing Kobayashi et al. (2007) and the TV program's explanation before presenting the task and experimental method. Guided by the teacher, students conducted the experiment using a 'smooth straight line' on a tennis court as the reference line and collected quantitative data (unit: cm). The results included Mean: 1.2 cm, Median: 1.2 cm, Mode: 13.5 cm, Interquartile Range: 3.0 cm, and Standard Deviation: 2.5 cm. For deeper analysis, data were categorized by dominant foot, right or left foot, and other attributes. Using the statistical learning software 'stathist' (developed by fourth author), stu-

dents analyzed their data, reported findings in PowerPoint, engaged in peer discussions, generated new problems, and conducted stratified analyses to draw further conclusions.

Student Conclusions and Model Creation in Lesson I

The use of statistical software enabled students to conduct smooth analyses and reach conclusions. Among them, 80.6% concluded that the theory was 'correct,' while 19.4% found it 'not necessarily correct' or 'incorrect'. Rather than a deterministic statement, some generated probabilistic conclusions such as 'Many people walk 1 cm further outward'. Regarding statistical models, 100% created histograms, 84.7% used the mean, 80.6% used the median, 59.2% used the mode, and 38.8% used relative frequencies. Other models were also observed. For instance, one student noted that the histogram showed the most frequent intervals were -0.5 cm to 1 cm and 1 cm to 2.5 cm, supporting the hypothesis. They also cited the total relative frequency (exceeding 0.536), the mean (1.18 cm), and the median (1.2 cm) as evidence aligning with the theory.

Generation of New Problems by Students in Lesson I

All students generated one or two of following four scientific problems, and most of these problems ware related to the experimental factors, such as 'Is there a difference in gap distance depending on foot dominance? or on gender?'. These problems were often based on personal experiences, as illustrated by one student's comment: 'I often hit my non-dominant foot'. Some students generated problems detached from personal experiences, such as 'How far is each foot from the reference line?'. Most students successfully derived conclusions for their newly created problems within the allotted time. Additionally, reflections such as 'Do trends differ by generation?', and 'Since we hit our toes mid-air, we should conduct the experiment with moving feet' indicated an increased awareness of problem generation and critical thinking.

Overview of Lesson II

Lesson II was conducted in December 2020 over two sessions at another middle school (School B), targeting a single grade 7 class of 35 students. The teacher introduced the task similarly to Lesson I. However, in this case, students planned the experiment themselves before conducting it. Since the floor of the experiment site had multiple lines, the variation in the collected data was larger than in Lesson I, with the center tendency slightly shifting toward higher class intervals. Students organized their conclusions on posters. Many reflections connected the experiment to daily life, such as 'From now on, I will try to walk 2 cm further inward'.

Student Planning of the Experiment in Lesson II

After the task was introduced, a discussion between the teacher and students led to various refinements in the experimental method. Some suggested concealing the feet to prevent visual bias, while others proposed representing displacement numerically. This dialogue helped finalize the method, aligning with Lesson I. Matsuzaki (2020) noted that this discussion prepared students for generating models and problems, supporting the gradual process of mathematization.

Student Conclusions and Critical Thinking in Lesson II

Among the nine student groups, three concluded that the hypothesis was correct, while six found it incorrect or not necessarily correct. One group questioned data reliability, suggesting misinterpretations of the straight lines may have affected results. Matsumoto (2020) observed that during the poster creation phase in Lesson II, students engaged in the interrogative cycle involving criticizing

(Wild & Pfannkuch, 1999). Some posters included statements such as 'Two-fifths of participants fall within a specific class interval' and '73.5% walked with their feet positioned outward', demonstrating the use of relative frequency despite no prior formal instruction. Matsumoto (2020) suggested that this task could support instructional units by building on students' informal statistical understanding to foster new knowledge.

Generation of New Student Problems in Lesson II

After discussing their conclusions, the teacher asked students if there were other aspects they wanted to investigate. In response, Students generated nine scientific problems, most of which were not related to the collected data. Students said: What happens in everyday life, not in an experiment? How do our toes collide with things? Why do our toes sometimes collide and sometimes not? What is the positional difference between the right and left foot? What if we used your big toe? How do our feet move when turning a corner? Is there a difference in awareness when they collide and when they don't? How can we avoid colliding? What would happen if we conducted an experiment while walking?

Matsuzaki (2020) interpreted the teacher's problem as prompting students to compare their findings with the original context, fostering deeper engagement. Moreover, all nine problems were rooted in real-world experiences, aligning with the concept of mental representation of the situation (MRS) (Borromeo Ferri, 2006).

Conclusion

This study demonstrated that the "Little Toe Gap" task effectively integrates statistical modelling with scientific inquiry. The task was implemented in two lessons, revealing the impact of varying levels of freedom in problem generation. In Lesson I, students followed a more structured approach, generating problems based on the experimental data and using statistical models like histograms, mean, median, and mode. This approach helped students connect real-world experiences to statistical concepts and fostered foundational statistical reasoning. In contrast, Lesson II allowed students greater freedom to generate problems unrelated to the data, encouraging creative thinking and broader scientific inquiry. The increased freedom led to a wider variety of student-generated problems, promoting critical thinking and deeper engagement with the topic. These differences suggest that higher freedom in problem generation stimulates creativity and critical thinking, while a more structured approach strengthens statistical analysis skills. Both approaches have complementary strengths, with Lesson I building statistical foundations and Lesson II fostering broader scientific inquiry. The task effectively taps into students' informal knowledge, supporting both statistical reasoning and scientific inquiry skills. Challenges remain in ensuring accurate data collection in uncontrolled environments, but future studies can explore the impact of varying levels of freedom on learning, potentially using other everyday experiences.

References

Barquero, B. & Jessen, B. E. (2020). Impact of theoretical perspectives on the design of mathematical modelling tasks. *Avances de Investigación en Educación Matemática, 17*, 98-113.

Blum, W. & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects - state, trends and issues in mathematics instruction. *Educational Studies in Mathematics, 22(1)*, 37-68.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process, *ZDM, 38(2)*, 86-95.

Garfield, J. (2011). Statistical literacy, Reasoning, and thinking. In Lovric, M. (Eds.), *International Encyclopedia of Statistical Science* (pp.1439-1442). Springer Publishers.

Gravemeijer, K., Stephan, M., Julie, C., Lin, F. & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future?. *International Journal of Science and Mathematics Education, 15*, 105-123.

Fujiwara, D. (2020). Each aspect of mathematics lesson study in lower secondary school based on modelling cycle and PPDAC cycle. *Proceedings of the 8th Spring Research Conference*, 179-186. (in Japanese).

Kobayashi, Y., Mine, Y. & Fujimoto, H. (2007). The study about position sense of human feet. *Transactions of the Japan Society of Mechanical Engineers, C, 73(725)*, 274-279 (in Japanese).

Kuntze, S., Aizikovitsh-Udi, E. & Clarke, D. (2017). Hybrid task design connecting learning opportunities related to critical thinking and statistical thinking. *ZDM, 49*, 923-935.

Matsumoto, M. (2020). A Consideration on mathematics lesson study in lower secondary school based on PPDAC cycle. *Proceedings of the 8th Spring Research Conference*, 195-202. (in Japanese).

Matsuzaki, A (2020). A Consideration on mathematics lesson study in lower secondary school based on modelling cycle: Focus on planned lesson and implemented lesson of 'Little Toe Gap'. *Proceedings of the 8th Spring Research Conference*, 187-194. (in Japanese).

Niss, M. and Blum, W（2020). *The learning and teaching of mathematical modelling*. Outledge.

Pfannkuch, Ben-Zvi, D. & Budgett, S. (2018). Innovations in statistical modelling to connect data, chance and context. *ZDM, 50,* 1113-1123.

Watson, A. & Ohtani, M. (Eds.) (2015). *Task design in mathematics education: An ICMI study 22*. Springer.

Wild, C. J. & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review, 67 (3),* 223-248.

TSG2-1-15

A PROBABILITY MODELING APPROACH TO PROMOTE STUDENTS' CONNECTIONS ACROSS THREE PERSPECTIVES OF PROBABILITY

Minh Kiem Tran
Hue University of Education, Vietnam, tkminh@hueuni.edu.vn

Sinh Thi Nguyen
The University of Da Nang, Vietnam, ntsinh@ued.udn.vn

Tien Huu Nhanh Nguyen
Hue University of Education, Vietnam.

Thanh Ngoc Nguyen
Hue University of Education, Vietnam, thanhbinh11569@gmail.com

Coordinating different perspectives in teaching and learning probability is essential for fostering students' deeper understanding of probability. This paper presents a probability modeling approach that integrates the subjective, frequentist, and classical perspectives, treating each as a distinct probability model embedded within a workspace. Drawing on epistemological and curricular analyses, we designed a learning situation that allows students to engage with and make connections among these models. These connections were analyzed using the theory of mathematical working spaces. Initial findings indicate that students successfully formed meaningful connections, enhancing their understanding of probability. This study contributes to existing probability modeling approaches by emphasizing the articulation and integration of three probability perspectives in school probability education.

Keywords: probability modeling, probability model, mathematical working spaces, connected working spaces.

Introduction

Probability is traditionally understood through three key perspectives. The classical perspective (theoretical probability) defines probability as the ratio of favorable outcomes to the total number of possible outcomes in an experiment. The frequentist perspective (experimental probability) views probability as a stable frequency that emerges when a random experiment is repeated independently under the same conditions, converging toward a theoretical limit. The subjective perspective (subjective probability) interprets probability as an individual's degree of confidence in an event, which evolves with empirical data and leads to conditional probability.

Understanding probability requires connecting these perspectives, particularly the classical and frequentist views. Studies suggest that probability modeling can help students connect these perspectives, particularly the classical and frequentist approaches (Kazak & Pratt, 2021; Park & Kim, 2023). Many mathematics education researchers advocate teaching probability as a modeling tool, using technology-based simulations to enhance student engagement (Kazak & Pratt, 2021; Pro-

dromou, 2014). To contribute to this research direction, our study adopts a modeling approach to probability, incorporating the concept of connected working spaces (Minh & Lagrange, 2016). We conceptualize each probability perspective as a model embedded within a mathematical working space (Kuzniak, 2022). The study aims to (a) design learning situations that promote students' coordination of the theoretical, frequentist, and subjective perspectives of probability and (b) analyze how students connect different probabilistic working spaces in their mathematical reasoning.

Theoretical Background

Probability Modeling

According to Park and Kim (2023), probability modeling involves estimating the true probability of an event by constructing a model for the probabilistic situation. Probability modeling based on theoretical probability derives from the inherent nature of events and involves modeling situations using sample spaces. Probability modeling based on experimental probability involves generating data through repeated random experiments to organize and interpret statistical information.

In this study, we adopt Park and Kim's (2023) perspective on probability modeling and extend it by incorporating probability modeling based on subjective probability, which is understood as assigning individual beliefs to each elementary event within a given sample space. We integrate these three views of probability modeling into probability learning situations to foster students' development of modern scientific thinking. This process includes making a priori predictions using subjective probability modeling, exploring and analyzing frequency data through experimental probability modeling to inform decision-making, and ultimately verifying or calculating probabilities with theoretical probability modeling.

Connected Working Spaces

A mathematical working space (MWS) is an abstract framework designed to support mathematical work within a specific domain (Kuzniak et al., 2016; Kuzniak, 2022). Kuzniak introduced the concept of MWS to highlight the mathematical work proposed by educational institutions and to describe how individuals engage with mathematical tasks in educational settings. The MWS theory has been widely applied in research, including studies on the interaction between multiple WS. In this study, we adopt Minh and Lagrange's (2016) idea of connected working spaces to explore the articulation and integration of three probabilistic working spaces: frequentist, theoretical, and subjective probability. Mathematical work within a given workspace is characterized by three dimensions: semiotic, instrumental, and discursive. The semiotic dimension involves the use of symbols, graphics, and concrete objects as signs. The instrumental dimension pertains to the use of artifacts for constructing, experimenting, and problem-solving. The discursive dimension relates to reasoning and proving using theorems and mathematical rules. By embedding multiple probability models within distinct WS, we facilitate connections between them. This approach supports our hypothesis that linking the three probabilistic WS enables students to deepen their probabilistic knowledge by interacting with different models and gaining a more comprehensive understanding of probability.

Methods

Our teaching experiment involved two twelfth-grade classes from different high schools—one in Hue City and one in Da Nang City, Vietnam. Each class comprised 30 students. We conducted four classroom observation sessions per class between December 2024 and January 2025. These ses-

sions aimed to enhance students' understanding of probability concepts, relative frequency estimation, and conditional probability through learning situations that integrated the three perspectives on probability with the support of Microsoft Excel spreadsheets.

Through epistemological and institutional analyses, we identified three probability models embedded in distinct probabilistic working spaces. Based on these, we designed four learning situations to promote articulation and connection among the workspaces. In each experimental session, students engaged with one learning situation. After that, at each high school, we conducted task-based interviews with three selected student pairs. Below is the learning situation presented in the third experimental session.

We have two fair four-sided dice with each die's four vertices numbered from 1 to 4. Consider the following random experiment: Rolling both dice simultaneously and observing the sum of two numbers on the two top-facing vertices.

1) Three probability models (three predictions) have been proposed for this experiment:

Sum	2	3	4	5	6	7	8
Model 1	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
Model 2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
Model 3	0.05	0.15	0.2	0.2	0.2	0.15	0.05

Based on your intuition, which probability model best fits the above random experiment?

2) Use the RANDBETWEEN(1,4) function in Excel to simulate this random experiment 500 times. Then, use the COUNTIF() function to count the number of times each sum appears and calculate the relative frequencies of sums "2", "3", "4", "5", "6", "7", and "8". Determine which probability model best fits the experiment based on the observed relative frequencies, and explain your reasoning.

3) Find the theoretical probabilities for each possible sum in this experiment. Compare these theoretical results with the probability model you selected in Task 2.

With this learning situation, we expect students to engage with each probabilistic working space and make connections among these workspaces to develop a deeper understanding of probability.

Findings

Due to space limitations, we analyze only one case (the student pair Hu-Ca) to illustrate the connections between the three probability models and, in doing so, highlight the linkages among the three workspaces.

Connections Between the Experimental Probability WS and the Subjective Probability WS

The student pair Hu-Ca intuitively rejected Model 1, believing the outcomes of this random experi-

ment could not have equal probabilities. After running 500 simulations in Excel, they calculated the relative frequencies of each sum, which led them to select Model 2 correctly. Working with Excel provided them with empirical evidence, bridging the frequentist and subjective models. From the perspective of connected working spaces, this also linked the experimental and subjective probability workspaces. In this process, the instrumental dimension (Excel use) and semiotic dimension (numerical symbols, formulas) were mobilized in the students' mathematical work.

Connections Between the Experimental Probability WS and the Fheoretical Probability WS

For Task 3, the student pair Hu-Ca applied the multiplication rule for independent events and the addition rule for probabilities to determine the probability of each sum, confirming the validity of Model 2. The connection between the frequentist and classical models is evident in the following interview excerpt:

> *Interviewer:* Based on your experience with Excel in this learning situation, how would you distinguish between relative frequency and probability? *Hu-Ca:* Relative frequency is the number of times an event occurs divided by the total number of trials, whereas probability is a fixed value. *Interviewer:* When do we need to conduct an experiment to determine these values?
>
> *Hu-Ca:* Relative frequency requires an experiment or existing data, while probability can be calculated theoretically using paper and pencil. *Interviewer:* What is the relationship between relative frequency and probability? *Hu-Ca:* If the number of simulations is large enough, relative frequency approximates probability. *Interviewer:* Did you observe this in your 500 simulations with Excel? *Hu-Ca:* Yes.

This exchange demonstrates that the students connected relative frequencies of outcomes, obtained through Excel simulations, with theoretical probabilities from a classical perspective. Running 500 simulations helped them recognize how relative frequency converges toward theoretical probability as the number of trials increases. These findings highlight the linkages between the experimental probability workspace and the theoretical probability workspace in their problem-solving process. The instrumental dimension (Excel use) and the discursive dimension (reasoning, application of probability rules) were both essential in supporting their learning.

Discussion and Conclusion

In this paper, we present an approach to probability modeling that distinguishes students' activities across three different probability models and emphasizes the connections among these models. This modeling approach is inspired by a new modeling perspective proposed by Lagrange (2021), which highlights the plurality of models and the organization of work that enables students to engage with them. By adopting the perspective of connected working spaces (Minh & Lagrange, 2016), we analyzed the mathematical work of a representative student pair and identified how they created connections among the three probabilistic working spaces. These connections reflect the benefits of our probability modeling approach in fostering students' articulation across three probability perspectives, leading to a deeper understanding of probability. Additionally, this study expands on Park and Kim's (2023) work by incorporating the subjective perspective into learning situations, alongside the theoretical and frequentist perspectives, further enriching students' conceptual engagement with probability.

References

Kazak, S., & Pratt, D. (2021). Developing the role of modelling in the teaching and learning of probability. *Research in Mathematics Education*, *23*(2), 113-133.

Kuzniak, A. (2022). The Theory of Mathematical Working Spaces - Theoretical Characteristics. In Kuzniak, A., Montoya-Delgadillo, E., Richard, P.R. (Eds.), *Mathematical Work in Educational Context. Mathematics Education in the Digital Era*, vol 18, (pp. 3-31). Springer.

Kuzniak, A., Tanguay, D. & Elia, I. (2016). Mathematical Working Spaces in schooling: an introduction. *ZDM - Mathematics Education, 48*, 721-737.

Lagrange, J. B. (2021). Les espaces de travail connectés: une perspective nouvelle pour la modélisation dans le secondaire? *Canadian Journal of Science, Mathematics and Technology Education, 21*, 9-28.

Minh, T. K., & Lagrange, J. B. (2016). Connected functional working spaces: a framework for the teaching and learning of functions at upper secondary level. *ZDM - Mathematics Education, 48*, 793-807.

Park, J., & Kim, D. W. (2023). Facilitating factors and obstacles in the coordination of theoretical probability and relative frequency estimates of probability. *Educational Studies in Mathematics, 114*, 439–460.

Prodromou, T. (2014). Developing a Modelling Approach to Probability Using Computer-Based Simulations. In Chernoff, E., Sriraman, B. (Eds.), *Probabilistic Thinking. Advances in Mathematics Education* (pp. 417- 439). Springer.

TSG2-1-16

MATHMATICS CLASSROOM MANAGEMENT IN EL SALVADOR ELEMENTARY SCHOOLS AND ITS RELATIONSHIP WITH STUDENTS' ATTITUDE AND ACADEMIC PERFORMANCE

Félix Abraham Guevara Menjívar
Naruto University of Education, Japan, felixabrahamguevara@gmail.com

Hiroki Ishizaka
Naruto University of Education, Japan, hishizaka@naruto-u.ac.jp

Since 2018 the Ministry of Education of El Salvador (MoE) has implemented a mathematics learning strategy in all levels. After implementing this strategy during 2018-2023, there is evidence about problems with students' academic achievement and with classroom management. This study aims to determine classroom management styles applied by teachers and analyze its relationship with students' attitude and academic performance. The analysis was based on a sample of 20 primary schools in San Salvador, the capital of El Salvador, specifically in fifth grade. Classroom Management Styles Assessment Protocol, Attitude Towards Mathematics Learning and Mathematics test were applied to collect the data. The findings revealed a tendency to apply a Non-interventionist and Interventionist style by teachers, but significant differences regarding students' academic performance were found above all with Interactionist style and the combination between latter and Interventionist style. No statistical evidence was found to suggest a relationship between the classroom management style employed by teachers and students' attitudes toward learning mathematics.

Keywords: Mathematics learning, classroom management, elementary school, academic performance

Background

In order to improve students' mathematics learning, MoE carried out a project together with Japan International Cooperation Agency, called ESMATE (MoE, 2022). This strategy is based on three components: the effective use of textbooks, designed with a problem-solving approach, the role of teachers as learning facilitators and the active learning by students (MoE, 2019).

Although the initial implementation of ESMATE reported positive results in students' learning (Maruyama & Kurosaki, 2021), recent research has revealed ongoing challenges with students' learning (UNESCO, 2021; Mejía & Ishizaka, 2021; MoE, 2022). According to UNESCO (2021), in Regional Comparative and Explanatory Study 2019 (ERCE), 50.1% of 3rd grade Salvadoran students scored above the Minimum Proficiency Level (MPL), which is similar to regional average of 52.3%. However, in 6th grade, only 6.6% exceeded the MPL, far below the regional 17.4%.

The MoE (2022) states that ESMATE strategy has not been sufficient to address the learning crisis in mathematics. The 2022 report on visits to 133 schools by the MoE (2023) reveals that only 45.1% of students solve mathematics problems independently, and 78.1% of teachers spend less than five minutes monitoring student work. These findings show serious issues related to classroom management. Because there is limited evidence about it, further classroom-level research is necessary (Mejía & Ishizaka, 2021). This study analyzed the following research questions:

1. What classroom management style is applied by mathematics teachers in elementary schools?
2. What is the relationship between teachers' classroom management style and the students' attitude towards the learning of mathematics?
3. What is the relationship between teachers' classroom management and students' academic performance in mathematics?

Literature Review

Classroom Management Styles and Dimensions

Classroom management, introduced in 1961, focuses on the teacher's role in addressing classroom situations (Brown & Phelps, as cited in Vairamidou & Stravacou, 2019). Gordana and Snezana (2012) define it as creating a conducive environment, while Sieber (2015) emphasizes its aim to help students reach their potential. Leadership styles include Interventionist (teacher-centered, strict control) (Gordana & Snezana, 2012, citing Walfgang & Glickman, 1980), Non-interventionist (learner-centered, minimal control) (Gordana & Snezana, 2012), and Interactionist (teacher-student collaboration) (Nchelem & Gladys, 2020). According to Gordana and Snezana (2012), citing Martin and Baldwin (1993), classroom management comprises three dimensions: personality (teachers' abilities and attitudes shaping classroom climate), teaching (organizing effective learning environments), and discipline (enforcing behavioral standards).

Attitude Towards Mathematics Learning

It includes liking or disliking mathematics, a tendency to engage with or avoid it, and beliefs about one's ability or its usefulness (Boz & Çalışkan, 2018). Tapia & Marsh (2000) developed the Attitude Towards Mathematics Inventory with four dimensions: self-confidence (students' belief in their ability), enjoyment (pleasure in learning math), motivation (interest and desire to learn), and value (perception of mathematics' importance in life and the future).

Methodology

This study employed a mixed-method approach involving 20 fifth-grade mathematics teachers from 20 schools in San Salvador, the capital of El Salvador, along with their 333 students. To assess the classroom management styles employed by the teachers, each was observed once using the Protocol for Classroom Management Styles Assessment (PCMSA), developed by Gordana and Snezana (2012) and grounded in the frameworks of Wolfgang and Glickman (1980) and Martin and Baldwin (1993). The original 20-item instrument was adapted to align with the Ministry of Education's (2019) mathematics guidelines. The final version consisted of 18 items: 4 related to personality, 11 to teaching, and 3 to discipline.

Student attitudes toward mathematics were assessed using the Attitude Towards Mathematics Inventory (ATMI), a Likert-scale questionnaire originally developed by Tapia and Marsh (2004). This study used the shortened version proposed by Lip and Chapman (2013), which is more suitable for time-limited settings. The ATMI consists of 19 items across four components: 5 pertaining to enjoyment, 4 to motivation, 5 to self-confidence, and 5 to value.

Student academic performance was evaluated through a mathematics test consisting of nine items covering three cognitive domains: knowledge, application, and reasoning. The items were adapted from mathematics materials developed by the Ministry of Education (2019).

To examine the relationships between classroom management styles, student attitudes, and academic performance, statistical analyses were conducted using SPSS software, including the Chi-Square test, Kruskal-Wallis test, and One-Way ANOVA.

Findings

The research identified four classroom management styles: Interventionist (A) (25 %), Interactionist (B) (20 %), Non-interventionist (C) (40 %), and a combination of Interventionist and Interactionist (D) (15 %). The Non-interventionist style was the most common, followed by the Interventionist style, indicating that most teachers either provide minimal control or adopt a teacher-centered approach, while the collaborative approach was applied less frequently. Findings varied by each dimension, but definitely Interventionist and Non-interventionist predominated in all the dimensions, as shown in Figure 1.

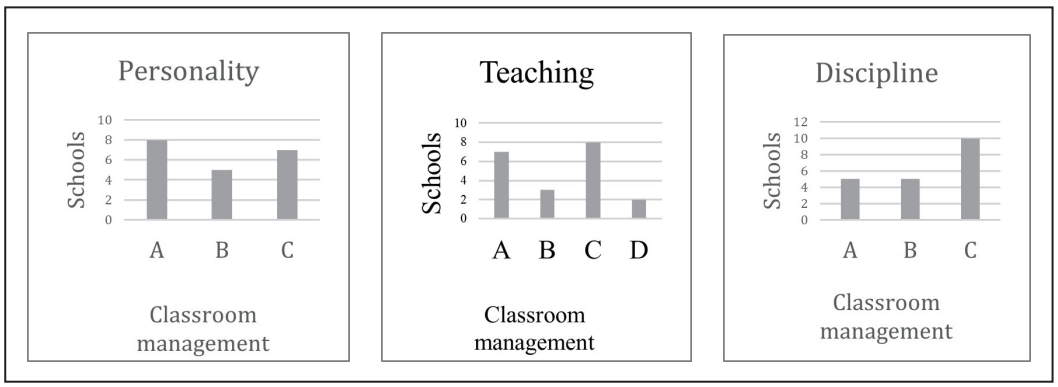


Figure 1. Classroom management styles by dimension

For the second research question, the relationship between classroom management styles and students' attitudes was examined. The null hypothesis proposed that the distribution of students' attitudes would not vary across the different classroom management style categories. Statistical analysis using a One-Way ANOVA test ($F(3, 291) = 2.262$, $p = .081$) revealed no significant evidence to support a relationship between teachers' classroom management styles and students' attitudes toward learning mathematics. See left graph of Figure 2.

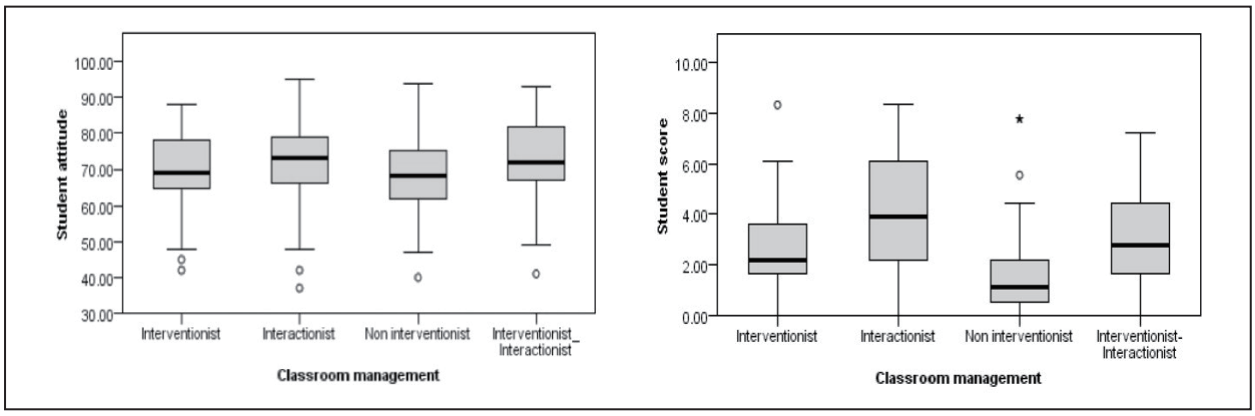


Figure 2. Classroom management and students' attitude and academic performance

The third research question examined the relationship between classroom management styles and students' academic performance. The null hypothesis stated that the distribution of students' scores would be consistent across the categories of classroom management styles. The Kruskal-Wallis test yielded statistically significant differences ($H(3) = 81.24$, $p = 0.000$), indicating that at least one classroom management style group differed significantly from the others. Further analysis using the Bonferroni post-hoc test revealed that the Interactionist style was consistently associated with the relationships where significant differences were observed, as shown in right graph of Figure 2. A similar trend was observed when the analysis was conducted by cognitive domain -knowledge, application, and reasoning- with the same pattern emerging across all domains.

Additionally, an analysis of students' means scores across the classroom management styles revealed the relationship: M(Interactionist) > M(Interventionist-Interactionist) > M(Interventionist) > M(Non-Interventionist).

Discussion

This study provides insights into classroom management styles in El Salvador's elementary schools and their impact on student attitude and academic performance. Despite the Ministry of Education's promotion of active learning (MoE, 2023), the widespread use of Non-interventionist and Interventionist styles suggests a gap in implementing balanced, Interactionist approaches. The study found that schools with Interactionist and Interventionist-Interactionist styles had the best student performance, aligning with previous research (Gordana & Snezana, 2012; Nchelem & Gladys, 2020). Unlike Adedigba and Sulaiman (2020), this study did not find a significant relationship between classroom management styles and students' attitudes toward mathematics.

Conclusion

Four prevalent classroom management styles were identified: Interventionist, Interactionist, Non-interventionist, and a combination of Interventionist and Interactionist. While the initial framework considered three styles, lesson observations revealed that teachers often applied a mix of styles, leading to the identification of a fourth style.

No statistically significant difference was found between classroom management style and students' attitudes toward learning mathematics. However, significant differences in academic performance were observed, with the Interactionist style resulting in the highest student performance across three cognitive domains (knowledge, application, and reasoning). In contrast, the Non-interventionist and Interventionist styles were associated with lower performance outcomes. These findings align with previous research, which suggests that collaborative and balanced classroom management styles promote better academic outcomes.

Overall, this study highlights the importance of interactive and balanced classroom management approaches, which have been shown to positively influence student achievement.

Limitations

This study was conducted with a small sample of 20 teachers, each observed during a single lesson. Such limited scope may restrict the depth and variability of the data, as it captures only a single moment rather than consistent patterns in teacher behavior.

References

Adedigba, O. & Sulaiman, F. (2020). Influence of teachers' classroom management style on pupils' motivation for learning and academic achievement in Kwara State. *International Journal of Educational Methodology 6*(2) 471-480. https://doi.org/10.12973/ijem.6.2.471

Boz, H., & Çalışkan, M. (2018). Developing an attitude observation form towards mathematics. *Research on Education and Psychology, 2*(1), 95–110.

Gordana, D., & Snezana, S. (2012). Protocol for classroom management styles assessment designing. *Procedia Social and Behavioral Sciences 45*, 65–74. https://doi.org/10.1016/j.sbspro.2012.06.543

Lim, S. Y., & Chapman, E. (2013). Development of a short form of the Attitudes Toward Mathematics Inventory. *Educational Studies in Mathematics, 82*(1), 145–164. https://doi.org/10.1007/s10649-012-9414-x

Martin, N., & Baldwin, B. (1993, April). Validation of an inventory of classroom management style: Differences between novice and experienced teachers. *Paper presented at the Annual Conference of the American Educational Research Association*, Atlanta, GA, United States.

Maruyama, T., & Kurosaki, T. (2021). Developing textbooks to improve student math learning: Empirical evidence from El Salvador. *JICA Ogata Sadako Research Institute for Peace and Development, 217*, 1–56. https://bit.ly/3URzlFZ

Mejía, F., & Ishizaka, H. (2021). Diagnostic analysis of the mathematics lessons considering the influence of teaching style and lesson plan adherence in the new textbooks implementation in El Salvador. *NUE Journal of International Educational Cooperation, 14*, 1–12.

Ministry of Education of El Salvador. (2019). *Guía metodológica de Matemática, 5to grado [Mathematics teacher guide, 5th grade].* Ministry of Education of El Salvador.

Ministry of Education of El Salvador. (2022). *ESMATE systematization phase 1 [Country report].* https://www.mined.gob.sv/esmate/

Ministry of Education of El Salvador. (2023). *Informe de las visitas pedagógicas 2022 [Pedagogical visits report 2022].* Ministry of Education of El Salvador.

Nchelem, R. G., & Gladys, I. C. (2020, October 20–21). Impact of teachers' classroom leadership style on students' attitude towards learning of mathematics in Rivers State senior secondary schools, Nigeria [Paper presentation]. *South Africa International Conference on Education*.

Tapia, M., & Marsh, G. (2000). Students' attitude towards mathematics: An investigation with middle school students. *Annual Meeting of the Mid-South Educational Research Association.* https://files.eric.ed.gov/fulltext/ED404165.pdf

UNESCO. (2021). *Curriculum analysis: Regional comparative and explanatory study (ERCE 2019).* https://www.unesco.org/es/articles/estudio-regional-comparativo-y-explicativo-erce-2019

Vairamidou, A., & Stravacou, P. (2019). Classroom management in primary and secondary education: Literature review. *Journal of Education and Human Development, 8*(2), 49–59. https://doi.org/10.15640/jehd.v8n2a7

Wolfgang, C. H., & Glickman, C. D. (1980). *Solving discipline problems: Strategies for classroom teachers.* Allyn and Bacon.

TSG2-2-11

RESULTS OF TEACHER QUESTION ANALYSIS IN A MATHEMATICS RESEARCH LESSON

Ganbaatar Tumurbaatar
Mongolian National University of Education, Mongolia, ganbaatar@msue.edu.mn

Khajidmaa Otgonbaatar
Mongolian National Institute for Educational Research, Mongolia, otgonbaatar@mier.mn

Densmaa Jargalsaikhan
Mongolian National Institute for Educational Research, Mongolia, densmaa@mier.mn

Itgel Miyejav
Mongolian National Institute for Educational Research, Mongolia, itgel@mier.mn

The quality of education is closely linked to the quality of individual lessons, making lesson analysis a fundamental component of educational research. In Mongolia, lesson study has evolved from simple classroom observation into a research-based approach incorporating video analysis, lesson transcripts, and preparatory materials. This study focuses on the role of teacher questioning in shaping classroom discourse, particularly within a mathematics lesson conducted using the Problem-Based Learning Approach (PBLA). Using transcript-based lesson analysis (TBLA), we examined the types and functions of teacher questions. Questions were coded into two categories and analyzed quantitatively. The findings indicate that closed-ended and content-focused questions dominated, while supportive, stimulating, and attention-grabbing questions were infrequent. These results highlight the need for more balanced and intentional use of questioning strategies to enhance student engagement and promote deeper thinking. The study offers implications for teacher professional development and instructional design in mathematics education.

Keywords: TBLA-Transcript Based Lesson Analysis, Teacher's question, lesson analysis, quality of education

Introduction

The quality of education is fundamentally determined by the quality of individual lessons. As such, lesson analysis plays a central role in evaluating and improving educational practices. In recent years, methods of lesson study and lesson analysis have become increasingly sophisticated, encompassing various tools such as transcript-based evaluations, video observations, and analysis of instructional planning.

Classroom interaction—particularly the dialogue between teachers and students—is a key element of effective teaching. The quality and type of teacher questions not only shape classroom discourse but also influence students' cognitive engagement and learning outcomes. Prior research has shown that both the form and function of teacher questioning significantly affect the nature of student responses.

This study builds on our ongoing research into lesson quality. Specifically, we investigate whether the quality of a lesson can be meaningfully assessed through a detailed analysis of the quantity and nature of teacher questions. Over the past few years, we have employed lesson study, action research, and transcript-based lesson analysis (TBLA) to explore this question, with the goal of identifying practical strategies to enhance teaching and learning.

Literature Review

The foundation for lesson study in Mongolia was established through the "Development of Teaching Methodology to Support Student Learning" project, implemented jointly by the Ministry of Education, Culture and Science and JICA between 2006 and 2009. This initiative introduced structured approaches to lesson analysis and promoted reflective teaching practices.

Transcript-Based Lesson Analysis (TBLA) is a method designed to uncover the sequence and pedagogical meaning of classroom events by interpreting detailed lesson transcripts (Matoba, 2013). Originating in Japan, TBLA complements the core philosophy of lesson study, which emphasizes attention to the learning needs of individual students (Shigematsu, 1961; Matoba, 2017). Within this framework, Shibata proposed segmenting the lesson into discrete events or activities and visualizing their structure through diagrammatic representations. These models—often composed of arrows linking various instructional elements—aid in examining the interconnectedness of classroom interactions. This body of literature provides a theoretical basis for evaluating the role of teacher questioning within lessons, offering tools to analyze both the form and function of instructional dialogue.

Design and Methodology

The purpose of this study is to evaluate the quality of a mathematics lesson by analyzing the nature and frequency of teacher questions. The analysis employs the Transcript-Based Lesson Analysis (TBLA) method, which involves examining a full transcription of a regular classroom lesson. Through this approach, the study seeks to identify patterns in teacher questioning and their potential implications for lesson quality.

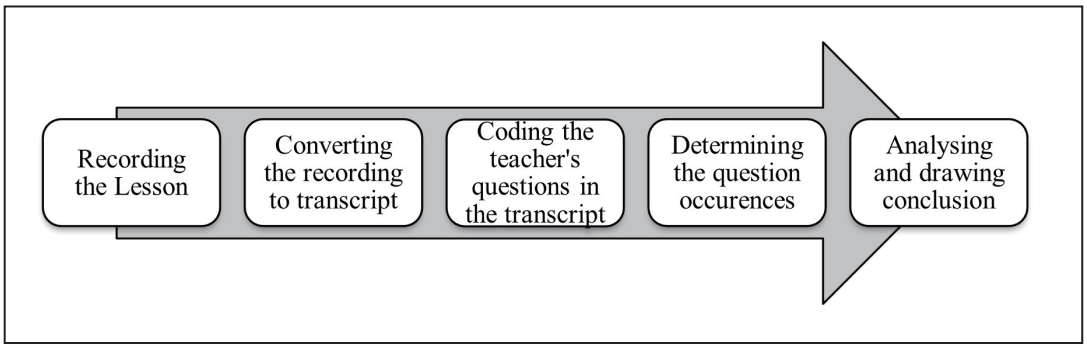


Figure 1. Design of the research

Data Collection

We collaborated with elementary school teachers to design and deliver a regular mathematics lesson aligned with the Problem-Based Learning Approach (PBLA). The lesson was observed in a research setting, during which a complete video recording was made. Following the lesson, the recording was transcribed into detailed written notes to support systematic analysis.

Development of the Coding Scheme: To analyze the teacher's questions, we developed a coding scheme consisting of two distinct classification categories. These categories were applied to the lesson transcript to examine both the form and pedagogical function of the questions posed during instruction.

Table 1. Type 1 of questions

Code	Question type
QO	Open question
QC	Closed question
QT	Lesson organizing question

Table 2. Type 2 of questions

Code	Question type	Definition
Q1	Confirming and reminding	Questions that remind or recall previously learned material or reference prior discussion.
Q2	Asking about content of the lesson	Questions directly related to the lesson content.
Q3	Facilitating and supporting	To support student responses and facilitate answering.
Q4	Motivating and attract attention	To attract students' attention and motivate interaction.

The lesson transcript was documented using Microsoft Word. A one-hour lesson resulted in approximately six pages of transcription in A4 format. Below is an excerpt from the transcript, illustrating the structure and content of the teacher-student interaction.

Findings

The mathematics lessons were analyzed using a transcript-based lesson analysis (TBLA) approach. In this study, the teacher's questions were categorized into two types, and conclusions were drawn based on the frequency and nature of these questions.

The analysis revealed that the majority of questions posed by teachers were closed-ended and content-related. Teachers asked questions frequently throughout the lessons; however, most questions required only brief responses—typically one or two words—and were predominantly low-level in cognitive demand.

Overall, the findings suggest that the teachers' ability to formulate and use effective questioning strategies is limited, indicating a need for further professional development in this area.

Discussion and Conclusion

The use of the Transcript-Based Lesson Analysis (TBLA) method in regular classroom settings enables a comprehensive examination of teacher-student interactions and provides direct insights into potential improvements in lesson delivery.

These findings underscore the importance and necessity of conducting further studies that explore diverse analytical approaches to better understand and enhance instructional practices.

The study also emphasizes the value of encouraging a wider variety of question types during lessons, as this can significantly contribute to enriching students' learning experiences and cognitive engagement.

In conclusion, the study revealed that teachers mainly used closed, low-level questions, limiting student engagement. Applying TBLA proved effective in identifying these patterns and can support future efforts to improve questioning techniques in the classroom.

References

Blosser, P. E. (2000). How to ask the right questions. National Science Teachers Association.

Chin, C. (2007). Teacher questioning in science classrooms: Approaches that stimulate productive thinking. Journal of Research in Science Teaching, 44(6), 815–843.

Chin, C., & Langsford, A. (2004). Questioning students in ways that encourage thinking. Teaching Science, 50(4), 16–21.

Christenbury, L., & Kelly, P. P. (1983). Questioning: A path to critical thinking. National Council of Teachers of English.

Elliot, J. (2017). Editorial review: Lesson study as curriculum analysis in action and the role of the teacher as a researcher. International Journal for Lesson and Learning Studies, 6(1), 2–9.

Ganbaatar, T. (2014). Lesson study and its future directions. Teaching Methods Supporting Child Development, 24.

Hargie, O. D. (1978). The importance of teacher questions in the classroom. Educational Research, 20(2), 99–102.

Kayima, F. (2016). Question classification taxonomies as guides to formulating questions for use in chemistry classrooms. European Journal of Science and Mathematics Education, 4(3), 353–364.

Matoba, M. (2013). Methods and issues in lesson analysis. In M. Matoba & Y. Shibata (Eds.), Lesson study and creation of lessons (pp. 5–20). Keisui-sya.

Nakashima, Y. (2014). How Japanese teachers build a learning community. Paper presented at WALS 2014 Conference.

Shibata, Y. (2002). A study on the integration of quantitative and qualitative methods in lesson analysis. Kazama-shobou.

Shigematsu, T. (1961). Method of lesson analysis. Meijitosyo.

Smart, J. B., & Marshall, J. C. (2013). Interactions between classroom discourse, teacher questioning, and student cognitive engagement. Journal of Science Teacher Education, 24(2), 249–267.

Tofade, T., Elsner, J., & Haines, S. T. (2013). Best practice strategies for effective use of questions as a teaching tool. American Journal of Pharmaceutical Education, 77(7), 155.

Wilen, W. W. (1991). Questioning skills for teachers. National Education Association.

TSG2-2-12

TEACHERS INTERPRETING STUDENT WORK IN GEOMETRY: USING CURRICULAR TEXT TO SUPPORT INSTRUCTIONAL EXCHANGE

Soobin Jeon
University of Michigan, USA, sbjeon@umich.edu

Understanding students' mathematical ideas and responding appropriately are essential to teaching (Jacobs et al., 2010; Sherin, 2011). However, making sense of student thinking from their work is complex (Wallach & Even, 2005), requiring teachers to balance mathematical goals, student learning, and external performance pressures (Cohen, 2011). This paper reports on a study examining how secondary mathematics teachers interpret student work in geometry and how curriculum texts—whether designed for teachers or students—shape their interpretations. Using a simulation-based interview, 12 experienced geometry teachers engaged with learning goals from a curriculum while analyzing student work. Findings illustrate how semiotic systems in texts influence teachers' reasoning, revealing distinctions in their approaches to identifying and responding to students' mathematical thinking. This study contributes to understanding how curriculum materials mediate teacher decision-making and the ways in which different semiotic representations inform teachers' interpretation of students' mathematical ideas.

Keywords: Teacher Noticing, Student Work, Curriculum, Geometry

Background and Theoretical Framing

Teachers engage in instructional exchanges, managing the unequal representation of knowledge—such as students' emergent ideas and the mathematical goals articulated in the curriculum (Herbst & Chazan, 2012). These exchanges are shaped by the didactical contract (Brousseau, 1997), a tacit set of responsibilities that holds teachers accountable for guiding students toward knowledge acquisition while positioning students as responsible for engaging in activities that demonstrate their learning. Managing an instructional exchange requires teachers to not only apply the didactical contract to the task at hand but also identify what students need to do and determine which aspects of the mathematical task highlight the knowledge at stake (Herbst & Chazan, 2012). Balancing responsiveness to students' ideas with maintaining the integrity of mathematical knowledge is central to this process (Ball, 1993).

One end of the instructional exchange I focus on is what teachers notice about students' mathematical ideas through their work. Interpreting student work is a fundamental aspect of pedagogical content knowledge (Hill, Ball, & Schilling, 2008; Ko & Herbst, 2020) and a key component of managing instructional exchanges (Chazan, Herbst, & Clark, 2016). Teachers must discern what students understand, where they may be struggling, and how to respond appropriately. This process aligns with the broader framework of teacher noticing (Mason, 2002; Jacobs, Lamb, & Philipp, 2010; Sherin, Jacobs, & Philipp, 2011), which examines how teachers attend to, interpret, and decide how to respond to students' mathematical thinking. It is a deliberate, individual process (Jacobs, Lamb, & Philipp, 2010) that can be studied and improved (Gibson & Ross, 2016)—an important consideration given teachers' profession-specific ways of "seeing and understanding" (Goodwin,

1994). However, hearing what students are saying, showing, and doing is challenging (Wallach & Even, 2005). Students' mathematical expressions—including written responses, diagrams, and verbal explanations—often contain opaque signs that make their reasoning difficult to discern (Herbst & Chazan, 2012, p. 606). The mathematical register adds another layer of complexity, as it includes specialized language, symbolic notation, and diagrammatic representations that require careful interpretation (Halliday, 1978).

On the other end of the instructional exchange is the articulation of the mathematical goal in the curriculum or the knowledge at stake. Clearly specifying mathematical learning goals is essential for various aspects of teaching, particularly in building a knowledge base for mathematics instruction (Jansen, Bartell, & Berk, 2009). This skill is closely related to mathematical knowledge for teaching, which Ball, Thames, and Phelps (2008) describe as requiring teachers to "hold unpacked mathematical knowledge because teaching involves making features of particular content visible to and learnable by students" (p. 400). Furthermore, the curriculum serves as a key mediator in instructional exchanges (Remillard, 2005; Stein et al., 2008). The way knowledge is structured in the curriculum—often broken down into text (Kang & Kilpatrick, 1992)—shapes how teachers and students interact with mathematical content. However, one challenge is the lack of direct interaction between teachers and students when working with curriculum texts (Rezat, 2011; Remillard, 2005). Teachers must navigate differences between the student's goal (how students are expected to engage with mathematical ideas) and the teacher's goal (how teachers are expected to facilitate and assess student engagement). Linguistic analysis of curriculum texts highlights how the semiotic systems in text designed for the teacher and student may impact the students' opportunity to learn and pose challenges in the instructional exchange between teacher and student (Jeon & Herbst, 2023).

To investigate how secondary mathematics teachers interpret student work in geometry, I draw on the teacher noticing framework and Systemic Functional Linguistics (SFL) (Halliday & Matthiessen, 2004) to understand the meaning teachers ascribe to student work. This study explores the following research questions:

1. What do teachers attend to, and how do they interpret students' mathematical ideas across different groups of teachers?
2. How do teachers' interpretations and responses align with the learning goals?

Methods

Participants and Study Setting

Twelve experienced high school geometry teachers participated in a simulation-based interview, averaging 15 years of mathematics teaching experience. They were randomly assigned to either the Student-Goal Group or the Teacher-Student Goal Group. Before the interview, participants completed a consent form and a pre-test on Qualtrics to assess their mathematical knowledge for teaching geometry (MKT-G). Conducted in November 2024, the simulation-based interviews used contextual inquiry (Liaqat et al., 2018), allowing participants to engage with tasks while explaining their reasoning.

The interview began with a curriculum intervention in which all participants received a lesson narrative. The Student-Goal Group viewed only student-facing learning goals, while the Teacher-Student Goal Group saw both teacher-facing and student-facing learning goals, encountering linguistic discrepancies in curricula (Jeon & Herbst, 2023). Participants then observed four simulated class-

room scenarios, each from a different geometry unit. In each scenario, they watched a teacher pose a geometry task and analyzed four pieces of hypothetical student work. They interpreted the work and provided feedback in response to the interviewer's prompts: "What feedback would you provide for this student?" and "How do you see this student's work matching the objective?"

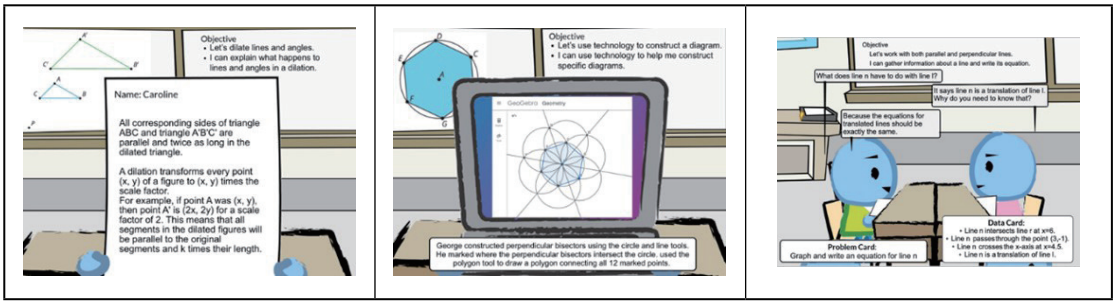


Figure 1. Student work examples shown during simulation-based interviews

Data Analysis

I analyzed the interview transcripts using the teacher noticing framework and Systemic Functional Linguistics (SFL) to examine how teachers interpret and respond to student work. I also examined the level of detail in teachers' attention to student thinking—whether they attended closely to mathematics ideas and goals. Drawing on professional noticing of children's mathematical thinking (Jacobs et al., 2010), I coded three components: attending (what teachers notice in student strategies), interpreting (how they reason about these strategies in relation to mathematical concepts), and responding (the instructional decisions they make based on these interpretations).

To examine how teachers constructed meaning through language, I applied transitivity analysis from SFL (Halliday & Matthiessen, 2004), which traces how language fulfills its experiential metafunction (i.e., representing the world). Transitivity analysis focuses on the use of grammar to uncover meaning within the clause by analyzing the process and circumstances. Four processes types were used: Mental (e.g., thinking, comprehending), Material (e.g., drawing, constructing), Relational (e.g., defining, representing), and Verbal (e.g., explaining, stating) (Halliday & Matthiessen, 2004). By applying the professional noticing framework alongside SFL, I was able to trace how teachers' interpretations and feedback were constructed through both their reasoning and their language.

Findings

Teachers' attention to and interpretation of student work varied depending on whether they received only student-facing learning goals or both student-facing and teacher-facing learning goals. Teachers in the Student-Goal Group, who received student-facing goals emphasizing material processes, primarily attended to procedural accuracy and observable features in student work. Their interpretations often relied on visual confirmation, descriptions of student actions, and general judgments about whether students had met the objective. In contrast, teachers in the Teacher-Student Goal Group, who had access to both student-facing and teacher-facing learning goals that emphasized mental processes as well, demonstrated a broader range of noticing. They attended not only to procedural correctness but also to the reasoning and justifications underlying students' responses. Their interpretations were more analytical, focusing on conceptual understanding, precision in mathematical language, and the logical coherence of students' explanations.

Teachers' responses aligned with the linguistic processes emphasized in their respective learning goals. In the first geometry task on Dilation, for example, the Student-Goal Group's attention on visual features corresponded with the material-based student-facing goals, whereas the Teacher-Student Goal Group's attention to reasoning and justification aligned with the mental-based teacher-facing goals. Similarly, in the third task on Side-Side-Angle Congruence, the Teacher-Student Goal Group's noticing of precise language and skepticism toward transformation-based reasoning reflected the material processes emphasized in the teacher-facing goals. In contrast, the Student-Goal Group, whose goals emphasized mental processes, attended more to informal reasoning and counterexamples. Across the four tasks, teachers with access to teacher-facing materials were more likely to structure responses that deepened mathematical engagement, whereas those with only student-facing goals were more likely to verify correctness without further exploration.

Conclusion

This study illustrates the variability in how teachers interpret student work, highlighting differences in what they attend to and how they reason about students' mathematical ideas. Managing instructional exchanges requires teachers to navigate the tension between students' emergent thinking and the mathematical knowledge at stake, yet our findings suggest that teachers differ in how they balance these demands. Some teachers focused more on surface-level correctness, while others engaged more deeply with students' reasoning, suggesting that interpretation is shaped by individual noticing practices as well as the instructional context. The curriculum, as a key mediator, plays a role in structuring the knowledge at stake and shaping opportunities for teachers to recognize and respond to student ideas. Differences in how knowledge was framed based on linguistic processes appeared to align with how teachers attended to student work, suggesting that linguistic structures in curriculum may contribute to patterns in instructional exchanges. These findings point to the potential for curriculum design to support teachers in aligning their interpretations and responses with mathematical goals while remaining responsive to students' developing ideas.

References

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The elementary school journal*, 93(4), 373-397.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special?. *Journal of Teacher Education*, 59(5), 389-407.

Brousseau, G. (1997). *Theory of didactical situations in mathematics: Didactique des mathematiques* 1970–1990 (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Eds. and Trans.). Dordrecht: Kluwer.

Chazan, D., Herbst, P. G., & Clark, L. M. (2016). Research on the teaching of mathematics: A call to theorize the role of society and schooling in mathematics instruction. In D. H. Gitomer & C. A. Bell (Eds.), *Handbook of research on teaching* (pp. 1039–1097). https://doi.org/10.3102 /978-0-935302-48-6_17

Cohen, D. K. (2011). Human Improvement. In *Teaching and Its Predicaments* (pp. 4–23). Harvard University Press. http://www.jstor.org/stable/j.ctt2jbtnr.5

Gibson, S. A., & Ross, P. (2016). Teachers' professional noticing. *Theory into Practice*, 55(3), 180-188. https://doi.org/10.1080/00405841.2016.1173996

Goodwin, C. (1994). Professional vision. *American Anthropologist*, 96(3), 606 –633. doi:10.1525/aa.1994.96.3.02a00100

Halliday, M. A. K. (1978, March). *Language as social semiotic*.

Halliday, M. A. K., & Matthiessen, C. M. I .M. (2004). *An introduction to functional grammar* (3rd ed.). Hodder Arnold.

Herbst, P., & Chazan, D. (2012). On the instructional triangle and sources of justification for actions in mathematics teaching. *ZDM Mathematics Education*, 44, 601-612. https://doi.org/10.1007/s11858-012-0438-6

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for research in mathematics education*, 39(4), 372-400. https://doi-org.proxy.lib.umich.edu/10.5951/jresematheduc.39.4.0372

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for research in mathematics education*, 41(2), 169-202.

Jansen, A., Bartell, T., & Berk, D. (2009). The role of learning goals in building a knowledge base for elementary mathematics teacher education. *The Elementary School Journal*, 109(5), 525-536.

Jeon, S. & Herbst, P. (2023). Dilating perspectives: A comparison of teacher and student facing texts on a unit on similarity. Lamberg, T., & Moss, D. (2023). *Proceedings of the Forty-fifth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 273-281). University of Nevada, Reno. Kang, W., & Kilpatrick, J. (1992). Didactic transposition in mathematics textbooks. *For the learning of Mathematics*, 12(1), 2-7.

Ko, I., & Herbst, P. (2020). Subject matter knowledge of geometry needed in tasks of teaching: relationship to prior geometry teaching experience. *Journal for Research in Mathematics Education*, 51(5), 600-630. https://doi-org.proxy.lib.umich.edu/10.5951/jresematheduc-2020-0163

Liaqat, A., Axtell, B., Munteanu, C., & Epp, C. D. (2018, March). Contextual inquiry, participatory design, and learning analytics: An example. In *Companion Proceedings 8th International Conference on Learning Analytics & Knowledge* (pp. 1-5).

Mason, J. (2002). *Researching your own practice: The discipline of noticing.* Routledge.

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of educational research*, 75(2), 211-246. https://doi.org/10.3102/00346543075002211

Rezat, S., Fan, L., & Pepin, B. (2021). Mathematics textbooks and curriculum resources as instruments for change. *ZDM Mathematics Education*, 53, 1189-1206. https://doi.org/10.1007/s11858-021-01309-3

Sherin, M., Jacobs, V., & Philipp, R. (2011). Using Video to Improve Preservice Mathematics Teachers' Abilities to Attend to Classroom Features: A Replication Study. In *Mathematics Teacher Noticing* (pp. 147-163). Routledge.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical thinking and learning*, 10(4), 313-340. https://doi.org/10.1080/10986060802229675

Wallach, T., & Even, R. (2005). Hearing students: The complexity of understanding what they are saying, showing, and doing. *Journal of Mathematics Teacher Education*, 8, 393-417. https://doi.org/10.1007/s10857-005-3849-2

TSG2-2-13

A STUDY ON ASPECTS OF GRASPING QUANTITATIVE RELATIONS IN WORD PROBLEMS INVOLVING INVERSE CALCULATION: FOCUSING ON THE PROCESS OF DRAWING THE DIAGRAMS

Maoko Nakahata
Graduate School of Education, Shimane University, JAPAN, e249356@matsu.shimane-u.ac.jp

This study examines instruction to help children properly grasp quantitative relations in word problems involving inverse calculation, which they often find difficult. In this paper, a survey was conducted with third-grade students, focusing on the process of drawing relationship diagrams in word problems involving inverse calculation, and an investigation was conducted on how children grasp quantitative relations from the word problems. As a result of the investigation, I was able to identify a characteristic feature of the discussion that supports grasping quantitative relations.

Keywords: word problems involving inverse calculation, relationship diagram

Introduction

Many children find word problems difficult in educational settings. Among them, word problems involving inverse calculation are particularly difficult for many children. The large-scale academic achievement surveys of fifth- and sixth-grade primary school students in Japan have also shown children's difficulty with word problems involving inverse calculation (e.g., National Institute for Educational Policy Research, 2015; National Institute for Educational Policy Research, 2024). Based on these findings, I believe that the reason for children's difficulty with word problems is that they read the words and numbers literally and formulate equations. Similarly, Nesher and Teubal (1975) pointed out that many children only look for numbers and key words when solving word problems, ignoring the context in which they are used. Therefore, this study aims to offer suggestions for effective instruction to support children's understanding of quantitative relations when solving word problems involving inverse calculation.

Theoretical Framework

In mathematical problem solving, Mayer (1992) shows that children go through two stages before arriving at an answer to a word problem: problem representation and problem solution. Problem representation involves converting the words and pictures of the problem into an internal representation, while problem solving involves moving from a mental representation of the problem to a final answer. In other words, problem representation refers to the stage where quantitative relations are properly grasped from the text. And problem solving refers to the stage where the grasped relations are formulated into an equation and the answer is derived. Children's difficulty at the stage of problem representation has been pointed out, but the transformation of their understanding of proper quantitative relations is not detailed (Mayer, 1992). Therefore, to understand how children

grasp quantitative relations, this study focuses on the process of drawing diagrams and aims to capture the transformation of their understanding. This study traces children's thinking by using a diagram called a *relationship diagram*, which is used in Japan (Figure 1). A relationship diagram is a diagram that uses arrows to visually and clearly show how quantities in a problem statement are related and is said to be a diagram that supports arithmetic judgments and equation planning based on these relationships (Keirinkan, 2024).

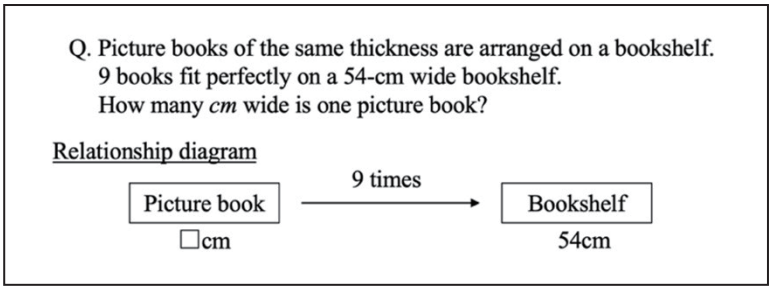


Figure 1. Example of relationship diagram
(Created by the author based on Shimizu et al., 2012, p. 42)

Research Question

Based on the above, this study focuses on the stage of problem representation. It aims to identify situations in which children were able to grasp quantitative relations properly in word problems involving inverse calculation and to analyze the factors that led to the transformation. The research question is set as follows:

［*R.Q.*］ *At the stage of problem representation, what were the factors that enabled children to grasp quantitative relations properly in word problems involving inverse calculation?*

Method

Survey Design

An experimental lesson was conducted for third grade elementary school students. Students who showed a change in the way of drawing relationship diagrams were selected based on the data obtained during class, and the factors that led to the change were examined. The subjects were 19 third grade students in a Japanese public elementary school. The experimental lesson consisted of solving the two problems shown in Figure 2 by drawing relationship diagrams. The data collected in this study were worksheets written by the students, VTR recordings, and still image recordings.

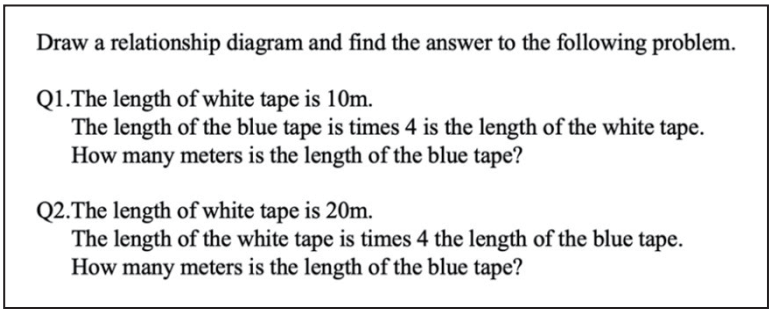


Figure 2. Worksheet used in class (partial)

Result

In the lesson conducted in the study, students first solved the problems on the worksheet (Figure 2) individually and discussed in groups of three or four what relationship diagrams they had drawn. After the group discussion, the students solved the problems on the worksheet (Figure 2) again. This study focuses on IU who changed the way of drawing the relationship diagram, and discusses the factors that led to this change. The symbols used in the transcript are "IU (initials)" and "HA (initials)" for a specific child.

How IU Drew the Relationship Diagram when Thinking Individually (Q2)

When thinking individually, IU read the word problem and drew the relationship diagram in the following way (Figure 3). Based on the relationship diagram IU drew, IU wrote the equation 20 × 4 = 80 and recorded the answer as 80m. From this, it is considered that, at the stage of problem representation, IU did not properly grasp the quantitative relation described in the text, and therefore the equation was set to 20 x 4.

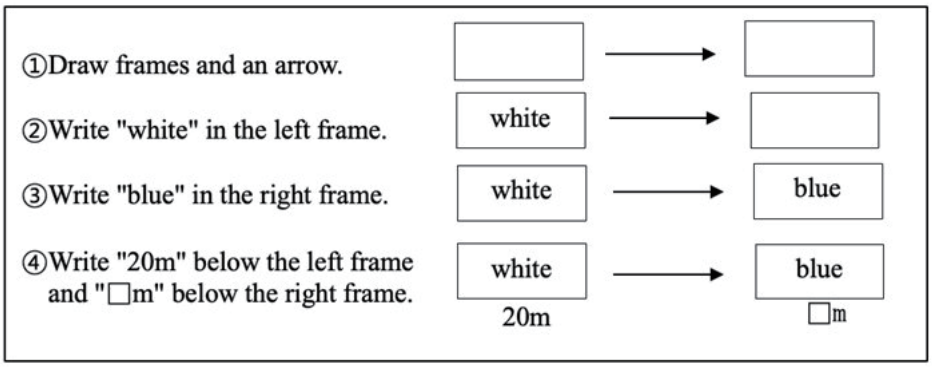


Figure 3. *The process of drawing IU's relationship diagram (when thinking individually: Q2)*

How IU Drew the Relationship Diagram After the Group Discussion (Q2)

After the group discussion, IU solved the same problem again. This time, IU read Q2 and drew the relationship diagram in the following way (Figure 4). Based on the process of drawing the relationship diagram, it is considered that, at the stage of problem representation, IU was able to properly grasp the quantitative relation described in the text.

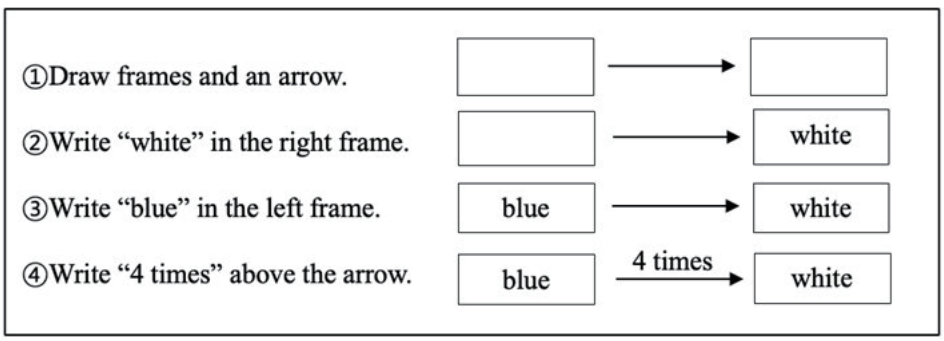


Figure 4. *The process of drawing IU's relationship diagram (after the group discussion: Q2)*

Group Discussion in Practice

This section presents the discussion between IU and HA from the same group.

C01 (IU): [Pointing to the diagram IU has drawn] The white tape is 20 meters long, so the blue tape is 4 times longer than the white tape. So, it is 4 times longer than the white tape, we set the length of the tape to 20 x 4.

C02 (HA): Can I just say something? This is 20 meters of white tape, so far so good.

C03 (HA): But the length of the white tape is four times as long as the length of the blue tape [HA traces the sentence in Q2 with their own finger]. The white tape is four times as long as the blue tape.

C04 (IU): Wow, I should have read the equation.

C05 (HA): It's not an equation. It's a word problem.

C06 (IU): No, it's not. It was a word problem. This one [pointing to Q1] is four times as long as the white tape. So, this one [pointing to Q2] is division.

C07 (HA): Correct.

IU points to the diagram IU drew and explains why it is 20 × 4 (C01). In response to IU's explanation, HA acknowledges that IU reads from the problem description that the white tape is 20 meters long (C02). Subsequently, HA traced the description in Q2 with their own finger and said twice, "The white tape is four times as long as the blue tape" (C03). This suggests that HA was encouraging IU to focus on the sentence. After HA's advice, IU appears to notice that the base and the quantity to be compared shown in IU's relationship diagram was different from the sentence in the word problem (C04). Then IU confirmed the difference between the description in Q1 and Q2, and confirmed that the operation in Q2 is division, not multiplication (C06).

Based on the above analysis of IU, the contributing factor was clarified. During group discussions, it became evident that HA's support helped IU to properly grasp the quantitative relation. In particular, it was important for HA to encourage IU to refocus on the sentences specifically describing the relationship between the blue tape and the white tape in the text.

Concluding Remarks

The purpose of this study was to identify the factors that enabled children to properly grasp quantitative relations in word problems involving inverse calculation during the stage of problem representation. The results of the analysis revealed that encouraging children to refocus on the relational sentence was effective in helping them properly grasp quantitative relations during the problem representation stage.

However, since the result of this study is limited, it is possible that new insights could be obtained by increasing the number of cases. Therefore, increasing the number of cases is an issue that remains to be addressed.

References

Mayer, R. E. (1992). *Thinking, problem solving, cognition (2nd ed.)*. W. H. Freeman.

National Institute for Educational Policy Research. (2015). *2015 national survey on school achievement and learning status, explanatory material: Elementary school arithmetic* (in Japanese). Author.

National Institute for Educational Policy Research. (2024). *2024 national survey on school achievement and learning status, explanatory material: Elementary school arithmetic* (in Japanese). Author.

Nesher, P., & Teubal, E. (1975). Verbal cues as an interfering factor in verbal problem solving. *Educational Studies in Mathematics, 6*, 41−51.

Shimizu, S., Funakoshi, S., Aoyama, K., Iijima, Y., Ieda, H., Ishida, J., Imai, T., Uda, H., Ebashi, N., Ota, M., Okabe, Y., Oguchi, Y., Obara, Y., Kageyama, K., Kageyama, S., Kamiyama, S., Kameoka, M., Kurosaki, T., Koto, R., ... Watabe, M. (2012). *Fun with math 3A for elementary school (T. Baba, M.* Teragaito, H. Ishii, N. Naka, & Japan Translation Center, Trans.). Shinko Shuppansha Keirinkan.

Keirinkan. (2024). *Waku-Waku Sansu 3: Teachers' guide, Part 2 detailed explanation, supplement 2 research materials section* (in Japanese). Shinko Shuppansha Keirinkan.

Acknowledgements

I would like to express our sincere gratitude to the teachers and students at public elementary schools in Shimane Prefecture, Japan, who cooperated in the research of this study.

TSG2-2-14

STREAMING IN SECONDARY MATHEMATICS: A VIEW FROM AUSTRALIA

Elena Prieto-Rodriguez
University of Newcastle, Australia, elena.prieto@newcastle.edu.au

Ability grouping, or streaming, is a classroom practice that separates students by perceived academic abilities. Proponents argue it allows tailored instruction, while critics say it perpetuates inequalities. Internationally, its impact varies. In the U.S. and the U.K., some educators believe that streaming may improve outcomes for high-achievers but widen the achievement gap for lower socioeconomic students. Japan and South Korea have traditionally rejected rigid grouping, achieving top international rankings with equitable education. In Australia, streaming in a widespread practice, raising concerns about its impact on achievement and equity. This paper examines reasons for streaming in Australian schools, based on a 2023 survey of Mathematics Head Teachers. Findings show streaming is justified by the need for differentiation, school culture, academic achievement, classroom management, and teacher comfort, with flexibility seen as beneficial for student engagement and self-esteem.

Keywords: streaming ability grouping, academic achievement, equity

Introduction

The practice of ability grouping, commonly known as streaming, involves organising students into classes based on their perceived or actual academic abilities. This approach is prevalent in many educational systems worldwide, particularly in subjects like mathematics.

Internationally, the implementation and impact of ability grouping vary significantly. In the United States, for instance, since the early 2010s, there has been a resurgence of ability grouping and studies indicate that while some educators believe that streaming benefits high-achieving students by providing appropriately challenging material, the overall effectiveness of this practice remains contentious (Loveless, 2013). In the United Kingdom, research suggests that while streaming can lead to improved outcomes for high-achieving students, it may also contribute to widening the achievement gap, particularly disadvantaging students from lower socioeconomic backgrounds (Connolly et al., 2019).

In contrast, countries like Japan and South Korea have adopted educational practices that reject rigid ability grouping. These nations have consistently attained top rankings in international assessments, such as PISA, without relying on streaming. The emphasis in these countries is on providing equitable educational opportunities for all students, regardless of their perceived abilities (Alam & Mohanty, 2023). In South Korea, the practice of ability grouping, or streaming, in mathematics education has been a subject of considerable debate (Hwang, 2013). While some educators advocate for this approach to provide tailored instruction that meets the specific needs of students, research indicates that it may not always lead to improved academic outcomes. For example, a study examining the effects of ability grouping on middle school students' academic self-efficacy found that such practices can influence students' perceptions of their academic abilities. This research suggests that while ability grouping may offer benefits to higher-achieving students, it could potentially impact the self-efficacy of students in lower-achieving groups (Hwang, 2013).

In recent years, Australia has seen a worrying decline in the number of students completing higher-level mathematics courses in Years 11 and 12, despite the increasing importance of STEM fields for future job markets and economic development. This decline is paralleled by lower performance in international assessments, such as the National Assessment Program – Literacy and Numeracy (NAPLAN) and the Programme for International Student Assessment (PISA). Some argue that one contributing factor to this issue is the widespread practice of streaming, which is commonly employed in mathematics classrooms (Johnson et al., 2025). While often justified as a way to tailor teaching to students' abilities, the effectiveness of ability grouping in improving student outcomes remains contentious.

This paper explores the reasons provided by Mathematics Head Teachers to justify the need (or otherwise) for streaming in their schools.

Methodology

Data were gathered at a Head Teacher event held in Sydney, Australia, in 2023. An anonymous survey was administered on-site using the Mentimeter web application, targeting the 400 attendees present at the event. Out of these, 336 participants provided responses via the application. Three of the questions in the survey were quantitative in nature and have been analysed using descriptive statistics. The fourth question, which was qualitative, has been examined through thematic analysis. This process involved systematically coding the responses to identify recurring patterns and categories, thereby enabling a nuanced interpretation of the participants' perspectives and experiences.

Results and Discussion

The first question in the survey asked Head Teachers whether ability streaming is used in their school. A total of 336 participants responded, of which 317 (94%) answered that they do.

The following two questions asked when the streaming begins, and who they think streaming benefits the most. The distribution of responses can be found below in Figures 1 and 2 respectively.

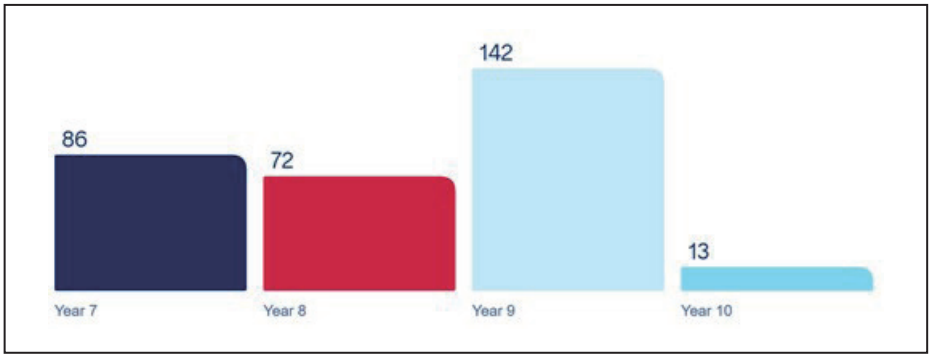


Figure 1: Distribution of responses to the question "When do you start streaming?"

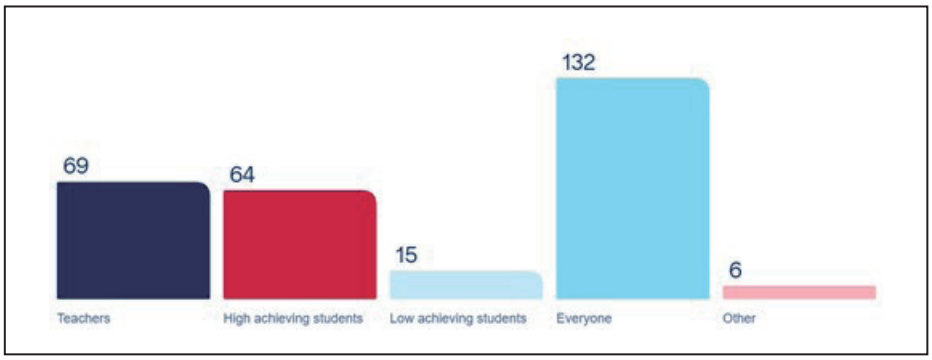


Figure 2: Distribution of responses to the question "Who do you think streaming helps the most?"

The last question asked the reason why streaming was chosen in their schools. The analysis of 432 responses from 313 participants highlights the multifaceted nature of ability grouping in education.

The coding of the *responses* is presented in Table 1 below. It is important to note that 49 quotes were coded against more than one category.

Category	Subcategory	Number of Quotes	Percentage
Catering to Student Needs	Differentiation	180	41.67%
	Targeted Teaching	122	28.24%
	Total	**302**	**69.91%**
School Culture and Policy	Embedded Practices	50	11.57%
	Historical Context	20	4.63%
	Total	**70**	**16.20%**
Student Outcomes	Academic Achievement	156	36.11%
	Behaviour Management	60	13.89%
	Total	**216**	**50.00%**
Teacher Perspectives	Ease of Teaching	126	29.17%
	Professional Comfort	20	4.63%
	Total	**146**	**33.80%**
Student Engagement and Self-Esteem	Learning Environment	64	14.81%
	Avoiding Neglect	20	4.63%
	Total	**84**	**19.44%**
Flexibility and Fluidity	Movement Between Groups	26	6.02%
	Pathway Preparation	20	4.63%
	Total	**46**	**10.65%**

The most prominent theme was catering to student needs through differentiation and targeted teaching, which allows educators to tailor lessons to varying abilities, thereby extending higher-ability students and providing additional support to those with lower abilities. This finding aligns with previous research indicating that ability grouping can enhance academic achievement and manage

classroom behaviour by reducing disruptions from students who may struggle with the content (Webel et al., 2021). School culture and policy also played a significant role, with ability grouping often being an embedded practice driven by senior directives or longstanding traditions. This is consistent with studies that suggest historical entrenchment and cultural acceptability as reasons for the persistence of ability grouping in schools (Johnson et al., 2025). From the teachers' perspective, instructing homogeneous classes provided ease and professional comfort, allowing for more focused and effective teaching. This perspective is supported by research indicating that ability grouping can facilitate differentiation and assist in managing the intensification of teaching (Johnson et al., 2025). Additionally, grouping students with similar abilities benefited student engagement and self-esteem, fostering a positive learning environment and ensuring that both high and low achievers received appropriate challenges and support. Lastly, the flexibility and fluidity of ability grouping allowed for movement between groups as students' abilities developed and facilitated preparation for specific pathways in senior courses, aligning current learning with future academic pursuits. This flexibility is crucial for adapting to students' evolving needs and ensuring that they are adequately prepared for future academic challenges (Baines, 2019).

Conclusion

Overall, the findings underscore the complex reasons prompting schools to perform ability grouping in education, highlighting its benefits in catering to diverse students, supporting teachers, and fostering positive outcomes. However, it is essential to consider the potential drawbacks and strive for equitable practices that ensure all students have access to high-quality learning opportunities.

References

Alam, A., & Mohanty, A. (2023). Cultural beliefs and equity in educational institutions: exploring the social and philosophical notions of ability groupings in teaching and learning of mathematics. *International Journal of Adolescence and Youth*, *28*(1), 2270662.

Baines, E. (2019). Ability Grouping. UCL Discovery.

Connolly, P., Taylor, B., Francis, B., Archer, L., Hodgen, J., Mazenod, A., & Tereshchenko, A. (2019). The misallocation of students to academic sets in maths: A study of secondary schools in England. *British Educational Research Journal*, *45*(4), 873-897.

Hwang, Y. J. (2013). Effects of ability grouping on middle school students' affective outcomes. In *Korean education in changing economic and demographic contexts (pp. 127-149). Singapore: Springer Singapore.*

Johnston, O., Spina, N., Macqueen, S., & Spooner-Lane, R. (2025). Why do schools continue to use between-class ability grouping? *PROSPECTS*, 54, 559-576.

Loveless, T. (2013). The resurgence of ability grouping and persistence of tracking. *Part II of the 2013 Brown Center Report on American Education, the Brookings Institution.[2]*, *1*.

Webel, C., Dames, B., Woldruff, J., Lindaman, L., & Daugherty, K. (2021). Field-Based Perspectives on Enacting Alternatives to Ability Grouping in Elementary Mathematics Instruction. *Mathematics Teacher Education and Development*, 23(3), 111-131

MEMO

KOREA 2025

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 3:

The Essence of Mathematics Education in Assessment and Evaluation

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 3
(Day 2)

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

TSG3-01

THE RELATIONSHIP BETWEEN MIDDLE SCHOOL STUDENTS' MATHEMATICS SELF-EFFICACY, MATHEMATICS ANXIETY, AND MATHEMATICAL LITERACY

Yi-Jhen Li
National University of Tainan, Taiwan, yijhen30803@gmail.com

Su-Wei Lin
National University of Tainan, Taiwan, swlin0214@mail.nutn.edu.tw

This study utilized the Programme for International Student Assessment [PISA] 2022 dataset from Taiwan to analyze the relationship between 15-year-old students' mathematics self-efficacy, mathematics anxiety, and mathematical literacy through descriptive statistics and Hierarchical Linear Modeling [HLM] analysis. The results showed a positive correlation between mathematics self-efficacy and mathematical literacy, while a negative correlation was observed between mathematics anxiety and mathematical literacy. The two research variables explained 16.4% of the within-school variance in mathematical literacy and reduced the between-school variance by 1.5%. These findings highlight the importance of the relationship between the research variables and students' mathematical literacy.

Keywords: Self-Efficacy, Mathematics Anxiety, Mathematical Literacy, HLM, PISA 2022

Research Motivation and Objectives

In 1977, Bandura introduced the term "self-efficacy," emphasizing its role in shaping an individual's fundamental behavior. When self-efficacy influences a person's thoughts, emotions, motivation, and actions, individuals tend to perform in alignment with their beliefs in specific areas. Literature indicates a positive correlation between students' mathematical literacy and mathematics self-efficacy (Al-Hija & Araidy, 2023). Therefore, the researcher included mathematics self-efficacy as one of the variables in studying students' mathematical literacy.

Although mathematics is designated as one of the core courses in Taiwan's national education curriculum, many students still experience anxiety toward learning mathematics. According to the results of PISA 2022, the mathematics anxiety levels of 15-year-old students in Taiwan exceeded the average and ranked among the top three globally (Organization for Economic Cooperation and Development [OECD], 2023). Anxiety can inhibit brain function, reduce attention, and prevent learners from fully engaging in solving mathematical problems. Previous research has found a negative correlation between mathematics anxiety and mathematical literacy (Kuo, 2024; Liu et al., 2017), highlighting the significant relationship between the two. Therefore, this study also includes mathematics anxiety as one of the research variables.

The research design of PISA adopts a multistage sampling method, where schools serve as the primary sampling units, and students are randomly selected from the sampled schools. This sampling approach results in a nested data structure, with students nested within schools. If this structure's influence is ignored and a general linear model analysis is directly applied, the effects of variables may be underestimated or overestimated. To prevent this issue, this study employs HLM for analysis (Lin, 2019).

This study uses PISA 2022 data to explore how background, self-efficacy, and anxiety affect 15-year-old Taiwanese students' math literacy at both student and school levels.

1. To analyze the relationships among background variables, mathematics self-efficacy, mathematics anxiety, and mathematical literacy at the student level.
2. To explore the explanatory power of influencing variables at different levels on the variation in mathematical literacy among 15-year-old Taiwanese students.

Literature Review

Self-efficacy

Bandura defines self-efficacy as an individual's belief in their ability to successfully accomplish a specific task, and this belief influences whether the individual is willing to exert effort and persist in the task (Bandura, 1977). Academic self-efficacy includes mastery experiences, vicarious experience, social persuasion, and physiological states. Math self-efficacy positively correlates with mathematical literacy, with both influencing each other significantly. (Bong & Skaalvik, 2003; Butz & Usher, 2015; Chen et al., 2018; Ho et al., 2022).

Mathematics Anxiety

Numerical anxiety shows that people with normal intelligence can feel distress with math, leading to poor performance and avoidance, sparking further research into its emotional and physical effects. (Dreger & Aiken, 1957; Fennema & Sherman, 1976; Richardson & Suinn, 1972). Mathematics anxiety in middle school students can be attributed to three factors: general mathematics anxiety, mathematics test anxiety, and mathematics concept anxiety. These anxieties lead to issues such as lack of focus, confusion, and avoidance of learning math. The emergence of mathematics anxiety is linked to the learner's self-perception of mathematics. Reducing negative attitudes can improve the situation. Previous studies show a negative correlation between mathematics anxiety and mathematical literacy. (Chen et al., 2024; Kuo, 2024; Lin & Kung, 2022; Weissgerber et al., 2022).

Research Methods

Research Framework

This study divides independent variables into two levels: student level (gender, socioeconomic status, mathematics self-efficacy, and mathematics anxiety) and school level (average school socioeconomic status). The ten plausible values of students' mathematical literacy from PISA 2022 are used as the dependent variable, with each value analyzed separately, and the average results serve as the basis for discussion.

Research Subjects and Research Instruments

The research subjects are 15-year-old Taiwanese students who participated in the PISA 2022 assessment, consisting of 182 schools and 5,857 participants, including 3,005 male students and 2,852 female students. The research tools include the socioeconomic status, mathematics self-efficacy, and mathematics anxiety indicator scores calculated by PISA 2022. These indicator scores are standardized based on the status of OECD countries or regions (with a mean of 0 and a standard deviation of 1). A higher score represents higher socioeconomic status, higher mathematics self-efficacy, and higher mathematics anxiety, respectively.

Data Processing and Analysis

Missing values in the data were imputed using Bayesian estimation with 5,000 iterations in MPLUS 8.11. At the student level, explanatory variables included background, imputed socioeconomic status, mathematics self-efficacy, and mathematics anxiety indicators. At the school level, the average socioeconomic status was used. SPSS 21.0 calculated correlations between explanatory variables and mathematical literacy to achieve the first research objective. MPLUS 8.11 was used to calculate the Intraclass Correlation Coefficient [ICC] to ensure data suitability for HLM, achieving the second research objective.

Research Results and Discussion

Correlation between Background Variables, Experimental Variables, and Mathematical Literacy of 15-Year-Old Students in Taiwan

Table 1 summarizes the correlation between mathematical literacy [ML] and background variables, as well as the two research variables for 15-year-old students in Taiwan. Results show no correlation with gender, a moderate positive correlation with socioeconomic status [ESCS] and mathematics self-efficacy [MSE] (with the highest correlation for self-efficacy), and a negative correlation with mathematics anxiety [MANX].

Table 1. The correlation between different levels of MSE, MANX, and ML of 15-Year-old students in taiwan (N=5857)

	ML	Gender	ESCS	MSE
Gender	.00*			
ESCS	.38*	-.01*		
MSE	.53*	.10*	.30*	
MANX	-.20*	-.13*	-.06*	-.33*

*p< .05

The Explanatory Power of Mathematics Self-Efficacy and Mathematics Anxiety on Mathematical Literacy Among 15-Year-Old Students in Taiwan

From Table 2, the ICC of .425 indicates that the data's hierarchical structure should be considered (Lee, 2000), making HLM appropriate. Model 1 examines the explanatory power of student gender, ESCS, and school socioeconomic status [SESCS] on ML variance at both levels. Models 2 and 3 add MSE and MANX at the student level to assess their explanatory power. Model 4 includes both research variables, along with demographic factors, to explore their explanatory power at both levels. The equations for student and school levels are provided:

Null model:

Student level model: $M_{ij} = \beta_{0j} + \gamma_{ij}$ ……(1)

School level model: $\beta_{0j} = \gamma_{00} + \mu_{0j}$ ……(2)

Model 1 includes background variables at the student level:

Student level model: $M_{ij} = \beta_{0j} + \beta_{1j} \times (\text{gender}) + \beta_{2j} \times (\text{ESCS}) + \gamma_{ij}$ ……(3)

Model 2 includes background variables and MSE at the student level:

Student level model: $M_{ij} = \beta_{0j} + \beta_{1j} \times (\text{gender}) + \beta_{2j} \times (\text{ESCS}) + \beta_{3j} \times (\text{MSE}) + \gamma_{ij}$ ·····(4)

Model 3 includes background variables and MANX at the student level:

Student level model: $M_{ij} = \beta_{0j} + \beta_{1j} \times (\text{gender}) + \beta_{2j} \times (\text{ESCS}) + \beta_{4j} \times (\text{MANX}) + \gamma_{ij}$ ·····(5)

Model 4 is the final model:

Student level model: $M_{ij} = \beta_{0j} + \beta_{1j} \times (\text{gender}) + \beta_{2j} \times (\text{ESCS}) + \beta_{3j} \times (\text{MSE}) + \beta_{4j} \times (\text{MANX}) + \gamma_{ij}$ ····· (6)

The school-level models for Models 1 to 4: $\beta_{0j} = \gamma_{00} + \gamma_{0j} \times (\text{SESCS}) + \mu_{0j}$ ·····(7)

Quotes M_{ij} represents the ML of the *i-th* student in the *j-th* school. β_{0j} indicates the average ML in the *j−th* school, β_{1j} represents the gender coefficient, β_{2j} represents the ESCS coefficient, β_{3j} represents the MSE coefficient, β_{4j} represents the MANX coefficient. γ_{00} represents the average ML across all schools, μ_{0j} represents the random effect of the *j−th* school, γ_{ij} represents the random effect for the *i-th* student in the j-th school. Here, $i = 1,2,3,\cdots n_j$, represents the number of students in the *j-th* school, and $j = 1,2,3,\cdots$,j represents the total number of schools.

In Model 2, it can be observed that when the MSE variable is included at the student level, the correlation between ESCS and ML at the student level is reduced, with MSE having the highest correlation with ML. In Model 3, it is found that when MANX is included at the student level, it also reduces the correlation between ESCS and ML, but to a lesser extent than in Model 2. In this model, MANX ranks second in correlation and shows a negative correlation.

In Model 4, MSE shows the highest correlation with ML, while MANX still has a negative correlation. This suggests that MSE improves ML, while excessive MANX may lower it. ESCS correlation with ML decreases after including the two research variables. At the school level, SESCS remains strongly correlated with ML, with high inter-school variation, indicating that students of similar ability tend to be concentrated in the same school. This is likely due to Taiwan's 10th-grade school allocation based on entrance exam scores, resulting in low intra-school and high inter-school variation.

Table 2. HLM results of ML, MSE, and MANX for taiwanese students (N=5857)

Fixed effects	Null model	Model 1	Model 2	Model 3	Model 4
	Coefficient *t* value	Coefficient *t* value	Coefficient *t* value	Coefficient *t* value	Coefficient *t* value
Intercept	530.37 94.26***	569.20 149.96***	573.54 148.73***	571.95 149.01***	584.065 113.40***
Student level					
Gender		0.68 0.24ns	-8.02 -2.95**	-4.75 -1.73 ns	-9.81 -3.65***
ESCS		19.48 9.94***	12.04 6.88***	18.45 9.64***	12.18 6.98***
MSE			28.17 24.33***		25.85 22.15***
MANX				-17.42 -12.21***	-8.06 -5.94***
School level					
SESCS		126.48 18.01***	125.94 17.64***	126.23 17.89***	125.84 17.60***
random effect					
Inter-school variance	5607.22	1471.10	1551.19	1500.51	1560.81
Intra-school variance	7594.38	7392.83	6212.26	7022.64	6149.22
ICC	0.425				
Inter-school R^2		73.7%	72.3%	73.2%	72.2%
Intra-school R^2		2.7%	18.2%	7.5%	19.1%

ns p> .05, *p< .05, **p< .01, ***p< .001

Conclusion

The HLM analysis revealed a strong positive correlation between mathematical self-efficacy and math literacy, while math anxiety had a negative effect. Educators are advised to provide positive feedback and use differentiated instruction to help students build confidence, reduce anxiety, and improve self-efficacy, ultimately enhancing math literacy.

The strong association between socioeconomic status and mathematical literacy may arise from the high school entrance examination in Taiwan. Promoting cross-school collaboration and resource sharing could help reduce educational inequality.

This study did not explore mediating or moderating variables affecting math literacy, which could be examined in future research.

References

Al-Hija, F. N. A., & Araidy, R. Z. (2023). Correlations Between Achievements in Mathematics, Student Characteristics, Classroom Climate and Self-Efficacy. *Journal of Higher Education Theory and Practice*, 23(8), 252-264. https://doi.org/10.33423/jhetp.v23i8.6079

Bandura A. (1977). *Social Learning Theory.* Englewood Cliffs, NJ: Prentice-Hall.

Bong, M., & Skaalvik, E. M. (2003). Academic self-concept and self-efficacy: How different are they really? *Educational Psychology Review, 15*(1), 1–40. https://doi.org/https://doi.org/10.1023/A:1021302408382

Butz, A. R., & Usher, E. L. (2015). Salient sources of early adolescents' self-efficacy in two domains. *Contemporary Educational Psychology, 42*, 49–61. https://doi.org/10.1016/j.cedpsych.2015.04.001

Chen, C. W., Chung, C. Y., Lin, R. J., & Deng, J. W. (2018). A Study of the Relationship Between Taiwan Students' Self Efficacy and Learning Achievements. *Journal of Educational and Multicultural Research,18*, 77-108. https://doi.org/10.3966/207802222018110018003

Chen, Y. J., Tso, T. C., & Liu, C. J. (2024). Construction and Validation of a Mathematics Anxiety Scale for Secondary School Students. *Contemporary Journal of Science Education, 32*(2), 207-231. https://doi.org/10.6173/CJSE.202406_32(2).0004

Dreger, R. M., & Aiken Jr, L. R. (1957). The identification of number anxiety in a college population. *Journal of Educational Psychology, 48*(6), 344–351. https://doi.org/10.1037/h0045894

Fennema, E., & Sherman, J. A. (1976). Fennema-Sherman mathematics attitudes scales: Instruments designed to measure attitudes toward the learning of mathematics by females and males. *Journal for Research in Mathematics Education, 7*(5), 324–326. https://doi.org/10.2307/748467

Ho, Y. C., & Yang, K. H. (2022). Effects of a Machine Learning-Based Digital Game-Based Learning Approach on Elementary School Students' Mathematics Learning Performance. *International Journal on Digital Learning Technology, 14*(1), 79-101. https://doi.org/10.53106/2071260X2022011401004

Kuo, P. H. (2024). Meta-Analysis of the Correlation Between Mathematics Anxiety and Mathematics Learning Achievement Among Students in Taiwan. *School Administrators,* (153), 107-137. https://doi.org/10.6423/HHHC.202409_(153).0005

Lee, V. E. (2000). Using hierarchical linear modeling to study social context: The case of school effects. *Education Psychologist, 35*, 125-141. https://doi.org/10.1207/S15326985EP3502_6

Lin, S. W. (2019). Relationship between Reading Strategies and Mathematical Literacy. *Psychological Testing, 66*(3), 213-248. https://www.airitilibrary.com/Article/Detail?DocID=16094905-201909-201910080015-201910080015-213-248

Liu, Y. C., Chen, P. L., & Hung, C. H. (2017). Math Academic Resilience as a Moderator: A Study of Junior High School Student's Math of Test Anxiety on Math Academic Achievement. *Curriculum & Instruction Quarterly, 20*(4), 151-176. https://doi.org/10.6384/CIQ.201710_20(4).0006

Organisation for Economic Co-operation and Development. (2023). *PISA 2022 results (Volume I): The state of learning and equity in education*. https://doi.org/10.1787/53f23881-en

Richardson, F. C., & Suinn, R. M. (1972). The mathematics anxiety rating scale: Psychometric data. *Journal of Counseling Psychology, 19*(6), 551–554. https://doi.org/10.1037/h0033456

Weissgerber, S. C., Grünberg, C., Neufeld, L., Steppat, T., & Reinhard, M. (2022). The interplay of math anxiety and math competence for later performance. *Social Psychology of Education: An International Journal, 25*(4), 977-1002. https://doi.org/10.1007/s11218-022-09700-y

TSG3-02

AN EXPLORATORY STUDY OF MIDDLE SCHOOL STUDENTS' PERFORMANCE AND TYPICAL ERRORS IN MODELING TASKS

Qianhui Zhao
East China Normal University, China, 52275500047@stu.ecnu.edu.cn

Yang Ren
East China Normal University, China, 51255500114@stu.ecnu.edu.cn

Shuhui Li
East China Normal University, China, shli@math.ecnu.edu.cn

Mathematical modeling has gradually become an important approach for students to apply mathematics in exploring real-world problems. This study is an explorative research study aimed at investigating middle school students' performance in modeling tasks, with a particular focus on their modeling competency and typical errors. Data were collected from twelve eighth-grade students in Hangzhou, China, through two modeling tasks. The results reveal that students' modeling competency was generally at low or medium levels. A total of ten typical errors were identified. Among them, errors related to "number consideration" were particularly prominent, and students often provided "no answer" during the "interpreting" and "validating" steps. Limitations and future directions are discussed at the end.

Keywords: modelling, error analysis, secondary school mathematics, Chinese mathematics education

Introduction

Mathematical modeling, as advocated for in many national mathematics curricular standards (e.g., MOE, 2022), is increasingly valued as a cyclical process carried out by transferring real-life problem situations to the world of mathematics (Blum, 2011). Previous modeling studies focused mainly on high school, with comparatively less attention on middle school students. Existing studies with middle school students usually focus on modeling competence (e.g., Greefrath, 2022) and affective domain (e.g., Yang, 2024), and little is known about students' errors in the modeling process, which may provide insights for the teaching and learning of modeling in middle schools. Therefore, our research question is: How do middle school students perform in mathematical modeling tasks?

Analytical Framework

This study established an analytical framework to explore students' overall modeling competency and typical errors in mathematical modeling tasks. Regarding overall modeling competency, the study adopts the three-level scheme for adequacy proposed by Lu & Kaiser (2022) to evaluate students' performance in modeling tasks into high, medium, and low levels. Concerning typical errors, oriented towards the seven-step modeling cycle (Blum & Leiß, 2007), Blum (2011) showed some typical examples of students' difficulties in the seven modeling steps: constructing, simplifying/structuring, mathematising, working mathematically, interpreting, validating, and exposing. In-

spired by Ulu's (2017) list of error types of fourth-grade students when solving modeling problems, we categorize students' errors in seven modeling steps into: missing critical information (MCI), incorrect relations/structure (IR/S), number consideration (NC), lack of understanding of application (LUA).

Methods

This study was conducted in January 2025 with 12 eighth-grade students (3 girls and 9 boys) from Hangzhou, China, abbreviated as S1 to S12. The students had almost no prior experience with modeling activities. The test included two modeling tasks (see Fig. 1) with a time limit of 40 minutes. They were adapted from modeling tasks in published mathematical modeling competition manuals, books, and conference papers. Both tasks were pre-tested by two graduate students majoring in mathematics education and then reviewed by an expert in mathematics education to ensure validity. For data analysis, two researchers coded the 24 task responses and rated them as high, medium, or low, with a consistency of 92%. Then, content analysis was used to identify students' typical errors, where two researchers coded errors shown in each modeling step and ultimately categorized them into four major error types, with a consistency of 97%. For the inconsistent coding, the two researchers reached an agreement after discussion.

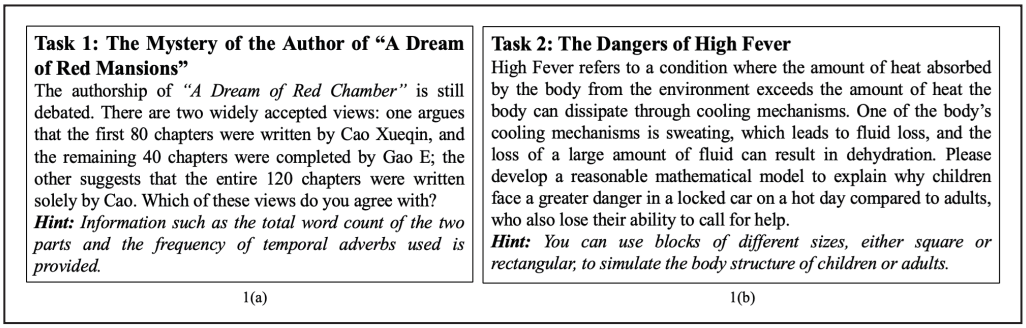


Figure 1. Modeling tasks

Results

Table 1 shows students' modeling competency in Task 1 and Task 2. Overall, the students' modeling competency was at low or medium levels. Students performed slightly better on Task 1. Only one student reached a high level in Task 1, whereas none did in Task 2, likely because Task 2 contained more distracting information that led students to focus on irrelevant details rather than core elements. Five students reached a medium level in Task 1, and three in Task 2.

Table 1. Overall modeling competency

No./level	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
Task 1	H	L	M	L	M	L	M	L	L	N/A	M	M
Task 2	M	L	L	L	L	M	L	L	L	L	L	M

Note. "H" refers to High; "M" refers to Medium; "L" refers to Low; and "N/A" refers to No Answer.

Table 2 summarizes the ten typical errors that the students made in different modeling steps: constructing, simplifying/structuring, mathematising, working mathematically, interpreting, and validating, with examples in Figure 1. Specifically, number consideration errors account for the largest proportion, followed by incorrect relations/ structure, missing critical information, and lack of understanding of application in descending order.

Table 2. Typical errors

Modeling step	Specific errors	Type
Construct	Lack of organization of the known information (7)	MCI (11)
	Incorrect identification of key information (4)	
Simplify /Structure	Feel hard to quantify indicators in real-life problems using math (16)	IR/S (22)
	The problem-solving plan deviates from the math perspective (6)	
Mathematise	Failure to construct a math model, or the constructed model does not align with the understanding of the problem (10)	NC (57)
	Failure to write or incorrect writing of variables used for modeling (15)	
	The constructed models, charts, data, etc., lack reliable evidence (10)	
Work mathematically	Failure to use rigorous math methods/rules in solving the problem (22)	
Interpret	Ignoring the conversion of math results into real-world outcomes (4)	LUA (10)
Validate	Neglecting to reflect on the model or modeling steps (6)	

Note. "MCI" refers to missing critical information, "IR/S" refers to incorrect relations/structure, "NC" refers to number consideration, and "LUA" refers to lack of understanding of application.

Step 1 & 2: Constructing & Simplifying/Structuring

Five students demonstrated a lack of attention or limited understanding of the problem context, leading to inefficient identification of core elements. For instance, the core issue was incorrectly identified by S11 as "analyzing the high mortality rate of children at certain ages". Additionally, seven students deviated from the mathematical perspective in their plan. In Task 2, S2 suggested a solution involving "creating a 1:1 human model and placing it in a constant-temperature box for heat dissipation simulation", which belongs to the realm of scientific experiments. It might stem from students' prolonged exposure to well-structured, closed-ended problems, which appears to hinder their ability to evaluate the relevance of given conditions or assess potential interference when solving ill-structured, open-ended tasks.

Step 3 & 4: Mathematising & Working Mathematically

As shown in F2(a), S6 did not present mathematical variables, such as the ratio of surface area to volume, to characterize the "faster heat dissipation" process. Furthermore, there is a widespread error of "lack of reliable sources for the constructed models or charts", as seen the body temperature

imbalance points and other elements in F2(b). It is also worth noting that eleven students, during the "working mathematically" step, tended to make empirical statements, such as "children dissipate heat slower, adults dissipate heat faster", but did not employ rigorous mathematical methods or rules to solve the problem.

Step 5 to 7: Interpreting, Validating & Exposing

Six students provided "no answer" results in the *interpreting* and *validating* steps, suggesting that they may lack substantive consideration of the application value of mathematics. Additionally, no typical error was identified in the *exposing* step.

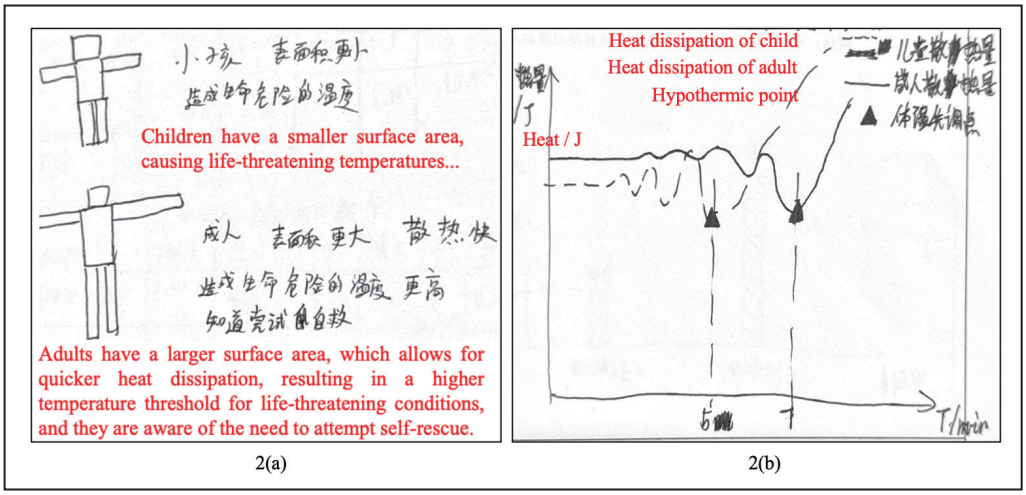


Figure 2. Sample errors

Concluding Remarks

This study examines twelve eighth-grade Chinese students' overall modeling competency in two modeling tasks and typical errors in seven modeling steps. The main findings are listed as follows.

First, the eighth-grade students' modeling competency was at low or medium levels. This result corroborates Lu & Kaiser (2022)'s finding that high school students, due to a lack of sufficient mathematical knowledge and application skills, do not exhibit significant advantages in the adequacy of the modeling process when handling complex tasks; however, this does not undermine the originality of the ideas they have demonstrated.

Second, students exhibited ten typical errors, which can be classified into four major types. Among them, number consideration errors were particularly prominent, exposing the challenges students faced in the *mathematising* and *working mathematically* steps. However, no errors related to calculation were identified, which may be directly associated with the allowance of calculators in the test. This also, from another perspective, confirms that modeling activities place more emphasis on evaluating the appropriateness of the modeling procedures rather than their correctness and minor calculation errors (Lu & Kaiser, 2022). In addition, students often neglected to convert mathematical results into real-world outcomes, lacked reflection, and provided "no answer" during the *interpreting* and *validating* steps. Therefore, instructors can build targeted scaffolding for students based on their common errors (Ulu, 2017). For example, instructors can provide step-by-step guidance in the form of a problem chain to help students overcome the difficulty of mathematization.

This study has certain limitations. Only twelve students participated, and the limited sample size may affect the generalizability of the findings. Future studies should include a larger sample size and address diverse participants to enhance the generalizability of the findings. In addition, students' prior modeling experience was not fully considered. It will be necessary to delve deeper into the participants' backgrounds to obtain a more comprehensive research perspective and conclusions.

References

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 15-30). Springer.

Blum, W., & Leiß, D. (2007). How do students and teachers deal with mathematical modeling problems? The example of Sugerloaf. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling: Education, engineering and economics—ICTMA12* (pp. 222-231). Horwood.

Greefrath, G., Siller, H.-S., Vorhölter, K., & Kaiser, G. (2022). Mathematical modelling and discrete mathematics: Opportunities for modern mathematics teaching. *ZDM – Mathematics Education*, 54(4), 865–879.

Lu, X., & Kaiser, G. (2022). Can mathematical modelling work as a creativity-demanding activity? An empirical study in China. *ZDM – Mathematics Education, 54*(1), 67-81.

Ministry of Education of the People's Republic of China. (2022). *The mathematics curriculum standards for compulsory education*. Beijing Normal University Publishing Group.

Ulu, M. (2017). Errors made by elementary fourth grade students when modelling word problems and the elimination of those errors through scaffolding. *International Electronic Journal of Elementary Education*, *9*(3), 553-580.

Yang, K.-L., Krawitz, J., Schukajlow, S., Yang, C.-C., & Chang, Y.-P. (2024). German and Taiwanese secondary students' mathematical modelling task value profiles and their relation to mathematical knowledge and modelling performance. *European Journal of Psychology of Education*, 39(3), 2969-2989.

TSG3-03

EXAMINING MASTERY OF COGNITIVE ATTRIBUTES IN MATHEMATICS USING SOUTH KOREAN NATIONAL ASSESSMENT OF EDUCATIONAL ACHIEVEMENT DATA

Hye-Yun Jung
Gongju National University of Education, South Korea, hy0501@gjue.ac.kr

Jihyun Hwang
Korea National University of Education, South Korea, jihyun-hwang@knue.ac.kr

This study analyzed and categorized the types of mathematical academic achievement in order to provide students with customized feedback for each type Tto develop a method for analyzing the type of mathematical academic achievement, a comprehensive investigation into the theoretical background and analytical methodologies were conducted. Based on this, to examine the type of mathematical academic achievement, the latent profile analysis and cognitive diagnostic model analysis were conducted using the results of the 2021 National Assessment of Educational Achievement(NAEA). The key findings are as follows. First, based on the cognitive diagnostic model analysis, five cognitive attributes were extracted and their hierarch structure was confirmed. Second, the analysis revealed that the types of mathematical academic achievement could be divided into 6 distict groups. By categorizing and describing the names and achievement characteristics of each group, this study demonstrated the potential for providing customized feedback based on these categorization.

Keywords: cognitive diagnostic models, cognitive attributes, Q -matrix, mathematics academic achievement type

Introduction

Recently, the purpose of assessment moved from assessing studnets' academic level to identifying students' strengths and weaknesses and supporting their learning. Cognitive diagnostic models can help students and teachers identify students' strengths and weaknesses revealed in their learning performance using cognitive attributes and Q-matrix. In summary of this discussion, in this study, we tried to analyze and categorize the students' types of mathematical academic achievement. To this end, we firstly extracted the cognitive attributes reflected in the test items, developed a Q-matrix, and categorized students according to their types of mathematical academic achievement.

Literature Review

Most previous literatures (e.g., Kim et al., 2008) the derived cognitive attributes presented five or six attributes, including calculation, selection, modeling, analysis, recognition, and reinvention. According to the previous literatures, it is necessary to show the process of deriving cotnitive attributes and Q-matrix.

Method

This study analyzed the results from the 2021 NAEA in South Korea, specifically focusing on the high school mathematics assessment. Data from 931 students, some of the participants, were included in the analysis for this study.

The research was conducted through a three-stage process: identification of cognitive attributes, development of the Q-matrix, and categorization of academic achievement types. Two authors of this study and a working group consisting of four high school teachers participated in each stage. In the first stage, the identification of cognitive attributes was carried out through a thorough review of assessment materials, previous research, and repeated consultations with mathematics education experts. The second stage involved the development of the Q-matrix, which was validated through both quantitative and qualitative methods, including the review of DINA model parameter estimates (Kim et al., 2012) and expert qualitative evaluation (Kim et al., 2019). The third stage focused on categorizing academic achievement types.

Results

Identified Cognitive Attributes

The initial identification of cognitive attributes resulted in the identification of 16 attributes: arithmetic calculations, routine algebraic calculations, recall, distinction, understanding of concepts and properties, heuristic reasoning, deductive reasoning, reflection on the reasoning process, mathematical representation, derivation of mathematical models, algebraic problem solving, reflection on the problem-solving process, internal connectivity, external connectivity, data interpretation, and information utilization.

Based on the expert reviews and subsequent discussions within the working group, five cognitive attributes—recall, distinction, reflection on the reasoning process, derivation of mathematical models, and reflection on the problem-solving process—were excluded, resulting in a refined set of 11 cognitive attributes: arithmetic calculations, routine algebraic calculations, understanding of concepts and properties, heuristic reasoning, deductive reasoning, mathematical representation, algebraic problem solving, internal connectivity, external connectivity, data interpretation, and information utilization. And after solving the test items, several attributes removed or divided into two distinct attributes.

Consequently, nine attributes was established: routine algebraic calculations, understanding of concepts and properties, deductive reasoning, Mathematical Representation 1, Mathematical Representation 2, algebraic problem solving, internal connectivity, external connectivity, and data interpretation.

Development of the Q-matrix

The initial Q-matrix was developed by linking the cognitive attributes with the test items. The validity of this Q-matrix was then assessed, specifically by examining the sum of the cognitive attributes matched to each item and the sum of items reflecting each cognitive attribute. Through the repeated Q-matrix validation work, several attributes were removed and new attributes were presented. Table 1 shows the revised cognitive attributes.

Table 1. Revised cognitive attributes

Cognitive attributes	Name	Description
Cognitive attribute 1 (CA1)	Understanding concepts and performing calculating	Understands basic mathematical concepts and performs calculations on given simple expressions
Cognitive attribute 2 (CA2)	Reasoning	Students can independently derive mathematical facts or propositions, distinguish between true and false statements, and logically describe all or part of the problem-solving process.
Cognitive attribute 3 (CA3)	Representing information mathematically	Expresses or interprets real-life data or information presented in the problem using symbols, expressions, graphs, etc., and applies the results.
Cognitive attribute 4 (CA4)	Transforming mathematical representation	Expresses and utilizes given mathematical representations (such as symbols, expressions, or graphs) in other mathematical forms depending on the purpose.
Cognitive attribute 5 (CA5)	Establishig and executing problem-solving strategies	Identifies relationships between mathematical concepts and domains to establish and execute strategies for solving problems.

Students' Mastery of Cognitive Attribute

Based on the revised version of the Q-matrix, a total of 14 latent groups were identified (see Table 2). In the notation of the latent groups, "0" means that the corresponding cognitive attribute was not mastered, while "1" means that the cognitive attribute was mastered. For example, a student in latent group 00100 has mastered CA3, but has not mastered the other attributes.

Table 2. The frequency and percentage of each latent group

Mastery pattern	Number of students	%	Mastery pattern	Number of students	%
00000	122	13.22	01101	3	0.33
00100	116	12.57	10110	1	0.11
10100	11	1.19	10101	4	0.43
01100	2	0.22	11110	66	7.15
11000	26	2.82	11101	18	1.95
10001	5	0.54	10111	14	1.52
11100	92	9.97	11111	443	48.00

Subsequently, the probability of mastering a cognitive attribute j given that another cognitive attribute i has mastered was also checked and the hierarchy structure of cognitive attributes was constructed. The lowest stage is CA3,followed by CA1 and CA2, then followed by CA4, and the highest stage is CA5.

Discussion

Based on the results, students' mastery patterns was categorized into seven groups: the group that no cognitive attributes were mastered (00000), the group that mastered only CA3 (00100), the groups that mastered CA1 and CA3 (10100) or CA2 and CA3 (01100), the group that mastered CA1, CA2, and CA3 (11100), the group that mastered CA1, CA2, CA3, and CA4 (11110), and the group that mastered all the cognitive attributes (11111). In the notation of the latent groups, "0" means that the corresponding cognitive attribute was not mastered, while "1" means that the cognitive attribute was mastered. For example, a student in latent group 01100 has mastered cognitive attributes 2 and 3, but has not mastered the other attributes. This hierarchical approach underscores the importance of addressing specific cognitive needs in mathematics education. Furthermore, the categorized types of mathematical academic achivement imply the necessity for providing studetns with customized feedback for each type.

References

Kim, H, Han. J., Choi, S. K., & Kim, B. (2012). *The application of cognitive diagnostic model for achievement profile analysis and score report*. Korea Institute for Curriculum and Evaluation.

Kim, H., Kim, W., Kim, S., Jung, H., Kim, M., & Kim, S. (2019). *Improvement of National Assessment of Educational Achievement scoring system and strategies for enhancing the utilization of the NAEA results*. Korea Institute for Curriculum and Evaluation.

Kim, S., Kim, S., & Song, M. (2008). Using cognitive diagnosis theory to analyze the test results of mathematics. *School Mathematics, 10*(2), 259-277.

TSG3-04

FROM ICT USE TO CREATIVE THINKING: THE MEDIATION OF ATTITUDES TOWARDS CREATIVE THINKING AMONG EAST ASIAN STUDENTS

Si Chen
Beijing Normal University, China, 51205500042@stu.ecnu.edu.cn

Qihang Guo
East China Normal University, China, 52275500045@stu.ecnu.edu.cn

Information and communication technology (ICT) plays an increasingly vital role in fostering creative thinking among students. In this process, the role of students' internal characteristics needs further investigation, particularly the creativity-related dispositions. This study examined the relationship between ICT use and creative thinking among East Asian students, with a specific focus on the mediation of creative self-efficacy and openness to intellect, after controlling for gender and economic, social and cultural status (ESCS). Based on PISA 2022 data and analyzed using structural equation modeling (SEM), the results indicated a significant positive association between ICT use and creative thinking, even after accounting for the two attitudinal mediators. Additionally, a significant positive mediation effect through openness to intellect was found, whereas the path through creative self-efficacy was not statistically significant. These findings suggested that educational interventions should target not only ICT integration but also the cultivation of students' openness to intellect to enhance their creative thinking.

Keywords: creative thinking, ICT use, creative self-efficacy, openness to intellect, East Asia

Introduction

Creative thinking has gained increasing importance in education in response to rapid globalization and technological developments (Alencar, Fleith, & Pereira, 2017). In this context, information and communication technology (ICT) is essential for developing creative thinking among students in the 21st century (Angarita & Chiappe, 2019). PISA 2022 was conducted in the context of the COVID-19 pandemic, during which students experienced remote learning, highlighting the quality of ICT integration in education. Given the under-researched area of enhancing creative thinking in school education (Karunarathne & Calma, 2024), it is important to investigate student-related factors, particularly their creative dispositions, to better explain and understand these differences.

Accordingly, this study aims to examine the influence of ICT-related factors on East Asian students' creative thinking performance, as well as the mediation of attitude-related factors, addressing the following two research questions: (1) to what extent does ICT use influence East Asian students' creative thinking? (2) Is the relationship between ICT use and creative thinking among East Asian students mediated by creative self-efficacy and openness to intellect, after controlling for gender and economic, social and cultural status?

Literature Survey

Defining Creative Thinking

Creative thinking in PISA 2022 refers to the cognitive processes required to engage in creative work, and is defined as "the competence to engage productively in the generation, evaluation and improvement of ideas that can result in original and effective solutions, advances in knowledge and impactful expressions of imagination" (OECD, 2023, p. 144). Similarly, the Australian Council for Educational Research (ACER) defines creative thinking as "the capacity to generate many different kinds of ideas, manipulate ideas in unusual ways, and make unconventional connections in order to outline novel possibilities that have the potential to meet a given purpose elegantly" (Ramalingam et al., 2020, p. 5). Both definitions suggest that creative thinking is an individual skill that can be learned and developed, and they emphasize cognitive processes (Karunarathne & Calma, 2024; OECD, 2023).

Role of ICT in Creative Thinking, Creative Self-efficacy and Openness to Intellect

Numerous studies have indicated that ICT use has a positive impact on the development of creative thinking (e.g., Angarita & Chiappe, 2019; Riley & Ahlberg, 2004; Allegra et al., 2001; Guan et al., 2024). The learning environment enhanced by innovative technologies plays a crucial role in fostering creative thinking (Beghetto & Kaufman, 2014; Fan & Cai, 2022; Wang & Burdina, 2024), which is often attributed to creative collaboration enabled by the interactive nature of digital tools (Wishart, 2014). Educational tools that have been shown to significantly enhance students' creative thinking include augmented reality (AR) (Li, 2024) and gamified virtual classroom learning environments (Wannapiroon & Pimdee, 2022).

While no significant correlation was found between ICT usage and overall personality traits (Hassanzadeh et al., 2012), a recent meta-analysis using subgroup analysis revealed that the use of various ICT tools was significantly associated with the Big Five personality traits. Among these traits, openness showed the strongest correlations, particularly with education-oriented and information-oriented ICTs (Joshi et al., 2023). Moreover, ICT was found to facilitate students' creative self-efficacy, as evidenced by interventions such as VR-supported projects (Lin & Wang, 2021). In addition, creative self-efficacy has been shown to mediate the relationship between mobile phone use and creative ideation (Guan et al., 2024).

Roles of Openness to Intellect and Creative Self-efficacy in Creative Thinking

A positive relationship has been consistently observed between creative self-efficacy and creative thinking (Smith, 2022; Tang et al., 2022; Puente-Diaz & Cavazos-Arroyo, 2023), and several studies have demonstrated a significant mediation of creative self-efficacy (Yang et al., 2020; Zhang et al., 2020; Li & Wu, 2011; Liu et al., 2024). However, some research has reported a negative association between creative self-efficacy and mathematical creative thinking (Suherman & Vidákovich, 2024).

The trait 'openness to intellect', as defined in PISA 2022 (OECD, 2023), has been found to correlate particularly with creative achievement in the sciences rather than the arts (Kaufman, 2013; Kaufman et al., 2016), corresponding to the term 'openness/intellect' used in existing literature (e.g., DeYoung, 2014; Kaufman et al., 2016). Some studies have shown that intellect exhibits either non-significant or marginally negative relationships with divergent thinking (Chuang & Hsu, 2025)—a core component of creative thinking—and is not significantly related to creative activities and achievements in any domain (Pi et al., 2019; Goulet-Pelletier & Cousineau, 2024). Conversely, other studies have reported a positive influence of intellect on originality scores and ratings of the best idea (Puente-Diaz & Cavazos-Arroyo, 2023).

Method

This study used secondary data from PISA 2022, focusing on five East Asian regional subsamples, referring to communities or educational systems sharing Confucian cultural heritage (Leung & Revina, 2023), including Hong Kong, Macao, Chinese Taipei, Korea, and Singapore; Japan was excluded due to its non-participation in the creative thinking assessment. The original dataset comprised 29,208 students, and after excluding cases with missing data on any examined variables, the final analytic sample consisted of 26,864 students (48.9% female, 51.1% male). PISA 2022 assigned ten plausible values (PV1CERTH to PV10CERTH) to estimate student performance on the creative thinking test. Additionally, relevant student-perceived variables were selected from the PISA 2022 questionnaires (OECD, 2024).

The ICT-use related factors examined in this study include: *Support or feedback via ICT (ICTFEED), Subject-related ICT Use During Lessons (ICTSUBJ), Use of ICT in enquiry-based learning activities (ICTENQ), Use of ICT for school activities outside of the classroom (ICTOUT).*

Structural Equation Modeling (SEM) using Mplus was conducted to examine the mediation of openness to intellect and creative self-efficacy in the relationship between ICT use and East Asian students' creative thinking, while controlling for two student background variables - economic, social and cultural status (ESCS) and gender (coded as 1 = female, 2 = male).

Results

The reliability and convergent validity of latent variables were first assessed and the results of ICT use (CR = 0.762, AVE = 0.456) indicated acceptable levels of reliability and convergent validity. The structural equation model demonstrated a good fit to the data (CFI = 0.961, TLI = 0.955, RMSEA = 0.067, SRMR = 0.073).

The direct and indirect effects of ICT use on creative thinking are shown in *Fig. 1*. The total effect of ICT use on creative thinking was statistically significant ($\beta = 0.129$, $p < 0.001$, 95% CI = [0.115, 0.144]), indicating a positive relationship.

The relationship between ICT use and creative thinking through multiple pathways was tested using a bias-corrected bootstrap method with 5,000 resamples. Two indirect pathways revealed differential mediation patterns: The path through openness to intellect was significant ($\beta = 0.020$, $p < 0.001$, 95% CI= [0.015, 0.026]), whereas the pathway through creative self-efficacy was not ($\beta = -0.002$, $p = 0.481$, 95% CI = [-0.007, 0.003]). Those results suggested that ICT use affects creative thinking primarily via openness to intellect rather than via creative self-efficacy, significantly accounting for 15.6% of the total effect. The direct effect of ICT use on creative thinking remained significant ($\beta = 0.111$, $p < 0.001$, 95% CI = [0.095, 0.127]), even after accounting for both mediators, suggesting that the mediation is partial rather than full.

After controlling for gender and ESCS, both covariates exhibited statistically significant associations with creative thinking. Male students showed significantly lower creative thinking scores than female students ($\beta = -0.136$, $p < 0.001$, 95% CI = [-0.146, -0.125]). ESCS showed a strong relationship with creative thinking ($\beta = 0.393$, $p < 0.001$, 95% CI = [0.382, 0.403]), suggesting that students from higher ESCS backgrounds tended to have better creative thinking performance.

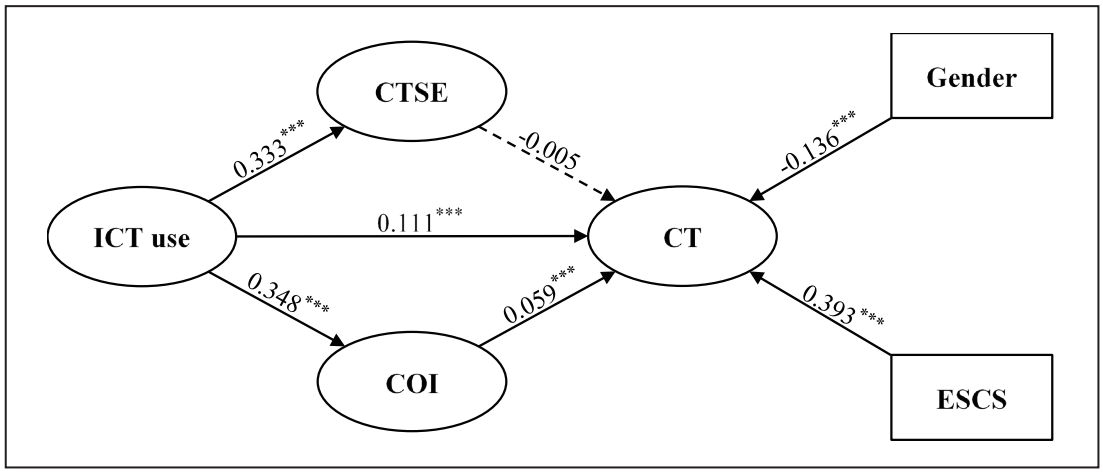


Figure 1. Path model for the effects of students' ICT use on Creative Thinking [1]

Conclusion and Implication

This study concluded that ICT use significantly enhanced students' creative thinking in East Asian. A significant mediation of openness to intellect was also observed, highlighting the dual role of ICT in the development of creative thinking—both by enabling interactive learning environments and by fostering students' openness to intellect. This suggested that pedagogical integration of ICT may more effectively equip students with opportunities to enhance creativity, particularly by stimulating intellectual openness in science-related learning. Further research should explore cross-national differences in educational cultures to provide more context-specific explanations.

References

Alencar, E. M. S., Fleith, D., & Pereira, N. (2017). Creativity in higher education: Challenges and facilitating factors. *Trends in Psychology*, *25*(2), 553-561.

Allegra, M., Chifari, A., & Ottaviano, S. (2001). ICT to train students towards creative thinking. *Journal of Educational Technology & Society*, *4*(2), 48-53.

Angarita, L. M., & Chiappe, A. (2019). Are ICT good partners for the development of creativity? A systematic review of literature. *International Journal of Arts and Technology*, *11*(3), 231-248.

Beghetto, R. A., & Kaufman, J. C. (2014). Classroom contexts for creativity. *High Ability Studies*, *25*(1), 53–69.

Chuang, S. C., & Hsu, K. Y. (2025). Beyond the intellect/openness model? Relationships between Chinese openness and cognitive performance. *Psychology of Aesthetics, Creativity, and the Arts*.

Chulvi, V., Mulet, E., Felip, F., & García-García, C. (2017). The effect of information and communication technologies on creativity in collaborative design. *Research in Engineering Design*, *28*, 7-23.

1 Note: The numbers correspond to standardized coefficients. ***$p < 0.001$.

DeYoung, C. (2014), Openness/intellect: a dimension of personality reflecting cognitive exploration. In M. Cooper, & R. Larsen(Eds.), *APA Handbook of Personality and Social Psychology: Personality Processes and Individual Differences* (pp. 369-399), American Psychological Association.

Fan, M., & Cai, W. (2022). How does a creative learning environment foster student creativity? An examination on multiple explanatory mechanisms. *Current Psychology*, *41*(7), 4667-4676.

Goulet-Pelletier, J. C., & Cousineau, D. (2024). Exploring Creative Spaces Predict Domain-Specific Creative Achievements. *The Journal of Creative Behavior*, *58*(4), 592-612.

Guan, J., Yang, Y., Ma, W., Li, G., & Liu, C. (2024). The relationship between mobile phone use and creative ideation among college students: The roles of critical thinking and creative self-efficacy. *Psychology of Aesthetics, Creativity, and the Arts*.

Harlen, W. & Deakin, R. (2003). *A systematic review of the impact on students and teachers of the use of ICT for assessment of creative and critical thinking skills*, EPPI-Centre, University of London.

Hassanzadeh, V., Gholami, R., Allahyar, N., & Noordin, N. (2012). Motivation and personality traits of TESL postgraduate students towards the use of information and communications technology (ICT) in second language teaching. *English Language Teaching*, *5*(4), 74-86.

Joshi, A., Das, S., & Sekar, S. (2023). How big five personality traits affect information and communication technology use: a meta-analysis. *Australasian Journal of Information Systems*, *27*.

Karunarathne, W., & Calma, A. (2024). Assessing creative thinking skills in higher education: deficits and improvements. *Studies in Higher Education*, *49*(1), 157-177.

Kaufman, S. B. (2013). Opening up openness to experience: A four-factor model and relations to creative achievement in the arts and sciences. *The Journal of Creative Behavior*, *47*(4), 233-255.

Kaufman, S. B., Quilty, L. C., Grazioplene, R. G., Hirsh, J. B., Gray, J. R., Peterson, J. B., & DeYoung, C. G. (2016). Openness to experience and intellect differentially predict creative achievement in the arts and sciences. *Journal of personality*, *84*(2), 248-258.

Leung, F. K., & Revina, S. (2023). The Influence of Culture on Students' Mathematics Achievement in East Asia. In W. Lee, P. Brown, A. L. Goodwin, & A. Green(Eds.), *International Handbook on Education Development in the Asia-Pacific* (pp. 1463-1479). Springer Nature Singapore.

Li, C. H., & Wu, J. J. (2011). The structural relationships between optimism and innovative behavior: Understanding potential antecedents and mediating effects. *Creativity Research Journal*, *23*(2), 119-128.

Li, T. (2024). The influence of information technologies on creative and critical thinking of students. *Culture and Education*, *36*(3), 571-601.

Lin, Y. J., & Wang, H. C. (2021). Using virtual reality to facilitate learners' creative self-efficacy and intrinsic motivation in an EFL classroom. *Education and Information Technologies*, *26*(4), 4487-4505.

Liu, G., Zhang, X., Liu, Z., & Zhang, J. (2024). Staged and gendered pathways from parenting behaviors to Chinese students' creative thinking: The roles of autonomous motivation and creative self-efficacy. *Psychology of Aesthetics, Creativity, and the Arts*.

OECD (2023), *PISA 2022 Assessment and Analytical Framework.* Paris. OECD Publishing.

OECD (2024), *PISA 2022 Technical Report.* Paris. OECD Publishing.

Pi, Z., Hong, J., & Hu, W. (2019). Interaction of the originality of peers' ideas and students' openness to experience in predicting creativity in online collaborative groups. *British Journal of Educational Technology*, *50*(4), 1801-1814.

Puente-Diaz, R., & Cavazos-Arroyo, J. (2023). The contribution of openness and intellect to the examination of the idea generation and selection processes: an exploration with business students. *Creativity Research Journal*, *35*(2), 189-195.

Ramalingam, D., Anderson, P., Duckworth, D., Scoular, C., & Heard, J. (2020). *Creative thinking: Definition and structure.* Australian Council for Educational Research.

Riley, N. R., & Åhlberg, M. (2004). Investigating the use of ICT-based concept mapping techniques on creativity in literacy tasks. *Journal of computer assisted learning*, *20*(4), 244-256.

Smith, J. E. (2022). Creative self-efficacy: An essential transition skill for students With learning disabilities. *Intervention in School and Clinic*, *57*(4), 256-261.

Suherman, S., & Vidákovich, T. (2024). Role of creative self-efficacy and perceived creativity as predictors of mathematical creative thinking: Mediating role of computational thinking. *Thinking Skills and Creativity*, *53*, 101591.

Tang, C., Duan, Q., & Long, H. (2022). How do parents influence student creativity? Evidence from a large-scale survey in China. *Thinking Skills and Creativity*, *46*, 101134.

Wang, A., & Burdina, G. (2024). Developing students' creative thinking using innovative education technologies. *Interactive Learning Environments*, *32*(7), 3490-3500.

Wannapiroon, N., & Pimdee, P. (2022). Thai undergraduate science, technology, engineering, arts, and math (STEAM) creative thinking and innovation skill development: a conceptual model using a digital virtual classroom learning environment. *Education and Information Technologies*, *27*(4), 5689-5716.

Wishart, J., Eagle, S. (2014). The Development of a Scale to Assess Creative Collaboration via Online Tools. In D. K. W. Chiu, M. Wang, E. Popescu, Q. Li, & R. Lau(Eds.), *New Horizons in Web Based Learning*: *ICWL 2012* (pp. 320-329). Springer.

Yang, Y., Xu, X., Liu, W., & Pang, W. (2020). Hope and creative self-efficacy as sequential mediators in the relationship between family socioeconomic status and creativity. *Frontiers in Psychology*, *11*, 438.

Zhang, H., Sun, C., Liu, X., Gong, S., Yu, Q., & Zhou, Z. (2020). Boys benefit more from teacher support: Effects of perceived teacher support on primary students' creative thinking. *Thinking skills and creativity*, *37*, 100680.

TSG3-05

FACTORS PREDICTING EAST ASIAN STUDENTS' PERCEIVED QUALITY OF MATHEMATICS INSTRUCTION: A MACHINE LEARNING APPROACH

Xu Wei
Beijing Normal University, China, weixu99@mail.bnu.edu.cn

Kan Guo
Beijing Normal University, China, guokan@bnu.edu.cn

This research examines the factors predicting East Asian students' perceived quality of mathematics instruction using machine learning techniques. Utilizing data from PISA 2022, we focus on four East Asian regions: Chinese Taipei, Hong Kong, Korea, and Macao. Our machine learning model, based on XGBoost, achieved relatively high R-squared values in predicting perceived instructional quality, with $R^2 = 0.5977$ on the training set and $R^2 = 0.3520$ on the test set.. The study identifies student fondness in mathematics and teacher support as the most significant predictors of perceived instructional quality. Additionally, we employ SHAP (SHapley Additive exPlanations) values to enhance model interpretability, revealing the top 10 features that most impact perceptions. These findings suggest that enhancing student engagement and teacher support could substantially improve the perceived quality of mathematics instruction. The results of this study provide valuable insights for educational policymakers and practitioners seeking to enhance student academic performance in the East Asian context.

Keywords: Perceived Instructional Quality, Machine Learning, PISA 2022

Introduction

Perceived instructional quality is a significant determinant of educational effectiveness, comprising a range of teacher behaviors and activities, such as teaching-modelling, questioning, and management of time (Liu et al., 2024). It is also influenced by student and environmental characteristics. Perceived instructional quality has garnered attention in international large-scale assessments due to its capacity to effectively predict learning outcomes (OECD, 2023; Scherer et al., 2016). In East Asian countries or regions participating in PISA 2022 (Chinese Taipei; Hong Kong; Korea; and Macao), the correlation coefficient between perceived quality of mathematics instruction and mathematics achievement was 0.293 ($p < 0.01$), which is higher than the average of PISA 2022. Therefore, in the East Asian region, exploring factors that predict perceived quality of mathematics instruction holds practical significance for enhancing student academic performance.

Most researchers typically employ classical statistical methods such as linear regression and hierarchical linear models when examining the determinants of a dependent variable (Wang et al., 2023). However, in the social sciences, where the number of potential influencing factors is vast, classical statistical methods are inevitably prone to issues such as endogeneity, which can introduce biases into the results. Moreover, these methods tend to focus on explanation rather than prediction. In fact, in the field of education, prediction is often the primary objective. Therefore, machine learning methods, which prioritize predictive accuracy, have gained increasing attention in recent years. The

fundamental principles of machine learning are rooted in statistics and computation. Unlike classical methods, machine learning can automatically uncover complex structures beyond linear patterns and does not require researchers to make a priori assumptions. Additionally, machine learning techniques enhance the accuracy and generalizability of results through practices such as splitting data into training and testing sets or employing cross-validation (Yarkoni & Westfall, 2017).

Theoretical Underpinnings: Dynamic Model of Educational Effectiveness

The Dynamic Model of Educational Effectiveness (DMEE) provides a comprehensive perspective for examining the interactions between instructional quality and student characteristics (Kyriakides et al., 2020). Perceived instructional quality can also be regarded as an outcome of student learning. Therefore, this model not only highlights the impact of teacher behaviors on instructional quality but also underscores the influence of student background characteristics and environmental factors on perceived instructional quality (Creemers & Kyriakides, 2008). Consequently, in the selection of variables for this study, we adopted a data-driven approach while also integrating factors from multiple levels, including student, teacher, and school contexts.

Methods

Sample

This study utilized data from PISA 2022. Four East Asian economies—Chinese Taipei; Hong Kong; Korea; and Macao—were included. Japan was excluded due to extensive missing data across multiple variables, which would have compromised the training effectiveness of machine learning models. The sample comprised 22,602 fifteen-year-old students (Chinese Taipei = 5,857; Hong Kong = 5,907; Korea = 6,454; Macao = 4,384). The gender distribution was 48.5% female and 51.5% male.

Measures

Independent Variables

Prior research has demonstrated that proximal variables, such as student-perceived classroom characteristics and teaching practices, are more significant predictors of student learning outcomes compared to distal variables at the school and system levels (OECD, 2023). Therefore, this study focused solely on variables from the student background questionnaire. After excluding independent variables with substantial missing data and conducting feature selection through feature engineering (filter method) in machine learning, a total of 549 independent variables were ultimately included in the analysis.

Dependent Variable

The Perceived quality of mathematics instruction was measured using student subjective ratings, ranging from 1 (lowest) to 10 (highest).

Data Analysis

Machine Learning: XGBoost

XGBoost (eXtreme Gradient Boosting) is an efficient machine learning algorithm based on the gradient boosting framework and is widely applied in data mining and machine learning tasks. It integrates multiple decision tree models, optimizes the objective function using gradient descent,

and incorporates regularization terms (such as Lasso and Ridge) to prevent overfitting. XGBoost controls model complexity through L1 and L2 regularization, reducing the risk of overfitting. Additionally, it can automatically handle missing values and sparse features in the data (Chen & Guestrin, 2016).

Results

This study employed a 5-fold cross-validation approach with a maximum of 50 iterations. The model achieved a Mean Squared Error (MSE) of 2.1387 and an R-squared value of 0.5977 on the training set, and an MSE of 3.6060 with an R-squared value of 0.3520 on the test set, indicating solid predictive performance.

Machine learning models have traditionally been regarded as "black boxes" due to their complexity and difficulty in interpretation. However, the introduction of SHAP (SHapley Additive exPlanations) methods has significantly enhanced the transparency of these models. SHAP values, derived from the concept of Shapley values in cooperative game theory, are used to explain the contribution of each feature to the model's predictions, thereby facilitating an understanding of how individual features impact the model output (Lundberg & Lee, 2017). Based on SHAP values, Figure 1 presents the top 10 features with the greatest impact on perceived quality of mathematics instruction. The specific meanings of these features can be found in the PISA 2022 Technical Report. The color intensity represents the magnitude of the feature values, with SHAP values greater than zero indicating a positive influence on the dependent variable. Figure 2 displays partial dependence plots of the dependent variable against some important features. The plots reveal that the influence of these features on the dependent variable is monotonic. However, the slope distribution and curvature of each plot vary, indicating that the nature of the relationship between each feature and the dependent variable is distinct.

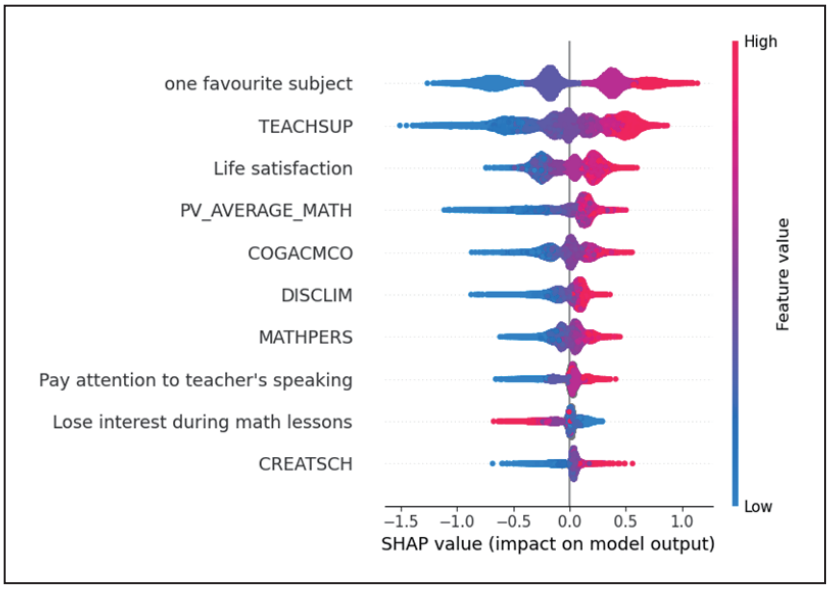


Figure 1. SHAP summary plot for top 10 features

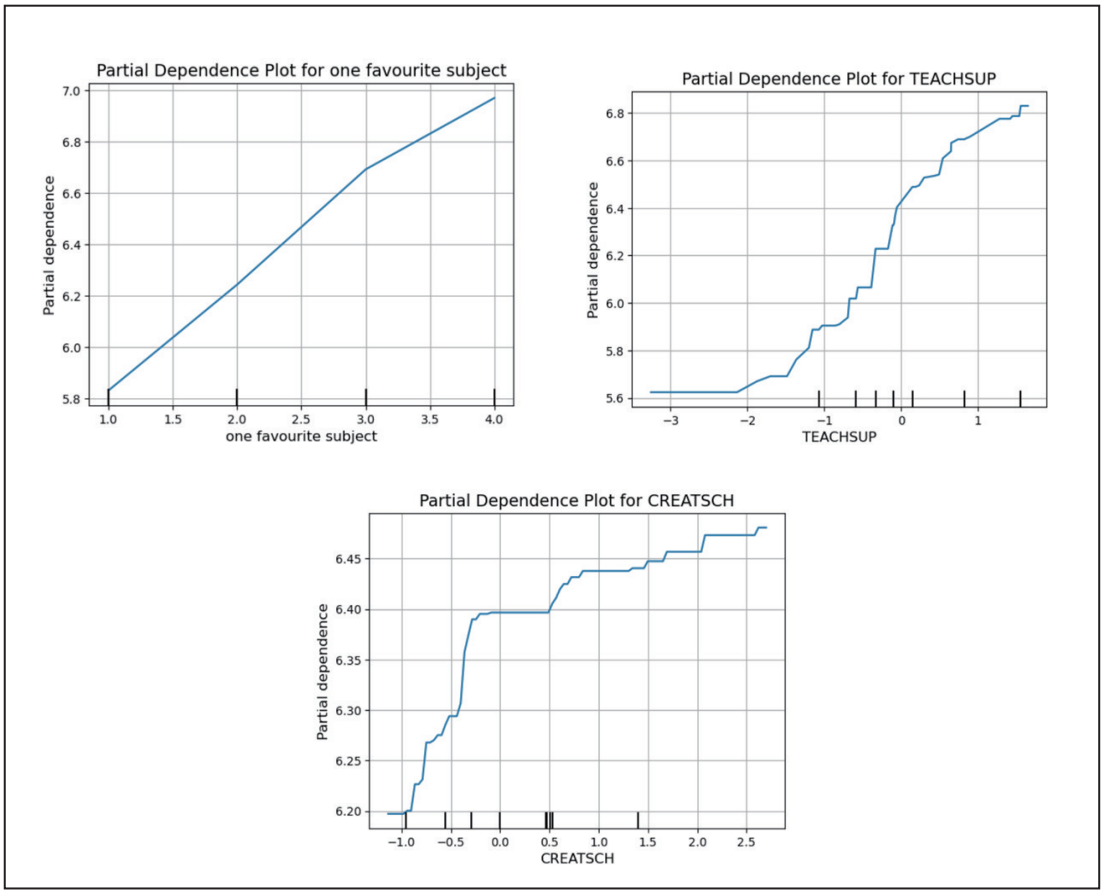


Figure 2. Partial dependence plots for top 10 features

Discussion

The application of machine learning to predict students' perceived quality of mathematics instruction has demonstrated a high explanatory power. Our findings indicate that students' fondness for mathematics is the most predictive factor of their perception of instructional quality. This could be attributed to the fact that when students have an interest in mathematics, they tend to focus more in math classes, consequently gaining more from the lessons. Additionally, teacher support has been identified as a significant factor. In PISA 2022, teacher support was characterized by teachers' efforts to assist students and persist in explaining until students understand. It is evident that the more support students receive from teachers, the better they are able to comprehend and master all the content within the curriculum. Lastly, a creative school and classroom environment also contributes to enhancing students' perceived quality of mathematics instruction. This suggests that when teachers value students' ideas and provide opportunities for exploration, it is equally important for the perceived quality of mathematics instruction. Moreover, we can find that the relationships between these predictors and the dependent variable are all nonlinear, which goes beyond the traditional linear model.

References

Chen, T., & Guestrin, C. (2016). XGBoost: A scalable tree boosting system. *Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 785–794.

Creemers, B., & Kyriakides, L. (2008). The dynamics of educational effectiveness: *A contribution to policy, practice and theory in contemporary schools*. Routledge.

Kyriakides, L., Creemers, B. P., Panayiotou, A., & Charalambous, E. (2020). Quality and equity in education: *Revisiting theory and research on educational effectiveness and improvement*. Routledge.

Liu, X., Hansen, K. Y., Valcke, M., & De Neve, J. (2024). A decade of PISA: Student perceived instructional quality and mathematics achievement across european countries. *ZDM – Mathematics Education*, 56(5), 859–891.

Lundberg, S. M., & Lee, S.-I. (2017). A unified approach to interpreting model predictions. *Proceedings of the 31st International Conference on Neural Information Processing Systems*, 4768–4777.

OECD. (2023). PISA 2022 assessment and analytical framework. OECD Publishing.

Scherer, R., Nilsen, T., & Jansen, M. (2016). Evaluating individual students' perceptions of instructional quality: An investigation of their factor structure, measurement invariance, and relations to educational outcomes. *Frontiers in Psychology*, 7.

Wang, F., King, R. B., & Leung, S. O. (2023). Why do east asian students do so well in mathematics? A machine learning study. *International Journal of Science and Mathematics Education*, 21(3), 691–711.

Yarkoni, T., & Westfall, J. (2017). Choosing Prediction Over Explanation in Psychology: Lessons From Machine Learning. *Perspectives on Psychological Science*, 12(6), 1100–1122.

TSG3-06

CREATIVITY IN MATH ASSESSMENTS

Ali Bicer
Texas A&M University, USA, alibicer@tamu.edu

Tugce Aldemir
Texas A&M University, USA, taldermir@tamu.edu

Geoff Krall
University of Wyoming, USA, tgkrall1@uwyo.edu

Fay Quiroz
University of Wyoming, USA, fsb24@uwyo.edu

Scott Chamberlin
University of Wyoming, USA, scott@uwyo.edu

Jana L
Nelson, University of Wyoming, USA, jnels107@uwyo.edu

Yujin Lee
Kangwon National University, Korea, ylee@kangwon.ac.kr

Hyunkyung Kwon
Texas A&M University, USA, kwonx133@tamu.edu

This study investigates whether U.S. standardized tests provide opportunities for students to demonstrate creative thinking in mathematics assessments. The analysis of two widely used national standardized tests (PARCC and SBAC) reveals that some creativity-directed tasks, such as multiple solution problems, are present. However, these tasks are not consistently included across all grade levels. The findings highlight the role of assessments in shaping instructional practices and underscore the need for assessment materials that foster creative mathematical thinking. In addition, the current study emphasizes that creativity is a crucial skill for success in Science, Technology, Engineering, and Mathematics (STEM) fields and should be more prominently featured in standardized assessments to encourage critical thinking and problem-solving abilities among students.

Keywords: Mathematical creativity, Assessment for creativity, Creativity-directed tasks

Introduction

Mathematics is a foundational subject for STEM fields, and fostering creativity in mathematics is essential for economic competitiveness and innovation. Despite this, traditional assessments often

emphasize procedural knowledge over creative problem-solving. Given the impact of standardized assessments on instructional practices, this study examines the extent to which creativity-directed tasks are integrated into the Partnership for Assessment of Readiness for College and Careers (PARCC) and Smarter Balanced Assessment Consortium (SBAC) mathematics assessments. The study aims to highlight the existing gaps in assessment design and encourage a shift toward more creativity-driven evaluations to better prepare students for real-world challenges.

Students benefit from creativity-driven mathematics instruction, yet standardized assessments primarily focus on procedural knowledge. Creativity in mathematics involves multiple solution pathways, problem-posing, visualization, and interdisciplinary connections. Studies have demonstrated that students who engage in creativity-enhancing mathematical tasks develop stronger analytical and reasoning skills (e.g., Bicer et al., 2021). However, past assessments have rarely measured creativity explicitly, leading to an overemphasis on rote memorization and algorithmic problem-solving. Recent efforts suggest that incorporating creative elements into assessments can enhance mathematical understanding, engagement, and student motivation.

Methodology

The study employs a framework based on creativity-directed tasks, including open-ended problems, mathematical connections, visualizations, extensions, and reflective communication. A total of 1,430 publicly available items from PARCC (n=999) and SBAC (n=431) were analyzed for their potential to promote creative thinking. The coding process achieved high interrater reliability, ensuring consistency in categorization. The study examined each test item in depth, identifying the prevalence of creative problem-solving elements and their alignment with broader educational objectives. Additionally, expert reviews were conducted to validate the findings and ensure that the evaluation framework accurately reflects essential components of mathematical creativity.

Results

In terms open-ended tasks, both PARCC and SBAC include multiple-solution and modeling tasks, but these are more prevalent in lower grades for PARCC and in upper grades for SBAC. The uneven distribution suggests a need for greater consistency across all grade levels to ensure sustained engagement in creative thinking.

In terms of visualization, PARCC includes more tasks that encourage visualization, particularly through technology and hands-on activities. SBAC lacks technology-integrated and hands-on visualization tasks across all grades, limiting opportunities for students to develop spatial reasoning and conceptual understanding.

In terms of mathematical connections, tasks that require interdisciplinary and real-life connections are underrepresented in both assessments, though within-mathematics connections are more frequent. Encouraging the inclusion of real-world applications can help students appreciate the relevance of mathematics beyond the classroom.

In terms of extensions, both assessments include tasks that require generalization and solving more complex problems, but their distribution is inconsistent across grade levels. Extension problems play a crucial role in fostering mathematical creativity by encouraging students to think beyond standard problem-solving approaches.

In terms of communication, reflective tasks are present but not uniformly distributed, with some grades lacking these tasks entirely. Providing students with opportunities to articulate their reasoning and approach can deepen their understanding and promote metacognitive skills.

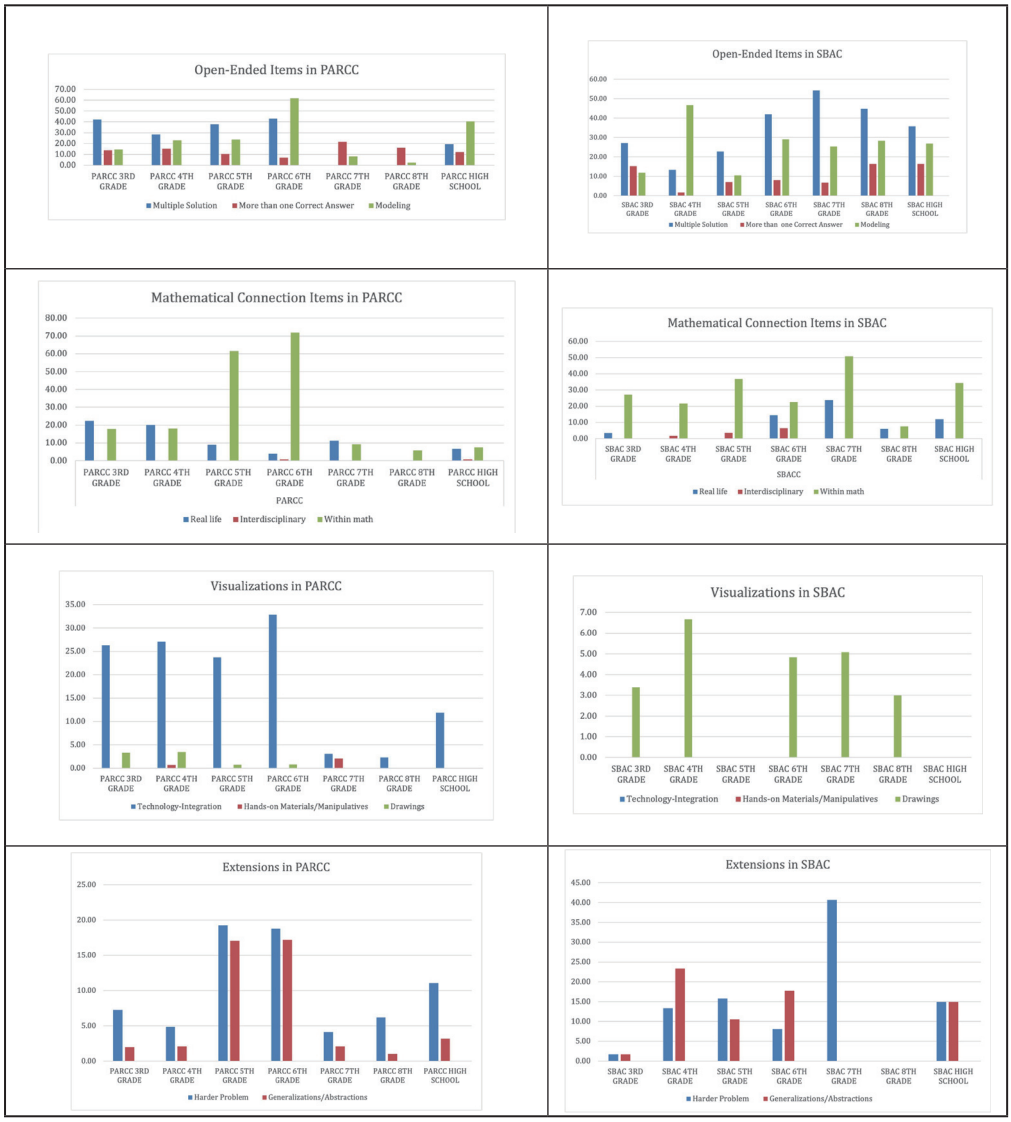


Figure 1. Percentages of subcategories

Discussion

While both PARCC and SBAC offer some opportunities for students to engage in creative mathematical thinking, inconsistencies in task distribution may send mixed signals to educators about the importance of creativity in mathematics. Aligning standardized assessments with creativity-driven instructional practices can enhance students' problem-solving abilities and conceptual understanding. Future assessments should strive for a more balanced inclusion of creativity-directed tasks across all grade levels to support the development of students' mathematical creativity consistently. Additionally, test designers should consider integrating emerging educational technologies to create dynamic, interactive assessment items that better capture creative problem-solving processes.

The study also highlights the broader implications of assessment practices on curriculum design and instructional strategies. Since teachers often structure their lessons based on standardized tests, incorporating more creativity-driven tasks into assessments can influence classroom instruction positively. Schools and policymakers should collaborate to ensure that assessments reflect a balanced approach to mathematical learning, combining procedural fluency with creative exploration.

Conclusion

This study highlights the need for standardized tests to incorporate more creativity-driven tasks systematically. By doing so, assessments can better support students' ability to think creatively, ultimately fostering deeper mathematical understanding and innovation. Future research should explore additional standardized assessments and develop methodologies to ensure creative mathematical thinking is consistently evaluated and encouraged. Furthermore, educators and policymakers should advocate for assessment reforms that prioritize not only accuracy and efficiency but also creativity and innovation. Addressing these gaps in assessment design will help cultivate a generation of students who are not only proficient in mathematics but also adept at solving complex, real-world problems through creative and analytical thinking.

References

Bicer, A., Marquez, A., Colindres, K. V. M., Schanke, A. A., Castellon, L. B., Audette, L. M., ... & Lee, Y. (2021). Investigating creativity-directed tasks in middle school mathematics curricula. *Thinking Skills and Creativity, 40*, 100823.

Bicer, A., Aldemir, T., Krall, G., Quiroz, F., Chamberlin, S., Nelson, J., Lee, Y., & Kwon, H. (2024). Exploring creativity in mathematics assessment: An analysis of standardized tests. Thinking Skills and Creativity, 101652.

Acknowledgements

This study is a revised version of the research conducted by Bicer et al. (2024).

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 3
(Day 3)

TSG3-07

A VALIDATION STUDY MEASURING INTERDISCIPLINARY MATHEMATICAL PROBLEM SOLVING ABILITY

Zhongtian Ji
Beijing Normal University, China, jizt@mail.bnu.edu.cn

Kan Guo
Beijing Normal University, China, guokan@bnu.edu.cn

Problem solving in mathematics education and research on interdisciplinary mathematical problem solving (IdMPS) are becoming increasingly important. Therefore, the authors developed a new test, "the Interdisciplinary Mathematical Problem Solving Ability Test (Id-MPST)". The Id-MPST measures potential for IdMPS ability in knowledge pre-exploration, exploration and application tasks in the interdisciplinary contexts. In the current study, the test was administered to 179 middle school students in China. A Rasch partial credit analysis showed that the items demonstrated satisfactory psychometric properties. The Id-MPST can be used as an objective measure.

Keywords: interdisciplinary, mathematical problem solving, educational assessment

Introduction

Students need to practice interdisciplinary skills in order to prepare the twenty-first century (English, 2023). Problem solving has long been recognized as a promising technique in mathematics education (Jacinto & Carreira, 2017). Nowadays, students ought to be prepared and trained in advanced problem solving, such as interdisciplinary mathematical problem solving (IdMPS). They should learn translate the non-mathematical contexts into a series of mathematical processes, and be able to combine mathematical knowledge and domain-general strategies to deal with interdisciplinary situations (Schukajlow et al., 2023). Therefore, problem solving in mathematics education and research on IdMPS are becoming more and more significant. To evaluate the effectiveness of interdisciplinary teaching approach such as project-based learning, and the development of students' competence, the interdisciplinary mathematical problem solving ability test (Id-MPST) was designed. The current study describes the test instrument and reports the results of the validation of the Id-MPST scores.

Development of the Id-MPST

Theoretical Framework

While both mathematical modelling and interdisciplinary problem-solving involve iterative refinement of real-world phenomena into mathematical structures, the former emphasizes formalization and validation of abstract representations (e.g., equation systems or algorithmic frameworks), whereas the latter prioritizes contextual application and synthesis across disciplinary boundaries. In this study, Blum and Leiß's (2007) modelling cycle was adapted as the conceptual

framework to describe students' IdMPS process (see Figure1). First, a situation model should be constructed from a real situation and also be able to explain the problem in real life. Then, this model is simplified to a problem in interdisciplinary contexts by making the necessary assumptions. During the mathematization process, students use mathematical knowledge and problem solving strategies to solve the problem and obtain mathematical results. Finally, students consider the mathematical results in the context of the interdisciplinary situation and discuss the validity of the results. In some ways, stressing more the process of problem solving, the power of modelling has been downplayed in this current study.

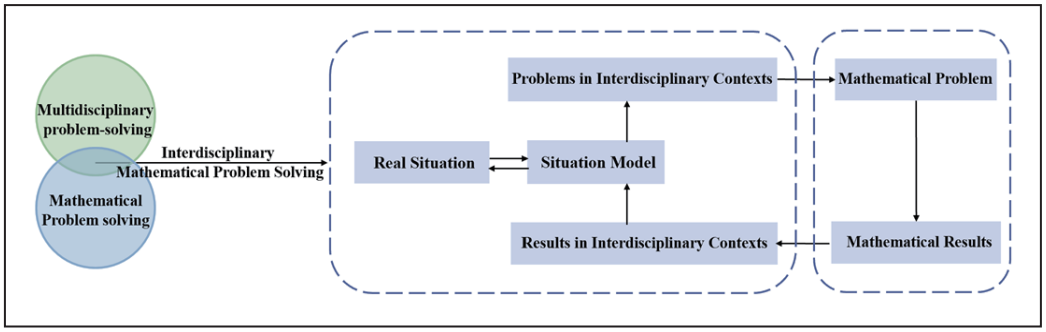


Figure 1. The interdisciplinary mathematical problem solving

However, most tests focused on monodisciplinary learning and transdisciplinary affective domains, but few empirical studies on interdisciplinary assessment (Gao et al., 2020). As a contribution to interdisciplinary assessment, a team of researchers developed the knowledge integration theory. According to Linns' (2006) theory, knowledge integration is defined as the process of adding, distinguishing, evaluating, and sorting out accounts of phenomena, situations, and abstractions in science. Following this theory, several researchers developed special batteries of tests on interdisciplinary topics (Lee & Liu, 2010). Based on key aspects of IdMPS as well as on the following considerations, a new test, the Id-MPST, was developed with three problem types:

(1) Pre-exploration. Knowledge pre-exploration refers to the understanding of interdisciplinary knowledge in simple contexts. Students may not know the interdisciplinary knowledge needed to solve the problems. Assessment in this problem type requires careful consideration of how much additional interdisciplinary knowledge related to the new situation model that a student needs to understand before solving the problem (Shen, Liu & Sung, 2014).

(2) Exploration. Knowledge exploration refers to understanding the application of the interdisciplinary knowledge involved in pre-exploration in simple contexts. That is, this problem type involves a set of related input and output variables (Weise, Greiff & Sparfeldt, 2022). Students are required to use mathematics and interdisciplinary knowledge to obtain mathematical results.

(3) Application. Knowledge-application refers to applying the interdisciplinary knowledge in complex contexts. Problems in this type involved two or more input and output variables. The authors suggest that multiple variables may make the problem more difficult. Students with advanced problem solving skills perform better in handling and coordinating the effects of multiple variables (Teig, 2024).

Description of the test

Based on the above points, special test items were developed. Through the joint efforts of educational experts, mathematics teachers and graduate students majoring in mathematics education, the authors developed three items targeting students' IdMPS ability (Table 1). Each item contains three questions corresponding to knowledge pre-exploration, exploration and application. The authors identified the contexts suitable for combining interdisciplinary knowledge and mathematical knowledge by reading textbooks of other subjects. Meanwhile, the authors tried to use textbook language to avoid possible confusion caused by discipline-specific jargon.

Table 1. Overview of the Id-MPST

Item	Id-MPST subtest	Discipline context
1	Time zone	Geography
2	Ouroboros	Music
3	Character	Biology

Scoring

Each question was scored using the holistic scoring rubric. In this rubric, students' performance was scored on a scale of 0 to 2 for each question, where 0=totally wrong or no answer; 1= not entirely correct; 2=totally correct.

Methods

Participants

A total of 179 middle school students from public schools in China participated in the study. Among them, 159 seventh-grade students participated in the paper-pencil test, and 20 eighth-grade students participated in the think aloud test.

Instrument

A survey adapted from Wilson et al. (2021) was used in this study. It asked students to provide examples of creative thinking, application, and problem solving in everyday life. The authors scored each response based on the complexity of the response and the quality of the evidence provided. This involved coding each answer response on a scale of 0 to 2, with 0 indicating that the answer was irrelevant to the topic, 1 indicating that the student gave an appropriate example without much elaboration, and 2 indicating that the answer was elaborative with analysis, or further explanation.

Analysis

The Rasch partial credit model was used to analyze the data. In this study, the authors used ConQuest to estimate student ability and SPSS to calculate correlation coefficients. The outfit statistic was used to indicate the item fit. In addition, internal consistency (Cronbach's α estimates) and Rasch reliability (EAP/PV reliability) were computed.

Results

All 20 students understood the questions and completed the test. The test questions were proved to be readable and able to observe students' thinking process in solving the problem.

Psychometric Properties of the Instrument

The assessment showed good reliability (Cronbach's α=0.77, EAP/PV reliability=0.74). All items showed good fit statistics (Figure 2), with the values ranging from 0.72 to 1.30. Overall, the data showed a satisfactory fit to the Rasch analyses. In addition, the Wright Map showed the problems are quite difficult for students, and the spread of the item difficulties well covers the major distribution of the ability of the students.

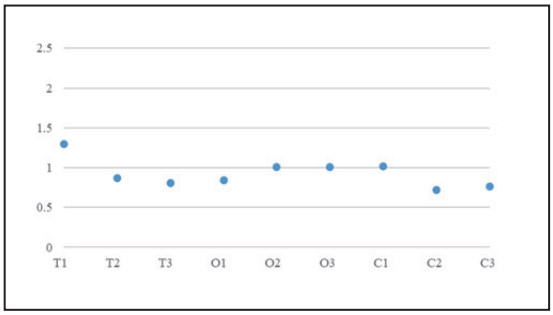


Figure 2. Item outfit statistic

Correlation between Id-MPST and related skills

The authors examined how the IdMPS performance correlated with students' creative thinking, application and problem solving ability. The results also show that those related ability were all positively associated with students' IdMPS performance.

Table 2. Correlation matrix

	IdMPS	Creative thinking	Application	Problem solving
IdMPS	1	0.317**	0.250**	0.167*
Creative thinking		1	0.303**	0.203*
Application			1	0.545**
Problem solving				1

*Note. *p<0.05, **p<0.01*

Conclusion

The IdMPS ability has become a crucial feature in mathematics education. Nevertheless, there exist problems related to the tests and its validity exists. The Id-MPST was developed to evaluate the IdMPS ability and the effectiveness of related approaches. Overall, the instrument showed satisfactory quality in measuring students' IdMPS performance. The Id-MPST scores had good reliability

and the content validity was ensured by the team of experts. Student responses also supported that the assessment items were clear and the instructions were easy to follow. The results of this study also provide insights for the assessment development of interdisciplinary teaching practices. In retrospect, student work was the focus of the interdisciplinary teaching assessment (Quigley& Herro, 2016). This test provides new perspectives and contributes to interdisciplinary mathematics education.

References

Blum, W., & Leiß, D. (2007). How do students and teachers deal with mathematical modelling problems? The example sugarloaf and the DISUM project. In C. Haines, P. L. Galbraith, W. Blum & S. Khan (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics* (pp. 222–231). Horwood.

English, L. D. (2023). Ways of thinking in STEM-based problem solving. *ZDM – Mathematics Education*, *55*(7), 1219–1230.

Gao, X., Li, P., Shen, J., & Sun, H. (2020). Reviewing assessment of student learning in interdisciplinary STEM education. *International Journal of STEM Education*, *7*(1), 24.

Jacinto, H., & Carreira, S. (2017). Mathematical Problem Solving with Technology: The Techno-Mathematical Fluency of a Student-with-GeoGebra. *International Journal of Science and Mathematics Education*, *15*(6), 1115–1136.

Lee, H.-S., & Liu, O. L. (2010). Assessing learning progression of energy concepts across middle school grades: The knowledge integration perspective. *Science Education*, *94*(4), 665–688.

Linn, M. (2006). The knowledge integration perspective on learning and instruction. In R.K. Sawyer (Eds.) *The Cambridge Handbook of the Learning Sciences* (pp. 243–264). Cambridge University Press.

Quigley, C. F., & Herro, D. (2016). "Finding the Joy in the Unknown": Implementation of STEAM Teaching Practices in Middle School Science and Math Classrooms. *Journal of Science Education and Technology*, *25*(3), 410–426.

Schukajlow, S., Krawitz, J., Kanefke, J., Blum, W., & Rakoczy, K. (2023). Open modelling problems: Cognitive barriers and instructional prompts. *Educational Studies in Mathematics*, *114*(3), 417–438.

Shen, J., Liu, O. L., & Sung, S. (2014). Designing Interdisciplinary Assessments in Sciences for College Students: An example on osmosis. *International Journal of Science Education*, *36*(11), 1773–1793.

Teig, N. (2024). Uncovering Student Strategies for Solving Scientific Inquiry Tasks: Insights from Student Process Data in PISA. *Research in Science Education*, *54*(2), 205–224.

Weise, J. J., Greiff, S., & Sparfeldt, J. R. (2022). Focusing on eigendynamic effects promotes students' performance in complex problem solving: A log-file analysis of strategic behavior. *Computers & Education*, *189*, 104579.

Wilson, H. E., Song, H., Johnson, J., Presley, L., & Olson, K. (2021). Effects of transdisciplinary STEAM lessons on student critical and creative thinking. *The Journal of Educational Research*, *114*(5), 445–457.

TSG3-08

THE EFFECTS OF TECHNOLOGICAL INNOVATION IN NATIONAL MATHEMATICS ASSESSMENT

Hye-Yun Jung
Gongju National University of Education, Korea, hy0501@gjue.ac.kr

Chang-Geun Song
Cheongju National University of Education, Korea, cgsong@cje.ac.kr

Young-Jun Kim
Seoul National University, Korea, astre0422@snu.ac.kr

Kyeong-Hwa Lee
Seoul National University, Korea, khmath@snu.ac.kr

The medium for large scale assessment of mathematics has been converted from paper-based test (PBT) to computer-based test (CBT). We aim to analyze the effect of technological innovation reflected in large scale assessment of mathematics. Data were collected from the results of PBT, CBT, and internet-based test (IBT) conducted in South Korea. Findings indicated that when the medium was converted from paper to computer, mode effect was more outstanding in items with higher mathematical complexity and cognitive loads.

Keywords: computer-based test, large-scale assessment, innovative item, mathematical complexity

Introduction

Recently, technological innovation in education has drawn the attention of large-scale assessment of mathematics (LSAM) designers who are interested in assessing higher-order thinking (Hoogland & Tout, 2018). This study analyzed the effect of technological innovation reflected in LSAM. We classified the mathematical complexity of each item of LSAM based on its mathematical thinking level and compared the correct answer rate based on the level of mathematical complexity. We then explored the effects of technological innovation on higher-order thinking assessments by comparing the correct answer rate at each level of complexity in each medium.

Background

Technological Innovation in LSAM

Recent computer-based test (CBT) in LSAM has involved *innovative items* that could not be implemented in PBT because of their specificity in item stems or response format (Drasgow & Mattern, 2006). CBT also includes technology-enhanced items of which the form of a given response or response action differs from the traditional test method. However, there have been limitations in the literature in analyzing the effects of technological innovation in assessment.

In this study, we provide a wider perspective on the effects of technological innovation, drawing on the frameworks of Bennett (2015). Bennett conceptualized the innovation of technology-based testing in terms of developmental stages. The focus of first-generation technology-based testing is primarily on delivering a new test medium and on substituting paper based testing (PBT) by resembling its content. In the second generation, traditional item formats are augmented and modified to assess new constructs by including advanced response action, media content, and so forth. For the third generation, traditional PBT has been reinvented and technology has been used to assess students' multidimensional characters (cognitive, metacognitive, and affective; Shute et al., 2016). This preliminary study focuses on the effects of first- and second-generation technology-based LSAM, although we admit third-generation LSAM has much strength.

Mathematical Complexity of Tasks

While the mathematics task framework (Stein & Smith, 1998) is popular for analyzing complexity of instructional tasks given in curricular materials, the framework of mathematical complexity proposed by the National Assessment of Education Progress (NAEP) is better suited for designing and analyzing the complexity of assessment items, especially for LSAM (NAGB, 2019). The framework determines an item's complexity by its demands on student thinking, which is divided into three levels. Low-complexity items require students to recall or recognize particular concepts or procedures. Moderate-complexity items involve more flexibility of thinking and choice among various alternatives than low-complexity items. Finally, high-complexity items ask students to use higher-order thinking skills such as reasoning, planning, analysis, judgment, and creative thinking.

Methods

Participants

The participants were those students who participated in the 2020 The National Assessment of Educational Achievement (NAEA) PBT, CBT, and internet-based tests (IBT). IBT was simply converted from PBT to a computer format, while CBT switched some items of IBT to computer-specific items (e.g., drag and drop). All participants were in their second year of high school. A two-stage stratified sampling process resulted in 10,526 students participating in PBT being selected. After selecting 3% of high schools in Korea as sampling schools, two classes were randomly selected from among the general classes belonging to the sampling schools. 473 students were selected for IBT through the same process. CBT was implemented about 2 months after converting some items presented in IBT into computer-specific types of items. Considering the school's academic achievement and CBT implementation environment, a school was sampled from one large city and one small city. 373 students from the two schools participated in the evaluation.

Materials

Each type of test consisted of 20 items, reflecting the full range of mathematics subjects commonly learned by first-year high school students in Korea. Table 1 shows detailed information on the items constituting PBT, IBT, and CBT.

Table 1. The characteristics of items by type of test

Type of test		PBT	IBT	CBT
Number of selected response items		15	15	13
Number of constructed-response items	Short-answer item	3	3	6
	Extended-response item	2	2	1
Number of innovative items		0	0	8

Data collection and analysis

The data included the items presented in PBT and IBT, the correct answer rate for each item for each test type. Data analysis was carried out following three steps. First, the mathematical complexities of the 20 identical items presented in PBT and IBT were analyzed. Each of the four researchers wrote down the level of mathematical complexity of each item and their reason for their evaluation. When the evaluations of the researchers were different, the mathematical complexity was readjusted through iterative discussion between the researchers. Second, based on the level of mathematical complexity, the difference in the correct answer rate of the identical items presented in PBT and IBT was analyzed. Difference statistical (regarding PBT and IBT) tests and a one-way ANOVA were conducted using Excel and SPSS ver. 26, producing indicators of the mode effect for each item (Ebrahimi et al., 2019; Fishbein et al., 2018). Third, the difference in correct answer rates based on the use of technological functions in the computer mode were analyzed (IBT and CBT).

Results

Analysis of the items in PBT and IBT showed that there were 4 high-complexity items, 8 moderate-complexity items, and 8 low-complexity items. Figure 1 shows the mean of the correct answer rate for each level of mathematical complexity in PBT and IBT, and the mean of the differences in the correct answer rate between PBT and IBT by mathematical complexity. It also shows that the mean of difference in the correct answer rate in PBT versus IBT was highest in high-complexity and lowest in low-complexity.

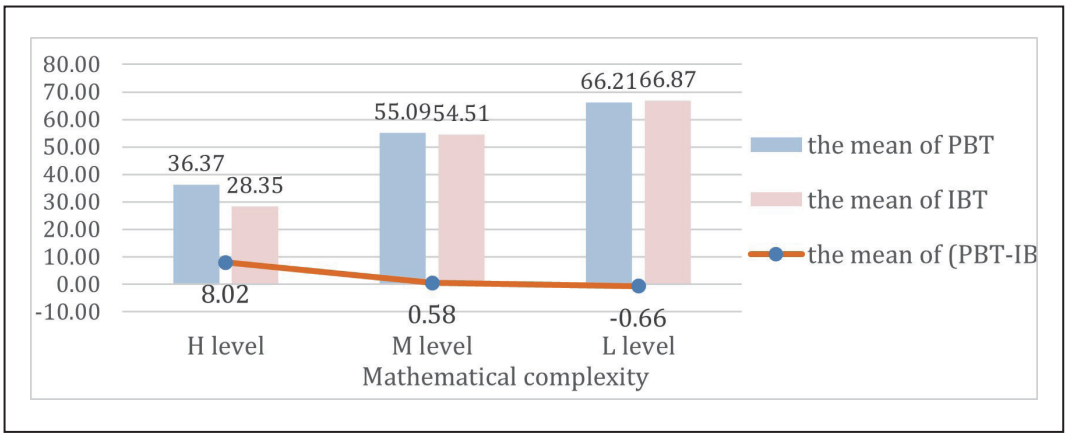


Figure 1. The mean of the correct answer rate in PBT, IBT, (PBT-IBT) based on the level of mathematical complexity (unit: %)

The results of one-way ANOVA between the mathematical complexity and the means of PBT − IBT, shows that the *p*-value for the differences in the correct answer rate between PBT and IBT by mathematical complexity is 0.008, which reveals a statistically significant difference at the significance level of .05. A Tukey post-hoc test revealed significant differences in the difference in the concrete answer rate between high-complexity and moderate-complexity (*p*-value = 0.017), and high-complexity and low-complexity (*p*-value = 0.008). PBT − IBT of high-complexity was significantly higher than those of moderate- and low-complexity, meaning that as the medium changes from paper to computer, the score of high-complexity decreases significantly compared to those of moderate- and low-complexity. Moreover, this result shows that the mode effect can be more evident with high-complexity items than with moderate- and low- complexity items.

Table 2 shows the change in the correct answer rate of items based on the use of technological functions. The correct answer rate in CBT increased or decreased depending on the type of technological function. The result implies that while the pulldown and video functions may have contributed to improving student achievement in CBT, the checkbox feature may not have had the same effect.

Table 2. Correct answer rates of items with technological function in IBT and CBT

IBT item	CBT item	Technological function (response format)	Technological function (item stem)	Correct Answer rate (%)	
				IBT	CBT
3	3	Pull down	-	46.03	58.07
9	8	Radio button	Multimedia (video)	58.56	65.33
12	10	Check box	-	61.10	39.47

Discussion

The findings confirmed that when the medium was converted from paper to computer, mode effect was more outstanding in items with higher mathematical complexity (Result 1). It was also confirmed that there was a difference in the results of IBT and CBT based on the technological functions used in CBT items (Result 2). Adding an analysis of mathematical complexity, our study expands upon prior studies that reported significant mode effects in CBT (Eid, 2005; Hargreaves et al., 2004; Poggio et al., 2005; Wang et al., 2007) and those that did not (Aspiranti et al., Backes & Cowan, 2019; Bennett et al., 2008; Fishbein et al., 2018). The main implication of this result is that first-generation technology-based testing may have a limited contribution in assessing higher-order thinking (Bennett, 2015). Result 1 suggests that the cognitive loads an item invokes may be prominent when the item of the direct PBT-translation type is highly mathematically complex. Meanwhile, Result 2 implies that second-generation technology-based testing can complement this limitation, but not in a simple way (Bennett, 2015). Specifically, Result 2 suggests that applying innovative items can reduce cognitive loads by mode conversion, thereby contributing to valid assessment of higher-order thinking. Notably, some innovative items using pull down and video functions may be useful in reducing cognitive loads and testing higher-order thinking, but others were not. Our results imply that technologically refined CBT items have advantages in avoiding cognitive overload and facilitating higher-order thinking if they are used appropriately. This view coincides with Hoogland and Tout's (2018), but our results also indicate that technology's effects on students' cognitive engagement are not only determined by advanced technology but also subordinate to its relationship with task complexity.

References

Aspiranti, K. B., Henze, E. E., & Reynolds, J. L. (2020). Comparing paper and tablet modalities of math assessment for multiplication and addition. *School Psychology Review, 49*(4), 453–465.

Backes, B. & Cowan, J. (2019). Is the pen mightier than the keyboard? The effect of online testing on measured student achievement. *Economics of Education Review, 68*(1), 89–103.

Bennett, R. E. (2015). The changing nature of educational assessment. *Review of Research in Education, 39*(1), 370–407.

Bennett, R. E., Braswell, J., Oranje, A., Sandene, B., Kaplan, B., & Yan, F. (2008). Does it matter if I take my mathematics test on computer? A second empirical study of mode effects in NAEP. *Journal of Technology, Learning, and Assessment, 6*(9).

Drasgow, F., & Mattern, K. (2006). New tests and new items: Opportunities and issues. In D. Bartram, R. K. Hambleton, & R. Hambleton (Eds.) *Computer-based Testing and the Internet: Issues and Advances* (pp. 59–75). John Wiley & Sons Ltd.

Ebrahimi, M. R., Toroujeni, S. M., & Shahbazi, V. (2019). Score equivalence, gender difference, and testing mode preference in a comparative study between computer-based testing and paper-based testing. *iJET, 14*, 128–143.

Eid, G. K. (2005). An investigation into the effects and factors influencing computer-based online math problem-solving in primary schools. *Journal of Educational Technology Systems, 33*(3), 223–240.

Fishbein, B., Martin, M. O., Mullis, I. V., & Foy, P. (2018). The TIMSS 2019 item equivalence study: examining mode effects for computer-based assessment and implications for measuring trends. *Large Scale Assessments in Education, 6*(11).

Hargreaves, M., Shorrocks-Taylor, D., Swinnerton, B., Tait, K., & Threlfall, J. (2004). Computer or paper? That is the question: Does the medium in which assessment questions are presented affect children's performance in mathematics? *Educational Research (Windsor), 46*(1), 29–42. http://dx.doi.org/10.1080/0013188042000178809

Hensley, K.K. (2015). *Examining the effects of paper-based and computer-based modes of assessment of mathematics curriculum-based measurement*. Unpublished PhD thesis, University of Iowa, Iowa.

Hoogland, K., & Tout, D. (2018). Computer-based assessment of mathematics into the twenty-first century: pressures and tensions. *ZDM, 50*(1), 675–686.

National Assessment Governing Board (NAGB). (2019*). Mathematics framework for the 2019 National Assessment of Educational Progress*. Retrieved from https://www.nagb.gov/content/dam/nagb/en/documents/publications/frameworks/mathematics/2019-math-framework.pdf

Poggio, J., Glasnapp, D., Yang, X., & Poggio, A. (2005). A comparative evaluation of score results from computerized and paper & pencil mathematics testing in a large scale state assessment program. *The Journal of Technology, Learning, and Assessment, 3*(6).

Ponce, H., Mayer, R., Sitthiworachart, J., & López, M. (2020). Effects on response time and accuracy of technology-enhanced cloze tests: An eye-tracking study. *Educational Technology Research and Development, 68*(5), 2033–2053.

Shute, V. J., Leighton, J. P., Jang, E. E., & Chu, M.-W. (2016). Advances in the science of assessment. *Educational Assessment, 21*(1), 34–59.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268–275.

Wang, S., Jiao, H., Young, M. J., Brooks, T., & Olson, J. (2007). A meta-analysis of testing mode effects in grade K-12 mathematics tests. *Educational and Psychological Measurement*, *67*(2), 219–238.

TSG3-09

STUDENTS' SELF-REFLECTION IN A GEOMETRY LESSON: PERSPECTIVES AND QUALITY

Satoshi Kusaka
Naruto University of Education, Japan, kusaka@ksm.world

Hiroki Ishizaka
Naruto University of Education, Japan, hishizaka@naruto-u.ac.jp

Hiroki Takemura
Tosayama Gakusha, Japan, hiroki1313@g.kochinet.ed.jp

Taisei Miyamoto
Naruto University of Education, Japan, 24813293@naruto-u.ac.jp

This study examines self-reflection in elementary mathematics to promote deep learning and autonomy. Analyzing sixth-grade students' reflections on lessons of enlargement and reduction, it categorizes them into learning content, problem-solving process, and awareness of learning. Most students described factual learning but struggled to connect concepts. While some reflected on problem-solving, few analyzed methods deeply. To enhance reflection, structured sheets, exemplary reflections, and group discussions are recommended. Teachers play a key role in fostering deeper self-reflection. Future research should explore diverse cases and teacher feedback's impact on improving reflection quality.

Keywords: self-reflection, self-regulation, metacognition, self-assessment

Introduction

In recent years, promoting learner autonomy and deep learning has become increasingly emphasized in education. Self-reflection activities in lessons play a significant role in this context. The OECD's "Learning Compass 2030" envisions not only equipping learners with the skills and knowledge necessary for the future but also fostering the ability to learn autonomously and contribute to the sustainable development of society (OECD, 2019). In Japanese education, similar to the OECD's vision, achieving 'proactive, interactive, and authentic learning' is a stated goal. The Ministry of Education, Culture, Sports, Science and Technology (MEXT) emphasizes the importance of students understanding learning objectives, creating plans, and advancing their learning while reflecting on the process (MEXT, 2017). On the other hand, some cases have been reported where reflection activities become formalized and fail to function effectively as opportunities for deep learning and self-regulation, or where teachers are unable to effectively utilize students' reflections (Furuya, 2022).

Literature Review

The Importance of Self-Reflection and Self-regulation

In Japanese education, self-assessment is positioned as an essential means to promote autonomous learning. The MEXT Course of Study emphasize cultivating students' ability to reflect on the knowledge and skills they acquire during the learning process and set goals for subsequent learning. In practice, self-assessment is widely implemented in schools, with activities such as recording 'what I learned today', 'what was difficult', and 'goals for the next lesson' at the end of lessons being common. These activities provide learners with opportunities to reflect on the lesson's content and organize their learning (e.g., Sakakura & Moriwaki, 2022). This self-assessment process not only raises learners' awareness of their understanding and challenges but also serves as valuable feedback for teachers. Zimmerman (2002) classified self-regulation into three domains: cognitive regulation, motivational regulation, and behavioral regulation. Cognitive regulation involves abilities related to processing and retaining learning content, including strategies such as summarization, self-questioning, and mind mapping. Motivational regulation encompasses maintaining and enhancing learning motivation, including goal setting, progress management, and sustaining a positive attitude. Behavioral regulation pertains to managing the learning environment, time, and resources to make the learning process feasible, involving activities like scheduling, organizing the environment, and preparing materials. These regulatory abilities interact and play an essential role in constructing a foundation for learners to deepen their learning autonomously and continuously.

Self-reflection and Self-regulation in Mathematics Education

One characteristic of self-reflection in Mathematics is that answers and procedures are concrete and clear. Since it is relatively easy to judge whether an answer is correct, learners can more readily reflect on the accuracy of their answers and the appropriateness of their procedures. While mathematics often has only one correct answer, there are typically multiple methods to arrive at that answer. Through deep questions such as 'Why did I choose this method?' or 'What other methods could I have used?' learners can understand their thinking patterns and explore efficient and effective solutions. Self-reflection allows learners to compare different methods and evaluate their advantages and efficiencies.

Research Objective and Questions

The objective of this study is to identify the perspectives and characteristics of students' self-reflection in mathematics lessons and to derive implications for effective reflection activities. To achieve this objective, the study sets the following two research questions: (1) What kind of analytical framework can be developed to evaluate students' reflections in mathematics learning?, and (2) What are the characteristics of students' self-reflections?

Methodology

Development of the Analytical Framework

Based on previous studies mentioned earlier, an analytical framework was constructed to classify reflection perspectives into three categories: 'Learning Content', 'Problem-Solving Process', and 'Awareness of Learning' (Table 1).

Table 1. Framework for evaluating children's reflections

Perspectives	Levels	Criteria
Learning content	1	Describe learned content as facts but lacks specificity or deep understanding.
	2	Explain key points of the learned content specifically, demonstrating understanding.
	3	Reflect deeply on learned content, relating it to other concepts or discovering new perspectives.
Problem-solving process	1	Describe problem-solving steps or methods factually but lacks awareness of improvement.
	2	Reflect on steps and methods, recognizing successes and challenges.
	3	Analyze causes of challenges and proposes concrete improvements or new methods.
Awareness of Learning	1	Mention impressions or future goals but lacks specificity.
	2	Describe attitudes toward learning or future goals specifically, showing autonomous learning.
	3	Reflect deeply on learning attitudes and proposes concrete plans to achieve future goals.

Classroom Practice and Reflection Writing

The study targeted a sixth-grade arithmetic unit on "enlargement and reduction," consisting of five lessons. At the end of each lesson, students were given five minutes to write reflections freely, without specific instructions. This study focused on reflections from the fourth lesson in the unit.

Analysis Method

Students' reflection texts were analyzed using the three perspectives and criteria outlined above. Each sentence in a reflection was further divided into multiple phrases if it included more than one perspective. Each phrase was categorized based on the relevant perspective and depth. Therefore, the total number of phrases analysed did not match the total number of original sentences.

Results and Discussion

Discussion of Reflection Results

The analysis results of students' reflections are shown in Table 2. Regarding learning content, most students' reflections fell into Level 1 or Level 2, where they described what they had learned as factual information. While students were able to accurately reflect on their acquired knowledge, they did not sufficiently connect it to new situations or relate it to different concepts. Regarding the problem-solving process, many students' reflections contained specific descriptions of the steps and methods they used. Although many students attempted to improve their processes, they did not

provide detailed explanations of their methods. Only one student reached Level 3, reflecting on the method they used and proposing to continue using it in the future after evaluating its effectiveness. In terms of learning awareness, multiple students set future goals and outlined specific plans to achieve them by demonstrating clear actions plan towards achieving the set goal, reflecting a high level of self-regulation.

Table 2. Analysis of Students' Reflections

Perspectives	Levels	Example Statements
Learning content	1	I drew an enlarged figure for the first time, and I was able to do it correctly. (5)
	2	I couldn't draw the reduced figure, next time I want to perfect it.(4)
	3	None (0)
Problem-solving process	1	I realized I could draw them using a ruler, protractor, and compass. (1)
	2	I understood that I could draw enlarged and reduced figures if I knew the base length and angles B and C. (3)
	3	I found that by measuring the base length with a ruler, measuring the angles with a protractor, marking the points, and connecting them with lines, I could explain and understand the process more clearly. I want to use this method when drawing reduced and enlarged figures. (1)
Awareness of learning	1	Next, I want to try drawing a 1.5x enlarged figure. (2)
	2	I forgot other methods, so I want to try drawing enlarged figures using a compass or other tools. (2)
	3	At first, I didn't understand the new terms, but as I deepened my thinking, I recalled what I had learned and wanted to explore further. (3)

Note: The number in parentheses represents the number of statements belonging to that level.

Implications for Future Reflection Activities

To guide students toward Level 3 self-reflection, it is essential to move beyond mere descriptions of facts and impressions. One effective approach is to introduce a structured framework for deepening reflections. Specifically, using a reflection sheet that prompts students to describe 'What I learned', 'Reasons for difficulties', 'Plans for improvement next time', and 'Potential applications of what I learned' can help them organize their thoughts and consider future actions more effectively. Additionally, sharing exemplary Level 3 reflections in class can help create an environment where other students can easily refer to and learn from high-quality reflections. Furthermore, incorporating group reflection activities can encourage students to gain new insights through peer discussions. Asking thought-provoking questions such as 'What other possible causes could explain the challenges identified?' , 'How could this be applied in different contexts?, and 'Why do you think this approach was effective?' can provide opportunities for students to deepen their thinking.

Conclusion

This study analyzed reflection activities in elementary school arithmetic lessons, focusing on perspectives and depth to enhance learning quality and self-regulation. The findings confirmed that reflection activities are critical for capturing learning content, evaluating problem-solving processes, and raising awareness for future learning. To deepen the quality of learning, reflection activities must extend beyond simple descriptions to include analysis of challenges and proposals for improvement. Teachers play a crucial role in fostering autonomy by designing structured reflection sheets and sharing exemplary reflections. However, this study's scope was limited to specific grades and units, highlighting the need for further research on diverse cases. Future research should explore specific instructional methods and the role of teacher feedback to improve reflection quality and enhance learning outcomes.

References

Furuya, T. (2022). Analysis of the impact of reflection on learners' academic performance. *Bulletin of the Faculty of Education, University of Yamanashi*, 72, 45–55. (In Japanese)

MEXT. (2017). *Course of Study (2017 Revision).* Retrieved from https://www.mext.go.jp/

OECD. (2019). *OECD Future of Education and Skills 2030: OECD Learning Compass 2030.* Retrieved from https://www.oecd.org/education/2030-project/

Sakakura, I., & Moriwaki, T. (2022). A study on reflection practices to foster a sense of agency. *Bulletin of the Faculty of Education, Mie University*, 73, 25–38. (In Japanese)

Zimmerman, B. J. (2002). Becoming a Self-Regulated Learner: An Overview. *Theory Into Practice*, *41*(2), 64–70. https://doi.org/10.1207/s15430421tip4102_2

TSG3-10

COMMUNICATION SKILLS PERFORMANCE IN QUANTITATIVE RELATED LITERACY AMONG ELEMENTARY SCHOOL STUDENTS

Chia-Jui Hsieh
National Taipei University of Education, Taiwan, paris@tea.ntue.edu.tw

Yun-Chi Tsou
Southern Illinois University Carbondale, US, yunchi.tsou@siu.edu

Lin-Chieh Tsai
Taipei Municipal Muzha Vocational High School, Taiwan, p327harry@gmail.com

The PISA2022 assessment framework (OECD, 2020) has identified communication skills as one of the essential 21st-century skills. This study designed an original assessment tool focused on quantitative literacy to systematically analyze student performance in three sub-elements of communication skills (sub-skills): access/retrieve, integrate/interpret, and represent/express. Using a questionnaire survey, data were collected from 696 sixth-grade students in northern Taiwan. A multidimensional item response theory (MIRT) analysis was conducted to estimate item parameters and assess the students' abilities in these communication skills. The results indicate that student performance in the access/retrieve dimension is significantly higher, whereas no significant differences were observed between the integrate/interpret and represent/express dimensions. Further analysis reveals significant correlations among the three sub-skills, with distinct error types observed in each. These findings offer valuable insights for future instructional strategies, particularly regarding targeted guidance and enhancement of student performance across the various dimensions.

Keywords: communication skills, multidimensional IRT, PISA assessment framework, quantitative related literacy

Introduction

Mathematical communication skills play an important role in the reform of mathematics education. As early as 1996, the National Council of Teachers of Mathematics (NCTM) published the monograph *Communication in Mathematics, K-12 and Beyond*, which deeply analyzed the current state of mathematical communication education and its future prospects (Elliott & Kenney, 1996). The core of mathematical communication lies in the generation and transmission of mathematical concepts. By its nature, communication serves a dual role as both a tool and an object of learning. This makes representation a key foundation for mathematical communication.

In the development of quantitative literacy in elementary schools in Taiwan, mathematical communication skills are particularly important. They not only relate to students' understanding of quantitative concepts but also impact their performance in problem-solving and collaborative learning. Observations in educational settings have shown that Taiwanese students often struggle to grasp the intent of a problem or, even when they arrive at the correct answer, fail to articulate their reasoning process clearly. Therefore, how to assist elementary students in constructing a deep understanding

of mathematical concepts and enhancing their communication skills has become an important issue in educational research.

The purpose of this study is to focus on elementary students' performance in mathematical communication skills within the context of quantitative literacy and systematically explore their performance levels through the development of assessment tools.

Literature Review

Mathematical communication refers to the ability to understand and express mathematical ideas. It involves not only conveying mathematical information to others but also comprehending the mathematical perspectives expressed by them (Niss & Højgaard, 2011). Based on this definition, mathematical communication includes both understanding mathematical information and using mathematical language to express one's opinions. An important aspect worth discussing is the cognitive process involved in moving from understanding information to expressing ideas. This form of communication is not limited to verbal expression; it also encompasses the use of symbols, images, and other representations to convey mathematical thinking—a process that can deepen one's understanding of mathematical concepts (Skemp, 1995). This reflects the connection between internal and external representations. According to Skemp (1995), understanding is essentially based on assimilation.

Furthermore, mathematical communication is closely linked to problem-solving. Research indicates that effectively using mathematical language to communicate can enhance problem-solving abilities (Ahdhianto et al., 2020; Maulyda et al., 2019). In other words, students must develop strong mathematical communication skills to clearly articulate their problem-solving processes and solutions. Accordingly, the NCTM (1989) emphasized that communication skills involve expressing mathematical ideas through oral, written, and visual means; thus, communication skills directly influence problem-solving performance. When solving problems, students must integrate various types of information, which may be presented through different representations such as text, graphics, and tables. Through problem-solving and student writing, educators can better understand how students interpret and integrate information (Freeman et al., 2016; Yang et al., 2016). Therefore, the meaningful integration and interpretation of information presented in different formats during problem-solving can also be considered an essential element of mathematical communication.

Moreover, drawing on the PISA reading literacy framework—which divides the reading process into accessing and retrieving, integrating and interpreting, and reflecting and evaluating (OECD, 2009)—this review highlights that mathematical communication is a multi-layered competency essential to learning mathematics. Building on these insights, this study categorizes mathematical communication into two primary components: understanding and expression. The "understanding" component is further divided into two sub-skills: accessing/retrieving and integrating/interpreting, while "expression" is defined as the sub-skill of representing/expressing.

Methodology

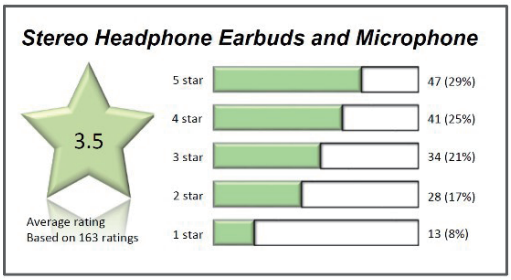
Research Design: This study employs an open-ended questionnaire survey method to assess students' performance in mathematical communication skills. The questionnaire is designed based on three sub-skills of mathematical communication: accessing/retrieving (A), integrating/interpreting (I), and representing/expressing (E). Detailed descriptions of the sub-skills are provided in Table 1.

Table 1. Detailed descriptions of the sub-skills

Sub-skill	Description
accessing/retrieving	The ability to identify clearly presented information from the given problem.
integrating/interpreting	The ability to meaningfully integrate or explain information across different sections.
representing/expressing	The ability to select appropriate representations to concretely express mathematical ideas or results.

Participants, Instruments, and Data Collection

In the pilot stage, this study selected five elementary school classes with a total of 114 students. The questionnaire consisted of 13 sets of 32 questions. Based on the pilot results, the questionnaire items were optimized and revised. One example of the questions is as follows:

<table>
<tr>
<td>PURCHASING DECISION
Andrea is shopping online for a new pair of headphones. She has identified a pair that she likes. However, she notices that even though the total number of reviews is small, the product received many poor reviews: a total of 25% 1- and 2-star reviews.
 

Source: OECD (2018), PISA 2021 Mathematics Framework (Draft). Retrieved from https://www.oecd.org/pisa/sitedocument/PISA-2021-mathematics-framework.pdf</td>
<td>To help with her decision to buy the product or not, Andrea studied the comments for the 1- and 2-star reviews and noticed that some of the reviews have nothing to do with the quality or the functioning of the product.
She grouped the responses for the 1- and 2-star reviews and summarised her findings in the table.
<table>
<tr><th>REASON</th><th>Number</th></tr>
<tr><td>Headphones arrived late</td><td>13</td></tr>
<tr><td>Headphones did not arrive at all</td><td>4</td></tr>
<tr><td>Cable was damaged or missing</td><td>7</td></tr>
<tr><td>One or both earbuds were broken</td><td>4</td></tr>
<tr><td>Packaging was unattractive</td><td>5</td></tr>
<tr><td>Wrong rating (good review, bad rating)</td><td>8</td></tr>
</table>
Source: OECD (2018), PISA 2021 Mathematics Framework (Draft). Retrieved from https://www.oecd.org/pisa/sitedocument/PISA-2021-mathematics-framework.pdf</td>
</tr>
<tr>
<td>Question 1: Based on the information provided above, please determine what percentage of the total ratings are two-star ratings.---(For accessing/retrieving skill)</td>
<td>Question 2: Please calculate the number of “wrong rating” for Andrea and explain your reasoning.---(For integrating/interpreting skill)</td>
</tr>
<tr>
<td colspan="2">Question 3: If you were a headphone salesperson, how would you use this data to persuade others to buy these headphones? Please explain your approach in detail.---(For representing/expressing skill)</td>
</tr>
</table>

The formal survey was conducted with a Balanced Incomplete Block (BIB) design. Comment Block included 13 questions. There are two blocks, Block I and Block II, each containing 4 to 5 question situations, with a total of 9 questions. The final valid sample size was 696, covering multiple schools and regular class groupings, allowing for an in-depth exploration of students’ performance in mathematical communication skills. The validity of the research instruments was reviewed by seven mathematics education experts, who evaluated the appropriateness of each question’s design and indicators. Based on the experts’ suggestions, necessary revisions were made to the questionnaire, ensuring the content validity of the instruments in three sub-skills of communication.

Analysis

For data processing, four expert scholars collaboratively developed the scoring guidelines and instructions for coding student responses. Due to the inclusion of multiple-select and open-ended constructed response questions, this study adopted the Partial Credit Model (PCM) for analysis. To ensure consistency and objectivity in scoring, each response was reviewed by two scorers. If the results from the two scorers differed, a third expert was consulted for further evaluation. After coding all responses, multidimensional item response theory (MIRT) was applied for analysis, using the tam package in R for data analysis. Finally, performance levels for students' communication skills were established based on the analysis results, and the differences between these levels were interpreted.

Results and Discussion

IRT Model Fit Indices and Item Parameters

The EAP reliabilities for the three sub-skills accessing/retrieving(**A-skill**), integrating/interpreting(**I-skill**), and representing/expressing(**E-skill**) were .815, .846, and .823, respectively, all of which reached ideal levels. Notably, the reliability for the **I-skill** was higher, indicating a strong measurement stability for that skill. These results provide valuable insights into students' performance across different communication skill dimensions. In terms of the **difficulty parameters** (xsi) of the test items, the difficulty range was from -2.7786 to 1.8133, indicating that the assessment could effectively differentiate students at various ability levels. Standard errors (se_xsi) shown that most items had standard errors within a reasonable range, indicating high item stability.

Student Communication Skills Performance Analysis

To make the results more understandable, the EAP scores from the IRT analysis were multiplied by 10 and then added to 60, converting them into a more intuitive numerical range, as shown in Table 2.

Table 2. Student communication skill performance analysis

Communication Skill	Accessing/Retrieving (A-skill, 9 items)	Integrating/Interpreting (I-skill, 11 items)	Representing/Expressing (E-skill, 11 items)
Average Score	67.95	53.68	55.00

Students' average score for the **A-skill** (67.95) was significantly higher than for the **I-skill** (53.68) and **E-skill** (55.00). This indicates that students are stronger in Accessing/Retrieving information, but there is room for improvement in integration, explanation, and expression. This result suggests that in the teaching process, more focus should be placed on strengthening students' abilities in integration and expression to promote their overall skill development. This study also found that students' performances across different mathematical communication sub-skills were highly intercorrelated, the result shown in Table 3, indicating that strength in one sub-skill tends to support and enhance performance in others. This finding suggests that instructional strategies should focus on the integrated development of these sub-skills to promote a more comprehensive improvement in students' overall mathematical communication abilities.

Table 3. Correlation among communication subskills

Subskill	A-skill	I-skill	E-skill
A-skill	1.00	.944	.932
I-skill	.944	1.00	.945
E-skill	.932	.945	1.00

Conclusion

This study's in-depth analysis of students' communication skills performance revealed a relative advantage in the Accessing/Retrieving skill, while indicating potential for improvement in the Integrating/Interpreting and Representing/Expressing skills. Further analysis reveals significant correlations among the three sub-skills, with distinct error types observed in each. These findings offer valuable insights for future instructional strategies, particularly regarding targeted guidance and enhancement of student performance across the various dimensions.

References

Ahdhianto, E., Marsigit, Haryanto, & Santi, N. N. (2020). The effect of metacognitive-based contextual learning model on fifth-grade students' problem-solving and mathematical communication skills. *European Journal of Educational Research, 9*(2), 753-764. https://doi.org/10.12973/eu-jer.9.2.753

Elliott, P. & Kenney, G. (1996). *Communication in mathematics, K-12 and beyond: 1996 yearbook.* NCTM.

Goldin, G., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in school mathematics: 2001 yearbook* (pp. 1-23). NCTM.

Freeman, B., Higgins, K. N. & Horney, M. (2016). How students communicate mathematical ideas: An examination of multimodal writing using digital technologies. *Contemporary Education Technology, 7*(4), 281-313. https://doi.org/10.30935/cedtech/6178

Maulyda, M. A., Annizar, A. M., Hidayati, V. R., &Mukhlis, M. (2019). Analysis of students' verbal and written mathematical communication error in solving word problem. *Journal of Physics: Conference Series, 1538*(1), 012083. https://doi.org/10.1088/1742-6596/1538/1/012083

National Council of Teachers of Mathematics (1993). *Curriculum and evaluation standards for school mathematics.* NCTM.

Niss, M., & Højgaard, T. (Eds.). (2011). *Competencies and mathematical learning: Ideas and inspiration for the development of mathematics teaching and learning in Denmark*. Roskilde University.

OECD (2009), *PISA 2009 Assessment framework: Key competencies in reading, mathematics and science*. OECD.

OECD (2020). *PISA 2022 Assessment and Analytical Framework.* Retrieved from https://www.oecd.org/pisa/publications/pisa-2021-assessment-and-analytical-framework.htm

Skemp, R. (1995) *The psychology of learning mathematics.* Routledge. https://doi.org/10.4324/9780203396391

Yang, E. F. Y., Chang, B., Cheng, H. N. H., & Chan, T. W. (2016). Improving pupils' mathematical communication abilities through computer-supported reciprocal peer tutoring. *Educational Technology & Society, 19*(3), 157-169.

TSG3-11

IN BUNDLENUMBER-MATH YOU JUST ASK THE BUNDLEBUNDLE BOARD

Allan Tarp
MATHeCADEMY.net, Denmark, Allan.Tarp@gmail.com

*The fourth United Nations Development Goal wants all youth and most adults to achieve numeracy. In 'essence-based' mathematics using one-dimensional line-numbers without units, numeracy is at the beginning and calculus is at the end. But the opposite is the case within the alternative 'existence-based' paradigm built on the numbers children bring to school, two-dimensional bundle-numbers with units, where 2 3s is short for 2 bundles with 3s per. Here calculus is needed in grade one for next-to addition of 2 3s and 4 5s as 8s since they add by their areas. Here, adding is preceded by counting leading to a recount-formula, T = (T/B)*B, with T and B for Total and Bundle, used to change units in STEM, and here recounting between icon-numbers and tens leads to a division and a multiplication table as well as to solving equation. Recounting physical units leads to per-numbers becoming fractions with like units and trigonometry when recounting the sides in a bundle-stack. So, changing from essence-based to an existence-based curriculum may allow reaching the UN Development Goal and exploring innovative assessment models and practices that move beyond traditional methods.*

Keywords: early childhood mathematics, numeracy, special education, curriculum, arithmetic

Background, the 4th United Nations Sustainable Development Goal

The fourth of the 17 UN Sustainable Development Goals defines quality education as 'ensuring inclusive and equitable quality education and promoting lifelong learning opportunities for all.' Here the subgoal 4.6 wants to "By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy". However, different definitions of 'numerate' seem to exist. The English Oxford Dictionary defines it as being "competent in the basic principles of mathematics, esp. arithmetic". In contrast, the American Merriam-Webster dictionary defines it as "having the ability to understand and work with numbers."

The difference in the definitions is interesting. The English uses the passive term 'competent' where the American uses the active term 'work'. The word 'competent' is a predicate, a non-action word, I cannot 'competent' something, I can only be judged as competent by someone who is competent. In contrast, 'work' is an action word, a verb, since with my body and mind I can work on something.

Also, there is a difference between the words 'mathematics' and 'numbers.' Again, mathematics is a non-action word, I cannot 'mathematics' or even 'math' a thing. In contrast, 'number' is both a verb and a noun since I can number different degrees of Many to produce a number for later calculations.

Finally, many exists in the outside world where humans see and name it differently. In contrast, mathematics does not do so, it is an institutionalized essence that is socially constructed as inside abstractions from outside examples, or as inside examples from inside abstractions. To understand these differences, we now consult the three grand theories, philosophy and sociology and psychology. Philosophy may be able to illuminate the different nature of predicates and verbs. Sociology may be able to illuminate the different inter-human power effects coming from using predicates in-

stead of verbs. And psychology may be able to illuminate the different learning results coming from listening to predicates or practicing verbs.

Grand Theory looks at Mathematics Education

Within philosophy, Existentialism holds that 'existence precedes essence' (Sartre, 2007) so that in a judging is-sentence, an existing subject is being colonized by its predicates. 'Many' thus should be seen ontologically in itself, instead of epistemologically, how some may perceive and verbalize it.

Within psychology, Piaget (1969) sees learning as adapting to outside existence, whereas a Vygotsky (1986) sees learning as adapting to inside institutionalized socially constructed essence.

Within sociology, a structure-agent debate discusses if institutions should be obeyed or negotiated between peers. Here, a Weberian viewpoint (1930) asks if SET is a rationalization gone too far by leaving Many de-enchanted and leaving learners in an 'iron cage'. As to the goal, Bauman (1990) suggests that, by institutionalizing mastery of Math as the means to reach mastery of Many, 'essence-math' has created a 'goal displacement' making the means a goal instead. Then, with the end goal, mastery of Many or existence-math, essence-math may be a means, but are there other means also?

Essence-math sees Many as an example of 1D linear cardinality always able to absorb one more by being built on the assumption that '1+1 = 2'. In contrast, humans see Many as a union of 2D stacks coming from numbering singles, bundles, and bundles of bundles, e.g., T = 345 = 3*BB + 4*B + 5*1. Essence-math sees mastery of math as its goal. A difference could see mastery of Many as the goal to be reached directly by using the bundle-numbers with units that children bring to school and that allows learning core math automatically (Tarp, 2018, 2024). Essence-math could be more meaningful by de-modelling it (Tarp, 2020) built on the French poststructuralist version of Existentialism where deconstruction is used to replace predicates with verbs (Derrida, 1991). As to differences, Difference Research (Tarp, 2018) searching for differences making a difference may design micro curricula to test with Design Research and building on the observation that when asked "How many years next time?" a 3-year-old child will say "That is not 4, that is two 2s" if seeing the 4 fingers held 2 by 2.

Counting Many with Bundles, Children Deserve a Bundle-Number Curriculum

A01. Bundle-counting with units and snap-cubes or a ten-by-ten BundleBundleBoard, two 3s is two bundles with 3s per bundle. The per-number 3s exists in space, the counting-number two in time. The Algebra Square reunites unlike and like counting- & per-numbers (table 1). Polynomials use bundle-counting with units. 43 = 4*B+3*1 = 4B3 tens, and 543 = 5BB4B3 tens. Bundle-numbers falsify '1+1 = 2' with two V-signs where one 1s + one 1s = one 2s and two 1s + one 2s = one 4s, and not three 3s.

A02. Flexible Bundle-counting in space. Space-count five and ten fingers in 2s, 3s, 4s and 5s. 5 = 1B3 = 3B-1 = 2B1 = 1BB0B1 2s, and Ten = 2BB0B2 = 1BBB0BB1BB0 2s. And T = 38 = 3B8 = 2B18 = 4B-2. T = 35+46 =3B5+4B6 = 7B11 = 8B1. T = 6*28 = 6*2B8 = 12B48 = 16B8 = 168. And T = 4507 = 4BBB 5BB 0B 7, T = 4*B^3 + 5*B^2 + 0*B + 7*1. Place value and carrying is unneeded.

A03. Time-counting fingers in ½B, "1,2,3,4,5,6" no!, "0B1, 0B2, 0B3, 0B4, 0B5 or ½B0, ½B1". Time-count from 88 to 100: "8B8, 8B9, 8Bten or 9B0, …, 9B9, 9Bten or tenB0 or 1BB0B0".

A04. Add and subtract 1digit numbers counted in half-bundles. T = 6+7 = ½B1 + ½B2 = 1B3 = 13. T = 4+7 = ½B-1 + ½B2 = 1B1 = 11. T = 3+4 = ½B-2 + ½B-1 = 1B-3 = 7. And T = 8-6 = ½B3 - ½B1 = 3-1 = 2. T = 6-4 = ½B1 - ½B-1 = 1 - -1 = 2 (so, - - = +). T = 6-8 = ½B1 - ½B3 = 1-3 = -2

A05. Digits are icons. 4 strokes as a 4-icon: ❙❙❙❙ → IIII → 4. And 5 as a 5-icon: ❙❙❙❙❙ → IIIII →5.

A06. Operations are icons also. Push-away and -back to lift to stack, (division & multiplication), 6/2 means 'from 6 push-away 2s to lift', so 6 = 3x2 = (6/2)x2, T = (T/B)xB (the recount-formula). Pull-away and -back (minus and plus) to get decimals, fractions and negatives. 7 = 3B1 = 3½B = 4B-1 2s.

A07. Recounting between icon and tens. "? 5s gives 40": u*5 = 40 = (40/5)*5, so u = 40/5, i.e., "To Opposite Side with Opposite Sign". 6 7s = ? 8s and 6 7s = ? tens leads to division and multiplication tables where 6 7s = 6*7 = (B-4)*(B-3) = From BB, pull-away 3B & 4B and pull-back the 4*3 pulled-away twice = 3B12 = 4B2 = 42. So (B-4)*(B-3) = BB - 3B - 4B + 4*3. Here, minus * minus is +.

A08. Recounting physical units gives per-numbers as 2$/5kg. 20kg = (20/5)*5kg = (20/5)*2$ = 8$. Meter = (meter/second)*second = speed*second. Fractions with like units: 2$/5$ = 2/5. Trigonometry in a stack: height = (height/base)*base = tan(Angle)*base. Pi = n*tan(180/n) for n high enough.

A09. Bundle-bundles are squares. 3 3s= 1BB 3s= 1BB2B1 2s= 1BB-2B1 4s. So, 1BB2B1 = next BB.

A10. Squaring stacks. T = 6 4s = 1BB where B=√(6*4). Guess 1: '(6-1) (4+1)s' or '5 5s', since ½(6-4) = 1. The empty 1-corner needs two 't 4s' stacks, and t*4 = ½ gives t = 1/8. Guess 2: '4.9 4.9s'.

A11. Solving quadratics. A (u+3) square has two squares and stacks: (u+3)^2 = u^2 + 3^2 + 2*3*u = u^2 + 6*u + 9. If u^2 + 6*u + 8 = 0, all disappears but 1, so, (u + 3)^2 = 1, so u = -4 or u = -2.

A12. Adding next-to and on-top or reversed. 2 3s + 4 5s = ? 8s. Here integral calculus adds areas, and recounting change units. 2 3s + ? 5s = 3 8s. Here ? = (T2-T1)/5 = ΔT/5 roots differential calculus.

A13. Adding per-numbers & fractions. 2kg at 3$/kg+ 4kg at 5$/kg=(2+4)kg at (3*2+5*4)/(2+4) $/kg. And, 2 apples with ½ red + 3 apples with 2/3 red = (2+4) apples of which (2*½+3*2/3)/(2+3) red. Integral calculus adds piecewise or locally constant per-numbers.

A14. The Algebra square. The 'Algebra Square' reunites unlike and like unit- & per-numbers

Table 1. The algebra square shows the ways to reunite unlike and like unit- and per-numbers

Calculations unite/*split Totals in*	Unlike	Like
Unit-numbers m, s, kg, $	T = a + n $T - n = a$	T = a * n $T/n = a$
Per-numbers m/s, $/100$ = %	T = ∫ f dx $dT/dx = f$	T = a^b $\sqrt[b]{T}=a$ $loga(T) = b$

A15. Fact, fiction & fake models. Fact 'since-then' models quantify and predict predictable quantities by using factual numbers and formulas. Typically modeling the past and the present, they need to be checked for truth and units. Fiction 'if-then' models quantify and predict unpredictable

quantities by using assumed numbers and formulas. Typically modeling the future, they need to be supplied with scenarios building on alternative assumptions. Fake 'what-then' models quantify and predict unpredictable qualities by using fake numbers and formulas or by adding without units (Tarp, 2001).

Conclusion, Recommendation and Discussion with a Reviewer

"By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy". We may reach this UN Development Goal by replacing an essence-based curriculum with an existence-based curriculum answering the question 'how many?' by counting and recounting totals before adding them. Here, digits and operations both become icons representing concrete things and actions. A BundleBundleBoard makes visible the children's own 2D bundle-numbers with units, just waiting to be recounted to teach the children about division and multiplication tables and solving linear and quadratic equations, as well as how to add stacks next-to by areas or on-top with like units. Thus, both proportionality and trigonometry occur in primary school, as does the Algebra Square showing how to reunite unlike and like unit- and per-numbers. Later, it becomes a coordinate system allowing also change formulas to be visible. The BBBoard may thus be used for self-instruction and for self-assessment, allowing the instructor to encourage the students to use their numerate number-language to make number-tales about things and actions within STEEM, Science, Technology, Engineering, Economics and Math. Thus, an existence-based curriculum will finally allow a communicative turn within the number-language as within the word-language in the 1970s (Widdowson, 1978). Using children's own flexible bundle-number with units thus represents a paradigm shift (Kuhn, 1962) that opens new areas for research and innovation; as well as self-organized pre-service and in-service teacher training asking the subject on a BundleBundleBoard instead of the instructor as exemplified on the MATHeCADEMY.net website.

Reviewer: *Are there empirical studies such as, e.g., teaching experiments examining the development of learners' number sense and mathematical reasoning under different number representation systems. What are the implications of the bundle number approach for assessment and evaluation?*

Answer: Grade one class A has an essence-math curriculum with unit-free 1D line-numbers and the traditional order: addition, subtraction, multiplication, division, power. And only few will hear about calculus. And class B has an existence-math curriculum with 2D Bundle-numbers with units and the opposite order. And, since counting and recounting precedes adding, they will not meet addition until they in part 12 meet the core of math directly, calculus and linearity. What will happen if a class B student changes to class A, or the other way around? In short, how ethical is it to test essence-math against existence-math in an ordinary school? Or in special education where the students will return to normal education far ahead. Ethical testing is for teacher education and home education. As to evaluation, line-numbers are in the students' heads and can only be evaluated through oral answers. Whereas the Bundle-numbers are on a BundleBundleBoard where the students' 'ability to understand and work with numbers' can be observed both in actions, individual reporting and group discussions.

References

Bauman, Z. (1990). *Thinking sociologically.* Blackwell.

Derrida, J. (1991). *A Derrida reader: between the blinds.* P. Kamuf (ed). Columbia Uni. Press.

Kuhn T.S. (1962). *The structure of scientific revolutions.* University of Chicago Press.

Piaget, J. (1969). *Science of education of the psychology of the child.* Viking Compass.

Sartre, J.P. (2007). *Existentialism is a humanism.* Yale University Press.

Tarp, A. (2001). Fact, fiction, fiddle - three types of models. in J. F. Matos, W. Blum, K. Houston & S. P. Carreira (Eds.). Modelling and mathematics education: ICTMA 9: Applications in Science and Technology. *Proc. 9th Int. Conf. on the Teaching of Mathematical Modelling and Applications* (pp. 62-71). Horwood Publishing.

Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: From inside-inside to outside-inside understanding. *Ho Chi Minh City Univ. of Education Journal of Science 17*(3), 453-466.

Tarp, A. (2024). *Many before math, math decolonized by the child's own bundle bundle-numbers with units.* https://youtu.be/uV_SW5JPWGs.

Vygotsky, L. (1986). *Thought and language.* MIT press.

Weber, M. (1930). *The protestant ethic and the spirit of capitalism.* Unwin Hyman.

Widdowson, H. G. (1978). *Teaching language as communication.* Oxford University Press.

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 3
(Day 4)

TSG3-12

A STUDY ON PERSPECTIVES FOR EVALUATING VISUAL REPRESENTATIONS IN MATHEMATICS LESSONS

Naoya Miwa
Graduate School of Comprehensive Human Sciences, University of Tsukuba, Japan,
miwa.naoya.fn@un.tsukuba.ac.jp

This study aimed to identify the perspectives that teachers use to evaluate visual representations in mathematics lessons and to develop a lesson plan for congruences. As a result, four perspectives were identified: cognitive appropriateness, mathematical validity, instructional usefulness, and the suitability of media and representation modes. Furthermore, a lesson plan for congruences was developed utilizing seating arrangements based on periodic grid diagrams.

Keywords: visualization, visual representations, perspectives for evaluating, congruences

Background and Purpose of Study

In mathematics instruction, visual representations are indispensable for understanding abstract concepts. Numerous previous studies have emphasized that visualization plays a crucial role in comprehending abstract mathematical ideas (e.g., Bishop, 1989). Arcavi (2003), synthesizing prior research on visualization in mathematics education, defined visualization as follows. This study adopts his definition as a basis for further investigation.

> Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings. (Arcavi, 2003, p.217)

For example, when connecting the concept of addition to concrete situations, there are various types of visual representations corresponding to a single mathematical concept—such as situations involving combination and those involving increase. It is therefore desirable that clear criteria be established for evaluating visual representations when teachers select or refine them. In this study, we aim to identify the criteria by which teachers evaluate visual representations in mathematics lessons. Furthermore, the goal is to develop a lesson plan on congruences based on these identified criteria.

Perspectives for Evaluating Visual Representations

Bishop (1989) stated that technology would dramatically advance research on visualization. In light of this, this study focuses on the didactical tetrahedron, which positions technology as a fundamental component within the didactical system (Ruthven, 2012). Referring to the four vertices of the didactical tetrahedron—student, content, teacher, and technology—this study establishes the following criteria for evaluating visual representations.

1. Cognitive appropriateness
2. Mathematical validity
3. Instructional usefulness
4. Suitability of media and representation modes

Cognitive appropriateness refers to the perspective that evaluates whether a visual representation is suitable for the learner's cognitive characteristics. This includes not only considerations of cognitive load, such as whether the amount of information is appropriate and whether the learner's developmental stage and learning experience are taken into account, but also whether the representation promotes intuitive understanding.

Mathematical validity refers to the perspective that evaluates whether a visual representation is mathematically appropriate. This includes not only whether the mathematical concept is accurately represented but also whether the representation can be applied to a wide range of problem situations beyond specific cases. Additionally, it considers whether the representation remains consistent when converted into symbolic or verbal expressions.

Instructional usefulness refers to the perspective that evaluates whether a visual representation can be effectively utilized in learning activities. This includes whether the representation is easy for teachers to explain and whether it supports students' exploratory and interactive learning activities.

Suitability of media and representation modes refers to the perspective that evaluates the technical and design characteristics of visual representations. Key aspects include whether the information is well-organized and has high visual clarity, as well as whether the representation effectively relates to gestures and technological tools.

Selection and Refinement of Visual Representations in Lessons on Congruences

Lesson plan for congruences

In the Japanese high school mathematics curriculum, congruences are classified as advanced content. However, they can also be a valuable topic for helping students understand the intriguing properties of integers. Assuming that four 50-minute lessons are available, a lesson plan on congruences was developed as shown in Table 1.

Table 1. Lesson plan for congruences

Lesson	Lesson content
1 & 2	Congruences are useful for analyzing seating arrangements! - Understand the definition and properties of congruences. - Learn how seating arrangements can be used to understand congruences.
3 & 4	How to create an exciting original subtraction game? - Understand the properties of congruences and learn how to solve congruence equations. - Understand the mechanism of the 105 subtraction method and create an original subtraction game.

As a topic for a lesson applying congruences, the 105 subtraction method was chosen. As shown in Figure 1, 105 subtraction is an arithmetic method used to determine a natural number n based on the remainders when divided by 3, 5, and 7. It is introduced as one of the traditional Japanese mathematical techniques known as "wasan". This method was also featured in "Jinkōki", a mathematical book written by Mitsuyoshi Yoshida during the Edo period, and it remains one of Japan's well-known number-guessing games. By understanding the mechanism of 105 subtraction, students can create their own original number-guessing game by assigning any numbers to the positions of 3, 5, and 7. Furthermore, since systems of congruences allow for a more logical understanding of the method, Lessons 1 and 2 were designed to cover systems of congruences.

1. Find the remainders a, b, c when a natural number n (less than or equal to 100) is divided by 3, 5, and 7, respectively.
2. Multiply the remainder a by 70, b by 21, and c by 15, then calculate the sum of these values.
3. Subtract 105 repeatedly from the sum until the result is a natural number less than or equal to 100.

Figure 1. Procedure of 105 subtraction

Selection and refinement of visual representations in lesson on congruences

Schoenherr & Schukajlow (2023) proposed "mode" and "similarity" as a framework for classifying visual representations and argued that "similarity" can be further divided into two types: "physical similarity" and "structural similarity." They defined "structural similarity" as a similarity in which the essential structure inherent in the visualized subject aligns with the mathematical structure of the visual representation. This study focuses on visual representations of congruences that possess "structural similarity."

A review of previous research reveals two types of visual representations for congruences: periodic grid diagrams and periodic circle diagrams (see Figure 2). A periodic grid diagram is a representation in which periodic data is arranged in a linear, grid-like pattern, commonly found in monthly calendars and the periodic table of elements. On the other hand, a periodic circle diagram arranges periodic data in a circular, looped structure, similar to analog clocks, circular train route maps, and the PDCA cycle.

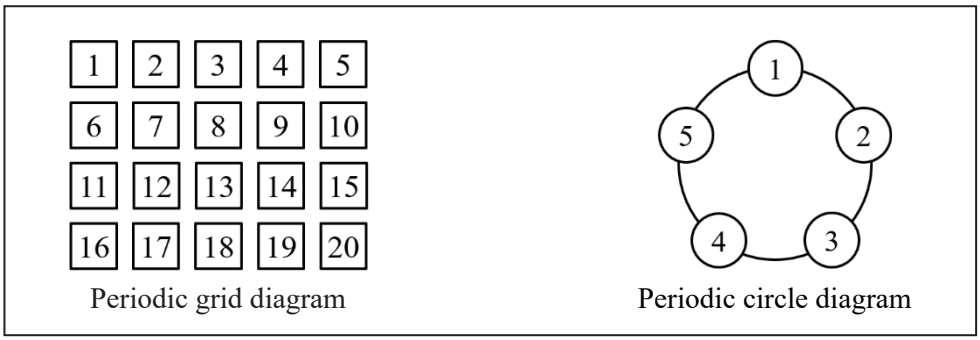


Figure 2. Visual representations of congruences

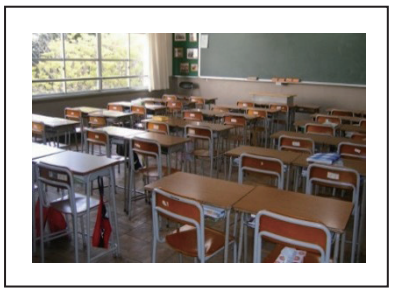


Figure 3

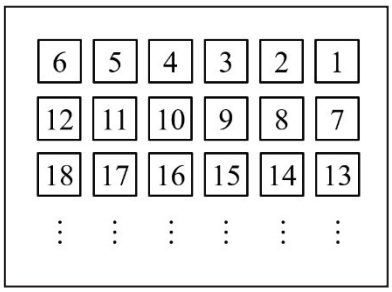


Figure 4

In this study, a periodic grid diagram is used as a visual representation for learning congruences. However, instead of using a standard periodic grid diagram as shown in Figure 2, we utilize the classroom seating arrangement illustrated in Figure 3. In Japan, it is common for mathematics lessons to be conducted in such classroom settings, where seats are typically numbered sequentially from the front-right corner. The following section explains how the visual representation was selected or refined based on the established criteria for evaluating visual representations.

When selecting visual representations, the first consideration is mathematical validity. In the case of a periodic grid diagram such as that shown in Figure 4, one notable feature is that integers beyond 7 are also displayed, making many of the numbers relevant to the exploration explicitly visible within the diagram. For example, when solving the system of congruences $\begin{cases} x \equiv 1 \pmod 2 \\ x \equiv 2 \pmod 3 \end{cases}$, students can identify the solution by locating the number that appears in the first column of a two-column seating arrangement and in the second column of a three-column arrangement. This serves as a clue to determine that the solution to the system is $x \equiv 5 \pmod 6$. Whereas a periodic circle diagram focuses on the number of operations involved, a periodic grid diagram highlights the retention of numbers beyond one full cycle. In the planned lesson on congruences, the periodic grid diagram is considered to function more effectively. These advantages also overlap with those identified from the perspective of instructional usefulness.

When refining visual representations, attention should be paid to cognitive suitability as well as the appropriateness of the medium and mode of representation. From the perspective of cognitive suitability, the learning environment itself can serve as a teaching aid, and the periodic grid diagram can be refined into a classroom seating arrangement that is familiar to students. For example, by assigning numbers to students based on a seating layout like that in Figure 4, the meaning of the congruence $x + 2 \equiv 1 \pmod 6$ can be interpreted as: “Who is the person whose number, when 2 is added, corresponds to a number in the first column?” Through this type of thinking, the solution $x \equiv 5 \pmod 6$ can be intuitively discovered, and students can then discuss whether properties such as transposition also hold in congruences, using the characteristics of congruence relations as justification.

From the perspective of suitability of media and representation modes, the ability to use gestures makes this approach effective. For example, after identifying the students whose seat numbers satisfy $x + 2 \equiv 1 \pmod 6$, they can all raise their hands simultaneously, allowing the entire class to verify the result together. One effective aspect of this approach is that students themselves become part of the visual representation, enabling them to physically embody the solution to the congruence equation.

Acknowledgments

This study was partially supported by the 2025 Grant-in-Aid for Scientific Research (Encouragement Research).

References

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, *52(3)*, 215-241.

Bishop A, J. (1989). Review of research on visualization in mathematics education. *Focus on Learning Problems in Mathematics*, *11(1)*, 7-17.

Presmeg, N. C. (2006). Research on visualization in learning and teaching mathematics. *Handbook of Research on the Psychology of Mathematics Education* (pp.205-235).

Ruthven, K. (2012). The didactical tetrahedron as a heuristic for analysing the incorporation of digital technologies into classroom practice in support of investigative approaches to teaching mathematics. *ZDM-Mathematics Education*, *44*, 627-640.

Schoenherr, J. & Schukajlow, S. (2023). Characterizing external visualization in mathematics education research. *ZDM-Mathematics Education*, *56*, 73-85.

TSG3-13

THE ROLE OF ORIGINAL PROBLEMS IN MATHEMATICAL PROBLEM POSING ASSESSMENTS

Ziyi Jin
Hiroshima University, Japan, d230164@hiroshima-u.ac.jp

In an information-driven society, the ability to identify and formulate essential problems is crucial, aligning with Mathematical Problem Posing (MPP). MPP assessments are crucial for evaluating students' mathematical creativity, yet challenges persist in designing effective assessments. This study addresses an underexplored issue: the role of original problems within structured task situations. By proposing a novel categorization based on problem extension and task structuring, this research offers a systematic framework that bridges theoretical understanding with practical implementation. Future empirical research will clarify how these categorizations affect students' problem-posing behaviors, thus enhancing assessment effectiveness.

Keywords: Mathematical Problem Posing, assessment, original problems, task situations

Introduction

In a highly information-driven society, it is impractical to master all advanced technologies. Instead, the ability to identify and formulate essential problems becomes crucial. This aligns with the broader concept of problem-posing (Baba, 2024), which has received increasing attention in mathematics education. Within this domain, Mathematical Problem Posing (MPP) refers to the process of generating mathematical problems, which has been a topic of active research for the past few decades. One fundamental focus is understanding the types of problems students pose (Cai et al., 2015), leading to the importance of MPP assessment as a research area.

Despite its significance, MPP assessment faces practical challenges, particularly in developing effective assessment tasks. One key issue is the largely overlooked role of original problems, which are specific problems provided within structured MPP assessment settings. Original problems may influence how students generate new problems, but their exact impact remains unclear (Baumanns & Rott, 2020). Neglecting the specific characteristics of original problems may prevent researchers and educators from accurately evaluating students' genuine creative contributions in MPP assessments. To address this gap, this study aims to define and categorize original problems while analyzing their role in MPP assessments to enhance students' creativity and problem-posing abilities.

Literature Review

In MPP assessments, researchers typically design a problem situation and ask students to pose problems that can be solved using the given information (Cai et al., 2015). Such assessments evaluate mathematical creativity by examining dimensions like fluency, flexibility, originality, and organization (Shriki, 2013), since posing problems itself is the basis of creativity. Most MPP assessments reveal key differences across four intrinsic dimensions: task situations, prompts, types

of posed problems, and criteria indicators. The first two dimensions relate to the task itself (Cai, 2022), while the latter focus on students' responses and their evaluation. Figure 1 presents the MPP assessment model. Further research is needed to understand how the characteristics of task situations affect the problems posed by students (Cai et al., 2015).

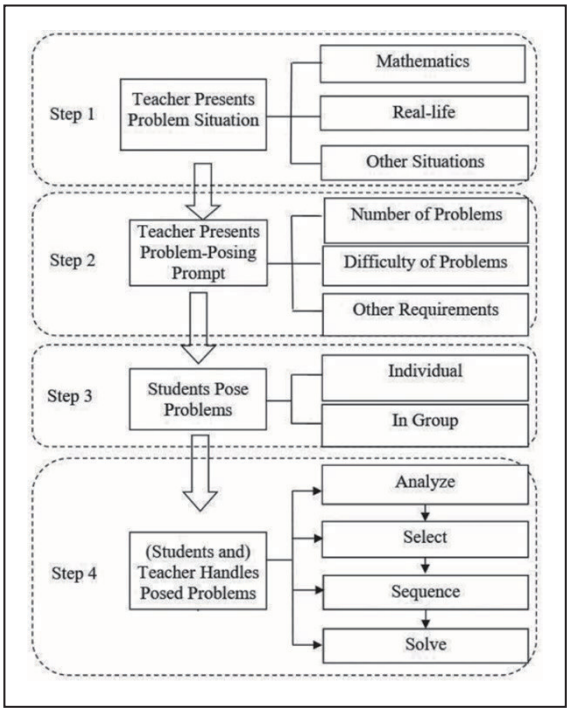


Figure 1. A problem-posing task-based instructional model (Cai, 2022)

Task situations provide the context and data that students may draw from to craft problems (Cai & Hwang, 2023). The variations of task situation significantly affect student responses, particularly the use of original problems. In some task situations that are quite structured, students are asked to pose further problems based on a specific problem, e.g. by varying its conditions. The term "original problem" refers to these specific problems. Sometimes it's called "given" or "initial" problems, and the impact on student-posed problems remains understudied.

Methods

The current study adopts a three-phase research design. Initially, a systematic literature review was conducted to develop the theoretical framework underlying original problems and task situations in MPP assessments. Subsequently, an MPP assessment prototype will be developed and validated in practical settings. Finally, future empirical research will examine the impact of this framework through data collection and analysis. The present paper primarily addresses the first phase, establishing theoretical foundations, categorizing original problems, and clarifying their relationships with task situations to guide subsequent empirical validation.

Results and Discussion

The Definition of Original Problems

The concept of the original problem was first introduced in Silver's (1994) definition of problem posing, where problem posing is described as generating new problems and reformulating given problems before, during, or after problem-solving. In this context, the "given problem" serves as the prototype of the original problem. Baumanns and Rott (2020) define "initial problems" as specific problems provided in structured task situations to encourage students to create new problems by altering the conditions of original problems. Although Baba (2024) does not provide a specific definition of the original problem, he points out that when the problem is to pose problems by changing the original problem, solving the problem involves understanding the characteristics of the original problem structure and pose as many problems as possible through creativity. In this study, original problems are explicitly defined as structured mathematical problems provided in MPP assessment tasks, intended for student modification to encourage creative mathematical thinking. These problems typically have a defined structure that can be altered by adjusting conditions to produce new mathematical problems.

The Categorizations of Original Problems

This study adopts the categorization of original problems based on their degree of extension, inspired by Cai and Hwang's (2002) original classification for student-posed problems: non-extended problems are confined to the given conditions in the task situation, whereas extended problems require modifications, expansions, or the addition of new conditions.

This distinction is also grounded in mathematical creativity. Leung (1993) emphasizes that MPP involves the "reformulation of a given problem," underscoring that extending an original problem is fundamentally a creative act. Distinguishing between non-extended and extended problems reflects two qualitatively different cognitive processes: reproducing a known problem versus creating a variation. While non-extended problems follow a schema-driven approach, extended problems require introducing new conditions, demanding greater creativity and cognitive effort.

It is important to note that this study adopts different reference criteria to distinguish the extension of original problems and student-posed problems. The extension of original problems is evaluated relative to the initial scope of the task situation, whereas the extension of student-posed problems is assessed with reference to both the initial scope and the original problem, which becomes part of the task situation once presented. This two-stage classification approach enables a more accurate evaluation of students' creative MPP performance, particularly when the original problem itself involves extended conditions, thereby reducing misjudgment in creativity assessment.

The Categorizations of Task Situations

Given that original problems are embedded within task situations, and their impact on student-posed problems may vary across different task situations, further discussion on the categorization of task situations is necessary.

<u>According to the level of mathematization of task situations</u>

Task situations are generally categorized into mathematical and simulated situations (step 1 in Figure 1). Mathematical situations involve pure mathematical concepts, structures, or processes, while simulated situations model real-world scenarios, often in simplified or idealized forms.

According to the Degree of Structuring of Task Situations

Stoyanova and Ellerton (1996) propose one of the most widely recognized categorizations of task situations, dividing them into Free, Semi-Structured, and Structured, while Baumanns and Rott (2020) argue for simplifying this distinction into structured and unstructured categories for clarity. Building upon these frameworks, task situations are categorized as structured and unstructured. Structured situations provide an original problem clearly to scaffold student problem posing, while unstructured situations don't provide specific original problems, requiring students to create entirely new problems.

Integrated Categorizations of Task Situations Considering Original Problems

Integrating the classifications outlined in Sections 4.2 and 4.3, this study proposes an integrated categorization framework consisting of six distinct types of task situations (Figure 2). Each type of task situation frames the role and function of original problems, providing a nuanced understanding of how task and original problem characteristics influence student creativity and cognitive engagement in MPP assessments.

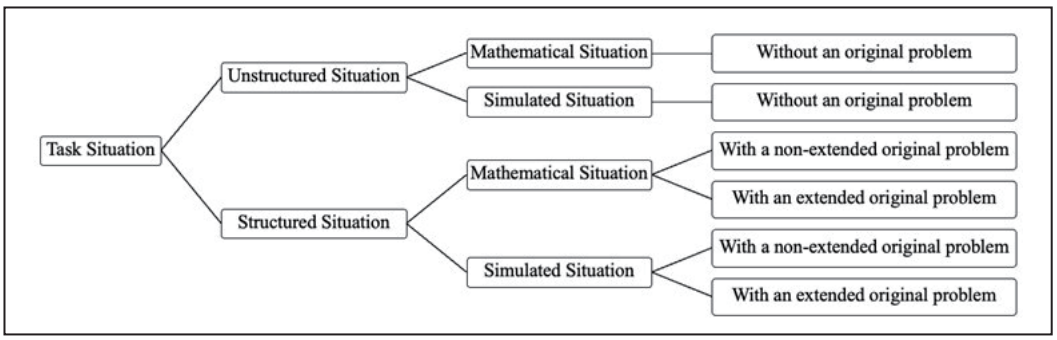


Figure 2. Integrated categorizations of task situations considering original problems

Conclusion

This study explores the definition of original problems and their role within MPP assessment. A systematic approach is introduced to categorize task situations and original problems. The refined classification distinguishes between structured and unstructured, as well as mathematical and simulated task situations, and refines the categorization of original problems based on their degree of extension. By explicitly identifying six distinct task situations (Figure 2), the framework offers a nuanced basis for evaluating how original problems influence student-posed problems. Future empirical studies based on this framework will further clarify how variations in task situations according to original problems influence students' cognitive processes and creativity.

References

Baba, T. (2024). Openness of Problem Solving in the 21st Century: Mathematical or Social? *Proceedings of the 14th International Congress on Mathematical Education*, 17-32. https://doi.org/10.1142/9789811287183_0002

Baumanns, L., & Rott, B. (2020). Rethinking Problem-Posing Situations: A Review. *Investigations in Mathematics Learning, 13*(2), 59–76. https://doi.org/10.1080/19477503.2020.1841501

Cai, J. (2022). What research says about teaching mathematics through problem posing. *Education & Didactique, 16*, 31–50.

Cai, J., Hwang, S., Jiang, C., Silber, S. (2015). Problem-Posing Research in Mathematics Education: Some Answered and Unanswered Questions. In: Singer, F., F. Ellerton, N., Cai, J. (eds) *Mathematical Problem Posing. Research in Mathematics Education* (pp. 3-34). Springer. https://doi.org/10.1007/978-1-4614-6258-3_1

Cai, J., & Hwang, S. (2002). Generalized and generative thinking in us and Chinese students' mathematical problem solving and problem posing. *Journal of Mathematical Behavior, 21* (4), 401–421. https://doi.org/10.1016/S0732-3123(02)00142-6

Cai, J., & Hwang, S. (2023). Making mathematics challenging through problem posing in the classroom. In R. Leikin (Ed.), *Mathematical challenges for all* (pp. 115–145). Springer. https://doi.org/10.1007/978-3-031-18868-8_7

Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist, 23* (2), 167–180. https://doi.org/10.1207/s15326985ep2302_6

Leung, S. S. (1993). On the role of creative thinking in problem posing. *ZDM Mathematics Education, 29*(2), 81–85. https://doi.org/10.1007/s11858-997-0004-9

Newell, A., & Simon, H. A. (1972). *Human problem solving.* Prentice-Hall.

Shriki, A. (2013). A model for assessing the development of students' creativity in the context of problem posing. *Creative Education, 4*(7), 430-439. https://doi.org/10.4236/ce.2013.47062

Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students' problem posing. In P. Clarkson (Ed.), *Technology in mathematics education* (pp. 518-525). Melbourne: Mathematics Education Research Group of Australasia.

Wickelgren, W. A. (1974). *How to solve problems*. W. H. Freeman; Company.

TSG3-14

SCAFFOLDED INTERVIEW TASKS BUILD ON WRITTEN TASKS TO REVEAL ALGEBRAIC THINKING

Catherine Pearn
The Faculty of Education, The University of Melbourne, Australia, cpearn@unimelb.edu.au

This paper initially examines the results from 470 upper primary students' responses to three reverse fraction tasks given on a written paper-and-pencil test. Reverse fraction tasks are those where students are given the number of objects representing a specific fractional part and then need to find the number of objects representing the unknown whole. Seventeen of these students, successful with two of the three written reverse fraction tasks, were individually interviewed using tasks that used the same fractions as on the paper-and-pencil test but had different quantities representing those fractions. Each interview task was divided into three separate parts moving from changes in quantity to a more generalizable task of 'any quantity' representing the given fraction. The careful scaffolding of the interview tasks allowed students to describe their thinking which highlighted the move from using arithmetic calculations to algebraic thinking.

Keywords: fractional competence, interviews, algebraic thinking

Researchers, teachers, and students agree that fractions are difficult to teach and learn. Kieren (1988) provided five interpretations of fractions: part-whole relations, measures, operators, quotients, and ratios. Behr and Post (1988) stated that students need to be competent in the four operations of whole numbers, along with an understanding of measurement, to enable them to understand rational numbers. Empson, Levi and Carpenter (2010) also argue that the key to learning algebra meaningfully is to help students: "to see the continuities among whole numbers, fractions and algebra" (p. 411). Siegler et al.'s analysis (2012) of longitudinal data from both the United States and United Kingdom showed that competence with fractions and division in fifth or sixth grade is a uniquely accurate predictor of their attainment in algebra and overall mathematics performance five or six years later when controlling other factors such as whole number arithmetic, intelligence, working memory, and family background.

Pearn (2019) investigated the links between fractional competence and algebraic thinking as middle-years students solved mathematical tasks using whole numbers, fractions, decimals and pronumerals. Fractional competence included understanding fraction size and relationships, demonstrating understanding of fraction concepts and basic arithmetic competence with simple fractions. Algebraic thinking included students' understanding of equivalence, transformation using equivalence, and the use of generalizable methods (Stephens & Ribeiro, 2012). In particular, algebraic reasoning for this research is defined in terms of students' capacity to identify an equivalence relationship between a given collection of objects and the fraction this collection represents of an unknown whole. This paper focuses on a very small part of the larger research and investigates the strategies the 470 Year 5 and 6 students used to solve written tasks compared to the more generalized interview tasks given to 17 of these students. The interview tasks were developed to determine evidence of algebraic thinking rather than a reliance of arithmetical thinking as shown in the written paper and pencil tests.

The Study

Participants

Four hundred and seventy Australian students from Years 5 and 6 (10 – 12 years) completed two paper and pencil assessments: The Fraction Screening Test and the Algebraic Thinking Questionnaire and 17 students were interviewed using a semi-structured interview (Pearn, 2019). Students were chosen to be interviewed if they had been successful with at least two of the reverse fraction tasks in the written Fraction Screening Test. Reverse fraction tasks are those where students are given the number of objects representing a given fractional part and then need to find the number of objects representing the unknown whole. As for the reverse fraction tasks in the written Fraction Screening Test the interview tasks focused on the fractions two-thirds, four-sevenths and seven-sixths.

The Assessment tasks

In this paper the results for the three written reverse fraction tasks from the Fraction Screening Test will be discussed before then focusing on selected results from the Structured Interview. The three reverse fraction tasks from the Fraction Screening Test are shown in Figure 1. All three tasks asked students to determine the number of objects representing the whole group if given the quantity of objects for a fractional part of that group. Students were expected to include explanations for the solution of all three tasks. Reverse Fraction Tasks A and C were worded tasks with diagrams (left-hand side and right-hand side of Figure 1) while Reverse Fraction Task B was a worded task but had no diagram (middle Figure 1).

Reverse Fraction Task A	Reverse Fraction Task B	Reverse Fraction Task C
This collection of 10 counters is 2/3 of the number of counters I started with. How many counters did I start with?	Susie's CD collection is 4/7 of her friend Kay's. Susie has 12 CDs. How many CDs does Kay have? Show all your working.	This collection of 14 counters is 7/6 of the number of counters I started with. How many counters did I start with? Explain how you decided that your answer is correct.

Figure 1. The three reverse fraction tasks (Fraction Screening Test, Pearn, 2019)

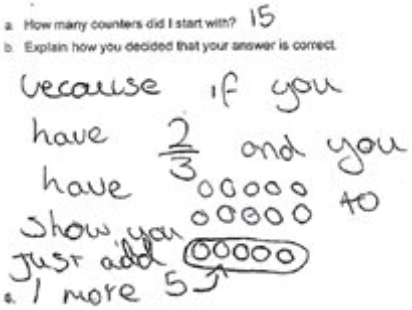
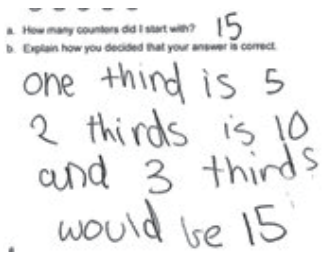
The Structured Interview included the same three fractions as used in the Fraction Screening Test, but changes were made to the number of objects (*quantity)* before introduced to unspecified quantities (*any quantity*). Figure 2 shows the three variations used in the interview for the fraction two-thirds. The initial task is similar to Reverse Fraction Task A (left-hand side of Figure 1) but has no diagram. This is followed by a *change of quantity* with the final task being more open-ended and asked about *any quantity*. These suggestions are in line with research (Marton et al., 2004) which showed that varying numbers in mathematical tasks can foster generalization.

Initial interview task	*Change of quantity*	*Any quantity*
Imagine that I gave you 12 counters which is $^2/_3$ of the number of counters I started with. How many counters did I start with? Explain your thinking.	If I gave you 18 counters, which is $^2/_3$ of the number of counters I started with, how would you find the number of counters I started with?	If I gave you any number of counters, which is also $^2/_3$ of the number I started with, what would you need to do to find the number of counters I started with?

Figure 2. The interview tasks for the fraction two-thirds (Structured Interview, Pearn, 2019)

Results

After a detailed analysis of the results for the three reverse fraction tasks (Figure 1) in the larger study (Pearn, 2019) a *Classification Framework for Reverse Fraction Tasks* was developed with examples given for each of the three reverse fractions shown in Figure 1 (Pearn, 2019). This framework was used to analyse both the responses for the written reverse fraction tasks and several of the interview tasks. Figure 3 includes the strategies, the percentage of students using each of these strategies and an example for Reverse Fraction Task A (Figure 1). For example, 7% of the students were deemed to be Diagram Dependent as they used explicit partitioning of diagrams before using additive strategies. 40% of the students used a partially multiplicative strategy for Reverse Fraction Task A, while 17% used a fully multiplicative strategy. Please note that 21% of students did not include a detailed solution, thus it was impossible to determine the strategy they actually used to solve these tasks. This confirmed the need for an interview protocol to be developed.

Explanation	Example for Reverse Fraction Task A
Diagram Dependent (7%): Students use explicit partitioning of diagrams before using additive strategies	
Additive/Subtractive (3%): Students use additive methods without explicit partitioning of a diagram. Students find the number of objects needed to represent one-third and then use counting or addition e.g. "add another 5" to find the number of objects that represents the whole.	

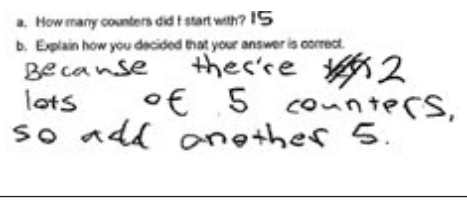
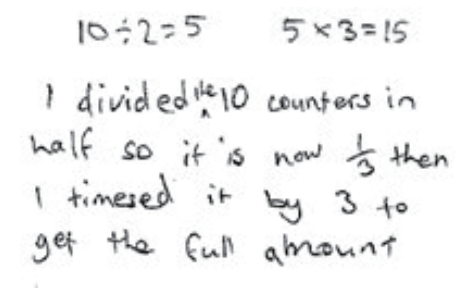
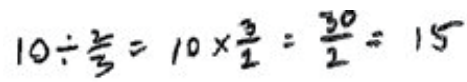
Partially Multiplicative (40%): Students use both multiplicative and additive methods. They calculate the missing fractional part (one-third) and then add it onto the original quantity (two-thirds).	
Fully Multiplicative (17%): Students use fully multiplicative methods. Students find the quantity representing one-third by dividing the quantity representing two-thirds by two. They then multiply the quantity representing one-third (5) by three to find the whole (15).	
Advanced multiplicative (2%): Students use more advanced multiplicative methods to solve the reverse fraction questions. These included appropriate algebraic notation to find the whole, or a one-step method as shown here to find the whole by dividing the given quantity (10) by two-thirds.	

Figure 3. The Classification framework for reverse fraction tasks for the fraction two-thirds

The 17 students' interview responses were also analyzed using the *Classification Framework for Reverse Fraction Tasks* (Figure 3). As for the written responses, the students' responses to the interview tasks varied from a reliance on computational methods (diagram dependent and additive/subtractive) to multiplicative responses which indicate algebraic reasoning. Eight students consistently used the same strategy for all questions of the Structured Interview. Two used *Partially Multiplicative* strategies, four used *Fully Multiplicative* strategies while two students used *Advanced Multiplicative* strategies. The other students used a variety of strategies which relied on arithmetic calculations – diagrams and/or addition and subtraction strategies.

Conclusion

Students' responses to three written reverse fraction tasks and the six of the subsequent interview tasks were analyzed using the *Classification Framework for Reverse Fraction Tasks* (Figure 3). In the written responses to Fraction Task A (left-side Figure 1) many students appeared to use diagrams to assist with their solution but used a variety of strategies to solve the other two reverse fraction tasks from the Fraction Screening Test. As shown in Figure 3 the detailed analysis of students' responses to the three written reverse fraction tasks revealed the use of different types of arithmetical strategies. These strategies ranged from a dependence on diagrams to fully or advanced multiplicative strategies.

The interview tasks were designed to build on the written tasks previously completed by the students. These interview tasks used the same three fractions and were carefully scaffolded to move

from *changes in quantity* to a more generalized task of *any quantity*. The use of the generalized task as the final task for each of the three fractions ensured that students were forced to move from a focus on arithmetic calculations to the more generalized algebraic thinking. The tasks used in this research demonstrate that tasks can be developed that allow students to move beyond using additive strategies to the more generalized multiplicative strategies.

References

Behr, M. J., & Post, T. R. (1988). Teaching rational number and decimal concepts. In T. R. Post (Ed.), *Teaching mathematics in grades K-8* (pp. 201-248). Allyn & Bacon

Empson, S. B., Levi, L., & Carpenter, T. P. (2010). The algebraic nature of fractions: developing relational thinking in elementary school in J. Cai and E. Knuth (Eds), *Early Algebraization: Cognitive, Curricular and Instructional Perspectives*. New York: Springer

Kieren, T. E. (1988). Personal knowledge of rational numbers: Its intuitive and formal development. In J. Hiebert and M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 162-181). Erlbaum.

Marton, F., Runesson, U., & Tsui, A. (2004). The space of learning. In F. Marton, A. Tsui, P. Chik, P. Ko, & M. Lo (Eds.), *Classroom discourse and the space of learning* (pp. 43–62). Lawrence Erlbaum.

Pearn, C. (2019). *Investigating connections between fractional competence and algebraic reasoning in the middle years*. https://minervaaccess.unimelb.edu.au/handle/11343/237473

Siegler, R., Duncan, G. Davis-Kean, P. Duckworth, K. Claessens, A. Engel, M. Susperreguy, M. & Chen, M (2012) *Early Predictors of High School Mathematics Achievement*.

Stephens, M., & Ribeiro, A. (2012). *Working towards Algebra: The importance of relational thinking*. Revista Latinoamericano De Investigacion En Matematica Educativa, 15(3), 373–402.

Acknowledgement

The large research study was doctoral research supervised by Dr Max Stephens and Associate Professor Robyn Pierce of the Faculty of Education, The University of Melbourne.

TSG3-15

ENHANCING ADAPTIVE MATHEMATICS ASSESSMENT THROUGH GENERATIVE AI: INTEGRATING THE COGNITIVE DISTRACTOR GENERATION MODEL (CDGM) INTO THE GENERATIVE AI-BASED MULTI-STAGE ASSESSMENT (GAMSA) FRAMEWORK

Jin Min Chung
University of Iowa, United States, jinmin-chung@uiowa.edu

Adaptive learning systems are becoming increasingly important in mathematics education, with a growing emphasis on assessment approaches that support personalized learning. This study explores the integration of the Cognitive Distractor Generation Model (CDGM) within the Generative AI-based Multi-Stage Assessment (GAMSA) framework to improve assessment accuracy and diagnostic feedback. By leveraging AI-driven distractor generation, this research addresses the challenges of traditional multi-stage testing, including limited feedback personalization. Finding suggest that CDGM-enhanced GAMSA assessments are expected to improve diagnostic precision and enhance student learning outcomes through real-time, tailored feedback.

Keywords: Adaptive Assessment, Generative AI, Cognitive Distractors, Multi-Stage Testing, Personalized Learning

Introduction

Mathematics assessment has evolved beyond traditional summative evaluations to include formative, competency-based, and AI-enhanced approaches (Black & Wiliam, 2010; Cheung & Slavin, 2013; Kretzschmar, Sailer, Wertenauer, & Seitz, 2024; Xu, 2024). In East Asia, where rigorous academic assessment plays a critical role in student learning, integrating advanced AI technologies into assessment design can address limitations of current evaluation models (Payadnya, Putri, Suwija, Saelee, & Jayantika, 2025).

This study introduces the Generative AI-based Multi-Stage Assessment (GAMSA) framework, which incorporates the Cognitive Distractor Generation Model (CDGM) to enhance diagnostic precision and feedback personalization. While GAMSA maintains the structured progression of MST, it offers an improved feedback mechanism by analyzing student responses and dynamically generating distractors tailored to specific misconceptions (Choi, 2024). This advancement provides more meaningful insights into student thought processes, fostering deeper conceptual understanding and engagement.

Theoretical Background and Significance

Assessment plays a crucial role in mathematics education, influencing both instructional practices and student learning outcomes. Existing literature highlights the importance of well-structured distractors in multiple-choice assessments to reveal student misconceptions (Mislevy, 2018). However, manual distractor creation is labor-intensive and prone to inconsistencies. Generative AI models such as CDGM offer a scalable solution by automating distractor generation, improving assessment reliability, and enabling real-time personalized feedback.

The GAMSA framework enhances traditional MST by integrating AI-driven item and distractor generation, mitigating common challenges such as item exposure, limited feedback (Choi, 2024). By leveraging student response data, GAMSA improves feedback precision and diagnostic accuracy, promoting deeper conceptual understanding and engagement.

Research Objectives

This study aims to:

1. Develop a methodology for integrating CDGM into the GAMSA framework to enhance adaptive mathematics assessments.
2. Evaluate the impact of AI-generated distractors on diagnostic accuracy and personalized feedback.
3. Assess student learning outcomes resulting from GAMSA-based assessments compared to traditional MST.

Methodology

This study employs a mixed-methods research design, integrating both quantitative and qualitative approaches to comprehensively evaluate the integration of the Cognitive Distractor Generation Model (CDGM) within the Generative AI-based Multi-Stage Assessment (GAMSA) framework.

Study Design

The research consists of an experimental component and qualitative analysis to assess the impact of AI-driven distractor generation on assessment quality and learning outcomes. The participants include first-year middle school students in South Korea, with a total sample of approximately 200 students. They will be divided into two groups: a control group utilizing traditional MST-based assessments and an experimental group employing the GAMSA framework enhanced with CDGM-generated distractors.

Data Collection

In this study, data will be collected through three primary methods to comprehensively evaluate the impact of AI-generated distractors within the Generative AI-based Multi-Stage Assessment (GAMSA) framework.

First, assessment performance analysis will be conducted by comparing student responses across the different experimental groups. This analysis aims to assess the diagnostic accuracy of the generated distractors and their effectiveness in identifying students' cognitive understanding. By evaluating response patterns and error types, the study will determine how well each distractor generation model supports accurate diagnosis of student learning gaps.

Second, student feedback surveys will be administered to gather qualitative insights regarding the perceived effectiveness of personalized feedback. These surveys will allow students to express their opinions on whether the feedback provided within the GAMSA framework was helpful in enhancing their understanding of the assessed concepts. The collected responses will contribute to assessing the impact of AI-generated distractors on students' learning experiences and engagement.

Finally, educator evaluations will be conducted through expert reviews. Experienced educators and assessment specialists will evaluate the objectivity and consistency of AI-generated distractors.

Their assessments will focus on determining whether the distractors appropriately reflect common misconceptions, maintain fairness, and align with best practices in assessment design. These expert reviews will provide an additional layer of validation regarding the quality and educational value of the generated assessment items.

Through this multi-faceted data collection approach, the study aims to provide both quantitative and qualitative insights into the effectiveness of AI-generated distractors in adaptive assessment.

Data Analysis

The collected data will be analyzed through a structured approach to evaluate the effectiveness of AI-generated distractors within the Generative AI-based Multi-Stage Assessment (GAMSA) framework. This analysis will focus on three key aspects: diagnostic accuracy, feedback effectiveness, and learning outcomes.

First, diagnostic accuracy assessment will be conducted to examine how well the distractors generated by the Cognitive Distractor Generation Model (CDGM) identify and address student misconceptions. The performance of CDGM-generated distractors will be compared with those manually created by educators to determine their effectiveness in diagnosing learning gaps. This comparison will involve analyzing student response patterns, error types, and the frequency with which AI-generated distractors correctly pinpoint conceptual misunderstandings. The goal is to assess whether AI-enhanced distractors contribute to a more precise identification of students' cognitive difficulties.

Second, feedback effectiveness analysis will focus on measuring student engagement and their perceived improvements in conceptual understanding. This analysis will be based on student survey responses regarding their experiences with AI-generated feedback. By examining self-reported levels of engagement, confidence, and comprehension, the study aims to determine whether the personalized feedback provided within GAMSA contributes to more meaningful learning experiences. The effectiveness of AI-generated feedback will also be compared with traditional methods to evaluate its added value in formative assessment.

Lastly, learning outcomes evaluation will be performed through statistical comparisons between the control and experimental groups. Various statistical methods will be used to assess knowledge retention and skill development among students exposed to different distractor generation models. Key indicators such as test scores, response times, and improvement rates will be analyzed to determine whether AI-generated distractors lead to measurable gains in learning outcomes.

By adopting this research approach, the study seeks to provide empirical evidence on the efficacy of AI-enhanced assessment frameworks in improving personalized learning experiences. The findings will offer valuable insights into how AI-generated distractors contribute to more effective diagnosis, feedback, and learning progression.

Findings and Implications

Preliminary findings suggest that CDGM-integrated GAMSA assessments offer higher diagnostic precision and personalized feedback compared to conventional MST. AI-generated distractors successfully capture common student misconceptions, leading to improved feedback loops and greater student engagement. These results highlight the potential of AI-driven assessments in supporting adaptive mathematics education in East Asia.

The implications of this research extend to policy-makers, curriculum designers, and educators seeking to optimize assessment strategies through AI-driven methodologies. Future research should explore large-scale implementations of GAMSA across diverse educational settings.

Conclusion

This study contributes to the ongoing discourse on AI-driven assessment models by demonstrating the effectiveness of CDGM within the GAMSA framework. The integration of generative AI into mathematics assessment presents an opportunity to enhance learning outcomes through personalized, real-time feedback. As educational technology continues to evolve, future research should focus on refining AI-based assessment tools and exploring their broader applications in personalized learning environments.

References

Black, P., & Wiliam, D. (2010). Inside the black box: Raising standards through classroom assessment. Phi delta kappan, 92(1), 81-90.

Cheung, A. C., & Slavin, R. E. (2013). The effectiveness of educational technology applications for enhancing mathematics achievement in K-12 classrooms: A meta-analysis. Educational research review, 9, 88-113.

Choi, J. (2024). Generative AI-based Multi-Stage Assessment (GAMSA) - Tatsuoka Model. Retrieved from https://ai.cafalab.com/F322B055-FDFE-4C6F-A84D-7506667ED710/

Kretzschmar, V., Sailer, A., Wertenauer, M., & Seitz, J. (2024). Enhanced Educational Experiences through Personalized and AI-based Learning. International Journal on Studies in Education (IJonSE), 6(2).

Mislevy, R. J. (2018). *Sociocognitive considerations in educational assessment*. Cambridge University Press.

Payadnya, I. P. A. A., Putri, G. A. M. A., Suwija, I. K., Saelee, S., & Jayantika, I. G. A. N. T. (2025). Cultural integration in AI-enhanced mathematics education: insights from Southeast Asian educators. Journal for Multicultural Education, 19(1), 58-72.

Xu, Z. (2024). AI in education: Enhancing learning experiences and student outcomes. Applied and Computational Engineering, 51(1), 104-111.

MEMO

MEMO

KOREA 2025

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 4:

The Essence of Mathematics Teacher Education

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 4
(Day 2)

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

TSG4-1-1

EXPLORING LESSON STUDY TEAM COLLABORATION IN TEACHING POSITIVE AND NEGATIVE NUMBERS TO GRADE 7 STUDENTS

Noppasorn Boonsena
Khon Kaen University, Thailand, noppasorn.nb@kkumail.com

Nabhatsorn Saithong
Bannongkungwittayakarn school, Thailand, napatsornsait@gmail.com

Nisakorn Boonsena
Khon Kaen University, Thailand, nisabo@kku.ac.th

Narumon Changsri
Khon Kaen University, Thailand, changsri_crme@kku.ac.th

This research examines the collaborative work of Lesson Study teams in teaching through problem situations related to positive and negative numbers in Grade 7. Teams from two schools designed and implemented teaching activities using an Open Approach to enhance students' understanding of the addition and subtraction of integers. Data were collected through classroom observations, students' ideas, and Lesson Study teams' reflections. The findings reveal that all the students could engage in problem situations. Approximately 78% of the students were able to write additional sentences, and many articulated their reasoning based on the total movement. However, challenges related to the use of mathematical symbols and filling in tables were observed. This study highlights the importance of collaborative planning and the improvement of mathematics instruction within Lesson Study teams.

Keywords: Collaborative Learning, Lesson Study, Mathematics Problem Situations

Introduction

Most teachers typically work alone and separately from their colleagues. Those who collaborate often find themselves unsure about what to do or how collaboration can enhance teaching, and they tend to focus on everything except improving classroom lessons (Stigler, J. W. & Hiebert, J., 1999). Teaching conducted by a single teacher limits perspectives and lacks an understanding of the students' contexts. If the teacher does not understand the students' context, the lesson plans may reflect only the teacher's perspective, which may not align with the needs and readiness of the students (EEFI, 2022).

Lesson Study incorporated with Open Approach helps to develop teachers' professional skills and improve student learning. Lesson Study is an innovation used to enhance collaboration among teachers, while teaching with the Open Approach focuses on student problem-solving to help students think independently. Therefore, designing mathematics activities using open-ended problems allows the problem situations to accommodate a wide range of students' ideas. Anticipating students' ideas to the problem situations created, observing their thinking in the classroom, and re-

flecting on their ideas are crucial steps. These processes cannot be carried out by one teacher alone, which is why it is necessary for teachers to engage in these activities together (Inprasitha, 2004).

Research Purpose

The purpose of this research was to study the collaboration of Lesson Study teams and the interactions of these teams during a learning activity in problem situations involving positive and negative numbers in Grade 7.

Research Methodology

The target group consisted of two Lesson Study Teams from two secondary schools in Khon Kaen, Thailand, referred to as School A and School B. School A has been forming Lesson Study team since 2019 and has 6 years of experience in learning together through lesson study. School B just started forming lesson study team this year.

The first team from School A, composed of the school principal, in-service mathematics teachers, and four pre-service mathematics teachers, totaling six people, with pre-service teachers serving as both instructors and lesson planners. The principal led the lesson planning, while the in-service teachers and pre-service mathematics worked together to plan, observe, and reflect.

The second team was from School B, composed of 7 in-service teachers, 2 of mathematics teachers, 2 of science teachers, 1 of foreign language teacher, 1 of Thai language teacher, and 1 of social studies teacher. The in-service mathematics teachers serve as both instructors and leaders in lesson planning. Teachers in both teams worked collaboratively to plan, observe, and reflect. Data collection encompassed lesson plans, students' worksheets, video recordings, and cameras. Analysis focused on Lesson Study Teams' interactions during three steps of the Lesson Study, according to the framework of Lesson Study incorporated with Open Approach (Inprasitha, 2006, 2014, 2022).

Results

The research findings are presented according to the three steps of the Lesson Study cycle: Co-Plan, Co-Do, and Co-See, focusing on both the collaboration of Lesson Study teams and student learning.

The first step of collaboratively design a research lesson (Co-Plan), the Lesson Study teams from both schools collaborated to analyze the mathematics textbook for junior high school that use in the TLSOA school project, to define the lesson plan, select activities, and set the lesson objectives. The objectives were for students to be able to write addition sentences using mathematical symbols and to compute the sum. Each school's Lesson Study team designed problem situations, instructions, and anticipated student ideas based on their own school context. Subsequently, the two Lesson Study teams discussed the design of problem situations that would align with the students' context. The activity "Treasure Hunt" was designed as a board game involving moving pieces, and the instructions were created to engage students in solving the problem by following the rules of the game. Players on each team took turns drawing one card: if the card showed +2, they would move 2 spaces toward the goal; if the card showed -3, they would move 3 spaces away from the goal; and if the card showed 0, no movement would occur. Additionally, students were asked to fill in the blanks in a table with addition sentences to calculate the total of the movable spaces. The pre-service teacher, who taught this lesson, anticipated that students would be able to participate in the

game and write the addition sentences. Meanwhile, the in-service teacher anticipated that students might struggle when not starting from 0, as well as with writing the plus sign and the order of addition. Additionally, members of the Lesson Study team noted that students might be able to move the pieces but struggle with filling in the table, or they might be able to fill in the table using the results from the piece movements.

The second Step of Collaboratively observe the research lesson (Co-Do), each school's Lesson Study team implemented the collaboratively developed lesson plan to teach in their schools using the Open Approach as follows:

Figure 1. Lesson study team of primary school and secondary school

Step 1 Posing open-ended problem, classroom observations showed that all of the students were actively engaged with the problem situations. Then, the students worked together to understand the rules and instructions.

Step 2 Students' self-learning, School A, before exploring the starting points. Then each team moved the pieces according to the number shown on the card. Afterward, a group representative filled in the numbers in the table. Some groups of students were able to move the pieces but were unsure of how to fill the table, so they needed to revisit the rules. Regarding the writing of addition sentences, some students wrote without parentheses, -5 - 4 = □. Other students wrote the sentence with parentheses, (+3) + (-4) = □. At School B, students worked together to explore the different card numbers and images. Some students were confused about how to move the pieces and how to write the addition sentences. Some groups forgot to write the equal sign and the blank space, Some students wrote addition sentences without equal signs or parentheses, such as +3 + 2. From both schools, there was a misunderstanding in filling in the blanks in the table. Most students only filled in the blanks for some rounds of the game.

Figure 2. Problem situations involving positive and negative numbers in Grade 7

Figure 3. Classroom activity of school A and school B

Step 3 Whole class discussion, each group presented their diverse addition sentences, which included both positive and negative numbers. While one group used parentheses and others did not, students were still able to discuss their reasoning based on the total movement and collaboratively determine the winning team.

Step 4 Summarize through connecting students' mathematical ideas emerged in the classroom, the teacher and students collaboratively summarized the use of parentheses in addition sentences and piece movements. For example, if the card revealed +2, it meant moving 2 steps toward the goal (or with +1 meaning 1 step forward), and if the card revealed -6, it meant moving 6 steps away from the goal (or with -3 meaning 3 steps backward)

Third step of Collaboratively discuss and reflect on the research lesson (Co-See), the Lesson Study teams from both schools discussed the ideas that emerged in the classroom from the collaboratively designed problem situations. The pre-service teacher, who was the teacher for the lesson, noted that the familiar game format facilitated student engagement with the problem situation. However, the school principal observed initial confusion among some students regarding the movement instructions, particularly concerning the number of spaces moved in the first and second rounds. The in-service teacher, a member of the Lesson Study team, highlighted challenges for some groups in

completing the table, evidenced by the need for repeated instruction review or trying to move the pieces 1–2 times before starting to play for real and filling in the blanks. The Lesson Study team agreed on the importance of developing teachers' skills in anticipating students' ideas and preparing questions related to students' ideas, or connecting with students' ideas.

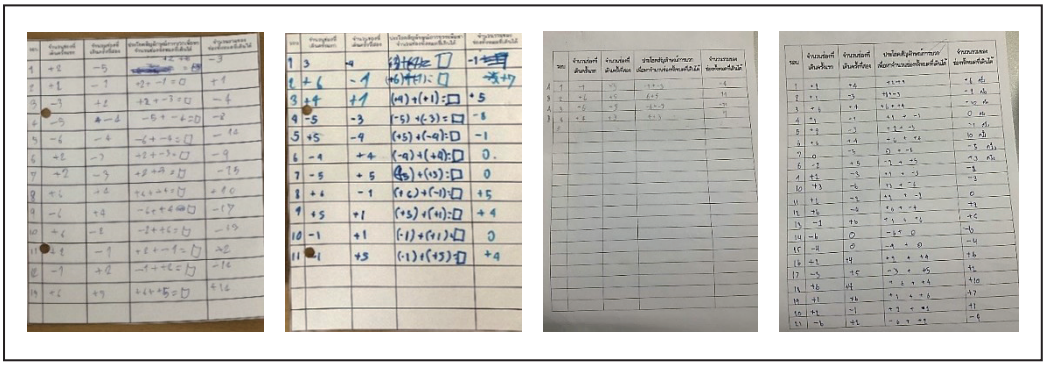


Figure 4. Example of students' worksheet

Discussion

The results of this study highlight the essential role of the collaboration of Lesson Study teams in improving teaching practices and supporting student learning. Consistent with Stigler and Hiebert's (1999) findings, collaboration during the lesson design phase enhanced teachers anticipate students' ideas and the process of learning than would have been possible individually. In alignment with the TLSOA model (Inprasitha, 2022), the Co-Do step showed that collaboration extended beyond planning into classroom practices. Observations revealed that teachers were more prepared to notice and respond to students' ideas because they had discussed these possibilities beforehand. The Co-See emphasized how collaborative reflection allowed teachers to identify necessary teaching adjustments. In addition to the ability to anticipate students' ideas was enhanced through collaborative design (Co-Plan) and cross-subject collaboration (in School B) presented both challenges and new perspectives.

Conclusion

The Lesson Study Team's collaboration demonstrably enhanced lesson design and fostered student learning of positive and negative numbers. While most students showed improved engagement and understanding, some persistent challenges highlight areas for continued attention, further scaffolding may be needed for symbolic representations. These findings underscore the critical role of evidence-based collaborative practices, encompassing design, observation, and reflection, are key to anticipating students' ideas more effectively, creating problem situations, and improving teaching based on observations of student learning.

References

Education Research Institute. (2022). *Understanding students' context through empathy*. https://research.eef.or.th/empathy/

Fernandez, C. and Yoshida, M. (2004). *Lesson study: a Japanese approach to improving mathematics teaching and learning.*

Inprasitha, M. (2022). *Process of Problem Solving in School Mathematics*. Khon Kaen: Penprinting.

_____. (2022). Lesson study and open approach development in Thailand: *a longitudinal study*. International Journal for Lesson and Learning Studies, Vol. 11 No. 5, pp. 1-15. Equitable

Nohda, N. (2000). *Teaching by Open-Approach Method in Japanese Mathematics Classroom*

Takahashi, A. (2014). *The role of the knowledgeable other in lesson study: Examining the final comments of experienced lesson study practitioners.* Mathematics Teacher Education and Development.

Stigler, J. W. & Hiebert, J. (1999). *The teaching gap*, New York: Free Press.

TSG4-1-2

COMPARISON OF LESSON PLANNING PROCESSES BY PRE-SERVICE MATHEMATICS TEACHERS: FROM THE PERSPECTIVE OF DESIGN RESEARCH

Hayato Hanazono
Miyagi University of Education, Japan, hanazono@staff.miyakyo-u.ac.jp

The purpose of this study is to determine how pre-service teachers in Japan develop the ability to plan lessons based on a theory. A comparison of lesson planning processes revealed that second-year graduate students used theory more appropriately than third-year undergraduate students. Second-year graduate students also developed important attitudes for theory-based lesson planning.

Keywords: professional development, pre-service mathematics teachers, planning lessons, design research

Background and Research Question

Lesson study, an activity in which teachers work collaboratively to improve the quality of their teaching, is widely practiced, especially in East Asia. In Japan in particular, this activity is called *jyugyou kenkyu* and is known as a following cyclic process: 1. Goal setting → 2. Lesson Planning → 3. Research Lesson → 4. Post-lesson Discussion → 5. Reflection → 1. Goal setting…(Fujii, 2017).

On the other hand, design research has been gaining attention as a cyclic activity related to teaching (Cobb et al., 2017). This activity is explained by the design aspect of classes, teaching materials, etc. through the coherent use of the theory and the refining and revising of theory based on empirical findings obtained in the process (Prediger et al., 2015). RME is a well-known theory of this kind. For example, Stephan (2015) conducts design research based on RME. The study describes a "design phase" with coherent use of RME, an "implementation phase" in which the plan is carried out, and an "analysis phase," for each.

A comparison of *jyugyou kenkyu* and design research reveals that theory is not explicit in the *jyugyou kenkyu* process. In fact, theory is not even drawn upon in the described model (Figure 1) that explains the details of the lesson planning process which is directly related to the "coherent use of theory."

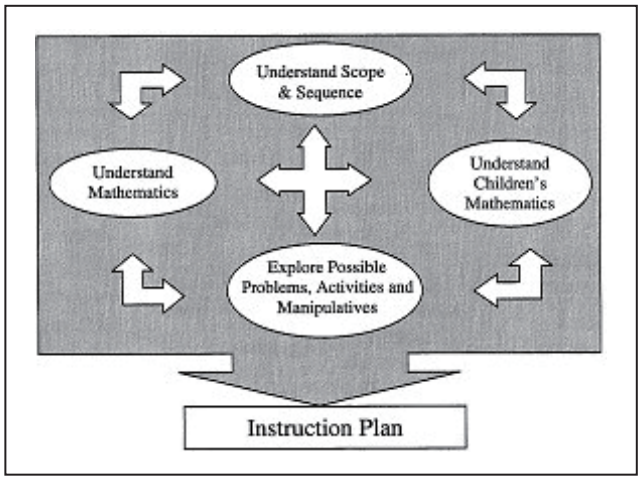


Figure 1. Lesson planning process (Watanabe et al., 2008, p.140)

Given the expansion of design research and the accumulation of theories in mathematics education, it is important to appropriately place theories in classroom research as well. Therefore, teachers need to have the ability to conduct lesson planning based on theory, as Stephan (2015) has done with the coherent use of RME. However, there is no finding on the extent to which professional development (PD) programs in Japan can develop the ability of teachers to carry out design research. This study addresses the following research questions, focusing on PD of pre-service mathematics teachers in Japanese colleges of education, especially on the ability of planning lessons based on theories (APLT) mentioned above: How do pre-service mathematics teachers in Japanese colleges of education grow in their APLT?

Methodology

This study adopts a method that describes and compares the processes of planning a lesson based on a theory of mathematics education conducted by pre-service teachers belonging to a certain college of education and its professional graduate school in Japan. The subjects are pairs of two students, each of whom are third year of their 4-year bachelor's degree (pair B), and second year of their 2-year master's degree students (pair M). All subjects are prospective mathematics teachers at the secondary level. In this faculty, the program on mathematics education begins in the second year, with one class in each of the two semesters of the year until the third year. in the fourth year, there is a seminar on mathematics education, where students learn the basics of how to conduct research. In the third and fourth years, there is a teaching practice. In the graduate school, there are two classes in each semester of the first year, and the seminar is held throughout the two years. In addition, students visit a school once a week to observe or teach a class. The graduate students who participate are graduates of this faculty and receive the continuous education.

In the study, Shimada (1995) was selected as the mathematics education theory to which the participants would refer. This theory is a relatively short volume that discusses a wide range of guidelines for planning lessons and was determined to be a theory that could withstand "the coherent use of the theory" (Prediger et al., 2015). Another reason for the selection was that none of the participants had read this literature in detail. In this book, the first half of the book explains its basic principles and guidelines for planning lessons, while the second half of the book provides examples of lessons

for different school stages. To avoid participants planning lessons from examples rather than theory, only this general theory part of the book was presented in the study. Otherwise, the study was designed as follows:

(i) The time available was set at 180 minutes so as not to be directly affected by differences in the participants' available time.

(ii) Participants were allowed to refer to several textbooks, the Course of Study, and web sits in addition to Shimada (1995).

(iii) The task was to plan a lesson on prime factorization for the first grade of junior high school. This content has recently been placed in junior high schools due to the revision of the Courses of Study, and the participants had never attended classes on this content as junior high school students.

(iv) The entire process was recorded with a video camera, an IC recorder, and the author's field notebook. Participants were also asked to write down their planned lesson.

(v) The study was conducted in the first semester.

For the analysis, first, the participants' conversations were transcribed in writing. Then, the textual information was coded by referring to the videos, fieldnotes, and participants' descriptions. Furthermore, by categorizing the codes obtained, the participants' lesson planning process was structured and compared to highlight its characteristics.

Results

Overview of the Lesson Planning process

Pair B finished reading the literature in about 52 minutes and spent the remaining time on lesson planning. They chose "how to classify" as their guideline. They began to consider lessons that used material not found in the textbook. However, they frequently referred to the textbook during the discussion process. Finally, they planned two lesson outlines.

Pair M finished reading the literature in roughly 56 minutes and spent the remaining time on lesson planning. They chose "how to find" as their guideline. First, they reviewed the textbook and anticipated the junior high school students' activities in that lesson. Ultimately, they did not develop an outline for the lesson, but they defined the task to be presented to the students at the beginning of the lesson and anticipated the details of the students' responses to that task.

Obtained Codes and Their Categories

A total of 15 different codes were obtained for the two pairs: A. Objectives, B. Evaluation, C. Learning Connections, D. Mathematics Structure, E. Students' Perspective, F. Preferred Idea, G. Natural Thought Processes, H. Task Characteristics, I. Environment and Progression, J. Organizing Ideas, K. Recalling Experience, L. Referencing Textbooks, M. Specifying the Theory in detail, N. Specifying the Theory, O. Fitting into the Literature. The number of times the codes appeared in the process for each pair is shown in Table 1. This value is the number of times the code was discussed in the participant's consideration; each discussion took a different amount of time.

Table 1. Number of codes appearance

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	total
Pair B	3	2	11	0	6	15	11	8	11	14	4	9	6	4	9	113
Pair M	5	0	7	7	21	5	27	20	0	12	0	1	2	2	4	113

What was more characteristic than what can be read from this table is that these codes often appeared alone in the process of considering Pair B, whereas multiple codes appeared simultaneously in the process of considering Pair M. This feature is particularly evident in the two codes M and N. Compared to pair B, the number of appearances of these codes in pair M is small. This means that pair M spent much of its time considering other codes with codes M and N in mind. These M, N, and O are codes that use theory to design the entire lesson and student thinking, and to examine whether the plan is coherent with the theory. In other words, while Pair B was able to plan the entire lesson from a variety of perspectives, these were less based on the coherent use of theory than was Pair M. Table 2 shows the categories based on the co-appearance of these codes except K to O. If a set of co-appearance codes is a subset of another set, it was merged into the larger set.

Table 2. Categorized codes (B: Pair B's categories; M: Pair M's categories)

Categories	A	B	C	D	E	F	G	H	I	J
B1: Tasks that elicit Preferred Idea	○	○				○	○	○		○
B2: Reasonable Evaluation		○	○			○	○	○		○
M1: Mathematics Educational Values	○			○	○	○				○
M2: Tasks that elicit Natural Thinking			○		○		○	○		
M3: Students' Mathematical Perspective					○	○	○			○

Discussion

As mentioned above, there was a clear qualitative difference in the "coherent use of the theory" between Pair B who were just beginning their study of mathematics education and Pair M who were in the final year of their PD program as pre-service teachers. To determine what experiences might result in such differences, it is necessary to analyze the lesson planning process by fourth-year undergraduates and first-year graduate students. It is worthwhile to examine the impact of the larger program of teaching practice and seminars. The qualitative difference between the two pairs can be read from the code categories. In particular, Pair M considered a balance between "what I want to teach" and "what the students think," while Pair B was biased toward the former. This is an important attitude in avoiding self-righteous judgments in theory-based lesson planning.

References

Cobb, P., Jackson, K., & Sharpe, C. D. (2017). Conducting design studies to investigate and support mathematics students' and teachers' learning. In J. Cai (Ed.) *Compendium for research in mathematics education* (pp. 208-233). NCTM Inq.

Fujii, T., (2017). Unifying lesson study with teaching mathematics through problem solving. In T. McDougal (Ed.), *Essential mathematics for the next generation: What and how student should learn* (pp. 85-103). Tokyo Gakugei University Press.

Prediger, S., Gravemeijer, K., & Confrey, J. (2015). Design research with a focus on learning processes: An overview on achievements and challenges. *ZDM Mathematics Education, 47*(6), 877-891. https://doi.org/10.1007/s11858-015-0722-3

Shimada, S. (Ed.). (1995). *The open-ended approach: A new proposal for teaching mathematics*. Toyokan Publishing Co., Ltd.

Stephan, L. M. (2015). Conducting classroom design research with teachers. *ZDM Mathematics Education, 47*(6), 905-917. https://doi.org/10.1007/s11858-014-0651-6

Watanabe, T., Takahashi, A., & Yoshida, M. (2008). Kyozaikenkyu: A critical step for conducting effective lesson study and beyond. In Arbaufh & Taylor (Eds.), *Inquiry into mathematics teacher education* (AMTE monograph series, vol. 5, pp. 131-142). Association of Mathematics Teacher Education.

TSG4-1-3

COLLABORATIVE RESEARCH LEARNING: COMMUNITIES OF PRACTICE AMONG PRE-SERVICE AND IN-SERVICE MATHEMATICS TEACHERS

Mailizar Mailizar
Syiah Kuala University, Indonesia, mailizar@usk.ac.id

Rahmah Johar
Syiah Kuala University, Indonesia, Rahmah.johar@usk.ac.id

Mukhlis Hidayat
Syiah Kuala University, Indonesia, Mukhlis.hidayat@usk.ac.id

Research is vital in higher education, fostering knowledge, collaboration, and personal growth. Communities of practice (CoPs) are recognized for enhancing collaborative learning and research. However, limited research explores how CoPs support research learning among educators at different stages of their professional journey, such as pre-service and in-service mathematics teachers. This study aims to examine how CoPs promote research skill development, knowledge-sharing, and problem-solving among teachers with varying levels of expertise. Using a concurrent mixed-methods approach, data were collected from a WhatsApp group and semi-structured interviews with seven participants—four pre-service and three in-service mathematics teachers. Quantitative analysis was conducted through social network analysis, while thematic analysis was used for qualitative data. This study shows that CoPs provide peer mentorship, promote collaboration, and create a supportive learning environment. Pre-service teachers offered practical assistance, while in-service teachers provided advanced guidance and leadership. Challenges such as scheduling conflicts were managed through flexible coordination and peer support. It suggests that educators should encourage the use of CoPs to enhance research learning across professional levels. By fostering collaboration and leadership within CoPs, teachers can benefit from shared knowledge, mentorship, and improved professional development.

Keywords: Communities of Practice (CoPs); Collaborative Research Learning; Pre-Service and In-Service Teachers Research Skill Development

Introduction

Research plays a crucial role in pre-service and in-service teacher education, offering numerous benefits to individuals and institutions. It helps enhance the understanding of educational practices and supports the growth of knowledge and innovation in the field (Basu, 2020). As part of this, the integration of teaching and research is essential, but requires strategies for its full potential to be realized (Tight, 2016). Research activities promote curiosity, social responsibility, and commitment among educators and academics, leading to personal growth and institutional prestige (Ng & Pemberton, 2013; Vallejo-Lopez & Rincón-Ríos, 2019).

In indonesia, both undergraduate and post-graduate program in mathematics education requires students to conduct research for their final project. One of research designs that is widely adopted by the students is design, also known as research and development (R&D). This research design poses

unique challenges for students who majority of them are pre-service and in-service mathematics teachers. These challenges are often related to curriculum design, engagement with research activities, and the development of strong research skills. Therefore, it highlights the need for structured development programs to enhance research quality without overburdening mentors or supervisors (Albertyn et al., 2018). To address these challenges, the concept of communities of practice (CoP) offers a promising approach by fostering collaboration and shared learning among pre-service and in-service teachers..

In particular, CoPs provide pre-service teachers with opportunities to engage in peer support, where they can learn from more experienced in-service teachers. This collaborative learning model helps bridge the gap in research knowledge and fosters the development of essential skills required for successful research projects. By working closely with their peers and mentors, teachers can better navigate the complexities of design research, ultimately leading to more meaningful research outcomes.

The aim of this paper is to explore how communities of practice (CoPs) support research learning among pre-service and in-service mathematics teachers during their study in undergraduate and post-graduate programs of mathematics education. It seeks to understand the role of these collaborative networks in enhancing teachers' research skills, knowledge-sharing, and problem-solving abilities. By analyzing the interactions within CoPs, this paper will highlight how these communities contribute to the overall development of teacher-researchers and foster a collaborative learning environment in academic settings.

This paper addresses the following key research questions:

a. How do communities of practice facilitate research learning among pre-service and in-service mathematics teachers?

b. What specific challenges are faced by teachers in CoPs, and how are these challenges addressed?

c. In what ways do CoPs enhance teachers' research skills, knowledge sharing, and problem-solving capacities?

d. How do pre-service and in-service mathematics teachers differ in their participation and contributions within CoPs?

Method

This research adopts a concurrent mixed methods design, integrating both qualitative and quantitative approaches to provide a comprehensive understanding of how communities of practice (CoPs) support research learning. This study involved seven participants, consisting of four pre-service and three in-service mathematics teachers. They are students at undergraduate and post-graduate programs in mathematics education at one of public universities in Indonesia. The participants were selected using purposive sampling, specifically targeting teachers interested in conducting design research or research and development (R&D) final project of their study.

Data for this study was collected through both quantitative and qualitative methods. The quantitative data was extracted from four months (September 2022–January 2023) of conversations in a WhatsApp group, allowing for the analysis of communication patterns and interactions among the participants. In addition, semi-structured interviews were conducted with all seven participants, offering deeper insights into their experiences and perspectives. Data analysis for this study involved

both quantitative and qualitative methods. The quantitative data was analyzed using descriptive analysis and social network analysis, with the latter conducted through Python software to examine communication patterns. For the qualitative data, thematic analysis was used to identify key themes from the semi-structured interviews

Results and Discussion

The findings of this study highlight the significant role that communities of practice (CoPs) play in supporting collaborative research learning among pre-service and in-service mathematics teachers. The results align with previous research showing that CoPs foster a collaborative environment where knowledge is shared, and skills are developed (Buysse et al., 2003). The participants in this study benefitted from both formal and informal discussions, as well as peer support, which is consistent with the idea that CoPs thrive on interaction and shared learning (Grout, 2022).

Challenges Faced by Teachers in CoPs

A key research objective was to identify the specific challenges encountered by teachers participating in CoPs. The data revealed several challenges. First, both pre-service and in-service teachers struggled with time management and scheduling conflicts, as many participants had to balance teaching responsibilities, coursework, and research activities. This finding is consistent with Albertyn et al. (2018), who emphasized the need for structured development programs to enhance research quality without overburdening mentors or supervisors. Second, some pre-service teachers reported difficulties in understanding advanced research concepts and methodologies, which sometimes led to feelings of inadequacy or hesitation to participate actively in discussions. Third, technological barriers, such as lack of internet access hindered effective communication within the WhatsApp group.

Despite these challenges, the CoP provided a supportive environment where members could discuss their difficulties and seek advice. For example, when a pre-service teacher faced challenges in designing research instruments, in-service teachers offered step-by-step guidance and shared their own experiences. This peer support helped to relieve anxiety and fostered a sense of belonging within the group, echoing the findings of Maritz et al. (2013) and Schulze (2009) regarding the importance of mentorship and community in overcoming research challenges.

Problem-Solving Capacities within CoPs

The study also found that participation in CoPs significantly enhanced teachers' problem-solving capacities. The collaborative nature of the group encouraged members to collectively address research-related issues. For instance, when participants encountered obstacles in data collection or analysis, they brainstormed solutions together, drawing on the diverse experiences of both pre-service and in-service teachers. This process not only led to practical solutions but also helped participants develop critical thinking and reflective skills, as highlighted by Nicolini et al. (2022).

Moreover, the CoP facilitated the development of problem-solving strategies. Teachers learned to navigate challenges such as limited resources or time constraints by sharing tips on efficient research practices, utilizing digital tools, and delegating tasks according to individual strengths. The regular exchange of feedback and constructive criticism further contributed to the group's collective problem-solving ability, supporting the findings of Calderwood and Klaf (2015) on the value of shared responsibility in professional growth.

Complementary Roles and Leadership

One of the key insights from this study is the complementary relationship between pre-service and in-service teachers in the CoP. Pre-service teachers provided practical help, while in-service teachers offered more advanced research guidance. The structured environment of the CoP, along with the peer mentorship from in-service teachers, helped pre-service teachers develop their research skills and navigate challenges, as supported by Nicolini et al. (2022), who emphasized the role of CoPs in enhancing professional development.

The social network analysis of the WhatsApp group revealed a central participant who took on leadership roles, coordinating meetings and guiding group activities. This mirrors findings from Ciampa and Wolfe (2023), who found that leadership within CoPs often emerges naturally and supports group cohesion and progress. The group's strong collaboration on video projects and consistent feedback loops also align with research by Calderwood and Klaf (2015), who noted that CoPs facilitate professional growth and enhance project outcomes through shared responsibility.

Recommendations

The study's findings offer practical recommendations for educators and institutions aiming to support collaborative research learning. CoPs should be encouraged as a valuable framework for promoting research skills, particularly through peer support, shared learning experiences, and collective problem-solving. Institutions can foster CoPs by providing structured opportunities for teachers to engage in both formal and informal interactions, similar to the settings described in this study, such as classrooms and online platforms (Grant, 2020).

Moreover, educators should emphasize the importance of both leadership and collaboration within CoPs. As seen in this study, teachers who take on leadership roles can help guide group activities and ensure progress, creating a more effective learning environment (Ciampa & Wolfe, 2023). Additionally, integrating CoPs into the curriculum through group projects, collaborative writing, and peer mentorship could further enhance teachers' academic and professional development, as suggested by Grant (2020).

Conclusion

In conclusion, this study highlights the important role that communities of practice (CoPs) play in fostering collaborative research learning among pre-service and in-service mathematics teachers. By creating spaces where teachers can share ideas, provide feedback, and solve problems together, CoPs help them improve their research skills and knowledge. The study shows that teachers benefit not only academically but also in developing teamwork, leadership, and problem-solving skills. The support they receive from each other, both in formal and informal settings, makes the research process smoother and more productive. This collaborative approach also helps the teachers stay motivated and meet deadlines, even when faced with challenges like time constraints, technological barriers, and the complexity of research tasks. By actively engaging in problem-solving within the CoP, teachers develop adaptive strategies that enhance their professional growth and research capacity.

References

Albertyn, R., van Coller-Peter, S., & Morrison, J. (2018). A multi-level researcher development framework to address contrasting views of student research challenges. *South African Journal of Higher Education*, *32*(1), 13–30. https://doi.org/10.20853/32-1-1639

Basu, M. (2020). Importance of research in education. *Social Science Research Network*. https://doi.org/10.2139/ssrn.3703560

Buysse, V., Sparkman, K. L., & Wesley, P. W. (2003). Communities of practice: Connecting what we know with what we do. *Exceptional Children*, *69*(3), 263–277. https://doi.org/10.1177/001440290306900301

Calderwood, P. E., & Klaf, S. (2015). Mentoring within a community of practice for faculty development: Adding value to a CTL role. *To Improve the Academy*, *34*(1–2), 290–318. https://doi.org/10.1002/tia2.20022

Ciampa, K., & Wolfe, Z. M. (2023). From isolation to collaboration: Creating an intentional community of practice within the doctoral dissertation proposal writing process. *Teaching in Higher Education*, *28*(3), 487–503. https://doi.org/10.1080/13562517.2020.1822313

Grant, M. (2020). Communities of practice. In *Encyclopedia of community: From the village to the virtual world* (p. 73). SAGE Publications, Inc. https://doi.org/10.4135/9781412952583.n91

Grout, G. (2022). What are communities of practice? *Nursing Older People*, *34*(2), 15. https://doi.org/10.7748/nop.34.2.15.s6

Maritz, J., Visagie, R., & Johnson, B. (2013). External group coaching and mentoring: Building a research community of practice at a university of technology. *Perspectives in Education*, *31*(4), 155–167. https://hdl.handle.net/10520/EJC151337

Ng, L. L., & Pemberton, J. (2013). Research-based communities of practice in UK higher education. *Studies in Higher Education*, *38*(10), 1522–1539. https://doi.org/10.1080/03075079.2011.642348

Nicolini, D., Pyrko, I., Omidvar, O., & Spanellis, A. (2022). Understanding communities of practice: Taking stock and moving forward. *Academy of Management Annals*, *16*(2), 680–718. https://doi.org/10.5465/annals.2020.0330

Schulze, S. (2009). Mentoring novice researchers in higher education: A communities of practice perspective. *Koers: Bulletin for Christian Scholarship*, *74*(1–2), 117–137. https://hdl.handle.net/10520/EJC59410

Tight, M. (2016). Examining the research/teaching nexus. *European Journal of Higher Education*, *6*(4), 293–311. https://doi.org/10.1080/21568235.2016.1224674

Vallejo-Lopez, Lcda. A., & Rincón-Ríos, T. (2019). Analysis of the importance of research in students in the health area. *International Journal of Research -Granthaalayah*, *7*(8), 245–249. https://doi.org/10.29121/granthaalayah.v7.i8.2019.663

TSG4-1-4

TRANSFORMING LESSON REFLECTION MEETING USING 'LEARNER OBSERVATION SHEET'

Naoki Yachimoto
Hokkaido University of Education, Japan, yachimoto.naoki@a.hokkyodai.ac.jp

Hiroshi Ishii
Hokkaido University of Education, Japan, ishii.hiroshi@h.hokkyodai.ac.jp

The purpose of this study was to examine the depth of discussion in lesson reflection meetings and the changes in teachers' awareness through lesson observation using the Learner Observation Sheet. Two fifth-grade math lessons were observed, and the discourse transcripts from the lesson reflection meeting were analyzed using AI text mining. Additionally, a questionnaire survey was conducted to examine changes in the participants' attitudes.

The results indicated that the use of the Learner Observation Sheet helped focus discussions from the outset, prompting participants to explore solutions to their concerns. The questionnaire survey also revealed that sharing the same perspectives as the teacher and collaborators made it easier for the teacher to open the lesson to a wider audience.

Keywords: Lesson Reflection Meeting, Learner Observation Sheet

Research Background and Objective

Since the 1990s, Japanese Lesson Study has been spreading globally as "Lesson Study" and has gained popularity in mathematics education as a method for improving teaching and fostering teacher development. In particular, Japanese mathematics lesson study has the following characteristics:

- It is based on an iterative approach that bridges theory and practice, focusing on students' learning (a "how-to-learn" type of lesson study).
- Participation in lesson study is considered an effective means of enhancing teachers' teaching skills.

On the other hand, although there are opportunities to observe lessons and participate in lesson study groups, the desirable way of conducting lesson reflection meetings has not been clarified. As a result, lesson observations may lead to variations in observers' perspectives, which can be shaped by individual interpretations and values. This study aims to examine the depth of discussion in post-lesson reflection meetings and the changes in teachers' perspectives through lesson observations using the "Learner Observation Sheet."

Purpose and Significance of Lesson Study

Hirabayashi (1999) states, "In Japan, much research is conducted on the nature of teaching itself," emphasizing that the primary goal of lesson study in mathematic is to examine the essence of teach-

ing. Baba (2005) defines lesson study as "a method in which teachers, in collaboration with their colleagues, critique and examine each other's lessons to progressively improve their own teaching." Similarly, Fujii (2014) describes lesson study as "a form of teacher training that serves as a platform for teachers to conduct practical research aimed at the qualitative improvement of their teaching practices." Based on these perspectives, lesson study, in a broad sense, can be defined as "what teachers do to improve their teaching."

The movement to make the components of lesson study more visible gained momentum following the publication of *The Teaching Gap* (2002). For example, Fujii (2021) outlines the stages of lesson study, identifying five key stages that clarify its components and processes. The fourth stage, known as the "Lesson Reflection Meeting," is particularly important in this context. In Japan, lesson study often concludes with a post-lesson reflection meeting, and the revised lesson is rarely implemented immediately. Therefore, reflection meetings play a crucial role as an opportunity for teachers to reflect on and enhance their teaching skills. Lewis et al. (2015) highlights that lesson study fosters teachers' professional development throughout its cycle and strengthens the community among educators. Given this, it becomes essential to consider how to position Fujii's (2014) "5. Reflection and Summary" to maintain a collaborative learning relationship among teachers.

Learner Observation Sheet Utilized in This Study

In this study, we adopted the TRU Summary Rubric as a lesson observation framework, based on the model developed by Schoenfeld et al. (2014, 2016). This decision was made for two main reasons: first, the five dimensions emphasized in the TRU framework align with key discussion points in lesson study; second, evaluating these five dimensions provides a structured approach to lesson observation, helping teachers find common ground in their analyses.

To effectively apply the Learner Observation Sheet, we adapted the framework to better suit elementary and junior high school settings. Table 1 presents the revised Learner Observation Sheet, originally developed by the authors. Each of the five dimensions includes a predefined evaluation criterion (corresponding to a score of 3) and is rated on a five-point scale. The observer assigns a score along with a justification for their rating.

Table 1. "Learner observation sheet" prepared by the authors

Perspective	1.Objectives of the lesson and mathematical viewpoints	2.Problem and focusing on "questions" through teacher's inquiries	3.Assessing learning conditions and supporting differences in proficiency levels	4.Collaborative solution	5.Problem-solving based on individual ideas and reflecting on learning
Evaluation Criteria 3	Students solve problems using their own thinking and perspectives related to the lesson content.	Students engage in learning activities while focusing on the "question" through the teacher's inquiries and prompts.	Students develop their own approach to problem-solving, guided by the teacher's advice to pursue their own ideas.	Students share their ideas and learn from others through learning activities structured by the teacher.	Students solve problems and reflect on their learning based on their own ideas, encouraged by the teacher throughout each situation.

1	Students have a limited understanding of the concepts and properties being learned.	Students are unable to focus on the "question,".	Students' progress through the learning activity without achieving understanding.	Students proceed with the learning activity without sharing their ideas with others.	Students follow the indicated problem-solving method and complete the activity without reflecting on it.
Examples of 5 (In Addition to Criteria 3)	Students are aware of mathematical viewpoints and apply them to improve problem-solving.	Students clarify the key questions to be solved independently, making connections with prior learning.	Students demonstrate flexibility in problem-solving by choosing a learning approach that suits them.	Students refine their own ideas by comparing and analyzing different perspectives.	Through independent problem-solving, Students gain new insights from the lesson.

Outline of Lesson Practice Using the Learner Observation Sheet

This lesson practice was conducted on October 26 and November 9, 2023, in different units for 5th-grade public elementary school class. The instructor was Teacher A, who was in his third year of teaching, and five collaborating teachers supported him in designing and reflecting on the lesson.

At the conclusion of the first lesson refraction meeting, the Learner Observation Sheet was distributed, and detailed instructions were provided on how to interpret the perspectives and record scores. Additionally, it was emphasized that this session was intended as a lesson observation and reflection meeting—focusing on understanding the collaborators' perspectives on the lesson by observing students' learning processes—rather than an evaluation of the teacher.

Actual Lesson Reflection Meeting and Learner Observation Sheet Scores

Based on discourse recordings from the reflection meeting held after the first lesson observation, two key characteristics of the discussion were identified:

i) Each collaborator shared their questions, opinions, and impressions in turn.

ii) The opinions expressed by the collaborators did not necessarily align with what the participating teachers wanted to know or learn.

The discussion allowed participants to exchange diverse questions, opinions, and impressions. However, it also highlighted concerns regarding the moderator's ability to guide the conversation effectively. Additionally, because a clear observation framework had not been provided beforehand, the discussion did not lead to resolving the instructor's specific concerns.

After the second lesson observation, the discussion began with participants stating their Learner Observation Sheet scores along with the reasons for their evaluations. Table 2 below presents the scores for each perspective. Two key findings emerged from this analysis:

i) Scores varied across perspectives, indicating differences in how each teacher perceived the lesson.

ii) While the instructor tended to give lower scores, some collaborators provided similar ratings.

The participants gradually aligned a consensus on their perspectives of the lesson observation by providing reasons for their scores based on facts about the students' conditions. Notably, by centering the discussion on Perspective 4 (Collaborative Solution), they also touched on students' learning experiences related to other perspectives.

Table 2. Recording scores from each perspective

	Perspective 1	Perspective 2	Perspective 3	Perspective 4	Perspective 5
Teacher A	2	3	2	2	3
Teacher B	4	4	4	3	4
Teacher C	3	3	4	3	2
Teacher D	2	5	2	3	3
Teacher E	3	4	2	2	3
Teacher F	5	5	4	4	4
Variance	1.14	0.67	1	0.47	0.47
Standard deviation	1.07	0.82	1	0.69	0.69

Analysis of the Discourse Recordings of the Lesson Reflection Meeting

Table 3 below shows the co-occurring keywords in the discourse transcripts from the first reflection meeting, based on the statistical process known as the TF-IDF method using AI text mining. The following Table 3 summarizes the co-occurrence relationships of the constituent words within the classified categories.

Table 3. Categories classified by co-occurring keywords (First)

Class	Main words that make up the composition	Discourse record in reflection meeting
A	mathematics, every time, difficult, review [5 words]	Difficulty of mathematics and how to conduct a lesson
B	per, study, fast, many, compare [5 words]	Relation between the study (comparison of speeds) in the lesson and what has already been learned
C	perspective, unit, learn, show, ask, want [9 words]	How to teach mathematics and motivate students
D	first, question, ask, give, hesitate [6 words]	How to present the problem and grasp the question

It was found that in categories A-D, there were discussions and information sharing among teachers related to the teaching of mathematics in general. On the other hand, since each category is independent, it can be confirmed that the discussion was not focused. Table 4 below summarizes the discourse recordings from the second reflection meeting in the same way.

Table 4. Categories classified by co-occurring keywords (Second)

Class	Main words that make up the composition	Discourse record in reflection meeting
A	student, teacher, think, can, class [13 words]	How to carry out collaborative solutions and individual thinking
B	Thinking, vertical, horizontal, equal, divide [7 words]	Student's reaction to the learning content
C	Understanding, compensating, making use of, worrying, listening [7 words]	Resolving doubts and concerns about the learning content

In the second meeting, the edges of the co-occurring keywords are thick, and the nodes are large, which confirmed that the co-occurrence relationships between each category are strong. Additionally, the edge connecting categories B and C is dotted, indicating that these categories are not far apart in their co-occurrence relationship, as they share similar problems in teaching the content of the study.

Discussion

From the analysis of the transcript, it can be inferred that the perspectives of lesson observations became clearer, leading them to make connections to the content discussed during the meeting. In particular, the fact that the discussion was focused from the outset on how to facilitate learning and teaching, as well as how to address the instructor's concerns, can be seen as contributing to the strong and continuous support of the lesson study cycle (Fujii, 2021). Next, a questionnaire survey conducted in December 2023 using Google forms revealed the following responses regarding the lesson reflection meeting utilizing the "Learner Observation Sheet.":

- Focusing the perspective helped clarify the points for discussion with collaborators, making the process more effective. (Instructor)
- The teachers were able to discuss the situation and opinions of the lesson they observed based on the perspective of observing the lesson.

From these descriptions, it can be concluded that the use of the "Learner Observation Sheet" in the lesson observation meetings showed a certain degree of success. In particular, it became clear that the resistance to speaking up diminished for younger teachers, regardless of their years of experience.

References

Baba, T. (2005). Lesson study. *In Japan's educational experience: Considering educational development in developing countries* (pp. 271-283). Toshindo.

Fujii, T. (2014). A study on the review process of lesson plan in lesson study. *Journal of Japan Society of Mathematics Education*, 96(14), 2-13. https://doi.org/10.32296/jjsme.96.10_2

Fujii, T. (2021). Conceptual definition and value of lesson study. Journal of Japan Society of Mathematics Education (Ed.), *Handbook of research on mathematics teaching* (pp. 6-15). Toyokan Shuppansha.

Lewis, Catherine & Perry, Rebecca. (2015). A Randomized Trial of Lesson Study with Mathematical Resource Kits : *Analysis of Impact on Teachers'Beliefs and Learning Community*, 133-158.

Hirabayashi, I. (1999). On research lesson: What is born from them? *Japan Academic society of Mathematics Education, 10th Research Presentation Materials*.

Schoenfeld, A. H, Folden, R. E, & the Algebra Teaching Study and Mathematics Assessment Project. (2014). The TRU Math Scoring Rubric. Berkeley, CA & E. Lansing, MI: *Graduate School of Education, University of California, Berkeley & College of Education*, Michigan State University.

Schoenfeld, A. H, & the Teaching for Robust Understanding Project. (2016). An Introduction to the Teaching for Robust Understanding (TRU) Framework. Berkeley, CA: *Graduate School of Education*.

Stigler, J. W., & Hiebert, J. (2002). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom* (S. Minato, Trans.). Kyoiku Shuppan. (Original work published in 1999).

TSG4-1-5

SUSTAINING LESSON STUDY AS A PROFESSIONAL DEVELOPMENT MODEL: FROM THE TEACHERS' PERSPECTIVE

Edna G. Callanta
University of the Philippines National Institute for Science and Mathematics Education Development, Philippines, egcallanta@up.edu.ph

In this paper, lesson study as an innovative professional development model for teachers was utilized to introduce a teaching strategy called Teaching Mathematics through Problem Solving (TMtPS) to a group of teachers. After exposing them to the features of these two innovations, lesson study and TMtPS, through a teacher training program, they engaged themselves to lesson study in their respective schools. Since lesson study has just been introduced as a professional development model, this study would like to determine the teachers' perspective of its sustainability. Specifically, it would like to determine their perception of how it can be sustained considering the benefits gained and challenges faced in engaging to this practice.

Keywords: lesson study, professional development model, teaching mathematics through problem solving,

Introduction

Lesson study, as an innovative professional development model for teachers, was utilized to introduce a teaching strategy called Teaching Mathematics through Problem Solving (TMtPS) to a group of teachers. After being exposed to the features of both Lesson Study and TMtPS, through a teacher training program, the teachers engaged themselves to Lesson Study within their respective schools. Since Lesson Study is a relatively new approach to professional development, this study seeks to explore the sustainability of this practice from the teachers' perspective. Specifically, it aims to examine how teachers perceive the sustainability of Lesson Study, taking into account the benefits they gained and challenges they faced while engaging in this professional development model.

In the Philippines, the professional development of teachers typically involves attendance in in-service teacher training programs offered by teacher training institutions (Ulep & Reyes, 2013). One of the most common modalities for these training programs is a lecture by specialists and discussions with colleagues (Hidalgo, Reyes, Roxas-Soriano & Bayten, 2016). However, this model has some limitations, particularly in its inability to address specific concerns and needs of individual teachers. This limitation is in contrast with the view of Dail, Goodsite, and Sanders (2018), who found that professional development is most effective when teachers are actively involved in designing and implementing it.

Lesson Study, however, has gained recognition as an effective approach to introducing innovative practices in mathematics classroom teaching and learning (Ulep & Reyes, 2013). Its effectiveness is largely attributed to the nature of Lesson Study as a school-based and continuing professional development model. The Japanese students, who consistently outperformed their international counterparts particularly in international assessments like TIMSS has been linked to widespread use of Lesson Study in Japan (Stigler & Hiebert, 1999). Nevertheless, while high-quality professional

development is essential, it is not the only factor for determining the sustainability of educational reform. Several other elements must be considered, like the support of school administrators. They can either facilitate or hinder the success of reforms (Douglas, Rynearson, Yoon, and Diefes-Dux, 2016). In addition, the presence of supportive advocates is crucial to ensure the continuation and advancement of the initiatives (Comber & Lawson, 2013).

Methodology

Forty-three public elementary school teachers from seven schools in one of the island provinces in the Philippines participted in the study. The teachers took part in a professional development program that consisted of two phases, a Seminar-Workshop on Lesson Study and TMtPS (Phase 1) and the School-Based Implementation of Research Lessons (Phase 2).

In Phase 1, the participants were introduced to TMtPS and the features of Lesson Study as a professional development model for teachers. This phase provided them with foundational knowledge and skills necessary for engaging in Lesson Study.

Phase 2 involved the school-based implementation of the research lessons which consisted of two visits. Each visit included two lesson implementations by a different teacher and a different set of students in two successive days. For both visits, Post Lesson Reflection and Discussion (PLRD) followed each lesson implementation following the procedures modelled during Phase 1. During the PLRD, the lesson study team reflected on the lesson and made revisions based on the feedback and suggestions shared. The revised lesson plan was then implemented on the following day.

After the first implementation of the lesson during the second visit, members of the lesson study team were asked to complete a questionnaire. Each participant received a copy which was collected after the second implementation of the lesson. The questionnaire focused specifically on sustaining Lesson Study as a professional development model. The data gathered was analyzed using thematic analysis. According to Braun and Clarke (2006), thematic analysis involves identifying patterns or themes within a set of qualitative data. A theme represents a significant element of the data in relation to the research question, capturing recurring patterns of responses across the data set. Braun and Clarke outlined six phases for conducting thematic analysis: familiarizing with the data, generating initial codes, searching for themes, reviewing themes, defining and naming themes, and producing the report. These phases were followed to systematically analyze the data and identify key themes related to the sustainability of Lesson Study.

Results and Discussion

Lesson study, as a model for teacher professional development, is very much different from the approaches the teachers in the Philippines are typically exposed to. Having to sit down with colleagues to plan a lesson, share ideas, and agree or disagree on some matters regarding lesson development is atypical to some schools, more so observing other teachers deliver a lesson and reflecting on and critiquing the lesson implementation for the purpose of improving the teaching and learning. These are inherent in lesson study and uncommon practices in Philippine schools. Considering the challenges the teachers encountered and the benefits they gained from their experience with Lesson Study, this study explored how they perceived its sustainability as a professional development model in their schools.

Most of the teachers (71%) signified that lesson study could be sustained in their schools while 15% expressed doubts about its sustainability. The remaining teachers were either uncertain or did not provide a response. The reasons given by the teachers reflected their views of what is necessary for Lesson Study to thrive in their schools. Teachers who believed it could be sustained cited the benefits they gained from the practice, while those who felt it could not be sustained pointed to their limited knowledge of Lesson Study as a primary obstacle. Interestingly, several reasons for sustainability were shared across both groups.

Based on teachers' responses, the following elements were identified as essential for sustaining Lesson Study in schools.

Availability of Time

Most teachers considered time constraint as the most significant challenge they faced. They had a hard time finding a common time to meet, plan lessons, and discuss the research lesson. Other responsibilities such as holding remedial classes, performing administrative tasks, and varying vacant time contributed to this lack of time. Time emerged as a critical factor not only for conducting Lesson Study but also for its long-term sustainability.

Cooperation, Attitude and Commitment of Teachers

Spillane (1999) pointed out that "teachers are the key agents when it comes to changing classroom practice" (p.144). Lesson study entails a high level of collaboration. Teachers need to set aside some time for planning, lesson implementation, PLRD, and revising the lesson plan. With the introduction of TMtPS together with Lesson Study, requires teachers' willingness to embrace this new approach even if it means stepping out of their comfort zones. Teachers emphasized that a positive attitude is necessary for the sustainability of Lesson Study. However, readiness to collaborate was also considered a challenge. Not all teachers are not comfortable with working with a team when planning a lesson and considering the difficulty of finding a common time. Despite this, teachers agreed that with commitment and right attitude, the benefits of Lesson Study would be evident.

Support of the School Administrators

Teachers did not identify a lack of administrative support as a challenge during the Lesson Study process but they acknowledged its importance for its sustainability. The involvement of school administrators in the training and planning phases helped reduced possible difficulties. For Lesson Study to be sustainable, support in terms of resources (both material and human), guidance, and incentives is crucial (Ylonen & Norwich, 2013; Chen & Yang, 2013, in Zhang, 2015). School administrators play a key role in providing these resources and ensuring the continuation of the practice.

Inadequate Knowledge of Lesson Study

Another aspect which the teachers considered important was the need for a shared understanding of the features of Lesson Study. As a relatively new model of professional development, it is essential to have a common understanding of its principles and procedures. Given the limitations within the schools and challenges teachers face, support of the school community is necessary to ensure the long-term sustainability of Lesson Study.

Summary and Implications

For teachers who have recognized the benefits of Lesson Study, the implementation and adoption of this model as part of their professional development proved to be challenging. The most significant difficulty that they encountered was the lack of time. Given the collaborative nature of Lesson Study, finding a common time to plan the lesson was difficult as teachers had other responsibilities and tasks to attend to within the school. However, it is inevitable for challenges to arise when introducing something new. To ensure the sustainability of Lesson Study, strategies to address these challenges must be considered.

One possible way to sustain Lesson Study is to integrate it into the educational system. For instance, teachers could incorporate Lesson Study into their Learning Action Cell (LAC) sessions. By utilizing these established structures, there would be a greater chance for its sustainability.

The support of school administrators is crucial for a successful implemention of innovative practices like Lesson Study. Creating an environment where collaboration among teachers is encouraged and given time to develop is essential. This setting may bring about teachers who are willing to share their knowledge and learn from others. Learning more about TMtPS, a teaching strategy introduced to teachers together with lesson study, needs to be emphasized further, considering that coping with the new strategy is identified as a challenge for the teachers.

References

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77-101.

Dail, J. S., Goodsite, M., & Sanders, S. (2018). Teacher empowerment through partnerships: A sustaining model of professional development. *The English Journal, 107*(4), 20-25.

Douglas, K. A., Rynearson, A., Yoon, S. Y., & Diefes-Dux, H. A. (2016). Two elementary schools' developing potential for sustainability of engineering education. *International Journal of Technology and Design Education, 26*, 309–334. https://doi.org/10.1007/s10798-015-9313-4

Hidalgo, F., Reyes, W. S., Roxaz-Soriano, P., & Bayten, E. O. (2016). In Meleisea, E. (Ed.), *Case studies summaries: Philippines*. In *School and teaching practices for twenty-first century challenges: Lessons for the Asia-Pacific region* (pp. 69-71). United Nations Educational, Scientific and Cultural Organization.

Lawson, T., & Comber, C. (2014). Videoconferencing and learning in the classroom: The effects of being an orphan technology? *International Journal of Technologies in Learning, 20*(1), 69-79.

Spillane, J. P. (1999). External reform initiatives and teachers' efforts to reconstruct their practice: The mediating role of teachers' zones of enactment. *Journal of Curriculum Studies, 31*(2), 143–175.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. Free Press.

Ulep, S. A., & Reyes, R. L. (2013). NISMED's journey in adapting lesson study. In S. A. Ulep, A. E. Punzalan, R. L. Reyes, & M. B. Ferido (Eds.), *Lesson study: Planning together, learning together* (pp. 1-7). National Institute for Science and Mathematics Education Development.

Zhang, Y. (2015). Sustaining lesson study in schools with positive peer leadership: A case study in Hong Kong. *International Journal for Lesson and Learning Studies, 4*(2), 140-154. https://doi.org/10.1108/IJLLS-07-2014-0018.

TSG4-1-6

TEACHERS' EXPERIENCE OF REFLECTIVE PRACTICE IN THE OPEN CLASS

Narumon Changsri
Khon Kaen University, Thailand, changsri_crme@kku.ac.th

Maitree Inprasitha
Khon Kaen University, Thailand, imaitr@kku.ac.th

Nisakorn Boonsena
Khon Kaen University, Thailand, nisabo@kku.ac.th

Jitlada Jaikla
Khon Kaen University, Thailand, jitlja@kku.ac.th

Chanika Senawongsa
Khon Kaen University, Thailand, chansen@kku.ac.th

Open Class is a multi-level professional learning community in the context of Transformative lesson study incorporated with Open Approach (TLSOA). This study focused on the National Open Class (NOC) level. This research examined the influence of the NOC on enhancing teachers' reflective practices, particularly through engagement strategies in "fish tank" problem situations. Data were collected from 110 participants reflecting on the online platform and 37 participants in classroom observation sessions. Findings indicate that NOC participation systematically promotes self-reflection, fosters collaborative learning, shifts teaching paradigms, enhances professional development, and leverages expert facilitation. In analyzing responses to the fish tank problem situation, themes emerged highlighting students' ideas, coloring choice, artistic expression, classroom environment, and learning process.

Keywords: open class, reflective practices, TLSOA model

Introduction

Reflective practice in the 21st century goes beyond simply thinking about what happened in a lesson or situation (Sherwood, 2024). It's a continuous, active, and iterative process of critically examining one's experiences, actions, and beliefs to improve practice and adapt to the ever-changing landscape of education (NWS Government, 2023). Reflection is often enhanced through collaboration with peers, mentors, or coaches (Jovanna, 2021). Sharing experiences, discussing challenges, and receiving feedback from others can provide valuable insights and perspectives (The University of Edinburgh, 2024). The essence of experience lies in the interplay between manipulating an object and perceiving its effects. This dynamic interaction unifies introspective and contemplative thinking into a single reflective thinking process, focusing on understanding how action shapes perception (Emori, 2023). Finlay (2008) synthesizes the concept of reflective practice based on the work of various researchers; the term 'reflective practice' is characterized as a cyclical process of experiential learning, involving the critical examination of everyday practices and underlying as-

sumptions. This process necessitates self-reflection on responses to situations, enabling practitioners to gain new insights into themselves and their practices. In the Thai context, Inprasitha (2022, 2023) has implemented TLSOA model to cultivate the culture of reflective practice based on collaboratively Plan, collaboratively Do and collaboratively See. Participating teachers could reflect on their practice on the weekly cycle of learning. Moreover, two more cycles of reflection to make sure the Lesson Study group members have sufficient reflection chances, such as Open Class at the district level and National Open Class at the national level for the respective semester and yearly basis. Integrating Lesson Study with the Open Approach, as delineated in the TLSOA model, utilizes Open Class sessions to foster vibrant Professional Learning Communities (PLCs) (Inprasitha, 2023).

Research Methodology

The research examined the influence of the NOC on enhancing teachers' experience on reflective practices while exploring the specific engagement strategies employed during "fish tank" problem situations and the whole program. Specifically, a study seeks to answer the following questions: 1) How does participation in the National Open Class influence teachers' reflective practices? 2) How do teachers engage in reflective practices during the "fish tank" problem situation within the NOC?. Research design based on participation in the 18th NOC, organized on February 2025, and focused on the following reflective practices: 1) Reflection of experts' experiences 2) Classroom observation 3) reflection by TLSOA team and expert and 4) Reaction to the classroom overall by expert.

Data was collected from 110 participants, including sixteen pre-service mathematics teachers, eighty-two in-service teachers, five academic advisors, and seven teacher educators who reflected on the online reflection platform for the whole program, and 37 participants including in-service teachers, academic advisors and teacher educators who participated in the classroom observation and reflection session related to the fish tank problem situation. Content analysis was employed to analyze the reflection data. The reflection data was transcribed and organized, key words and phrases were identified and highlighted, categorized into themes to reveal broader trends, and validated with a second researcher. The lesson study team included three pre-service mathematics teachers, two in-service mathematics teachers, two art educators, and five mathematics educators. The team collaboratively designed the problem situation by creating context and conditions (Inprasitha,2023).

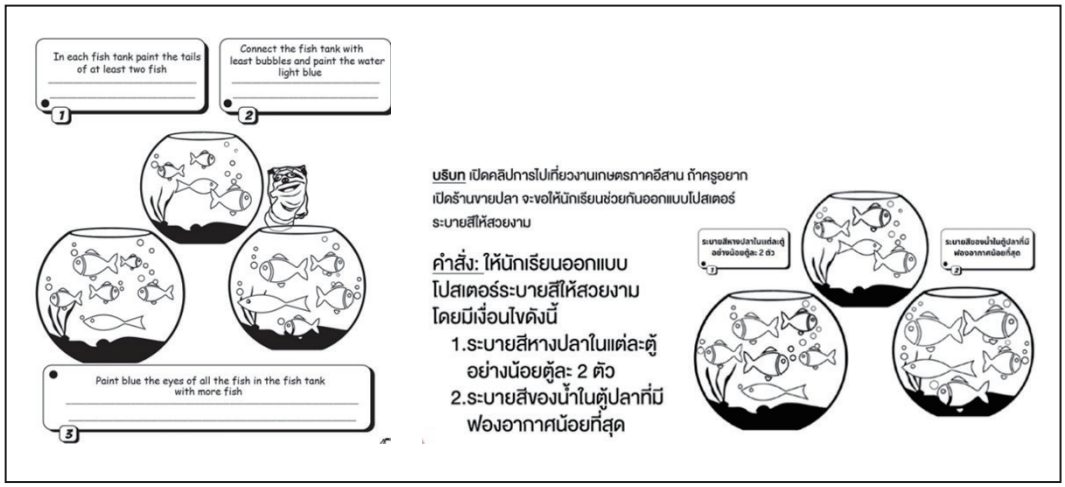


Figure 1. Illustrate original task and adapted to problem situation.

Figure 1 illustrates an original coloring task (Araya, 2019). The lesson study team adapted to design a problem situation. The adaptation involved translation into Thai, contextualization relevant to the students' experiences. The keywords on conditions were "at least two fish", "each", "least," and the addition of a "beautiful coloring" prompt to encourage creative expression. Furthermore, the third condition of the original problem was removed to align with the 60-minute instructional timeframe.

Research Results

Reflective Practices

Participation in the National Open Class significantly influences teachers' reflective practices by the following themes: 1) systematically promoting self-reflection: The structured program prompts teachers to regularly reflect on their own teaching, leading to more conscious and intentional adjustments to their practice. 2) Fostering collaborative reflection: The shared observation and discussion experiences facilitate peer learning and support, enriching the reflective process. 3) Shifting teaching paradigms: By encouraging the Open Approach, teachers are prompted to reflect on their roles, moving from knowledge delivery to facilitator of student discovery. 4) Enhancing professional development: The program directly addresses areas for skill development, and the feedback suggests that most teachers experience improvement in their anticipation, observation, and reflection capabilities. 5) Expert facilitation: The guidance of experienced facilitators plays a crucial role in shaping the discussions, ensuring a productive exchange of knowledge, and providing timely feedback. 6) Rich diversity: Participants with varied backgrounds and expertise create a dynamic learning environment where teachers can learn from each other's unique experiences and approaches. 7) Inspiration: NOC participation inspired teachers to embrace student-centered learning and develop professionally. The program fostered excitement for new techniques and a renewed commitment to student growth.

Reflective Practice on Fish Tank Problem Situation

Reflection on the topic Summary of Responses to the Fish Tank Problem Situation:

75.68% of participants could reflect on this topic related to classroom observation. The results revealed the theme as 1) Students' Ideas related to Instructions/Conditions such as shows varied interpretation of "at least" and "each tank.", highlights confusion about identifying the "least" and consistent tank ordering and directly addresses misunderstanding of key terms. 2) Coloring Choices and Artistic Expression, such as identifying a common choice, describes a specific artistic technique used, overall aesthetic assessment, combines adherence to conditions with artistic expression, and highlights individual artistic freedom. 3) Classroom Environment and Learning Processes such as positive observation about the learning environment, focus on the learning outcome of categorization, and point to a need for deeper processing and sharing of understanding.

Reflection on how the responses to the problem situation are consistent or have gaps with the context and condition in the problem situation.

Analysing from 51.35% of participants found several key themes. The reflections reveal a mixed bag. While some aspects of the lesson, such as the materials and potentially the overall design, were perceived positively, significant gaps existed in the clarity and consistency of the instructions, leading to a disconnect between the intended learning objectives and student actions. The emphasis on aesthetics over conceptual understanding also emerges as a key concern. The reflections themselves

also contain inconsistencies, suggesting that different teachers may have had different experiences or perspectives on the lesson's effectiveness.

Reflection related to the topic: Discuss ways to bridge the gaps

The result from 21.62% of participants found on the themes; 1) Clarity of conditions such as emphasize universal application, specify visual references, highlight key concepts and utilize specific language. 2) Structured teaching approach such as posing condition and organizing presentations and summaries. 3) Visual aids and engagement, such as incorporating visual supports and transforming worksheets for engagement and clarity in visual representation.

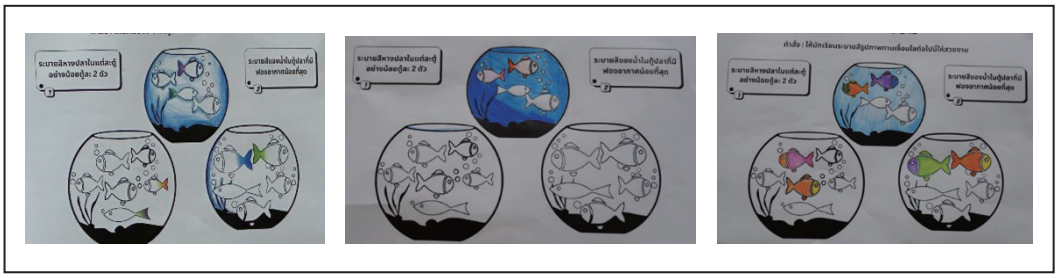


Figure 2 Illustrates students' worksheets that represent their ideas.

After the participants engaged in collaborative reflection, the lesson study team teacher, including the teacher educators and a mathematics education expert, facilitated a discussion highlighting the importance of student ideas, designing problem situations, the key point of the classroom, and potential challenges for future development. This process broadened the participants' perspectives on classroom observation, anticipating student ideas, and reflection.

Concluding Remark

NOC participation significantly enhanced teachers' reflective practices, fostering both self and collaborative reflection, shifting teaching paradigms towards student-centered learning, and promoting professional growth. Moreover, their reflections revealed valuable insights into student thinking, artistic expression, and classroom environment, highlighting areas for improved instruction clarity and a stronger focus on conceptual understanding. Collaborative reflection, consistent with Philipp's (2007) assertion that reflection allows teachers to re-interpret observations and notice previously unseen aspects. These emphasized the importance of student ideas and informed future pedagogical development, including articulating emotional experiences like inspiration, enjoyment, and surprise (Changsri & Inprasitha, 2024). In teacher education, participation in a PLC significantly enhances various teaching skills, including lesson planning, classroom practices, collaborative learning, innovative methods, and adaptability to change (Ohayon, 2023). Ultimately, these enhancements not only benefit teachers' professional growth but also significantly enrich the learning experiences of students, fostering a more dynamic and responsive educational environment.

Acknowledgment

This research was supported by the Fundamental Fund of Khon Kaen University and National Science, Research and Innovation Fund and CRME, Khon Kaen University.

References

Araya, R. (2019). Colorea Ideas. AutoMind Press. ISBN 978-956-401-279-7.

Changsri, N., & Inprasitha, M. (2024). *The impact of open class on teachers' perceived beliefs through reflection*. Paper presentation at *The 15th International Congress on Mathematical Education*, Sydney, Australia.

Emori, H. (2023). Introspective thinking and contemplative thinking in mathematical communication—The power of seeing the invisible to overcome a fragmented world. In *Proceedings of the Regional Conference of the International Group for the Psychology of Mathematics Education 2023*. Khon Kaen, Thailand: PME.

Finlay, L. (2008). Reflecting on 'Reflective practice'. Practice-based Professional Learning Paper 52, The Open University.

Inprasitha, M. (2022), "Lesson study and open approach development in Thailand: a longitudinal study", *International Journal for Lesson and Learning Studies*, Vol. 11 No. 5, pp. 1-15. https://doi.org/10.1108/IJLLS-04-2021-0029

Inprasitha, M. (2023). Learning to think mathematically in Thai classrooms using Thailand lesson study incorporated with Open Approach (TLSOA). In Inprasitha et al., (Eds.). *Proceedings of the Regional Conference of the International Group for the Psychology of Mathematics Education 2023* (pp.1-8). Khon Kaen, Thailand: PME.

Jovanna G.M. (2021). Action research and collaborative reflective practice in English language teaching, Reflective Practice, DOI: 10.1080/14623943.2021.1982688

NWS Government. (2023). Using reflective practice to drive improvement in your service. Retrieved from https://education.nsw.gov.au/early-childhood-education/leadership/news/using-reflective-practice-to-drive-improvement-in-your-service--.

Ohayon, A. (2023). *Influence Of Teachers Participation In Professional Learning Community On Their Teaching Skills*. https://doi.org/10.15405/epes.23056.40

Philipp, A. (2007). Mathematics Teachers' Beliefs and Affect. In Lester, F.K. (Ed.). *Second Handbook of Research on Mathematics Teaching and Learning. National Council of Teachers of Mathematics*: United State of America.

The University of Edinburgh. (2024). Reflection Toolkit. Retrieved from https://reflection.ed.ac.uk/reflectors-toolkit/producing-reflections/ways-reflecting/

Sherwood G. (2024). Reflective practice and knowledge development: Transforming research for a practice-based discipline. *International journal of nursing sciences*, *11*(4), 399–404. https://doi.org/10.1016/j.ijnss.2024.08.002

TSG4-2-1

INVESTIGATING PRESERVICE TEACHERS' CURRICULAR NOTICING ABILITY THROUGH TEXTBOOK ANNOTATION ACTIVITIES

Siyu Zuo
Beijing Normal University, China, zuosyu@mail.bnu.edu.cn

Chunxia Qi
Beijing Normal University, China, qichxia@126.com

This study investigates preservice teachers' curricular noticing ability through textbook annotation activities. Using the framework developed by Dietiker et al. (2018), the study examines how PSTs engage with e-textbooks, marking areas of attention, interpreting their instructional significance, and responding pedagogically. The research aims to explore the current state of PSTs' curricular noticing ability as they interact with curricular materials during lesson planning. By analyzing these activities, the study sheds light on how PSTs notice key elements in textbooks and their understanding of curriculum design in the context of teacher preparation.

Keywords: preservice teacher, curricular noticing, textbook annotation, teacher education

Introduction

In teacher education, developing preservice teachers' (PSTs) ability to engage with and modify curricular materials is a critical aspect of professional growth. One key skill in this process is curricular noticing, which refers to the ability to attend, interpret, and respond to elements of a curriculum during lesson planning. Previous research has highlighted the importance of curricular noticing for lesson planning and student learning (Amador et al., 2017; Dietiker et al., 2018; L. Males et al., 2015). However, few studies have explored how structured activities(Lee & Lee, 2024), such as textbook annotation, can facilitate the development of this skill. Textbook annotation activities have been shown to support the development of various teaching skills. Despite these benefits, there remains a gap in the literature regarding the specific role of textbook annotation activities. This study aims to fill this gap by investigating how PSTs' engagement with e-textbooks through annotation activities can promote their curricular noticing ability.

Literature Review

PSTs' Curricular Noticing

Recent research has focused on strategies to enhance PSTs' curricular noticing(Cho, 2021; Christopher W. et al., 2023; Kang & Ellis, 2024; Lee & Choy, 2017; L. M. Males & Setniker, 2019; Nicol & Crespo, 2006; Roth McDuffie et al., 2018). Studies have demonstrated that specific interventions, such as guided analysis of curricular materials (Krupa et al., 2017), structured feedback activities (Cavanagh & Tran, 2023), and controlled tasks like problem sequencing (Lee & Lee, 2024), sig-

nificantly improve PSTs' ability to identify, interpret, and respond to key curricular elements. These approaches highlight the importance of explicit scaffolding in supporting the development of PSTs' curricular noticing skills.

However, much of the existing research has concentrated on specific mathematical topics, such as fractions and pattern generalization (Lee & Lee, 2024). Moreover, limited research has explored how structured textbook annotation activities can serve as both a development tool and an assessment for curricular noticing. Given the growing use of digital tools in education, this study examines how e-textbook annotation activities can help develop PSTs' curricular noticing abilities.

Textbook Annotation Activities

Textbook annotation activities have been explored as an effective strategy for enhancing PSTs' lesson planning skills(Jin & Lu, 2018; Nicolas, 2016; Remillard, 2000). Research has shown that annotated lesson plans help PSTs prepare for teaching diverse student populations (Hughes, 2014), and that such annotations support the understanding and application of instructional methods (Zaragoza et al., 2023). Additionally, annotation activities have been found to assist PSTs in constructing lesson plans by encouraging critical engagement with original curriculum materials (Lim et al., 2018). These studies suggest that textbook annotation can play a crucial role in developing PSTs' ability to engage with and modify instructional materials, ultimately leading to more effective lesson planning.

Despite the potential benefits, there is a lack of research focusing on the use of annotation activities to develop PSTs' curricular noticing skills. Therefore, this study aims to fill this gap by examining PSTs' curricular noticing through their performance of annotation activities on e-textbooks. The research questions were as follows: 1. What elements do PSTs attend to while annotating the textbook for lesson planning? 2. How do PSTs interpret these elements in relation to the function of mathematics teaching? 3. How do PSTs respond to these elements in textbook for their lesson plans?

Method

Participants

A total of 26 PSTs participated in this study, consisting of 8 males and 18 females. These participants were enrolled in a course titled "A Study of Mathematics Curriculum and Teaching Materials" at a normal university in China. Each participant selected two lessons, one from the trigonometry chapter and one from the solid geometry chapter, in the Chinese high school mathematics textbook published by People's Education Press. For the convenience of PSTs to annotate, an electronic version is available for them to annotate using their computers.

Data Collection

The data for this study consisted of the annotations made by participants on e-textbooks. These textbook annotation activities were guided by prompts designed by the researchers, which were divided into three parts. In the first part, PSTs were asked to use digital tools to identify sections of the textbook that they would consider for teaching materials. In the second part, PSTs were asked to explain in their annotations why they selected these specific sections of the textbook as teaching materials. In the third part, PSTs were asked to respond to the following prompt: "If you were teaching this lesson, how would you use the selected content in your classroom?" After completing all three parts of the textbook annotation activities, PSTs uploaded the electronic textbook files with their personalized annotations for further analysis.

Data Analysis

The data analysis was conducted in alignment with the three research questions. For RQ1, we categorized the elements based on prior research on textbook structures (definitions, theorems, explorations, worked examples, reading materials, introductions, exercises, formulas, and diagrams). For RQ2, we analyzed PSTs' explanations using Jacobs' (2010) framework. Additionally, each explanation was assigned one or more thematic codes: curriculum, students, and mathematics. For RQ3, each annotation was coded based on Brown's (2009) framework. The coding categories included offloading, adapting, and improvising.

Results

A total of 223 annotations were uploaded by 26 PSTs, including 114 on trigonometry and 109 on solid geometry. Based on each of the three research questions, the study presents the results as shown in the tables below. As shown in the table 1, PSTs' attention to the nine elements of the textbook varied widely. In the trigonometry, the majority of PSTs paid attention to both definitions (73.1%) and worked examples (76.9%), but only a few PSTs found the theorems (7.7%) and reading material (15.4%) in the textbook noteworthy as well. The solid geometry had similar results, with 96.2% of PSTs attending the worked examples, but only two (7.7%) PSTs finding the diagrams noteworthy.

Table 1. PSTs' curricular attending of the elements in textbook

Topic	Attending	Def	Int	The	For	Exa	Exp	Dia	Exe	Rea
trigonometry	Number of teachers n=26	19	15	2	7	20	17	7	14	4
	Frequency n=114	23	15	2	7	23	18	7	15	4
solid geometry	Number of teachers n=26	7	12	17	4	25	12	2	13	7
	Frequency n=109	8	12	22	4	29	12	2	13	7

Note. Def: definitions; Int: introductions; The: theorems; For: formulas; Exa: worked examples; Exp: explorations; Dia: diagrams; Exe: exercises; Rea: reading materials.

Among the 223 interpretations of the PSTs, 93(41.7%) interpretations lacked evidence, 110(49.3%) had limited evidence, and 20(9.0%) had robust evidence. The PSTs provided a variety of interpretations regarding the elements, all of which were related to one or more of the themes in the students, mathematics, and the curriculum. As shown in the table 2, PSTs were more likely to make interpretations related to the learning of mathematics, followed by those for the development of students' abilities, and finally the structure of the curriculum. Their interpretations usually relate to one or two themes, and less frequently to all three.

Table 2. PSTs' curricular interpreting of textbook

Topic	Interpreting	S	M	C	SM	SC	CM	SCM
trigonometry	Number of teachers n=26	8	12	4	20	5	8	6
	Frequency n=114	13	30	7	42	6	8	8
solid geometry	Number of teachers n=26	12	13	1	20	1	3	7
	Frequency n=109	18	24	1	49	2	3	12

Note. S: students; M: mathematics; C: curriculum; SM: students and mathematics; SC: students and curriculum; CM: curriculum and mathematics; SCM: students, curriculum and mathematics.

Among the 223 responses made by the PSTs, 111 responses used content directly from the textbook without any changes, 97 responses adapted content from the textbook, and 15 responses added content created by the PSTs to it. For elements in textbooks as shown in the table 3, PSTs improvised them but in very small quantities, not even in formulas and explorations. For the elements of definitions, theorems, diagrams, and reading material, there was little difference between adapting and offloading. However, for the three elements of introduction, worked examples and exercises, PSTs were more likely to offload rather than adapting them.

Table 3. PSTs' curricular responding of the elements of textbook

Topic	Responding	Def	Int	The	For	Exa	Exp	Dia	Exe	Rea
trigonometry	Offloading	14	10	1	5	15	9	3	10	1
	Adapting	19	3	1	2	7	9	4	4	1
	Improvising	1	2	0	0	1	0	0	1	2
solid geometry	Offloading	5	6	10	1	17	3	1	7	4
	Adapting	3	3	17	3	11	9	0	6	3
	Improvising	0	2	3	0	1	0	1	1	0

Conclusion

This study explored the potential of textbook annotation activities in fostering preservice teachers' curricular noticing abilities. The results suggest that engaging with e-textbooks through structured annotation tasks provides PSTs with a focused approach to identifying key curricular elements, interpreting their instructional relevance, and formulating pedagogical responses. This method also revealed the current state of PSTs' curricular noticing of the elements in textbooks, offering valuable insights into their strengths and areas for development.

References

Amador, J. M., Males, L. M., Earnest, D., & Dietiker, L. (2017). Curricular Noticing: Theory on and Practice of Teachers' Curricular Use. In E. O. Schack, M. H. Fisher, & J. A. Wilhelm (Eds.), Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks (pp. 427–443). Springer International Publishing. https://doi.org/10.1007/978-3-319-46753-5_25

Brown, M. W. (2009). The Teacher–Tool Relationship: Theorizing the Design and Use of Curriculum Materials. In Mathematics Teachers at Work. Routledge.

Cavanagh, M., & Tran, D. (2023). Pre-service teachers' curricular noticing when providing feedback on peers' lesson plans: Annual Conference of the International Group for the Psychology of Mathematics Education (46th : 2023). Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education, 2023, 2, 171–178.

Cho, M. K. (2021). Preservice Elementary Mathematics Teachers' Curricular Noticing: Focusing on the Lesson Planning for Rate. Education of Primary School Mathematics, 24(2), 83–102.

Christopher W., P., Rachel B., S., & Mark A., C. (2023). Investigating how secondary prospective teachers plan to launch cognitively demanding tasks. Journal of Mathematics Teacher Education, 26(3), 395–423. https://doi.org/10.1007/s10857-022-09534-7

Dietiker, L., Males, L. M., Amador, J. M., & Earnest, D. (2018). Research Commentary: Curricular Noticing: A Framework to Describe Teachers' Interactions With Curriculum Materials. Journal for Research in Mathematics Education, 49(5), 521–532. https://doi.org/10.5951/jresematheduc.49.5.0521

Hughes, C. (2014). Annotated Lesson Plans: The Impact on Teacher Candidate Preparation for Emergent Bilingual Students. All Faculty Scholarship for the College of Education and Professional Studies. https://digitalcommons.cwu.edu/cepsfac/4

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional Noticing of Children's Mathematical Thinking. Journal for Research in Mathematics Education, 41(2), 169–202. https://doi.org/10.5951/jresematheduc.41.2.0169

Jin, T., & Lu, X. (2018). A Data-Driven Approach to Text Adaptation in Teaching Material Preparation: Design, Implementation, and Teacher Professional Development. TESOL Quarterly, 52(2), 457–467. https://doi.org/10.1002/tesq.434

Kang, R., & Ellis, A. B. (2024). Preparing middle and high school teachers for teaching mathematical modeling: Building curriculum vision and trust through curricular and professional noticing. Journal of Mathematics Teacher Education. https://doi.org/10.1007/s10857-024-09673-z

Krupa, E. E., Huey, M., Lesseig, K., Casey, S., & Monson, D. (2017). Investigating secondary preservice teacher noticing of students' mathematical thinking. In Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks (pp. 49–72). Springer.

Lee, M. Y., & Choy, B. H. (2017). Mathematical Teacher Noticing: The Key to Learning from Lesson Study. In E. O. Schack, M. H. Fisher, & J. A. Wilhelm (Eds.), Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks (pp. 121–140). Springer International Publishing. https://doi.org/10.1007/978-3-319-46753-5_8

Lee, M. Y., & Lee, J.-E. (2024). Investigating Pre-Service Teachers' Problem-Solving Ability and Their Curricular Noticing Ability Through Pedagogical Sequencing Activities. International Journal of Science and Mathematics Education. https://doi.org/10.1007/s10763-024-10506-x

Lim, W., Son, J.-W., & Kim, D.-J. (2018). Understanding Preservice Teacher Skills to Construct Lesson Plans. International Journal of Science and Mathematics Education, 16(3), 519–538. https://doi.org/10.1007/s10763-016-9783-1

Males, L., Earnest, D., Dietiker, L., & Amador, J. (2015). EXAMINING K-12 PROSPECTIVE TEACHERS' CURRICULAR NOTICING.

Males, L. M., & Setniker, A. (2019). Planning with curriculum materials: Interactions between prospective secondary mathematics teachers' attention, interpretations and responses. International Journal of Educational Research, 93, 153–167. https://doi.org/10.1016/j.ijer.2018.09.016

Nicol, C. C., & Crespo, S. M. (2006). Learning to Teach with Mathematics Textbooks: How Preservice Teachers Interpret and Use Curriculum Materials. Educational Studies in Mathematics, 62(3), 331–355. https://doi.org/10.1007/s10649-006-5423-y

Nicolas, N. (2016). Exploring playful annotations in interactive textbooks: Engaging the teacher and the learner in an active learning process. Malmö högskola/Kultur och samhälle. https://urn.kb.se/resolve?urn=urn:nbn:se:mau:diva-21640

Remillard, J. T. (2000). Can Curriculum Materials Support Teachers' Learning? Two Fourth-Grade Teachers' Use of a New Mathematics Text. The Elementary School Journal. https://doi.org/10.1086/499645

Roth McDuffie, A., Choppin, J., Drake, C., & Davis, J. (2018). Middle school mathematics teachers' orientations and noticing of features of mathematics curriculum materials. International Journal of Educational Research, 92, 173–187. https://doi.org/10.1016/j.ijer.2018.09.019

Zaragoza, A., Seidel, T., & Santagata, R. (2023). Lesson analysis and plan template: Scaffolding preservice teachers' application of professional knowledge to lesson planning. Journal of Curriculum Studies, 55(2), 138–152. https://doi.org/10.1080/00220272.2023.2182650

TSG4-2-2

MATHEMATICS TEACHERS' TEXTBOOK UNDERSTANDING: A MEDIATION EFFECT ANALYSIS

Qiaohong Cheng
Fujian Normal University, China, 1679037559@qq.com

Xiaoying Chen
China West Normal University, China, cherrylaw39@163.com

Qinqiong Zhang
Fujian Normal University, China, qqzhang922@126.com

This study investigates the specific circumstances of mathematics teachers' textbook understanding through a questionnaire survey and explores the relationships and internal mechanisms among the main factors influencing this understanding using mediation effect analysis. The results show that mathematics teachers grasping curriculum standards significantly positively impacts their textbook use. Moreover, the overall interpretation of textbooks plays a full mediating role between the grasp of curriculum standards and textbook use.

Keywords: mathematics textbooks; mathematics teachers; textbook understanding; curriculum standards; mediation effect

Introduction

Mathematics teachers' textbook understanding is a prerequisite for educational practice, shaping classroom content and goals (Remillard, 2013). However, in practice, this understanding is often vague (Fan et al., 2021), lacking clear frameworks and methods. Existing research is limited, often focusing on macro-level analyses without fully exploring the detailed processes and steps involved in mathematics teachers' textbook understanding. This study aims to fill this gap by examining the internal mechanisms and relationships among the factors influencing mathematics teachers' textbook understanding. The main research questions are: (1) What are the relationships among the factors influencing mathematics teachers' textbook understanding? (2) What is the internal mechanism of this understanding?

Literature Review

Teachers' Textbook Understanding

Mathematics textbooks, designed by educational authorities to support teaching and learning, are central to teachers' understanding. This understanding, defined as comprehension of curriculum texts (Remillard, 2018) or an interactive process involving curriculum standards, textbooks, and students, is a complex, multi-level interaction. Specifically,Mathematics teachers' understanding of instructional materials refers to the process by which teachers, through the dynamic interaction

among “curriculum standards - textbooks - teachers” , engage in the rational construction of textbook content and creatively apply it in classroom instruction.

Hierarchical Interaction Model of Teachers' Textbook Understanding

According to the TIMSS three-tier curriculum, Building upon the curriculum levels proposed by American scholar Shulman（Shulman, 1987）, Mathematics teachers' textbook understanding involves the hierarchical transformation of textbook forms and the strategies for interacting with different textbook forms. Teachers convert ideal textbooks into operational textbooks through personalized interpretation, guided by curriculum standards. This process involves creatively adapting textbook content to meet students' learning characteristics. Based on this, a hierarchical interaction model of teachers' textbook understanding is proposed (Figure 1).

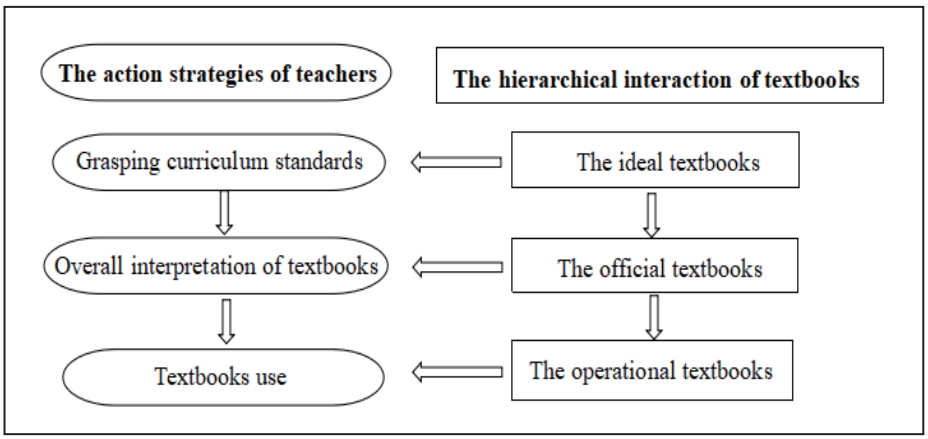


Figure 1. Hierarchical interaction model of mathematics teachers' textbook understanding

According to this model, mathematics teachers' textbook understanding can be divided into three steps: a) Understanding ideal textbooks by grasping curriculum standards; b) Interpreting official textbooks to understand the design intentions; c) Generating operational textbooks by transforming “textbooks” into “learning materials.” Teachers' overall interpretation of textbooks is in the middle location(Rezat et al., 2021), influenced by teachers grasping curriculum standards and impacting textbook use. Thus, the hypothesis is proposed: Teachers' overall interpretation of textbooks mediates the relationship between their grasp of curriculum standards and textbook use.

Methods

Research Tools

A questionnaire on mathematics teachers' textbook understanding was developed based on the mathematics teachers' textbook understanding. Refer to some widely recognized questionnaires,The questionnaire covers the following areas: basic information (5 items), mathematics teachers' understanding of the curriculum standards (8 items), overall interpretation of the instructional materials (17 items), and specific practices in the use of textbooks (16 items).The internal consistency test showed a total Cronbach's α of 0.86, with subscale α coefficients ranging from 0.783 to 0.831, indicating good reliability. The content validity was ensured by expert evaluation, with 12 experts in curriculum and instruction confirming the content effectiveness of the items.

Participants

A random sampling method was used to distribute the questionnaire in F and Z Province in China, targeting mathematics teachers. A total of 437 valid questionnaires were collected, with a gender distribution of 69 males (15.8%) and 368 females (84.2%).

This study utilized SPSS 26.0 software to conduct differential tests and correlation analyses. Subsequently, a mediation analysis was performed with mathematics teachers grasping curriculum standards as the independent variable, overall interpretation of textbooks as the mediator, and textbooks use as the dependent variable.

Results

Common Method Bias Test and Differential Test

To address potential common method bias, Harman's single-factor test was conducted. The results showed that the first factor explained 27.03% of the variance, below the critical value of 40%, indicating no significant common method bias. ANOVA results revealed significant gender differences in teachers grasping curriculum standards (X), overall interpretation of textbooks (M), and textbook use (Y). Gender was thus controlled in subsequent mediation effect analyses.

Mediation Effect test

Correlation analysis among the variables are shown Teachers grasping curriculum standards was positively correlated with their overall interpretation of textbooks ($r=0.54$, $p<0.01$) and textbook use ($r=0.22$, $p<0.01$). The overall interpretation of textbooks was also positively correlated with textbook use ($r=0.16$, $p<0.01$).Based on this, conduct multiple regression analysis for the variables. Regression analysis results (Table 1) showed that teachers grasping curriculum standards significantly positively impacted their overall interpretation of textbooks ($\beta=0.239$, $p<0.001$) and textbook use ($\beta=0.122$, $p<0.001$). The overall interpretation of textbooks also significantly positively impacted textbook use ($\beta=0.083$, $p<0.001$). Thus, the mediation effect of teachers' overall interpretation of textbooks between their grasp of curriculum standards and textbook use was significant.

Table 1. Regression analysis of variables

Item	Y		M		Y	
	β	t	β	t	β	t
Intercept	2.13	23.04***	2.43	38.44***	1.931	9.957***
X	0.122	4.61***	0.239	13.28***	0.102	3.257***
M					0.083	1.179***
R^2	0.047		0.289		0.0496	
F	21.248***		176.388***		11.3285***	

*Note. * indicates p<0.05, ** indicates p<0.01, *** indicates p<0.001 (two-tailed test).*

Further mediation effect analysis using Hayes' Process v4.2 Model 4 with Bootstrap method (5000 samples, 95% CI) confirmed that teachers grasping curriculum standards significantly impacted textbook use through the mediation of their overall interpretation of textbooks (Table 2). The total effect of curriculum standards on textbook use was 0.122 (SE=0.026), with a mediation effect of 0.020 (SE=0.017) and a 95% CI of [0.012, 0.056]. The direct effect was 0.102 (SE=0.031) with a 95% CI of [-0.044, 0.113]. Since the CI of the direct effect included 0, this indicated a full mediation effect.

Table 2. Decomposition of total, direct, and mediation effects

Item	Effect	Boot SE	Boot LLCI	Boot ULCI
Total Effect (X → Y)	0.122	0.026	0.070	0.174
Direct Effect (X → Y)	0.102	0.031	-0.044	0.113
Mediation Effect (X → M → Y)	0.020	0.017	0.012	0.056

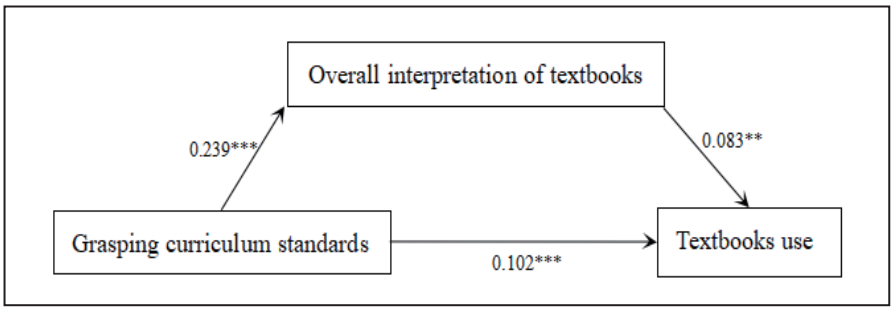


Figure 2. Mediation effect Model

Conclusions and Discussion

Significant Positive Impact of Understanding Curriculum Standards and Overall Interpretation of Textbooks on Textbook Use

Mathematics teachers grasping curriculum standards and their overall interpretation of textbooks both significantly and positively impact textbook use, aligning with prior research findings (Davis & Krajcik, 2005; Ball & Cohen, 1996). New curriculum standards serve as the foundation for understanding textbooks, influencing the entire process of textbook interpretation. Teachers should deeply study these standards, accurately grasp changes in curriculum goals and content structure, and use them as a benchmark for instructional design and implementation. They should also interact deeply with textbooks by examining their organization from the perspective of curriculum standards, focusing on textbook characteristics and changes, and considering the entire textbook (Pepin, Gueudet, & Trouche, 2013), including activities and evaluation systems, to creatively construct its meaning and uncover implicit values.

Complete Mediating Role of Overall Interpretation of Textbooks

Mediation analysis shows that mathematics teachers' understanding of curriculum standards impacts textbook use through overall interpretation of textbooks. Curriculum standards, which are abstract and complex, are more effectively implemented through textbooks that are concrete and closely linked to teaching practice. Teachers should adopt a holistic approach, analyzing textbooks within the broader curriculum system and understanding the relationship among curriculum standards, textbooks, and teachers (Remillard, 2005).

This study constructs a mediation model linking mathematics teachers grasping curriculum standards, overall interpretation of textbooks, and textbook use, providing empirical evidence for effective textbook understanding and use, while highlighting the need for larger sample sizes and deeper qualitative insights in future research.

References

Fan, L., Cheng, J., Xie, S.etal. (2021). Are textbooks facilitators or barriers for teachers' teaching and instructional change? An investigation of secondary mathematics teachers in Shanghai, China. *ZDM–Mathematics Education, 53*, 1313-1330.

Remillard, J. T. (2013). Examining resources and re-sourcing as insights into teaching. *ZDM–Mathematics Education, 45*, 925-927.

Remillard, J. T. (2018). Examining teachers' interactions with curriculum resource to uncover pedagogical design capacity. In L. Fan, L. Trouche, C. Qi, S. Rezat, & J. Visnovska (Eds.), *Research on mathematics textbooks and teachers' resources: Advances and issues* (pp. 69–88). Springer.

Rezat, S., Fan, L., & Pepin, B. (2021). Mathematics textbooks and curriculum resources as instruments for change. *ZDM–Mathematics Education, 53*(6), 1189-1206.

Ball, D. L. & Cohen, D. K. (1996). Reform by the book: What is—or might be—the role of curriculum materials in teacher learning and instructional reform? *Educational researcher, 2*5(9), 6-14.

Davis, E. A. & Krajcik, J. S. (2005). Designing educative curriculum materials to promote teacher learning. *Educational Researcher, 34*(3), 3-14.

Pepin, B., Gueudet, G., & Trouche, L. (2013). Re-sourcing teachers' work and interactions: A collective perspective on resources, their use and transformation. *ZDM–Mathematics Education, 45*, 929-943.

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211–246.

Shulman,L.(1987). Knowledge and teaching:Foundations of the reform.Harvard Educational Review,29(7):4-14.

TSG4-2-3

SUPPORTING EARLY CAREER TEACHERS' NOTICING: A FACILITATOR'S INTENT IN LESSON STUDY

Hyomin Kang
Graduate School of Comprehensive Human Sciences, University of Tsukuba, Japan,
s2030317@u.tsukuba.ac.jp

This study explores how a facilitator's intent shapes early career teachers' noticing in school-based lesson study in Japan. Using non-interventional participant observation, data was collected through lesson study recordings (transcribed), artifacts, and interviews with four ECTs and one facilitator in a public elementary school. Focusing on a unique case where the facilitator also served as the research lesson instructor, the analysis examines both ECTs' noticing and the facilitator's intent. The facilitator intentionally designed tasks to create space for discussion and leveraged unexpected situations to enhance learning in the geometry lesson. Findings highlight the facilitator's role in supporting ECTs' noticing and its significance for teacher education.

Keywords: Lesson Study, Teacher Noticing, Facilitator, Early Career Teacher

Introduction

Lesson study fosters teacher collaboration, but its implementation does not guarantee improved teacher noticing—the process of perceiving, interpreting, and making instructional decisions (Choy, 2025; Lewis, 2016). For early career teachers (ECTs), these processes remain challenging (Blömeke et al., 2015; Yun et al., 2024). In school-based lesson study (SBLS) in Japan, teachers refine lessons within their school practices (Lewis, 2016). Unlike external experts, facilitators work closely in the same school, mentor ECTs, guide discussions, and support teacher learning. Facilitators may also lead research lessons themselves, taking on multifaceted roles. Despite their critical contributions, facilitators remain underexamined in SBLS (e.g., Borko, 2004).

At primary level, perpendicularity and parallelism can be conceptually demanding for both students and ECTs (Sinclair & Bruce, 2015). While students recognize individual geometric shapes, understanding relationships between line segments is newly introduced, making instruction challenging.

This study explores how a facilitator's intent shapes ECTs' noticing in SBLS, focusing on a research lesson where the facilitator also served as the instructor. The study explores how facilitators influence noticing through live lesson observations, where unpredictable classroom events make real-time noticing more complex. With the background described above, the research questions are: How does a facilitator's intent influence ECTs' noticing in SBLS?

Literature Review and Conceptual Framework

The PID model (Perceiving, Interpreting, Decision-making) explains teacher noticing as the process of perceiving classroom events, interpreting them, and making instructional decisions (Blömeke et al., 2015). Within SBLS, noticing occurs through the four stages of lesson study: goal setting, plan-

ning, research lessons, and post-lesson discussions (Choy, 2025; Lewis, 2016). Teachers anticipate key moments through task design in the planning phase and align research lessons with long-term goals. ECTs refine noticing by validating observations with colleagues and reflecting on their interpretations in post-lesson discussions (Maison, 2002).

This study examines ECTs' noticing and facilitators' intent through two perspectives: *broadened perception*, where teachers recognize previously unnoticed classroom aspects, and *deepened interpretation and decision-making*, where they reinterpret familiar situations with greater nuance and make more informed instructional choices. In particular, it explores how facilitators intentionally create opportunities that trigger ECTs' noticing, shaping the conditions for instructional reflection and teacher development.

The facilitator's intent refers to deliberate efforts to make space for teachers' noticing by structuring discussions, setting focal points and designing tasks in this study. In primary school geometry, students often use informal language or omit key attributes when defining parallelism and perpendicularity, making conceptual assessment challenging. Drawings are visual tools for analyzing student thinking, enabling teachers to identify misconceptions and refine instructional responses (Sinclair & Bruce, 2015). This study examines explicitly how facilitators deliberately select the introductory lesson of an elementary geometry unit for research lessons, ensuring that it serves as a key moment for triggering ECTs' noticing and guiding their interpretation of classroom events.

Context and Methodology

This study is based on a long-term study spanning 18 months in a public elementary school in Japan, where the author conducted non-interventional participant observation and interviews. The data included four ECTs with 2–5 years of experience and the facilitator, Hiro, who had 18 years of teaching experience, including seven as a facilitator.

Data was collected through video recording and transcription of the lesson study process, including research lessons and post-lesson discussions. Additionally, pre- and post-lesson interviews with ECTs and Hiro were recorded and transcribed. Lesson plans and artifacts were also collected for analysis.

The study employed the constant comparative analysis (Corbin & Strauss, 2014), focusing on cases where ECTs showed *broadened perception* and *deepened interpretation and decision-making*. Hiro's statements and interview responses were closely examined to identify his intent in shaping these noticing shifts.

In September 2022, Hiro, serving as both facilitator and instructor, implemented a geometry lesson introducing perpendicularity and parallelism. The lesson aimed to develop students' understanding through real-life inquiry in a Grade 4 classroom. Instead of using the textbook, Hiro designed a task-based activity, prompting students to "create a user-friendly notebook." By discussing free-hand lines' limitations, students identified key terms like "straight line" and "parallel." Hiro intentionally omitted a direct review of prior learning, allowing students' responses to guide the lesson. Collaborative discussions facilitated shared learning. To define parallelism, students must first learn the concept of perpendicularity, which should be considered when structuring the lesson sequence within the unit.

In the post-lesson discussion, Hiro explained that he withheld pre-prepared notebook examples to encourage students to independently focus on spacing and parallelism. However, he noted that stu-

dents overlooked horizontal and vertical alignments, requiring further guidance on perpendicularity and right angles. The next lesson aimed to reaffirm "parallel" and "spacing" concepts through applied problem-solving. Discussions also addressed the timing and role of structured learning materials, with teachers exchanging perspectives on instructional refinement.

Findings

The ECTs demonstrated *broadened perception* by recognizing precise mathematical terminology, such as "straight line," which they had not previously considered, and by noticing Hiro's responsive teaching. All four ECTs reflected on this point, and one noted, "Students often say things like 'a straight one' because it is easier, but we need to teach accurate mathematical terminology consistently across all grade levels." This noticing had not emerged before the lesson study.

Regarding *deepened interpretation and decision-making*, their interpretation (I) evolved notably in instructional material analysis, but they remained hesitant in decision-making (D) about their own instructional choices. Discussions on multiple versions of prepared materials led to in-depth debates, marking a shift from relying on senior teachers to actively engaging in instructional reasoning.

Hiro intentionally selected the introductory lesson to have students engage with perpendicularity and parallelism before fully understanding them. Rather than aiming for mastery, he designed a task where students used these concepts without formal definitions, encouraging them to recognize what they could not yet define and anticipate what they needed to learn throughout the unit. This aligned with the SBLS theme of fostering students' ability to anticipate their learning process, promoting active engagement over passive learning. Before the research lesson, Hiro shared the following response during a pre-lesson interview with the author.

> (In Pre-interview with Hiro) I felt like lessons were too tied to the textbook and rigid, with little room for flexibility. Instead of teachers leading everything, I wanted a lesson where students could take an active role in their learning. That was my main goal. (…)I needed to clarify my intent for early career teachers—showing them, 'This is why we do it this way,' or 'It is okay to approach it differently.' When I was a young teacher, I was not sure what was right or wrong, so I wanted to model that for them. (…) The most important part was getting students to look at a poorly designed notebook and, in their own words, think about how to improve it using geometric concepts. That is what mattered most to me.

Hiro's words reflect his emphasis on creating an instructional setting where students explore and reason for themselves, rather than passively receiving knowledge. This approach also served as a model for ECTs, demonstrating how teachers can structure lessons to encourage student-driven inquiry.

As students struggled with misalignment and curvature, they naturally identified concepts such as "straight line" and "parallel." However, some misinterpreted parallelism as a single line, revealing conceptual gaps. Hiro leveraged these misconceptions as discussion points, using students' reasoning as opportunities for peer learning. For ECTs, observing students work with undefined concepts provided insight into how understanding develops. Hiro's use of incomplete definitions helped teachers see how students' reasoning evolved and how lesson sequences could be structured to support conceptual growth.

Discussion

This study analyzes data from ECTs and the facilitator to explore how a facilitator's intent shapes noticing in SBLS. Findings highlight students' engagement with mathematical concepts, ECTs' observation of the research lesson and the facilitator's responsive teaching.

First, the research lesson demonstrated the role of visualization in learning geometry. As students worked with freehand notebook designs, they recognized the limitations of imprecise drawings, leading to discussions on "straight line" and "parallel," though some misinterpreted parallelism as a single line. Students actively engaged in learning by incorporating a drawing activity, while ECTs' noticing of geometric concepts and relationships was enhanced (Sinclair & Bruce, 2015).

Second, Hiro's research lesson design fostered student-centered learning and teacher reflection. Using incomplete definitions and open-ended discussions, he guided students to anticipate key concepts while ECTs observed how reasoning develops. Observing key moments in lesson study is essential for developing teacher noticing (Choy, 2025). Discussions on equal spacing in defining parallelism highlighted the impact of precise definitions.

Third, Hiro's responsive lesson emphasized adjusting the lesson based on student responses rather than following a rigid plan. While ECTs often rely on structured lessons, Hiro designed tasks to model instructional flexibility, demonstrating dynamic coordination with students and creating opportunities for discussion in SBLS.

Concluding Remarks

This study sheds light on facilitators' support in ECTs' noticing, showing how intentional lesson design creates noticing opportunities. Unlike external mentors, facilitators support ECTs' learning by sharing school-based challenges and responding to students' needs within Japanese SBLS. These findings underscore the importance of facilitator support in teacher education. As a case study, the findings are not broadly generalizable but highlight facilitators' overlooked contributions. Future research should further explore their impact on teacher learning and instructional decision-making.

References

Blömeke, S., Gustafsson, J., & Shavelson, R. J. (2015). Beyond dichotomies: Competence viewed as a continuum. *Zeitschrift für Psychologie*, 223(1), 3–13.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational researcher*, 33(8), 3–15.

Choy, B. H. (2025). Noticing to learn from lesson study: the role of critical events. *Journal of Mathematics Teacher Education*, 1–18.

Corbin, J., & Strauss, A. (2014). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. Sage Publications.

Lewis, C. (2016). How does lesson study improve mathematics instruction?. *ZDM– Mathematics Education*, 48, 571–580.

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. Routledge.

Sinclair, N., & Bruce, C. D. (2015). New opportunities in geometry education at the primary school. *ZDM–Mathematics Education*, 47, 319–329.

Yun, H., Zhang, Q., Cao, W., & Zhang, X. (2024). Teacher noticing within the context of lesson study: a systematic review and prospective trends. *International Journal for Lesson & Learning Studies*, 13(4), 361–381.

TSG4-2-4

VISUAL PERCEPTION OF EXPERT AND NOVICE MATHEMATICS TEACHERS ACROSS STUDENT PROFILES

Rangmei Li
Beijing Normal University, China, rangmei_li@mail.bnu.edu.cn

Yiming Cao
Beijing Normal University, China, caoym@bnu.edu.cn

Nicole B. Kersting
University of Arizona, USA, nickik@arizona.edu

Gabriele Kaiser
University of Hamburg, Germany, gabriele.kaiser@uni-hamburg.de

Despite growing interest in teachers' professional noticing, research on how mathematics teachers perceive students with varying achievements and other aspects during instruction remains scarce. This case study employs mobile eye tracking to compare an expert and a novice mathematics teacher's visual perception across three student profiles—strong, intermediate, and struggling. Results show that the expert teacher distributed attention evenly, whereas the novice teacher focused primarily on strong students, followed by intermediate and struggling ones. Both teachers shifted focus within the same student profiles, but only the expert made significant transitions between strong and struggling or intermediate students, while the novice often returned to strong students after focusing on intermediate ones.

Keywords: visual perception, teacher noticing, student profiles, mobile eye tracking

Introduction

Effective mathematics teaching, which adeptly navigates the unpredictable nature of classrooms to address students' needs, depends on teachers' professional noticing abilities (Gibson & Ross, 2016). Perception as the first and foremost facet of teacher noticing, refers to the process of identifying noteworthy events in the classroom, which is known to be a prerequisite for interpreting relevant features of student learning and making accurate instructional decisions (Van Es & Sherin, 2008).

Although numerous studies have examined teachers' perception skills across experience levels, the ways in which mathematics teacher visually perceive specific situations during their classroom instruction remains unclear. Given that human eye movements are highly task-specific (Kaakinen, 2021), identifying the context-specific gaze patterns in mathematics teachers' instruction is essential for understanding teachers' selective attention in real time. Mobile eye-tracking technology has been increasingly utilized to study teachers' visual perception in classrooms (Cortina et al., 2015), but so far, few studies have explored the relation between teachers' perception and student profiles. Student profiles shaped by prior knowledge, emotional state, and learning needs are critical dynamic cues that mathematics teachers rely on to guide instruction (Pielmeier et al., 2018).

Thus, the current study aims to utilize a case study approach to explore in-depth differences in the visual perception of an expert and a novice mathematics teacher across various student profiles by mobile eye-tracking technology, specifically focusing on the distribution of their attention among students and the way they shifted their focus between different student profiles in authentic classroom settings. The research questions are therefore: (1) Which student profiles do the expert and novice mathematics teacher pay attention to during instruction? (2) How do the expert and novice mathematics teacher shift their attention between different student profiles during instruction?

Methodology

Participants and Context

Two female mathematics teachers from prominent middle schools in Beijing, China—one expert and one novice—were invited to participate in this study with their eighth-grade students. The expert teacher aged 46, with 25 years of teaching experience, is a senior educator who completed a bachelor's degree in education and received high peer recognition for her classroom instruction. In contrast, the novice teacher was 27 years old, with half a year of teaching experience; she had completed her compulsory teacher education during the three-year master's period with a degree of education. Each mathematics teacher conducted two consecutive lessons on the same topic for eighth-grade students over two days, based on their teaching plan selected at their convenience. The written consent was required from the mathematics teachers and their students before the data collection.

Instruments and Procedure

To capture teacher gaze during mathematics classes, we asked the expert and novice teacher to wear Tobii Pro Glasses 2, a mobile eye-tracking device with a sampling rate of 50 HZ. The scene camera on this device had a resolution of 1920 × 1080, offering a vision of 82° horizontally and 52° vertically. The mathematics teachers were asked to calibrate with the device using a one-point calibration system. After recording, the gaze sample rate of the eye-tracking videos from both teachers exceeds 85%, qualifying them for detailed analysis. After the lessons, a questionnaire was utilized to help identify student profiles as perceived by the mathematics teachers. The expert and novice teacher reported information about students' general cognitive abilities, mathematics pre-knowledge, self-concept of ability in mathematics, and interest in mathematics on a 5-point rating scale (1 = low, 2 = below average, 3 = average, 4 = above average, 5 = high) (Seidel, 2007).

Data Analysis

Considering teachers' perception may vary across teaching phases which involve distinct goals and intentions (Kaakinen, 2021), we selected three eye-tracking video clips for each mathematics teacher, representing different teaching phases: introduction (Intro), exploration (Expl), and extension (Ext). We created 49 AOIs for the expert and 46 AOIs for the novice to identify their gaze targets on the snapshot. These AOIs comprised 45 students for the expert teacher, 42 students for the novice teacher, and four additional objects that might also caught the teachers' attention in the classroom: blackboard, computer screen, interactive whiteboard, and others. This study primarily focused on students and only presented results related to them. Furthermore, three TOIs were created for each teacher to represent the interval of three teaching phases introduced before by selecting a start event and an end event. After manually mapping what the teacher was focusing on at the timestamp of each fixation onto the snapshot, we exported indicators from selected TOIs for all AOIs, including

recording timestamp, event, mapped eye movement type, mapped eye movement type index, fixation count, and AOI hit which represents whether the fixation is located inside the AOI.

To address the first question, we first utilized latent class analysis (LCA) with Mplus 8 to identify student profiles as perceived by the mathematics teachers. We then visually depicted the mean fixation counts of mathematics teachers for students in each profile during lessons and specific teaching phases. In order to further investigate the relationship between mathematics teachers' attention distribution and student profiles, we conducted a negative binomial regression model to examine and compare the impact of student profiles on the expert and novice mathematics teacher's fixation counts, given that the fixation count data exhibited overdispersion in this study. For the second question, we performed the lag sequential analysis (LSA) to explore and compare attention transition patterns of the expert and novice mathematics teacher during whole lessons. We filtered for the eye movement type "fixation" and removed the repeated mapped eye movement type index to obtain the fixation sequences of mathematics teachers based on the indicator of AOI hit. The data were analysed using LSA software (general sequential querier, GSEQ), which can provide frequency tables of fixation sequences and adjusted residual tables with z-scores. These results were used to construct teachers' attention transition paths and probability diagrams, thereby elucidating the attention transition patterns in the expert and novice mathematics teacher's classroom interactions.

Results

Expert and Novice Mathematics Teachers' Attention Distribution Among Students

In both classes, a three-profile solution was ultimately chosen based on LCA results, despite the non-significant LMR result for three profiles in the expert's class ($p = 0.37$) which had the lowest AIC and high entropy values, aligning well with students' grade points and providing clearer differentiation. Meanwhile, the profiles labeled were consistent across both classes: Strong students who exhibited high-grade points; Intermediate students who showed average performance in these four dimensions; and Struggling students who had below-average scores.

The results of teachers' attention distribution among profiles show that the expert mathematics teacher exhibited a balanced distribution of attention across different student profiles. In contrast, the novice mathematics teacher's attention showed a primary focus on strong students, followed by intermediate and struggling students. A similar trend was observed in specific teaching phases, with one exception: in the introduction phase, the expert directed more fixations to struggling students to diagnose students' existing knowledge at the beginning of the lesson.

The negative binomial regression model was further conducted to statistically assess the effect of student profiles on the expert and novice mathematics teacher's fixation counts. The results confirmed the descriptive differences mentioned above, but no significant impact of student profiles was found on the fixation counts of the expert mathematics teacher. However, during the introduction phase, a marginally significant effect was noted ($x^2 = 5.434$, $p = 0.066$), with the expert teacher paying more attention to struggling students than to strong and intermediate students. In contrast, the novice teacher's fixation counts were significantly influenced by student profiles across Lesson 1 ($x^2 = 11.900$, $p = 0.003$) and Lesson 2 ($x^2 = 11.306$, $p = 0.004$), and also during exploration ($x^2 = 21.957$, $p < 0.001$) and extension phases ($x^2 = 21.957$, $p < 0.001$), suggesting that strong students received the most attention, followed by intermediate students, with struggling students receiving the least.

Expert and Novice Mathematics Teachers' Attention Transition Across Students

Transitions with a z-score greater than 1.96 were considered statistically significant at the 0.05 level by conducting the LSA. The results reveal that both teachers frequently shifted their focus within student groups of the same profile—strong, intermediate, and struggling. Additionally, transitions from intermediate to strong students, and from struggling to intermediate students, were also observed during their lessons. However, there exist significant differences in attention transitions between the expert and novice mathematics teacher. For the expert teacher, attention transitions from strong students to intermediate or struggling students were identified, as well as transition from struggling students to strong students, which were not observed in the novice's lessons. In contrast, the novice teacher demonstrated a significant transition probability from intermediate to struggling students.

Summary and Discussion

In the present case study, we employed mobile eye-tracking technology to explore how an expert and a novice mathematics teacher allocate their attention and shift their focus across different student profiles in authentic classroom settings. The results indicated that the expert teacher distributed attention more evenly among all groups of students, whereas the novice teacher focused mainly on strong students. These findings align with prior studies showing that expert teachers had a broader horizon across the classroom than novice teachers (Cortina et al., 2015; Van den Bogert et al., 2014). The novice teacher directed her attention primarily toward strong students with high achievement and confidence in the subject, who participate more verbally and actively in class. Additionally, both teachers were able to shift their fixations within the same profile groups. By gathering behavioural cues from students with same profiles, teachers can address homogeneous needs and provide timely feedback more effectively (Pielmeier et al., 2018). However, significant transitions between strong and struggling students, as well as between strong and intermediate students, were observed only in the expert's lessons. This reflects the expert teacher's balanced and systematic approach to monitoring the classroom to address students' diverse needs.

This study is limited by the case study approach which restricts generalizability, and the potential influence of teaching topics on teachers' attention. Nevertheless, it provides insights into student-centred attention in mathematics classrooms, emphasizing the observation of students' cognitive, emotional, and social needs during instruction. Future research should further investigate the mechanisms shaping teachers' visual perception patterns in teaching contexts.

References

Cortina, K. S., Miller, K. F., McKenzie, R., & Epstein, A. (2015). Where low and high inference data converge: Validation of CLASS assessment of mathematics instruction using mobile eye tracking with expert and novice teachers. *International Journal of Science and Mathematics Education*, *13*(2), 389–403.

Gibson, S. A., & Ross, P. (2016). Teachers' professional noticing. *Theory Into Practice*, *55*(3), 180–188.

Kaakinen, J. K. (2021). What can eye movements tell us about visual perception processes in classroom contexts? Commentary on a special issue. *Educational Psychology Review*, *33*(1), 169–179.

Pielmeier, M., Huber, S., & Seidel, T. (2018). Is teacher judgment accuracy of students' characteristics beneficial for verbal teacher-student interactions in classroom? *Teaching and Teacher Education*, *76*,

255–266.

Seidel, T. (2007). The role of student characteristics in studying micro teaching–learning environments. *Learning Environments Research*, *9*(3), 253–271.

Van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education*, *24*(2), 244–276.

Van den Bogert, N., van Bruggen, J., Kostons, D., & Jochems, W. (2014). First steps into understanding teachers' visual perception of classroom events. *Teaching and Teacher Education*, *37*, 208–216.

Acknowledgement

Research reported here was funded by the Humanity and Social Science Youth Foundation of Ministry of Education of China (No. 24YJC880075).

TSG4-2-5

EXAMINING ADAPTIVE EXPERTISE OF EMERGING AND EXPERIENCED INSTRUCTIONAL COACHES THROUGH NOTICING

Sze Looi Chin
Massey University, New Zealand, s.l.chin@massey.ac.nz

Generosa Leach
Massey University, New Zealand, g.leach@massey.ac.nz

Jodie Hunter
Massey University, New Zealand, j.hunter1@massey.ac.nz

This study presents two vignettes that highlight adaptive expertise from an emerging- and an experienced-instructional coach in a study conducted in New Zealand. As instructional coaching becomes more prevalent internationally, there is a need for research to explore ways that coaches enact their roles, and the expertise involved in effective coaching. The findings of this study demonstrate that experience in teaching sets the foundation for effective coaching, but adaptive expertise develops and becomes more consistent with experience.

Keywords: instructional coaches; coaching; adaptive expertise; noticing

The growing interest in mathematics education research on *coaching* (Sutton et al., 2011) highlights the value of providing continued professional learning to in-service teachers (Russell et al., 2020). However, effective coaching requires knowledge across domains and expertise for both noticing student and teachers' thinking (Amador, 2022). Hatano and Ingakai (1986) distinguish two types of expertise: routine expertise and adaptive expertise. In our context, *routine expertise* refers to coaches employing rehearsed coaching strategies, such as following a routine questioning protocol to ensure teachers attend to specific aspects of the lesson. In contrast, *adaptive expertise* is often dynamic and unrehearsed yet specific, and responsive to the changing context. Experienced teachers who are known to be effective in their teaching are often sourced as coaches, however, Chval et al. (2010) ask whether effective teachers immediately make for effective coaches. We aim to add to the current literature by examining how emerging and experienced instructional coaches enact their roles when working with teachers in classrooms. The research question that guides this study is: *How do emergent and experienced instructional coaches interpret and respond to in-service teachers in-the-moment during mathematics lessons?*

Methodology

Data in this paper are drawn from a wider project, *Developing Mathematical Inquiry Communities* (DMIC) (Hunter et al., 2018). As part of the project, coaches provided in-the-moment instructional coaching to in-service primary and secondary mathematics teachers during lessons that consisted of launching a task, small group discussions, a big group discussion, and a connect. During lessons,

the coach or the teacher called *pauses,* instructional moves (Kazemi et al., 2009) that provided opportunities for co-construction of the lesson through reflective discussions, seeking information, modelling instruction, providing direction, and evaluating teachers' instruction. Lessons were video-recorded, then uploaded onto a video-sharing platform where the coaches annotated each pause, their purpose, and their perceived outcome of the pause. The data here are drawn from two coaches, Evan and Malia (pseudonyms). While both were experienced teachers, Evan was an emerging coach in his second year of coaching, and Malia was an experienced coach in her ninth year of coaching. Four videos were randomly selected from a collection of eight and coded in two stages by two to four researchers. Firstly, for the type of pause (Kazemi et al., 2009) and the pedagogical dimension (Hunter et al., 2024). Secondly, each pause was identified as demonstrating routine or adaptive expertise depending on how the coach attended to, interpreted, and responded to in-the-moment classroom interactions (Table 1).

Table 1. Expertise in coaching through noticing

	Adaptive Expertise	**Routine Expertise**	**Unclear**
Attending	Awareness of students' and teachers' thinking relevant to mathematical, cultural, and social pedagogical dimensions, based on observations and evidence of situation	Limited awareness of students' and teachers' thinking relevant to mathematical, cultural, and social pedagogical dimensions, based on observations and evidence of situation	Unaware of a need to call a pause, even upon reflection
Interpreting	Interpretation of students' or teachers' thinking connected across multiple pedagogical dimensions that aligns with observations	Superficial interpretation of students' or teachers' thinking that aligns with observations	Misinterpretation of teachers' and students' thinking, lacking in evidence of the situation
Responding	Effective communication of specific observations with interpretation linked directly to response that has the potential to extend students' or teachers' thinking	Communication with limited specificity to observations and interpretation, with the use of a rehearsed protocol that is unlikely to extend students' and teachers' thinking beyond a single pause	Calling a pause that is not helpful in supporting teachers' learning, may not align with the project framework

Vignette 1

Evan regularly initiated directive pauses and used scripted responses in his interactions with teachers. He often focused on attending to pedagogical dimensions such as teacher actions, mathematical practices, lesson structure, inclusivity, engagement, and social norms, with some instances of responding dynamically to teacher-initiated pauses. We highlight an episode of emerging in-the-moment adaptive expertise from a lesson with students aged 9-10-years-old working in groups on a task to find 12 possible distances toy cars could have travelled between 3.45 m and 3.46 m.

Evan's Vignette

Evan and the teacher noticed a group had used a number line, marked in tenths, to record the possible distances the cars may have travelled. The teacher pointed to the number line, asking the students to think about the numbers between two points. Noticing one student's confusion, Evan called his first pause, telling the teacher "Sometimes [the students] find it easier if you indicate... exactly halfway between." Subsequently, the teacher immediately asked the student, "What would that one be called?" indicating to the space between 3.451 and 3.452. The student responded, "That would be 3.4515." Pursuing this line of discussion, the teacher asked, "Can you prove that?" and turned to Evan saying, "So, he has got it." Following on, Evan and the teacher discussed how "from struggling and thinking", the students were pressed to think beyond whole number and discrete decimal increments, towards infinitely smaller increments. Although the point identified was exactly half-

way between 3.451 and 3.452, the teacher noticed that through making conjectures, her students extended their initial reasoning leading towards an "infinity idea" and she could "elicit that out of the conversation". Evan prompted the teacher to consider how this could address the same misconception others may have, "to think hmm, where maybe some of those others are 3.4515 and maybe deal with that misconception of theirs ... it's good what you're doing because you've got your main group that you want to share but you want them to pull in [other's reasoning]." Evan called multiple pauses as he engaged in cycles of noticing. He attended to the students' response to the teacher's instructions, and interpreted it as an opportunity to resolve a potential misunderstanding while also raising the teacher's awareness to it. In the subsequent cycle, he attended to the teacher's actions, interpreted them as insufficient for addressing the misconception, and directed the teacher to be more specific. Then, he attended to the student response and the teacher, interpreting this as sufficient and pressed the teacher to consider how this group's reasoning would support other groups' understanding during the connect. His actions demonstrated adaptive expertise as he noticed potential learning opportunities to support the teacher to extend students' mathematical reasoning. The multiple cycles of interaction demonstrated how Evan's specificity through his explicit identification led to the teacher supporting the students to productively work through misconceptions. Furthermore, his effective communication and elaboration on what he attended to resulted in a genuine co-construction of learning.

Vignette 2

Malia frequently initiated reflective discussions linked to student thinking and the teachers' actions. She often attended to pedagogical dimensions relevant to teachers' actions, mathematical practices, challenging tasks, and substantive conversations, interpreted teachers' thinking and needs while upholding their status, and responded dynamically yet unobtrusively through enactments and extended discussions. This vignette exemplifies Malia's adaptive expertise in coaching. It is drawn from one episode with students aged 7-8-years-old working in groups on a mathematical task to determine if a spinner game with eight segments (four pink, two yellow, two blue) was fair.

Malia's Vignette

The teacher asked if it was fair that if the spinner landed on pink, Player 1 would win, and if it landed on any other colour, Player 2 would win, and the group of students nodded. The teacher prompted her students to discuss their thinking with their peers, then chose one student to explain their thinking and another student to repeat the first student's sharing. Noticing there was ten minutes remaining of the lesson, Malia spontaneously asked the class, "is it fair if she only got the yellows? Why?" The teacher repeated the question, and the students correctly identified it would not be fair, as well as if Player 1 had only the blue segments. Malia focusing on mathematical practices asked the group "Why? Why not?" to which the teacher repeated the question and students responded, "because that's only one quarter and then it becomes…?". Noticing an opportunity to further extend students' thinking, Malia asked the students, "so what is the percentage of a half?", to which the teacher said, "Oh yeah, what's the percentage? Remember percentage is of a hundred." This form of questioning continued, initiated as an enactment from Malia, followed by the teacher interacting with the students directly and Malia stepping back to allow the teacher to lead the discussion. After two more cycles of this, Malia brought the teacher aside and asked the teacher, "So, how do you think you went today with highlighting mathematical practices?" and the teacher shared how she was developing her questioning and prompts, particularly for engaging all students in her class to participate. Across the vignette lasting approximately seven minutes, Malia called six pauses,

each attending to student responses and the teacher's attempts to engage students in mathematical practices through justification, assessing whether the teacher's prompts had the potential to extend students, and modelling questioning to co-construct extending the group's thinking. Malia's actions demonstrated adaptive expertise as she noticed opportunities to extend the students' mathematical practices and to extend the teachers' actions in making students' mathematical practices explicit, ending with a final pause as a reflective discussion. Malia moved between attending to student thinking, teachers' actions, and connecting the key mathematical concept of fairness to percentage, which led to co-constructing the learning episode with the teacher, while still upholding their status by modelling actions and then allowing the teacher to take control back of the learning.

Conclusion

Effective instructional coaches need expertise to notice both students' and teachers' thinking (Amador, 2022), yet, there is a general assumption that "effective teachers will be effective coaches and that these teachers will need little support as they transition into their new roles" (Chval et al., 2010, p. 192). However, when individuals enter the role of instructional coaches, they should not be expected to be immediately adaptive in their coaching. These two vignettes illustrate adaptive expertise enacted by two coaches at different stages of their development. Both Evan and Malia were experienced teachers and had undoubtedly developed pedagogical content knowledge relevant to their tasks over their extensive teaching careers. Although Evan's pauses were generally routine across his lessons, this vignette demonstrated his developing adaptive expertise through engaging in multiple cycles of noticing. Malia illustrated her adaptive expertise characterized by her in-the-moment responsiveness to extend both students' and teachers' thinking. While emerging coaches generally employ rehearsed questioning, they can interpret and respond to students and teachers in adaptive ways. The findings from our study suggest that the differences in Evan and Malia's noticing of teachers' thinking were likely linked to their experiences in coaching, specifically in how they respond to the teachers' actions. We acknowledge this is a small snapshot of two coaches at differing phases, however, it illustrates the possibilities for adaptive expertise to be present and that effective teachers may have moments of effective coaching, but it may not be consistent. Further research is required to investigate what support emerging coaches may require to fully develop consistent adaptive expertise. We wonder, how can we support emerging coaches to *transition* to effectively notice in-the-moment, so that they are capable of enacting and developing their adaptive expertise?

References

Amador, J. M. (2022). Mathematics teacher educator noticing: Examining interpretations and evidence of students' thinking. *Journal of Mathematics Teacher Education*, *25*(2), 163–189. https://doi.org/10.1007/s10857-020-09483-z

Chval, K. B., Arbaugh, F., Lannin, J. K., van Garderen, D., Cummings, L., Estapa, A. T., & Huey, M. E. (2010). The Transition from Experienced Teacher to Mathematics Coach: Establishing a New Identity. *The Elementary School Journal*, *111*(1), 191–216. https://doi.org/10.1086/653475

Hatano, G., & Inagaki, K. (1986). TWO COURSES OF EXPERTISE. *RESEARCH AND CLINICAL CENTER FOR CHILD DEVELOPMENT Annual Report*, *6*, 27–36.

Hunter, R., Hunter, J., Anthony, G., & McChesney, K. (2018). Developing mathematical inquiry communities: Enacting culturally responsive, culturally sustaining, ambitious mathematics teaching. *Set: Research Information for Teachers*, *2*, 25. https://doi.org/10.18296/set.0106

Russell, J. L., Correnti, R., Stein, M. K., Bill, V., Hannan, M., Schwartz, N., Booker, L. N., Pratt, N. R., & Matthis, C. (2020). Learning From Adaptation to Support Instructional Improvement at Scale: Understanding Coach Adaptation in the TN Mathematics Coaching Project. *American Educational Research Journal*, *57*(1), 148–187. https://doi.org/10.3102/0002831219854050

Sutton, J. T., Burroughs, E. A., & Yopp, D. A. (2011). Coaching Knowledge: Domains and Definitions. *Journal of Mathematics Education Leadership*, *13*(2), 12–20.

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 4
(Day 3)

TSG4-1-7

AN ANALYSIS OF MATHEMATICS TEACHER'S SUPPORT IN HIGH SCHOOLERS' INQUIRY: FOCUSING ON THE MOMENT OF SUPERVISIONS IN SRP

Toshihiko Ogawa
Waseda University, Japan, toshi.k2@fuji.waseda.jp

Takeshi Miyakawa
Waseda University, Japan, tmiyakawa@waseda.jp

This study aims to deepen the understanding of mathematics teacher support characteristics in inquiry-based lessons. Based on the Anthropological Theory of the Didactic, we investigated the teacher support in the moment of supervision in a Japanese high school setting. Specifically, the dialectic of media and milieu was used to categorize the teacher's support, and questions and answers that emerged in a particular session were presented in a list format. As a result, many teacher's support was classified as "promoting interaction with milieu", with a notable emphasis on the provision of "deeper questions". These results suggest that being able to provide such questions is related to teacher expertise.

Keywords: mathematics teacher support, inquiry-based learning, teacher supervision, Study and Research Paths (SRP)

Introduction and Theoretical Framework

Inquiry-based learning (IBL) has recently attracted significant attention in international mathematics education research (Artigue & Blomhøj, 2013). In IBL lessons, teachers are required to adopt roles that differ significantly from those in ordinary mathematics lessons (e.g., Dorier & Maass, 2020). However, the specific roles expected of teachers in IBL lessons, as well as the precise characteristics of their support, do not remain sufficiently explored. Therefore, this study aims to deepen the understanding of mathematics teacher support characteristics in IBL lessons. To achieve this, we focused on the moment of teacher supervision observed in IBL lessons conducted in a Japanese high school context, and analyzed the teacher's support in this moment using the *Anthropological Theory of the Didactic* (ATD), with emphasis on the *dialectics of inquiry* (cf. Chevallard & Bosch, 2019).

Within ATD, the process and structure of inquiry are conceptualized as *Study and Research Paths* (SRP). SRP is a sequence of activities that unfold within a learning environment, using all available resources (called *media* in ATD). Typically, an inquiry begins with a lively generative initial question (Q_0), from which other questions are progressively derived. Questioning is a fundamental concept in SRP. The mechanism through which inquiry deepens is further explained using the *dialectics of inquiry*. This concept refers to the iterative interplay between two polar constraints that have a significant effect on inquiry (cf. Chevallard & Bosch, 2019). Among the various dialectics proposed, the *dialectic of media and milieu* is particularly important (Kidron et al., 2014). This shows how inquiry progresses through the complementary interplay between interaction with media to collect needed information and with milieu to advance inquiry using the acquired information.

The aim of this study, by relying on ATD, can be articulated through the following research question: How did the teacher support the dialectic of media and milieu during each supervision session within the SRP process?-

Methods

The IBL lessons, as reported in Ogawa & Miyakawa (2025), were carried out in a newly introduced subject entitled *Period for Inquiry-Based Cross-Disciplinary Study* at a Japanese high school. This year-long subject comprised 53 lessons, each lasting 50 min. The lessons were collaboratively designed by a teacher and a researcher. Twenty-two Grade 12 students participated in this class and were divided into seven groups, each selecting a different theme (Q_0). In this paper, we focused on the moment of teacher supervision during the first and second semesters. Each supervision lasted about 10 min and was a key opportunity for each group to receive the teacher's support. During these sessions, groups presented the progress of their inquiry, and the teacher provided comments, posed questions, and collaboratively explored future directions. Additionally, the teacher occasionally delivered lectures to support the acquisition of mathematical knowledge and tools. In other words, during the supervision sessions, each group and the teacher discussed *about* its inquiry.

In this analysis, we first regarded the discussion during each session as an inquiry and identified the QAs. The QAs were classified by the teacher or student group. The initial question in each session was set as follows: "What is the progress of the inquiry, and what activities have been addressed at this point?" Subsequently, we classified the teacher's support from a macro perspective. Specifically, we identified one or two *adopted teacher's QAs* during each session. These were defined as the teacher-initiated QAs that the student groups subsequently addressed after the sessions. Then, in this paper, we focused on the dialectic of media and milieu and categorized them based on whether they promoted interaction with milieu or media. *Promoting interaction with milieu* refers to teachers co-constructing milieu alongside students. This involves providing support for students in furthering their investigation-related information. In contrast, *promoting interaction with media* refers to teachers functioning as part of media, for example, by delivering lectures on mathematical knowledge or tools required. Finally, among the categories commonly identified in the macroanalysis, we provided both the teacher's and the group's QAs from a particular session.

Result

The supervision sessions were conducted 15 times in the first semester and 21 times in the second semester, with each group participating approximately five supervision sessions over the course. A total of 47 *adopted teacher's QAs* were identified, and each was categorized in Table 1.

Table 1. Categorization of adopted teacher's QAs based on the dialectic of media and milieu

Teacher support	First semester (15 sessions)	Second semester (21 sessions)
Promoting interaction with *media*	6	9
Promoting interaction with *milieu*	18	14
Total	24	23

As shown in Table 1, a significant number of supervision sessions were categorized as "promoting interaction with *milieu*." This characteristic was particularly prominent in the first semester when many adopted teacher's QAs provided in the first session for all groups were classified under this category. These QAs primarily focused on supporting the groups in deepening their inquiry. For instance, teacher's questions (Q_T) such as "Q_T: How can the length of a circumference be approximated using an inscribed regular polygon?" and "Q_T: How can the mass and speed of rain on the internet be measured?" were posed. In contrast, in the second semester, there was an increase in sessions categorized as "Promoting interaction with media." This shift highlights the progression of their inquiry, which increasingly required the acquisition of mathematical knowledge and tools that the students had not previously encountered or utilized. For instance, the teacher provided specific support through activities such as "A_T: Lecture on how to calculate integrals in polynomials" and "A_T: Introduction to a paper analyzing penalty kicks" (A_T is teacher's answer).

To gain deeper insights, we analyzed a specific session categorized as "promoting interaction with milieu". This session lasted approximately 12 min and was conducted in the 10th lesson, following the students' selection of their themes. This was the second session for Group D, whose inquiry theme was "Q_0: Do Mersenne primes exist infinitely?" Initially, Group D investigated Mersenne numbers and tools for testing prime numbers using the internet. During their first session, the teacher facilitated the students to investigate Mersenne primes in detail and to explore the mechanism of the tools.

The details of this supervision session were summarized in Table 2. In this table, the teacher's QAs are denoted by the subscript T, and the students' QAs are denoted by the subscript S. The adopted teacher's QA during this supervision was "Q_T: How to determine whether a specific three-digit integer is a prime number, for example, is 157 a prime number?". The question was posed because the teacher recognized that Group D lacked a comprehensive understanding of the method for determining prime numbers based on the definition they had previously investigated. The question promoted Group D to verify this method concretely by testing it with a specific three-digit integer. In response, Group D dedicated the final 6 min of the session to specifically calculate whether 157 could be divided into other integers or prime numbers. Following this supervision, Group D continued to determine whether 157 is prime. This led them to explore more concise methods for prime number judgement.

Discussion and Conclusions

A central characteristic of the teacher's support in "promoting interaction with milieu" was their use of questions directed at groups that did not critically evaluate the information obtained from media. These questions served to verify the accuracy of the information and to encourage students and groups fully understanding them. Within ATD, such questions are referred to as *side questions* (Chevallard, 2015). Two distinct types of side questions have been identified: those aimed at going *deeper* and those aimed at *wider* (Rasmussen, 2016). In this case, the teacher posed deeper questions to direct

Table 2. Discussion in the second supervision of Group D

QAs by the teacher and the students (Group D) in the second supervision
Q_T: What is the progress of the inquiry, and what activities have been addressed at this point?
A_S: Confirmation of the definition of Mersenne primes
A_S: Investigation of the mechanisms of the prime number tester and the exponents calculator
A_S: Confirmation of the values of the first ten Mersenne primes
Q_T: How were the Mersenne primes specifically calculated, for example, what is the value of the eighth Mersenne number, and is it a prime number?
A_S: The value is 255, and it is not a prime number because it is divisible by 5
Q_T: Why is 255 divisible by 5?
A_S: Investigation of a method for determining whether a number is a multiple of 5
A_S: Investigation of a method for determining whether a number is a prime number based on the definition of prime numbers
Q_T: How to determine whether a specific three-digit integer is a prime number, for example, is 157 a prime number?
A_S: 157 is not divisible by any integer less than 12
A_S: 157 is not divisible by any prime number less than 53
Q_T: What are the content and direction for further inquiry?
A_T: Confirmation of the values of Mersenne prime numbers after the 11th
A_T: Testing whether 157 is a prime number or not

attention to specific content and activities, thereby fostering a more thorough understanding. For instance, during the second session of Group D, the teacher provided a three-digit integer and posed a deeper question intended to prompt an activity where students were required to determine whether the integer was prime. By incorporating specific examples, these questions effectively promoted interaction with milieu and were also fulfilled the sufficient function of deeper questions. Therefore, *raising deeper questions using specific examples* can be considered one aspect of teacher support in IBL lessons. Furthermore, we consider that being able to conduct such support is part of teacher expertise in IBL lessons.

Finally, although this case can be considered a typical example of *Period for Inquiry-Based Cross-Disciplinary Study* in Japan, the supervision sessions were unique to this context. However, in other cases, similar forms of interaction between students and teachers regarding students' inquiry often take place (e.g., mid-term presentations), and the teacher's practices observed in this case can be seen as analogous to the support typically provided in such situations. Further research is needed regarding the extent to which the teacher's practices identified in this case can be generalized.

References

Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. ZDM Mathematics Education, 45, 797–810. https://doi.org/10.1007/s11858-013-0506-6

Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *The proceedings of the 12th international congress on mathematical education* (pp. 173–187). Springer. https://doi.org/10.1007/978-3-319-12688-3_13

Chevallard, Y., & Bosch, M. (2019). A short (and somewhat subjective) glossary of the ATD. In M. Bosch, Y. Chevallard, F. García, & J. Monaghan (Eds.), *Working with the anthropological theory of the didactic in mathematics education: A comprehensive casebook* (pp. xxiv–xxxvii). Routledge. https://doi.org/10.4324/9780429198168

Dorier, J., & Maass, K. (2020). Inquiry-based mathematics education. In S. Lerman (Ed.), Encyclopedia of Mathematics Education (pp. 384–388). Springer. https://doi.org/10.1007/978-3-030-15789-0_176

Kidron, I., Artigue, M., Bosch, M., Dreyfus, T., Haspekian, M. (2014) Context, Milieu, and Media-Milieus Dialectic: A Case Study on Networking of AiC, TDS, and ATD, In Bikner-Ahsbahs & A., Prediger, S. (Eds.), *Networking of Theories as a Research Practice in Mathematics Education* (153–177). Springer. https://doi.org/10.1007/978-3-319-05389-9_10

Ogawa, T., & Miyakawa, T. (2025, February, 4–8). *Mathematics teacher practice in inquiry-based lessons: A case study in a Japanese upper secondary school* [Paper presentation]. The 14th Congress of the European Society for Research in Mathematics Education, Bozen-Bolzano, Italy.

Rasmussen, K. (2016). The direction and autonomy of interdisciplinary study and research paths in teacher education. *REDIMAT*, *5*(2), 158–179.

TSG4-1-8

TOWARDS REFASHIONING TEACHERS' TASK DESIGN IN TRIGONOMETRY: A PROFESSIONAL DEVELOPMENT MODEL

Jake B. Garnace
Ateneo de Manila University and University of Northern Philippines Philippines,
jake.garnace@student.ateneo.edu

Catherine P. Vistro-Yu
Ateneo de Manila University, Philippines, cvistro-yu@ateneo.edu

This study aims to model a professional development (PD) program to refashion teachers' task design in trigonometry. It investigates the praxeologies of teachers in developing and organizing their mathematical tasks, determines the priorities of teachers when designing mathematical tasks, and explores how these priorities influence the design of their tasks. Under the realm of the Anthropological Theory of Didactics, the study will highlight how didactical situations, and the socio-cultural contexts can make explicit the application of Variation Theory in designing tasks. This investigation also examines how teachers adapt and design mathematical tasks that fit specific educational contexts through pedagogical discourses. It is hoped that the proposed PD model will ultimately foster teachers' Profound Understanding of Fundamental Mathematics.

Keywords: mathematics teachers, professional development, Profound Understanding of Fundamental Mathematics, task design, trigonometry

Introduction

Over the past decades, the gap between theory and practice has been of primary interest to researchers (Cardona, 2023; Korthagen, 2007; Malara & Zan, 2008; Østergaard, 2013; Smith, 2000). da Ponte, et al (2023), strongly emphasized that the gap between research (knowledge derived from the work of mathematics education researchers) and practice (the work of mathematics teachers, curriculum developers, teacher educators, and many others) is particularly evident in mathematics classrooms as it is often a struggle to translate theoretical advancements in pedagogy into effective classroom practices.

Central to mathematics pedagogy are classroom practices such as task design. Designed tasks must accurately reflect mathematical principles and challenge students in ways that promote a deep understanding of mathematical concepts. Watson and Ohtani (2015) contend that task design remains a fundamental issue in research because the details and presentation of tasks are often described inadequately and without sufficient justification, notwithstanding whether the research takes place through clinical interviews or authentic classroom settings Clarke et al (2014) recognizes the importance of tasks in the socio-didactical tetrahedron where task design is situated at the heart of the triadic relationship between mathematics, students, and teachers (or the socio-didactical triangle). Task design synthesizes the socio-didactical triangle into actionable instructional activities. Therefore, teachers must be skillful in developing and organizing tasks in mathematical instruction.

Teacher quality (teaching experience, educational attainment, and communicative competence) is believed to influence student achievement in schools (Siagian & Artha, 2023). In a meta-analytic review of Lopez-Martin, et. al (2023), they found that student performance is affected by the characteristics and competencies of their teachers. Hou (2024) argued that the cornerstone to sustain and enhance the quality of mathematics education is the professional development of high school mathematics teachers, and highlighted the pivotal role that professional development trainings play to alleviate teachers' difficulties in adapting to the dynamic landscape of education.

These issues call for capacitating teachers with tools that will help them in refashioning their mathematics education practices in the classrooms, specifically task design. Researchers claim that teacher professional development is vital towards improving the quality of education in mathematics (Kwon, et al., 2017; Chiew & Lim, 2017). As Mercado and Cabuquin (2023) argued, exposing teachers to professional activities through a well-planned professional development program is essential as it contributes to their advancement, sense of responsibility to their profession, and abilities in assisting students to become independent learners

One of the promising theoretical frameworks in optimizing the expertise of teachers in designing tasks is Variation Theory (VT). Orgill (2012) states that VT is essentially a framework that aids in understanding how a learner comes to "see, understand, or experience a given phenomenon in a certain way" (p. 3391). VT suggests that for learners to experience a phenomenon in a specific way, they need to be aware of the critical aspects of that phenomenon and simultaneously focus on them. In addition, Marco and Palatnik (2024) claimed that varying tasks could help teachers supplement their knowledge of problem posing, thereby helping them compose or adjust their tasks and achieve their pedagogical goals. The researchers call for further studies to utilize VT as a theoretical framework for learning mathematics and investigate how adjustments made by teachers on problems they pose contribute to their learning process.

Most of the research on the use of Variation Theory as a tool for student learning focus on multiple graphical representations (Johnson, 2022), comparing fractions (Kiko, 2017), or dynamic geometry environment (Leung, 2003). Variation Theory has also been applied in improving the ability of teachers to identify levels of cognitive demands of tasks (Bodochi, 2020), enhancing the art of questioning among teachers (Mokotedi, 2023), and in supplementing the teaching of mathematics even when traditional methods are employed (Mhlolo, 2013). While these studies have focused on the teaching of algebra and geometry concepts, the current study uses VT as a tool in designing tasks in Trigonometry. Trigonometry is important in mathematics because it has its wide range of applications across multiple fields (Bornstein, 2020; Bekene Bedada & Machaba, 2022). Despite its importance in the field of mathematics, it remains a challenge for both teachers and learners. As Spangenberg (2021) argued, the insufficient pedagogical content knowledge of teachers causes them difficulty in teaching the subject, emphasizing the call for an all-inclusive understanding of trigonometry. In relation to the insufficiency mentioned, research have highlighted students' difficulties in learning trigonometry that include the following: having procedural knowledge but not conceptual mastery (Rohimah & Prabawanto, 2020), struggling to understand mathematical statements and choosing the correct trigonometric concept to solve the problems (Nanmumpuni & Retnawati, 2021), and misconceptions on angle and angle measure, trigonometric functions and inverse trigonometric functions and their graphs, trigonometric identities, and solutions of triangles (Ancheta, 2022). With these concerns, task design could pave a way for teachers to help students think more deeply about their learning (Taylor, 2018) by providing opportunities to engage in varied representations and problem-solving contexts and by encouraging conceptual mastery that address misconceptions.

It is upon these premises that the current study incorporates VT into professional development for designing tasks in trigonometry with the hope that it can help teachers develop a more nuanced understanding of mathematical concepts by highlighting different aspects and features. This study also delves into the pedagogical discourses (Sfard, 2008) of the teacher-participants during the PD program to investigate how they unpack their design of trigonometry tasks. By constantly practicing the art of teaching and engaging in professional discussions and collaborative exchange of ideas with colleagues, teachers will ultimately foster their Profound Understanding of Fundamental Mathematics. The current study explores how teachers develop and organize mathematical tasks, model a professional development program for teachers, as well as determine how multiple iterations of the professional development program impact teachers' task design and their profound understanding of fundamental mathematics.

Theoretical Bases

This study is anchored on Anthropological Theory of Didactics (ATD). Bosch and Gascon (2014) explain that the ATD offers a way to analyze the practices of teaching and learning, which comprises its components, how they change, and the conditions that shape them within the context of the educational setting. The ATD is used in analyzing the practices of teachers in the development and organization of tasks, as influenced by factors such as their institution or personal biases. In addition, a proposed professional development model will utilize Variation Theory (Marton & Booth, 1997; Marton & Tsui, 2004). Variation Theory will allow teachers to create tasks that present mathematical concepts through varied examples and contexts, highlighting the critical aspects of the object of learning. Given the complexities of task design, commognitive theory (Sfard, 2008) will be used to understand the engagement of teachers in designing mathematics tasks through pedagogical discourses and the use of discursive elements. This study also considers the notion of Profound Understanding of Fundamental Mathematics (Ma, 1999) as the ultimate end-goal in designing the professional development program.

Methodology

This study adopts a qualitative approach using design-based research (Amiel and Reeves, 2008) as the methodological framework, and case study (Creswell, 2013) as the research design. Key-informant interviews and focus-group discussions will be conducted to ground the modeling of the professional development program for refashioning teachers' task design in trigonometry. There are two iterations in the study. In the first iteration, four teachers from the selected schools in Ilocos Sur will be involved. The second iteration will aim for twelve teacher-participants from selected schools in Marikina City. The PD program will be subjected to experts' validation and subsequently be implemented for the first iteration. Microanalyses will be done in between the PD sessions and a macroanalysis will be conducted at the end of each iteration. Thematic analysis (Clarke & Braun, 2017) will be used to identify, analyze and interpret themes in qualitative data from the interviews and FGDs.

Anticipated End Results of the Study

Several studies have shown that using Variation Theory indeed serves as a supplemental tool in improving student learning outcomes (Hino, 2017; Johnson, 2022). What is unknown is its role in assisting teachers refashion their task design and in unlocking their profound understanding of fun-

damental mathematics (Ma, 1999). This study will contribute to the greater theory in mathematics education on how a Variation Theory-informed professional development program for mathematics teachers on task design will aim to understand the teachers' processes, principles, and purposes of experiencing trigonometry tasks. Moreover, it is anticipated that the results of this study will unpack critical features of a PD program that will provide avenues for teachers to engage in a professional learning community of task designers, providing a space to continuously refine and improve their task design practices. By modelling a PD program tailored for the specific needs of teachers, newer perspectives on strategies and tools in task design will be introduced and teachers' ways of experiencing tasks will be broadened and refashioned, allowing opportunities for reflection, innovation, and continued professional growth.

References

Amiel, T., & Reeves, T. C. (2008). Design-based research and educational technology: rethinking technology and the research agenda. *Educational Technology & Society, 11*(4), 29–40.

Ancheta, C. D. (2022). An error analysis of students' misconceptions and skill deficits in pre-calculus subjects. *Journal for Educators, Teachers, and Trainers, (13)*5, 283-295. https://doi:10.47750/jett.2022.13.05.026

Bekene Bedada, T., & Machaba, F. (2022). The effect of GeoGebra on STEM students learning trigonometric functions. *Cogent Education, 9*(1), 5-18. https://doi.org/10.1080/2331186X.2022.2034240

Bodochi, M. (2020). Educating awareness: Using Variation Theory in task design (Doctoral dissertation, University of Oxford).

Bornstein, N. 2020. Teaching transformations of trigonometric functions with technology. Journal of Interactive Media in Education, 2020(1): 15, pp. 1–9. DOI: https://doi.org/10.5334/jime.503

Bosch, M., & Gascón, J. (2014). Introduction to the Anthropological Theory of the Didactic (ATD). *Networking of theories as a research practice in mathematics education*, 67-83.

Cardona, R. S. (2023). Bridging the gap between theory and practice: the research productivity and utilization of research outcomes among secondary mathematics teachers. *Mathematics Teaching Research Journal*, *15*(4), 164-183.

Chiew, C. M., & Lim, C. S. (2017). Profiling mathematics teacher professional development in Malaysia. *Professional Development of Mathematics Teachers: An Asian Perspective*, 77-88.

Clarke, D., Strømskag, H., Johnson, H. L., Bikner-Ahsbahs, A., & Gardner, K. (2014). Mathematical tasks and the student. In *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education.*

Clarke, V., & Braun, V. (2017). Thematic analysis. The journal of positive psychology, 12(3), 297-298.

Cresswell, J. (2013). Qualitative inquiry & research design: Choosing among five approaches. California, USA: SAGE Publications, Inc.

da Ponte, J. P., Aguilar, M. S., Vondrová, N., Zehetmeier, S., Seleznyov, S., & van Bommel, J. (2023, July). Bridging the research-practice gap–a panel report. In *Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)*. Alfréd Rényi Institute of Mathematics; ERME.

Hino, K. (2017). Improving teaching through variation: A Japanese perspective. In Teaching and Learning Mathematics through Variation (pp. 301-320). Brill.

Hou, Y. (2024). Research on the Approaches and Strategies for the Professional Development of High School Mathematics Teachers. *The Educational Review, USA, 8*(6), 857-862.

Johnson, H. L. (2022). Task design for graphs: Rethink multiple representations with Variation Theory. *Mathematical Thinking and Learning, 24*(2), 91-98.

Korthagen, F. A. J. (2007). The gap between research and practice revisited. *Educational Research and Evaluation, 13*(3), 303–310. https://doi.org/10.1080/13803610701640235

Kwon, O. N., Park, J. H., Park, J. S., & Park, J. (2017). A community-based, practice-oriented teacher professional development program: changes in teaching culture in Korea. *Professional Development of Mathematics Teachers: An Asian Perspective*, 141-157.

Leung, A. (2003). Dynamic Geometry and The Theory of Variation. *International Group for the Psychology of Mathematics Education, 3*, 197-204.

López-Martín, E., Gutiérrez-de-Rozas, B., González-Benito, A. M., & Expósito-Casas, E. (2023). Why do teachers matter? A meta-analytic review of how teacher characteristics and competencies affect students' academic achievement. *International Journal of Educational Research, 120*(2023), 1-14.

Ma, L. (1999). Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Mahwah, NJ: Lawrence Erlbaum.

Malara, N., & Zan, R. (2008). The complex interplay between theory in mathematics education and teacher's practice: Reflections and examples. *Handbook of international research in mathematics education*, 535-560.

Marco, N., & Palatnik, A. (2024). Teachers pose and design context-based mathematics tasks: what can be learned from product evolution?. *Educational Studies in Mathematics, 115*(2), 223-246.

Marton F., and Booth, S., (1997). *Learning and Awareness.* Mahwah, NJ: Lawrence Erlbaum Associates, Inc

Marton, F. & Tsui, A. B. M. (2004). Classroom discourse and the space of learning, Mahwah, NJ, Lawrence Erlbaum Associates

Mercado, H. A. R., & Cabuquin, J. C. (2023). Professional development vis-a-vis teaching effectiveness of public secondary mathematics teachers. *Jurnal Pendidikan Progresif, 13*(2), 568-579.

Mhlolo, M. (2013). The merits of teaching mathematics with variation. *Pythagoras, 34*(2), 1-8.

Mokotedi, L. B. (2023). Teachers' questions matter: exploring the attributes of mathematics teachers' questions within the lesson study context (Doctoral dissertation, University of Pretoria (South Africa).

Nanmumpuni, H. P., & Retnawati, H. (2021). Analysis of senior high school student's difficulty in resolving trigonometry conceptual problems. *Journal of Physics: Conference Series, (1776)*1, 1-8. https://doi:10.1088/1742-6596/1776/1/012012

Orgill, M. (2012). Variation Theory. Encyclopedia of the sciences of learning, 3391-3393.

Østergaard, K. (2013, April). Theory and practice in mathematics teacher education. In *Proceedings of the IVth international congress on the anthropological theory of didactics (ATD)* (pp. 1-22).

Rohimah, S. M., & Prabawanto, S. (2020). Students' difficulties in solving trigonometric equations and identities. *Journal of Physics: Conference Series, 1521(3)*, 1-6. https://doi:10.1088/1742-6596/1521/3/032002

Sfard, A. (2008). Thinking as communicating: Human development, the growth of discourses and mathematizing. Cambridge University Press.

Siagian, R., & Artha, B. (2023). The influence of teacher quality, teacher characteristics, and teaching experience on school quality that impacts student performance in bandung: evidence from high school. *Jurnal Pendidikan West Science*, *1*(03), 184-192.

Smith, T. (2000). Bridging the research-practice gap: Developing a pedagogical framework that promotes mathematical thinking and understanding. *Mathematics Teacher Education and Development*, *2*(1), 4-16.

Taylor, J. (2018). Using variation to develop deep conceptual understanding of trigonometry [Master's thesis]. University of Oxford.

Watson, A., & Ohtani, M. (2015). Task design in mathematics education: An ICMI study 22 (p. 339). Springer Nature.

TSG4-1-9

REFLECTING IN-SERVICE ELEMENTARY TEACHERS' MATHEMATICAL KNOWLEDGE FROM TEACHER PROFESSIONAL EDUCATION PROGRAM

Zetra Hainul Putra
Universitas Riau, Indonesia, zetra.hainul.putra@lecturer.unri.ac.id

Shindy Lestari
Yogyakarta State University, Indonesia, shindylestari.2022@student.uny.ac.id

Hartono Tjoe
The Pennsylvania State University, U.S.A., hht1@psu.edu

Ayman Aljarrah
Acadia University, Canada, ayman.aljarrah@acadia.ca

A teacher professional education program is an educational program to prepare 21st century teaching professionals. In Indonesia, it is called as PPG aiming to preparing pre-service and in-service teachers to enhance skills and teaching competences. The present study aims to explore elementary teachers' mathematical knowledge through the end of the program. It uses a case study method to investigate teachers' mathematical knowledge through the framework of praxeology from the anthropological theory of the didactic. The findings indicate that teachers tend to focus on the practical block, namely a type of task given to students and procedural techniques used to solve such tasks. Meanwhile, the logos part, technologies and a theory, to justify the praxis often does not appear in the teaching process. Thus, teacher professional education programs in Indonesia still need to reform some courses in order to support teachers' mathematical praxeologies.

Keywords: mathematical knowledge, praxeologies, teacher professional education program

Introduction

Teacher quality is an essential factor for student success in the 21st century. It is determined by teacher educational programs prepared by institutions to support teacher competences. In Indonesia, the government presently supports teacher competences through a teacher professional education programs *("Pendidikan Profesi Guru* (PPG) in Indonesian terminology), which aim to prepare professional teachers (Chang et al., 2024). These programs are divided into two tracks, one for pre-service teachers who have never taught before and one for in-service teachers who have had several years of teaching experience. Therefore, it might be assumed that in-service teachers have sufficient content and pedagogical knowledge because they have been involved in teaching experiences at school.

In the present study, we would like to focus on a specific knowledge type, namely teachers' mathematical knowledge. This knowledge relates not only to teachers' mathematics proficiency in calcu-

lations, but also to teachers' competences in explaining common rules and mathematical procedures to students, evaluating their answers and justifications, and visually representing mathematical concepts and procedures using pictures or diagrams. Especially at the elementary school level, many studies show a gap in teachers' mathematical knowledge (Putra, 2019). Teachers tend to focus on students' procedural knowledge rather than conceptual knowledge.

A teachers' mathematical knowledge has a significant impact on the teaching process in the classroom and students' success in learning mathematics. A study conducted by Hill et al. (2005) has indicated that teachers' lack of mathematical knowledge creates difficulties in teaching mathematics and has an impact on a student's ability to understand concepts in mathematics (Kaufmann & Ryve, 2023). In many cases, teachers only actively supply information and use one-way interactions in mathematics learning (Ayuwanti et al., 2021). The teachers focus on procedural methods used by students to solve mathematical problems without paying attention to the mathematical reasoning behind those methods. As a result, students have a propensity to memorize and think mechanically, which indicates low proficiency.

The PPG programs could prepare teachers to address these difficulties in their teaching practice, and it could support transformative and exploratory mathematics (Su et al., 2023). Teachers' needs and experiences must be taken into consideration when creating PPG programs that impact their mathematical knowledge and teaching practices. Therefore, the present study aims to evaluate and reflect on PPG programs' effect on elementary teachers' mathematical knowledge. We will investigate teachers' knowledge through their lesson plan design and teaching practices.

Teachers' Mathematical Knowledge

Teachers' mathematical knowledge can be understood from the anthropological theory of the didactic (ATD) (Chevallard, 2006). ATD understands mathematics as a human activity that can be evaluated from a praxeology. A praxeology consists of "praxis" (a "practical block") and "logos" (a "theoretical block"). The practical block is unified by a type of task (T) and at least one technique (τ) to solve the task. The theoretical block consists of a technology (θ) to explain, prove, or even design the technique, and the theory (Θ) justifying the underlying technology (Putra, 2019). For example, one type of task is addition of two fractions, an example being to add $\frac{1}{2}+\frac{1}{3}$. To solve such a task, techniques can be applied by students such as transforming the two fractions into equivalent fractions having a common denominator. To go beyond the technique alone, a teacher needs to ask students for clarification and explanation, so the students can come to some technologies and possibly a theory. A technology can be a reasoning behind the need to change the fractions into a common denominator, such as two fractions needing to have a common unit, and equivalent fractions can be a theory to justify it. To implement this process, a teacher needs to orchestrate the learning process well and to engage the classroom into a discussion.

Teachers' mathematical knowledge is not only about mathematical praxeology (MP) but also didactic praxeology (DP). An example of didactic types of task in relation to the mathematical task such as to teach pupils how to add two fractions (Putra, 2019). Then, didactic technique is a strategy used by a teacher to support students' learning adding two fractions. It depends on the teacher's mathematical technique, such as using an algorithm to solve the task or use diagram representation. Meanwhile, an argumentation behind a chosen technique is the technological discourse. For instance, the teacher uses the algorithm because she believes that students need to master algorithm in order to solve the task practically. Thus, a theory to justify it is a direct instructional learning (Putra, 2019).

Methods

This study applies a case study method to evaluate in-service elementary teachers' mathematical knowledge. This study focuses on an in-service elementary teacher (EL, pseudonym) who took a performance test in order to receive teacher professional certificates from PPG programs in a teacher institution in Pekanbaru, Riau province. In that test, the teacher has to submit 3 documents, a lesson plan, a video of teaching, and a case study report, to be evaluated by teacher educators. Meanwhile, the present study focuses on analyzing the teacher' mathematical knowledge from the lesson plan and her teaching practice.

Before taking the test, in-service elementary teachers have to taken asynchronous courses provided by the teacher institution. The program runs for 6 months, and the contents of the courses are more on general philosophy and pedagogy. Therefore, there is no such a specific course on teaching and learning mathematics. Mostly, the participants choose mathematics contents when designing an instructional learning based on their willingness.

The praxeology is used as a framework to analyze the type of tasks and techniques appearing in the video recording and supported by the lesson plan. Meanwhile, the logos part is evaluated from interactions between a teacher and students, as well as among the students themselves. Through the data analysis, we reflect teachers' mathematical knowledge from the PPG program in order to understand and evaluate the effectiveness of the program.

Results

Praxeological Analysis of Lesson Plan

The first teacher, EL was a first-grade teacher who taught subtraction up to 20. The aim of this lesson was to help students be able to subtract numbers using concrete objects up to 10. The teacher gave a type of task based on a diagram representation. It can be stated as follows:

$T_{1,EL}$: represent the diagram representations into a symbolic representation of subtraction.

This type of task was elaborated into 5 mathematical tasks, and one of them is given as an example of the task. As presented in Figure 1, the diagram representation can be used by the students to solve the problems, and also the students can follow the given example to solve the other problems.

Praxeological Anaylsis of Teaching Experiences

EL started the lesson by asking how many students were in the class. The students gave a correct answer of 15 students in the class. Some students used the mathematical technique of counting one-by-one to find the answers. Next, EL provoked the students with a mathematical task of subtraction. She asked students if 3 students did not come, how many students would be in the class. The students gave the correct answer of 12 students. They were able to use number sense to find the correct answer.

Before starting the lesson of subtraction, EL recalled the previous lesson about addition. Additionally, EL also provoked students to use sticks and fingers as a learning tool to learn addition and subtraction. However, EL still emphasized counting one-by-one. For instance, EL asked students to count how many fingers they have, and then a student started counting one-by-one and found that the answer was 10. After that, EL gave the following mathematical task:

EL : Bilqis has 8 cakes, and she ate 3 cakes. How many cakes left?

All students gave the correct answer of 5. EL did not elaborate on this task or how students came to a correct answer.

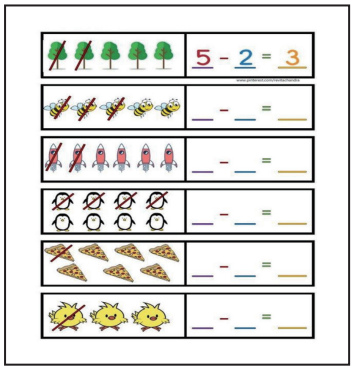


Figure 1. Student worksheet of mathematical tasks for subtraction up to 10

The teacher continued the lesson by presenting a video to the students. The video introduced a child named Edo who helped his parent to pick oranges in their farm. There were six oranges, and 3 of them were rotten. The video presented the problem of determining how many oranges were good via a mathematical technique of counting one-by-one. First, Edo counted all the oranges, then counted the rotten oranges, and then finally the remaining oranges. However, we observed that the students actually knew the answer was 3 without counting one-by-one anymore. After the video was finished, EL asked the students about what they learned. The students were able to answer that the video was about subtracting numbers. They were also able to give answers to the mathematical task given by the teacher from the video regarding 7 eggs, 3 of which were broken, leaving 4 intact eggs left. EL did not elaborate on how students came to their answers, but we observe that the students demonstrated good number sense regarding numbers up to 10.

EL continued the learning activities by giving the worksheet (Figure 1) to the students in groups of 5. EL moved from one group to another group to guide the students to find the answers. Then, each group presented their work and no questions from other groups. Therefore, we cannot observe what techniques were used by students to find the answers. Furthermore, we did not observe a discussion of logos by the teacher and students during the lesson. EL mostly focused on the finding the correct answers using counting techniques.

Discussion and Concluding Remarks

The present study only focuses on a praxeological analysis of an elementary teacher. Therefore, we cannot come to a general conclusion regarding teachers' mathematical knowledge from the PPG program. However, we can reflect from this study how should we support teachers for having sufficient praxeology in teaching mathematics in primary school for the future.

This study showed that the teacher was only concerned with the practical block and did not elaborate upon students' thinking and reasoning to support the logos block. This finding has been confirmed by several studies that concluded that teachers in Indonesian do not have sufficient mathematical and didactic knowledge (Putra, 2019). The sustainability of the PPG program tends to be merely "*lip service*", due to the absence of evaluation to determine whether a teacher is professional

or not. They mostly focus on instructing students with procedural techniques that rely on algorithm and counting strategies. Teachers' incomplete mathematical knowledge has an impact on the lack of quality of learning plans and mathematics teaching. Therefore, it is important to reflect on the Teacher Professional Education program activities to complete praxeology as a solution to the gap in teachers' mathematical knowledge in order to support students' mathematics learning needs.

References

Ayuwanti, I., Marsigit, M., & Siswoyo, D. (2021). Teacher-student interaction in mathematics learning. *International Journal of Evaluationan and Research in Education*, *10*(2), 660667. https://doi.org/http://doi.org/10.11591/ijere.v10i2.21184

Chang, C.-F., Annisa, N., & Chen, K.-Z. (2024). Pre-service teacher professional education program (PPG) and Indonesian science teachers' TPACK development: A career-path comparative study. *Education and Information Technologies*. https://doi.org/10.1007/s10639-024-13160-6

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.), *Proceedings of the IV Congress of the European Society for Research in Mathematics Education* (pp. 21–30). La Pensée Sauvage. https://doi.org/10.1093/0198248601.003.0007

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement. *American Educational Research Journal*, *42*(2), 371–406. https://doi.org/10.3102/00028312042002371

Kaufmann, O. T., & Ryve, A. (2023). Teachers' framing of students' difficulties in mathematics learning in collegial discussions. *Scandinavian Journal of Educational Research*, *67*(7), 1069–1085. https://doi.org/10.1080/00313831.2022.2115134

Putra, Z. H. (2019). Danish pre-service teachers' mathematical and didactical knowledge of operations with rational numbers. *International Electronic Journal of Mathematics Education*, *15*(1), 619–632. https://doi.org/10.29333/iejme/5775

Su, C. S., Díaz-Levicoy, D., Vásquez, C., & Hsu, C. C. (2023). Sustainable Development Education for Training and Service Teachers Teaching Mathematics: A Systematic Review. *Sustainability*, *15*(10), 8435. https://doi.org/10.3390/su15108435

TSG4-1-10

THE DEVELOPMENT OF DIDACTIC PRAXEOLOGIES IN LESSON DESIGN THROUGH SOCIAL RESOURCE INTERACTIONS: EVIDENCE FROM TWO NOVICE MATHEMATICS TEACHERS

Xingfeng Huang
Shanghai Normal University, China, xfhuang@shnu.edu.cn

Jiawen Zhu
Shanghai Normal University, China, JiawenSHNU@163.com

Luc Trouche
École Normale Supérieure de Lyon, France, luc.trouche@ens-lyon.fr

This study examines how teacher collaboration influenced novice mathematics teachers' professional development during a Shanghai project to create digital resources for global primary mathematics education. Using the dual frameworks of the Documentation Approach to Didactics (DAD) and the Anthropological Theory of the Didactic (ATD), we analyzed how two teachers' interactions with social resources shaped their didactic praxeologies in lesson design. Data included teaching resources, lesson materials, and four interviews over seven months. Findings show that social resources and culturally dominant didactic praxeologies effectively catalyzed changes in teachers' praxis, but transforming their logos in lessons was more challenging. From Lakatos's epistemological perspective, neither teacher developed substantial praxeologies to foster student engagement in mathematical proofs and refutations. While both gained techniques through instrumentalization of social resources, their development, shaped by Chinese didactic praxeology, was hindered by limited instrumentation, affecting the transformation of their logos.

Keywords: Documentation Approach to Didactics, Anthropological Theory of the Didactic, Social resources, Cultural didactic praxeology·

Introduction

Teaching resources play a critical role in novice mathematics teachers' professional development (Gueudet & Trouche, 2009; Huang et al., 2023; Pepin et al., 2017). This study investigates how novice teachers develop their teaching knowledge by using and transforming resources. While prior research has often studied the practical and cognitive dimensions of this process separately (Adler, 2000; Brown, 2009), this study employs a dual theoretical lens to explore their interconnection.

In 2020, UNESCO-TEC launched a project in Shanghai to design electronic resources for global primary mathematics teaching. This initiative highlighted the challenge of analyzing how teachers' resource adaptation and professional knowledge development intersect. Traditional approaches to resource use (Remillard, 2005) often fail to capture this complex relationship. To address this, we combine the Documentation Approach to Didactics (DAD) (Gueudet & Trouche, 2009) and the Anthropological Theory of the Didactic (ATD) (Chevallard, 2019). DAD focuses on resource adaptation and implementation, while ATD examines the interplay of teaching practices and theoretical

knowledge construction. Together, these frameworks enable a nuanced exploration of how novice teachers develop didactic praxeologies through resource engagement.

This study examines two novice teachers participating in the UNESCO-TEC project, investigating how their interactions with resources facilitated professional growth and shedding light on the relationship between teaching resources and professional knowledge development.

Theoretical Framework

Documentational Approach to Didactics (DAD)

DAD emphasizes how teachers interact with resources, focusing on instrumentalization (adapting resources for teaching) and instrumentation (how resources influence teachers' thinking and actions). These processes drive documentation genesis, through which teachers develop professional knowledge. Collaborative engagement transforms social resources into opportunities for reflection and innovation, enhancing teaching techniques (Gueudet & Trouche, 2009). While， the DAD does not reveal how teachers develop knowledge during the processes of instrumentalization and instrumentation.

Anthropological Theory of the Didactic (ATD)

Teaching and learning are social and cultural practices embedded in institutional settings (Bosch & Gascón, 2014; Chevallard, 2019). The Anthropological Theory of the Didactic (ATD) highlights the dialectical relationship between individuals and institutions, where individuals shape and are shaped by institutions, influencing the development of mathematical and didactic knowledge.

In the ATD framework, the media-milieu dialectic is key to knowledge construction (Chevallard & Bosch, 2019). Media refers to any system that provides statements (e.g., textbooks, teachers, websites, or even peers offering ideas). These statements, although possibly credible, are treated as conjectures needing verification. Conversely, milieu is an "adidactic" setting that responds to a learner's actions without trying to teach, thus inviting exploration and feedback. ATD uses praxeology to analyze practices, comprising praxis (tasks and techniques) and logos (technology and theory) (Chevallard et al., 2015). Mathematical praxeology focuses on problem-solving in mathematics, including tasks, techniques, technology, and theory (Chevallard & Sensevy, 2014), while didactic praxeology examines how teachers facilitate mathematical learning through didactic tasks, techniques, technology, and theory (Bosch & Gascón, 2014; Chevallard, 2019). These praxeologies are shaped by and embedded within institutions, reflecting distinct cultural, historical, and social conditions (Artigue & Winsløw, 2010; Chevallard, 2019). This is particularly visible in Chinese mathematics education as *Chinese Didactic Praxeology* (Fan et al., 2004, 2015; Huang & Li, 2017).

Integrating ATD and DAD

The DAD emphasizes how teachers interact with resources, shaping their professional knowledge (Gueudet & Trouche, 2009), while the ATD structures the analysis of mathematical and didactic knowledge embedded in these interactions (Chevallard, 1999). Theoretical networking connects complementary perspectives to address educational complexities (Artigue, 2019; Kidron et al., 2018), with frameworks like DAD and ATD sharing a focus on how knowledge transforms through practice and resource interaction. Prior research (Chevallard & Bosch, 2020a, b; Trouche et al., 2020) highlights the epistemological and didactic dimensions of teaching, extending to teachers' documentation as a key growth driver. This integrated approach enables nuanced analysis via the

media-milieu dialectic in the ATD, aligning with instrumentalization and instrumentation in the DAD. Instrumentalization involves appropriating institutionalized praxeologies, while instrumentation refines and transforms resources into a supportive milieu, occurring within broader institutional and social contexts. The concept of schemes of usage bridges the DAD and ATD, analyzing how teachers' documentation evolves. Schemes, as cognitive structures (Vergnaud, 1998), align with the ATD's praxis and logos blocks, representing practical and theoretical dimensions. By tracing the evolution of didactic praxeologies, we can understand how teachers' professional knowledge develops through social resource interactions.

Research Questions

This study seeks to address the following research questions:

RQ1: What didactic praxeologies do novice teachers design and develop through interactions with social resources?

RQ2: What conditions and constraints influence novice teachers in designing didac tic praxeologies when supported by social resources?

Methodology

This study examines two novice teachers, Lichun and Qiufeng, with strong educational backgrounds and under five years of experience, as they develop Grade 5 math e-resources. From August 2022 to February 2023, data collected included: ①Lesson plans (initial and revised), ②PowerPoint slides and materials, and ③Four interviews tracking changes in praxis and logos. The ATD framework analyzes knowledge organization, transformation, and dissemination in teaching. (Chevallard, 1985; Bosch & Gascón, 2006). Using Lakatos's perspective, the study focuses on iterative knowledge refinement through conjectures, refutations, and revisions (Lakatos, 1976). Analysis occurred in four phases. Phase 1 explored interviews to understand teachers' backgrounds and contextual influences. Phases 2-4 analyzed lesson materials (e.g., plans, slides) and interview data, focusing on mathematical praxis (tasks, techniques, and content alignment) and didactic techniques (Chevallard, 1999; Barbé et al., 2005). Interviews revealed reasoning behind material modifications, tracing iterative refinement of didactic praxeologies. Teachers' interactions with social resources (media and milieu) were analyzed through instrumentation and instrumentalization (Trouche, 2004; Drijvers et al., 2010). The analysis integrated cultural didactic praxeology and Lakatos's model, showing how media and milieu shaped teachers' instructional strategies and the interplay between mathematical and didactic practices. This framework situates teaching strategies within contextual factors and social interactions, highlighting their iterative evolution. Collaborative coding ensured rigor and reliability, with coders resolving discrepancies through discussion to maintain consistency and validity.

Results

Lichun refined her mathematical techniques through interactions with curriculum resources and colleagues. For example, she revised her geometric representations of decimal multiplication to address feedback on mathematical rigor. However, these changes were constrained by institutional norms, such as the requirement to build on students' existing knowledge. While Lichun's collaboration with colleagues demonstrated elements of proof and refutation, this approach was not fully

translated into her classroom practice. From DAD perspective, Lichun's interactions with social resources facilitated instrumentalization, as she adapted institutional techniques to her lessons. However, instrumentation was limited, as her theoretical understanding did not evolve significantly.

Qiufeng primarily relied on institutional resources, such as textbooks and online videos, making minimal modifications to her lesson plans. While she incorporated new techniques, such as unit conversion and decimal composition, these changes remained procedural and aligned closely with institutional norms. Collaborative feedback prompted task sequencing changes and the introduction of new techniques, but Qiufeng's engagement with resources remained largely at the level of instrumentalization. From an ATD perspective, Qiufeng's didactic praxeology was shaped by cultural and institutional constraints, limiting her ability to innovate beyond established practices.

In response to RQ1: Drawing on the experiences of the two novice teachers, the findings show that social resources functioning as media or milieu tend to enhance teaching praxis primarily by refining technical aspects (e.g., geometric representations, decimal decom position, or lesson sequencing). While these resources introduce limited opportunities for reflecting on the rationale (logos) behind these techniques, the teachers' overarching didactic theories remain relatively intact, reflecting what Lakatos terms "monster-barring" (1976, pp. 18–20). Even when external suggestions prompt small-scale adjustments to tasks or classroom structures, these modifications typically remain surface-level. Docu mentation work supported by social resources concentrates on creating, revising, and organizing lesson plans and slides, showing instrumentalization (selective integration of institutional requirements, such as variation task and progressive complexity) and only partial instrumentation (adopting techniques to address issues through resource-based adaptations without achieving the theoretical depth required to reshape their designed didactic praxis fundamentally). Thus, although the teachers refine tasks and incorporate new elements, they rarely move beyond the constraints of familiar frameworks.

In response to RQ2: From an ecological perspective, multiple conditions and constraints shape the teachers' designed didactic praxeologies in the Chinese context where culturally dominant didactic praxeologies prevail. Firstly, textbooks and fixed teaching progressions exercise considerable authority, prompting teachers to stick closely to established content and pacing, thereby limiting deeper forms of inquiry or "refutation–revision." Secondary, collaborative processes offer channels for sharing experiences and refining teaching sequences; however, these same processes prioritize "collective synergy" and "incremental improvements," leaving little room for radical critique or full-scale reconfiguration of existing didactic praxeologies. This ecosystem encourages modest, localized adaptations rather than transformative shifts, continuously shaping how teachers conceptualize and enact their work.

Discussion

Findings reveal that while both teachers engaged with resources, their theoretical development (logos) remained limited due to shallow instrumentation. Lichun's adaptations enriched her praxis but lacked deep theoretical grounding, whereas Qiufeng's adherence to institutional norms restricted critical reflection and innovation. Culturally dominant praxeologies provided structure but constrained opportunities for teaching proof and refutation. Professional development must prioritize deep instrumentation, balancing institutional requirements with innovation. By fostering reflective practices and strengthening theoretical foundations, teachers can navigate cultural and institutional constraints, enabling meaningful adaptation and pedagogical growth.

Conclusion

The comparative analysis of the two cases highlights both the complexity of developing didactic praxeology and the pivotal role of instrumentation in fostering teachers' professional growth. The findings emphasize that a key challenge lies in how teachers engage with social resources and transform institutional requirements into meaningful tools that enhance instructional practice.

References

Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education, 3*(3), 205–224. https://doi.org/10.1023/A:1009903206236

Artigue, M. (2019). Reflecting on a theoretical approach from a networking perspective: The case of the documentational approach to didactics. In L. Trouche, G. Gueudet, B. Pepin (Eds.), *The 'resource' approach to mathematics education*. Advances in mathematics education (pp. 89–118). Springer. https://doi.org/10.1007/978-3-030-20393-1_ 5

Artigue, M., & Winsløw, C. (2010). International comparative studies on mathematics education: A view point from the anthropological theory of didactics. *Recherches en didactique des mathématiques*, 30(1), 47–82. Retrieved April 6, 2025, from https://revue-rdm.com/2010/international-comparative studies/

Barbé, J., Bosch, M., Espinoza, L., & Gascón, J. (2005). Didactic restrictions on the teacher's practice: The case of limits of functions in Spanish high schools. *Educational Studies in Mathematics*, *59*(1–3), 235–268.

Bosch, M., & Gascón, J. (2006). Twenty-five years of the didactic transposition. *ICMI bulletin*, *58*(58), 51-65.

Bosch, M., & Gascón, J. (2014). Introduction to the anthropological theory of the didactic (ATD). In A. Bikner-Ahsbahs & S. Prediger (Eds.), *networking of theories as a research practice in mathematics education* (pp. 67–83). Springer. https://link.springer.com/chapter/10.1007/978-3-319-05389-9_5

Brown, M. W. (2009). The teacher-tool relationship: Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 17–36). Routledge.

Chevallard, Y. (1985). La transposition didactique: Du savoir savant au savoir enseigné. *Recherches en Didactique des Mathématiques*, 4(2), 131–170.

Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en Didactique des Mathématiques*, *19*(2), 221–266.

Chevallard, Y. (2019). Introducing the anthropological theory of the didactic, an attempt at a principled approach. *Hiroshima Journal of Mathematics Education, 12*, 71-114. https://doi.org/10.24529/hjme.1205

Chevallard, Y., & Bosch, M. (2020a). Anthropological theory of the didactic (ATD). In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 53–61). Springer. https://doi.org/10.1007/978-3-03015789-0_100034

Chevallard, Y., & Bosch, M. (2020b). Didactic transposition in mathematics education. In S. Lerman (Ed.), Encyclopedia of mathematics education (pp. 214–218). Springer. https://doi.org/10.1007/978-3-030-15789-0_48

Chevallard, Y., & Sensevy, G. (2014). Anthropological approaches in mathematics education, French perspectives. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 38-43). Springer. https://link.springer.com/referenceworkentry/10.1007/978-94-007-4978-8_9

Chevallard, Y., Bosch, M., & Kim, S. (2015). What is a theory according to the anthropological theory of the didactic? In K. Krainer & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (CERME9, 4-8 February 2015) (pp. 2614-2620). Charles University in Prague, Faculty of Education and ERME.

Drijvers, P., Godino, J. D., Font, V., & Trouche, L. (2010). Theoretical frameworks and research on digital technologies in mathematics education. *Mathematics Education Research Journal*, *22*(2), 1–17.

Fan, L., Wong, N. Y., Cai, J., & Li, S. (Eds.). (2004). *How Chinese learn mathematics: Perspectives from insiders*. World Scientific. https://doi.org/10.1142/5629

Fan, L., Wong, N. Y., Cai, J., & Li, S. (Eds.). (2015). *How Chinese teach mathematics: Perspectives from insiders*. World Scientific. https://doi.org/10.1142/8542

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics, 71*(3), 199–218. https://doi.org/10.1007/s10649-008-9159-8

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics, 71*(3), 199–218. https://doi.org/10.1007/s10649-008-9159-8

Huang, R., & Li, Y. (Eds.). (2017). *Teaching and learning mathematics through variation: Confucian heritage meets western theories*. SensePublishers. https://doi.org/10.1007/978-94-6300-782-5_1

Huang, X., Huang, R., & Trouche, L. (2023). Teachers' learning from addressing the challenges of online teaching in a time of pandemic: A case in Shanghai. *Educational Studies in Mathematics*, 112(1), 103–121. https://doi.org/10.1007/s10649-022-10172-2

Kidron, I., Bosch, M., & Monaghan, J. (2018). Theory usage and theoretical networking in mathemat ics education research. *ZDM-Mathematics Education*, 50(4), 529–541. https://doi.org/10.1007/ s11858-018-0947-3

Lakatos, I. (1976). *Proofs and Refutations: The Logic of Mathematical Discovery*. Cambridge University Press.

Pepin, B., Gueudet, G., & Trouche, L. (2017). Refining teacher design capacity: Mathematics teachers' interactions with digital curriculum resources. *ZDM Mathematics Education, 49*(5), 799-812. https://doi.org/10.1007/s11858-017-0870-8

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of educational research*, *75*(2), 211-246. https://doi.org/10.3102/00346543075002211

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning*, *9*(3), 281–307.

Vergnaud, G. (1998). Toward a cognitive theory of practice. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics education as a research domain: A search for identity* (pp. 227–241). Springer.

TSG4-1-11

A STUDY ON INSTRUCTIONAL REPRESENTATIONS IN DIFFERENT TEACHING SCENARIOS: INTRODUCING TEXT MINING TECHNIQUES

Ling-Yi Huang
National Taipei University of Education, Taiwan, lilith28lilith28@gmail.com

Kuan-Yu Chen
National Taipei University of Education, Taiwan, tonny556123@gmail.com

Chia-Jui Hsieh
National Taipei University of Education, Taiwan, paris@tea.ntue.edu.tw

This study used a questionnaire to collect responses from 107 pre-service elementary school teachers facing authentic mathematics teaching scenarios. Combining content analysis with automated word frequency analysis (text mining), we examined the types of representations employed. Our findings classify these representations into three categories: Concrete Visual Representations (CVR), Classroom Utilization Representations (CUR), and Formal Abstract Representations (FAR). CVR were found to effectively facilitate students' conceptual understanding and stimulate interest, while narrative-based representations, although engaging, offered limited support for comprehension. Moreover, significant differences emerged based on academic background; teachers with mathematics expertise showed a lower preference for CUR and FAR. Text mining of teacher narratives further revealed distinct keyword patterns, providing new insights into the use of instructional representations in elementary mathematics teaching.

Keywords: instructional representations, mathematics teaching scenarios, text mining

Background and Aims

Teachers' decisions in instructional settings are guided by their internal knowledge structures and experiences. Prior studies have emphasized the critical role of representations in mathematics learning and problem solving (Goldin & Kaput, 1996; Kaput, 1991). In mathematics instruction, the selection of different forms—such as concrete visual images, real-life examples, symbols, and abstract formulations—directly impacts students' conceptual understanding and motivation. Given the importance of representations for effective communication and knowledge transmission, this study focuses on pre-service elementary teachers' choices of representations, examining differences between concrete and abstract forms.

Traditional qualitative methods often face limitations in sample size and efficiency. Recently, the integration of content analysis and text mining (Chen, 2014) has enabled efficient analysis of large-scale textual data. Based on this approach, our study employs text mining to analyze teacher narratives, revealing the underlying connotations and patterns in their instructional representation choices. This study aims to investigate the different forms of representations adopted by pre-service elementary mathematics teachers when faced with authentic teaching scenarios, such as mathematics content preparation, concept introduction and student problem handling, as well as their effects

on students' conceptual understanding and learning interest. Simultaneously, text mining techniques are applied to analyze the key linguistic features in the teachers' natural language.

Literature Review

Instructional Representations

The mathematics education community widely recognizes that representations are not only a means of presenting concepts but also a critical tool for internalizing knowledge. The NCTM (2000) Standards for School Mathematics explicitly indicate that representations play a pivotal role in facilitating the internalization of mathematical concepts. Goldin (1998) posits that representations involve reinterpreting abstract concepts through symbols, images, or text, with their effectiveness varying according to the context and communicative purpose. English and Halford (1995) further note that by using appropriate external representations, teachers can guide students to form internal representations, thereby enhancing knowledge transmission. Taken together, the literature underscores that appropriate instructional representations have a profound impact on students' conceptual understanding and problem-solving abilities.

Text Mining Techniques

Traditionally, text mining relies on natural language processing and machine learning techniques to automatically analyze latent information in texts, and it has been widely applied to data sources such as news articles, social media posts, and classroom recordings (Blake, 2011; Chen, 2014). In recent years, text mining in educational research has evolved rapidly and exhibits several trends. For instance, in digital learning and emerging environments, Tsai (2023) used document and association analysis to explore self-directed learning patterns in the metaverse, demonstrating the potential of automated text analysis in new digital contexts and offering fresh insights for analyzing educational texts.

In processing Chinese texts, despite challenges such as polysemy and the complexity of proper nouns, Yen (2021) illustrated the potential of text mining in Chinese environments by analyzing online course forum posts, and Chi et al. (2022) used text analysis to uncover underlying leadership trends in educational research abstracts, further broadening the scope of text mining applications in education. Overall, recent literature from the past five years not only broadens the application scenarios of text mining techniques but also demonstrates the theoretical and practical value of integrating deep learning and sentiment analysis in educational evaluation and research.

Methodology

Participants and Design

This study employed a questionnaire survey to collect teaching responses from 107 pre-service elementary school teachers at National Taipei University of Education as they engaged in authentic mathematics teaching scenarios. The sample included 51 participants from the department of mathematics education and 56 from non-mathematics-related departments, allowing for an exploration of how different backgrounds influence the selection of instructional representations. Participants were divided into the department of mathematics education and non-mathematics-related departments because elementary mathematics teachers in Taiwan are not typically trained through specialized programs, and non-mathematics majors' pedagogical content knowledge has long been a weakness. Each participant completed one instructional representations questionnaire for mathe-

matics teaching. The items of the questionnaire were designed as open-ended questions, and each participant provided at least 100 words for each item.

Instruments and Data Collection

The questionnaire was designed around three major dimensions of instructional scenarios:

Mathematics Content Preparation: Questions such as "Which representations help students understand mathematical concepts when starting a new unit?" and "Which representations can stimulate students' interest in learning?" are included.

Concept Introduction: For example, "Please describe in approximately 100 words how you would introduce a concept in the first five minutes before starting a new unit."

Student Problem Handling: For instance, "If you notice that most students appear confused during the explanation of a mathematical concept, how would you respond?"

The draft questionnaire was revised based on content validity checks by at least two mathematics education experts to ensure the appropriateness and research value of the items.

Data Coding and Analysis

To quantify teachers' use of various representations, the representations were classified into three categories. Based on Greeno's (1987) assertion, the responses to open-ended questions were coded into three categories (see Table 1) with different scores assigned. The questionnaire comprised 37 items, and the coding yielded a Cronbach's alpha of 0.598, indicating acceptable reliability for a pilot study.

Table 1. Classification of instructional representations, examples, and coding scheme

Representation Type	Example	Code (points)
CVR	images, everyday objects, metaphors, examples, real-life items	2
CUR	definitions, numerical examples, proofs	1
FAR	narrative-based, abstract symbols, formulas	0

Note: CVR: Concrete Visual Representations; CUR: Classroom Utilization Representations; FAR: Formal Abstract Representations.

For the text mining component, the study employed the Chinese Word Segmentation System (CKIP) provided by the Academia Sinica to perform word segmentation and used the TF-IDF algorithm to build a term-document matrix. A set of keywords relevant to the representations (e.g., "example," "life," "object") was selected for frequency analysis, and an independent samples t-test was conducted to examine significant differences in keyword usage between teachers with different academic backgrounds.

Results and Discussion

Effects of Instructional Representations on Student Understanding and Learning Interest

According to the statistical results (see Table 2), pre-service teachers generally believe that concrete visual representations (e.g., using examples, real-life cases, images, or demonstrations with nearby

objects) are effective in promoting students' understanding of mathematical concepts, while also positively influencing their interest in learning. In contrast, formal abstract representations (e.g., using formulas or abstract symbols) were deemed less helpful for student understanding. In particular, "narrative-based representations" achieved an 88.24% approval rate for stimulating interest but only a 19.61% support rate for enhancing understanding, indicating that the relationship between interest and comprehension is not entirely consistent.

PARS: Marked by a primary use of abstract representations with significantly lower use of concrete representations.

Table 2. A summary of the scores of the two groups

Item	Type	Enhances Understanding	Enhances Interest in Math
Using examples to present concepts	CVR	77.45%	53.92%
Using real-life objects to present concepts	CVR	75.49%	95.10%
Using metaphors to present concepts	CVR	72.55%	67.65%
Demonstrating concepts with nearby objects	CVR	67.65%	86.27%
Using drawings to present concepts	CVR	64.71%	78.43%
Using numerical exam to present concepts	CUR	64.71%	7.84%
Using formal methods (e.g., definitions)	CUR	60.78%	0.00%
Using proofs to present concepts	CUR	55.88%	2.94%
Using formulas to present concepts	FAR	35.29%	0.00%
Using narratives to present concepts	FAR	19.61%	88.24%
Using abstract symbols to present concepts	FAR	12.75%	0.00%

Text Mining Analysis

The approach adopted in this study is based on corpus linguistics. First, a lexicon of words related to representations was established as a reference corpus. This corpus was then used to analyze the language used by the participants in their responses, particularly in two key items (referred to as "tag" analysis), to identify important keywords that distinguish between teachers with data-related expertise and those from other fields. We illustrate this with the example Q2A.

In the text mining analysis of Q2A: "Please describe in approximately 100 words how you would introduce a concept in the first five minutes before starting a new unit"—keywords such as "unit," "life," and "motivation" appeared with high frequency, indicating that teachers emphasize real-life connections and motivational factors when introducing concepts. Furthermore, a comparison of keyword usage between teachers with data-related expertise and those with other backgrounds revealed that teachers with data-related backgrounds more frequently used terms such as "object" and "solution" (with statistically significant differences), suggesting that different academic backgrounds may influence the emphasis and language choices in teachers' descriptions of instructional representations.

Conclusion and Recommendations

Concrete and real-life representations enhance mathematical concept internalization and learning interest, whereas solely using abstract representations may hinder balance. Future research should explore optimizing cognitive and affective factors in instruction, integrating diverse representations based on teachers' backgrounds, and applying advanced text mining methods like deep learning and sentiment analysis to analyze instructional language.

References

Blake, C. (2011). Text mining. *Annual Review of Information Science and Technology*, *45*(1), 121-155.

Chen, S.-J. (2014). Text mining for social studies: Meaning-based document classification and its problems. *Journal of Humanities and Social Sciences, 27*(4), 683-718.

English, L. D. & Halford, G. S. (1995). *Mathematics education: Models and processes*. Lawrence erlbaum associates.

Goldin, G. A. (1998). Representational systems, learning, and problem solving in mathematics. *Journal of Mathematical Behavior*, *17*(2), 137-165.

Goldin, G. A., & Kaput, J. J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), *Theories of Mathematical Learning* (pp. 397-430). Erlbaum.

Greeno, J. G. (1987). Instructional representations based on research about understanding. In A.H. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education* (pp. 61-88). Erlbaum.

NCTM (2000). *Principles and standards for school mathematics.* Author.

Shih, J.-J., Shen, C.-H., & Tsai, C.-T. (2022). The inquiry into the trend of educational leadership during the implementation of the grade 1-9 curriculum in elementary schools-applying text mining. *Ebscohost, 138*, 74-104.

Tsai, Y.-C. (2023). Learner-centered analysis in educational metaverse environments: Exploring value exchange systems through natural interaction and text mining. *Journal of Metaverse, 3*(2), 121-132.

Yen, C.-H. (2021). Exploring asynchronous online interaction via text mining techniques. *Journal of Management and Information, 26*, 53-72.

TSG4-1-12

THE ROLE OF PROFESSIONAL DEVELOPMENT IN ENHANCING MATHEMATICAL CREATIVITY: A CASE STUDY OF NOVICE TEACHERS

Yujin Lee
Kangwon National University, Korea, ylee@kangwon.ac.kr

Ali Bicer
Texas A&M University, USA, alibicer@tamu.edu

This comparative case study examines how two novice mathematics teachers facilitate students' creative thinking using Rhodes' (1961) 4P model—person, product, process, and press. The study investigates how professional development (PD) influences creativity-directed instructional practices. Teacher A, who did not receive PD, and Teacher B, who did, were observed to compare their instructional methods. Results indicated that PD significantly enhanced teacher's ability to integrate process-oriented strategies, fostering richer creative engagement among students. The study suggests that tailored PD can support novice teachers in implementing effective creativity-enhancing strategies in mathematics education. These findings contribute to a broader understanding of how instructional support and professional learning opportunities can be leveraged to foster creativity in mathematics classrooms.

Keywords: professional development, creativity, novice teacher

Introduction

Creativity in mathematics extends beyond product-based assessments, encompassing students' problem-solving strategies, reasoning, and engagement. Traditional research often focuses on the product dimension, evaluating fluency, flexibility, and originality. Rhodes' (1961) 4P model—person, process, press, and product—offers a comprehensive framework for understanding creativity. The person encompasses the teachers and students involved in the creative act, while the process pertains to the cognitive strategies used to generate ideas. The press dimension emphasizes the supportive environment that encourages creative thinking, and the product refers to the outcomes generated during the process. In mathematical creativity research, emphasis has predominantly been on the product dimension, measured through components such as fluency, flexibility, originality, and elaboration (Molad et al., 2020). However, focusing on the product has often come at the expense of exploring the process dimension, which deals with the strategic and cognitive methods students use. This limited emphasis on the process dimension has created a gap in understanding how teachers, especially novice teachers, support students' creative thinking process in mathematics classrooms. Consequently, there is limited insight into the strategies teachers employ to foster and respond to students' generation of creative ideas. This study seeks to bridge this gap by exploring how two novice teachers facilitate students' creative thinking in mathematical classrooms and how their practices differ when one receives professional development (PD) and ongoing support while the other operates independently. Through this comparison, we aim to highlight the importance of PD in promoting strategic processes that support mathematical creativity.

Methodology

A comparative case study approach was adopted to explore creativity-directed instructional practices in two novice fifth-grade mathematics teachers. Teacher A, who did not participate in a PD program, and Teacher B, who received structured PD, were selected from the same school district. Classroom observations, teacher interviews, lesson plans, and student problem-solving artifacts were analyzed using a four-level framework aligned with Rhodes' 4P model. Collected data was coded based on the presence of creativity-directed practices and the extent of teacher-student and student-student interactions. This mixed-methods approach provided a nuanced understanding of how different instructional strategies and external supports shape teachers' ability to foster creativity in mathematics classrooms.

Results

Analysis results indicated notable differences in creativity-directed practices between the two teachers. Teacher A's classroom placed greater emphasis on the press dimension, creating a positive and mistake-free learning environment but with limited engagement in process-oriented strategies. Collective creativity was observed infrequently, and instructional methods leaned towards direct instruction rather than inquiry-based learning. Students were encouraged to participate in discussions, but they were not frequently given opportunities to generate and explore multiple solution paths independently. On the other hand, Teacher B, who received PD, implemented a variety of strategies supporting creativity. Her classroom demonstrated a higher frequency of creativity-directed practices, particularly in the process and product dimensions. Key strategies included modeling, promoting exploratory discussions, incorporating real-life applications, and encouraging multiple solution pathways. These methods allowed students to engage more deeply in mathematical concepts, fostering both individual and collective creativity. By structuring lessons around open-ended problems and encouraging students to think through their reasoning processes, Teacher B was able to create a more dynamic and interactive learning environment.

Creativity-Directed Practices

A key difference between the two classrooms was the approach to problem-solving. Teacher B frequently incorporated mathematical modeling and real-life contexts to help students make sense of mathematics. For example, in a lesson on dividing fractions, students used physical models of graham crackers to visualize and explore division concepts. This hands-on approach promoted deeper conceptual understanding and encouraged students to articulate their reasoning. The use of concrete materials and real-world scenarios provided students with tangible ways to understand abstract mathematical principles, reinforcing both conceptual understanding and creative problem-solving skills. In contrast, Teacher A's classroom was structured around more conventional problem-solving exercises, where students followed step-by-step instructions rather than exploring open-ended questions. While Teacher A fostered a supportive learning environment, the absence of structured creativity-directed practices limited students' opportunities for divergent thinking and self-directed problem-solving. Without the structured support of PD, Teacher A relied more on traditional teaching methods, leading to fewer instances where students were challenged to engage deeply with mathematical problems.

Professional Development Impact

Interviews with Teacher B revealed that PD played a crucial role in shaping her instructional approach. She credited the PD sessions for helping her facilitate open-ended discussions, encourage risk-taking, and guide students without immediately providing answers. The PD program emphasized inquiry-based instruction, encouraging teachers to allow students to explore multiple representations of problems. The structured nature of PD sessions provided Teacher B with opportunities to reflect on her teaching practices, receive targeted feedback, and experiment with new instructional strategies in a supportive setting. Teacher B reported that, prior to PD, she was unsure of how to foster mathematical creativity effectively. However, through structured PD sessions that included collaborative problem-solving, reflective practice, and iterative feedback, she gained confidence in implementing strategies that enhanced student engagement. Teacher A, without such support, demonstrated less flexibility in instructional methods, relying more on procedural teaching. The contrast between the two teachers illustrates how PD can equip novice educators with the necessary tools to implement more effective and engaging instructional strategies.

Discussion

This study highlights the importance of PD in equipping novice teachers with strategies that foster mathematical creativity. The findings suggest that without targeted PD, novice teachers may struggle to implement creativity-directed practices, even if they recognize their value. The support and guidance provided by structured PD sessions can play a pivotal role in helping teachers bridge the gap between theory and practice, allowing them to integrate research-backed strategies into their classrooms effectively. The study also expands the concept of mathematical creativity by redefining the product dimension to include mathematical reasoning, sense-making, and conceptual understanding. Traditional measures of mathematical creativity often focus solely on fluency, flexibility, and originality. However, this research demonstrates that fostering a broader definition of creativity in mathematics classrooms can help students develop a deeper and more meaningful understanding of mathematical concepts. By incorporating inquiry-based learning and open-ended problem-solving opportunities, teachers can create environments that encourage students to engage in higher-order thinking and innovative mathematical exploration.

Moreover, the results emphasize the interconnectedness of the 4P dimensions. While the person dimension (teacher characteristics) and press dimension (learning environment) play critical roles in fostering creativity, the process and product dimensions—heavily influenced by instructional strategies—are where PD can make the most significant impact. This study suggests that well-designed PD programs should focus on these dimensions to enhance creativity in mathematics education. Future research should explore how sustained and iterative PD initiatives can further support novice teachers in refining and expanding their creativity-directed teaching practices.

Conclusion

Professional development plays a vital role in helping novice teachers implement creativity-directed instructional strategies. Teacher B, who received PD, demonstrated a significantly higher use of process-oriented practices that encouraged student engagement and mathematical reasoning. In contrast, Teacher A's instruction, though supportive, lacked the structured strategies necessary to foster deep creative engagement. The ability of PD to provide teachers with concrete, actionable strategies highlight its critical role in improving mathematics education.

The findings suggest that integrating creativity-directed instructional practices into PD programs can enhance novice teachers' ability to cultivate students' mathematical creativity. Future research should explore expanding PD initiatives and examine their long-term impact on instructional practices. Supporting novice teachers through sustained professional development can help bridge the gap between theoretical understanding and practical application, ultimately enriching mathematics education for students. Through targeted training and continuous support, educators can be empowered to implement innovative teaching practices that foster mathematical creativity in diverse learning environments.

References

Molad, O., Levenson, E. S., & Levy, S. (2020). Individual and group mathematical creativity among post–high school students. *Educational Studies in Mathematics*, *104*(2), 201-220.

Rhodes, M. (1961). An analysis of creativity. *Phi Delta Kappan, 42*(7), 305–310.

TSG4-1-13

INTERVENTIONS TO DEVELOP PRE-SERVICE MATHEMATICS TEACHERS' COMPETENCE IN DIFFERENTIATED INSTRUCTION: A SCOPING REVIEW

Iden Rainal Ihsan
National Taiwan Normal University, Taiwan, irainalihsan@ntnu.edu.tw

Kai-Lin Yang
National Taiwan Normal University, Taiwan, kailin@ntnu.edu.tw

Situated within the DI-Quest framework and Shulman's model of pedagogical reasoning, this scoping review synthesizes findings from 62 peer-reviewed studies published between 2006 and 2025 to identify and analyze recommended interventions to prepare pre-service mathematics teachers in implementing differentiated instruction. Four central intervention types are identified: (1) simulation-based diagnostic training; (2) video-based professional vision development; (3) content-focused scaffolding strategies; and (4) adaptive lesson planning guided by formative assessment and collaborative analysis of student work. Additionally, mentorship and peer collaboration are shown to reinforce reflective practice and support the transfer of DI principles into classroom enactment. A 16-week course model is proposed, offering a coherent framework for systematically integrating these interventions into teacher education programs, thereby equipping future mathematics teachers to plan and enact equitable, adaptive instruction.

Keywords: Differentiated instruction, Learning interventions, Mathematics teacher education, Pre-service mathematics teacher

Introduction

The growing diversity in student abilities, interests, and learning profiles necessitates instructional approaches that move beyond the traditional "one-size-fits-all model" (Bondie et al., 2019). Differentiated instruction (DI) has been widely advocated as a framework for designing teaching practices that are responsive to individual student needs (Tomlinson, 2014). In mathematics education, and specifically within mathematics teacher education, DI offers a promising alternative by addressing the unique learning gaps encountered in both pre-service training and in-service professional development (Bondie et al., 2019).

Students come to the classroom with varying levels of prior knowledge, cognitive abilities, and learning preferences, making it challenging for uniform teaching methods to meet every learner's needs (Marks et al., 2021). Despite its theoretical appeal, the practical implementation of DI in mathematics remains challenging (Bobis et al., 2021). This is especially evident in mathematics teacher education, where pre-service and novice teachers frequently encounter difficulties in translating DI theory into classroom practice due to limited exposure and insufficient training in differentiated methods (Bondie et al., 2019; Prast et al., 2018). These challenges highlight the pressing need for a clear synthesis of empirically supported pedagogical strategies tailored to mathematics teacher education. Pre-service teachers encounter unique challenges when attempting to implement DI. Teacher preparation programs often provide limited exposure to practical differentiation

strategies, leaving new teachers underprepared for the realities of diverse classroom environments (Herner-Patnode & Lee, 2021; van den Kieboom & Groleau, 2022). This study investigated how 46 pre-service teachers (PSTs. This study aims to identify and analyze interventions recommended to prepare pre-service mathematics teachers to implement differentiated instruction. Thus, we formulated the following research question: What interventions are recommended to develop pre-service mathematics teachers' competence in implementing differentiated instruction?

Theoretical Framework

Tomlinson (2014) defines DI as a pedagogical approach in which teachers modify curriculum content, proactively develop varied teaching strategies, and continually revisit learning outcomes so that each student can connect with prior understanding and build knowledge effectively. Additionally, research indicates that pre-service teachers who cultivate high self-efficacy and a growth mindset are more inclined to implement innovative, student-centered instructional practices (Roth & Walshaw, 2019; Xie & Cai, 2021). The DI-Quest model (Coubergs et al., 2017) operationalizes these ideas by identifying key teacher variables: two DI philosophies (growth mindset and ethical curriculum orientation), two DI instructional principles (flexible grouping and "output=input" formative assessment), and the core outcome of adaptive teaching, instruction tailored to students' readiness, interests, and learning profiles. Adaptive teaching closely resonates with Shulman's model of Pedagogical Reasoning and Action (Shulman, 1987), in which teachers transform subject matter knowledge into pedagogically powerful forms that are adaptive to the diverse needs of students. In Shulman's model, effective teaching involves an ongoing cycle of comprehension, transformation, instruction, evaluation, reflection, and new comprehension, allowing teachers to make informed pedagogical decisions that accommodate student diversity. These adaptive practices require the integration of diagnostic teaching strategies, as emphasized by Black and Wiliam (1998) and reaffirmed by Swan (2015), where formative assessment evidence is actively used to adjust teaching to meet student needs in real time. Consequently, preparing Pre-service mathematics teachers for DI demands cultivating their pedagogical reasoning capacities, grounding their practice in DI-Quest principles, and equipping them with adaptive and diagnostic teaching.

Methodology

This scoping review follows the methodological framework of Arksey and O'Malley (Arksey & O'Malley, 2005), as refined by Levac et al. (2010), and guided by the PRISMA-ScR guidelines (Tricco et al., 2018). The review process consists of five stages: defining research questions, identifying relevant studies, selecting studies, charting data, and synthesizing results (see figure 1). The literature search was conducted in the Scopus database, covering peer-reviewed education research. Our search strategy used two groups of keywords. The first group (searching within article title, abstract, and keywords) centered on "differentiated" combined with terms such as strategy, teaching, and learning, and included related terms such as "adaptive" and "diagnostic" to broaden the search. The second group (searching within all fields) included terms equivalent to "pre-service mathematics teacher" (e.g., prospective mathematics teacher, mathematics teacher candidate). This strategy aimed to capture studies focusing on DI in mathematics teacher education across pre-service and student teacher populations. Study selection followed a structured process with predefined inclusion and exclusion criteria.

Table 1. Study selection criteria

Inclusion Criteria	Exclusion Criteria
• Peer-reviewed articles published between 2006 and 2025 • Studies focusing on DI in mathematics education • Research on teacher preparation, professional development, or competencies related to DI • Involving pre-service, in-service, or student mathematics teachers	• Studies not published in English • Articles not focused on mathematics education • Opinion pieces, editorials, or nonempirical studies • Studies without a clear focus on DI

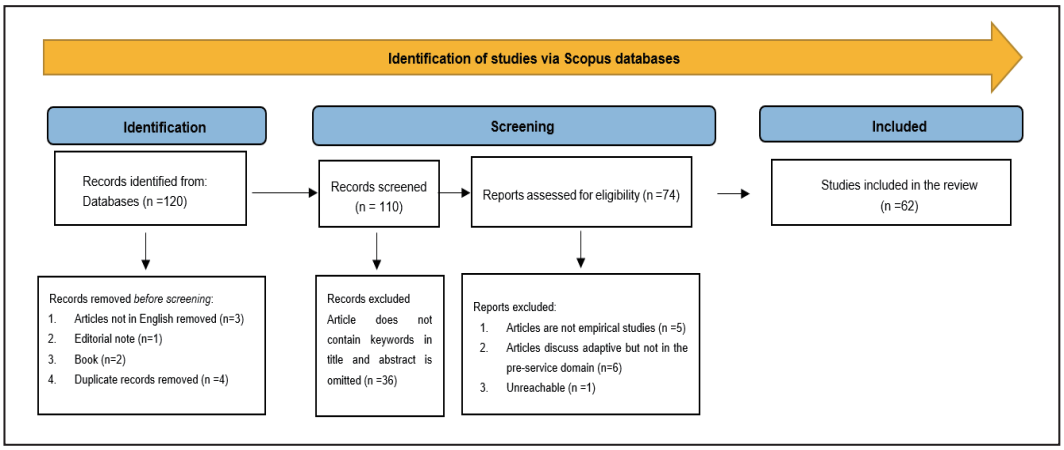


Figure 1. PRISMA-ScR flow diagram in this study

Results and Findings

This scoping review identified four categories of pedagogical interventions that contribute to developing pre-service mathematics teachers' competence in implementing DI, i.e., (1) simulation-based diagnostic training, (2) video-based professional vision development, (3) content-focused scaffolding strategies, and (4) adaptive lesson planning and formative decision making. These interventions support the instructional dimensions of the DI-Quest model (Coubergs et al., 2017), which emphasizes the integration of teacher beliefs, instructional principles, and adaptive teaching. Together, these interventions address both the perceptual and decision-making demands of DI by building pre-service mathematics teachers' capacity to diagnose learner needs and respond with pedagogically sound, flexible moves.

Simulation-based diagnostic training provides pre-service teachers with low-risk opportunities to practice identifying and interpreting student misconceptions or difficulties through authentic and dynamic interactions. Whether through AI-based diagnostic chatbots (Lee & Yeo, 2022), scripted student interviews (Marczynski et al., 2022), or argumentation-focused simulations (Codreanu et al., 2022), these experiences foster foundational diagnostic reasoning. Video-based professional vision development complements this by helping Pre-service teachers learn to notice relevant student responses in complex classroom contexts. Through guided analysis of classroom videos

(Reuker & Künzell, 2021; Schnitzler et al., 2020), Pre-service teachers refine their ability to interpret mathematical understanding and connect it to appropriate instructional responses. Content-focused scaffolding strategies, i.e., designing a tiered task (Dröse & Prediger, 2023), using enabling and extending prompts (Hackenberg et al., 2021), and crafting anticipatory questions (Hubbard & Livy, 2021), equip Pre-service teachers with tools to plan instruction that adapts content delivery to varying readiness levels while preserving mathematical rigor. Finally, adaptive lesson planning and formative decision making empower pre-service teachers to revise their instruction responsively. Collaborative planning and student work analysis (Jordan et al., 2017; Lupiáñez et al., 2024) enable pre-service mathematics teachers to align learning goals with real-time formative insights, reinforcing the cyclical relationship between diagnosis and instructional adjustment. In addition, several studies emphasized the role of sustained mentorship and peer collaboration in supporting these interventions. Pairing pre-service teachers with experienced mentors who model differentiated practices and offer targeted feedback throughout teaching rehearsals was found to enhance the transfer of DI principles to actual classroom enactment (Gervasoni et al., 2021; Herner-Patnode & Lee, 2021). Likewise, structured peer review during lesson study cycles further supported reflective decision-making, allowing PSTs to critically evaluate their own and others' adaptive moves.

Conclusion

Based on these findings, we recommend designing a 16-week DI preparation course as a foundational experience prior to pre-service teachers' participation in school-based internships. This course should integrate all four identified intervention types to systematically build diagnostic and adaptive teaching skills. Early weeks may focus on video analysis and simulation-based diagnosis to cultivate noticing and interpretive expertise. Mid-semester modules should engage pre-service mathematics teachers in designing tiered mathematical tasks and developing strategic scaffolding practices. In later sessions, pre-service mathematics teachers can plan and revise lessons through formative assessment analysis and engage in peer microteaching with structured feedback cycles. This design offers a research-informed framework for preparing future mathematics teachers to enact differentiated instruction with confidence, responsiveness, and pedagogical intentionality.

References

Arksey, H., & O'Malley, L. (2005). Scoping studies: towards a methodological framework. *International Journal of Social Research Methodology*, *8*(1), 19–32. https://doi.org/10.1080/1364557032000119616

Black, P., & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education: Principles, Policy & Practice*, *5*(1), 7–74.

Bobis, J., Russo, J., Downton, A., Feng, M., Livy, S., McCormick, M., & Sullivan, P. (2021). Instructional moves that increase chances of engaging all students in learning mathematics. *Mathematics*, *9*(6), 1–19. https://doi.org/10.3390/MATH9060582

Bondie, R. S., Dahnke, C., & Zusho, A. (2019). How Does Changing "One-Size-Fits-All" to Differentiated Instruction Affect Teaching? *Review of Research in Education*, *43*(1), 336–362. https://doi.org/10.3102/0091732X18821130

Codreanu, E., Huber, S., Reinhold, S., Sommerhoff, D., Neuhaus, B. J., Schmidmaier, R., Ufer, S., & Seidel, T. (2022). Diagnosing Mathematical Argumentation Skills: A Video-Based Simulation for Pre-Service Teachers. *Learning to Diagnose with Simulations: Examples from Teacher Education and Medical Education*, 33–47. https://doi.org/10.1007/9783030891473_4

Coubergs, C., Struyven, K., Vanthournout, G., & Engels, N. (2017). Measuring teachers' perceptions about differentiated instruction: The DI-Quest instrument and model. *Studies in Educational Evaluation*, *53*, 41–54.

Dröse, J., & Prediger, S. (2023). Prospective Teachers' Diagnostic Thinking on Students' Understanding of Multi-Digit Multiplication: A Content-Related Analysis on Unpacking of Knowledge Elements. *Journal Fur Mathematik-Didaktik*, *44*(1), 1–28. https://doi.org/10.1007/s13138-022-00214-w

Gervasoni, A., Roche, A., & Downton, A. (2021). Differentiating Instruction for Students Who Fail to Thrive in Mathematics: The Impact of a Constructivist-Based Intervention Approach. *Mathematics Teacher Education and Development*, *23*(3), 207–233.

Hackenberg, A. J., Creager, M., & Eker, A. (2021). Teaching practices for differentiating mathematics instruction for middle school students. *Mathematical Thinking and Learning*, *23*(2), 95–124. https://doi.org/10.1080/10986065.2020.1731656

Herner-Patnode, L., & Lee, H. J. (2021). Differentiated instruction to teach mathematics: Through the lens of responsive teaching. *Mathematics Teacher Education and Development*, *23*(3), 6–25.

Hubbard, J., & Livy, S. (2021). Self-study of a mathematics learning consultant: Supporting teachers to plan lessons for implementing differentiation in the classroom. *Mathematics Teacher Education and Development*, *23*(3), 148–165.

Jordan, R., DiCicco, M., & Sabella, L. (2017). "They sit selfishly." Beginning STEM Educators' Expectations of Young Adolescent Students. *RMLE Online*, *40*(6), 1–14. https://doi.org/10.1080/19404476.2017.1320065

Lee, D., & Yeo, S. (2022). Developing an AI-based chatbot for practicing responsive teaching in mathematics. *Computers and Education*, *191*(February), 104646. https://doi.org/10.1016/j.compedu.2022.104646

Levac, D., Colquhoun, H., & O'Brien, K. K. (2010). Scoping studies: advancing the methodology. *Implementation Science*, *5*(1), 69. https://doi.org/10.1186/1748-5908-5-69

Lupiáñez, J. L., Olivares, D., & Segovia, I. (2024). Examining the role played by resources, goals and orientations in primary teachers' decision- making for problem-solving lesson plans. *ZDM - Mathematics Education*, 1153–1167. https://doi.org/10.1007/s11858-024-01614-7

Marczynski, B., Kaltefleiter, L. J., Siebeck, M., Wecker, C., Stürmer, K., & Ufer, S. (2022). Diagnosing 6th Graders' Understanding of Decimal Fractions: Fostering Mathematics Pre-Service Teachers' Diagnostic Competences with Simulated One-on-One Interviews. *Learning to Diagnose with Simulations: Examples from Teacher Education and Medical Education*, 49–62. https://doi.org/10.1007/9783030891473_5

Marks, A., Woolcott, G., & Markopoulos, C. (2021). Differentiating instruction: Development of a practice framework for and with secondary mathematics classroom teachers. *International Electronic Journal of Mathematics Education*, *16*(3), em0657.

Prast, E. J., Van de Weijer-Bergsma, E., Kroesbergen, E. H., & Van Luit, J. E. H. (2018). Differentiated instruction in primary mathematics: Effects of teacher professional development on student achievement. *Learning and Instruction*, *54*, 22–34.

Reuker, S., & Künzell, S. (2021). Learning diagnostic skills for adaptive teaching–a theoretical foundation. *Cogent Education*, *8*(1). https://doi.org/10.1080/2331186X.2021.1887432

Roth, W. M., & Walshaw, M. (2019). Affect and emotions in mathematics education: toward a holistic psychology of mathematics education. *Educational Studies in Mathematics*, *102*(1), 111–125. https://doi.org/10.1007/s10649-019-09899-2

Schnitzler, K., Holzberger, D., & Seidel, T. (2020). Connecting Judgment Process and Accuracy of Student Teachers: Differences in Observation and Student Engagement Cues to Assess Student Characteristics. *Frontiers in Education*, *5*(December), 1–28. https://doi.org/10.3389/feduc.2020.602470

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1–22. http://people.ucsc.edu/~ktellez/shulman.pdf

Swan, M. (2015). Designing Formative Assessment Lessons for Concept Development and Problem Solving. *North American Chapter of the International Group for the Psychology of Mathematics Education*.

Tomlinson, C. A. (2014). *The differentiated classroom: Responding to the needs of all learners*. Ascd.

Tricco, A. C., Lillie, E., Zarin, W., O'Brien, K., Colquhoun, H., & Levac, D. (2018). PRISMA extension for scoping reviews (PRISMA-ScR): checklist and explanation. 2018; 169 (7): 467-73. *DOI: Https://Doi. or/10.7326/M18-0850» Https://Doi. or/10.7326/M18-0850*.

van den Kieboom, L. A., & Groleau, S. V. (2022). Pre-service teacher planning for differentiation of instruction in mathematics classrooms. *Educational Studies in Mathematics*, *111*(2), 225–252. https://doi.org/10.1007/s10649-022-10149-1

Xie, S., & Cai, J. (2021). Teachers' Beliefs about Mathematics, Learning, Teaching, Students, and Teachers: Perspectives from Chinese High School In-Service Mathematics Teachers. *International Journal of Science and Mathematics Education*, *19*(4), 747–769. https://doi.org/10.1007/s10763-020-10074-w

TSG4-2-6

NAVIGATING STATISTICS AND PROBABILITY INSTRUCTION THROUGH MKT: A CONTENT ANALYSIS OF TEACHER REFLECTIONS

Joynarose Permentilla
Department of Education, Schools Division of Lipa City, Philippines, joynarose.permentilla@deped.gov.ph

Ella Ansay
Department of Education Central Office, Ateneo de Manila University, Philippines, ella.ansay@deped.gov.ph, ella.c.ansay@student.ateneo.edu

Joseph Ma. Steven Cabalo
Ateneo de Manila University, Department of Education, Schools Division of Lipa City, Philippines, joseph.cabalo@deped.gov.ph, joseph.cabalo@student.ateneo.edu

Jade Hizon
Nueva Ecija University of Science and Technology, Philippines, jadehizon@neust.edu.ph

Resty Catinoy
National University, Department of Education Schools Division of Lipa City, Philippines, rscatinoy@nu-lipa.edu.ph, resty.catinoy@deped.gov.ph

Diana Sandoval
Department of Education, Schools Division of Lipa City, Philippines, diana.sandoval@deped.gov.ph

Christy Sandoval
Department of Education, Schools Division of Lipa City, Philippines, christy.sandoval@deped.gov.ph

Marco Gonzalvo
Department of Education, Schools Division of Lipa City, marco.gonzalvo@deped.gov.ph

This qualitative content analysis examines a senior high school teacher's Daily Lesson Logs (DLL) over five weeks, focusing on a Statistics and Probability subject. The series of DLL served as the main data source, recording the teacher's day-to-day instructional strategies, observed challenges, and reflective insights. Anchored in the Mathematical Knowledge for Teaching (MKT) framework, findings underscore strong Common Content Knowledge, Specialized Content Knowledge, and Knowledge of Content and Teaching. However, limited Horizon Content Knowledge and curriculum constraints hinder deeper conceptual connections. The study highlights how reflective practice illuminates teaching strengths and points to targeted professional development needs, particularly in fostering diagnostic tools and connecting probability concepts to broader mathematics contexts.

Keywords: reflection, mathematics knowledge for teaching, daily lesson logs, statistics and probability

Introduction

Mathematics teachers' reflections on their own learning experiences significantly shape their teaching approaches and student outcomes. Such reflective practices are crucial for teachers to effectively adapt their teaching methods to meet students' needs. Teachers face resource constraints, curriculum issues, and professional development needs. Still, reflection helps them navigate these challenges and enhance teaching (Luzano, 2024), while out-of-field teaching compels teachers to adapt and reflect for quality instruction (Nob & Tañola, 2024). Moreover, inclusive education in the Philippines demands personalized pedagogy and ongoing reflection to meet diverse student needs (Culajara & Culajara, 2023).

Considering that teaching approaches are influenced by teacher reflections on context teaching, strategy selection, and instructional challenges (Dhungana, 2023; Aghakhani et al., 2023; Tatira & Boateng, 2023), many teachers lack structured guidance in reflection, leading to superficial analyses that do not spur meaningful change (Huynh, 2022). Moreover, some teachers may struggle to recognize the core issues in their teaching practices, which can hinder their ability to implement effective changes (Liu, 2024), highlighting the need for targeted support and feedback in writing teacher reflections. Thus, this study aims to: (1) examine the use of innovative materials, common instructional challenges, and strategies highlighted in a teacher's reflections; and (2) use Mathematics Knowledge for Teaching to classify teachers' subject matter knowledge and pedagogical content knowledge based on their reflections.

MKT is a specialized knowledge form that integrates mathematical content and pedagogical strategies, explicitly tailored for teaching mathematics. According to MKT, teachers need six different kinds of knowledge for teaching mathematics (Koponen et al., 2019). It is a comprehensive framework with subdomains that help teachers plan lessons, anticipate student responses, and connect advanced mathematical concepts to teaching practices (Sapkota & Hayes, 2024; Pincheira & Alsina, 2021). On the other hand, the Department of Education (DepEd) DLLs are structured tools educators use to plan and document their daily teaching activities. Through DepEd Order No. 42, series of 2016 (Department of Education, 2016), these logs are designed to enhance the teaching-learning process by providing a consistent framework for lesson delivery, including entries for teacher reflections.

Methodology

This research employed a qualitative content analysis approach to explore the reflective practices of a senior high school mathematics teacher who taught "Statistics and Probability" to Grade 11 students during the third quarter of School Year 2024–2025. The primary dataset consisted of the teacher's DLLs spanning five consecutive weeks. Specifically, qualitative data were collected from the written teacher reflections in each DLL's final section. These reflections were guided by three open-ended questions prompting the teacher to examine their instructional practice—namely: (1) whether their teaching strategies were effective, (2) what difficulties they encountered during instruction, and (3) what innovations they discovered and wished to share with fellow educators. The reflective responses provided rich insights into the teacher's pedagogical reasoning, adaptive strategies, and professional learning over time. Following a content analysis framework, the study proceeded in iterative phases: (1) The researchers thoroughly retyped in a separate document and read all DLL entries to gain an overall sense of the teacher's reflective comments; (2) An initial coding was performed to identify recurring ideas, such as common instructional strategies, specific student

misconceptions, and resource-related challenges; (3) these codes were clustered aligned with the six dimensions of MKT; (4) the categories were systematically reviewed and interpreted within a content analysis framework, highlighting key insights into how reflective practice informed teaching methods and revealed strengths and gaps in pedagogical content knowledge.

Results and Discussion

The teacher's reflections reveal a consistent emphasis on procedural fluency, with the majority of learners achieving high scores in discrete random variables and normal distribution tasks as seen in Table 1. By using immediate feedback, personalized materials, and remediation, the teacher demonstrates strong CCK, SCK, and KCT. However, limited attention to broader mathematical structures highlights a gap in HCK. Despite effective lesson adjustments, some learners still require targeted support, especially where resource constraints (e.g., calculators) persist. This pattern underscores how small group instruction and timely interventions bolster student mastery of foundational concepts.

Table 1. Clustered codes and written reflection responses aligned with the six dimensions of MKT.

Domains of MKT	Sample Codes	Sample DLL Written Responses
CCK	- High percentage of students scoring ≥80%	*"Plato-34, Kohlberg-30, Emerson-30 [students] … scored above 80% on the formative assessment."*
SCK	- Representations - Customized materials	*"Personalized PPT [PowerPoint] —tailor-made according to the current skills acquired by the students."*
KCT	- Frequent use of immediate marking and corrections - Small-group remediation and recognition of who needs remediation - Adjusting instructional strategies based on quick feedback	*"Peer tutoring—students who struggled [sic] partnered with students who grasps [sic] easily." and "Teachers/Advisers might help me with the absences … Ask the advisers to monitor the attendance."* *"Immediate marking, corrections, groupings for remediation—these methods work well."* *"Immediate markings—to be able to gauge the students' current knowledge."* *"How to resolve the problem on how [sic] students can have their own calculator?"*
KCC	- Time and resource concerns - Curriculum pacing issues	*"Time allotted for the subject (45 mins is too short)" [need to cover sampling distributions and normal curve interpretations within the schedule]*

The clustered codes in Table 1 aligned with the six domains of MKT based on the nature and instructional purpose of each teacher's reflections. For CCK, the teacher's report of high student scores demonstrates fluency in teaching basic concepts and procedures, which reflects the teacher's grasp of fundamental statistical content. SCK is evident in creating tailored instructional materials, such as personalized PowerPoint presentations, which indicate a deliberate adaptation of content for specific learning needs—an essential feature of SCK. A peer tutoring strategy reflects an attempt to respond to diverse learner performance, seen as an indicator of KCT, since it demonstrates a general instructional approach rather than an insight into specific student conceptions or misconceptions. KCT is further supported by reflections on using immediate feedback and flexible grouping—hallmarks of teaching strategies grounded in formative assessment. KCC is justified through the teacher's concern about instructional pacing and resource gaps, aligning with the teacher's ability to plan within the structural limits of the curriculum. Notably, the absence of any reflection data linked to HCK and KCS underscores a gap in connecting current lessons to broader mathematical themes and the teacher's knowledge of how students learn specific lessons.

Moving forward, the teacher can strengthen KCS through more diagnostic tools to uncover misconceptions before they impede comprehension. Crooks and Alibali (2014) argued that students who lack explicit experience of mathematical applications tend to struggle with knowledge transfer to new settings, which decreases their problem-solving capacity. Additionally, refining KCC will help address time constraints by aligning lesson pacing with required competencies. Engaging students in higher-level connections between probability topics can enhance HCK, ensuring that learners see the relevance of concepts across multiple domains. Such holistic integration fosters deeper understanding and retention. Ultimately, the teacher's reflective practice and targeted professional development provide a structured pathway for continuous improvement in mathematics instruction delivery. Collective efforts from administrators can aid.

Conclusions and Recommendations

Analysis of the teacher's five-week daily lesson logs reveals a solid grounding in Common Content Knowledge, as students generally performed well on formative tasks. However, the teacher's emphasis on procedural accuracy underscores gaps in Specialized Content Knowledge. Though immediate feedback demonstrates strengths in Knowledge of Content and Teaching, limited diagnostic assessment and planning for resource constraints point to weaknesses in Knowledge of Content and Curriculum. On these findings, the researchers recommend the following for future research directions. First, integrate real-world applications to deepen conceptual understanding. Second, structured diagnostic tools may be implemented to address misconceptions promptly. Third, establish contingency plans for resource shortages, ensuring lesson continuity and quality. Finally, reflective practice may be maintained through targeted self-assessment, focusing on both lesson effectiveness and conceptual progression.

References

Aghakhani, S., Lewitzky, R. A., & Majeed, A. (2023). Developing Reflective Practice among Teachers of Mathematics. International Electronic Journal of Mathematics Education, 18(4). https://files.eric.ed.gov/fulltext/EJ1408828.pdf

Crooks, N. M., & Alibali, M. W. (2014). Defining and measuring conceptual knowledge in mathematics. *Developmental review*, *34*(4), 344-377.

Dhungana, H. L. (2023). Journeying Towards Transformative Mathematics Teacher through Critical Self-Reflection. *Academic Journal of Mathematics Education, 6*(1), 71-80.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for research in mathematics education, 39*(4), 372-400.

Huynh, H. T. (2022). Promoting professional development in language teaching through reflective practice. *Vietnam Journal of Education*, 62-68. https://vje.vn/index.php/journal/article/download/126/88

Koponen, M., Asikainen, M. A., Viholainen, A., & Hirvonen, P. E. (2019). Using network analysis methods to investigate how future teachers conceptualize the links between the domains of teacher knowledge. *Teaching and Teacher Education, 79*, 137-152.

Liu, X. (2024). Investigating written reflection and its relation to language teaching strategy change: A self-study. *Studies in English Language Teaching, 12*(1), 30–47. https://doi.org/10.22158/selt.v12n1p30

Luzano, J. F. (2024). Optimizing Ipsative Assessment in the Philippines: A Narrative Review on the Experiences of Mathematics Teachers. *Diversitas Journal, 9*(2).

Nob, J. M. R., & Tañola, M. (2024). Ang Magturo ay Di Biro: A HERMENEUTIC PHENOMENOLOGICAL STUDY ON THE LIVED-EXPERIENCES OF MATH EDUCATORS IN OUT-OF-FIELD TEACHING. *EPRA International Journal of Research & Development, 9*(2), 131–141. https://doi.org/10.36713/epra15810

Pincheira, N., & Alsina, Á. (2021). Teachers' mathematics knowledge for teaching early algebra: A systematic review from the MKT perspective. *Mathematics, 9*(20), 2590.

Sapkota, B. (2024). Investigating Preservice Teachers' Conceptualizations of Mathematical Knowledge for Teaching through Video Analysis. *International Journal of Education in Mathematics, Science and Technology, 12*(4), 836-860.

Tatira, B., & Boateng, S. (2023). Teaching Reflections of Secondary Mathematics Final-Year Student Teachers. *International Journal of Social Science Research and Review, 6*(7), 61-74.

Acknowledgements

The researchers would like to acknowledge the Philippine Council of Mathematics Teacher Educators, Inc. (MATHTED), Center for Integrated STEM Education, Inc. (CISTEM), and the Department of Education for providing an avenue of collaboration among the researchers.

TSG4-2-7

MANIFESTATIONS OF TEACHER AGENCY IN MATHEMATICS EDUCATION: PERSONAL, SOCIAL, AND CONTEXTUAL INSIGHTS FROM A GRADUATE COURSE ON TEACHING STATISTICS AND PROBABILITY

Nhoriel I. Toledo
Centro Escolar University & Ateneo de Manila University, Philippines, nitoledo@ceu.edu.ph

Teacher agency, which is influenced by personal, social, and contextual factors, is an important basis of instructional decision making and professional identity. This research investigates the teacher agency of 15 junior and senior high school mathematics teachers who are studying a graduate-level course on Teaching Statistics and Probability. A validated questionnaire was used to collect data. The findings of the research revealed that the course enhanced the professional identities of the teachers, improved their pedagogical strategies and encouraged collaboration. Reflection was identified as a key process in changing the focus of the teachers from procedural to conceptual learning, and hence towards student-centered teaching. However, there were some constraints such as workload and resource limitations that hindered the full exercise of agency. These findings suggest the need for specific and systematic professional development, collaboration, and institutional support in the Philippine setting to enable teachers to teach Statistics and Probability more effectively.

Keywords: teacher agency, graduate course, professional development, teaching statistics and probability

Introduction and Research Background

Teacher agency consists of the professional beliefs, identities, and reflections that educators develop throughout the course of their professional journey (Jenkins, 2020), in the specific case of this research, a graduate course. During the process of guiding graduate students through the Teaching Statistics and Probability course, the researcher gains insights into how their beliefs, identities and reflections influence their teaching practices. Hence, the researcher had the motivation of investigating how these educators experience and develop their agency, and to what extent personal, social and contextual factors are implicated in their professional learning on graduate school.

Personal beliefs and identities are some of the major key concepts to teacher agency. Biesta et al. (2015) have established that when teachers recognize their professional agency, they become more reflective and intentional in their instructional decisions. Moreover, Izadinia (2013) explains that self-reflection enhances professional identity which in turn enhances agency.

Teaching is a social work where educators spend time collaborating with their peers, sharing ideas, modifying their methods, and reassured about their position. According to Kaplan and Garner (2017), the formation of identity is a continuous process during professional communication. The researcher also observes this in his own class where discussions and peer exchanges make teachers to critically examine their approaches and broaden or change their perspectives.

Apart from the personal and social factors, the context of the teaching environment is also a key factor that affects the agency. Some factors include workload, resources, and support from the insti-

tution can either enhance or limit the teachers' agency. Aoyama (2021) explained that professional contexts determine teacher identity, while Jackson (2019) noted that rigid policies specifically on curriculum limit agency by controlling the decision-making authority of teachers. This supports the idea of teachers needing and having both agency and support to succeed.

This research investigates teacher agency dimensions through an evaluation of personal elements (self-related) and social elements (graduate class interactions) and contextual elements (each teacher's classroom environment). The findings are intended to inform the development of empowering professional development programs specifically designed for educators in the Philippine context.

Methodology

This research focuses on 15 junior and senior high school mathematics teachers from three different public schools in Manila, who are also graduate students in their first semester in the Master of Arts in Mathematics Education program of a private university in Manila. They attended 18 3-hour class sessions which covered topics such as Descriptive Statistics, Basic Concepts in Probability, Inferential Statistics and Applications of Statistics and Probability as well as discussing teaching strategies to use with their respective students. To determine how their experiences in the course affected their teacher agency, the researcher used a validated researcher-made questionnaire as the main data collection tool. The questionnaire was checked by graduate school coordinators and some of the researcher's colleagues to check its reliability and validity. This research also guaranteed the anonymity of the responses so that the integrity of the research would not be compromised.

The questionnaire started by gathering basic information about the teacher including their name and school and their current grade level and specific mathematics subjects they teach including Statistics and Probability. The main content of the questionnaire evaluated teacher agency through three distinct sections which study personal elements such as confidence levels and statistical belief systems and teaching practice reflections and social elements including colleague collaboration and professional dialogue effects and contextual elements including resource availability and teaching environment and workload and prior professional development opportunities. It also contained sections that investigate agency expression through curriculum compliance and teaching practice adaptation and open-ended questions to explore innovative approaches and resistance experiences and personal insights and application strategies and required support needs.

Results and Discussion

The analysis of teacher reflections covering different grade levels provides important information about teacher agency development because teachers experience different effects from the graduate course based on their teaching context statistical complexity.

Personal Insights

The "Teaching Statistics and Probability" course improved significantly the teachers' confidence, beliefs, and professional identities. At the beginning, the teachers were different in terms of confidence where some of them were uncertain in their view, since the subject matter was rather complicated; while others were rather confident, but they knew that there was much they could not do. After the semester, most of the teachers stated that they had enhanced confidence, knowledge, and teaching strategies.

This course also helped them to gain a deeper understanding of Statistics and Probability and made them change their perspective from the calculation to the concepts and the application in the real

world. For instance, one of the teachers mentioned this: *"Through this course, my identity as a mathematics teacher has evolved from focusing primarily on computation to emphasizing the conceptual understanding of Statistics and Probability."* This way, teachers also changed their professional identity, from teachers who teach students how to do procedures to the teachers who teach students how to think critically and how to understand and use data. In support of this, one teacher said this: *"I realized that if I won't be able to cover the topics using statistical tool properly it affects their understanding of the competencies in Practical Research 2. I also realized that knowing your data well can guide you in the appropriate statistical tool to use."* Despite the negative factors such as time limitation and lack of resources, they adopted more innovative and student-centered teaching strategies. In the end, the course helped them to teach Statistics and Probability more effectively, and the focus was on meaningful learning.

Social Insights

In the context of social factors, the results showed that collaboration and professional development have a positive impact on teachers' teaching effectiveness in Statistics and Probability. As one of them said, *"[by s]haring of idea[s], sharing of solutions and quicker ways of solving[, w]e are also able to learn from those who have been teaching Statistics and Probability in the higher level."*

Before the course, collaboration was varied - some shared resources and strategies quite often, others had short informal interactions, and still, others taught simpler topics. During the course, the collaborative activities, peer discussions and the mentorship increased the knowledge and capacity of the teachers in teaching and learning. Many got to learn new ways of teaching from their colleagues, how to incorporate technology into teaching and learning and how to solve problems with other teachers. To support this, one of the teachers cited this: *"Engaging in group activities allowed me to experience firsthand how collaborative problem-solving can enhance understanding, as we worked together to analyze data, interpret results, and discuss different approaches."* After the course, most of the teachers indicated a very high willingness to collaborate through discussions, workshops, and peer mentoring. They emphasized the requirement of more systematic ways, such as Learning Action Cells sessions and the technology to support them.

Contextual Insights

The contextual factors (resources, workload and teaching environment) are the major problems and scenario to be considered in teaching Statistics and Probability. The availability of resources is quite different, where better performing schools have been equipped by the government; whereas other public schools have had to contend with poor or broken calculators, restricted access to software, and inadequate technological support, which makes interactive teaching very difficult. Take for example that one of them say this: *"Access to additional teaching resources, such as specialized software or interactive tools, would enhance lesson delivery and student engagement."*

Professional development before this graduate course was often inconsistent and not very profound, missing some advanced statistical methods. However, the structured format of this specific course provided useful training in the use of tools like JAMOVI and basic data analytics on Microsoft Excel, enhancing the teachers' self-efficacy and pedagogical techniques. Nevertheless, many of them mentioned that previous training was rather general mathematics, not specific to Statistics and Probability as one of them pointed out the *"[r]egular workshops or training focused on advanced statistical methods, data analysis, and innovative pedagogical approaches."* In the same manner, workload is another factor that hinders or enhances teachers' application of new strategies in teaching and learning. Some of them were able to balance their other duties while others were not be-

cause of the heavy workloads that did not permit them to try out new pedagogical strategies. As one of them said that *"[t]ime constraints and heavy workloads are also factors affecting the compliance with the curriculum."*

Other collaborative practices such as peer mentoring, interdisciplinary projects, and shared lesson planning were also considered as important. Also, the effectiveness of teaching and learning could be enhanced through adjustments to the curriculum such as well-focused content and regulations on the teaching pace to the rate of student learning.

Synthesis, Conclusions and Recommendations

The graduate course on "Teaching Statistics and Probability" enhanced teacher agency through increased confidence and reflective practice (shifting focus to conceptual understanding) (Suphasri & Chinokul, 2021) and technology integration (Masiello et al., 2023) such as Excel and JAMOVI which transformed teachers' professional identity (Brodie, 2013). The structured peer mentoring and discussions proved vital for collaboration which aligned with community of practice principles (Smith et al., 2017). However, challenges such as resource limitations, heavy workloads, and lack of support hindered the full implementation of new strategies. It is recommended to increase resources, provide targeted training, reduced workload, and improved collaboration to enhance teaching effectiveness in Statistics and Probability in the Philippines.

References

Aoyama, R. (2021). Language Teacher Identity and English Education Policy in Japan: Competing Discourses Surrounding "Non-native" English-speaking Teachers. *RELC Journal*, *54*(3), 788–803.

Biesta, G., Priestley, M., & Robinson, S. (2015). The role of beliefs in teacher agency. *Teachers and Teaching*, *21*(6), 624–640.

Brodie, K. (2013). The power of professional learning communities. *Education as Change*, *17*(1), 5–18.

Izadinia, M. (2013). A review of research on student teachers' professional identity. *British Educational Research Journal*, *39*(4), 694–713.

Jackson, J. (2019). Balancing prescriptiveness and flexibility in the school curriculum. *NSW Curriculum Review*.

Jenkins, G. (2020). Teacher agency: The effects of active and passive responses to curriculum change. *The Australian Educational Researcher*, *47*(1), 167–181.

Kaplan, A., & Garner, J. K. (2017). A complex dynamic systems perspective on identity and its development: The dynamic systems model of role identity. *Developmental Psychology*, *53*(11), 2036–2051.

Masiello, I., Fixsen, D. L., Nordmark, S., Mohseni, Z. (Artemis), Holmberg, K., Rack, J., Davidsson, M., Andersson-Gidlund, T., & Augustsson, H. (2023). Digital transformation in schools of two southern regions of Sweden through implementation-informed approach: A mixed-methods study protocol. *PLOS ONE*, *18*(12), e0296000.

Smith, S. U., Hayes, S., & Shea, P. (2017). A Critical Review of the Use of Wenger's Community of Practice (CoP) Theoretical Framework in Online and Blended Learning Research, 2000-2014. *Online Learning*, *21*(1), Article 1.

Suphasri, P., & Chinokul, S. (2021). Reflective Practice in Teacher Education: Issues, Challenges, and Considerations. *PASAA: Journal of Language Teaching and Learning in Thailand*, *62*, 236–264.

TSG4-2-8

LESSON PLANNING AS A REFLECTION OF PRE-SERVICE MATHEMATICS TEACHERS' MODELING TEACHING COMPETENCE

An Thi Tan Nguyen
Hue University, Vietnam, nttanan@hueuni.edu.vn

Phuong Thi Minh Ta
Hue University, Vietnam, ttmphuong@hueuni.edu.vn

Phuong Thi Minh Ho
Quynhon University, Vietnam, htmphuong@qnu.edu.vn

Thuy Thi Mai Nguyen
The University of Da Nang – University of Science and Education, Vietnam, ntmthuy@ued.udn.vn

This study developed a teacher training program that integrates mathematical modeling and examined its impact on pre-service mathematics teachers' modeling teaching competence through their lesson plans. Based on the literature, a 10-criteria rubric was designed and applied to analyze 20 lesson plans from pre-service math teachers. The results revealed both strengths and weaknesses in their instructional design. The rubric proved to be a valuable tool for both research and practice-oriented reflection. Several recommendations for the teacher training program were proposed to enhance pre-service teachers' competence in teaching mathematical modeling.

Keywords: mathematical modeling, modeling teaching competence, lesson plan, pre-service mathematics teachers.

Introduction

In recent decades, the integration of mathematical modeling into school curricula has gained increasing support as a means to enhance mathematics education in a more practical and applicable way (Kaiser, 2020). Recognizing the importance of connecting learning with real-world contexts, Vietnam's 2018 education reform emphasized that mathematics instruction should go beyond knowledge transmission and focus on developing students' ability to apply mathematics into real-life problem-solving. As part of this reform, modeling competence has been identified as one of the five core mathematical competencies that students must develop in school. This shift presents new challenges for teacher education, particularly in preparing future mathematics teachers to effectively incorporate modeling into their teaching practices (Ferri & Blum, 2009).

A key aspect of effective teaching is lesson planning, which is not only a routine responsibility but also a critical indicator of professional competence. Lesson plans reflect a teacher's ability to structure instruction, design meaningful learning experiences, and implement pedagogical content knowledge in practice (Tillema, 2009; Kitsantas & Baylor, 2001; Baylor, 2002; Yildirim, 2003). However, research on lesson planning as a specific skill among pre-service teachers remains limited (König et al., 2020). This study examines how mathematics pre-service teachers apply their peda-

gogical content knowledge of modeling to design lesson plans after participating in a training program. Furthermore, a specialized analytical tool has been developed to assess the quality of lesson plans focused on modeling-based instruction.

Theoretical Framework

Pedagogical Content Knowledge to Teach Modeling

Modeling is the process of using mathematics to solve real-world problems, in order to gain a deeper understanding of a situation, explore potential solutions, explain phenomena, and support decision-making (Niss et al., 2007). Researchers have represented the modeling process through various cycles, but common stages typically include analyzing and reconstructing the real-world situation, selecting an appropriate mathematical model, working within a mathematical environment, interpreting and evaluating mathematical results in context, refining the model, and iterating the process until a satisfactory solution is achieved.

Teacher knowledge is a key component of professional competence and has been a significant focus of research in recent decades (Baumert et al., 2010). It encompasses various domains, including content knowledge, pedagogical knowledge, and pedagogical content knowledge. Pedagogical content knowledge, in particular, refers to the use of illustrations, examples, explanations, and instructional strategies that help students overcome difficulties and misconceptions, making learning more effective (Shulman, 1987). In other words, pedagogical content knowledge includes the knowledge teachers need to design lesson plans, interact effectively in instructional settings, and understand curriculum structures (Kaiser et al., 2017). Research indicates that teachers with strong pedagogical content knowledge are better equipped to engage students cognitively and enhance their learning experiences (Kunter et al., 2013).

The concept of pedagogical content knowledge highlights the importance of providing pre-service teachers with opportunities to experience mathematical modeling as active learners while equipping them with the necessary knowledge and skills to teach modeling through structured learning tasks. Ferri (2018) identified four key aspects of pedagogical content knowledge essential for teaching modeling:

- Knowledge of the modeling cycle, its role in teaching mathematics, the criteria of the modeling task.
- Knowledge and skills to solve modeling tasks, and anticipate potential difficulties students may face during the process.
- Knowledge and skills in planning lessons that incorporate modeling and providing appropriate guidance and support throughout the modeling process.
- Knowledge and skills to diagnose students' difficulties in different stages of the modeling cycle, as well as evaluate students' performance on modeling tasks.

By fostering these competencies, teacher education programs can better prepare pre-service teachers to integrate modeling effectively into their instructional practices.

Lesson Planning – a Part of Teacher's Professional Competence

The primary responsibility of teachers is instruction, making the development of their professional competence synonymous with enhancing their teaching skills (Kaiser et al., 2017). Teacher training programs often emphasize lesson planning to help pre-service teachers reflect on what to teach, how to teach, and how to assess student learning. Effective lesson planning also allows them to

anticipate potential challenges in the classroom and prepare appropriate support strategies (Baylor, 2002; Yildirim, 2003). Ruys et al. (2012) demonstrated that well-structured lesson plans not only positively impact student achievement by creating more learning opportunities but also enhance the quality of teachers' instructional practices. Tillema (2009) suggests that analyzing lesson plans is a valuable approach for gaining insights into teachers' professional competence. However, empirical studies on lesson planning skills and the assessment of lesson plans among mathematics education students remain limited (König et al., 2020).

During the lesson planning process, teachers need consider several key elements, including learning content, lesson objectives, student characteristics, teaching and assessment methods, and appropriate instructional resources (Mutton et al., 2011). One of the most critical aspects of lesson planning is selecting and designing learning tasks that structure student activities in the classroom, as these tasks integrate various instructional decisions (Kang, 2017). The learning tasks chosen for a lesson reflect how its objectives are defined—what knowledge, skills, or competencies students are expected to develop. Moreover, these tasks should be designed to stimulate student cognition (Neubrand et al., 2013). The way teachers incorporate learning tasks into lesson plans provides valuable insight into their pedagogical flexibility and adaptability.

Methodology

The study was conducted over three semesters, from February 2020 to February 2021, involving 80 third-year students from the Mathematics Department at Hue University of Education, Vietnam. We integrated Ferri's (2018) four aspects of pedagogical content knowledge for teaching mathematical modeling into four math teaching methods courses in the third and fourth years of the teacher training program. Specifically, students engaged in examining real-world problems in textbooks from various countries, understanding the modeling cycle, analyzing and solving modeling tasks, designing modeling tasks, planning integrated modeling lessons, conducting modeling projects, and assessing modeling products.

Data collection. In February 2021, students worked in groups of four to develop lesson plans that integrated mathematical modeling, either to introduce new concepts or to apply previously learned knowledge. Each group selected a high school-level lesson and incorporated pedagogical content knowledge for teaching modeling into their instruction. A total of 20 lesson plans were collected during this phase.

Data analysis. Based on the comprehensive literature review on the guidelines for teaching mathematical modeling proposed by Ferri (2018), we developed an assessment scale to evaluate pre-service teachers' ability to integrate modeling into their classrooms through their lesson plans. This process consisted of three stages: (1) Identifying key aspects - We determined the factors influencing the effectiveness of modeling instruction to establish evaluation criteria, including objectives (C7), task design (C1, C2, C3), the modeling process (C4, C10), diagnosis and guidance strategies (C5, C6), the organization of implementation (C8, C9); (2) Expert review and refinement - Four researchers with expertise in mathematical modeling and teacher education reviewed the content for validity and suggested revisions, leading to a near-final version; (3) Pilot analysis and final adjustments: Two researchers used the final version to analyze two sample lesson plans, discussing their scoring process and justifications. Minor refinements were made to the criteria descriptions, making them more specific and precise.

The final assessment scale consists of ten criteria (C):

- (C1) The modeling problem aligns with students' mathematical knowledge, allowing them to solve it using their prior understanding.
- (C2) The context is engaging, realistic, understandable, and relatable to students.
- (C3) The problem is authentic and clearly stated.
- (C4) The problem-solving process follows all stages of the modeling cycle.
- (C5) Anticipation of possible solution approaches and assessment strategies.
- (C6) Preparation of appropriate intervention and guidance strategies at different levels.
- (C7) The lesson achieves its intended objectives, such as mathematical understanding, applying mathematics, or improving specific modeling competencies.
- (C8) Adequate time is allocated for students to engage in problem-solving.
- (C9) The implementation is explicitly described.
- (C10) The lesson encourages students' metacognitive activities, such as reflection, validation, and exploring alternative solutions.

Evaluation is based on the presence and clarity of each criterion. Each criterion is scored on a 0–2 scale (0 = Not present; 1 = Needs improvement; 2 = Meets expectations)

This assessment framework provides a structured approach to evaluating pre-service teachers' competency in designing modeling-based lessons, ensuring alignment with effective instructional practices.

Results

Using the assessment scale for integrating mathematical modeling into teaching, as described earlier, we coded the student groups' lesson plans into three levels: 0, 1, and 2. The results are presented in Figure 1.

Based on the statistical results, all student groups scored above the average level for integrating mathematical modeling into teaching, with total scores of 10 or higher. Specifically, five groups achieved scores between 15 and 18 (out of 20), while the remaining 15 groups scored between 10 and 14. Additionally, students performed exceptionally well in criteria 1, 7, 5, and 8. However, their scores were notably low in criteria 10 and 6. For all other criteria, students achieved above-average performance. In the following section, we will analyze each criterion in detail, examining how student groups demonstrated their ability to integrate mathematical modeling into teaching through their lesson plans.

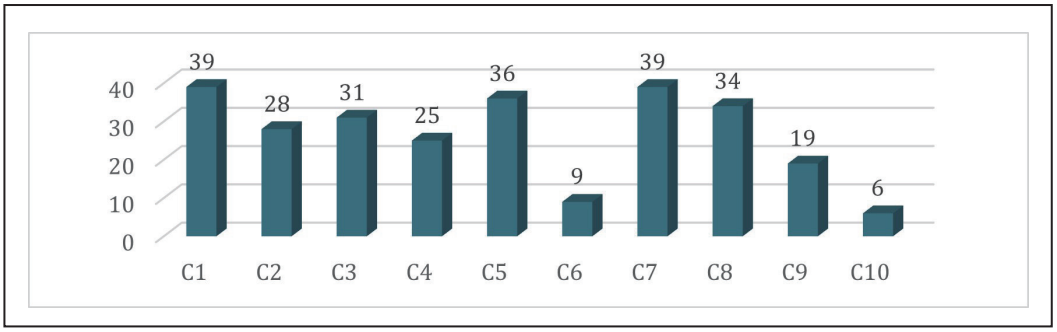


Figure 1. Scores for integrating mathematical modeling into teaching

Most student groups met Criterion 1 by proposing modeling problems aligned with students' mathematical knowledge and solvable using prior knowledge, except for Group 3, where the problem was more suitable for 8th-grade rather than 11th-grade students. For Criterion 2, all groups created engaging and relatable contexts, but only 8 out of 20 achieved the highest score for real-world relevance, while 12 groups scored lower due to unrealistic scenarios. For example, Group 3 overlooked door areas when calculating the painted wall area, and Group 5 framed the problem as a word problem, making it easier for learners to detach from the real-world context. In Criterion 3, 9 out of 20 groups scored only 1 point due to unrealistic, unclear problem statements, or reliance on mathematical rather than everyday language. For Criterion 4, only 5 out of 20 groups guided students through the entire modeling process, with teachers handling the model-building step, while the remaining 15 groups either started with a given mathematical model or omitted student reflection on mathematical results. In Criterion 5, 16 out of 20 groups earned full marks for proposing appropriate solutions, while Groups 3, 5, 10, and 14 scored lower due to incomplete or vague approaches. Most groups presented only one solution, except for Group 12, which proposed three. For Criterion 6, only Group 12 received full marks by providing structured guidance using thought-provoking questions, anticipating student responses to support flexible learning, while 12 groups scored 0 for lacking student support during problem-solving. In Criterion 7, most groups met the instructional goal of applying or introducing new knowledge, except for Group 3, which scored 1 point due to unrelated lesson content. Among the 19 groups that earned full marks, 5 used modeling to teach new concepts, while 14 applied it to real-world problem-solving, reinforcing the relevance of mathematics. For Criteria 8 and 9, most groups implemented modeling activities through group work, but some presented problems as word problems, causing students to disengage from real-world contexts and affecting time allocation. Group organization was often generic, lacking optimal group sizes for effective collaboration. In Criterion 10, no group provided questions or guidance for students to evaluate the accuracy or efficiency of their modeling process or explore alternative solutions, though six groups (1, 2, 3, 4, 12, and 13) considered the real-world validity of their results.

Conclusion

This study developed an assessment tool to analyze pre-service teachers' lesson plans on related to teaching mathematical modeling. This tool can also serve as a valuable resource for identifying strengths and weaknesses in modeling-based instruction within teacher training programs. The findings show that most pre-service teachers designed modeling problems aligned with students' knowledge and proposed appropriate problem-solving approaches, achieving the intended learning goals.

However, some challenges remain. Many struggled to create authentic modeling scenarios, often using unrealistic or overly mathematical problems. They also tended to guide students through only parts of the modeling process, lacked strategies to support students in overcoming difficulties, and gave little attention to student reflection on results. Addressing these issues in teacher training can enhance their ability to teach modeling effectively.

References

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., & Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*(1), 133–180.

Baylor, A.L. (2002) Expanding preservice teachers' metacognitive awareness of instructional planning through pedagogical agents. *Educational Technology Research and Development*, 50(2), 5–22.

Ferri, R. B. (2018). Instructional Competency: For Effective and Quality Lessons. In *Learning How to Teach Mathematical Modeling in School and Teacher Education* (pp. 77-102). Springer, Cham.

Ferri, R. B., & Blum, W. (2010). Mathematical modelling in teacher education–experiences from a modelling seminar. In *Proceedings of the sixth Congress of the European Society for Research in Mathematics Education* (pp. 2046-2055).

Kaiser, G., Blömeke, S., König, J., Busse, A., Döhrmann, M., & Hoth, J. (2017). Professional competencies of (prospective) mathematics teachers—Cognitive versus situated approaches. *Educational studies in mathematics*, *94*, 161-182.

Kaiser, G. (2020). Mathematical modeling and applications in education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 553-561). Springer.

Kang, H. (2017). Preservice teachers' learning to plan intellectually challenging tasks. *Journal of Teacher Education*, *68*(1), 55-68.

Kitsantas, A. & Baylor, A. (2001) The impact of the Instructional Planning Self-Reflective Tool on preservice teacher performance, disposition, and self-efficacy beliefs regarding systematic instructional planning. *Educational Technology Research and Development*, 49(4), 97–106.

König, J., Bremerich-Vos, A., Buchholtz, C., & Glutsch, N. (2020). General pedagogical knowledge, pedagogical adaptivity in written lesson plans, and instructional practice among preservice teachers. *Journal of Curriculum Studies*, 52(6), 800-822.

Kunter, M., Baumert, J., Blum, W., Klusmann, U., Krauss, S., & Neubrand, M. (2013). *Cognitive activation in the mathematics classroom and professional competence of teachers: Results from the COACTIV project*. Springer Science & Business Media.

Mutton, T., Hagger, H., & Burn, K. (2011). Learning to plan, planning to learn: The developing expertise of beginning teachers. *Teachers and Teaching*, 17(4), 399–416.

Neubrand, M., Jordan, A., Krauss, S., Blum, W., & Löwen, K. (2013). Task analysis in COACTIV: Examining the potential for cognitive activation in German mathematics classrooms. In M. Kunter, J. Baumert, W. Blum,U. Klusmann, S. Krauss, & M. Neubrand (Eds.), *Cognitive activation in the mathematics classroom and professional competence of teachers* (pp. 125–144). Springer.

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education: The 14th ICMI study* (Vol. 10, pp. 3-32). Springer.

Ruys, I., Keer, H. V., & Aelterman, A. (2012). Examining pre-service teacher competence in lesson planning pertaining to collaborative learning. *Journal of Curriculum Studies*, *44*(3), 349-379.

Shulman, L. S. (1987). Knowledge and teaching: foundations of the new reform. *Harvard Educational Research*, 57, 1–22

Tillema, H.H. (2009) Assessment for learning to teach: Appraisal of practice teaching lessons by mentors, supervisors, and student teachers. *Journal of Teacher Education*, 60, 155–167.

Yildirim, A. (2003) Instructional planning in a centralized school system: lessons of a study among primary school teachers in Turkey. *International Review of Education*, 49(5), 525–543.

TSG4-2-9

PERCEPTIONS AND PRACTICES OF FILIPINO JUNIOR HIGH SCHOOL MATHEMATICS TEACHERS ON PROBLEM-POSING

John Patrick Cultura
La Salle College Antipolo, Philippines, patrick.cultura@lsca.edu.ph

Flordeliza Ferrer
Taguig City University, Philippines, flordeliza.ferrer@tcu.edu.ph

Vanessa Zubieta
South Luzon State University, Philippines, vzubieta@slsu.edu.ph

Christine Nicole Victorio
Ateneo de Manila University, National University Philippines, christine.victorio@student.ateneo.edu

Bryan Ceasar Felipe
Central Luzon State University, Philippines, bryanceasar.felipe@clsu2.edu.ph

Lester Hao
Ateneo de Manila University, Philippines, lhao@ateneo.edu

Mary Jane Castilla
University of Sto. Tomas, Philippines, macastilla@ust.edu.ph

Rodolfo Calimlim Jr.
University of Baguio High School, Philippines, rfcalimlimjr@e.ubaguio.edu

Joseph Ma. Steven Cabalo
Department of Education Schools Division of Lipa City, Ateneo de Manila University, Philippines
joseph.cabalo@deped.gov.ph

Flordeliza Francisco
Far Eastern University, Philippines, ffrancisco@feu.edu.ph

The present paper reports on a survey administered to Filipino mathematics teachers on their problem-posing perceptions and practices. It is part of an ongoing larger project that aims to promote problem-posing in the classroom by designing and implementing a professional development program for Filipino mathematics teachers. Sixty-six (66) junior high school Filipino teachers based in Luzon responded to the online survey. Results show that the respondents have a good grasp of what problem-posing is; nonetheless, possible hindrances have been identified that limit their use of problem-posing in the classroom.

Keywords: problem-posing, perception, practice

Introduction

Mathematics education plays a crucial role in the development of students' creativity, critical thinking, and problem-solving (PS) skills. Calabrese et al. (2022) argued for problem-posing (PP) as one of the innovative approaches in developing the aforementioned. They reiterated how PP could be utilized as a pedagogical strategy by creating new problems or modifying existing ones, or as an assessment strategy by checking for students' understanding of concepts. Moreover, teaching supported by PP strategies has been shown to improve students' PP and PS skills, as well as their ability to approach tasks of varying structure and complexity (Divrik, 2023). In relation, Sangco et al. (2021) found that their sample of Filipino students who were exposed to PP performed much better than their counterparts who received traditional forms of instruction. Recognizing the potential of utilizing PP in Filipino mathematics classrooms, the Philippine Council of Mathematics Teacher Educators, Inc. (MATHTED) launched a special interest group (SIG) on problem-solving and problem-posing (Vistro-Yu, 2023), in which the authors are members. The SIG intends to promote problem-posing through professional development programs and research, to which the present paper reports on the initial survey conducted by the team to better understand the perceptions and practices of Filipino in-service teachers in the use of PP in teaching mathematics to determine gaps in resources and training to effectively support mathematics teachers in utilizing PP.

Methodology

An online survey was designed and administered by the authors through Google Forms. They invited several junior high schools in Luzon to participate in the study to which 66 teacher-respondents completed the survey. The survey encompassed two sections: (1) perceptions on PP, and (2) practice of PP. Respondents were first asked to indicate the type of institution they belong to (i.e., public, private, science high school). The section on perceptions featured fifteen (15) Likert-scale items which the respondents rated according to their strength of agreement (i.e., 1 as strongly disagree, 2 as disagree, 3 as agree, 4 as strongly agree). A sample statement is "Students generating their own mathematical problems can learn more about the topic being discussed." The section concluded with an open-response item that inquired what PP meant for them. Similarly, the section on practice featured fifteen (15) rating-scale items which the respondents rated based on frequency (i.e., 1 as never, 2 as sometimes, 3 as often, 4 as always). A sample statement in this section is "I explicitly teach students how to design their own questions or problems." Likewise, this section concluded with an open-response item in which respondents were asked whether they have incorporated PP in their classes, and the reasons for doing so or not.

Further, the questions in each section are divided into three clusters namely, (1) Instructional Design (ID), (2) Instructional Management (IM), and (3) Instructional Assessment (IA). These clusters correspond to the use of PP in these key areas of teaching and learning. The clusters highlight that teachers should view themselves as designers who facilitate learning through PP (Henriksen and Richardson, 2017), with PP tasks serving as an effective tool to assess mathematical comprehension skills (Lin, 2004). ID questions target to know the ability of teachers to plan effective strategies in incorporating PP in teaching mathematics. Meanwhile, IM questions inquire the manner in which teachers facilitate the use of PP in the classroom. Lastly, IA pertains to the practice of evaluating learning outcomes on PP. The Delphi technique (Sablatzky, 2022) was employed by the researchers to establish the content and face validity of the instrument. They also conducted a reliability test on the two sections showing high reliability using Cronbach Alpha on perception on PP (0.936) and on practices (0.946). After the conduct of the online survey, the mode of each scale item for both

sections was determined, whereas responses to the open-response items were qualitatively analyzed for interesting patterns.

Results and Discussion

The teacher-respondents represented various institutional types. The majority (65%) were from public junior high schools, while 33% were from private junior high schools, and 2% were from science high schools. Nonetheless, the following data reports only the survey responses of the entire sample (n = 66) in the three key areas (i.e., ID, IM, IA).

Teachers' Perceptions on Problem-Posing

Teachers generally rated the three (3) key areas with 'Agree'. This means that the teachers have a general understanding of what PP is and how it is designed, implemented, and assessed in teaching mathematics. In the ID of PP, the statement "I feel confident in designing tasks that allow students to create their own mathematical problems" garnered the highest frequency of agreement. This indicates that the respondents have the confidence in creating learning tasks and materials that allow students to generate their own problems. However, when it comes to collaboration in developing students' ability to generate their own problem, the teachers agreed the least to this statement. This could possibly mean that there is little to no opportunity for teachers to collaborate and incorporate PP in their learning design. Tichá and Hošpesová (2013, as cited in Papadopoulos et al., 2022) mentioned that in teacher training, PP is essential in pointing out flaws, mistakes, and misconceptions. Further, in terms of IM, most of the respondents agree that they are comfortable in employing teaching methods that tasks students to come up with their own mathematical problems. To add more confidence in designing and implementing tasks, it is deemed necessary to conduct workshops related to PP (Li, Sun & Cai, 2022). Nonetheless, it was noticeable that some teachers perceive employing PP tasks in teaching as time consuming. As for IA, teachers generally agree that they can be a more effective teacher if they allow students to generate their own problems but some of them perceive that they are not easy to assess.

The open-response item on their perceived understanding of PP revealed that most teacher-respondents view it as a tool that makes students take ownership of their learning. This highlights PP's potential to deepen conceptual understanding, enhance critical thinking and problem-solving skills, and serve as an effective assessment strategy. A public school respondent described PP as a method that encourages students to create their own mathematical problems, while a private school teacher emphasized its role in engaging students in designing problems to foster higher-order thinking. A science high school teacher defined PP as a means to assess student understanding through problem construction. These are parallel to their understanding of PP in each of the three key areas. However, some respondents interpret PP as simply identifying a problem, while others equate it to a routine classroom exercise. Practical concerns also emerged regarding its implementation.

Teachers' Practices on Problem-Posing

The teachers often practice the statements indicated in the three key areas. In ID, teachers often design open-ended problems that allow multiple solutions. Yet, monitoring of lesson plans with PP is not done often. In IM, teachers are used to transforming textbook problems into similar problems by allowing students to create their own by modifying existing ones. They sometimes explicitly teach students how to design their own questions or problems. A study conducted by Cai, et al. (2022) revealed that the teachers' most frequently cited challenge was students' inability to pose interest-

ing mathematical problems. Lastly, in IA, it is notable that most of them often include the creation of questions or problems in their formative or summative assessments. Likewise, they often ask students to reflect on their own mathematical problems. However, in practice, most of them find it sometimes difficult to assess these problems; this aligns with their perception that student-generated problems are difficult to assess.

In the use of PP in the classroom, most respondents have provided opportunities for students to generate their own mathematical problems. They noted that effective problem-posing tasks require students to draw on prior knowledge, apply real-life experiences, and collaborate with peers in designing and solving problems. Some respondents also viewed problem posing as a means to assess student learning, while others highlighted its role in enhancing engagement and improving topic comprehension. Teachers who frequently incorporate PP shared the strategies, criteria, and scaffolding techniques they use to support students. In contrast, those who rarely or never implement PP cited time constraints, gaps in students' mathematical abilities, and challenges related to students' proficiency in constructing questions in English as key barriers.

Conclusion

Based on our sample, it could be said that Filipino Junior High School teachers generally know what problem-posing is, its definition and possible functions inside the classroom. In terms of their perception and practice in instructional design, they feel confident in producing problems that allow multiple solutions. Yet, collaboration and monitoring of lessons with integrated PP tasks were not practiced as often. Teachers perceive that they are comfortable in employing PP tasks and can manage their classroom properly. The use of textbooks and examples are common practices of teachers that enable students to generate their own problems, yet generating problems remains a challenge. In terms of assessment, teachers believe that incorporating PP is an essential practice. Reflecting on their generated problem is also commonly done. Nonetheless, it was mentioned that assessing these problems is not always easy and is sometimes time consuming.

Teachers' perceptions and practices show promise in advancing PP in math instruction. The findings will guide the SIG in upskilling teachers to design, implement, and assess PP-infused learning. Capacity-building, with PP as a key strategy to boost engagement and understanding. Moreover, the group will likewise explore improving the survey instrument to help gauge more focused aspects of PP, such as assessment of student-generated problems.

References

Calabrese, J., Capraro, M. M., & Thompson, C. (2022). The relationship between problem posing and problem solving: A systematic review. *International Education Studies*, *15*(4), 1–8.

Cai, J., et al. (2020). Exploring the impact of a problem-posing workshop on elementary school mathematics teachers' conceptions on problem posing and lesson design. *International Journal of Educational Research,* 3–4.

Divrik, R. (2023). Effect of teaching mathematics supported by problem-posing strategies on problem-posing skills. *International Journal of Modern Education Studies*, *7*(2), 371–408.

Henriksen, D., & Richardson, C. (2017). *Teachers are designers: Addressing problems of practice in education.* Phi Delta Kappan, *99*(2), 60–64.

Li, X., Sun, X., & Cai, J. (2022). Integrating problem posing into the mathematics classroom: current advances and future directions of research. https://doi.org/10.1016/j.cobeha.2022.101213

Lin, P. J. (2004). Supporting teachers on designing problem-posing tasks as a tool of assessment to understand students' mathematical learning. In *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 3, 257–264.

Papadopoulos, I., Patsiala, N., Baumanns, L., & Rott, B., (2022). Multiple approaches to problem posing: Theoretical considerations regarding its definition, conceptualisation, and implementation. *CEPS Journal*, https://doi.org/10.26529/cepsj.878

Sablatzky, T. (2022). The Delphi Method. *Hypothesis: Research Journal for Health Information Professionals, 34*(1). https://doi.org/10.18060/26224

Sangco, S., Elpidang, E., & Sangco, R. M. (2021). Problem Posing Strategy: Effect on Students' Mathematical Performance and Anxiety. *Surigao del Sur State University Multidisciplinary Research Journal*, 9, 1–4.

Vistro-Yu, C. P. (Ed.). (2023). About MATHTED's special interest groups (SIGs). *Intersection, 12*(2), 31–33.

Acknowledgment

The authors would like to express their utmost gratitude to the Philippine Council of Mathematics Teacher Educators (MATHTED), Inc. for providing them the opportunity and platform to collaborate.

TSG4-2-10

TAIWANESE SECONDARY MATHEMATICS TEACHERS' PERSPECTIVES ON CALCULUS INSTRUCTION: INSIGHTS FROM THE CURRICULAR AND CONTEXTUAL FACTORS

Kai-Lin Yang
National Taiwan Normal University, Taiwan, kailin@ntnu.edu.tw

Pin-Chen Guo
National Taiwan Normal University, Taiwan, pcguo@ntnu.edu.tw

Teachers' perspectives act as a bridge between the official and operational curricula, yet their perspectives on calculus instruction remain underexplored in non-Western contexts and with the consideration of curricular and contextual factors. Guided by the curriculum enactment framework, this study conducted semi-structured interviews to investigate Taiwanese secondary mathematics teachers' perspectives on calculus instruction and employed thematic analysis to form various themes. The findings reveal that teachers' perspectives involve considering factors such as curricular tracks, cross-disciplinary structures, university entrance exams, textbooks, classroom identities, and students' outcomes and futures. These findings broaden our understanding of teachers' perspectives in the curriculum enactment process and provide a holistic view of the challenges and opportunities in calculus instruction. Future research could further explore how professional development programs and policy adjustments can support teachers in navigating these complexities.

Keywords: Teachers' Perspectives, Eastern Countries, Calculus Instruction, Curricular and Contextual Factors

Introduction and Theoretical Framework

In the process of curriculum enactment, teachers' perspectives on mathematics teaching and learning act as a bridge between the official and operational curricula (Remillard & Heck, 2014). With high school calculus becoming a central issue in mathematics education (Thompson & Harel, 2021), exploring secondary mathematics teachers' perspectives on calculus instruction holds significant implications for curriculum design and implementation. While previous studies (e.g., Eichler & Erens, 2014) have categorized teachers' orientations (or beliefs) into several types, they pointed out that their findings face limitations in generalizability due to a predominant focus on Western contexts, leaving a gap in understanding teachers' perspectives in Eastern settings. Furthermore, much of the existing research centers exclusively on teachers themselves, overlooking how curricular and contextual factors may shape their perspectives. Given that "all beliefs are situated—dialectical constructions, products of activity, context, and culture" (Hoyles, 1992, p. 40), this study aims to deepen the understanding of teachers' perspectives on calculus instruction in Eastern countries and shed light on how the curricular and contextual factors are considered in teachers' perspectives.

Teachers' perspectives can be counted as "an inter-related set of beliefs and intentions that gives direction and justification to our actions" (Pratt et al., 2001, p. 71). As instruction involves the

integration of instructional goals, teaching methods, and assessment, we aim to investigate how teachers think about these elements, particularly under the curricular and contextual factors. To do so, we adopt the comprehensive framework proposed by Remillard and Heck (2014), which describes how the curriculum is enacted and transitions from official policy to classroom practices. In their framework, the official curriculum includes curricular aims and objectives (e.g., national curriculum guidelines), the content of consequential assessments (e.g., entrance examinations), and the designated curriculum (e.g., approved textbooks); and the operational curriculum encompasses teacher-intended curriculum (e.g., teachers' decisions), the enacted curriculum (e.g., classroom interactions), and student outcomes (e.g., students' understanding). In particular, the teacher-intended curriculum connects the official and operational curricula and is influenced by teachers' knowledge, beliefs, and practices. Therefore, exploring teachers' perspectives through the lens of this framework has the potential to reveal how teachers formulate their perspectives in the interaction with curriculum policy and classroom practices within specific contexts.

Methodology

This study is part of a larger project investigating the written curriculum, students' performance, and teachers' perspectives on the teaching and learning of the derivative concept. To gain a broader understanding, we extended our interviews to explore teachers' perspectives on calculus instruction in general rather than solely on the derivative concept. Specifically, we conducted semi-structured interviews with 14 secondary high school mathematics teachers (seven male, seven female) from nine different schools. Three teachers came from top-tier schools, eight from upper-middle schools, and three from mid-tier schools. Using purposive sampling, we required all participants to have taught calculus in at least four different classes. Under this criterion, three teachers had less than 10 years of teaching experience, six had between 10 and 20 years, and five had over 20 years. Given the distinct tracks in Taiwan's high school mathematics curriculum (Ministry of Education, 2018), we also examined whether teachers had experience teaching students from science and from social science streams, with nine participants reporting such experience.

The interview addresses three main areas: the nature of concepts (with curriculum materials and goals), teaching methods, and assessment approaches. Some interview questions were adapted from Eichler and Erens (2014), e.g., teachers were asked to share their levels of agreement with the following statements: when teaching calculus, the most important is to enable students (1) to formulate calculus concepts through their thinking process; (2) to recognize the relevance of calculus concepts to their daily lives; (3) to understand concepts in a rigorous and formal manner; or (4) to become familiar with the procedures for differentiation and integration. The interviews were conducted by the second author and three secondary mathematics teachers, and the data were analyzed and discussed collaboratively by all interviewers under the first author's guidance. It was noted that teachers were interviewed without direct reference or guidance to specific curricular or contextual factors, allowing them to express their perspectives independently.

Findings

As demonstrated directly by one teacher's response "My level of agreement and the teaching approach I choose after understanding my school and my students would probably result in different ratings" (T076), teachers' perspectives may vary across various curricular and contextual factors:

Teachers' Perspectives Related to the Official Curriculum

First, regarding the curricular aims and objectives, the Taiwanese national curriculum guideline allows senior high school students to choose between mathematical tracks with different difficulties. Teacher interviews reveal that some teachers hold distinct expectations and teaching approaches for students in different tracks. For instance, for the rigorous understanding of concepts, one teacher said,

> It might depend on the academic track. For the science track, I think it's necessary. As for the social sciences track, of course, when I present it, it's rigorous and formal, but whether they accept it or not, I cannot force them (P137).

Apart from different learning tracks, the cross-disciplinary structure of the curriculum also influences teachers' perspectives. For example, regarding the statement, "When teaching the derivative, one can first learn the computational rules and then the underlying concepts," one teacher shared,

> Actually, I have two perspectives. If we consider the current placement of this topic in the 12th grade, I feel it's not appropriate, and I would give it a two (disagree). But under the current curriculum guidelines, especially in physics …… if we consider the curriculum more broadly—not just the mathematics curriculum but also other subjects—this approach does have its necessity (agree) (C055).

Second, regarding the consequential assessments, Taiwanese students face a critical university entrance exam shortly after the first semester of their 12^{th} grade, which is when calculus is taught. Many teachers reflected that the dual need to teach calculus while also preparing students for this exam creates significant time pressure, which in turn influences their perspectives on calculus instruction. It is noted that calculus is not included in the exam's scope, posing a challenge for instruction as students lack the motivation to engage with the subject. Some teachers mentioned that, without the constraints of exams, they could emphasize conceptual development, but under current conditions, they are forced to prioritize computational skills due to the limited time available.

Finally, regarding the designated curriculum, most teachers referenced textbooks approved by the Ministry of Education as their primary teaching resource. For example, in terms of the application of calculus in real life, several teachers expressed, "If it's in the textbook, I'll teach it," highlighting how different instructional materials influence teaching practices. Even when interviewers encouraged teachers to step outside the bounds of the official curriculum by asking questions like, "Apart from what's written in the textbooks or curriculum guidelines, what do you personally think?" many teachers still responded with remarks such as, "I just teach according to the textbooks." This indicates that textbooks significantly shape teachers' instructional perspectives.

Teachers' Perspectives Related to the Operational Curriculum

First, regarding the enacted curriculum, teachers mentioned that while rigor is important, overemphasizing it in the classroom might make the lessons feel tedious. As one teacher stated,

> I think this is very important, but …… if you emphasize the rigorous and formal approach too much, students might actually find it boring. …… For teachers, it's more important. Teachers need to understand in a very rigorous and formal way. For students, I think that's not as crucial (H040).

This excerpt also illustrates how differing identities (teachers or students) may shape their perspectives on understanding mathematics. Thus, the challenge lies in how teachers reconcile these contrasting perspectives in the classroom, as two teachers shared different perspectives on the importance of the procedural calculation: "I personally would rate it as a three, but for the students,

it should probably be a four or even a five" (T084) and "The fortunate thing for me is that I don't have to emphasize this; the students will pursue it on their own" (J128).

Second, regarding student outcomes, teachers pointed out the benefits of mastering procedural calculations, which include "the key to earning scores" to fostering a "sense of achievement in learning" (M142) and the fact that "you still need a high level of accuracy in calculations, because even if your concepts are clear and you know how to approach the problem, you might still feel frustrated if the result is incorrect" (Y147). In other words, while previous (Western) literature often views an exclusive emphasis on procedural calculations as a negative, teachers in the Taiwanese context seem to argue that emphasizing procedural accuracy can have positive effects on students, both in terms of academic performance and emotional engagement.

Furthermore, it is notable that some teachers extended their thoughts about students' future career aspirations. For example, many teachers mentioned the difference in attitude between students pursuing mathematics-related majors and those not intending to pursue math-related fields when discussing rigor. As one teacher shared, "I believe every student can have their own different attitude towards calculus" (K093), and he elaborated with examples of students planning to pursue academic paths in science, engineering, or business. This shows how teachers' perspectives about the curriculum also intersect with their consideration of students' future academic and career paths.

Discussion and Conclusion

This study provides a nuanced understanding of Taiwanese secondary mathematics teachers' perspectives on calculus instruction, influenced by curricular and contextual factors including curricular tracks, cross-disciplinary structures, university entrance exams, textbooks, classroom identities, and students' outcomes and futures. These findings broaden the scope of the influential factors summarized by Remillard and Heck (2014) and extend beyond Eichler and Erens' (2014) categorizations of teachers' beliefs towards teaching calculus by demonstrating how teachers' perspectives may vary depending on specific conditions. By integrating teachers' perspectives on official and operational curricula, the study provides a holistic view of the challenges and opportunities in calculus instruction. Future research could further explore how professional development programs and policy adjustments can support teachers in navigating these complexities.

Acknowledgement

We would like to thank the three interviewers: Yun-Ting Lai, Ya-Wen Lo, and Min-Xiang Zhang. This paper, part of a research project funded by the National Science and Technology Council (NSTC 113-2410-H-003-028-MY3), reflects the author's views, not necessarily those of the NSTC.

References

Eichler, A., & Erens, R. (2014). Teachers' beliefs towards teaching calculus. *ZDM, 46*(4), 647-659.

Hoyles, C. (1992). Mathematics teaching and mathematics teachers: A meta-case study. *For the Learning of Mathematics, 12*(3), 32-44.

Ministry of Education. (2018). *Curriculum guidelines for the 12-year basic education elementary school, junior high school, and upper secondary school: The domain of mathematics*. https://cirn.moe.edu.tw/Upload/file/40548/112074.pdf

Pratt, D. D., Arseneau, R., & Collins, J. B. (2001). Reconsidering “good teaching” across the continuum of medical education. *Journal of Continuing Education in the Health Professions, 21*(2), 70-81.

Remillard, J. T., & Heck, D. J. (2014). Conceptualizing the curriculum enactment process in mathematics education. *ZDM, 46*(5), 705-718.

Thompson, P. W., & Harel, G. (2021). Ideas foundational to calculus learning and their links to students’ difficulties. *ZDM, 53*(3), 507-519.

TSG4-2-11

CONCEPTIONS OF MATH TEACHING AND LEARNING IN CONVERSATIONS ABOUT CURRICULAR CHANGE

Grace A. Chen
New York University, USA, grace.a.chen@nyu.edu

Nurdan Turan
New York University, USA, nt2124@nyu.edu

Jasmine Y. Ma
New York University, USA, j.ma@nyu.edu

This study explores how teachers connect math teaching and math learning while implementing curricular change. Using social practice theory and interaction analysis methods, we analyzed math teachers' planning meetings and found that learning was always described as dependent on teaching. Teaching and learning could proceed discontinuously, continuously, or in a spiral, but were most often described in highly-sequential ways. Findings have implications for how teachers are supported in understanding and enacting curricular change.

Keywords: curriculum implementation, teacher collaboration, teachers' conceptions, teaching and learning

Research has shown that when teachers conceive of their content as highly sequential, or organized in a fixed order, it is both harder for them to adapt to curricular change and more likely that they view students through deficit lenses; this is particularly true for math teachers (Horn, 2007; Stodolsky & Grossman, 2000). Our study extends this work to explore how teachers connect math teaching and math *learning*– including math content and also how pedagogy and socio-emotional needs affect teaching and learning– in the context of curricular change. Specifically, as teachers adopt problem-based curricula intended to cultivate flexible mathematical thinking while also navigating district demands for more rigid systems, what conceptions of math teaching and learning arise in their conversations? How sequential are these conceptions, and what are the implications?

Theoretical Framework

In social practice theory, learning is defined as an inherently social and collective phenomenon which is "an aspect of changing participation in changing 'communities of practice' everywhere" (Lave, 1996, p. 150). This theory of learning recognizes people's histories and the socially and dialectically constructed nature of practices and identities as people shift their participation. Social practice theory informs this research in two ways. First, Lave (1996) argues that teaching is not always necessary to produce learning; learning is always happening wherever people engage in doing things, even if what is being learned is different from what the teacher intends. Therefore, understanding math education requires analytical attention to teachers' ideas about what and how stu-

dents need to be explicitly taught to learn math. Second, social practice theory suggests that methodologically, teachers' conceptions of math teaching and learning can be studied through teachers' participation and practices (such as in collaborative planning meetings) and not just by looking for their individual underlying beliefs. Focusing on teachers' participation and practices allows for examination of the ideas generated through interaction and the consequences they have on subsequent interactions– and ultimately, on student learning.

Research Methods

City Middle School (CMS; all names are pseudonyms) is a public school located in one of the largest and most diverse cities in the United States. Any student in the city can attend regardless of academic background, and students are often drawn to its arts focus and intimate size (~200 students in grades 6-8, roughly ages 12-14). In 2023, CMS's district mandated the adoption of Illustrative Mathematics (IM), a problem-based curriculum that encourages students to discuss and make sense of mathematical concepts in small groups. The district has offered IM-aligned professional development (PD) for teachers, but students (and teachers) are still evaluated on state and local standardized tests with traditional, mostly-procedural questions.

In 2024, the district also mandated the Multi-Tiered System of Supports (MTSS), a program that aims to support struggling students by dividing them into tiers based on test results. Soon after the MTSS mandate, the CMS principal, Hathaway, and one math teacher, Nancy, attended a district MTSS PD. Then, they met with another math teacher, Frank, to plan how to share PD content with other CMS teachers. One researcher attended and video-recorded this virtual meeting. The research team then reviewed the video and chose three episodes in which ideas linking math teaching to math learning arose in relation to implementing IM and/or MTSS. The team used methods of interaction analysis (Jordan & Henderson, 1995) to collaboratively watch and rewatch the episodes to find the assumptions underlying these ideas and how they were taken up, modified, or rejected.

Findings

CMS educators presented the relationship between math teaching and learning in three ways: *discontinuous*, where discrete objectives must be taught and learned before subsequent objectives (Figure 1a); *continuous*, where math objectives were taught and learned sequentially but with connections between them (Figure 1b); and as a *spiral*, where multiple math skills could be taught and learned simultaneously and revisited over time (Figure 1c). All three portrayed students' math learning as dependent first and foremost on math teaching (see Discussion for Figure 1d). Next, we explain these three conceptions of the relationship between math learning and teaching.

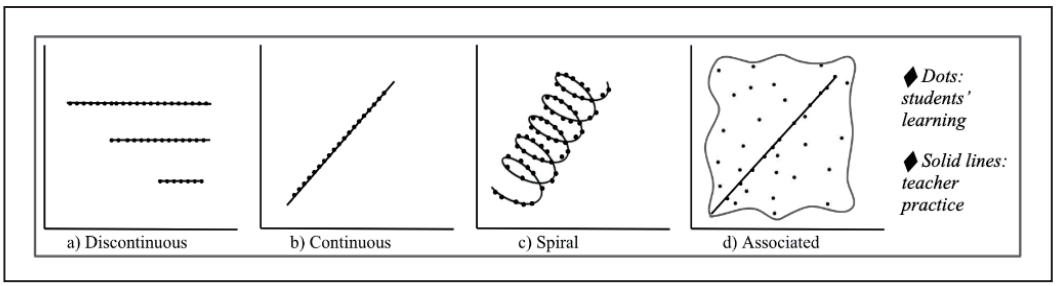


Figure 1. A rough sketch of different conceptions of math teaching and learning

At the start of the meeting, Nancy introduced Frank to key ideas of the MTSS PD (Clip 1):

> But with MTSS what we're saying is, everyone gets Tier 1 instruction. That's what happens during the core instructional time. [Tier 2 is] about for 8 kids who has, who might have additional needs. And we're trying to see how we can support them so that they can access the core content better. And then [Tier 3] might be for your 1, 2, or 3 kids who, uh, might not even [pause] work on the– they would use that extra time on something more basic like number sense.

Even though MTSS, as Nancy is interpreting it, calls for distinguishing instruction based on students' needs, it presents a *discontinuous* conception of math teaching and learning. Learning is classified into three tiers with the assumption that a certain range of test results will predict a group of students' generalized instructional needs. Nancy also explained that teachers need to know "the prerequisite standards leading up to [each lesson]. All the way maybe back from 3rd grade, whatever far back we need to go for each kid." This *continuous* conception suggests that learning requires a sequence of prior knowledge that builds over time; a student who lacks this prerequisite knowledge cannot learn, and a teacher who does not know what the prerequisites are cannot teach.

In Clip 2, Frank expressed concern with MTSS's strategy of starting with Tier 1, then Tier 2, "and then if that doesn't work, go down to Tier 3." Citing three students who had failed the previous year, he proposed greater and more immediate instructional support for their most struggling students. Hathaway responded at length (nearly 3 min.) by emphasizing "the attendance factor" and the importance of building relationships prior to providing instructional support, using language like "once you can pull them in…" and "even if you are making yourself available 24/7 they would not have entered…" This indicates a *discontinuous* and sequential ordering of conditions for students' math learning– first attendance, then emotional connection, then academic instruction– where latter supports would have no effect if the prior ones had not been achieved, even if offered simultaneously. Several minutes later, Frank asked, "what more could I have done?" describing a widening of concurrent instructional and emotional support for students (*spiral*).

In Clip 3, Nancy asked for "bigger picture" planning time as a math department, explaining "bigger picture" as attending to "the scope of the unit," including IM's desired outcomes, anticipated student difficulties, extensions, and opportunities for feedback, in contrast to nitty-gritty lesson logistics. This was necessary because in a *spiral* conception of math teaching and learning, student learning results not just from a single day of instruction but from the teacher integrating concepts and ideas across an entire several-week unit of instruction. It requires teachers' understanding of what comes after a particular lesson in addition to what comes before. By way of example, Nancy continued:

> This is Lesson 3, but in Lesson 8, they're going to get to go over it again. And if you know that, then you don't need to reteach that, you don't need to pull that kid out. You don't need to freak out, and the kid doesn't need to freak out.

As previously discussed, a *continuous* conception of math teaching and learning is sequential: students would not be able to effectively learn Lessons 4, 5, 6, or 7 without first succeeding in Lesson 3, so students who struggle in Lesson 3 would require reteaching, potentially creating anxiety for both students and teacher. By contrast, in a *spiral* conception, and in Nancy's understanding of how the IM curriculum is designed, a student who struggled with Lesson 3 could still be successful in Lesson 8. After Nancy's explanation, however, the educators' conversation returned to strategies for implementing MTSS.

Looking across these clips illustrates how different conceptions of math teaching and learning are brought into conversation with each other as educators collaborate. Importantly, in discussions of both instructional planning and a holistic view of math learning that includes students' socio-emotional needs, CMS educators were drawn to continuous and spiraled conceptions (arguing for "bigger picture planning" and "more" simultaneous supports). As their discussion unfolded, however, discontinuous conceptions took precedence (returning to MTSS tiers). This mirrors the district's actions in first mandating the IM curriculum, which is designed to spiral between prior knowledge and "new" content, and then the following year, mandating MTSS with its discontinuous conception of math teaching and learning.

Discussion

Identifying distinct conceptions of the relationship between math learning and teaching extends existing literature on teachers' conceptions of math content, both of which are consequential for teaching practices. For example, problem-based pedagogy becomes prohibitively time-consuming if students must be taught and must learn all the component skills first (as in *discontinuous* or *continuous* conceptions of math content); some students may never be given opportunities for rich mathematical exploration if teachers feel their emotional needs must be addressed first (as in *discontinuous* or *continuous* conceptions of math learning). Future research can explore relationships between the sequentiality of teachers' conceptions of math content and their conceptions of the relationship between math teaching and learning.

Our findings align with existing literature showing that teacher collaboration can be conservative as well as transformative, especially in reproducing deficit narratives about students (e.g., Horn, 2007; Yoon, 2016). We expand on it by identifying a different mechanism for this conservatism: discontinuous and highly-sequential conceptions of math teaching and learning in conversations about curricular change. Such conceptions reflect local culture and politics: academic tracking is culturally pervasive in US schools (Lave, 1996); accountability regimes encourage teachers to sort children into discontinuous tiers (Horn, 2018); and the district-mandated MTSS PD in our study explicitly promoted discontinuous conceptions of math teaching and learning. Their contrast with *spiral* conceptions of math teaching and learning, such as in the mandated IM curriculum, may create tension or incoherence for teachers (Nolan, 2018).

Furthermore, in the spirit of teachers' impulses towards *spiral* conceptions of math teaching and learning, our theoretical framework suggests that math learning can be conceptualized as "fundamental to all participation and all participants in social practice" (Lave, 1996, p.157). We represent this possibility in Figure 1d where student learning does not depend solely on teaching practices, even though what students learn is certainly associated with teachers' practice. Future research might explore how such conceptions may be linked to more expansive and transformative versions of math education.

References

Horn, I. S. (2007). Fast kids, slow kids, lazy kids: Framing the mismatch problem in mathematics teachers' conversations. *Journal of the Learning Sciences*, *16*(1), 37–79.

Horn, I. S. (2018). Accountability as a design for teacher learning: Sensemaking about mathematics and equity in the NCLB era. *Urban Education*, *53*(3), 382–408.

Jordan, B., & Henderson, A. (1995). Interaction analysis: Foundations and practice. *Journal of the Learning Sciences*, *4*(1), 39–103.

Lave, J. (1996). Teaching, as learning, in practice. *Mind, Culture, and Activity, 3*(3), 149–164.

Nolan, K. (2018). The lived experience of market-based school reform: An ethnographic portrait of teachers' policy enactments in an urban school. *Educational Policy*, *32*(6), 797–822.

Stodolsky, S. S., & Grossman, P. L. (2000). Changing students, changing teaching. *Teachers College Record*, *102*(1), 125–172.

Yoon, I. H. (2016). Trading stories: Middle-class white women teachers and the creation of collective narratives about students and families in a diverse elementary school. *Teachers College Record*, *118*(2), 1–54.

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 4
(Day 4)

TSG4-1-14

TEACHERS' INTENTION TO IMPLEMENT BLENDED LEARNING IN ALGEBRA: INSIGHTS FROM A PILOT STUDY

Nursaadah Jailani
The National University of Malaysia, Malaysia, saadahjailani@gmail.com

Roslinda Rosli
The National University of Malaysia, Malaysia, roslinda@ukm.edu.my

Blended learning is increasingly recognised as an effective approach to teaching mathematics, particularly in algebra. However, its successful implementation depends on teachers' intention to integrate digital tools into their instructional practices. This study investigates the factors influencing secondary school mathematics teachers' intention to implement blended learning in algebra. Using the Unified Theory of Acceptance and Use of Technology (UTAUT) and the Technological Pedagogical Content Knowledge (TPACK) framework, this study examines how professional knowledge, self-efficacy, and external factors shape teachers' adoption tendencies. A pilot test was conducted to validate the measurement instrument and assess key predictors of blended learning adoption. Findings reveal the results from Partial Least Squares-Structural Equation Modeling (PLS-SEM) confirmed internal consistency, convergent validity, and discriminant validity, supporting the instrument's use in future large-scale research and professional development planning. This study contributes to mathematics teacher education by offering insights into professional development strategies that foster effective blended learning implementation.

Keywords: Blended Learning, Mathematics Education, Teacher Intention, TPACK, UTAUT

Introduction

Blended learning combines traditional face-to-face instruction with digital learning tools, offering greater flexibility and engagement in mathematics education (Chin et al., 2019; Helsa et al., 2021). Algebra, as a foundational topic, benefits significantly from blended learning strategies that promote problem-solving and conceptual understanding (Lyakhova & Joubert, 2022; Rudyanto et al., 2019). However, teachers' intention to implement blended learning varies based on their professional knowledge, technological readiness, and institutional support (Al-Ayed & Al-Tit, 2021). This study aims to examine the factors influencing secondary school mathematics teachers' intention to adopt blended learning in algebra. The research is grounded in the UTAUT and TPACK frameworks, which provide a comprehensive understanding of technology adoption in education. To address this objective, the following research questions guide this study:

1. To what extent does the developed instrument demonstrate internal consistency and convergent validity in measuring mathematics teachers' intention to implement blended learning in algebra?
2. What are the findings related to the discriminant validity of the instrument based on UTAUT, TPACK, and teacher self-efficacy?

3. What are the contributions of the validated instrument to the professional development of mathematics teachers within the context of blended learning implementation?

Literature Review

Mathematics teacher plays a crucial role in ensuring successful technology integration in classrooms (Aliyu et al., 2021). The UTAUT model identifies performance expectancy, effort expectancy, social influence, and facilitating conditions as key predictors of technology adoption (Venkatesh et al., 2012). Meanwhile, the TPACK framework emphasizes the intersection of technological, pedagogical, and content knowledge necessary for effective digital learning implementation (Koehler et al., 2013; Landry, 2010). Previous studies indicate that while teachers acknowledge the benefits of blended learning, challenges such as a lack of technological proficiency and institutional support hinder its adoption. This study addresses these gaps by examining the interplay between professional knowledge, self-efficacy, and external support in blended learning implementation.

Methodology

A pilot study was conducted with secondary school mathematics teachers to validate the research instrument and assess key predictors of blended learning adoption. The study employed a quantitative survey approach, collecting data through structured questionnaires measuring UTAUT constructs such as performance expectancy, effort expectancy, social influence, and facilitating conditions. Additionally, TPACK constructs, including technological knowledge, pedagogical knowledge, content knowledge, and their intersections, were measured. The study also assessed self-efficacy constructs related to student engagement, instructional strategies, and classroom management. A total of 96 secondary school mathematics teachers from 20 school participated in the pilot study. Data analysis was conducted using Partial Least Squares-Structural Equation Modeling (PLS-SEM) to examine the relationships among constructs.

Findings

The pilot test aimed to validate the measurement instrument before the full-scale study. The results confirm the instrument's reliability and its ability to measure factors influencing teachers' intention to implement blended learning.

Internal Consistency and Convergent Validity

The developed measurement instrument demonstrated strong internal consistency and reliability across all constructs. Cronbach's Alpha values ranged from 0.816 to 0.966, indicating high internal consistency. Composite Reliability (CR) values exceeded the threshold of 0.70 for all constructs, further confirming the robustness of the instrument. Convergent validity was assessed through Average Variance Extracted (AVE), where most constructs achieved values above the recommended minimum of 0.50 as shows in Table 1. However, the AVE for the Teacher Self-Efficacy and Technological Pedagogical Content Knowledge (TPACK) construct was slightly below this threshold. Rather than removing indicators immediately, it is important to assess how their removal affects composite reliability, content coverage, and convergent validity. Indicators with outer loadings between 0.40 and 0.70 should only be considered for deletion if doing so clearly improves the values of Composite Reliability or AVE (Hair et al., 2017). In this study, the items were retained.

Table 1. Convergent validity, reliability and significance of the loadings

Construct	Cronbach's α	CR	AVE
Teacher Self-Efficacy	0.898	0.893	0.420
Performance expectancy	0.942	0.941	0.801
Effort expectancy	0.920	0.920	0.697
Intention to implement blended learning	0.966	0.966	0.906
Facilitating conditions	0.816	0.807	0.517
Pedagogical content knowledge	0.849	0.848	0.664
Pedagogical knowledge	0.858	0.858	0.514
Pedagogical content knowledge	0.888	0.887	0.616
Social influence	0.924	0.924	0.709
Technological knowledge	0.883	0.863	0.530
Technological content knowledge	0.904	0.903	0.652
Technological pedagogical knowledge	0.833	0.832	0.554
Technological pedagogical content knowledge	0.833	0.823	0.455

Discriminant Validity

The instrument demonstrated strong convergent and discriminant validity across constructs derived from the UTAUT, TPACK, and teacher self-efficacy frameworks. Discriminant validity was examined using the Heterotrait-Monotrait Ratio of Correlations (HTMT). All HTMT values between constructs were below the conservative threshold of 0.85 (Hair et al. 2019) or 0.90, indicating that each construct was empirically distinct and did not exhibit problematic overlap as shows in Table 2. This confirms that the constructs reliably capture different conceptual dimensions of mathematics teachers' readiness and intention to implement blended learning in algebra.

Contributions to Mathematics Teacher Education

The validated instrument ensures that professional development programs are designed based on empirical evidence. The findings emphasize the need for targeted training to enhance teachers' technological and pedagogical competencies. The validated instrument offers a reliable tool for assessing mathematics teachers' readiness to implement blended learning in algebra. By encompassing constructs from UTAUT, TPACK, and teacher self-efficacy, the instrument can guide the design of professional development programs. It enables education stakeholders to identify specific areas for growth, such as technological integration or pedagogical adaptation, and provides a framework for evaluating the effectiveness of interventions.

Table 2. Discriminant analysis result

	TE	PE	EE	FC	IBL	CK	PK	PCK	SI	TK	TCK	TPK
PE	**0.335**											
EE	0.277	**0.465**										
IBL	0.591	0.533	**0.550**									
FC	0.351	0.567	0.740	**0.580**								
CK	0.445	0.117	0.242	0.402	**0.328**							
PK	0.576	0.315	0.437	0.689	0.640	**0.649**						
PCK	0.615	0.218	0.403	0.448	0.459	0.501	**0.753**					
SI	0.470	0.66	0.712	0.669	0.817	0.298	0.585	**0.437**				
TK	0.448	0.243	0.359	0.423	0.522	0.397	0.788	0.668	**0.412**			
TCK	0.690	0.364	0.537	0.559	0.565	0.402	0.711	0.732	0.540	**0.641**		
TPK	0.592	0.461	0.412	0.643	0.519	0.310	0.739	0.718	0.565	0.597	**0.797**	
TPACK	0.670	0.420	0.557	0.600	0.646	0.432	0.871	0.854	0.635	0.830	0.874	**0.875**

Note. The values in the diagonals cells (bold) are the square root of the AVE while the un-bolded values are the correlations.

A total of 68 items were adapted from existing literature and modified to suit the Malaysian educational context. The structural model also explained 96.2% of the variance in teachers' intention to implement blended learning ($R^2 = 0.962$). The validated instrument ensures that professional development programs are designed based on empirical evidence. The findings emphasize the need for targeted training to enhance teachers' technological and pedagogical competencies. The validated instrument will be used in the full-scale study to assess more comprehensive trends in teachers' adoption of blended learning.

Conclusion

This pilot study validates a research framework that examines the factors influencing teachers' intention to implement blended learning in algebra. The results emphasize the importance of professional development, self-efficacy, and external support in fostering technology adoption in mathematics education, especially in algebra context. Future research will extend this study to a larger sample to further refine the model and develop targeted interventions for improving blended learning implementation in algebra.

References

Al-Ayed, S. I., & Al-Tit, A. A. (2021). Factors affecting the adoption of blended learning strategy. *International Journal of Data and Network Science*, *5*(3), 267–274. https://doi.org/10.5267/j.ijdns.2021.6.007

Aliyu, J., Osman, S., Daud, M. F., & Kumar, J. A. (2021). Mathematics teachers' pedagogy through technology: A systematic literature review. *International Journal of Learning, Teaching and Educational Research*, *20*(1), 323–341. https://doi.org/10.26803/IJLTER.20.1.18

Chin, C. K., Munip, H., Miyadera, R., Thoe, N. K., Ch'ng, Y. S., & Promsing, N. (2019). Promoting Education for Sustainable Development in teacher education integrating blended learning and digital tools: An evaluation with exemplary cases. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(1), 1–17. https://doi.org/10.29333/ejmste/99513

Hair, J. F., Hult, G. T. M., Ringle, C. M., & Sarstedt, M. (2017). A primer on partial least squares structural equation modeling (PLS-SEM). Thousand Oaks.

Hair, J. F., Risher, J. J., Sarstedt, M., & Ringle, C. M. (2019). When to use and how to report the results of PLS-SEM. European business review, 31(1), 2-24.

https://doi.org/10.1108/EBR-11-2018-0203

Helsa, Y., Darhim, Juandi, D., & Turmudi. (2021). Blended learning in teaching mathematics. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *10*(2), 733–743. https://doi.org/10.24127/ajpm.v10i2.3447

Koehler, M. J., Mishra, P., & Cain, W. (2013). What Is Technological Pedagogical Content Knowledge (TPACK)? *Journal of Education*, *193*(3), 13–19.

Landry, G. A. (2010). Creating and Validating an Instrument to Measure Middle School Mathematics Teachers' Technological Pedagogical Content Knowledge (TPACK) [University of Tennessee]. In *Tesis PhD*. http://trace.tennessee.edu/utk_graddiss/720

Lyakhova, S., & Joubert, M. (2022). Post-16 Further Mathematics blended learning: Learner self-regulation, mathematical resilience and technology. *Teaching Mathematics and Its Applications*, *41*(1), 51–68. https://doi.org/10.1093/teamat/hrab005

Rudyanto, H. E., Marsigit, Wangid, M. N., & Gembong, S. (2019). The use of bring your own device-based learning to measure student algebraic thinking ability. *International Journal of Emerging Technologies in Learning*, *14*(23), 233–241. https://doi.org/10.3991/ijet.v14i23.11050

Venkatesh, V., Thong, J. Y. L., & Xu, X. (2012). Consumer Acceptance and Use of Information Technology: Extending the Unified Theory of Acceptance and Use of Technology. *MIS Quarter*, *36*(1), 157–178.

TSG4-1-15

INNOVATION IN MATHEMATICS TEACHER EDUCATION THROUGH A STEAM-BASED GLOBAL PROFESSIONAL LEARNING COMMUNITY MODEL

Jungin Heo
Samjung High school, Korea, gjwjddls32@daum.net

Jihwa Noh
Pusan National University, Korea, nohjihwa@pusan.ac.kr

This study introduces a new educational model for mathematics teacher education. It aims for qualitative innovation. The model combines the interdisciplinary approach of STEAM education with principles of professional learning communities. A review of literature and policy reports was conducted. The analysis showed that teachers must address local, global, and cultural issues. Teachers need to adopt diverse perspectives. They must interact positively with different cultures. Proficiency in foreign languages and digital tools is essential. The model systematically integrates these competencies into teacher education programs. Finally, a framework to boost teacher self-efficacy and student outcomes was validated by various experts.

Keywords: STEAM Education, Global Professional Learning Community, Global Competencies, Mathematics Teacher Education, Innovation Model.

Introduction

In contemporary society, the rapid acceleration of digital innovation and international interconnectedness has highlighted the limitations of traditional teacher education systems. In the field of mathematics education, there is an increasing demand for an integrated approach that goes beyond mere theoretical instruction, enabling students to analyze real global issues and exercise creative problem-solving skills. In response, this study aims to develop an innovative educational model that supports teachers in thoroughly exploring a range of local and global challenges, adopting multicultural perspectives, and internalizing the core competencies necessary for effective cross-cultural communication. The proposed model offers a systematic framework that enhances teacher expertise through organized support, international collaboration, and continuous feedback within teacher education programs, thereby fostering transformative changes in curriculum design and implementation. Moreover, this approach is expected to play a pivotal role in preparing future mathematics teachers to effectively address complex global issues.

Research Method

This study was conducted in three primary phases. In the first phase, a comprehensive analysis of domestic and international academic literature and policy reports was undertaken to define the global competencies essential for mathematics teacher education and to identify their constituent elements. In this process, key elements were found to include the ability of teachers to explore and analyze local, global, and cultural issues, to understand and respect diverse perspectives and worl-

dviews, and to communicate effectively with individuals from different cultural backgrounds. In the second phase, various categories and subcomponents were examined to systematically integrate the identified core competencies into teacher education programs. Specifically, the study investigated whether global competencies are officially embedded within educational organizations, how they can be concretely reflected in the curriculum, and the necessity for continuous refinement and transformation through international exchanges and collaboration. This analysis led to the development of specific implementation strategies, including a multidisciplinary approach through problem-based learning, student-driven participation in lesson design, and the incorporation of multicultural content. In the final phase, the study aimed to clarify the mechanism by which professional learning communities enhance teacher self-efficacy and improve student learning outcomes through a shared educational philosophy, a collaborative learning environment, distributed leadership, and continuous feedback. This systematic interaction structure was depicted as a pyramid-shaped framework, illustrating how teacher expertise and classroom practices are substantially improved through organizational support, collaborative reflection, and international exchanges. The strategic approach was validated for its feasibility and practical applicability through written surveys and online meetings with 42 domestic and international experts.

Research Results

The findings of this study can be summarized as follows. First, three main elements constitute the global competencies required of mathematics teachers. Teachers must possess the ability to explore and analyze issues and situations of local, global, and cultural significance, while internalizing an attitude that values and respects diverse perspectives and worldviews. Moreover, the study confirmed that the capacity to engage in positive interactions with individuals from different national, ethnic, religious, social, or cultural backgrounds, as well as the ability to communicate effectively using foreign languages and digital tools, is essential. Second, these global competencies must be systematically integrated into teacher education programs. At the organizational level, the concept of global competencies should be formally embedded within teacher education curricula, with appropriate personnel assignments and structural support in place. At the curriculum level, diverse ideas, pedagogical approaches, and content related to global competencies need to be explicitly reflected, while institutional measures—facilitated by international exchanges and collaboration—should support the internalization of evolving teacher roles and future orientations. These elements were found to be interconnected through the five categories of organization, curriculum, internationalization, the need for change, and instructional practice. Third, five specific strategies for practical implementation in mathematics teacher education were derived. Teachers should gain practical experience in resolving real global issues through a multidisciplinary, problem-based learning approach and incorporate content that reflects multicultural perspectives and global understanding to broaden students' horizons. Additionally, enhancing self-directed learning through student-centered instructional design, establishing international collaborative networks to share diverse educational practices, and ensuring systematic implementation through supportive educational policies and institutional backing are all essential. Fourth, several core principles essential for the effective operation of professional learning communities were identified. It is crucial that teachers collaborate under a shared educational philosophy and clear objectives, focus on the learning of each student, foster a collaborative learning environment, and practice distributed leadership by sharing responsibility for students facing challenges. The implementation of these principles was clearly manifested in four outcome areas: improvements in student learning performance, shifts in teachers' attitudes and beliefs, enhancements in pedagogical methods and professional knowledge, and better class-

room practices. Furthermore, the integrated framework is underpinned by a shared educational philosophy and vision that permeates the entire school, thereby providing a foundation for teachers to work collaboratively toward common goals. A supportive school structure enables teachers to conduct classes confidently within a stable environment, while a collaborative culture—where teachers observe each other's lessons and exchange feedback—leads to continuous reflection and improvement. Ultimately, the gradual enhancement of teachers' professional knowledge and instructional methods, coupled with a significant increase in self-efficacy, demonstrates that these interconnected elements create a stable and innovative educational environment capable of nurturing students' creative problem-solving abilities.

Implications and Applications

The educational model presented in this study provides a concrete foundation for future mathematics teachers to effectively cultivate the core competencies required in the field. It emphasizes that global competencies in educational practice go beyond mere knowledge transmission, requiring teachers to explore and analyze issues and situations that are significant at local, global, and cultural levels, while internalizing an attitude that values and respects diverse perspectives and worldviews. Moreover, it is essential for teachers to engage in positive interactions with individuals from different national, ethnic, religious, or social backgrounds and to communicate effectively using foreign languages and digital tools. These competencies should be naturally integrated into teacher education programs, enabling educators to implement multidisciplinary approaches and creative collaborations in lesson planning and problem-solving. Furthermore, teacher education programs must address various aspects, such as ensuring that global competencies are officially incorporated at the organizational level, determining how these competencies are concretely reflected in the curriculum, and continuously enhancing them through international exchanges and collaborations. In practice, for teachers to internalize global competencies, it is crucial that they share their educational philosophies, continuously exchange feedback in a collaborative learning environment, and engage in shared responsibilities under distributed leadership. Such a structural approach is supported by specific strategies including problem-based learning, the incorporation of diverse cultural perspectives, student-centered lesson planning, and the establishment of international collaborative networks. In particular, the proposed model builds a supportive school structure and a collaborative reflective process based on shared values and visions within the school, encouraging teachers to continually update their professional knowledge and attitudes. These interactive processes directly contribute to enhancing teachers' self-efficacy, as well as improving students' creative problem-solving abilities and overall academic performance. Consequently, this model can serve as a valuable practical basis not only for pre-service teacher education but also for in-service teacher training, as well as for the formulation of educational policies and the improvement of school systems. This integrated approach is expected to significantly assist both domestic and international educational institutions and policymakers in establishing cooperative networks and promoting joint research and international exchanges.

Conclusion

In summary, the global competencies required of mathematics teachers can be distilled into three key components. First, teachers must be able to thoroughly explore and analyze issues and situations that are significant on local, global, and cultural levels. They are expected to internalize an attitude of understanding and respect for diverse perspectives and worldviews, while also engaging

positively with individuals from different backgrounds through effective communication using foreign languages and digital tools. Second, these competencies must be systematically integrated into teacher education programs. Educational organizations should formally incorporate the concept of global competencies, supported by appropriate staffing and structural frameworks, while the curriculum should reflect diverse pedagogical methods, content, and international exchanges to foster the internalization of evolving teacher roles and future orientations. Third, five specific implementation strategies for mathematics teacher education emerged: a multidisciplinary approach centered on problem-based learning, the incorporation of content that reflects multicultural perspectives and global understanding, student-centered instructional design, the establishment of international collaboration networks, and the provision of supportive educational policies and institutional measures. Furthermore, the effective operation of professional learning communities—through collaboration under a shared educational philosophy and clear objectives, distributed leadership, and a collective sense of responsibility—has been shown to yield positive changes in student outcomes, teacher attitudes and instructional methods, professional knowledge, and classroom practices. This integrated framework, grounded in a shared educational philosophy and vision that permeates the entire school, establishes a supportive organizational structure and a collaborative learning environment. In doing so, it enhances teacher professionalism and self-efficacy, ultimately creating a stable and innovative educational setting that fosters students' creative problem-solving skills.

References

Mansilla, V. B., & Jackson, A. W. (2022). *Educating for global competence: Preparing our students to engage the world.* ASCD.

Mattos, M. W., DuFour, R., DuFour, R., Eaker, R., & Many, T. W. (2016). *Learning by doing: A handbook for professional learning communities at work*(3rd ed.). Solution Tree Australia Pty Limited.

DuFour, R., & Eaker, R., & DuFour, R. (2005). *On common ground: The power of professional learning communities*. Solution Tree Press.

Fullan, M., Rincón-Gallardo, S., & Hargreaves, A. (2015). Professional capital as accountability. *Education policy analysis archives, 23*, 15-15.

Hall, D. T., & Hite, R. L. (2022). School-level implementation of a state-wide professional development model for developing globally competent teachers. *Teacher Development*, 26(5), 665-682.

Hardy, I. J., & Melville, W. (2019). Professional Learning as Policy Enactment: The Primacy of Professionalism. *Education Policy Analysis Archives*, 27(90), n90.

Hodson, D. (2003). Time for action: Science education for an alternative future. *International journal of science education*, 25(6), 645-670.

Hudson, C. (2024). A conceptual framework for understanding effective professional learning community (PLC) operation in schools. *Journal of Education*, 204(3), 649-659.

Kangas, M., & Rasi, P. (2021). Phenomenon-Based learning of multiliteracy in a Finnish upper secondary school. *Media Practice and Education*, 22(4), 342-359.

Karumanthra, A., & Buck, G. Becoming a Globally Competent Educator: A Self-Study of My Theoretical and Practical Understandings of Global Competency.

Mansilla, V. B., & Jackson, A. W. (2022). *Educating for global competence: Preparing our students to engage the world*. ASCD.

Nielsen, C. P., & Lockhart, A. K. (2020). Using a Professional Learning Community Framework to Assist Early Field Experience Students as They Move from Teacher Candidate to Teacher. Insight: *A Journal of Scholarly Teaching*, 15, 52-66.

PISA, O. (2018). Preparing our youth for an inclusive and sustainable world.

Olivier, D. F., & Huffman, J. B. (2016). Professional learning community process in the United States: Conceptualization of the process and district support for schools. *Asia Pacific Journal of Education*, 36(2), 30-317.

Shintaku, K., & Takahashi, T. Enhancing Global Citizenship Education in Japan's K-12 Curriculum: Exploring Lessons and Opportunities for Growth.

Stoll, L., Bolam, R., McMahon, A., Wallace, M., & Thomas, S. (2006). Professional learning communities: A review of the literature. *Journal of educational change*, 7(4), 221-258.

Vangrieken, K., Meredith, C., Packer, T., & Kyndt, E. (2017). Teacher communities as a context for professional development: A systematic review. *Teaching and teacher education*, 61, 47-59.

York, M. K., & Hite, R. (2021). Preservice science and mathematics teachers' intent to use classroom-based global collaboration (CBGC) in their future classrooms. *Teacher Education Quarterly*, 48(2), 45-68.

Zong, G. (2022). Integrating global sustainability into social studies teachers' education: a collaborative self-study. *Social Studies Research and Practice*, 17(1), 94-113.

TSG4-1-16

DEVELOPMENT OF A TEACHER TRAINING MODEL FOR ENHANCING AI DIGITAL TEXTBOOK UTILIZATION COMPETENCY

Seo Hyun Ahn
University of Ajou, Korea, aekdl78@ajou.ac.kr

Ho Kyoung Ko
University of Ajou, Korea, kohoh@ajou.ac.kr

This study develops a teacher training model that strengthens AI Digital Textbooks (AIDT) utilization by incorporating the TPACK framework, Cognitive Load Theory, and Learning Analytics. Additionally, the Forschendes Lernen (FL) approach is applied to foster inquiry-based learning, enabling teachers to explore and implement AI-driven teaching methods. The proposed model emphasizes hands-on AI teaching strategies, adaptive lesson design, and AI-assisted feedback mechanisms, overcoming the limitations of traditional training programs. This research provides a structured framework for AIDT-based teacher professional development and offers insights into AI integration in education. Future research should focus on empirical validation of the model's effectiveness and continuous refinement to align with emerging AI technologies.

Keywords: AI Digital textbooks, teacher training, Forschendes Lernen, TPACK model

Introduction

The rapid advancement of AI technology and the increasing demand for digital competencies in education necessitate the development of AI literacy and application skills among teachers. In response, the Korean Ministry of Education has introduced initiatives such as the Plan for Digital-based Education Innovationand the implementation of AI Digital Textbooks (AIDT), shifting teachers' roles from knowledge transmitters to facilitators. However, existing teacher training programs remain largely focused on traditional instructional methods, lacking structured frameworks for AI-based pedagogy. This study proposes a teacher training model that integrates the TPACK framework, Cognitive Load Theory, and Learning Analytics, while incorporating the Forschendes Lernen (Inquiry-Based Learning) approach to enhance AI-driven teaching practices. By enabling teachers to actively engage in instructional research and develop AI-supported pedagogical strategies, this model facilitates the practical application of AI-driven teaching and learning. The findings contribute to the development of AI-integrated educational environments and provide insights for refining teacher training policies and instructional design frameworks.

Theoretical Framework

Model of Technological Pedagogical Content Knowledge

Teacher training for AI Digital Textbook (AIDT) integration aims to enhance teachers' ability to effectively utilize AI-based digital textbooks, with the Technological Pedagogical Content Knowledge (TPACK) model providing the theoretical foundation (Mishra & Koehler, 2006).

The TPACK model focuses on the interaction of three core knowledge domains to optimize instructional design: Content Knowledge (CK), which pertains to the ability to utilize AI-driven personalized content; Pedagogical Knowledge (PK), which involves designing student-centered instructional strategies using AI-driven interactive feedback; and Technological Knowledge (TK), which includes AI-assisted grading and adaptive learning. The integration of these components forms Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), and Pedagogical Content Knowledge (PCK), all of which are essential for designing AI-enhanced lessons with AIDT.

The TPACK model ensures that teachers systematically integrate technological (TK), pedagogical (PK), and content (CK) knowledge to create effective AI-enhanced learning environments (Kim et al., 2020). Specifically, elements such as collaborative learning designs utilizing AI chatbots, personalized instruction informed by learning analytics, and AI-driven interactive feedback can be effectively implemented using the TPACK model (Park, 2023). Therefore, teacher training programs utilizing AIDT must be designed within the TPACK framework, shifting from mere tool utilization to comprehensive AI-based instructional design and learning analytics applications for teacher competency development .

Instructional Design for AIDT Based on Cognitive Load Theory

Instructional design for AIDT should incorporate Cognitive Load Theory (CLT) (Sweller, 1988) to minimize learners' cognitive burden and optimize learning outcomes. Sweller (1988) defines cognitive load as the total mental effort required for learning, emphasizing the need to manage this load effectively during instructional design.

According to CLT, cognitive load is classified into three main categories: Intrinsic Load, which arises from the inherent complexity of the learning task; Extraneous Load, which is generated by the instructional method and presentation format; and Germane Load, which reflects the cognitive effort dedicated to constructing meaningful learning schemas. When designing instruction, it is essential to consider the limitations of working memory capacity and develop learning materials accordingly. AI-enhanced instructional design can support this process by reducing extraneous load and facilitating more efficient cognitive processing (Mayer & Moreno, 2003). By applying CLT to AIDT-based instruction, teachers can design learning experiences that effectively manage cognitive load through AI-driven scaffolding, automated feedback, and adaptive content delivery, ultimately enhancing student learning efficiency and engagement.

Instructional Design for AIDT Utilizing Learning Analytics

AIDT incorporate Learning Analytics (LA) to analyze students' learning patterns in real time and provide personalized learning support. Learning analytics can be categorized into Diagnostic Analytics, Predictive Analytics, and Prescriptive Analytics (Romero & Ventura, 2020). Teacher training programs utilizing AI-based learning analytics should integrate automated grading, error pattern analysis, and AI-generated feedback to implement personalized learning strategies effectively. By leveraging these analytics-driven instructional strategies, teachers can enhance student engagement and achievement through data-informed decision-making and AI-driven adaptive learning interventions.

Teacher Training Model Applying Forschendes Lernen

The Concept of Forschendes Lernen

Forschendes Lernen (inquiry-based learning) is rooted in the Humboldtian educational philosophy, which emerged in early 19th-century Germany. Wilhelm von Humboldt viewed university education as a continuous process of scholarly participation, requiring active learner engagement (Huber, 2009; Huber & Reinmann, 2019). According to Huber (1983), academic learning is an ongoing pursuit of truth, necessitating active student involvement. In this context, pre-service teachers, through collaboration with in-service teachers, develop competencies that extend beyond academia to national and societal progress (Huber & Reinmann, 2019; Schleiermacher, 2010).

Huber (2009) defines *Forschendes Lernen* as follows:

> *" In contrast to other learning methods, inquiry-based learning is characterized by the fact that learners shape, learn and deliberate on the process of a research project, which is aimed at obtaining insights that are of interest to third parties, doing so throughout all the essential phases of said project; from developing questions and hypotheses, selecting and implementing the methods, through testing and presenting the results, either by working independently or in active collaboration with an overarching project. (Huber, 2009, p. 11, translated)*

The essence of *Forschendes Lernen* lies in direct engagement with scientific inquiry processes, allowing learners to internalize the significance of theoretical knowledge and acquire strategies for effectively applying it. Moreover, active participation in the creation and development of knowledge is emphasized. The outcome of this learning process is not merely the acquisition of static ("dead") knowledge but the development of practical competencies that can be applied to new contexts. Through this practice-oriented, inquiry-driven approach, teachers gain opportunities to critically reflect on their assumptions, refine their perspectives, and engage in deeper theoretical exploration. This process fosters a continuous cycle of questioning and inquiry, which is fundamental to Forschendes Lernen. At its core, this approach aims to cultivate a research-oriented mindset (forschende Grundhaltung), reinforcing teachers' ability to engage in lifelong professional learning and pedagogical innovation.

Development of a Teacher Training Model for AIDT Utilization Based on Forschendes Lernen

Inquiry Foundation Phase

In this study, the Inquiry Foundation phase consists of three key stages: topic exploration, question development, and information gathering. These stages lay the groundwork for inquiry-based activities by fostering:

- **Problem Recognition and Exploration:** Identifying fundamental issues and exploring relevant themes related to AI Digital Textbook (AIDT) integration.
- **Formulation of Key Questions:** Developing essential research questions that drive the inquiry process.
- **Collection of Inquiry Materials:** Gathering relevant resources, including theoretical frameworks, empirical studies, and AI-based teaching tools, to support informed investigation.

Inquiry Method Development Phase

In this study, the Inquiry Method Development phase consists of two key stages: method acquisition and inquiry design development. This phase focuses on selecting appropriate research methods and establishing a structured inquiry plan.

- **Method Acquisition:** Teachers explore and acquire suitable methodologies for AI-integrated instruction, such as AI-driven learning analytics, adaptive learning strategies, and data-informed instructional design.
- **Inquiry Design Development:** Teachers formulate a detailed inquiry plan, outlining research objectives, AI tools to be utilized, and evaluation criteria for assessing instructional effectiveness.

Inquiry Execution and Reflection Phase

In this study, the Inquiry Execution and Reflection phase consists of three key stages: conducting inquiry activities, compiling and presenting results, and reflecting on the overall process. This phase involves the practical implementation of inquiry activities, the generation and sharing of research findings, and critical self-assessment.

- **Conducting Inquiry Activities:** Teachers apply the selected methodologies to explore AI Digital Textbook integration in actual or simulated educational settings. This may include AI-driven lesson planning, adaptive learning implementation, and real-time learning analytics utilization.
- **Compiling and Presenting Results:** Teachers document their findings, analyze AI-supported instructional effectiveness, and present their results through reports, discussions, or workshops.
- **Reflecting on the Overall Process:** Teachers critically evaluate their inquiry experiences, identify challenges and successes, and refine their approaches for future AI-integrated teaching practices.

Conclusion

This study proposes a teacher training model to enhance teachers' AI and digital tool utilization competencies in AI Digital Textbook (AIDT)-integrated learning environments. Addressing the limitations of traditional teacher training, which primarily focuses on theoretical knowledge transfer, this model provides a structured framework for the practical application of AI-based pedagogy in real classroom settings. Grounded in the TPACK framework, Cognitive Load Theory, and Learning Analytics, the model optimizes AI-supported instructional design. Additionally, the Forschendes Lernen (Inquiry-Based Learning)approach is integrated to foster active teacher engagement in problem-solving and pedagogical development. By encouraging teachers to conduct instructional research and refine AI-integrated teaching strategies, this model cultivates a reflective, research-oriented professional mindset, contributing to the advancement of AI-enhanced educational environments.

References

Huber, L. (2009). Warum Forschendes Lernen nötig und möglich ist. In L. Huber, J. Hellmer, & F. Schneider (Eds.), Forschendes Lernen im Studium. Aktuelle Konzept und Erfahrungen (pp. 9-35). UVW.

Huber, L. (2014). Forschungsbasiertes, Forschungsorientiertes, Forschendes Lernen: Alles dasselbe? - Ein Plädoyer für eine Verständigung über Begriffe und Unterscheidungen im Feld forschungsnahen Lehrens und Lernens. In: Das Hochschulwesen. Forum für Hochschulforschung. -praxis und –politik, Heft1+2/2014. Bielefeld. p.22-29.

Huber, L. & Reinmann, G. (2019). Vom forschungsnahen zum forschenden Lernen an Hochschulen. Springer Fachmedien Wiesbaden.

Kim, Y. M., Ko, H. K., & Huh, N. (2020). A Study on Development of Integrating Mathematics and Coding Teaching & Learning Materials Using Python for Prime Factorization in 7th Grade. *Communications of Mathematical Education*, *34*(4), 563–585. https://doi.org/10.7468/JKSMEE.2020.34.4.563

Ko, H. K. (2020). A study on development of School Mathematics Contents for Artificial Intelligence (AI) capability. *Korean School Mathematics Society*, *23*(2), 223–237. https://doi.org/10.30807/ksms.2020.23.2.003

Mayer, R. E., & Moreno, R. (2003). Nine ways to reduce cognitive load in multimedia learning. Educational Psychologist, 38(1), 43-52.

Ministry of Education(2023). Digital-based education innovation plan

Ministry of Education(2023). AI Digital Textbook promotion plan

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. Teachers College Record, 108(6), 1017-1054.

Romero, C., & Ventura, S. (2020). Educational data mining and learning analytics: An updated survey. Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery, 10(3), e1355.

Schleiermacher, F. (2010) [1808]. Gelegentliche Gedanken über Universitäten im deutschen Sinn (1808). In Humboldt-Universität zu Berlin (Hrsg.),Gründungstexte. Johann Gottlieb Fichte, Friedrich Daniel Ernst Schleiermacher, Wilhelm von Humboldt(ed. R. v. Bruch) (S. 123–228). Berlin: Humboldt-Universität.

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. Cognitive Science, 12(2), 257-285.

TSG4-1-17

"AM I BEING TOO UNHELPFUL?": MATHEMATICS TEACHER DECISION-MAKING & RESPONSIBILITY

Grace A. Chen
New York University, USA, grace.a.chen@nyu.edu

Samantha A. Marshall
North Carolina State University, USA, smarsha3@ncsu.edu

Mathematics teachers make thousands of decisions every day, but research empirically examining teachers' instructional decisions has largely taken cognitive-psychological perspectives. To account for the role of social, cultural, and political contexts in teachers' decision-making, this paper uses a situated perspective to examine how mathematics teachers' instructional decisions are discussed in relation to their pedagogical responsibilities in collaborative conversations. Through qualitative analysis of a group of experienced mathematics teachers discussing videoclips of their own teaching, this paper finds that teachers justify their instructional decisions by constructing pedagogical responsibilities as competing. This analysis demonstrates that situated interactions can play a pivotal role in determining what and how teachers learn about their instructional decisions. Findings challenge a common assumption that teachers' pedagogical responsibilities drive their instructional decisions, with implications for supporting mathematics teacher learning.

Keywords: decision-making, pedagogical responsibility, teacher collaboration

Teachers' instructional decisions have academic, social, and political consequences for students' learning opportunities and outcomes in the mathematics classroom. Teachers' ideas about these consequences are tied to their pedagogical responsibilities: who and what they feel beholden to (Chen et al., 2021). If teachers made instructional decisions based on their obligations, then changing teachers' understandings of their pedagogical responsibilities would affect the actions they take in the classroom. In fact, this is an implicit mechanism assumed in many teacher education programs (Webel & Platt, 2015). This study, however, complicates the relationship between teachers' enacted instructional decisions and their articulated pedagogical responsibilities.

Theories of Teacher Decision-Making

Early cognitive-psychological literature on teacher decision-making in mathematics education has evolved into a robust literature on what teachers notice and how they interpret and respond to what they notice (e.g., van Es & Sherin, 2002; König et al., 2022). However, what is noticed is always situated in practice, including political contexts (Goodwin, 1994). Moreover, cognitive theories of decision-making tend to focus on teachers' deliberate decisions, despite acknowledging that the majority of teachers' pedagogical decisions are made instinctively or out of habit (Bishop, 1976).

To explore situated aspects of teacher decision-making, scholars have studied how societal norms create frames that influence what teachers notice (Louie et al., 2021) and how teachers' meaning-making is shaped by their on-the-job interactions (Coburn, 2001). Attending to "how contexts afford and constrain… negotiation of meaning" (Horn et al., 2013, p. 189) shows that although teachers may make instructional decisions on their own, in their classrooms, what those decisions *mean* gets co-constructed when teachers discuss them with colleagues. The relationship between teachers' collaborative conversations and their instructional decision-making has generated extensive research and practice on professional learning communities (Lefstein et al., 2020; Ronfeldt et al., 2015). Collaborative conversations create opportunities for teacher learning (Horn, 2005), shaping how teachers make decisions about students (Bannister, 2015). For example, our prior research found that when social inequalities were foregrounded in a collaborative conversation, teachers minimized their responsibilities to students' mathematics learning in favor of their responsibilities to love students and teach life skills (Chen et al., 2021).

Examining teachers' co-constructed pedagogical responsibilities in relation to pedagogical actions and pedagogical reasoning (Horn, 2021) is a useful way to conceptualize decision-making because it offers a way to consider how teachers' decisions may be informed by resources other than their cognitive-psychological noticings. Teachers' shared obligations, for example, may orient their instinctual or habitual responses to classroom situations, and these obligations themselves may be informed by the institutional, social, cultural, and historical contexts in which teachers work. Inquiry into teachers' pedagogical responsibilities, however, has largely been theoretical and philosophical (e.g., Stengel & Casey, 2013; Wahlstrom et al., 2023) and related empirical work linking teachers' responsibilities to their decision-making has relied largely on interviews with individual teachers (e.g., Santoro, 2016). But, given the importance of teachers' on-the-job interactions in their meaning-making, studying the relationship between teachers' decision-making and their pedagogical responsibilities requires attention to the micropolitics of how responsibilities and decisions are constructed, justified, and contested in collaborative conversation.

Research Methods

The data for this study come from a four-year design-based study of mathematics teacher learning in the United States (Horn & Garner, 2022). In this study, researchers videotaped classroom lessons from experienced secondary mathematics teachers selected for their proficiency and commitment to ambitious and equitable mathematics instruction, chose clips from these videos aligned with teachers' own inquiry questions about their classroom practice (e.g., "how do I respond to the questions that students ask me?"), and debriefed these clips with teachers and their colleagues within two days of videotaping. Each of these 36 debrief conversations was approximately 45-90 minutes long and contained dozens of episodes of teachers explaining, justifying, and evaluating their decisions. The videos of these debrief conversations, as well as supplemental data in the form of transcripts, classroom artifacts, and facilitation memos, formed the corpus for the current study.

We began this analysis by reviewing the corpus for episodes of pedagogical reasoning (Horn, 2005) where teachers 1) explained, justified, or evaluated instructional decisions that had already been made and 2) considered alternative instructional decisions that could have been made or could be made in the future. Then, we used methods drawn from critical discourse analysis (Fairclough, 2015) to trace the relationship between teachers' articulated pedagogical responsibilities and their discussion of instructional decisions. To identify teachers' pedagogical responsibilities, we coded for framings in their reasoning (Bannister, 2015): what did teachers present their decisions as a case

of? Was a decision to walk away from a group of students treated as a case of negligence or as a case of deliberate non-intervention? Next, we looked for grammatical or syntactical constructions that posited particular relations between responsibilities, such as opposition, incompatibility, or equivalence. Finally, we traced whether and how decisions, reasoning, underlying responsibilities, and/or outcomes were ratified, expanded, or contested.

Findings

Across the cases, teachers frequently linked their instructional decisions to their pedagogical responsibilities by explaining how their decisions fulfilled their responsibilities or failed to do so. Here, we highlight one case in which a teacher began by stating that his decision failed to fulfill a responsibility but through collaborative conversation, the decision became commended as fulfilling alternate responsibilities. This focal case illustrates how collaboration co-constructs a complex relationship between teachers' instructional decisions and pedagogical responsibilities.

In this focal case, four math teachers at the same high school (students ages 14-18), each with about 10-20 years of teaching experience, discussed a video clip from Brad's geometry class with two researchers. In the clip, students worked in small groups to find measures of center and spread. A girl asked Brad for help and Brad responded, "why don't you talk about that [to your group], okay?" and walked away. After watching this video clip, Brad expressed concern that his decision to walk away left unfulfilled his responsibility to acknowledge and affirm students' efforts:

Brad: I just ignored her [group laughs].
Researcher: You didn't ignore her.
Brad: … I ignored giving her a gratification that "yes that is awesome."
Greg: How did she respond?
Brad: Her face was like, "Perfect. If only I had a teacher that would help me once in a while."

In response, other participants cited a professional development that the teachers had attended and identified pedagogical responsibilities that Brad *did* fulfill by walking away, such as prompting students to rely on each other as resources rather than relying on the teacher. After some discussion, Brad repeated this language, "using each other as resources," and a colleague, Marisa, agreed: "I think I need to do more of that, like not answering questions. Because I have that problem too."

Initially, Brad expressed discomfort around leaving a responsibility unfulfilled when he walked away. His colleagues focused on the outcome of his decision, suggesting that it was okay— even desirable—that Brad had left unfulfilled the responsibility to acknowledge and affirm students, because it allowed for the fulfillment of an alternative responsibility: supporting students to rely on each other. Although justifying Brad's decision this way invited a conversation about student interdependence, it had several additional consequences. First, it foreclosed the possibility that another decision could have reconciled the apparent contradictions between pedagogical responsibilities by affirming students' efforts and also prompting them to turn to each other as resources; perhaps Brad could have done or said something that accomplished both. Second, emphasizing the responsibilities Brad's decision did fulfill foreclosed the consideration of other responsibilities that might have been overlooked. In addition to ignoring students' desires for affirmation, it is also possible that Brad's decision to walk away stemmed from feeling compelled to attend to more struggling students, or from a common gender bias in mathematics classrooms (Copur-Gencturk et al., 2020), but these possibilities remained unexamined. Finally, by saying that she wanted to emulate Brad's decision to not answer students' questions, rather than emulating the underlying responsibility of

supporting students to rely on each other, Brad's colleague implied that students relying on each other would be an inevitable outcome of a teacher's decision to walk away. As a result, her decision-making became more closely tied to pedagogical actions than to pedagogical responsibilities.

In this case, teachers appeared to construct competing responsibilities in order to justify instructional decisions that had already been made, rather than making decisions according to previously articulated responsibilities or seeking decisions that could fulfill felt responsibilities. This finding suggests that models of teacher decision-making cannot rely on the linear assumption that teachers' individually-held pedagogical responsibilities shape their instructional decisions; it is possible that teachers construct responsibilities in their collaborative conversations to justify decisions that were made out of habit or for other reasons, building on each other and on external resources to legitimize their justifications as a group.

Significance

Theories of affect and power in teacher education have posited that teachers' habits often differ from their articulated intentions and are difficult to change (Boler, 1999); if teachers are making decisions out of habit rather than according to their pedagogical responsibilities, then bringing responsibility into the conversation may actually lend moral and intellectual authority to habits that diverge from what teachers desire (Jaggar, 1989). This matters in the case of our prior research where teachers downplayed their responsibilities to students' mathematics learning in a collaborative conversation (Chen et al., 2021); by constructing alternate responsibilities—loving students and teaching them life skills—as being in competition with mathematics learning, teachers justified the minimizing of mathematics content rather than pursuing instructional decisions that could fulfill multiple pedagogical responsibilities at once.

Our findings have implications for teacher educators seeking to support teachers in making instructional decisions that are aligned with their values, goals, and responsibilities. Future research should explore how common this observed phenomenon is and also when and how teachers' pedagogical reasoning about their responsibilities does align with their instructional decisions. Whether teachers feel they can fulfill their moral responsibilities is linked to teacher retention and resistance (Santoro, 2018), suggesting that teachers' relationships to their responsibilities, and how they understand the possible decisions that might fulfill them, warrants further examination.

References

Bannister, N. A. (2015). Reframing practice: Teacher learning through interactions in a collaborative group. *Journal of the Learning Sciences*, *24*(3), 347–372.

Bishop, A. J. (1976). Decision-making, the intervening variable. *Educational Studies in Mathematics*, *7*(1-2), 41-47.

Boler, M. (1999). A pedagogy of discomfort: Witnessing and the politics of anger and fear. In *Feeling power: Emotions and education* (pp. 175–203). Routledge.

Borko, H., Roberts, S. A., & Shavelson, R. (2008). Teachers' decision making: From Alan J. Bishop to today. In P. Clarkson & N. Presmeg (Eds.), *Critical issues in mathematics education: Major contributions of Alan Bishop* (pp. 37–67). Springer.

Chen, G. A., Marshall, S. A., & Horn, I. S. (2021). "How do I choose?": Mathematics teachers' sensemaking about pedagogical responsibility. *Pedagogy, Culture & Society*, *29*(3), 379–396.

Coburn, C. E. (2001). Collective sensemaking about reading: How teachers mediate reading policy in their professional communities. *Educational Evaluation and Policy Analysis*, *23*(2), 145–170.

Copur-Gencturk, Y., Cimpian, J. R., Lubienski, S. T., & Thacker, I. (2020). Teachers' bias against the mathematical ability of female, Black, and Hispanic students. *Educational Researcher*, *49*(1), 30–43.

Fairclough, N. (2015). *Language and power*. Routledge.

Horn, I. S. (2005). Learning on the job: A situated account of teacher learning in high school mathematics departments. *Cognition and Instruction*, *23*(2), 207–236.

Horn, I. S. (2020). Supporting the development of pedagogical judgment: Connecting instruction to contexts through classroom video with experienced mathematics teachers. In G. M. Lloyd & O. Chapman (Eds.), *International Handbook of mathematics teacher education: Volume 3* (pp. 321–342). Koninklijke Brill.

Horn, I. S., & Garner, B. (2022). *Teacher learning of ambitious and equitable mathematics instruction: A sociocultural approach*. Routledge.

Horn, I. S., Nolen, S. B., & Ward, C. (2013). Recontextualizing practices: Situative methods for studying the development of motivation, identity, and learning in and through multiple contexts over time. In S. Volet & M. Vauras (Eds.), *Interpersonal regulation of learning and motivation: Methodological advances* (pp. 188–203). Routledge.

Jaggar, A. M. (1989). Love and knowledge: Emotion in feminist epistemology. *Inquiry*, *32*(2), 151–176.

König, J., Santagata, R., Scheiner, T., Adleff, A.-K., Yang, X., & Kaiser, G. (2022). Teacher noticing: A systematic literature review of conceptualizations, research designs, and findings on learning to notice. *Educational Research Review*, *36*, 100453.

Lefstein, A., Louie, N. L., Segal, A., & Becher, A. (2020). Taking stock of research on teacher collaborative discourse: Theory and method in a nascent field. *Teaching and Teacher Education*, *88*, 102954.

Louie, N., Adiredja, A. P., & Jessup, N. (2021). Teacher noticing from a sociopolitical perspective: The FAIR framework for anti-deficit noticing. *ZDM - Mathematics Education*, *53*(1), 95-107.

Munby, H. (1982). The place of teachers' beliefs in research on teacher thinking and decision making, and an alternative methodology. *Instructional Science, 11*(3), 201-225.

Ronfeldt, M., Farmer, S. O., McQueen, K., & Grissom, J. A. (2015). Teacher collaboration in instructional teams and student achievement. *American Educational Research Journal, 52*(3), 475–514.

Santoro, D. A. (2016). "We're not going to do that because it's not right": Using pedagogical responsibility to reframe the doublespeak of fidelity. *Educational Theory*, *66*(1–2), 263–277.

Santoro, D. A. (2018). *Demoralized: Why teachers leave the profession they love and how they can stay*. Harvard Education Press.

Stengel, B. S., & Casey, M. E. (2013). To "grow by looking": From moral perception to pedagogical responsibility. In H. Sockett & R. Boostrom (Eds.), *A moral critique of contemporary education* (pp. 116–135). Teachers College Press.

van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education*, *10*(4), 571–596.

Wahlström, N., Bossér, U., & Vogt, B. (2023). Teaching as a pedagogical responsibility: an introduction. *Journal of Curriculum Studies*, *55*(1), 1-7.

Webel, C., & Platt, D. (2015). The role of professional obligations in working to change one's teaching practice. *Teaching and Teacher Education, 47*, 204-217.

Acknowledgements

This material is based upon work supported by the United States National Science Foundation under Grant Nos. DRL-1620920 and DGE-1445197. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

TSG4-1-18

ANALYZING STUDENT REFLECTIONS IN TEACHING PRACTICUM TO ENHANCE MATHEMATICS INSTRUCTION COURSES

Hideyo Makishita
Yamato University, Japan, makishita.hideyo@yamato-u.ac.jp

Mahiko Takamura
Tokyo Polytechnic University, Japan, T3711@st.t-kougei.ac.jp

Hisao Oikawa
Shitennoji University, Japan, oikawa@shitennoji.ac.jp

This study analyzes student teachers' reflections on their teaching practicum to identify challenges and insights for improving mathematics instruction. During the practicum, they face challenges such as lesson planning, student engagement, and classroom management while making independent decisions quickly. Although they have studied theoretical aspects at university, they experience difficulties applying their knowledge in practice. This study uses text mining to analyze the "Achievements and Reflections" of 17 mathematics student teachers from University A, extracting common themes like building trust with students, time management difficulties, questioning techniques, and instructional material research. The findings reveal a gap between theoretical knowledge and practical skills, highlighting the need for a more integrated training program. To address these challenges, this study proposes incorporating mock lessons, and strategies that foster student autonomy. The research aims to bridge the gap between theory and practice by developing a more field-oriented and practical teacher education curriculum.

Keywords: Teaching Practicum, Lesson Design, Student Engagement, Classroom Management, Theory-Practice Gap

Introduction

During the teaching practicum, student teachers leave the university and conduct lessons independently at their assigned schools while receiving advice from supervising teachers. They are expected to encounter situations that cannot fully be addressed by the content learned in university courses, such as mathematics teaching methodology. Indeed, an analysis of the "Achievements and Reflections on the Teaching Practicum" in the practicum journals reveals that many student teachers mention the difficulties and challenges they faced. The author believes these records could directly improve teacher training curricula. The reflections of student teachers include elements such as decision-making in lesson planning, interactions with students, and responses to students' reactions during lessons—elements that are difficult to learn in traditional lecture-based university courses. Therefore, the authors believe that a detailed analysis of these reflections could provide valuable insights for improving mathematics teaching methodology courses to be more practical and field-oriented.

Purpose of This Study

In recent years, educational settings have increasingly required the utilization of ICT (Information and Communication Technology), the introduction of active learning, and the promotion of student-centered learning activities, necessitating instructional practices that differ significantly from traditional teaching methods. Consequently, student teachers must devise innovative lesson designs and teaching strategies. Furthermore, they often encounter a gap between theory and practice, facing difficulties in classroom management and engaging students. In addition, teaching practicum requires student teachers to develop effective questioning techniques, manage time allocation, and utilize ICT while fostering student autonomy, which emerges as a key challenge in lesson implementation during the practicum.

This study analyzes student teachers' reflections, identifies challenges, and explores possible solutions. Based on these findings, it seeks to improve teacher training programs, including mathematics instruction methodology. Through this research, we aim to explore instructional strategies that enable student teachers to adapt to difficulties in the classroom and develop practical teaching skills, ultimately fostering an organic connection between university education and hands-on teaching experience.

Research Methodology

In this study, we conduct a comprehensive analysis of the content written by 17 student teachers who participated in teaching practicums at the secondary school level, specifically in mathematics education. These student teachers were enrolled in middle or high school mathematics programs, and the data was collected from the "Achievements and Reflections on the Teaching Practicum" section of their practicum journals. The primary goal of this analysis is to explore their reflections and to extract valuable insights that will contribute to the improvement of mathematics teaching methodology courses within teacher education programs.

Through this detailed examination, we aim to identify common trends, challenges, and reflections that student teachers encounter during their practicum experiences. This, in turn, provides crucial feedback for refining and enhancing the effectiveness of teacher training curricula. By integrating the personal reflections of student teachers, we hope to better bridge the gap between theoretical knowledge gained in academic settings and practical teaching skills developed in real-world classroom environments.

Key Challenges Identified in Student Reflections

In analyzing the reflections and feedback of the student teachers, several key challenges emerged that highlight the complexities of teaching at the secondary school level. One prominent issue identified was the importance of building trust and rapport with students, which is seen as directly contributing to the success of the lessons. Many student teachers reflected on their experiences, noting statements such as, "I was able to understand the students' true feelings," "I learned the importance of engaging with students," and "I built trust through interactions during breaks and after school." These observations underscore the critical need for effective communication with individual students and the challenge of understanding and addressing their diverse needs in a classroom setting.

Additionally, a common challenge faced by many student teachers was managing time effectively during lessons. Numerous entries in the practicum journals revealed comments such as, "I could not

finish the planned content within the allotted time" and "Adjusting the pace of the lesson was difficult." These reflections highlight the crucial role of careful time management and pacing in lesson planning. It became evident that student teachers must develop more effective strategies for allocating time to different lesson sections to ensure that learning objectives are met within the given time frame.

Another difficulty that emerged from the analysis was related to questioning techniques in the classroom. Several student teachers expressed frustration with their attempts to engage students through questioning, with comments such as, "I incorporated questions but did not receive the expected responses" and "I need to devise ways to ask questions that make it easier for students to respond." This suggests a significant challenge in developing questioning techniques that effectively engage secondary school students and elicit meaningful responses. Student teachers need support in crafting questions that foster a productive dialogue and stimulate critical thinking among students.

These findings underscore the necessity for more targeted instruction and guidance in areas such as student engagement, classroom management, and the development of effective questioning strategies. Moreover, teacher training programs should emphasize these practical aspects of teaching, providing student teachers with the tools and strategies they need to succeed in real-world classroom settings.

Conclusion

The authors analyzed and examined the achievements and reflections extracted from the student teachers' "Achievements and Reflections on the Teaching Practicum" and obtained the following findings as the outcomes of this study. It should be noted that most of these findings are interrelated, requiring a cross-sectional approach. Based on the results of this study, we aim to utilize these insights to improve mathematics instruction methodology courses.

Findings:

1. Realize the importance of relationships with students.
2. Challenges in lesson planning and execution:
    a. Difficulty in time management.
    b. Crafting practical questions.
    c. Utilizing preparation and review effectively.
3. Importance of instructional material research and lesson preparation.
4. Self-growth through the practicum.
5. Impact of practicum experiences on the development of teaching philosophy.

Future Challenges

We have identified four key focus areas when considering improvements in mathematics instruction methodology courses. Moving forward, we plan to implement practical case studies addressing the following aspects:

1. Instructional Material Research
   a. Encouraging students to analyze real-world situations where mathematics is applied.
   b. Example: Relating mathematical concepts to students' specialized fields of study.
2. Support for Lesson Planning through Mock Lessons and Research Discussions
   a. Increasing the frequency of mock lessons.
   b. Establishing an evaluation system for mock lessons.
3. Utilization of ICT and Case Studies on AI Implementation
   a. Introducing dynamic geometry software such as GeoGebra and Cinderella.
   b. Implementing software like WolframAlpha and R.
4. Clarifying the Relationship with New Teaching Methodologies such as Active Learning
   a.Developing instructional materials for project-based learning in high schools.
   b. Utilizing university-high school collaboration materials, such as SSH (Super Science High School) programs.

We will further examine these aspects in detail and propose concrete recommendations.

References

Nakanishi, Y., Watanabe, K., & Iwata, K. (2004). "A Study on the Learning Effects of Teaching Practice in Arithmetic Education Methods." *Gifu Mathematics Education Research, Vol. 3*, 1-11.

Nagata, T. (2013). "Current Situation and Improvement of Lesson Practice in Teaching Practicum: Analyzing Student Teachers' Lesson Practice through Teaching Practicum Records'." *Aichi University Annual Report on Teacher Education, 2*, 69-82.

Nishinaka, N., Hanaki, R., & Funahashi, Y. (2015). "A practical study on fostering the trainee teachers' ability of using ICT in mathematics classroom." *Bulletin of Teacher Education Center for the Future Generation, 1*. 311-316.

Amano, K. (2016). "A Study on Improving the Course 'Educational System' in Teacher Education:

Lesson Design Integrating Experiential Learning through School Support Activities." *Shimonoseki City University Review, Vol. 59, No. 3*, 65-74.

Watanabe, K., & Aso, R. (2017). "A Study on Changes in Perception Towards the Profession of

Teaching as Seen in Student Teachers in Teaching Practice: Surveys Before, During, and After Practice." *Bulletin of the Comprehensive Center for Educational Practice, Faculty of Education, Oita University, No. 35*, 161-174.

TSG4-1-19

INVESTIGATING BEST PRACTICES OF HIGH SCHOOL MATH TEACHERS

Bryan Chao
Cal Poly University, Pomona, USA, bwchao@cpp.edu

Hyunkyoung Yoon
Cal Poly University, Pomona, USA, hkyoon@cpp.edu

This study examines how high school math teachers' academic backgrounds and teaching experience affect their instructional approaches. We interviewed two Algebra 1 teachers who have different degrees and years of teaching. Before conducting interviews, we administered the Mathematical Meanings for Teaching secondary mathematics (MMTsm) items to assess their mathematical meanings of slope. Findings reveal that a math degree does not ensure that productive mathematical meanings are being made. The teacher with a math degree expressed their mathematical meanings in multiple ways while the non-math major teacher demonstrated their usage of consistent examples.

Keywords: Mathematical meanings for teaching, Algebra 1, Slope

Introduction

High school math teachers have multiple ways of communicating their mathematical meanings. In the United States, a mathematics degree is not required to teach high school mathematics because certification can be obtained by completing a certification program and passing content exams. High school math teachers develop their understanding of mathematical meanings as they advance in mathematics, but math teachers who are not math majors do not get to experience that. This study investigates the relationships between teachers' academic major, and years of teaching, and their instruction. We seek to answer the following research question: *How do two teachers with different academic backgrounds and teaching experience convey their mathematical meanings to students?*

Background

Teachers who hold bachelor's or master's degrees in mathematics or science have been observed to be associated with higher student performance than teachers who hold those degrees in other subjects. This suggests that teachers who completed subject-specific training or education in their relative teaching area are most likely to positively affect the outcomes of their students in that class. Research has found that teachers who undergo specialized training in their subject areas are more likely to use highly effective instructional approaches. It has been argued that teachers with strong subject-specific training are better equipped to use instructional methods that promote conceptual understanding rather than rote memorization (Ball, Thames, & Phelps, 2008). According to Darling-Hammond (2000), teachers who are experts in their subject matter can create more engaging and challenging learning environments. We discuss productive and unproductive meanings that teachers hold when teaching the idea of slope. By productive meanings for teaching, we refer

to meanings a teacher holds that would be productive for students' long-term learning (Yoon & Thompson, 2020). For example, describing slope in terms of "slantiness" meanings during lessons might be less productive for students' understanding of negative slopes than emphasizing slope as a ratio of changes in x and y.

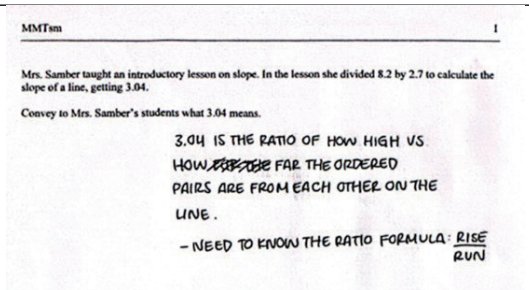
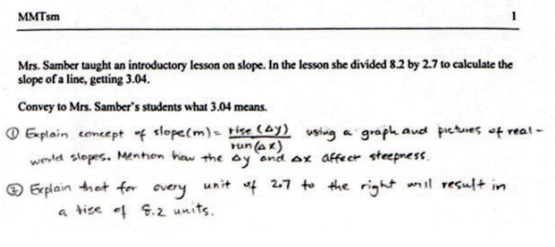
Method

The two high school math teachers volunteered to participate in this study. Both teachers taught algebra 1 at their schools in 2024. Students in these classes consist of around 30 students in each class and are in grade 9. The two teachers have different academic backgrounds and years of teaching. Morgan is a teacher who has a math degree and five years of experience teaching high school math. Penny is a teacher who has a political science degree and is in their first year of teaching high school math. We asked the teachers to take items for slope from a diagnostic test (MMTsm) before the interview to gauge their mathematical meanings (Thompson, 2016). We held initial interviews with both teachers to discuss their responses to the MMTsm items. In the pre-lesson interview, the participants answered open-ended questions about their academic major and how their instructional methods evolved over time. Data that was collected from the MMTsm items and interviews were the teacher's mathematical meanings and their understanding of best practices for teaching students. After the pre-lesson interview, the classroom observation took place. We video recorded the lesson to capture their delivery of the lesson and interactions with students. From the classroom observation, we witnessed how the teachers expressed their mathematical meanings to students.

Results

The two teachers expressed different sets of mathematical meanings for slopes in their MMTsm items, interviews and instruction. We present evidence from their MMTsm item responses, pre-lesson interviews, and instruction. During our MMTsm interview, Morgan expressed the meaning of the slope as rise over run. It seemed that Morgan was fixated on having slope represent the ratio of rise over run and the concept of distance. Table 1 depicts Morgan's and Penny's written response.

Table 1. Morgan's and penny's responses to the MMTsm item

Morgan's Response	Penny's Response
	

Morgan's response provides insight on their meaning of distance to conceptually define slope. Morgan expressed their meanings for slope in multiple ways, including descriptions for horizontal and vertical movement, and the ratio formula for slope which is "rise over run." Morgan seemed to closely tie the concept of slope to fractions as evident in their teaching approaches in the classroom observation. Penny expressed their meanings for slope as "rise over run" but emphasized the idea of

steepness. Penny was consistent in expressing their meaning of slope to their students in an attempt to lessen any confusions that students might have. We observed that Penny used their intended meanings expressed in the pre-lesson interview where they relate the concept of slope to steepness consistently when delivering their instruction. Responses to the MMTsm item provides insight into how Morgan and Penny perceive the meaning of slope. We now analyze dialogue captured from their classroom observations that shows how each teacher expressed their meaning to students.

Table 2. Morgan's expressed meanings of slope during lessons

Slope Meaning Expressed	Dialogue from Classroom Observation
"m" is slope	... writing the equation of a line we had the formula $y = mx + b$ and we said that m is what is considered our slope...
Rise over run explanation	Slope is a non-vertical line that passes through two points and is the ratio of the rise which is the y-value to the run which is the x-value.
Fraction indicating directions	... looking at slope okay, it's always going to be kind of like a fraction so whatever is on the top of the fraction, that tells you to go up a few points or to go down a few points and the bottom number, your denominator tells you to go left or right.

Table 2 shows how Morgan expressed their meaning of slope to their students. It seemed that Morgan utilized their background in mathematics to identify key concepts to use in relation to their meaning of slope. However, it was noted that Morgan's comment, "slope is a non-vertical line" is a misconception because slope can be a vertical line with the slope being undefined. It seemed that Morgan was trying to introduce multiple cases of slope but came across a misconception with their experience in university mathematics. There were multiple instances where Penny prioritized the idea of having students relate slope with steepness of objects in real life to keep personal familiarity. Penny constantly used this example to help students try to understand different types of slope, and they also used common math terms like "rise over run" to depict the concept of slope. In Table 3, Penny expressed their meaning of slope as the idea of slantiness.

Table 3. Penny's expressed meanings of slope during lessons

Slope Meaning Expressed	Dialogue from Classroom Observation
Slantiness	If I were to tell you to walk or run or something up one of these slopes. What would you prefer this bridge this hill or this cliff?
	Why would we say the bridge, it's flatter, it's flat you're not going to be climbing ...
	... slope and steepness right we're talking about different steepness...
	...in terms of rate of change, it's going to be a fraction that shows change in steepness...

Penny consistently expressed this idea to their students, which offered less variety in comparison to Morgan. It could imply that Penny did not have the proper background to express their meaning of slope in multiple ways like Morgan. Table 4 summarizes our findings from the two teachers and helps us answer the research question.

Table 4. Teacher demographics and their meaning for slope

	Morgan	Penny
Degree	Mathematics	Political Science
Years of Teaching	5 years	1 year
Meanings for Slope expressed in MMTsm, interviews, lessons	Rise over run, directional movement based on value of fraction, slope as distance between two points, rate of change, fractions relating to counting slope	Rise over run, Steepness
Instructional Approach	Expressed meanings in multiple mathematical ways	Used consistent real-world metaphors to reinforce conceptual clarity

Conclusions and Implications

Both teachers demonstrated a mix of productive and unproductive mathematical meanings for slope regardless of their academic major or teaching experience. The difference between the two teachers lies in the extent to which they expressed the meaning of slope in different ways to their students. Morgan expressed their meanings for slope in multiple ways as seen in their lessons, which may be attributed to their degree in mathematics and teaching experience. In contrast, Penny consistently used a limited but structured set of meanings for slope to their students, which might be influenced by their minimal teaching experience.

This study highlights that a teacher with a background in math might be better positioned to express their mathematical meanings in multiple ways. This finding aligns with previous research suggesting that teachers with the same academic background as the subject they are teaching are more likely to produce higher student performance and deep mathematical learning (Darling-Hammond, 2000). In addition, Morgan mentioned difficulties in transforming college-level mathematical knowledge into teachable content reflects the concept of "double discontinuity" introduced by Klein (1932) where secondary math teachers struggle to bridge college-level mathematics with high school instruction. This study provides insight into how teachers' backgrounds can shape their instructional practices. It also underscores the need for structured support to help teachers develop meaningful connections between secondary and tertiary mathematics. The findings suggest that high school math teachers' undergraduate degrees might impact their instructional practices, particularly in fostering students' deep mathematical understanding.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching:What Makes It Special? Journal of Teacher Education, 59, 389-407.

Darling-Hammond, L. (2000). Teacher Quality and Student Achievement. *Education Policy Analysis Archives*, *8*, 1.

Klein, F., Hedrick, E. R., & Noble, C. A. (1932). *Elementary mathematics from an advanced standpoint: Arithmetic, algebra, analysis*. New York: Macmillan.

Thompson, P. W. (2016). Researching mathematical meanings for teaching. In English, L., & Kirshner, D. (Eds.), *Handbook of International Research in Mathematics Education*, 435-461.

Yoon, H., & Thompson, P. W. (2020). Secondary teachers' meanings for function notation in the United States and South Korea. *The Journal of Mathematical Behavior, 60*, 100804.

TSG4-2-12

EVALUATION OF STUDENTS' COUNTEREXAMPLES BY PRE-SERVICE PRIMARY SCHOOL TEACHERS

Kayo Miura
Hiroshima University, Japan, d225226@hiroshima-u.ac.jp

This study examines how pre-service primary school teachers (PSTs) evaluate students' counterexamples and explanations using counterexamples. The analysis was performed in generic and explanatory perspectives, and it was found that the PSTs tended to focus more on the explanatory nature than the generic one. Additionally, it was assumed that differences among individuals exist in the PSTs' evaluation criteria due to a lack of education on counterexamples and explanations using counterexamples in teacher training.

Keywords: counterexamples, disproving, explanations

Introduction

Logically, it only takes one counterexample to show that a proposition (statement) does not always hold. However, pedagogically, counterexamples not only serve to prove (disprove) that a proposition is false, but also to explain why it is incorrect and in what cases a counterexample is produced (Peled & Zaslavsky, 1997). From the same point of view as Peled & Zaslavsky (1997), we have researched on counterexamples and explanations using counterexamples (Miura & Shinno, 2022; Miura, 2024).

In Japan, primary school students do not learn proofs and counterexamples to false statements. Primary school teachers also do not learn how to teach proofs and counterexamples to false statements at their training stage. In previous studies, it has been unclear how primary school teachers and pre-service primary school teachers (PSTs) perceive and evaluate students' counterexamples and explanations using counterexamples. Therefore, the following research question (RQ) is addressed to clarify how PSTs perceive and evaluate students' counterexamples and explanations using counterexamples. RQ: How do PSTs determine whether students' counterexamples and explanations using counterexamples are appropriate or not?

Theoretical Framework

Peled & Zaslavsky (1997) demonstrated that counterexamples arising in the context of disproving a false claim should provide an explanation as to why the claim is false and suggest a way to generate other counterexamples. Based on their survey, the counterexamples were classified into three types.

1. *Specific*: A counter-example which satisfies the task, as it provides a specific example contradicting the claim, yet does not give a clue to an underlying mechanism for constructing other (similar or related) counter-examples.

2. *Semi-General*: A counter-example which provides an idea about a mechanism for generating other (similar or related) counter-examples for the claim, yet does not tell "the whole story" and does not cover "the whole space" of counter-examples.
3. *General*: A counter-example which provides a "behind the scene story," and suggests a way to generate the entire counter-example space. (Peled & Zaslavsky ,1997, p53)

Peled and Zaslavsky (1997) classified counterexamples by focusing on the mechanisms by which counterexamples were generated. The classification is not only based on the logical point of view, but also on the pedagogical point of view, with the aim of understanding the counterexamples and encouraging new discoveries from the counterexamples. Zeybek (2017) also focused on explanations other than counterexamples, partially modifying Peled & Zaslavsky's (1997) classification and reestablishing three types. Zeybek's (2017) classification distinguishes between "specific counterexamples", which indicate that a statement is false, and "semi-generic counterexamples" and "general counterexamples", according to the explanatory level of the reason why a statement is false. Peled & Zaslavsky (1997) and Zeybek (2017) used the same name for their classification but their methods differed in whether they focused on the generic (general) nature of counterexamples or the explanatory nature of the reason for being false. In sum, there are two perspectives from which counterexamples and explanations using counterexamples can be viewed: the generic perspective and the explanatory perspective. I investigate how PSTs view these two points.

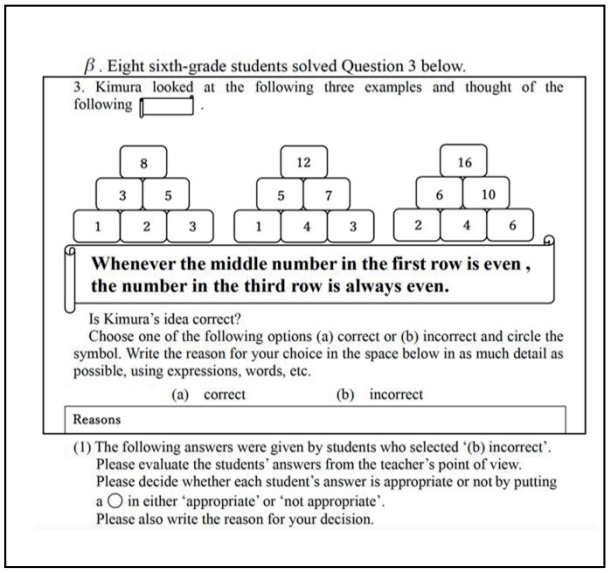
Methods

The survey period was June 2024, and the target PSTs were 13 fourth-year students in the Department of Education at the National University as described previously (Miura, 2024). The survey questions consisted of three parts: question 1, 2 and 3, and question α and β, and the questions on counterexamples.

This paper focuses on question β-1 about a false statement. Question β-1 is partly based on the answers to question 3, which was previously asked of students in the sixth grade. Question 3 is about the "number pyramid" and is based on NIER (2006) and Komatsu (2012). Question 3 asks students to decide whether the statement "Whenever the middle number in the first row is even, the number in the third row is always even." is correct or incorrect and to state the reason.

As shown in Figure 1, question β-1 evaluates whether the students' answers are appropriate or not and the reasons. In this paper, I analyze the PSTs' responses to the following students' answers 'I' (Figure 2, low generic and low explanatory), 'M' (Figure 3, high generic and low explanatory), 'N' (Figure 4, high generic and high explanatory), and 'P' (Figure 5, somewhat low generic and high explanatory) in terms of generic (general) and explanatory perspectives.

It is better that I deal with low generic and high explanatory answers, but there are no such students' answers. In fact, the students' answers tend to become more generic as the explanatory level increased. Therefore, I need to use 'P' for this research. 'P' describes how to generate counterexamples using specific numbers and words. 'P' is somewhat less generic (general) than 'N', which uses algebraic expressions and words. In addition, since students do not learn in school the reasons why even and odd become odd numbers and even and even become even numbers, I do not require explanations of such reasons for analysis in this study.


β. Eight sixth-grade students solved Question 3 below.

3. Kimura looked at the following three examples and thought of the following [ ].

Whenever the middle number in the first row is even, the number in the third row is always even.

Is Kimura's idea correct?

Choose one of the following options (a) correct or (b) incorrect and circle the symbol. Write the reason for your choice in the space below in as much detail as possible, using expressions, words, etc.

(a) correct (b) incorrect

Reasons

(1) The following answers were given by students who selected '(b) incorrect'. Please evaluate the students' answers from the teacher's point of view. Please decide whether each student's answer is appropriate or not by putting a ○ in either 'appropriate' or 'not appropriate'. Please also write the reason for your decision.

Figure 1. Question β *-1*

I

For example, if the numbers in the first row are 3, 4 and 2 from the left, the numbers in the second row are 7 and 6 from the left, and the number in the third row is 13, which cannot be an even number.

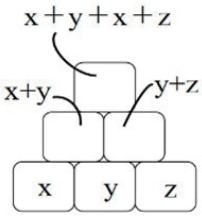
Figure 2. Question β *-1 'I'*

M

In all three cases, the numbers on both sides are either both odd or both even.
If we divide the numbers on both sides into odd and even, the answer will always be odd.
So, it is incorrect.

Figure 3. Question β *-1 'M'*

N

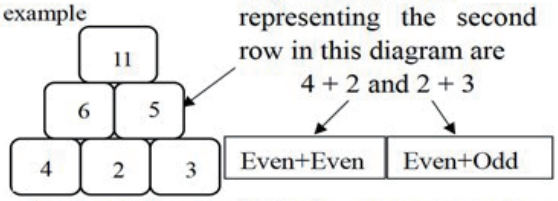


If the number on the left of the first row is x, the number in the middle of the first row is y and the number on the right of the first row is z, then the number in the third row is x+y+y+z.
If y is even and only one of x or z is odd, then the number in the third row is odd.
Therefore, it is incorrect.

Figure 4. Question β *-1 'N'*

P

example



An even + even number is always an even number and an even + odd number is always an odd number.
This means that the two numbers in the second row are even odd or odd even.
Consequently, when the numbers in the third row are produced, they will be even + odd or odd + even. This is incorrect because they will be odd either way.

Figure 5. Question β *-1 'P'*

Results

Table 1 presents the results of the appropriateness evaluations made by 11 PSTs who evaluated all the questions correctly in the questions on counterexamples. Most of the PSTs decided that the highly explanatory answers 'N' and 'P' were appropriate, regardless of their genericity. More than half of the PSTs considered the less explanatory and less generic answer 'I' to be appropriate, while less than half considered 'M' to be appropriate despite being more generic than 'I'.

Table 1. Appropriateness evaluations for each students' answer in the PSTs.

	I	M	N	P
Number of appropriate	6	4	9	9
Number of inappropriate	5	7	2	2

Additionally, the most common reasons for evaluating appropriateness were the presence (or absence) of a counterexample and the presence (or absence) of explanations. Furthermore, I explored the trends in each of the PST's reason for evaluating the appropriateness of the four students' answers and found three representative types.

1. Counterexample type, Evaluations are performed according to the presence of a counterexample: PST04 determined that 'I' and 'P' were appropriate because they contained concrete counterexamples; 'M' and 'N' were inappropriate because they contained no counterexamples and incomplete explanations of how the counterexamples were generated.
2. Explanation type, Evaluations are performed according to the presence of an explanation of why a counterexample is generated or how a counterexample is generated: PST02 determined that 'I' and 'M' were inappropriate due to insufficient explanation. PST02 decided that 'N' and 'P' were appropriate because 'N' used algebraic representations to explain that a counterexample was generated, and 'P' relied on the fact that the sum of odd and even numbers is always odd. PST08 considered 'I' inappropriate because it did not use general terms when the third row would not be even, while 'M', 'N' and 'P' were appropriate because they used written words when the third row would not be even.
3. Combined counterexample and explanation type, Type 3 consists of a combination of Type 1 and Type 2. Evaluations are performed according to the presence of a counterexample or an explanation or both: PST07 determined that all of 'I', 'M', 'N' and 'P' were appropriate because there was their counterexample for 'I', there were a counterexample and an explanation of why a counterexample is generated for 'P' and there were references to how a counterexample is generated for 'M' and 'N'.

Discussion and Conclusions

In assessing whether students' counterexamples and explanations using counterexamples were appropriate, the PSTs tended to emphasize the explanatory perspective of Zeybek's (2017) classification criteria over the generic perspective. There are two possible reasons why the generic perspective was not given much weight. Firstly, it is logically sufficient to provide a counterexample in a false statement. The number of PSTs who thought 'I' appropriate was greater than that of PSTs who 'M' appropriate. Secondly, the PSTs considered that students do not learn proofs involving algebraic expressions in primary school. This may be the reason why there was no difference between the numbers of PSTs who thought 'N' appropriate and PSTs who thought 'P' appropriate.

The results of the reasons for evaluating appropriateness demonstrated that most of the PSTs also accepted disproving other than counterexamples. Based on their understanding of counterexamples, the PSTs evaluated students' answers from a pedagogical point of view. However, the criteria for evaluation differed slightly from PST to PST. For example, some PSTs used a method for generating counterexamples and others stated the reasons why counterexamples can be generated. Several

PSTs considered the answers to be inappropriate because of insufficient explanation, even if students provided counterexamples. The differences among the PSTs are presumably due to the lack of education on counterexamples and explanations using counterexamples in teacher training. In addition, the PSTs targeted in this study are national university students and have relatively high academic ability. If PSTs from other universities are included, it is assumed that many PSTs will have insufficient understanding of counterexamples. Teacher training needs to include tasks that promote understanding of counterexamples, and tasks that require PSTs to generate and evaluate them.

References

Komatsu, K. (2012). Theory of Learning and Learner in Mathematics. In I. Ohtaka, & Y. Shimizu (Eds.), Theory and lesson of Subject Education II (pp. 71-84). kyodo shuppan. (In Japanese, translated by the author of this article.)

National Institute for Educational Policy Research (2006). *Research on Specific Issues: Mathematics and Arithmetic: Elementary and Secondary schools*. https://www.nier.go.jp/kaihatsu/tokutei/

Miura, K., & Shinno, Y. (2022). A Study on Students' Explanations by Counter-examples in Elementary School Mathematics: Characterising Students' Responses Based on a Survey Using "Number Pyramid" as a Task. *Journal of science education in Japan, 46, 3,* 243-257. (In Japanese with English abstract.)

Miura, K. (2024). Prospective Elementary School Teachers' Perception of Explanations by Counterexamples. *Journal of science education in Japan, 48, 3,* 271-282. (In Japanese with English abstract.)

Peled, I., & Zaslavsky, O. (1997). Counter-example that (only) prove and counter-example that (also) explain. *FOCUS on Learning Problems in Mathematics, 19, 3,* 49-61.

Zeybek, Z. (2017). Pre-service Elementary Teachers' Conceptions of Counterexamples. *International Journal of Education in Mathematics, Science and Technology, 5, 4,* 295-316.

Acknowledgment

This work was supported by JSPS KAKENHI Grant Number JP25H00071.

TSG4-2-13

SHORT-DURATION OBSERVATION INSTRUMENT FOR ELEMENTARY MATHEMATICS TEACHERS' COMPETENCE (SDOI-EMTC)

Ting-Chun Kuo
National Taipei University of Education, Taiwan, arthur258a@gmail.com

Chia-Jui Hsieh
National Taipei University of Education, Taiwan, paris@tea.ntue.edu.tw

Assessing whether a mathematics teacher possesses the required professional competencies is not only a fundamental responsibility of teacher education institutions but also a critical evaluation method for school recruitment. In practice, it is often necessary to complete a teacher competency assessment within a brief period (10–20 minutes), which calls for a concise yet reliable observation instrument. This study aims to develop such an instrument. Employing a survey research design, we began by adapting an existing mathematics classroom observation scale through expert deliberation. Empirical validation was then conducted by analyzing 10-minute teaching videos of 122 pre-service elementary mathematics teachers. By referencing external standards, we identified the core competency indicators essential for mathematics teachers, culminating in the development of a short-duration observation scale with demonstrated reliability and validity.

Keywords: elementary mathematics teachers, short-duration observation checklist, teacher professional competence

Introduction

Assessing a teacher's professional teaching competence can be approached in various ways. One common method involves evaluating their professional abilities through examinations. However, such assessments often lead teachers to present an idealized version of their knowledge, which may not accurately reflect their typical classroom performance. A more practical alternative involves observing their actual classroom teaching and utilizing a checklist to evaluate specific professional competencies. Yet, this approach can be overly time-consuming and is easily influenced by the variability of lesson content and students' responses.

Another critical consideration is determining which aspects of teaching should be observed. The National Council of Teachers of Mathematics (NCTM) established the Professional Standards for Teaching Mathematics (1991), which include six standards: experiencing good mathematics teaching, knowing mathematics and school mathematics, knowing students as learners of mathematics, knowing mathematical pedagogy, developing as a teacher of mathematics, and the teacher's role in professional development. However, some of these standards cannot be fully captured within a single lesson. For example, "developing as a teacher of mathematics" requires long-term experience and continuous feedback to foster improvement. In practical settings, there is often a need to complete an evaluation of a mathematics teacher's professional competence within a very short timeframe (10–20 minutes). To address these challenges, this study aims to develop an observation checklist capable of effectively assessing a teacher's professional competence within such a limited

period. This tool seeks to provide an efficient and objective means for evaluating teaching practices, thereby supporting the ongoing improvement of instructional quality.

Literature Review

Shulman (1986) initially identified three essential types of teacher knowledge—content knowledge, pedagogical content knowledge (PCK), and curricular knowledge—which form the foundation of effective teaching. Conceptual representation is fundamental to teaching and learning mathematics, with scholars such as Goldin and Kaput (1996) and Lesh et al. (1987) emphasizing the importance of diverse representational systems for understanding mathematical concepts. Bruner (1966) further proposed three interconnected modes of representation—enactive, iconic, and symbolic—highlighting the need to align representations with learners' developmental stages. Similarly, English and Halford (1995) stressed that selecting appropriate external representations facilitates the formation of internal ones, making representation selection a crucial professional competence for teachers. Likewise, guiding students in mathematical problem-solving is essential. Polya's (1945) four-step model and Schoenfeld's (1985) focus on metacognition and strategic planning illustrate that clear thinking, reflective practice, and self-monitoring are key to helping students develop mathematical understanding, reinforcing the central role of problem-solving skills in teachers' professional expertise.

Bromme (2001) further noted that effective teaching involves adapting one's skills to the ever-changing classroom environment. Complementing these perspectives, Hsieh (2012) developed a comprehensive framework for mathematics teaching competence in secondary schools by identifying 20 critical elements across dimensions of cognition, reasoning, and operational execution, thereby highlighting the multifaceted nature of professional competence in mathematics education. These twenty elements include mathematical literacy, mathematical thinking, mathematical language, mathematical misconceptions, mathematical representations, motivation for learning mathematics, assessment in mathematics, mathematics teaching materials, mathematics curriculum, mathematics teaching methods, classroom management in mathematics, values in mathematics, attitudes toward mathematics, attitudes toward learning mathematics, mathematics teaching process, instructional media and manipulatives in mathematics, mathematical problem-solving, cognitive understanding in mathematics, student abilities and conditions, and mathematical content knowledge.

The aforementioned elements are widely adopted in classroom observation practices in Taiwan. In alignment with the original framework, the researcher retained those elements that are feasible for observation within a limited instructional timeframe.

Methodology

Research Design

This study employs a survey research method to analyze observable teaching behaviors in recorded mathematics lessons. Recognizing that a teacher's professional knowledge is largely implicit, the study uses teaching videos as explicit manifestations of instructional competence. The aim is to develop and validate a short-duration observation checklist that efficiently captures essential teaching behaviors within a limited time frame.

Participants

The research sample consists of 122 pre-service elementary school mathematics teachers. Their 10-minute teaching videos serve as the primary data source for evaluating instructional competencies.

Instruments and Data Collection

The primary instrument is a refined version of the Junior High School Mathematics Classroom Teaching Observation Form originally developed by Hsieh (2008). To ensure its validity and appropriateness, a panel of ten experts in mathematics education provided detailed feedback and recommendations, leading to multiple rounds of revisions. The final checklist includes clearly defined scoring criteria that guide the systematic evaluation of teaching performance. Data collection involved recording the teaching sessions, and two experienced elementary school teachers—each with over 15 years of teaching experience—served as raters. Prior to the evaluation, these raters engaged in thorough discussions to achieve a shared understanding of the scoring criteria, thereby minimizing subjectivity in the ratings.

Data Analysis

For reliability testing, a random sampling method was used to select 50% of the total sample for inter-rater consistency analysis. When discrepancies between raters exceeded one point on any item, a mathematics education expert was consulted to adjust the scores accordingly. Cohen's kappa coefficient (Cohen, 1960) was employed to quantify the level of agreement between raters, ensuring the objectivity of the evaluation process. In addition, confirmatory factor analysis (CFA) was conducted to assess the construct validity of the observation checklist. Model fit indices and factor loadings were evaluated to confirm that the internal structure of the checklist aligns with its intended theoretical framework.

Results and Discussion

Development of the Teacher Professional Competence Observation Checklist

In this study, we began with Taiwan's existing "Mathematics Classroom Teaching Observation Form" (Hsieh, 2012). Following expert deliberation, we empirically validated the instrument by analyzing 10-minute mathematics teaching videos from 122 pre-service elementary school mathematics teachers. This process confirmed five essential and observable core competence categories for elementary mathematics teachers, which include: Expression/Attitude, Conceptual Understanding, Problem-Solving, Activity Arrangement, and Assessment. These five categories were further divided into 11 subcategories, with each subcategory comprising multiple evaluation items. Examples of the categories, subcategories, and evaluation items are presented in Table 1.

Table 1. Observation checklist for elementary mathematics teachers' professional competence

Observation Category	Sub-categories	Specific Items
Expression/Attitude	Expression	( ) Clear oral expression, ……
	Attitude	( ) Friendly demeanor, ……
Conceptual Understanding	Conceptual Representation	( ) Numerical examples, ……
	Concept Explanation	( ) Concise and easy to understand, ……
	Concept Organization	( ) Connects with prior experiences, ……
	Example Arrangement	( ) Well-paired concepts and examples, ……
Problem-Solving	Mathematical Problem-Solving	( ) Clear problem presentation, ……
	Mathematical Thinking	( ) Facilitates discussion, ……
Activity Arrangement	Teaching Methods	( ) Summarization, ……
	Motivation Elicitation	( ) Considers student comprehension, ……
Assessment	Formative Assessment	( ) Classroom problem-solving assessment, ……

Note: Items marked with an asterisk ( ) are observable within a 40-minute class session. Items marked with double asterisks ( ) relate to the course content; their implementation adds bonus points.

Preliminary Results from the Initial Compilation of the Observation Checklist

To evaluate the reliability of the developed observation scale, the *Cronbach's α* value for the 11 items was .859, indicating that the scale can consistently measure the professional abilities of elementary school mathematics teachers. The scale has a maximum score of 20, with an average score of 12.18, and the average score for the top 25% was 16.83. Furthermore, rankings based on the scores from this scale were compared with those from peers using the original observation scale, and a correlation analysis was conducted. The resulting correlation coefficient was .83, showing a strong positive correlation between the two, further confirming the validity and reliability of the scale.

Conclusion

This study aims to develop a validated observation checklist that serves as a reliable and practical tool for assessing mathematics teachers' professional competence. The checklist will assist teachers in objectively evaluating their instructional behaviors, identifying areas for improvement, and enhancing their teaching practices, ultimately improving student learning outcomes in mathematics.

The observation checklist developed in this study demonstrates high reliability and validity. Through expert validation and empirical testing, the checklist has shown stability and accuracy in measuring teachers' professional competence. The five core competence categories covered by the checklist not only allow for a comprehensive evaluation of teachers' instructional performance but also guide teachers in improving and developing specific areas. In the future, this checklist can serve as an important tool in professional development programs for teachers and can be further applied in various teacher training and educational research, promoting continuous growth for teachers and enhancing teaching effectiveness.

References

Bruner, J. S. (1966). *Toward a theory of instruction*. Harvard University Press.

Bromme, R. (2001). Teacher expertise. In N. J. Smelser & P. B. Baltes (Eds.), *International encyclopedia of the social & behavioral sciences* (pp. 15459–15465). Elsevier.

Cohen, J. (1960). A coefficient of agreement for nominal scales. *Educational and Psychological Measurement, 20*(1), 37–46.

English, L. D., & Halford, G. S. (1995). *Mathematics education: Models and processes.* Lawrence Erlbaum Associates.

Goldin, G., & Kaput, J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. In L. P. Steffe, P. Nesher, P. Cobb, B. Greer, & J. Hunting (Eds.), *Theories of mathematical learning* (pp. 397–430). Lawrence Erlbaum Associates.

Hsieh, F.-J. (2008). Research on the professional development of mathematics teaching competency for secondary school teachers (III). *National Science Council Research Project Final Report* (NSC 96-2522-S-003-008). National Taiwan Normal University, Taipei, Taiwan.

Hsieh, F.-J. (2012). Indicators of professional competence for mathematics teaching in secondary schools. *Secondary Eucation, 63*(3), 30–40.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33–40). Lawrence Erlbaum Associates.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Author.

Polya, G. (1945). *How to solve it: A new aspect of mathematical method.* Princeton University Press.

Schoenfeld, A. H. (1985). *Mathematical problem solving.* Academic Press.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

TSG4-2-14

KNOWLEDGE NEEDED FOR TEACHING MATHEMATICS TO STUDENTS WITH LEARNING DISABILITIES

Roslinda Rosli
The National University of Malaysia, Malaysia, roslinda@ukm.edu.my

Chai Xin Ni
The National University of Malaysia, Malaysia, p130728@siswa.ukm.edu.my

Sarah Irdina Misri
The National University of Malaysia, Malaysia, p141954@siswa.ukm.edu.my

Khairul Farhah Khairuddin
The National University of Malaysia, Malaysia, kfk@ukm.edu.my

Teaching mathematics to students with learning disabilities presents significant challenges. These challenges are often linked to a need for more knowledge and skills in teaching mathematics to this group of students. This study aimed to examine the level of teachers' knowledge and experience in teaching mathematics to students with learning disabilities in Sabah and Sarawak, Malaysia. A survey was conducted involving data collection through questionnaires administered to 61 special education teachers in these regions. The questionnaire consisted of 24 items covering demographics, knowledge of teaching mathematics and the level of teachers' knowledge and experience in mathematics instruction. Data were collected online via Google Forms and face-to-face using convenience sampling and were analyzed descriptively. The findings revealed that most teachers understood the importance of in-depth knowledge of mathematics, students' learning processes, and the specific challenges students with learning disabilities need to face. However, variations were observed in teachers' confidence regarding their existing knowledge and experience, with most respondents rating their experience as moderate. The study also found that a strong foundation in mathematical knowledge and the ability to understand individual student characteristics need to be further strengthened. These findings highlight the necessity of enhancing teacher professional development through focused workshops and ongoing development programs. Training that emphasizes the integration of effective teaching strategies, mathematical content knowledge, and assistive learning technologies is recommended to ensure teachers can effectively meet the unique needs of students.

Keywords: mathematics teaching, special education, learning disabilities, mathematical pedagogical content knowledge, teacher knowledge level

Introduction

Studies have shown that teaching mathematics to students with learning disabilities (LD) in special education setting is indeed challenging. The issue is closely linked to a lack of knowledge and skills in teaching mathematics to these students. Even though teachers possess existing knowledge and skills related to teaching strategies, implementing effective instruction in a special education classroom is challenging. Many teachers need help delivering effective teaching to students with LD (Pawlak & Gross, 2019) especially during mathematics instruction. Studies also showed that nu-

merous teachers still need more confidence or adequate training in adapting their teaching to meet the needs of students with LD (Kamarudin et al., 2022). In the case of teaching mathematics, more studies are needed because existing studies tend to focus more on teaching strategies in general for mainstream mathematics classrooms with learning difficulties (Liong, 2019). Therefore, the present study aims to identify teachers' perceptions regarding knowledge and experiences of teaching mathematics to students with LD.

Literature Review

Students with LD in special education face significant difficulties in specific areas of learning. Various factors, including developmental, neurological, or psychological disorders, can cause their LD. According to the American Psychiatric Association (APA, 2013), specific LD is classified as disorders that affect the ability to read, write, and perform mathematical calculations. As a result, these students need more support to understand basic mathematics conceptually. In learning mathematics, students with LD require teachers to have knowledge such as specific teaching approach to help them comprehend the concept better (Abol, 2023).

Researchers have slightly different ideas about the knowledge needed for teaching. For example, Carrillo et al. (2018) suggested that mathematics teaching content and pedagogical content knowledge can be divided into three key elements. Content knowledge includes knowledge of the topics, the structure of mathematics, and the practices within mathematics. On the other hand, pedagogical content knowledge involves knowledge of mathematics instruction, understanding the characteristics of mathematics learning, and knowledge of mathematics learning standards. A study by Delgado-Rebolledo and Zakaryan (2019) found that all of these types of knowledge are interconnected to ensure effective mathematics learning among students. For instance, teachers' instructional strategies should utilize visual, kinesthetic, and auditory aids to help students understand ideas in mathematics. Teachers are strongly advised to effectively use teaching aids and technology, such as manipulatives and interactive software, to explain abstract concepts such as fractions and integers. Scaffolding in stages is essential to provide gradual support and reduce guidance as students become more proficient (Vivi et al., 2019).

Methods

The data of the survey study was gathered through a questionnaire containing 24 items that was distributed to 61 teachers who teach mathematics in special education settings in selected schools in two states in Malaysia, Sabah and Sarawak. The instrument was adapted and translated from Sheppard and Wieman (2020) and has been validated by mathematics and special education experts. The questionnaire consisted of 10 items in section A related to demographic information, and seven items in section B comprised statements about the knowledge and experiences teachers should possess for teaching mathematics to students with LD. The respondents rated the statements on a five-point Likert scale: not at all important, a little important, moderately important, quite important, and extremely important. In section C, respondents choose their agreement on seven items based on their level of knowledge and experience in teaching mathematics according to the six-point Likert scale: Strongly disagree, disagree, somewhat disagree, somewhat agree, agree and strongly agree. The data from the questionnaire were then analyzed to obtain the mean, standard deviation, frequency, and percentage.

Results

Based on the data analysis, Item 3 received the highest percentage of agreement, with 86.0% of respondents indicating that it is important to know students individually. Additionally, item 2 and item 4 also received high agreement percentages regarding general knowledge about how students learn and the specific challenges they face in learning mathematics. We found 51 (83.6%) and 50 (82%) out of 61 teachers involved in this study considered knowing students during mathematics teaching important. The results from these three items show a consistent perception among teachers regarding the importance of understanding the characteristics of each student with LD, their learning styles, and the difficulties they face in learning abstract mathematical concepts. However, more than knowledge about students with LD is required for teachers to teach mathematics effectively in schools.

A total of 40 out of 61 respondents (65.6%) felt it was very important for a teacher to have solid mathematical knowledge so that the concepts taught are accurate. Although Items 5, 6, and 7 received somewhat lower agreement compared to the other items in the same category, the percentage of teacher agreement remained above 50% (majority). Regarding items related to teaching experience, 36 of 61 respondents (59%) agreed most with item 7, 'It is important to have experience teaching special education'. On the other hand, 35 (57.4%) respondents extremely agreed with item 5, which stated, 'It is important to have experience teaching in general''. The data showed that 31 (50.8%) respondents in this study extremely agreed that teachers should have experience teaching mathematics. Teachers with experience teaching students in special education settings are better equipped to develop effective teaching styles based on the characteristics of each student with LD in their classrooms.

The findings indicate that teachers are aware of the importance of knowledge and experience that teachers should have when teaching mathematics to students with LD. Teachers also recognize the significance of understanding the ways of learning and the specific challenges each faces in special education settings. While teachers acknowledge the importance of knowledge in teaching mathematics, data shows variation in their agreement regarding the knowledge teachers possess. There is a difference between awareness of the importance of knowledge and the confidence level in their existing knowledge. Teachers consider knowledge and experience to be essential elements in teaching mathematics to students who encounter learning disabilities, which supported findings from Sheppard and Weiman (2020).

Overall, this study emphasizes the importance of teachers' knowledge and experience in effective mathematics teaching for students with LD, particularly knowledge about the individual characteristics of students, their learning styles, and the challenges they face. Although most teachers recognize the importance of solid mathematical knowledge and appropriate teaching strategies, their confidence levels regarding their existing knowledge still vary. The results of this study suggest the need for improvement through ongoing professional training and support to equip teachers with the necessary knowledge and skills. Experience in teaching students with LD also proves crucial, but only a few teachers are confident in their experience with mathematics instruction for special education. Therefore, continuous training and professional development focused on effective teaching strategies are necessary among teachers to enhance teaching effectiveness and teacher confidence. This study provides a foundation for ongoing assessment of the effectiveness of current mathematics instruction for students with LD and opens up opportunities for more in-depth future research.

Acknowledgement

This study was conducted under the Fundamental Research Grant Scheme FRGS/1/2023/SSI07/UKM/02/1 funded by the Ministry of Higher Education Malaysia.

References

Abol, M.T. (2023). Keperluan menangani gaya pembelajaran visual, auditori dan kinestetik (VAK) murid berkeperluan pendidikan khas di program pendidikan inklusif sekolah menengah di Malaysia. *Journal of Human Capital Development*, 16(2), 1–17.

American Psychiatric Association. (2013). *Diagnostic and statistics manual of mental disorders* (DSM-5). American Psychiatric Association.

Delgado-Rebolledo, R. & Zakaryan, D. (2020). Relationships between the knowledge of practices in mathematics and the pedagogical content knowledge of a mathematics lecturer. *International Journal of Science and Mathematics Education, 18*(3), 567–587.

Kamarudin, N., Khairuddin, K. F. & Mansor, A. Z. (2022). Penggunaan bahan bantu mengajar guru Pendidikan Khas dalam meningkatkan kemahiran Matematik operasi darab. *Malaysian Journal of Social Sciences and Humanities*, 7(1), 175 - 183.

Pawlak, F. & Gross, K. (2019). Using classroom management to support inclusive chemistry learning. Science Teaching Processes, 359

Sheppard, M.E. & Wieman, R. (2020). What do teachers need? Math and special education teacher educators' perceptions of essential teacher knowledge and experience. *Journal of Mathematical Behavior*, 59.

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.

Vivi, A.N., Heribertus, S. & Budi, U. (2019.) Elementary school teacher's obstacles in the implementation of problem-based learning model in mathematics learning. Journal on Mathematics Education, 10 (2), 229-238.

TSG4-2-15

PRESERVICE MATHEMATICS TEACHERS' EMERGING PEDAGOGICAL CONTENT KNOWLEDGE IN A LESSON STUDY-MEDIATED PRACTICE TEACHING

Monalisa T. Sasing
University of the Philippines National Institute for Science and Mathematics Education Development, Philippines, mtsasing@up.edu.ph

Studies have shown that lesson study contributed to the development of the emerging pedagogical content knowledge of preservice teachers (Akerson, Pongsanon, Rogers, Carter, & Galindo, 2015; Baki & Arslan, 2015). However, as a relatively new professional development model in the country, most Philippine teacher education institutions (TEIs) have not yet utilized lesson study. Using a quasi-experimental research design, this study investigated the use of lesson study with microteaching in preservice teachers' practice teaching and how it affects their emerging pedagogical content knowledge. The study found that the use of the microteaching technique in introducing lesson study to preservice mathematics teachers significantly improved their perceived pedagogical content knowledge. It also helped them significantly enhance their ability to collaboratively develop lesson plans, made them more confident and prepared to teach the lesson during their practice teaching. Most improvements in the lesson were found to be related to assessment. Lesson study with microteaching also helped preservice teachers better understand lesson study as it enabled them to gradually assimilate the lesson study process.

Keywords: pedagogical content knowledge, practice teaching, lesson study, preservice mathematics teachers

Introduction

Studies have shown that lesson study contributed to the development of the emerging pedagogical content knowledge of preservice teachers (Akerson, Pongsanon, Rogers, Carter, & Galindo, 2015; Baki & Arslan, 2015). However, as a relatively new professional development model in the country, most Philippine teacher education institutions (TEIs) have not yet utilized lesson study. This study investigated the effects of using lesson study with microteaching on preservice teachers' emerging pedagogical content knowledge.

Pedagogical Content Knowledge, Preservice Teacher Education, and Lesson Study

Studies (Carpenter, Fennema, Peterson & Carey, 1988; Hill, Rowan, & Ball, 2005) revealed that PCK positively influences student learning. With this, such type of teacher knowledge is an important component of teacher competence. Kleickmann et al. (2013) recommended that educational reforms related to the development of PCK can be channeled through teacher education programs. Some educational studies also recommended the development of the PCK of preservice mathematics teachers (An, Kulm, & Wu, 2004; Turnuklu & Yesildere, 2007).

Preservice teachers are still learning and gaining knowledge and skills needed for teaching while in their preservice teacher education. This learning comes mostly from theories and from classroom experiences and discussions in courses they have taken. Thus, their PCK is still limited (Lee, Brown, Luft & Roehrig, 2007; Nason, Chalmers & Yeh, 2012; Wilson, Floden & Ferrini-Mundy,

2002, as cited in Hurrell, 2013). In fact, it is still emerging and the development of preservice teachers' PCK continues as they gain teaching experiences.

Although teaching experience plays a major role in the development of the PCK of preservice teachers, to be more beneficial, it should be supplemented with a reflection on the teaching that transpired (Park & Oliver, 2008). Park and Oliver (2008) further emphasized that teacher education institutions should provide preservice teachers with experiences where they "examine students' understanding, reasoning types, misconceptions, learning styles, motivation, etc." (p. 281).

Lesson study is one opportunity where teachers collaborate to learn from how students think and from interacting with one another. The lesson study cycle includes a post-lesson reflection and discussion session where teachers reflect on how the lesson implementation was done and share suggestions for improving the learning environment during the next round of lesson implementation for the attainment of the goals and sub-goals (Ulep, 2013). With this, lesson study enables teachers to be reflective of their own practice and that of their colleagues to improve teaching and student learning. Studies have shown that this teacher-led professional development model can serve as the vehicle to further increase the repertoire of PCK of preservice teachers (Baki & Arslan, 2015; Harsono, 2016; Murata, 2011). Thus, exposing preservice teachers to lesson study gives them opportunities to further develop their emerging PCK.

Lesson Study and Microteaching in Preservice Teacher Education

In lesson study, teachers learn from observations of lesson implementation and from the feedback of the lesson study group members and knowledgeable others. Dudley (2013) reported that participation in lesson study resulted to improvements in teaching and assessment. According to Fernandez and Yoshida (2004), in Japan, preservice teachers start learning about lesson study from collaboration with experienced university mentors, field practice mentors, and outside experts. They also reported that working collaboratively with experienced teachers enabled preservice teachers to gain a deeper understanding of the lesson study process. From Japan's experience, the efficient conduct of lesson study requires enculturation—a process which takes effort and dedication (Fernandez & Yoshida, 2004). Lesson study has not proliferated yet in preservice teacher education in the Philippines, mechanisms of enculturation still do not exist. Thus, studies which investigate possible mechanisms of enculturation is needed and timely.

On the other hand, microteaching is a widely utilized training technique in preservice teacher education programs (Remesh, 2013) involving the process of planning-teaching-observing-replanning-reteaching-observing. It can help preservice teachers plan, prepare to teach, and evaluate the lesson they developed and the teaching strategies they plan to use (Pauline, 1993). Studies (Elias, 2018; Remesh, 2013) also revealed that microteaching helped preservice teachers to be more confident because they see the possible problems they might encounter when they implement the lesson. Hatfield (1990) also commented that microteaching provides a simplified learning environment where preservice teachers can practice. Thus, microteaching has the potential of improving preservice teachers' lesson study experience.

Given these research gaps and the findings of previous studies, this study investigated the use of lesson study with microteaching and how such approach influenced the emerging pedagogical content knowledge of preservice mathematics teachers. It also discusses how the microteaching process helped preservice mathematics teachers gradually assimilate the lesson study process.

Methodology

The study used a quasi-experimental design. Fifteen preservice mathematics teachers who were at the practice teaching stage of their Bachelor of Secondary Education program major in Mathematics were randomly divided into two groups. Each group was randomly assigned to a lesson study (LS) approach—the lesson study approach without microteaching and the lesson study approach with microteaching prior to the research lesson implementation during their practice teaching. Two lesson study groups were formed in each group. The grouping was based on the chosen fourth quarter topic that they were assigned to teach. Cooperating teachers from the secondary school where the preservice mathematics teachers had their practice teaching and faculty members of mathematics education from the College of Education of the state university were involved as knowledgeable others in the research lesson planning and development and lesson implementation stages of the study.

Figure 1 illustrates the quasi-experimental design of the study, where O_1 and O_2 are the perceived pedagogical content knowledge (PCK) and the lesson plan scores, respectively, of the preservice mathematics teachers prior to their exposure to the lesson study approaches, while O_3 and O_4 are the perceived PCK and lesson plan scores of preservice mathematics teachers after exposure to the LS approaches. The variable X_1 on the other hand, represents the intervention assigned to LS Approach 1 Group which is the lesson study without microteaching, while X_2 is the intervention given to LS Approach 2 Group which is the lesson study with microteaching. The microteaching of a research lesson involved planning-teaching-observing-replanning of the research lesson prior to actual classroom implementation.

LS Approach 1 Group	R	O_1O_2	X_1	O_3O_4
LS Approach 2 Group	R	O_1O_2	X_2	O_3O_4

Figure 1. The experimental design of the study.

The *Pedagogical Content Knowledge Inventory* (PCKI) was adapted from Jang, Guan, and Hsieh's study in 2009 and gathered the self-assessment of preservice mathematics teachers of their teacher knowledge and practices which they likely think will happen in their class. The pilot test showed a reliability of .92. The *Lesson Study Planning and Instructional Rubric* (LSPIR) on the other hand, was adopted from the study of Mostofo (2013) to assess the quality of the research lessons developed by the lesson study groups. The LSPIR had a reliability of .99 (planning for instruction) and .96 (content knowledge and instructional strategies). Preservice mathematics teachers' feedback and reflections were also gathered to supplement the discussion of the results of the study.

Findings and Recommendations

Overall, preservice mathematics teachers' self-ratings of their pedagogical content knowledge significantly improved after the lesson study-mediated practice teaching with microteaching prior to actual classroom implementation. Ranks results indicated that 12 out of 15 preservice mathematics teachers had PCK posttest scores greater than their PCK pretest scores. Results by group showed that it is only in Lesson Study Approach 2 Group (with microteaching) where preservice mathemat-

ics teachers significantly improved in their PCK self-ratings. This implies that the microteaching session in the lesson study workshop may have helped preservice mathematics teachers become more confident and believed that their PCK improved. They also felt more prepared in their lesson implementation as expressed by one of the preservice mathematics teachers in his feedback regarding the workshop. According to one of them, the lesson study workshop helped her learn more from experienced teachers who served as knowledgeable others during the workshop. However, the study found no significant difference in the two groups' PCK scores in the posttest using Mann-Whitney U test.

In terms of lesson planning, which also requires pedagogical content knowledge, the study found that it was only for the LS Approach 2 Group that a statistically significant difference was found in the lesson plan scores of preservice mathematics teachers across the three versions of their research lessons using Friedman's 2-way analysis of variance by ranks. This implies that the inputs of the knowledgeable others and the revisions made on the lesson plans based on the formulated goals, may have helped improve the quality of the lesson in terms of planning for instruction, content, and strategies. This is true also with the revisions made based on the inputs of knowledgeable others and group members after the microteaching of the research lesson. Moreover, it was also found that those who underwent lesson study with microteaching (LS Approach 2 Group) had a significantly higher mean rank than those who underwent lesson study without microteaching. This implies that those whose research lessons were revised based on what were observed during the microteaching significantly improved in quality in terms of planning for instruction, content, and strategies.

With these findings, TEIs may introduce lesson study with microteaching through a workshop prior to practice teaching. It can serve as a support mechanism for their preservice mathematics teachers. Furthermore, the microteaching technique also helped preservice mathematics teachers better understand lesson study as it enabled them to gradually assimilate the lesson study process. Thus, it is a good strategy for TEIs and teacher training institutions to use it in introducing teachers to lesson study through a workshop.

References

Akerson, V., Pongsanon, K., Rogers, M.A.P., Carter, I., & Galindo, E. (2015). Exploring the use of lesson study to develop elementary preservice teachers' pedagogical content knowledge for teaching nature of science. *International Journal of Science and Mathematics Education*. doi: 10.1007/s10763-015-9690-x

An, S., Kulm, G. & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, *7(2)*, 145–172. doi: 10.1023/B:JMTE.0000021943.35739.1c

Baki, M. & Arslan, S. (2015). Examining the effect of lesson study on prospective primary teachers' knowledge of lesson planning. *Turkish Journal of Computer and Mathematics Education*, *6*(2), 209-229. doi: 10.16949/turcomat.02379

Ball, D. L., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of Research on Teaching,* (4th ed.), (pp. 433-456). New York: Macmillan. Retrieved from http://www-personal.umich.edu/~dball/chapters/BallLubienskiMewbornChapter.pdf

Carpenter, T. P., Fennema, E., Peterson, P.E., & Carey, D.A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education*, *19(5)*, 385-401. Retrieved from http://www.jstor.org/stable/749173

Dudley, P. (2013). Teacher learning in lesson study: what interaction-level discourse analysis revealed about how teachers utilised imagination, tacit knowledge of teaching and freshly gathered evidence of pupils learning, to develop their practice knowledge and so enhance their pupils' learning. *Teacher and Teacher Education, Teaching and Teacher Education*, 34 (2013) 107e121.

Elias, S.K. (2018). Pre-Service teachers' approaches to the effectiveness of micro-teaching in teaching practice programs. *Open Journal of Social Sciences*, 6, 205-224. doi: 10.4236/jss.2018.65016.

Fernandez, C., & Yoshida, M. (2004). Lesson study: A Japanese approach to improving mathematics teaching and learning. Mahwah, New Jersey: Lawrence Erlbaum Associates.

Harsono, M. (2016). *The impact of lesson study on primary school teachers' knowledge and skills in differentiating primary school mathematics instruction: a digital mixed methods approach* (Unpublished doctoral dissertation). Curtin University, Australia.

Hatfield, R. C. (1989). Developing a procedural model for the practice of microteaching. East Lansing, MI: Michigan State University. Retrieved from https://www.researchgate.net/publication/234674670_Developing_a_Procedural_Model_for_the_Practice_of_Micro_Teaching.

Hill, H. C., Rowan, B. & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42(2)*, 371-406. doi:10.3102/00028312042002371

Hurrell, D. P. (2013). What Teachers Need to Know to Teach Mathematics: An argument for a reconceptualised model. *Australian Journal of Teacher Education*, 38(11). doi: 10.14221/ajte.2013v38n11.3

Kleickmann, T., Richter, D., Kunter, M., Elsner, J., Besser, M., Krauss, S. & Baumert, J. (2013). Pedagogical content knowledge and content knowledge of mathematics teachers: The role of structural differences in teacher education. *Journal of Teacher Education*, 64, 90-106. doi: 10.1177/0022487112460398

Mostofo, J. (2013). Using lesson study with preservice secondary mathematics teachers: effects on instruction, planning, and efficacy to teach mathematics (Unpublished doctoral dissertation). Arizona State University, USA.

Murata, A. (2010) Teacher learning with lesson study. In P. Peterson, E. Baker, & B. McGaw, (Eds.) *The International Encyclopedia of Education*. Volume 7, 575–581. Amsterdam, The Netherlands: Elsevier.

Park, S. & Oliver, J. (2008). Revisiting the conceptualisation of pedagogical content knowledge: PCK as a conceptual tool to understand teachers as professionals. *Research in Science Education*. 38, pp. 261-284. doi: 10.1007/s11165-007-9049-6.

Pauline, R.F. (1993). Microteaching: an integral part of a science methods class. Journal of Science Teacher Education, 4(9). doi: 10.1007/BF02628852.

Remesh, A. (2013). Microteaching, an efficient technique for learning effective teaching. *Journal of Research in Medical Sciences*, 2013 Feb; 18(2): 158–163.

Shulman, Lee S. (1986). Those who understand: Knowledge growth in teaching. *Educational Research, 15(2)*, 4-14. Retrieved from http://www.jstor.org/

Turnuklu, E. B. and Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspective in turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal*, *1*, 1-13. Retrieved from http://www.eric.ed.gov.

Ulep, S. (2013). Implementing the lesson study process. In S. Ulep, A. Punzalan, R. Reyes, & M. Ferido (Eds). *Lesson study: planning together, learning together*, (pp. 7-12). Quezon City: University of the Philippines National Institute for Science and Mathematics Education Development.

Funding

This study was funded by the Outright Research Grant of the Office of the Vice Chancellor for Research and Development of the University of the Philippines Diliman.

TSG4-2-16

EXPLORING THE DEVELOPMENT OF PRIMARY MATHEMATICS PRE-SERVICE TEACHERS' TEACHING SELF-EFFICACY: A PRELIMINARY ANALYSIS

Hui Min Chia
University of Macau, Macau SAR, hmchia@um.edu.mo

This study explored the development of 33 primary school pre-service teachers' (PSTs) sense of self-efficacy in mathematics teaching who attended the primary mathematics curriculum and instruction course. A mixed-method approach was used to collect data from pre- and post-course surveys on the sense of self-efficacy, written reflections, feedback on classroom activities, and lesson plans. A paired sample t-test was used to test the differences between pre- and post-course survey scores. The results indicate an increase in the score of PSTs' self-efficacy in teaching mathematics. The preliminary qualitative analysis of written reflection reveals that mastery experiences contribute the most to PSTs' self-efficacy, followed by vicarious experiences, verbal persuasion, and psychological/affective responses. The findings suggest the importance of learning experiences and reflective practices for primary school PSTs' preparation and teaching self-efficacy. The study offers implications for mathematics pre-service teacher education and research.

Keywords: sources of self-efficacy, primary school, mathematics teacher education, reflective practice

Introduction

Improving teaching quality has been a focus of teacher education (Loughran & Menter, 2019). Teacher educators play an important role in facilitating pre-service teachers (PSTs) to learn about teaching. However, it is challenging for teacher educators to link theory to practice and craft meaningful learning experiences. This could be due to the teaching of teaching being a complex process that requires knowledge, skills, and abilities (Loughran & Menter, 2019). Mathematics teaching is uniquely demanding due to the abstractness of the mathematics subject and the prevalence of students' negative affect (such as attitudes and emotions) toward mathematics. These factors can undermine PSTs' mathematics teaching confidence. Hence, developing mathematics teaching self-efficacy, especially in primary education, is crucial to support PSTs in navigating instructional challenges. The researcher argues that examining PSTs' self-efficacy and self-efficacy sources can help to structure the learning content for teacher education courses.

Self-efficacy is a person's judgment about their ability to effectively carry out a task in a specific situation (Bandura, 1977). Teacher self-efficacy can affect teachers' wellbeing, including personal fulfilment, job satisfaction, and engagement (Granziera & Perera, 2019; Zee & Koomen, 2016). Teacher self-efficacy influences instructional decisions on classroom management, students' interests, and innovative strategies (Zee & Koomen, 2016). Studies suggest that the development of teacher self-efficacy is significant during teacher education (Bjerke & Solomon, 2020; Charalambous et al., 2008; Marschall, 2023). Once established, teacher self-efficacy tends to be relatively stable and resistant to change (Hoy & Spero, 2005). Besides, primary education PSTs perceived their competence in teaching mathematics differently from other subjects (Charalambous et al., 2008). Thus, research into early PSTs' learning experiences during teacher preparation in mathematics teaching is needed.

According to Bandura (1977), there are four sources of self-efficacy: mastery experiences (successful past experiences with a task), vicarious experiences (observing others succeed at a task), verbal persuasion (feedback from others), and physiological/ affective responses (positive or negative emotional states). Most studies investigating PSTs' teaching self-efficacy sources were conducted during their school teaching practicum (Bjerke & Solomon, 2020; Charalambous et al., 2008; Marschall, 2023). This could be because mastery experiences are often regarded as the most substantial source of efficacy (Bandura, 1977). Teaching practicum provides PSTs with their first real, robust teaching experiences and interactions with mentors and pupils that strengthen their self-efficacy (Charalambous et al., 2008).

Other researchers (Bjerke & Solomon, 2020; Gurvitch & Metzler, 2009) argued that simulation teaching or peer teaching during teacher preparation, which provides a supportive environment for teaching experimentation, is equally vital in enhancing PSTs' self-efficacy. Morris et al. (2017) suggested that the development of teacher self-efficacy is complex because mastery experiences can stem from multiple sources. This implies that PSTs' learning experiences at the university level, such as observing how their peers conduct the lesson (vicarious experiences), can contribute to the content and pedagogical content knowledge. Such vicarious experiences differ from the observation of their mentor during teaching practicum, whereby someone with a similar stage of development managed to use a different approach can enhance their self-efficacy to experiment with various teaching approaches (Gurvitch & Metzler, 2009). Yet, few studies examine how university training influences the development of primary mathematics PSTs' self-efficacy (Bjerke & Solomon, 2020). This study addressed the gap by exploring: 1) How do PSTs' teaching self-efficacy change at the end of the primary mathematics curriculum and instruction course? 2) What sources of the learning experiences contribute towards this change?

Methodology

Research Design and Context

This study is part of an ongoing self-study project researching the challenges faced by an early career mathematics educator. A mixed-method approach was used to investigate the complex phenomenon of the development of PSTs' self-efficacy in a primary mathematics curriculum and instruction course at a university in Macau, SAR. The course, conducted over 15 weeks, met twice weekly for approximately 2.5 hours and aimed to develop PSTs' knowledge and skills in teaching primary mathematics. The course content introduced various teaching approaches in primary mathematics, including storytelling approaches with hands-on activities in mathematics classrooms (vicarious experiences). PSTs were exposed to how to use storytelling in mathematics classrooms in the whole class setting and the hands-on activity in small groups (2-4) to plan for a travel trip. Such a setting developed PSTs' pedagogical content knowledge and learning via social interactions involving the exchange of ideas in the mathematics content specific to primary mathematics teaching. Then, as part of the coursework, PSTs collaborated in small groups (two to four persons) to design and teach a geometry lesson to their peers (mastery experience), who acted as students. During the lesson implementation, their peers may give feedback on their teaching approaches. The group could adjust and re-teach the parts they were unsatisfied with (verbal persuasion, vicarious experiences, and physiological/ affective responses). They recorded their teaching process during peer teaching and submitted the version of the video recording that they thought was the best as part of the assignment. At the end of the course, they had to present their lesson plan, extract video clips, and reflect on their experiences. During the presentation, the instructor and peers would provide

constructive feedback on the strength of the lesson and suggestions on how to improve the lesson (verbal persuasion, vicarious experiences, and physiological/ affective responses).

Data Collection

Data were collected from 33 third-year primary education PSTs (5 males, 28 females), mostly aged 20 to 24, with a few exceptions (one participant was above 40 years old, and three were below 19 years old). The majority were from Macau (n=28), while five were from mainland China. Quantitative data, the pre- and post-tests of PSTs' self-efficacy, were collected using the shortened and translated version of Teachers' Sense of Self-Efficacy (TSES) in instruction (Tschannen-Moran & Hoy, 2001). The answers ranged from 1 (nothing) to 5 (A great deal) to reduce the complexity, and participants were familiar with the 5-point Likert scale (Chen et al., 2015). Various qualitative data were collected, including assignments, course topics' feedback, and written reflection. This paper focused on PSTs' written reflections.

Data Analysis

Data analysis was conducted in two phases: the quantitative phase and the qualitative phase. A paired t-test was conducted for quantitative data analysis to determine the significant differences between the pre- and post-test scores. Qualitative data were analysed using deductive analysis based on the literature (Bjerke & Solomon, 2020; Marschall, 2023) on the four sources of self-efficacy (mastery experiences, vicarious experiences, verbal persuasion, and physiological/affective responses). To ensure consistency, the researcher reviewed and refined coded data iteratively.

Findings and Discussion

Quantitative Data

The results indicated that most of the scores of items increased but did not show statistically significant changes ($p > 0.1$), suggesting that participants' self-efficacy remained relatively stable throughout the course (Hoy & Spero, 2005). The increase is not statistically significant due to the course duration being only 15 weeks and the small sample size. Future studies can be conducted over a longer period, which might yield more significant results.

Qualitative Data

From the qualitative analysis, 19 participants reported mastery experiences, followed by vicarious experiences (9 participants), verbal persuasion (3 participants), and physiological/ affective responses (2 participants). This finding is consistent with existing literature, whereby mastery experiences are the primary source of self-efficacy (Charalambous et al., 2008; Gale et al., 2021). In the written report, a participant (Case 26) stated that their experiences of implemented group work and role-playing during peer teaching had promoted students' learning motivation, "I learned how to design interactive lessons that use group discussions and role-playing to promote student engagement".

Vicarious experiences appear to be another source of self-efficacy in this study, highlighting the importance of observational learning during teacher education (Marschall, 2023). As a participant (Case 22) described their observation during the presentation, their peers demonstrated different strategies, such as using song singing to consolidate students' learning during the peer teaching session as follows, "In this group activity, I saw some interesting classroom activities, such as [my friend used] singing a song to consolidate the characteristics of geometrical shapes".

PSTs' verbal persuasion reported in this study focused on the instructor's or their peers' feedback on improving the lesson. Participant (Case 20) wrote in the report, "After receiving feedback from the instructor, I realised that I needed to be more detailed when writing lesson plans". This implies that constructive feedback can potentially help PSTs develop their self-efficacy (Marschall, 2023; Morris et al., 2017). Regarding physiological/ affective responses, participants acknowledged that their negative emotions, such as nervousness and lack of confidence, affected their teaching performance. For instance, "During the teaching process, I was so nervous that I forgot what I was going to teach, which slowed down the teaching pace and affected the overall teaching performance" (Case 02). Such negative emotional states are commonly reported by PSTs in previous studies (Marschall, 2023; Morris et al., 2017).

In conclusion, the positive trend in self-efficacy scores reflects the potential benefits of structured learning activities, such as peer teaching, collaborative lesson planning, and reflective practices. The researcher argues that despite the significant role of mastery experiences, it is possible to provide observation opportunities (vicarious experiences) at the post-peer teaching session. Then, PSTs can have the chance to observe not only the instructor who demonstrates various teaching approaches during the lectures but also their peers who successfully implemented innovative teaching approaches. Furthermore, guiding PSTs to provide constructive feedback (verbal persuasion), such as the strength of the approaches and how they can do better, can be beneficial to developing their reflective practices. Due to this being an ongoing study, the researcher has yet to interview PSTs about their learning experiences, which could provide richer data to triangulate the findings further.

References

Bandura, A. (1977). Self-efficacy: Toward a unifying theory of behavioral change. *Advances in Behaviour Research and Therapy*, *84*(2), 191–215. https://doi.org/10.1016/0146-6402(78)90002-4

Bjerke, A. H., & Solomon, Y. (2020). Developing self-efficacy in teaching mathematics: Pre-service teachers' perceptions of the role of subject knowledge. *Scandinavian Journal of Educational Research*, *64*(5), 692–705. https://doi.org/10.1080/00313831.2019.1595720

Charalambous, C. Y., Philippou, G. N., & Kyriakides, L. (2008). Tracing the development of preservice teachers' efficacy beliefs in teaching mathematics during fieldwork. *Educational Studies in Mathematics*, *67*(2), 125–142. https://doi.org/10.1007/s10649-007-9084-2

Chen, X., Yu, H., & Yu, F. (2015). What is the optimal number of response alternatives for rating scales? From an information processing perspective. *Journal of Marketing Analytics*, *3*(2), 69–78. https://doi.org/10.1057/jma.2015.4

Gale, J., Alemdar, M., Cappelli, C., & Morris, D. (2021). A mixed methods study of self-efficacy, the sources of self-efficacy, and teaching experience. *Frontiers in Education*, *6*(September), 1–16. https://doi.org/10.3389/feduc.2021.750599

Granziera, H., & Perera, H. N. (2019). Relations among teachers' self-efficacy beliefs, engagement, and work satisfaction: A social cognitive view. *Contemporary Educational Psychology*, *58*(February), 75–84. https://doi.org/10.1016/j.cedpsych.2019.02.003

Gurvitch, R., & Metzler, M. W. (2009). The effects of laboratory-based and field-based practicum experience on pre-service teachers' self-efficacy. *Teaching and Teacher Education*, *25*(3), 437–443. https://doi.org/10.1016/j.tate.2008.08.006

Hoy, A. W., & Spero, R. B. (2005). Changes in teacher efficacy during the early years of teaching: A comparison of four measures. *Teaching and Teacher Education*, *21*(4), 343–356. https://doi.org/10.1016/j.tate.2005.01.007

Loughran, J., & Menter, I. (2019). The essence of being a teacher educator and why it matters. *Asia-Pacific Journal of Teacher Education*, *47*(3), 216–229. https://doi.org/10.1080/1359866X.2019.1575946

Marschall, G. (2023). Teacher self-efficacy sources during secondary mathematics initial teacher education. *Teaching and Teacher Education*, *132*, 1–12. https://doi.org/10.1016/j.tate.2023.104203

Morris, D. B., Usher, E. L., & Chen, J. A. (2017). Reconceptualizing the sources of teaching self-efficacy: A critical review of emerging literature. *Educational Psychology Review*, *29*(4), 795–833. https://doi.org/10.1007/s10648-016-9378-y

Tschannen-Moran, M., & Hoy, A. W. (2001). Teacher efficacy: Capturing an elusive construct. *Teaching and Teacher Education*, *17*(7), 783–805. https://doi.org/10.1016/S0742-051X(01)00036-1

Zee, M., & Koomen, H. M. Y. (2016). Teacher self-efficacy and its effects on classroom processes, student academic adjustment, and teacher well-being: A synthesis of 40 years of research. *Review of Educational Research*, *86*(4), 981–1015. https://doi.org/10.3102/0034654315626801

TSG4-2-17

A TEACHER'S DEVELOPMENT OF STUDENTS' METACOGNITION IN A MATHEMATICS CLASSROOM

Leng Low
Academy of Singapore Teachers, Ministry of Education, Singapore, low_leng@moe.gov.sg

Kit Ee Dawn Ng
National Institute of Education, Nanyang Technological University, Singapore, dawn.ng@nie.edu.sg

Mei Liu
National Institute of Education, Nanyang Technological University, Singapore, mei.liu@nie.edu.sg

As part of an interdisciplinary programmatic research, we worked with 18 case-study teachers to examine how a teacher professional development programme has impacted teachers' conception on metacognition and their practice. In this paper, we present one teacher case to illustrate her professional growth in instructional strategies to foster students' metacognition. The teacher's stance about being intentional in developing students' metacognition during mathematics lessons will also be described. Implications on future teacher education programmes, research on metacognition, and teacher facilitation in mathematics lessons will be drawn.

Introduction

The 21st century has been described as volatile, complex, and ambiguous where future employability depends on whether the person is a "valuable knowledge worker" (Brynjolfsson & McAfee, 2014). Lifelong learning is perceived as crucial in ensuring that a person remains open to "new developments and opportunities as they arise" (Scott, 2015, p. 12). Scholars (e.g., Scott, 2015) argue that learners are more successful at acquiring new skills and competencies when they are strong in ***metacognition***, because they can reflect on new concepts learnt, and consciously integrate new information into their existing schema of knowledge and skills.

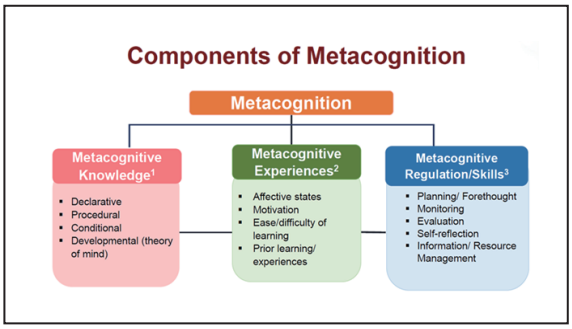
Metacognition is one of five key, inter-related components in attaining the goal of mathematical problem solving in Singapore's mathematics curriculum framework (Ministry of Education [MOE], 2023). Stillman et al. (2010) espoused that successful problem solvers typically employ metacognitive strategies (e.g., deliberate choice of approaches, careful planning, constant monitoring of the process, and regulation). Goal attainment requires an interplay of cognitive and metacognitive thinking processes. Students need to be taught how to activate these processes, especially metacognition.

Theoretical Framework on Metacognition

First defined by Flavell (1976), metacognition refers to "one's knowledge concerning one's own cognitive processes and products or anything related to them". Then, Flavell articulated two com-

ponents of metacognition: "active monitoring" and "consequent regulation" (p. 232) and that these processes are activated in synergy within a cognitive endeavour towards a goal. Subsequent discussions by Flavell (1979) and others unpacked the construct further. Figure 1 outlines the components of metacognition our programmatic research is based on, synthesised from literature.

Nonetheless, because metacognition exists as an internal process within oneself, it is often unobservable. It is also difficult to distinguish between cognition and metacognition, particularly during real-time lessons. Hence, researchers (e.g., Ng et al., 2021) call for intentional incorporation of metacognition through instructional strategies in lessons so that students' metacognition can be activated, explicit, and developed. Metacognitive guidance needs to be consistent over time, and embedded in the subject matter an appreciation of the role metacognition plays in learning. If done across subject domains, students may well become metacognitive as a habit of mind, setting the stage for lifelong, self-directed learning. Teachers can adapt existing practices to provide platforms for students' metacognition to be activated (e.g., allow students to make their thinking visible and/or audible, see Lee, 2008).



Note. This diagram was produced by Lee et al. (2024) as part of a programmatic research (02/21 LNH: Paving the Way Towards Lifewide and Lifelong Learning: Exploring and Fostering Metacognition for Learning and Transfer). From Lee et al. (2024, 30 June). *Examining metacognitive teaching strategies and practices for learning and transfer in Singapore classrooms.* Redesigning Pedagogy International Conference, Singapore. Copyright 2025, 02/21 LNH.

Figure 1. Components of metacognition synthesised from literature

Our study investigated how a series of teacher professional development workshops impacted teachers' conception of metacognition, their knowledge of instructional strategies to develop students' metacognition in lessons, and their practice in Singapore secondary schools. This paper will only present preliminary insights gleaned from one mathematics teacher case. Her practice has evolved with deepened knowledge about instructional strategies towards fostering students' metacognition. Implications on future teacher education programmes, research on metacognition, and teacher facilitation in mathematics lessons will be drawn.

Research Design

A multi-site, multi-case embedded case-study design (Yin, 2018) was carried out with the teachers together with their intact classes, in-situ, within their respective school culture and contexts. The 18 teacher-cases were selected from the sample in the first study of the programmatic research. They represented teachers from Singapore government schools with different number of years of teaching

experience, levels of knowledge on metacognition and instructional strategies to facilitate student's metacognition (see Tay et al., 2023, an earlier study in the programmatic research), teaching levels, and subject-based banding classes.

Prior to attending five professional development workshops on metacognition, participating teachers responded an open-ended survey. The workshops aimed to provide teachers with a deeper understanding of metacognition and how instructional strategies can be adapted to infuse metacognition in lessons. Each teacher was mentored by two research team members in planning and enacting a lesson to foster students' metacognition. The research team comprised experts in metacognition from the National Institute of Education, Singapore, as well as curriculum specialists and Master Teachers (i.e., pedagogical leaders) from Singapore Ministry of Education. The teachers also attended pre- and post-lesson interviews. Focused-group interviews were carried out with selected students representing different learning progress. All the teachers kept a learning log with reflection pieces throughout the research period.

Preliminary analysis involved at least two levels of coding. First level coding examined how teachers' conception of metacognition and their knowledge of instructional strategies infusing metacognition evolved during the research period (Apr-Nov 2023), challenges faced, and enacted strategies. We adapted the Resources-Orientation-Goal-Identity (ROGI) framework by Karsenty et al. (2023) for understanding teachers' practices and their decisions in second level coding. The ROGI framework was developed from Schoenfeld's (2010) ROG framework, with an additional component on identity. A teacher's decision-making is typically goal-oriented (e.g., develop conceptual understanding). Goals can come about from the teacher's orientation (e.g., beliefs about how students learn, preferences on the types of instructional approaches). A teacher's resources include access to internal and external sources. Teacher identity refers to the ways a teacher "recognize or position themselves or how they are recognized or positioned by others" (Karsenty et al., p. 31). An initial coding framework was developed from literature, but open coding was used to ensure that grounded data is captured. Constant comparative analysis was applied across data sets, and the coding framework stabilized after several rounds of refinement, with an inter-rater reliability of at least 92%.

Preliminary Findings from One Teacher Case

Teacher A has been teaching mathematics in Singapore secondary schools for 25 years. Though a highly competent and confident teacher, she was humble and willing to learn (identity). Her typical practice was to deliver teacher-centred lessons with direct instruction consisting of frequent scaffolding moves to guide students (orientation), but she has not been intentional in fostering metacognition in students. Through her reflections and discussions, we found that her conception of metacognition became more robust after the workshops. She also reinforced her understanding about how instructional strategies can be adapted to infuse metacognition. Initially, she had concerns about students of different profiles working on metacognition during limited class time. Nonetheless, she was open to changing her existing practice and constantly reflected on her attempts.

In the observed lesson, which was analysed together with her interviews, Teacher A taught the conditions for two triangles to be congruent as well as the tests for congruence (goal). Although it was still largely a teacher-directed lesson, she provided time for students to explore what the three conditions to identify a pair of congruent triangles were, instead of direct instruction. She gave cut-outs of triangles (resources) and used teacher questioning to encourage class discussions to make students' thinking visible, hoping to activate student metacognition such as awareness of triangle

properties, monitoring of reasoning, and regulation of approaches. Interviews with selected students showed one student explaining that the exploration allowed them "to think more deeply in maths", while another mentioned that "we use and apply methods that focus on properties of triangles to find the solutions based on what we had learned before". There was possibly some degree of *meta-cognitive awareness* manifested.

Reflecting after the enactment during an interview, the teacher felt that student learning of the content was not as rich as she had expected because each student only had time to try out one exploration using cut-outs instead of multiple explorations for generalisation to all the cases. She also surmised that there was a need be very clear about her intent in infusing metacognition in her facilitation and practice. Moving forward, she would like to enhance students' opportunities to make thinking visible during decision making about mathematical content so that metacognition can be further activated. One such instance which this could have been done was allowing students to draw separate diagrams arising from the given information and ask questions to assess whether the information given was useful, and if there were implicit information not visible in the given diagram by harnessing their prior knowledge (*awareness* and *monitoring* problem solving process). One of her key learning points from her research journey was to explore ways to allow for more students' voice during her lessons, and review her scaffolding moves so that students could be more self-directed in their learning, allowing for students' metacognition to be activated.

Conclusion and Implications

Teacher A was very clear of her content goals in the lesson. This may stem from her belief in her role as a teacher and her understanding of how students learn. Being a reflective teacher, she taps into her resources, particularly her inner resources (i.e., her knowledge of pedagogy, content, and students) to attain the content goals. In contrast, Teacher A, similar to many other teachers in our sample, needed prompting from mentors to articulate the metacognitive goals for the lesson. Teacher A had requested her students to check the reasonableness of an answer, encouraged them to learn from mistakes, and invited them to review their problem-solving process and think about how to seek help when necessary. However, she was one step short of explaining to her students why. It would be ideal if students know the purpose of monitoring and appropriate regulation to improve problem solving success. This highlighted a need to establish the metacognitive goals in the lesson alongside content goals as we make metacognitive instruction intentional.

More classroom-based research can be conceptualised to explore teaching practices that facilitate the development of metacognition in mathematics classrooms, with different teachers in different settings to enable constructions of accounts for promoting metacognition within existing teaching practices. Whilst metacognition instruction has to be intentional and explicit, setting metacognition goals are also important. There are many ways to achieve this, to be explored during teacher professional development programmes on metacognition.

References

Brynjolfsson, E., & McAfee, A. (2014). *The second machine age: Work, progress, and prosperity in a time of brilliant technologies*. W. W. Norton & Company.

Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. B. Resnick (Ed.), *The nature of intelligence* (pp. 231-235). Lawrence Erlbaum.

Flavell, J. H. (1979). Metacognition and Cognitive Monitoring: A New Area of Cognitive-Developmental Inquiry. *American Psychologist, 34*, 906-911. https://doi.org/10.1037/0003-066X.34.10.906

Karsenty, R., Pöhler, B., Schwarts, G., Prediger, S., & Arcavi, A. (2023). Processes of decision-making by mathematics PD facilitators: The role of resources, orientations, goals, and identities. *Journal of Mathematics Teacher Education*, *26*, 27–51. https://doi.org/10.1007/s10857-021-09518-z

Lee, N. H. (2008). *Enhancing mathematical learning and achievement of secondary one normal (academic) students using metacognitive strategies* [doctoral dissertation, Nanyang Technological University]. Singapore.

Lee, N. H., Lee, J., Begum, T., & Liu, M. (2024, June 30). *Examining metacognitive teaching strategies and practices for learning and transfer in Singapore classrooms* [Conference Presentation]. Redesigning Pedagogy International Conference, Singapore.

Ministry of Education [MOE]. (2023). *Mathematics syllabuses (secondary one to four): Express course, normal academic course (implementation starting with secondary one cohort 2020)*. Curriculum Planning and Development Division, Ministry of Education, Singapore.

Ng, K. E. D., Lee, N. H., & Safii, L. (2021). Facilitation of students' metacognition: Some insights gleaned from mathematics classrooms in Singapore secondary schools. In B. Kaur & Y. H. Leong (Eds.), *Mathematics instructional practices in Singapore secondary schools* (pp. 105-122). Springer Nature Singapore Pte Ltd. https://doi.org/https://doi.org/10.1007/978-981-15-8956-0_6

Schoenfeld, A. H. (2010). *How we think: A theory of goal-orientated decision making and its educational applications.* Routledge.

Scott, L. C. (2015). *Education and foresight working papers: The futures of learning 3 - what kind of pedagogies for the 21st century?* United Nations Educational, Scientific and Cultural Organisation (UNESCO).

Stillman, G., & Mevarech, Z. (2010). Metacognition research in mathematics education: from hot topic to mature field. *ZDM*, *42*(2), 145-148. https://doi.org/10.1007/s11858-010-0245-x

Tay, L. Y., Chan, M., Chong, S. K., Tan, J. Y., & Aiyoob, T. B. (2023). Learning of mathematics: A metacognitive experiences perspective. *International Journal of Science and Mathematics Education*, *22*(3), 561-538. https://doi.org/10.1007/s10763-023-10385-8

Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). SAGE.

TSG4-2-18

SHIFT IN TEACHER'S PERSPECTIVES IN MATHEMATICS BASED ON INTERVENTIONS ON INDIAN KNOWLEDGE SYSTEMS AND DISTINCTIONS BETWEEN EMPIRICAL AND FORMAL MATHEMATICS

Sanjeev Ranganathan
C3STREAM Land, SAIIER, India, sanjeev.r@auroville.org.in

Poovizhi Pachaiyappan
C3STREAM Land, SAIIER, India, poovizhi@auraauro.com

Sohela Sajjan
SAIIER, India, sohela.sajjan@gmail.com

India, like many colonized countries, has an education system that follows mathematical history that is not chronological, but philosophical. The philosophy emphasizes formal (faith-based) mathematics (maths) over empirical (fact-based/ normal) maths crediting the West for most mathematics and discrediting earlier work done as merely practical including that done by Indians. The cost of this emphasis is borne by children who fail to connect with formal mathematics across the world. This paper explores an intervention over six months of introduction to the Indian Knowledge Systems to teachers in India. This is done through comparisons of the scientific and mathematical aspects between the Christian calendar and the Indian Panchang. Further distinction between formal and empirical mathematics with materials, visualizations and projects were presented to them. We examine knowledge facilitators acquired and perspective shifts about IKS, their understanding of formal and empirical maths and changes in their interactions with students.

Keywords: IKS, empirical, formal, teacher perspective

Introduction

Math has been used as a tool to further cultural imperialism (Bishop, 1990). The education system in India was changed to follow the British education system after a revolt in 1857 (Macaulay, 1835) claiming "immeasurable superiority" of the European knowledge. Even now the Indian education system in math continues what was mainly started in colonial India (Faruque, 2024). Indian teachers heavily rely on textbooks (Sharma, 2024) and these lack IKS (Indian Knowledge Systems), a study on mentions of contributions by Indian Mathematicians in NCERT (National Council of Educational Research and Training) text-books revealed that in books from 6th to 10th grade only 6 Indian Mathematicians are discussed out of a total of 33 Mathematicians (Bondu, 2020). Indian National Educational Policy 2020 highlights the need for IKS and only recently through initiatives like Akhil Bhartiya Shiksha Samagam, in 2023 the Ministry of education is working on addressing curriculum in schools and higher education institutions, and to create appropriate faculty training (Ministry of Education, 2022).

India is given credit for 'zero' though it should be credited for decimal system, efficient arithmetic, scientific calendar, astronomy (including trigonometry), geometry and algebra (K. Ramasubramani-

an, Ministry of Culture, 2022). The presumed "immeasurable superiority" of European knowledge has been questioned by C.K Raju in his book *Indian Calendar: Scientific Aspects* (2024) by comparing what is used to record time - Western and Indian calendars for their science and mathematics. After all, superior maths should translate into a superior calendar. He also describes how credit to the work done by earlier civilisations including Indian was not given by stating that these were merely of practical value. Further independent discovery was claimed and the history of mathematics was changed from being chronological to being philosophical. The philosophy being that formal math is superior and empirical math is inferior. This allowed the West to claim credit for work centuries after it had already been done elsewhere (Raju, 2024).

Formal (faith-based) vs Empirical (fact-based) Maths

Formal (Faith-based) maths is perfect in an unreal (not abstract) world of axioms and proofs based on these. Formal maths should not be confused with abstraction or generalization. An example is the definition of an angle in the IXth class textbook (NCERT, 2022), is:

> "Angle is formed when two rays originate from the same end-point. The rays making an angle are called the arms of the angle, and the end point is called the vertex of the angle."

Empirical (Fact-based) maths is based on senses – students can see, feel, touch, taste and hear, can play with and verify themselves. It is imperfect in the real world and uses Zeroism: that an error that is small enough in a context can be ignored (Raju, 2012). The definition from India of angles is related to arc length. The angle in degrees is the ratio of arc length to circumference (where circumference is divided into 360 pieces). The angle in radians is the ratio of arc length to the radius of the circle. These definitions can be drawn outdoors or indoors, measured, verified and played with. Though these measurements are not possible with a toy like the pencil box.

Schools all over the world teach an odd mix of empirical and formal maths, due to which students cannot connect with mathematics. This is one reason that math is the most feared or disliked subject both in India (Gafoor, Kurukkan 2015; The Print, 2021) and across the world (Cuemath, 2021).

Intervention

In this paper, we look at an introduction to teachers about Indic knowledge systems (IKS) and India's contribution to the world in (Raju, 2024) maths by comparing the Christian calendar with the Indian calendar. Further, the teachers are exposed to empirical (fact-based) maths with materials e.g,. using the old definition of angles in India, hands-on activities with tokens related to the Bhramagupta rules of integers, use of polypad for visualization of mathematical concepts as well as Snap/ Scratch programming for projects. These sessions were made available to teachers through one-hour sessions in the morning, one or two times a week over six months. In this paper, we discuss the outcomes of this intervention. The teachers work with children from 2nd Grade to 10th Grade in Mathematics in rural schools of Tamil Nadu in India.

Research Questions

1. What knowledge have the facilitators acquired, and what have been the perspective shifts about IKS?
2. What explicit distinctions and examples about empirical and formal math led to perspective shifts in teachers?
3. Has there been any change in how the teachers are interacting with children?

Research Methodology

- A survey consisting of 12 questions asking about learning from the intervention, shift in perspective, and practices adopted by the teachers was shared with STEM Teachers at C3SL.
- 22 teachers participated in the survey.
- The responses were qualitatively analysed.

Findings

Knowledge Acquired and Perspective Shift Based on IKS After the Intervention:

Many teachers have gained a deeper appreciation for India's contributions to mathematics, recognizing its historical and practical significance. Key contributions highlighted include the development of the place value system, the decimal system, trigonometry, the sine series, the theory of infinitesimals, and infinite series, largely attributed to schools like the Kerala School of Mathematics. Indian mathematicians such as Aryabhata, Brahmagupta, and Bhaskaracharya were noted for their pioneering work in areas like algebra, astronomy, and the Indian calendar system, which integrates both solar and lunar calculations.

Participants also emphasized the practical and scientific approach of Indian mathematics, which connects to real-world applications, such as calculating time, angles, and heights using simple tools.

The Indian calendar, with its astronomical precision, was particularly praised for its accuracy and secular nature compared to the Gregorian calendar. Overall, there was a shift in perspective from viewing maths as primarily a Western achievement to recognizing India's profound global contributions. Some of the responses highlighting the learnings are:

> "my perspective on India's contributions to the world of mathematics has changed deeply. the contribution I see now, the lunar and solar calendar, concept of zero. I've come to appreciate more deeply the interconnected global contributions that have shaped mathematical development."

> "Over the last six months, my perspective on India's contribution to mathematics has indeed deepend. I come to know the concept of brahmaguptha, bhaskaracharya and lilavathi, integers.now I could able to notice the contribution of maths in many places around me in shops. Calculation of indian calendar by sun and moon ,What is realy angle mean, calculating the height of tree and building with small object I have and distance ,Radiant, pi , so on."

> "Yes, Initially, it was difficult to understand the Indian contributions, as we are already deep into European standards. A few notable points for the change are: the Gregorian calendar is based on Christian traditions; terms like A.D. and B.C. have religious origins, making the calendar non-secular; and Western education was controlled by the Church until 1870. I found that the Indian contributions are so accurate, as we have both solar and lunar calendars."

Distinction between formal vs empirical maths:

Participants felt that empirical (fact-based) maths is practical, and rooted in real-world observations, while formal (faith-based) maths is abstract, based on assumptions and often disconnected from practical applications. The teachers talk about the shift toward fact-based learning, using tools like visualization and hands-on activities, has made maths more accessible and meaningful as they are able to apply it in the real world. Some responses that encapsulate what the teachers have learnt are:

"Fact based mathematics is learning the story and experimenting how things evolved and derived without assumption. Faith based mathematics as taking assumptions and explaining the concepts without understanding the core value."

"I could apply the normal mathematics in real world and I could see and feel things like $\sqrt{2}$, π where as formal mathematics make it complicated. Though $\sqrt{2}$,π is a real number, when I think about its never ending decimal numbers it seems both real and unreal." Here "normal mathematics" stands for empirical mathematics.

"During my school days, I found math very difficult to understand, so I memorized the concepts and formulas. But now, with the materials available, I can create projects using STEMland resources, which helps me understand the logic behind them. This has been very useful for me in both my life and my environment."

Changes in Teachers' Engagement with Children:

Multiple teachers share a similar shift in their teaching approach—moving from traditional, textbook-based instruction to a more hands-on, interactive, and student-centered method, use of innovative teaching methods and tools, like Scratch and practical problem-solving, has made mathematics more accessible and engaging for learners. They emphasize real-world applications, conceptual understanding, and exploration through materials, games, and technology. One of the teachers mentioned using materials that enable children to see patterns and derive formulas on their own, which is a shift from following the rules of mathematics without having an understanding of them. Another teacher recalled that children are more engaged when they are exploring on their own.

Some responses that demonstrate the changes teachers have undertaken in their teaching methods are as follows:

"As I teach maths using materials, I look for utility and logical aspects in fact-based mathematics. I focus on how concepts are proved or used in real life. In my school days, I learned faith-based mathematics, where they would write things down, and I would memorize them without understanding the logic or real-life applications"

"When I work with children, I do some research and prepare how to introduce connecting with real life and use materials that help them to derive the formula or notice pattern."

Discussion

IKS supports teachers in India be more appreciative of the contributions of India to global mathematics. Teachers feel more confident about making math more accessible for children even though this is not how they learned maths. However, this work is not only for children in India, but for all children who have borne the cost of the philosophy of superiority of formal math over empirical maths. Once teachers can see the distinction between formal and empirical mathematics they take an effort to make math more accessible to children. This perspective transcends polarities that can arise from the history of Mathematics.

References

Bishop, A. J. (1990). Western mathematics: The secret weapon of cultural imperialism. https://bpb-us-e2.wpmucdn.com/sites.uci.edu/dist/f/4286/files/2020/12/Bishop_Cultural_Imperialism.pdf

Cuemath. (2021, December 1). UK maths anxiety the worst in the world, global survey reveals. FE News. https://www.fenews.co.uk/education/press-release-uk-maths-anxiety-the-worst-in-the-world-global-survey-reveals

Macaulay, T (April, 1847) A Speech Delivered In the House of Commons on the 18th of April 1847. Project Gutenberg. (n.d.). https://www.gutenberg.org/files/2170/2170-h/2170-h.htm#link2H_4_0031

Faruque, O. (2024). School Mathematics in Colonial India: Analyzing an Arithmetic Textbook from the Late Nineteenth Century . JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE, 15(2). https://journals.library.columbia.edu/index.php/jmetc/article/view/12958/6464

Gafoor, K. A., & Kurukkan, A. (2015). Why High School Students Feel Mathematics Difficult? An Exploration of Affective Beliefs. https://files.eric.ed.gov/fulltext/ED560266.pdf

National Council of Educational Research and Training. (2022) Chapter 6 - Lines and Angles. In Mathematics Textbook for class IX (pp. 69–78). https://ncert.nic.in/textbook.php?iemh1=6-12

Bondu, R. (2020). A Study on Indian Mathematicians and their Contributions Covered in NCERT Mathematics text books. Studies in Indian Place Names.

Raju, C.K. (2012) Decolonising Math and Science Education." In Decolonising the University, edited by Claude Alvares and U.S.M. Shad Faruqi, 162–195. Penang: and Citizens International, 2012

Raju, C. K. (2024). Indian Calendar: Scientific Aspects. Kant Academic Publishers.

Ramasubramanian, K. (2022). DHARA An ode to Indian Knowledge System. Ministry of Culture, Government of India https://www.aicte-india.org/sites/default/files/Maths%20Brochure%2021%20FEB%20FINAL%202%20PM_compressed.pdf

Sharma, N., & Bhardwaj, M. (2024). Challenges in Mathematics Education and solutions: Indian perspective. International Journal of Innovative Research in Technology, 11(5), 2069–2074. https://ijirt.org/Article?manuscript=168856

The Print. (2021, July 14). Students scared of maths in Indian Middle Schools, the fear only grows with time, survey says. https://theprint.in/india/education/students-scared-of-maths-in-indian-middle-schools-the-fear-only-grows-with-time-survey-says/695916

Acknowledgements

We acknowledge all the teachers at STEM Land for their receptivity, agency, offering Insights, and for applying their learnings to enrich the lives of children.

MEMO

MEMO

KOREA 2025

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 5:

The Essence of Mathematics Education in Learning and Cognition

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 5
(Day 2)

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

TSG5-01

ANALYSIS OF THIRD-GRADE STUDENTS' STRATEGIES FOR SOLVING MULTI-DIGIT MULTIPLICATION PROBLEMS

Johyeon Chang
Graduate school of Korea National University of Education, Korea, imjhchang@gmail.com

JeongSuk Pang
Korea National University of Education, Korea, jeongsuk@knue.ac.kr

In multi-digit multiplication, developing diverse strategies beyond the standard algorithm is crucial for fostering a deeper understanding of multiplication principles. This study redesigned a multiplication unit for a third-grade class, implemented lessons aimed at developing diverse problem-solving strategies, and analyzed the resulting students' understanding. Analysis of the pre-test, in-class worksheets, and post-test revealed that students employed a significantly wider variety of strategies. Notably, students demonstrated their understanding of multiplication principles by effectively using the decomposition strategy to solve multi-digit multiplication problems.

Keywords: multiplication strategy, multi-digit multiplication, decomposition strategy

Introduction

Using various strategies to solve arithmetic problems enhances students' mathematical understanding and flexibility (National Council of Teachers of Mathematics, 2014). The ability to use diverse strategies not only improves mathematical achievement but also supports knowledge retention and extension (Liu et al., 2015; Star et al., 2020). According to the 2022 revised mathematics curriculum in Korea, the second-grade textbook explicitly presents multiple methods for solving addition and subtraction problems, emphasizing the importance of learning through varied strategies (Ministry of Education, 2024). However, research on multi-digit arithmetic, particularly in multiplication and division, is still relatively limited compared to single-digit arithmetic (Hickendorff et al., 2019).

This study redesigned and implemented lessons that emphasized various multiplication strategies for third graders. Students' understanding was analyzed through the pre-test, post-test, and worksheets. The findings provide practical implications for effective instruction in multi-digit multiplication, particularly in fostering an understanding of the multiplication principle by using diverse strategies.

Literature Review

Solving arithmetic problems using various strategies enhances conceptual understanding and the ability to flexibly select among diverse approaches (Verschaffel et al., 2009). Consequently, it has been emphasized that students should be guided to develop and use their own strategies (Hickendorff et al., 2019). Specifically, when students solve multiplication problems using multiple strategies, they naturally uncover the derivation process of the standard algorithm and gain an intuitive

understanding of its underlying principles (Kamii & Dominick, 1998). This study analyzed and categorized multiplication strategies identified in prior research to explore effective ways to support students' understanding of multiplication principles. Based on this analysis, a classification of multiplication strategies was developed as follows.

Table 1. Reconstructed multiplication strategy classification

Strategy	Definition	Example 19 × 20	Theoretical basis
Direct Modeling	Using repeated addition methods, such as equal grouping, counting by groups, and skip counting	Adding 19 twice yields 38; adding 38 ten times produces 380.	Baek (1998) Van de Walle et al. (2022)
Decomposition-Addition	Decomposing numbers and representing them in the form of addition number sentence	(10+9) × 20 = (10×20) + (9×20) = 200 + 180 = 380	Baek (1998) Hickendorff et al. (2019) Van de Walle et al. (2022)
Decomposition-Multiplication	Decomposing numbers and representing them in the form of multiplication number sentence	19 × (5×4) = (19×5) × 4 = 95 × 4 = 380	
Compensation	Adjusting numbers by rounding them to the nearest multiple of 10 and then compensating for the rounding error to simplify calculations	(20×20) - (1×20) = 400 - 20 = 380	Baek (1998) Hickendorff et al. (2019) Van de Walle et al. (2022)
Hybrid	Combining two or more methods	First calculating 19×10 to get 190, then adding it twice	Sherin & Fuson (2005)
Standard Algorithm	Decomposing numbers by place value, multiplying each part to produce partial products, and then summing those partial products	Using the vertical algorithm format 1 9 × 2 0 1 8 0 2 0 0 3 8 0	Hickendorff et al. (2019) Van de Walle et al. (2022)

This study identified key instructional elements by reviewing prior research, analyzing third-grade textbooks, and examining fundamental concepts in mathematics education. These elements were categorized into three areas. First, "Expressing a multiplication sentence in equivalent forms" was taught through three main approaches: switching the order of the multiplier and multiplicand; decomposing numbers into addition sentences; and decomposing numbers into multiplication sen-

tence. Second, "Solving multiplication problems using various strategies" involved encouraging students to develop their own problem-solving approaches, utilize visual aids, and share their strategies with peers. Finally, "Connecting various strategies" focused on comparing different strategies and connecting them to the underlying principles of multiplication.

Methods

A third-grade class of 22 students from a school in D City was selected because the second semester was when students formally learn (three-digit) × (one-digit) and (two-digit) × (two-digit) multiplication. The multiplication unit was redesigned to focus on solving multiplication problems using diverse strategies and was structured into two phases. The first phase built a foundation for strategy development by reviewing second-grade multiplication and exploring different expressions using multiplication properties. The second phase focused on solving multiplication problems (132×3, 231×9, 12×20, 9×28, and 29×12) selected from 10 textbooks, prioritizing those that can be solved using multiple strategies. Students were encouraged to attempt problems independently and to use visual aids — such as base-ten blocks, number cards, arrays, open arrays, and grid paper — if needed. Throughout the lessons, students solved problems, shared and compared strategies through worksheets, and connected their approaches to the underlying multiplication principles.

The researcher observed and recorded each lesson, documenting teaching processes, student activities, and discussions for analysis. Pre- and post-tests (41×9 and 52×8) were administered, requiring students to solve problems using three different approaches. Students' multiplication strategies were analyzed by coding their responses from worksheets and tests, with a focus on changes in the number and types of strategies used over time. During the analysis, the strategy classifications were further refined: the Decomposition-Addition strategy was subdivided into two categories—cases where numbers were freely decomposed were coded as Decomposition-Addition (F), while cases where numbers were decomposed according to place value were coded as Decomposition-Addition (P).

Results

For (two-digit)×(one-digit) multiplication, students demonstrated a greater variety of strategies on the post-test compared to the pre-test (see Table 2). In the pre-test, all students initially used the Standard Algorithm, which remained the most common strategy in subsequent attempts, with Direct Modeling as the next most frequent approach. In the post-test, although the Standard Algorithm continued to be widely used, its dominance decreased as students employed a broader range of strategies. When solving the 21×39 problem, use of the Direct Modeling strategy decreased significantly, while use of the Decomposition-Addition (P) strategy increased noticeably. Notably, 9 out of 22 students applied the Compensation strategy.

Comparing the results of 231×9 to those for 132×3, the use of Direct Modeling decreased, while more students decomposed numbers by place value using the Decomposition-Addition strategy. Additionally, the use of both Direct Modeling and the Standard Algorithm declined. Notably, one student who had applied the Compensation strategy in the pre-test for 41×9 did not use it for 132×3 but reintroduced it for 231×9, likely because the multiplier was 9. Another student also employed this strategy.

Table 2. Students' use of strategies across multiplication problems

Strategy	132×3	231×9	12×20	9×28	29×12	Pre-test 41×9	Post-test 52×8	Post-test 21×39
Direct Modeling	24	7	8	1	2	17	11	3
Decomposition-A (F)	21	8	10	11	5	0	12	4
Decomposition-A (P)	3	13	4	10	9	4	13	20
Decomposition-M	0	0	5	4	1	0	2	0
Compensation	0	2	0	2	2	1	2	9
Hybrid	0	0	4	0	1	0	2	2
Standard Algorithm	11	8	11	6	8	32	19	17

Note. Decomposition-A (F) and Decomposition-A (P) are subcategories of Decomposition-Addition, where "A" denotes "Addition."

While students primarily used the standard algorithm to solve 12×20, some applied the Decomposition-Addition strategy by breaking 20 into 10 and 10, while others used Direct Modeling with base-ten blocks, number cards, and arrays. For the first time, students also demonstrated the Decomposition-Multiplication strategy as well as a Hybrid strategy combining multiplication and addition. In solving 9×28, the only visual aid provided was an open array. It was intriguing to observe students creatively applying the Decomposition-Addition strategy by partitioning the open array. Notably, a key difference from previous learning topics was the variety of ways students decomposed 28. Some represented 28 as 10+18, 19+9, or 9+9+9+1. Others divided it into four equal parts (7+7+7+7), and some halved it into 14+14.

In 29×12, all six strategies were observed, with a decreased reliance on Direct Modeling and an increased use of place value decomposition within the Decomposition-Addition strategy. As the numbers became more complex, students adopted more efficient Decomposition strategies. For instance, some decomposed 12 into 6×2, and applying the Decomposition-Multiplication strategy using an open array.

Discussion and Conclusion

Analysis of the pre- and post-test data, along with in-class worksheets, showed that students employed a wider range of strategies with increased use of the compensation strategy, particularly when solving multiplication problems with numbers ending in 9. These findings underscore the importance of using diverse strategies—especially the place value-based Decomposition-Addition (P) strategy—to deepen students' understanding of multiplication principles. Additionally, the integration of various visual aids appeared to foster exploration of alternative strategies beyond the standard algorithm. Finally, the study suggests that explicit instructional support is essential for helping recognize and apply the compensation strategy more effectively, as evidenced by its increased use in the post-test.

References

Baek, J. M. (1998). Children's invented algorithms for multidigit multiplication problems. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics* (pp. 151-160). National Council of Teachers of Mathematics.

Hickendorff, M., Torbeyns, J., & Verschaffel, L. (2019). Multi-digit addition, subtraction, multiplication, and division strategies. In A. Fritz, V. G. Haase, & P. Räsänen (Eds.), *International handbook of mathematical learning difficulties: From the laboratory to the classroom* (pp. 543-560). Springer. https://doi.org/10.1007/978-3-319-97148-3_32

Kamii, C., & Dominick, A. (1998). The harmful effects of algorithms in grades 1-4. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics* (pp. 130-140). National Council of Teachers of Mathematics.

Liu, R. D., Ding, Y., Gao, B. C., & Zhang, D. (2015). The relations between number property strategies, working memory, and multiplication in elementary students. *Journal of Experimental Education, 83*(3), 319–343.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics, Inc.

Ministry of Education (2024). *Mathematics 2–1* [in Korean]. Cheonjae.

Ministry of Education (2024). *Mathematics teacher's guidebook* for 2-1 [In Korean]. Cheonjae.

Sherin, B., & Fuson, K. (2005). Multiplication strategies and the appropriation of computational resources. *Journal for Research in Mathematics Education, 36*(4), 347-395.

Star, J., Rittle-Johnson, B., Durkin, K., Shero, M., & Sommer, J. (2020). Teaching for improved procedural flexibility in mathematics. In M. Gresalfi & I. S. Horn (Eds*.), The interdisciplinarity of the learning sciences: 14th International Conference of the Learning Sciences (ICLS) 2020, Volume 4* (pp. 2285-2288). International Society of the Learning Sciences.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2022*). Elementary and middle school mathematics: Teaching developmentally* (11th ed.). Pearson

Verschaffel, L., Luwel, K., Torbeyns, J., & Van Dooren, W. (2009). Conceptualizing, investigating, and enhancing adaptive expertise in elementary mathematics education. *European Journal of Psychology of Education, 24*(3), 335-359. https://doi.org/10.1007/BF03174765

TSG5-02

INVESTIGATING THIRD GRADERS' PLACE VALUE CONCEPT AND MULTI-DIGIT ADDITION THINKING STRATEGIES

Insook Chung
Saint Mary's College at Notre Dame, USA, ichung@saintarys.edu

Hannah Mortensen
Saint Mary's College at Notre Dame, USA, hmortensen01@saintmarys.edu

Forty-one third graders enrolled in two public schools in the USA participated in the study. The study examined whether they have acquired place value concepts and how they apply them to establish addition thinking strategies and solve double-digit addition problems. A researcher-made Place Value survey and a researcher-made worksheet (Close to 100) were used to examine students' understanding of the place value concept and addition thinking strategies. An open-ended question was included in the worksheet to have students explain a strategy to make the sum close to 100. The results from the collected data were qualitative and quantitative. The qualitative data from the third graders' responses to the addition thinking strategy and the quantitative data from the survey and worksheet were analyzed and discussed. The quantitative data showed a correlation between the place value concepts and double-digit addition thinking strategies. The T-test showed statistical significance. ($t = 0.0001$, $p < 0.05$).

Keywords: place value concept, multi-digit addition, base-ten system.

Introduction

One of the most important mathematical concepts to be understood by children during their elementary grades is that of place value. However, numerous research studies reported that many school children have difficulty understanding the place value system, particularly the essential characteristics of the place value system (Birgin, Gurbuz, & Memis, 2022; Fuson, 1990; Hibert & Wearne, 1992; Miura & Okamoto, 1989; Varelas & Becker, 1997). Place value is the position of a digit that represents its value or the value of a digit as determined by its position in a number. It is also a crucial organizing principle of place value numeration. For example, 5 in 53 is five groups of 10, an integral principle of base-10 numeration (Van De Walle et al., 2021). Successful place value understanding is one of the crucial factors for further developing arithmetic competencies, specifically in multi-digit addition tasks (Moeller et al., 2011; Sari & Olkun, 2019).

According to the Common Core State Standards (National Governors Association and CCSSO, 2010), second graders begin to understand groups of ten and connect them to the place value system. They are expected to solve addition problems with two- and three-digit numbers. The third graders use the place value concept to represent and show equivalent forms of whole numbers up to 10,000, to round two- and three-digit whole numbers to the nearest 10 or 100, and fluently add and subtract multi-digit whole numbers using strategies and algorithms. (Indiana Department of Education, 2023). Students also need to acquire a deep understanding of the operations and properties of the operations to establish addition and subtraction strategies so they can solve addition problems

by decomposing and composing numbers flexibly. (Van De Walle et al., 2021). Learning double-digit addition is focused on the 2nd graders in the USA; however, third- and fourth-graders still struggle to solve double-digit addition problems. The NAEP results (NCES, 2024) showed the US 4th graders achieved the basic level (score 214/500, among basic, proficient, and advanced levels), which is the lowest level that includes composing and decomposing skills with multi-digit whole numbers based on place value and adding and subtracting multi-digit whole numbers.

A study conducted by Sari and Olkun in 2019 reported that the relationship between fourth graders' place value concepts and arithmetic performance was closely related. The current study examines the conceptual structure of third graders' place value and strategies for solving multi-digit addition problems. The study's objectives focused on the following: 1) Identifying place value structural concepts; 2) Applying the concepts to make a sum Close to 100 using two double-digit addends; and 3) Justifying what addition thinking strategies are used to make the sum close to 100.

Methodology

Participants

The population for this study consisted of 41 students, 25 boys and 16 girls, from four third-grade classrooms at two elementary schools in the US public school system. The mean ages of the participants were 9 years and 3 months. Forty-nine third-graders recruited and participated in the study, but eight students' data were excluded since they did not complete the assessment tasks for the study. The State assessment of the ILEARN math achievement reported that the proficient levels of the two schools that participated in the study were 12.3% and 31.1%, while the statewide proficient level was 52.4% (Indiana Department of Education, 2024). The classrooms were selected by convenient stratified purposeful sampling (Patten, 2005) from two public school districts in the region of a mid-eastern area of the USA. The classroom teachers of the forty-one third graders distinguished students' academic ability levels. Twelve students were identified as high-, seventeen as average-, and twelve as low-ability students academically by their classroom teacher.

Instrument

A researcher-made Place Value (PV) survey with seven questions (e.g., How many groups of tens in 789?) was employed to check the third graders' understanding of place value. A "Close to 100" worksheet examines the third graders' conceptual understanding of place value and addition thinking strategies. It consists of five tasks that use the same format as the card game Close to 100. The game aimed to make a sum of nearly 100 using two 2-digit addends. The researcher-made worksheet contained an open-ended question as the sixth task. It requires the students to explain how they make the sum close to 100. The game "Close to 100" with a scoresheet and its worksheet were adopted from the curriculum, *Investigations in number, data, and space* (2008), and used after revising them for the study.

Procedure and Data Analysis

The researchers distributed a letter and a consent form containing information explaining the study's objectives to the students' homes to get a consent form signed by their parents/guardians. After collecting the parent/guardian consent form with the assistance of four third-grade classroom teachers, the students received an assent form and an additional explanation to sign prior to participating in the activities for collecting research data.

The PV survey, Close to 100 worksheets, scoresheets, and written and verbal directions were updated after a pilot study conducted with four third graders selected from another school not participating in the study to review the designs, questions specifically for an open-ended question in the Place Value survey, and the Close to 100 worksheets before the data collection date from the participants in the study.

The participants (n= 41) completed a Place Value survey before playing the "Close to 100" number card game. One of the researchers explained the directions for how to play the game. The students, in pairs, play the game five rounds following the researcher's instructions. The researcher who led the activity recorded the observations on how the students process the card game "Close to 100" and how they process and solve the addition problems while interacting with the students. Another researcher took digital photos to record the student's progress in playing the game and working on the worksheet, Close to 100. The task of the game was creating two double-digit numbers that sum up close to 100 using four cards from 6 number cards that the players randomly drew. After playing five rounds, the students with the lowest total score won the game and received an incentive package (prize). The other students who lost the game also received a smaller package, so they were still motivated to continue collecting data for the researchers. The score is the difference between 100 and the sum of two double-digit numbers the players created. After completing the game three times, the students work on the worksheet "Close to 100." The activities and data collection were conducted in February, the early second semester of the school year.

Quantitative data from students' Place Value surveys and game scores on the worksheets were analyzed using the T-test P-value to investigate how they are correlated. Students' responses about the strategies were categorized and qualitatively discussed using the Researcher-made rubric. Both researchers cross-examined the coding system and input data.

Results

Game scores ranged from 0 to 167 (mean: 32). Nine students (22%) had the scores 0 to 10. These students created two double-digit addends to make a total score (difference) for five problems that were less than 10 in total score. It means that the average difference to make 100 was 2 at most. Twelve students (29%) had 11 to 20, indicating the mean score for each problem was 4 at most. Four students (10%) had 21 to 30, five students (12%), four students (10%) had 31 to 40, four students (10%) had 41 to 50, and seven students (17%) obtained the score range 51 -167. The mean score was 32, which means each problem's score (the difference between 100 and the sum they made with two double-digit addends) had a mean score of 6.4.

Sixteen students (39.02%) added correctly to make a sum. Twenty-five students (60.98%) made calculation errors when adding two two-digit numbers. Among them, five students made significant errors, such as 35 + 65 not making a sum of 87, and the difference is not 7 to make 100, but 13. Another example is 65 + 51 = 84; the student wrote the difference 19. They were not able to add double-digit numbers correctly. Twenty students made minor errors, such as 97 + 34 = 121 instead of 131; the difference between 89 and 100 was 9. The T-test correlation between each student's scores from the Place Value survey and close to 100 was statistically significant since the T-test result was 0.0001 ($p < 0.05$).

Discussion and Conclusion

The principles and standards for school mathematics (NCTM, 2000) advocated that a balance and connection between conceptual understanding and computational proficiency are required for students to develop fluency in mathematics. The researchers observed that none of the students had trouble making two-digit numbers to make larger or smaller numbers while composing two double-digit addends using four number cards 0 to 9, looking at 10s and 1s places. They used a larger number card in the 10s place to make a bigger number, demonstrating they understood the base-10 system and achieved solid place value concepts.

When adding two double-digit numbers, twenty-one students (51%) drew diagrams of Base-10 Blocks (longs and units) to get the sum. However, only six students (29%) from twenty-one could regroup 10s and 1s to provide a correct sum. About six students used their fingers to add two single-digit numbers as if the numbers in the 10s place were the numbers in the 1s place. The students who made higher scores or differences (rounding numbers to 100) had trouble composing and decomposing numbers flexibly, whichever strategy they used to make a sum of 100. Making a sum of 100 by adding two double-digit numbers requires composing and decomposing numbers without difficulties. The students had expected place value concepts but struggled to compose and decompose fluently in adding and subtracting numbers. More than sixty percent (60.98%) of the third-graders did not calculate correctly to make a sum for the addition problems. The result indicates that most third-graders still make calculation errors when solving double-digit addition problems applying place value concepts. However, some calculation errors, especially for students who made minor errors, could be improved if the students had time to review the answers later. Since the researchers were allowed limited time to conduct the study, they did not have enough time for the participants to review the answers before submitting the worksheets.

The students who made significant errors should have more experience with concrete examples (e.g., Base-10 Blocks), working on the addition problems while manipulating the physical objects, and practicing regrouping strategies. For example, one long unit and nine units represent 19. Two longs and six units show 26. When adding these two sets, a new set becomes 3 longs and 15 units. By trading and regrouping this new set, the sum shows 4 longs and 5 units, which is 45. Sahin, Dixon, and Schoen's study (2020) investigated how students' strategy use related to mathematics achievement with the second graders in three-digit subtraction problems and reported the students who used the mixed strategy among standard, invented, mixed, and unclassified algorithms yielded significant highest mathematics achievement scores. The study's result implies that students should be able to compose and decompose numbers using invented or alternative algorithms and conventional algorithms to do well in mathematics.

One of the alarming findings from this study was that the students did not clearly explain the procedure for making the target number 100 for the task. Fifteen students (37%) of forty-one participants wrote responses strongly suggesting they might understand the meaning of the digits in the ten's place and one's place. For example, the answers were: I used bigger numbers for the tens and smaller numbers for the ones to get close to 100, and a student drew Base-10 Blocks and illustrated the procedure with diagrams. Twenty-one students (51%) tried to explain, but they could not successfully articulate their strategy in writing. Five students (12%) gave up, did not write anything, or stated I don't know. The findings indicated that students struggle to represent and communicate their knowledge and skills at an abstract level (written language). *The Curriculum and evaluation standards for school mathematics* (NCTM, 1989) advocated that describing how a problem was solved allows students to clarify their meaning and develop a deeper understanding of mathematics

(p. 26). This document claims that communication in mathematics is vital in the process of concept (Neil, 1996, p. 3). This study confirms why writing and communicating their understanding of mathematics content is integral. It should be encouraged when teaching mathematics to help students develop problem-processing and reasoning skills to reach correct answers, extending and applying their previous learning.

As mentioned earlier, 51% of students drew Base-10 Blocks on a scrap paper given to find a sum. It showed that the classroom teacher used this concrete manipulative to teach place value concepts, and they knew that a long stick (10s) is one group of ten and ten longs are one flat (100s). Some students could not quickly regroup to make a new group, so they seemed to rush to give their answers on the worksheet. The students could not solve double-digit addition problems by mentally processing the problem. So, they were provided scrap papers to write equations to solve the problems. Most of the children wrote the equations in a horizontal form (e.g., 26 + 74 = 100) rather than vertical ones after physically arranging number cards shown in a vertical form. Eighteen students (44%) used the horizontal form to calculate double-digit addition problems on the scrap paper. Based on the school's math curriculum, both horizontal and vertical forms to solve multiple-digit addition problems were taught in the second grade. A further study is recommended to explore why more than 50% of students used a horizontal form to solve the addition problems.

Another observation was that the students approached the researchers and asked what to write for the question, which required explaining how they made a sum of 100. The researchers rephrased the question and asked them to describe it. Right away, they started verbally explaining what they did. The researchers had to redirect them to write what they had just said. The researchers found out they did not write on the worksheet as they verbally explained. It is another area a further research study should be designed to observe their reactions to how they verbally describe the procedure or strategy, not only writing their explanation on the paper to examine if they can express their thoughts in words through an interview to see if they can verbally explain better.

References

Birgin, O., Gurbuz, R., & Memis, K. Z. (2022). Performance of second-grade elementary school students on counting, place value understanding, and addition operation in natural numbers. *International Journal of Mathematical Education in Science and Technology,* 53(12), 3377-3392.

Fuson, K. C. (1990). Issues in place value and multi-digit addition and subtraction learning and teaching. *Journal for Research in Mathematics Education, 21*, 273–280.

Hibert, J., & Wearne, D. (1992). Links between teaching and learning place value with understanding in first grade. *Journal for Research in Mathematics Education, 23*, 98–122.

Indiana Department of Education (2024). *Indiana ILEARN results*: *Most grade levels see a slight increase in English/Language Arts proficiency; Math proficiency slightly decreases following significant growth in recent years.* https://www.in.gov/doe/about/news/2024-ilearn-results-released/

Indiana Department of Education (2023). *Academic standards for mathematics*. Retrieved from https://www.in.gov/doe/students/indiana-academic-standards/

National Center for Education Statistics (2024). The NAEP mathematics achievement levels by grade. https://nces.ed.gov/nationsreportcard/mathematics/achieve.aspx

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: The Council.

National Governors Association & Council of Chief State School Officers (NGA & CCSSO) (2010). *Common Core State Standards Initiative.* www.corestandards.org

Miura, I. T., & Okamoto, Y. (1989). Comparisons of US and Japanese first grader's cognitive representation of numbers and understanding place value. *Journal of Educational Psychology, 81*,(1), 109-113.

Moeller, K., Pixner, S., Zuber, J., Kaufmann, L., & Nuerk, H. C. (2011). Early place-value understanding as a precursor for later arithmetic performance—A longitudinal study on numerical development. *Research in Developmental Disabilities*, 32(5), 1837-1851.

Patten, M. L. (2005). *Understanding research methods: An overview of the essentials* (5th ed.). Glendale, CA: Pyrczak Publishing.

Sahin, N., Dixon, J. K., & Schoen, R. C. (2020). Investigating the association between students' strategy use and mathematics achievement. *School Science and Mathematics, 120*, 325-332. https://doi.org/10.1111/ssm.12424

Sari, M. H., & Olkun, S. (2019). The relationship between place value understanding and mathematics achievement in general. *Elementary Education Online, 18*(2), 951-958.

Technical Education Research Centers (TERC) (2008). *Investigations in number, data, and space* (2nd ed.) Upper Saddles River, NJ: Pearson Education, Inc.

Varelas, M., & Becker, J. (1997). Children's developing understanding of place value: Semiotic aspects. *Cognition and Instruction, 15*(2), 15-22.

Ven De Walle, J. A., Karp, K. S., Bay-Williams, J. M. (2021). *Elementary and middle school mathematics: Teaching developmentally* (8th ed.). Upper Saddles River, NJ: Pearson Education, Inc.

TSG5-03

TEACHERS SEE 1 NUMBER WHERE KIDS SEE 3 NUMBERINGS IN 507, WHO IS NUMBER-BLIND?

Allan Tarp
MATHeCADEMY.net, Denmark, Allan.Tarp@gmail.com

*The fourth United Nations Development Goal wants all youth and most adults to achieve numeracy. In 'essence-based' mathematics using one-dimensional line-numbers without units, numeracy is at the beginning and calculus is at the end. But the opposite is the case within the alternative 'existence-based' paradigm built on the numbers children bring to school, two-dimensional bundle-numbers with units, where 2 3s is short for 2 bundles with 3s per bundle. Here calculus is needed in grade one for next-to addition of 2 3s and 4 5s as 8s since they add by their areas. Here, adding is preceded by counting leading to a recount-formula, T = (T/B)*B, with T and B for Total and Bundle, used to change units in STEM, and here recounting between icon-numbers and tens leads to a division and multiplication table, and to solving equations. Recounting physical units leads to per-numbers and fractions with like units, and trigonometry when recounting the sides in a bundle-stack. So, changing from essence-based to existence-based curriculum may allow reaching the UN Development goal and provide a divergent thinking to assist the learner in getting a conceptual understanding of math.*

Keywords: Early childhood mathematics, numeracy, special education, curriculum, calculus

Background, the 4th United Nations Sustainable Development Goal

The fourth of the 17 UN Sustainable Development Goals defines quality education as 'ensuring inclusive and equitable quality education and promoting lifelong learning opportunities for all.' Here the subgoal 4.6 wants to "By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy". However, different definitions of 'numerate' seem to exist. The English Oxford Dictionary defines it as being "competent in the basic principles of mathematics, esp. arithmetic". In contrast, the American Merriam-Webster dictionary defines it as "having the ability to understand and work with numbers."

The difference in the definitions is interesting. The English uses the passive term 'competent' where the American uses the active term 'work'. The word 'competent' is a predicate, a non-action word, I cannot 'competent' something, I can only be judged as competent by someone who is competent. In contrast, 'work' is an action word, a verb, since with my body and mind I can work on something.

Also, there is a difference between the words 'mathematics' and 'numbers.' Again, mathematics is a non-action word, I cannot 'mathematics' or even 'math' a thing. In contrast, 'number' is both a verb and a noun since I can number different degrees of Many to produce a number for later calculations.

Finally, Many exists in the outside world observed by humans. In contrast, mathematics does not, it is institutionalized essence that is socially constructed as inside abstractions from outside examples, or as inside examples from inside abstractions. To understand these differences, we now consult the three grand theories, philosophy and sociology and psychology. Philosophy may be able to illuminate the different nature of predicates and verbs. Sociology may be able to illuminate the different

inter-human power effects coming from using predicates instead of verbs. And psychology may be able to illuminate the different learning results coming from listening to predicates or practicing verbs.

Grand Theory looks at Mathematics Education

Within philosophy, Existentialism holds that 'existence precedes essence' (Sartre, 2007) so that in a judging is-sentence, an existing subject is being colonized by its predicates. 'Many' thus should be seen onto-logically in itself, instead of epistemologically, how some may perceive and verbalize it.

Within psychology, Piaget (1969) sees learning as adapting to outside existence, whereas a Vygotsky (1986) sees learning as adapting to inside institutionalized socially constructed essence.

Within sociology, a structure-agent debate discusses if institutions should be obeyed or negotiated between peers. Here, a Weberian viewpoint (1930) asks if SET is a rationalization gone too far by leaving Many de-enchanted and leaving learners in an 'iron cage'. As to the goal, Bauman (1990) suggests that, by institutionalizing mastery of Math as the means to reach mastery of Many, 'essence-math' has created a 'goal displacement' making the means a goal instead. Then, with the end goal, mastery of Many or existence-math, essence-math may be a means, but are there other means also?

Essence-math sees Many as an example of 1D linear cardinality always able to absorb one more by being built on the assumption that '1+1 = 2'. In contrast, humans see Many as a union of 2D stacks coming from numbering singles, bundles, and bundles of bundles, e.g., T = 345 = 3*BB + 4*B + 5*1. Essence-math sees mastery of math as its goal. A difference could see mastery of Many as the goal to be reached directly by using the bundle-numbers with units that children bring to school and that allows learning core math automatically (Tarp, 2018, 2024). Essence-math could be more meaningful by de-modelling it (Tarp, 2020) built on the French poststructuralist version of Existentialism where deconstruction is used to replace predicates with verbs (Derrida, 1991). And, Difference Research (Tarp, 2018) searching for differences making a difference may design micro curricula to be tested.

Children use 2D Bundle-numbers with Units, Teachers use 1D Line-numbers without Units

When asked "How many years next time?" a 3-year-old child will say "That is not 4, that is 2 2s" if seeing the 4 fingers held 2 by 2. This statement will change math education forever since, as educated, the essence, 4, is all we see. But as uneducated, the child sees what exists, bundles of 2s in space, and 2 of them when counted in time. Counting in tens instead of in 2s, we write 47 and say 'forty-seven' instead of '4 bundles with tens per bundle and 7 unbundled' to respect what exists and call it one number despite it is two numberings. 2 thus exists both in space and in time. In space, 2 exists as 2s, a space number, a bundle of 2s, a 2-bundle, which can be united with a 3-bundle. Horizontally to a (2+3) bundle, a 5-bundle, or vertically to a stack of 2B1 2s or a stack of 2B-1 3s with B for bundle. In time, 2 exists together with the unit that was counted, as 2 units, a time-number, or a counting-number. So, 2+3 is 5 only with like units. Without units, a counting-number is an operator to be multiplied with a unit to become a total that can be added with another total if the units are the same, or after the units are made the same by recounting the two totals in the same unit. And since we count bundles and singles, the unit should be included in the counting sequence: 0B1, 0B2, …, 0B9, 1B0. So, outside in the real world, 2 is a 'ghost-number'. What exists are 0B2 in time and 2s in space.

What kind of knowledge is mathematics when integrating two different phenomena, 0B2 in time and 2s in space, into one phenomenon, 2, defined as 1+1 = 2? Knowledge may be created individually (Piaget, 1969) and collectively (Grounded theory, 1967) both respecting that learning means differentiating a phenomenon into sub-phenomena with different properties. By doing the opposite, mathematics become a self-referring anti-science with inside descriptions of itself and not of the outside world where its foundation, 1+1=2, is falsified by children seeing that with an open and a closed V-sign on a hand, adding the 2 1s and 1 2s gives 1 4s, and not the 3 3s, as mathematics claims.Likewise, the school teaches both '2+3 = 5' and '2x3 = 6'. Here 2 3s always recounts as 6 1s, but 2weeks + 3days is 17days. So, even if both hold inside the school, outside 'multiplication holds, but addition folds.' Adding numbers without units may be called 'mathematism', true inside but seldom outside the school, whereas mathematics that add numbers with units may be called 'Many-math', using bundle-numbers with units as 2 3s and 4 5s that may be added next-to as 8s, or on-top after shifting the units. But adding areas and shifting units is called 'calculus' and 'proportionality', the core of mathematics. And here they occur in the first lessons where normally they enter very late.

To get a more precise definition of space-numbers as 2s and time-numbers as 0B2 we observe the following when using the time-number sequence '0B1, 0B2, …, 0B5' to count the fingers on a hand: After in time saying '0B1' when pulling up one finger we now in space have 1 1s. And, after in time saying '0B2' when pulling up one more finger we now in space have 2 1s that may bundle as 1 2s. Likewise with 0B3, 0B4, and 0B5. Then, after in time saying '0B6' when pulling up one more finger we now in space have 6 1s that may bundle as 1 6s, or that may bundle as ½B1 with ½B as 1 5s. We may repeat the same on a ten-by-ten Bundle-Bundle board that contains all the first Bundle-numbers.

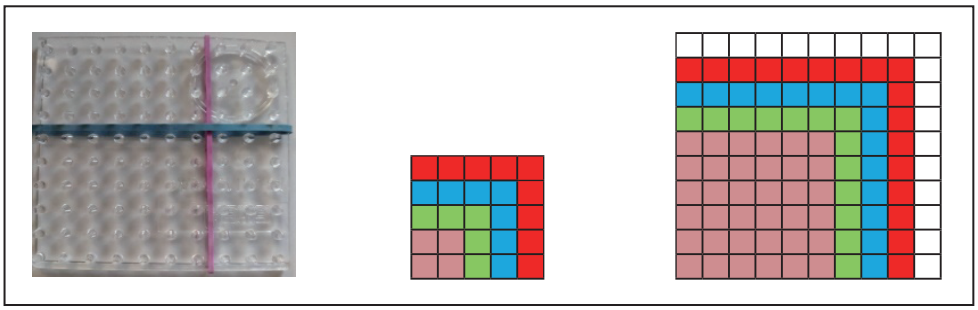


Figure 1. A BundleBundleBoard with 6 7s; and squares with 1BB2B1 or 1BB-2B1 as the neighbor

Overload	Underload	Overload	Overload
65 + 27	65 – 27	7 x 48	336 /7
6 B 5 + 2 B 7	6 B 5 – 2 B 7	7 x 4 B 8	33 B 6 /7
8 B12 9 B 2	4 B-2 3 B 8	28 B 56 33 B 6	28 B 56 /7 4 B 8
92	38	336	48

Figure 2. Bundles and over- and underload make place values and carrying unneeded

Counting in half-bundles later eases addition, subtraction and multiplication: 6+7 = ½B1 + ½B2 = 1B3 = 13; and 9-7 = ½B4 - ½B2 = 2; and 6*7 = 6* ½B2 = 3B12 = 4B2 = 42.

To see that also bundles may be bundled we count five fingers in 2s: Five = 0B5 = 1B3 = 2B1 = 3B-1 = 1BB0B1 2s. Holding two hands together we see that ten = 2BB0B2 = 1BBB0BB1B0 = 1010 2s. And that ten =1B0 = 10, hundred is 1BB0B0 = 100, and thousand = 1BBB0BB0B0 = 1000 tens.

But, before totals can be added they must be counted or recounted. Counting 8 in 2s, we push-away bundles of 2s to be lifted into a stack of 4 2s, which may be iconized by a broom and as a lift so that we have 8 = (8/2)x2, or T = (T/B)xB with T and B for Total and Bundle. This linear 'proportionality' recounting-formula to shift units now occurs in the first lesson when we bundle-count with units. And, it solves equations where '*u* x 2 = 8' is asking 'How many 2s in 8?', which is simply answered by '*u* = 8/2' since 8 is recounted in 2s as above, thus moving 'to opposite side with opposite sign'. Recounting 8 in 3s, we meet subtraction as a rope to pull-away the stack to find 2 unbundled that are included on-top of the stack as a decimal, 8 = 2B2 3s, or as a fraction when also counted in 3s as 2 = (2/3) x 3, 8 = 2 2/3 3s, or with a negative number telling how much is needed for an extra bundle or pulled-away from this, 8 = 3B-1 3s = 1BB0B-1 3s since the 3 bundles of 3s as one bundle of bundles.

Conclusion and Recommendations

Counting before adding thus leads to rectangular and squared bundle-numbers with units; and to decimals, fractions, and negative numbers; and to solving equations by recounting; and to proportionality needed to make units like when adding on-top; and to calculus when adding next-to as areas all carried out on a Bundle-Bundle Board. So, with Many-math's 'counting before adding' we have learned most mathematics almost before we begin. This will fulfill the UN Development numeracy goal. And it performs miracles in special education classes where the losers suddenly become users understanding and using numbers to count and calculate things and events in space and time. We therefore should stop teaching mathematics as 'mathematism' with its ghost-numbers replacing units with a place value system where carrying is needed when adding without units, and built on the rule that '1+1 = 2. Instead, we should use a 'counting before adding' curriculum to make children develop their innate number sense to allow a communicative turn in the number-language as the one that took place in the word-language around 1970 (Widdowson, 1970), so that the children can produce end enjoy number tales with its three genres, fact, fiction and fake (Tarp, 2001). Here, numeracy follows the American view as having the ability to understand and work with numbers. Everyday numbers carry units that often change. So, it is only natural that the children use and develop the Bundle-numbers with units they bring to school ready to be recounted to change the units. Existence-math is based upon Existentialism, the root of French post-structural thinking where Foucault's skepticism towards institutions and Derrida deconstruction of centrism is at the root of postmodern sociocultural thinking. Here we respect a social group that has had no voice until lately (Tarp 2024, 2025): children which Bundle-number language is colonized by institutionalized essence. Testing is ethical difficult since using bundle-units and overloads and underloads makes place values and carrying unneeded. But, without a score there will be no music. So, maybe a composer should be allowed to compose and leave it to others to play and evaluate the music.

References

Bauman, Z. (1990). *Thinking sociologically.* Blackwell.

Derrida, J. (1991). *A Derrida reader: between the blinds.* P. Kamuf (ed). Columbia Uni. Press.

Kuhn T.S. (1962). *The structure of scientific revolutions.* University of Chicago Press.

Piaget, J. (1969). *Science of education of the psychology of the child.* Viking Compass.

Sartre, J.P. (2007). *Existentialism is a humanism.* Yale University Press.

Tarp, A. (2001). Fact, fiction, fiddle - three types of models. in J. F. Matos, W. Blum, K. Houston & S. P. Carreira (Eds.). Modelling and mathematics education: ICTMA 9: Applications in Science and Technology. *Proc. 9th Int. Conf. on the Teaching of Mathematical Modelling and Applications* (pp. 62-71). Horwood Publishing.

Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: From inside-inside to outside-inside understanding. *Ho Chi Minh City Univ. of Education Journal of Science 17*(3), 453-466.

Tarp, A. (2024). *Many before math, math decolonized by the child's own bundle bundle-numbers with units.* https://youtu.be/uV_SW5JPWGs.

Tarp, A. (2025). 1+1 is 1 not 2 say the children in their declaration of independence. *Journal of Mathematics Education, 2025 Special Issue, forthcoming. Preprint on http://mathecademy.net/childrens-declaration-of-independence/.*

Vygotsky, L. (1986). *Thought and language.* MIT press.

Weber, M. (1930). *The protestant ethic and the spirit of capitalism.* Unwin Hyman.

Widdowson, H. G. (1978). *Teaching language as communication.* Oxford University Press.

TSG5-04

CHARACTERISTICS OF MATHEMATICAL NEGOTIATION THAT PROMOTE FRACTION CONCEPT FORMATION : AN ANALYSIS OF CLASS UTILIZING THE NUMBER LINE

Taketo Shimomura
Graduate School of Education, Shimane University, Japan, tshimomura@edu.shimane-u.ac.jp

Yuri Masutani
Graduate Student at the Graduate School of Education, Shimane University, Japan, e249303@matsu.shimane-u.ac.jp

Yasuyuki Okabe
Graduate School of Human Development and Environment, Kobe University, Japan, k0117107@gsuite.kobe-u.ac.jp

Third-grade elementary school students in Japan have an insufficient understanding of fractions. To address this issue, this study examines the characteristics of interactions that influence fraction recognition. In this context, we define mathematical negotiation as the intentional exchange of ideas aimed at constructing mathematical knowledge. Using an analytical framework based on Searle's speech act theory, we analyze the intentions behind students' utterances. The analysis of the survey results reveals that when students are about to reach a consensus based on misconceptions, teachers can guide them toward correct understanding through mathematical negotiation, such as making explicit requests for alternative opinions or questioning students about key concepts.

Keywords: mathematical negotiation, fraction, third grade of elementary school, number line

Introduction

Fractions have long been recognized as essential for the development of mathematics itself (Descartes, 1954), and learning fractions significantly refines learners' mathematical understanding (Lamon, 2020). However, in Japan, third-grade students often exhibit an insufficient understanding of fractions (Hasegawa, 2003). Furthermore, the study of fractions at the third-grade level serves as a critical foundation for subsequent mathematical learning and is considered a particularly significant component of fraction education (Nunokawa, 2019). With the aim of improving fraction learning among third-grade students, previous research has examined the interactions observed in classroom settings, focusing on student–student and student–teacher interactions that may influence the recognition of fractions. In this context, mathematical negotiation has been defined as the mutual exchange of intentions aimed at constructing mathematical knowledge, and learners' cognitive states have been analyzed based on an analytical framework grounded in Searle's speech act theory (Shimomura, 2019). However, prior studies have not yet fully explored how specific types of mathematical negotiations lead to transformations in students' understanding.

Therefore, this study aims to identify instances wherein students' understanding has changed and analyze the mathematical negotiations observed in those situations.

Theoretical Background

Based on a questionnaire survey, Lamon (2020) reported that many students who had taken only minimal mathematics courses in high school or feared mathematics began their poor relationship with mathematics immediately after learning fractions in elementary school. A considerable amount of research has been conducted on fractions. Studies on elementary schools have reported that student interactions, as well as interactions between teachers and students, effectively form fractional concepts (Fujimaki, 2011). However, the characteristics of the interactions that contribute to concept formation have not been discussed in these studies, and a deeper exploration of this aspect is necessary. Some studies have confirmed that learning using number lines contributes to concept formation among children who face difficulties in understanding fractions, and that the effects persist for seven weeks (Barbieri, 2020). Building on these findings, this study aimed to capture the actual process of concept formation in children, which is typically difficult to grasp, from the perspective of Searle's speech act theory, influenced by Wittgenstein[1)].

Research Question

This study analyzed mathematical negotiations observed in third-grade elementary school classrooms. The research questions were as follows:

[R.Q.] What are the characteristics of mathematical negotiations that promote the construction of fraction concepts in learning situations utilizing number lines?

Methodology

Survey design

This study conducted an experimental lesson on fractions for third-grade elementary school students, and mathematical negotiations were extracted from the data obtained in the lesson. The participants were 25 third-grade students of a public elementary school in Japan. The experimental lesson was designed to enable students to understand how to express fractions greater than one based on unit fractions. The experimental class was conducted using the worksheet developed in this study, as illustrated in Figure 1. The data collected in this study included worksheets written by the children, VTR recordings, and still image recordings.

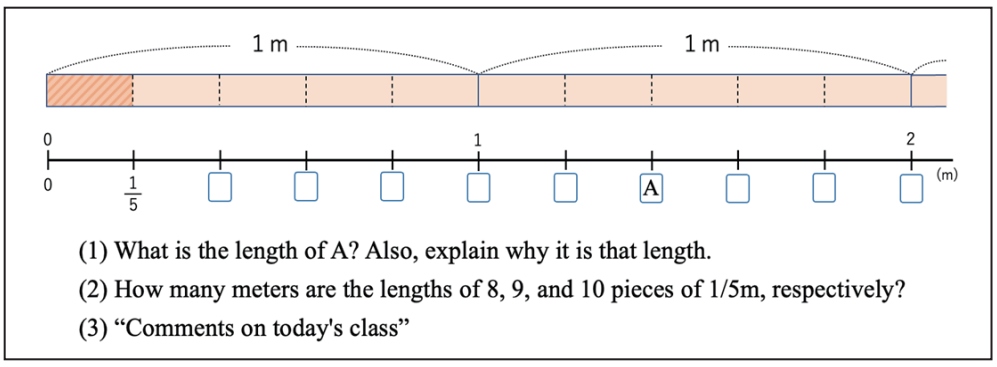


Figure 1: Worksheet used in class

1) Regarding the meaning of words in general, Wittgenstein argued that the use of language within a community is more important than the speaker's intention, while Searle (1969) asserted that social rules and the speaker's intention (intentionality) are crucial. These views reflect a significant difference in the interpretation of intention.
Wittgenstein, L. (2009). Philosophical investigations. Hoboken, NJ: John Wiley & Sons.

Analytical Framework

Searle calls the act of doing something else and generating a certain force by speaking an "illocutionary act" (Searle, 1969). He defined illocutionary acts by establishing four rules that constitute intraverbal acts: the propositional content rule, the prior rule, the sincerity rule, and the essence rule. Furthermore, he elaborately systematized the illocutionary act by classifying it into five types, and defining goals within these utterances (Searle, 1979). In this study, we developed a descriptive framework to capture the intention of utterances in mathematical negotiation by referring to Searle's rules, typologies, and goals of utterances (Shimomura et al., 2022). The study also considered the intention of utterances in mathematical negotiation by using the descriptive framework shown in Table 1.

Table 1: Descriptive framework for capturing the intent of speech

	Request / [command type]	*Assert* / [assertive type]	*Consent* / [action bound type]
・Propositional content	Future act A to H.	Any proposition p.	Past act A to H.
•Preparatory	1.H is able to do A. S believes H is able to do A. 2.It is not obvious to both S and H that H will do A in the normal course of events of his own accord.	1. S has evidence (reasons, etc.) for the truth of 2. It is not obvious to both S and H that H knows (does not need to be reminded of, etc.) p.	1.S has reason to believe that H is right about A. 2.It is not obvious to S whether the listener knows that the speaker also agrees.
・Sincerity	S wants H to do A.	S believes p.	S is convinced by past action A and has a
・Essential	Counts as an attempt to get H to do A.	Counts as an undertaking to the effect that p represents an actual state of affairs.	Counts as an attempt to show support for H
	Question / [command type]	*Advise* / [command type]	*Grant* / [command type]
・Propositional content	Any proposition or propositional function.	Future act A of H	Future act A of H
•Preparatory	1.S does not know 'the answer', i.e., does not know if the proposition is true, or, in the case of the propositional function, does not know the information needed to complete the proposition truly(but see comment below) 2.It is not ovbious to both S and H that H will provide the information at the time without being asked.	1. S has some reason to believe A will benefit H. 2. It is not obvious to both S and H that H will do A in the normal course of events.	1.S believes that H has the ability to do A. 2.It is not obvious that H does A in the normal course of events.
・Sincerity	S wants this information.	S believes A will benefit H.	S allows H to do A.
・Essential	Counts as an attempt to elicit this information from H.	Counts as an undertaking to the effect that A is in H's best interest.	Counts as an attempt by H to forgive A.

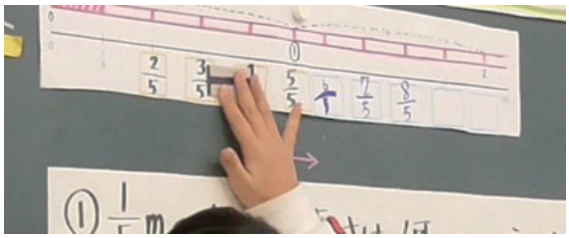
S : Speaker, H : Hearer, A : Action, p : proposition

Result

The following are the mathematical negotiations observed in the class. The intent of each utterance was identified by referring to Table 1. The agreement rate among the three raters was 88.2%. The symbols used in the transcript are "T" for teacher, "S (initials)" for a specific child, and "Ss" for multiple children.

1	T:	Ok? Then, YU, please answer what is 10 parts of 1/5m. **[Request]**
2	S(YU):	Let's see, 10/5m. **[Assert]**
3	S(GK):	I have a different opinion. **[Assert]**
4	T:	GK, you may speak. **[Grant]**
5	S(GK):	1/6m. **[Assert]**
6	S:	So yes, 1/6m. **[Consent]**
7	T:	GK, can you explain why you thought it was 1/6m? **[Request]**
8	S(GK):	Yes. If the numerator is 10, the denominator is carried forward by one, which I think is 1/6m. **[Assert]**

9	T:	Well, let's see if 1/5m for 9 really comes to 9/5m... Iwa please. **[Request]**
10	S(IW):	[while moving the scale tool] 1, 2, 3, 4, 5, 6, 7, 8, 9. **[Assert]**



11	T:	Is this one scale 1/5m correct? **[Question]**
12	Ss:	Yes. **[Consent]**
13	T:	Then, what if it is for 10 pieces, Ma please. **[Request]**
14	S(MA):	[while moving the scale tool] 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. **[Assert]**
15	T:	What was this one tick? **[Question]**
16	S:	1/5m. **[Assert]**
17	T:	Yes. WA, if there were ten of these, how would you describe them? **[Request]**
18	S(WA):	If you have 10 of these, you get 10/5m. **[Assert]**
19	Ss:	Good. **[Consent]**

The following are GK's comments after the class

GK's "Comments on today's class": *I thought at first that 1/5m for 10 was 1/6. I thought that the reason was that 1/5 was 10 times the amount of 1/6, but when I heard the reason for 10/5m, I thought it was 10/5.*

At the worksheet-writing stage, before this discussion took place, 14 of the 25 children (56%) responded with 1/6m. In addition, the teacher succeeded in eliciting further statements and agreement from the children through phrases intended as [requests] and [questions] (7,9,11,13,15,17). Furthermore, the post-lesson reflections showed that GK changed his perception through mathematical negotiation. These findings suggest that the teacher's purposeful questions and supporting tools played a key role in helping students see 1/5 as the base quantity.

Discussion and Conclusion

The results of the study indicate that, at the beginning of the class, Go, who initially held an incorrect understanding, revised his perception while listening to explanations from others (from GK's comments after the class). Additionally, the results confirm that, in discussions solely among third-grade children, there is a possibility of agreeing on an incorrect understanding of fractions. Therefore, it is crucial for teachers to ask questions with the intention of guiding students toward a correct understanding through requests or inquiries. These findings demonstrate that, when developing the concept of fractions using a number line, it is essential for the children to engage with one another and engage in mathematical negotiation with an entity possessing a higher level of discourse (in this case, the teacher).

References

Barbieri, C. A., Rodrigues, J., Dyson, N., & Jordan, N. C. (2020). Improving fraction understanding in sixth graders with mathematics difficulties: Effects of a number line approach combined with cognitive learning strategies. Journal of Educational Psychology, 112(3), 628.

Descartes, R. (1954). *The geometry of René Descartes* (D. E. Smith & M. L. Latham, Trans.). New York, NY: Dover Publications.

Fujimaki, Y. (2011). A foundational study on the formation of children's fractional knowledge through interaction. Joetsu Journal of Mathematics Education, 26, 45-58. (in Japanese)

Hasegawa, J. (2003). An investigation of fourth grade students' understanding of basic fraction concept. *Journal of Japan Society of Mathematical Education*, 85(10), 2-10. (in Japanese)

Lamon, S. J. (2020). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. London: Routledge.

Nunokawa, K. (2019). Descriptions of Patterns and Objectifications of Patterns in Learning Fractions in Textbooks. *Journal of Japan Society of Mathematical Education*, 101(12), 2-15. (in Japanese)

Searle, J. R. (1969). *Speech acts: An essay in the philosophy of language* (Vol. 626). Cambridge, UK: Cambridge university press.

Searle, J. R. (1979). *Expression and meaning: Studies in the theory of speech acts*. Cambridge, UK: Cambridge University Press.

Shimomura, T., Okabe, Y., Shimomura, S., & Saito, H. (2022). Characteristics of mathematical negotiation in the process of building the fractional concept: Based on the analysis of small group learning in quantity fraction. Journal of science education in Japan, 46(4), 283-298. (in Japanese)

Shimomura, T. (2019). Characteristics of negotiation in the composition of mathematical knowledge, *Proceedings of the 43th PME conference*, 4, 97.

Acknowledgements

This work was supported by JSPS KAKENHI (grant number: JP23K02341). I would also like to thank the teachers and students at public elementary schools in Shimane Prefecture for their cooperation in this study.

TSG5-05

"DIVIDING THE DENOMINATORS AND NUMERATORS SEPARATELY": SIXTH GRADERS' STRATEGIES OF THE MEASUREMENT FRACTION DIVISION

Jeongwon Kim
Bongmyeong Elementary School, South Korea, nymph019@hanmail.net

Fraction division is a critical yet often misunderstood domain in mathematics, even among upper elementary and secondary school students, despite its importance for their success in mathematics learning. Introducing the measurement interpretation of fraction division can provide an alternative approach to fostering students' conceptual understanding. This study explored how sixth-grade students reason the measurement model of fraction division problems and how they justify their strategies. The findings indicated that students developed strategies for fraction division within the measurement framework based on their prior mathematical knowledge. Additionally, students inductively discovered a strategy of dividing the denominators and numerators separately and provided further justifications for their reasoning. These findings offer pedagogical implications for instructional practices of fraction division to enhance students' conceptual understanding.

Keywords: fraction division, measurement interpretation, students' strategies, conceptual understanding

Introduction

Fraction division is a critical mathematical domain that integrates various concepts (Adu-Gyamfi et al., 2019). Despite its importance in school mathematics, research has consistently shown that students struggle to develop a conceptual understanding and tend to approach it procedurally (Lamon, 2007). As a result, students frequently rely on rote memorization of the invert-and-multiply algorithm without comprehending its underlying structure. To foster conceptual understanding, introducing the measurement model of division as a foundational approach to fraction division can help students build on their prior knowledge of whole number division (Zembat, 2015). For example, $\frac{8}{9}\div\frac{2}{9}$ can be interpreted as determining how many groups of $\frac{2}{9}$ are contained within $\frac{8}{9}$, analogous to the measurement interpretation of whole number division in 8÷2.

Measurement-based fraction division problems can be solved using the common denominator algorithm, which is algebraically expressed as $\frac{a}{b}\div\frac{c}{d}=\frac{ad}{bd}\div\frac{bc}{bd}=ad\div bc$. While prior research has examined students' performance in fraction division (e.g., Warrington, 1997), fewer studies have focused on students' understanding of the measurement conceptualization of faction division. Given this background, the present study aims to investigate sixth grade students' reasoning within the measurement model of fraction division. Specifically, this study addresses the following research questions: What strategies do students employ when reasoning through the measurement model of fraction division? And how do students justify their strategies?

Methods

Participants

This study involved students from a Year 6 class at a public elementary school located in a medium socio-economic metropolitan area in Korea. The participants consisted of 25 students aged 11-12 years (15 boys, 10 girls), who showed limited interest in mathematics, including fraction learning. Classroom observation revealed that most students were able to solve mathematics problems by applying algorithms or recalling memorization, however, they faced challenges in justifying such algorithms and mathematical principles. The student's parents provided consent for their participation in the study.

Procedures and Data Analysis

The fraction division unit was designed to enhance students' conceptual understanding of fraction division. The lessons incorporated various fraction division contexts, including measurement division, partition division, determination of a unit rate, and division as the inverse of a Cartesian product. Among them, this study specifically focuses on the measurement interpretation of fraction division. Figure 1 provides an overview of the lesson topics and key activities.

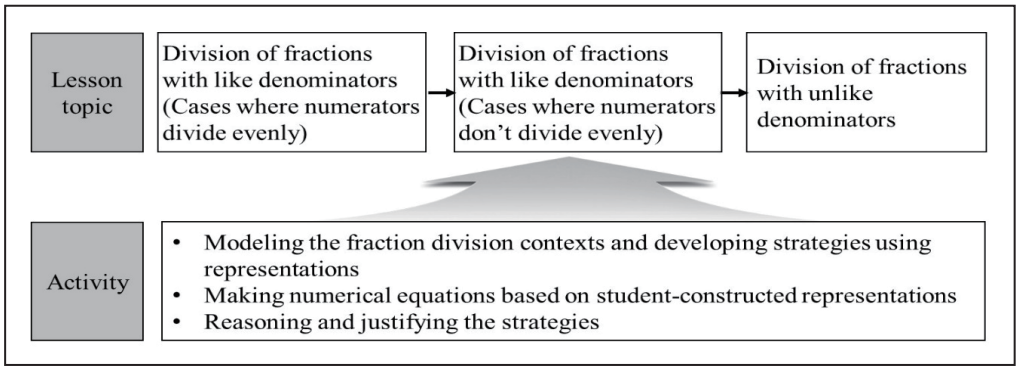


Figure 1. Lesson subjects and activities for the measurement model of fraction division

Four types of data were collected from the study: lesson recordings and transcripts, students' worksheets, interviews, and the classroom teacher's lesson planning and reflection notes. The data analysis focused on students' reasoning in the context of the measurement model of fraction division. Specifically, we examined the strategies students employed and how they justified these strategies. Accordingly, students' problem-solving strategies were categorized, and both the strategies and their corresponding justifications were systematically analyzed. Throughout the analysis, we aimed to identify the potentials and challenges that students encountered, thereby providing pedagogical implications for the teaching and learning of fraction division.

Results

Students' Exploration of Fraction Division Strategies

Students' strategies for the measurement model of fraction division were categorized into three types. First, for division problems involving fractions with like denominators, students employed the strategy of dividing the numerators, similar to how they approach whole number division. For example, when solving $\frac{5}{9} \div \frac{2}{9}$, they determined how many $\frac{2}{9}$s is in $\frac{5}{9}$, leading to the solution: $\frac{5}{9} \div \frac{2}{9} = 5 \div 2 = \frac{5}{2} = 2\frac{1}{2}$. For fractions with unlike denominators, students applied a strategy of first converting the denominators to a common denominator before dividing the numerators, a method known as the common denominator algorithm. Figures 2 illustrates a student's representation of this approach, demonstrating the use of fraction bars and numerical equations to model the process of creating a common denominator.

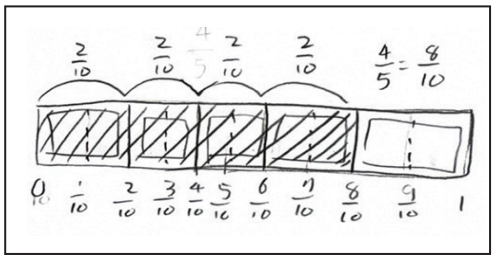


Figure 2. Problem-solving using common denominator algorithm

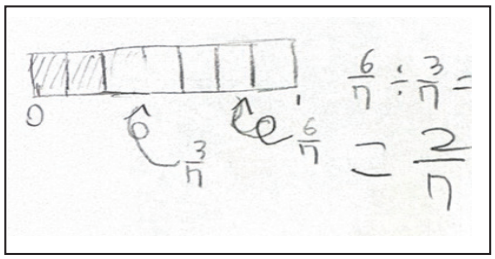


Figure 3. Incorrect solving-strategy

Second, some students exhibited errors in applying the common denominator algorithm. Although this method involves dividing the numerators after establishing common denominators, the denominators were incorrectly retained in the quotient instead of being disregarded. It appeared that students treated faction division similarly to fraction addition, keeping the common denominator unchanged and performing division only on the numerator, as in the example: $\frac{6}{7} \div \frac{3}{7} = \frac{6 \div 3}{7} = \frac{2}{7}$ (see Figure 3).

Lastly, students proposed a strategy of dividing the denominators and numerators separately. For instance, the problem $\frac{5}{9} \div \frac{2}{9}$ was solved as follows: $\frac{5}{9} \div \frac{2}{9} = \frac{5 \div 2}{9 \div 9} = \frac{5 \div 2}{1} = 5 \div 2 = \frac{5}{2} = 2\frac{1}{2}$. Students explained that fraction division could also be calculated by dividing the denominators and numerators separately, drawing an analogy to the fraction multiplication algorithm, where the numerators and denominators are multiplied separately.

Students' Justification of Fraction Division Strategies

After discussing fraction division strategies, the teacher asked students to explain why the strategies work and whether they apply to all fractions. A whole-class discussion followed regarding the third strategy above, which involves dividing the denominators and numerators separately.

T: S1 suggested that fraction division could be performed by dividing the denominators and numerators separately. What do you think about this strategy?

S2: I think it is correct. Since dividing the denominators results in 1, the calculation is ultimately the same as dividing only the numerators.

S3: I don't think so. While the answer may be correct, dividing the denominators seems unnatural.

T: What do you mean by unnatural?

S3: For example, when dividing a pizza, the denominator represents the number of slices the pizza is divided into. I don't see how it makes sense to divide the denominators.

S4: I think this method only works when the denominators are the same. If the denominators are different, the result would not be 1, so they cannot simply be divided.

To further scrutinize students' thinking, the teacher asked whether the equation $\frac{8}{3} \div \frac{4}{9} = \frac{8 \div 4}{3 \div 9}$ is valid and prompted students to explain their reasoning. Some students explained that this is possible because dividing the denominators after converting them to a common denominator always results in 1: $\frac{8}{3} \div \frac{4}{9} = \frac{24}{9} \div \frac{4}{9} = 24 \div 4 = 6$. Others justified their reasoning based on the interpretation of faction as quotients: "I agree that $\frac{8}{3} \div \frac{4}{9} = \frac{8 \div 4}{3 \div 9}$ Since $\frac{8 \div 4}{3 \div 9}$ is equal to $(8 \div 4) \div (3 \div 9)$, it becomes $2 \div \frac{1}{3} = 2 \times 3 = 6$." Students who doubted the validity of this strategy explained that the division of fractions with different denominators should be solved using the invert-and-multiply algorithm. They reasoned that since the denominators do not always divide evenly, it results in factions with factional denominators like $\frac{2}{\frac{1}{3}}$, making the strategy invalid.

Discussion

The results of the study indicate that students were able to reason division fraction strategies for the measurement model, supporting previous research that suggests the measurement model of fraction division provides conceptual affordances for students' understanding (Zembat, 2015). The study further revealed that students could justify their fraction division strategies based on related mathematical concepts and principles (e.g., whole number division, fraction as a quotient). Given students' difficulties and their tendency to rely on rote memorization without understanding in learning fraction division (e.g., Li & Kulm, 2008), the measurement model of fraction division can serve as a crucial resource in helping students deepen their comprehension of the meaning and principles behind fraction division.

Notably, the students inductively discovered a strategy of dividing the denominators and numerators separately, then further generalized it and applied it to solving fraction division problems. Park (2014) explains that, regarding the fraction division strategy of dividing the numerators, the common denominator algorithm justifies it in a transitive manner, whereas dividing the denominators and numerators separately is able to justify it directly. Although this strategy does not appear in the Korean elementary mathematics curriculum or textbooks and it is challenging to model using the measurement conceptualization of fraction division, students' ability to reason and justify its validity suggests the potential for utilizing the strategy for elementary school students. We acknowledge that the results are constrained by the small sample size of sixth-grade students in Korea and the narrow scope of fraction division models. Nevertheless, we hope the findings of this study will provide valuable instructional insights for teaching and learning fraction division with conceptual understanding.

References

Adu-Gyamfi, K., Schwartz, C. S., Sinicrope, R., & Bossé, M. J. (2019). Making sense of fraction division: domain and representation knowledge of preservice elementary teachers on a fraction division task. *Mathematics Education Research Journal, 31*, 507-528.

Lamon, S. (2007). Rational numbers and proportional reasoning: Towards a theoretical framework for research. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629-666). Information Age Publishing.

Li, Y., & Kulm, G. (2008). Knowledge and confidence of prospective mathematics teachers: The case of fraction division. *ZDM–The International Journal on Mathematics Education, 40*, 833–843.

Park, K. S. (2014). An analysis on processes of justifying the standard fraction division algorithms in Korean elementary mathematics textbooks. *Journal of Elementary Mathematics Education in Korea, 18*(1), 105-122.

Warrington, M. A. (1997). How children think about division of fractions. *Mathematics Teaching in the Middle School, 2*, 391–394.

Zembat, İ. Ö. (2015). An alternative route to teaching fraction division: Abstraction of common denominator algorithm. *International Electronic Journal of Elementary Education, 7*(3), 399-422.

Acknowledgment

The researcher wishes to thank the teacher, students, and parents for their participation in the study.

TSG5-06

HOW JAPANESE CHILDREN FOCUSING ON SPONTANEOUS PATTERNS CREATE PATTERNS?

Atsuya Fukuzawa
Chugoku Junior College, Japan, fukuzawa.hoiku@g.cjc.ac.jp

Nanae Matsuo
Chiba University, Japan, matsuo@faculty.chiba-u.jp

The purpose of this study was to clarify how young Japanese children, who had never received pattern education and spontaneously focused on patterns (SFOP), created patterns during their participation in a shape-making and pattern program. To this end, a pre- and post- survey on SFOP was conducted with 5- to 6-year-old children at a kindergarten in Chiba Prefecture, Japan, which did not adopt special teaching methods. Analysis was conducted on pattern making, focusing on Child 5 who showed the most pronounced tendency toward SFOP in the series of surveys. The results showed that Child 5 imitated and extended the pattern presented by the teacher during the pattern program and then independently created a new problem to continue the pattern. Child 5 not only extended the repetition of the pattern but also constructed a new unit composed of two units, symmetrically swapped their order, and created a continuous repeating pattern. This demonstrates an advanced structural category of AMPS, as the child generated a complex repeating pattern with a double structure.

Keywords: pattern, SFOP, AMPS, pattern program, early childhood education(ECE)

Introduction

In recent years, many countries have incorporated pattern recognition, the most fundamental ability in mathematics learning, into the content of early childhood education (ECE). The OECD (2018) has positioned pattern recognition as a fundamental ability in cognitive skills; its importance is not limited to mathematics education, but should be considered in the context of ECE. However, in Japan, ECE does not include instructional content related to patterns, as indicated by the current kindergarten education guidelines (Ministry of Education, Culture, Sports, Science and Technology [MEXT], 2017). Moreover, teacher training programs for ECE do not deal with patterns (Matsuo, 2017), research on patterns in Japanese ECE is minimal, and limited information is available regarding the actual situation of children's pattern recognition in Japan.

Pattern Recognition in Young Children

Mulligan et al. (2020) define mathematical patterns as predictable regularities, usually involving numerical, spatial, and logical relationships. The authors state that mathematical structures are most often expressed in the form of generalizations of numerical, spatial, and logical relationships that are always true in some domain. Young children often use strategies related to "repetition," "shape," "color," and "position" to identify patterns in their activities (Papic & Mulligan, 2005). It has also been shown that young children handle three types of patterns: repeating (e.g.,

○○△○○△○○…), spatial structure (e.g., ↓→↑→↓→↑→...), and growth patterns (e.g., 2, 4, 6, 8...) (e.g., Rittle-Johnson et al., 2013). Among these, repeating patterns are the easiest for young children to understand (Wijns, 2020). Older children are able to identify the smallest units (hereafter referred to as units) that form a repeating pattern (Blanton & Kaput, 2004; Mulligan et al. 2013). In the case of complex patterns, children are more likely to recognize patterns when they are represented through pictures or diagrams rather than through language or symbols (Michael et al., 2006).

Recognizing patterns requires attention to the structure of the object, which Mulligan and Mitchelmore (2013) define as Awareness of Mathematical Pattern and Structure (AMPS). Specifically, AMPS refers to the ability to recognize and use patterns and is generalizable across various mathematical concepts addressed in early childhood. In this series of studies, the responses of young children to tasks in early childhood were categorized into five categories with respect to pattern and structure. They are pre-structural, emergent, partial structural, structural, and advanced structural (Mulligan et al., 2015). Furthermore, Wijns et al. (2020) point out that AMPS explicitly presents pattern-related challenges to young children, thus guiding them into pattern-specific, non-routine contexts. Wijns et al. note that AMPS is an important concept for capturing young children's pattern recognition. However, they argue that AMPS cannot measure young children's tendency to attend to spontaneous patterns, given that preschool education in many European countries is delivered through natural learning environments and contexts embedded within society. Therefore, Wijns et al. (2020) proposed Spontaneous Focusing on Patterns (SFOP) as a concept that captures young children's tendency to spontaneously look for, notice, and create patterns, and positioned it as part of AMPS. Nowadays, Wijns et al. (2020) and others have repeatedly pointed out that spontaneous pattern-making in early childhood is critical for promoting later math learning (e.g., Rittle-Johnson et al, 2018).

The Purpose of This Research

As mentioned earlier, in Japan, patterns are not clearly positioned as part of the content of preschool instruction, including the kindergarten education guidelines. One reason is that Japan's ECE emphasizes "comprehensive instruction through play" and aims to provide instruction from a subject-integrated perspective, avoiding delivering specific knowledge related to subjects, especially mathematics. Therefore, in Japan, it is necessary to capture young children's pattern recognition in their independent activities, which is highly compatible with the SFOP proposed by Wijns et al. (2020). Therefore, the purpose of this study was to clarify how young Japanese children, who had never received instruction on patterns and focused on spontaneous patterns, created patterns during their participation in the pattern program.

Methods

Research Participants

The subjects of this study were six young children (5 years and 11 months to 6 years and 8 months) attending a kindergarten in Chiba prefecture, Japan. The target kindergartens were general Japanese ECE facilities that did not adopt special educational methods.

Procedure

The study involved a shaping and pattern program and a pre- and post-survey of the SFOP. The study period was from November 2024 to February 2025. Six children participated in the program for a total of approximately 200 minutes. The program consisted of one shaping session (approximately 80 minutes) and five repeating pattern sessions (approximately 170 minutes). In the shaping session, a picture book about castle redecoration was read to the children, who then created their own castles based on colored boards of triangles, rectangles, circles, and semicircles. In the pattern session, using a unit consisting of two or three boards related to circles, triangles, and squares, the children imitated the teacher's example, continued on their own, and individually made problems to guess the shape of the subsequent board, solved the problems they made, and checked their answers together. Before the program, immediately after the program, and one month later, Figure 1 was presented and the participants were asked, "Which of these do you like best? This question was conceived as an easy way to capture children's SFOP in a context not related to mathematics and patterns.

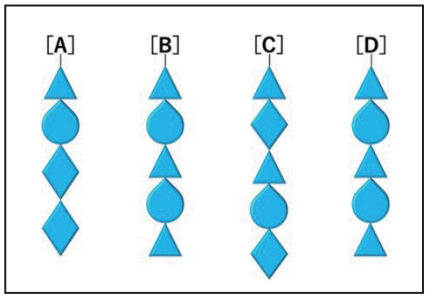


Figure 1. Decorations with a series of figures

Data Collection and Analysis

Based on responses to questions about pattern preferences, as well as videos of activities during the program and recordings made by the teacher, the characteristics of the activities of Child 5, the girl who exhibited the clearest SFOP tendency in the series of surveys, were analyzed.

Results and Discussion

In the pre-survey, Child 5 was presented with Figure 1 and chose C. She focused on the number of shapes that make up the ornament and the specific shape, saying, "C has two triangles and two rhombuses, but there is only one drop shape, so I am wondering if I should choose C." She also mentioned that ornaments B and D were the same and then selected the C ornament. Therefore, Child 5 was able to grasp the structure of the four ornaments based on the types of shapes and explain why she preferred the C ornament. In the post-survey, she again noted that the B and D ornaments were the same, and she preferred C after expressing that the C ornament had the same two triangles and two diamonds, but only one drop, which lacked the second identical shape. Thus, Child 5 did not prefer ornaments B and D but chose C because she focused on the type and number of shapes, demonstrating a tendency toward SFOP.

In the pattern program, Child 5 was able to imitate the two units presented by the teacher without difficulty in the activity, as well as more units. As the teacher had put the boards in order, it is possible that at this point Child 5 simply recognized the shapes and placed the boards in the same order

as the teacher. However, even when the teacher stopped at the third unit and considered the continuation, Child 5 immediately noticed the regularity and was seen to arrange the continuation. This demonstrates that Child 5 noticed the units from the sequence of boards presented by the teacher and saw them as a repeating pattern. Noticing the structure of the object (in this case, the unit) and making a pattern is precisely what allows us to see the pattern and structure in the sequence of the boards and determine that the AMPS category of Child 5's response is structural. In addition, in the problem of continuing to imitate and arrange the ○△□○△□ presented by the teacher, Child 5 spontaneously thought of an answer the opposite of □△○□△○. This suggests that Child 5 was not only imitating the board sequence presented by the teacher but was also able to recognize the pattern accurately. Furthermore, Child 5 was increasing the number of board shapes that she could arrange independently and was working on creating more complex patterns. At this point, we can conclude that her abilities align with the advanced structural AMPS category.

In particular, Child 5, in a situation where she created her own question about the continuation of the pattern, asked, "What is the next step after ○△□○△□?" Child 5 considered "○△□○△□" as a single unit and arranged it in reverse order, creating a "□△○□△○" arrangement intended to be symmetrical. The reason for this was that she believed that a different answer from what other children might give would be more interesting. She also mentioned that "□△○□△○" followed by "○△□○△□" could be considered two units forming one unit and assumed a repetitive pattern in which the two units repeatedly alternated. This means that she was adding symmetry to the repeating pattern and creating a further double-structured repeating pattern, which is the advanced structural AMPS category as well. This suggests that her performance may be prompted not only when working on pattern-related tasks but also by the context and purpose of the activity.

Conclusion

Considering challenges in teaching patterns to young children, Lüken (2023) points out the difficulty of explaining the concept of pattern and the fact that the explicit use of the word pattern during a young child's activity can change the purpose of the activity. Therefore, it is important to assess the tendency toward SFOP and consider methods of teaching that encourage it. In this study, children with a tendency toward SFOP showed highly structured aspects of AMPS in a program on patterns. This suggests that even in environments not explicitly focused on teaching patterns, such as in Japan, young children with SFOP tendencies may demonstrate pattern recognition when repeatedly exposed to patterns. The next step is to expand the number of children in the survey and examine the differences in the effects of using ICT.

References

Blanton, M. L., & Kaput, J. J. (2004). Elementary grades students' capacity for function thinking. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education,* 135-142.

Lüken, M. M. (2023). Young children's self-initiated pattern-making during free play. *Mathematical Thinking and Learning*, 1-21.

Matsuo, N. (2017). Issues concerning the training and development of teachers for preschool mathematics education. T. Ikeda (Ed.), *Proceedings of the 5th Spring Conference of the Japanese Society for Mathematics Education*, 27-28. (In Japanese).

Michael, S., Elia, Iliada., Gagatsis, A., Theoklitou, A., & Savva, A. (2006). Levels of understanding of patterns in multiple representations. *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education, 4*, 161-168.

Ministry of Education, Culture, Sports, Science and Technology (MEXT) (2017). The kindergarten national curriculum standard. MEXT. Chrome extension://efaidnbmnnnibpcajpcglclefindmkaj/https://www.mext.go.jp/content/1384661_3_2.pdf

Mulligan, J. T., Mitchelmore, M. C., English, L. D., & Crevensten, N. (2013). Reconceptualizing early mathematics learning: The fundamental role of pattern and structure. In L. D. English & J. T. Mulligan (Eds.), *Reconceptualizing early mathematics learning*, (pp. 47-66). Springer.

Mulligan, J. T., Mitchelmore, M.C., & Stephanou, A. (2015). Pattern and structure assessment (PASA): An assessment program for early mathematics (F-2). *Camberwell, Victoria: Australian Centre for Educational Research (ACER),* (pp. 15-16).

Mulligan, J., Oslington, G., & English, L. (2020). Supporting early mathematical development through a 'pattern and structure' intervention program, *ZDM, 52,* 663-676.

Papic, M. & Mulligan, J. (2005). Pre-schoolers' mathematical patterning. *Mathematics Education Research Group of Australasia, 1*, 609-616.

Rittle-Johnson, B., Fyfe, E. R., McLean, L. E., & McEldoon, K. L. (2013). Emerging understanding of patterning in 4-year-olds. *Journal of Cognition and Development, 14(3),* 376-396.

Rittle-Johnson, B., Zippert, E. L., & Boice, K. L. (2018). The roles of patterning and spatial skills in

early mathematics development. *Early Childhood Research Quarterly, 46*, 166-178.

Wijns, N. (2020). *On the hunt for regularities. An investigation of children's early patterning competencies* [Doctoral dissertation]. KU Leuven. LIRIAS.

Wijns, N., De Smedt, B., Verschaffel, L., & Torbeyns, J. (2020). Are preschoolers who spontaneously create patterns better in mathematics? *British Journal of Educational Psychology, 90(3),* 753-769.

OECD (2015). Skills for social progress: *The power of social and emotional skills.* OECD Publishing. *http://dx.doi.org/10.1787/9789264226159-en*

Acknowledgement

The authors are grateful to the children and teachers of the kindergarten who cooperated in this survey. This study was funded by JSPS Kaken-hi (Grant number 21K18487).

TSG5-07

A VISUALIZATION OF THE STRUCTURE OF KNOWLEDGE BASED ON THE CONNECTIONS OF MATHEMATICAL CONCEPTS

Ye-Jin Lee
Graduate School of Seoul National University of Education, South Korea, pkllj@naver.com

Hyewon Chang
Seoul National University of Education, South Korea, hwchang@snue.ac.kr

This study aims to visualize cognitive structures through the connections between mathematical concepts. Among these connections, transversal connections are defined as relationships where mathematical concepts transcend domain boundaries based on structural similarities, facilitating conceptual transfer. To this end, 88 mathematical concept nodes were identified from Korea's elementary mathematics curriculum, and 28 were found to be interconnected through transversal connections related to the idea of equal partitioning. Using NetMiner for network analysis, the structure of these conceptual connections was visualized, and centrality indicators revealed that fractions and division function as hub concepts, while ratio and rate exhibited high closeness and betweenness centrality. The knowledge structure presented in this study illustrates potential transfer pathways, enabling learners to organically connect and comprehend concepts centered on the meaning of equal partitioning.

Keywords: transversal connections, mathematical concepts, network analysis, knowledge structure, transfer of learning

Introduction

In mathematics education, meaningful learning extends beyond acquiring isolated concepts to recognizing and structuring relationships among concepts. Such learning is facilitated by forming and reorganizing learners' cognitive structures, enabling transfer to novel problem situations (Bruner, 1973; Rumelhart & Norman, 1976). In particular, the ability to connect concepts across different domains of mathematics fosters integrated understanding and serves as a critical factor in constructing a stable knowledge structure. From this perspective, transversal connections—which link concepts across domains such as number, operation, measurement, probability on structural similarities—offer a valuable framework for exploring the inherent connectivity within mathematical concept structures.

The significance of connections in mathematics is explicitly emphasized in international curricula, official educational documents, and models of teachers' knowledge structures. The National Council of Teachers of Mathematics (NCTM) highlighted connections as a core process standard, underscoring the importance of recognizing relationships among mathematical ideas and procedures and integrating them as a central goal of learning(NCTM, 2000). Similarly, the Mathematics Teachers' Specialized Knowledge (MTSK) model identifies knowledge of structure (KSM) as a key component, wherein transversal connections are explicitly acknowledged (Carrillo-Yañez et al., 2018; Montes et al., 2013).

Visualizing cognitive structures based on such connections is essential, as it reveals the orderly relationships among mathematical concepts and renders them analytically accessible (Lynch, 1988). While network analysis has the potential to examine and visualize cognitive structures, its current application remains confined mainly to analyzing research trends (Yığ, 2022). Therefore, this study seeks to visualize the knowledge structures that both teachers and learners should construct by focusing on transversal connections within elementary mathematical concepts, thereby laying a foundation for enhancing connection-oriented mathematics instruction.

The research question guiding this study is as follows:

How are mathematical concepts structured and visualized based on transversal connections?

Theoretical Background

Knowledge Structure from a Connection Perspective

A knowledge structure, or schema, refers to a network of interrelationships generally assumed to exist among conceptual components (Rumelhart & Ortony, 1977). Skemp (1987) defined schema as a network of interconnected concepts, while Marshall (1995) emphasized that the essence of schema lies in these connections. The denser the network of relationships among components within a structure, the more stable the knowledge structure becomes (Russell, 1999), enhancing the efficiency of memory retrieval (Siew, 2020). Such structured knowledge embodies general principles that facilitate transfer (Bruner, 1973), highlighting the necessity of considering both structural similarities and differences between concepts for successful transfer of learning (Thorndike, 1922; Osborne, 1976;). This study defines a knowledge structure as a network of connections between concepts. Although conceptual structures can be represented in various ways, the organization of concepts can generally be categorized into vertical and horizontal connections, which are exemplified by hierarchy and web (Hiebert a& Carpenter, 1992) . As illustrated in *Figure 1,* hierarchy and web differ in several aspects, including the unit and nature of nodes, the composition of sets, the characteristics of links, and the scope of those links (Novick & Hurley, 2001).

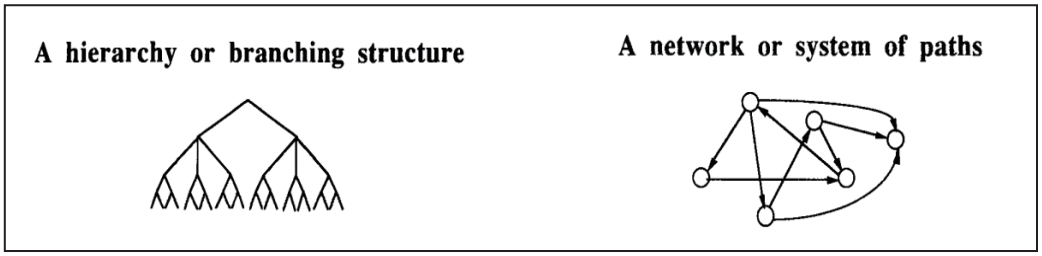


Figure 1. Types of knowledge structures (Novick, Hurley, & Francis, 1999, p. 290)

Transversal Connections and Mathematics Learning

The MTSK model, which describes mathematics teachers' specialized knowledge, identifies transversal connections as a key category within knowledge of structure (Carrillo-Yañez et al., 2018; Gamboa et al., 2022). For instance, concepts like limits, derivatives, integrals, and continuity are interconnected through the overarching idea of infinity (Carrillo-Yañez et al., 2018). Transversal connections are established when relationships are formed across multiple topics with common fea-

tures, grounded in the fundamental similarities between concepts. As a result, domain boundaries within the curriculum are disregarded

Methods

The overall research process followed five stages as shown in *Table 1*: selection of concept nodes, establishment of connection criteria, extraction of relevant nodes, analysis of relational structures, and network analysis with visualization.

Table 1. The overall research process

Selection of concept nodes	⇒	Establishment of connection criteria	⇒	Extraction of relevant nodes	⇒	Analysis of relational structures	⇒	Network analysis with visualization

Network analysis, which conceptualizes ideas as nodes and their relationships as links, is a tool to visualize and quantify knowledge structures. NetMiner, recognized globally for integrating visual and quantitative analyses, was employed to visualize the conceptual structure and analyze its structural characteristics (Huisman & Van Duijn, 2005; Zhang, 2010).

A total of 88 mathematical concept nodes were derived from the 2022 revised Korean elementary mathematics curriculum. Concepts were organized into three hierarchical levels based on two selection criteria, and validity was ensured through review by three mathematics education experts holding doctoral degrees.

Criteria for identifying transversal connections were established based on theoretical review, focusing on three aspects: heterogeneity (connections across domains), structural similarity, and extensibility through indirect links which are characteristics of a web. Structural similarities are represented as ‘big ideas’. Among several big ideas recognized in mathematics education (Baroody et al., 2004; Hurst & Hurrell, 2014; Siemon, 2006), equal partitioning was selected as the central idea. Concepts meeting these criteria were extracted, and content validity was verified by 20 experts using Lawshe’s (1975) Content Validity Ratio (CVR), yielding a value of 0.9.

The finalized conceptual relationships were visualized using NetMiner (v.4.5.1) in 2D and 3D spring maps. Centrality measures—including eigenvector centrality, closeness centrality, and betweenness centrality—were calculated to identify hub concepts and assess their relative importance.

Results

The transversal connection map is structured by direct and indirect connections. The concepts directly connected include fractions and division while ratio, rate, and mean are indirectly connected. Moreover, concepts specifically connected to fractions include angle and time, and nodes connected to the division of natural numbers include factors and even numbers. Additional hierarchical connections, including those between rate and pi, and rate and chance, were observed beyond those identified during concept extraction. The transversal connection network comprised 28 concept nodes and 30 links, with a network density of 0.079 and an average degree of 2.14. The network diameter was 5, indicating that most concepts were connected within approximately three steps.

Centrality analyses revealed that fractions, division, and rate held the highest centrality values, with angle and mean also exhibiting notable eigenvector and closeness centrality scores. The visualization of the knowledge structure is shown in *Figure 2*.

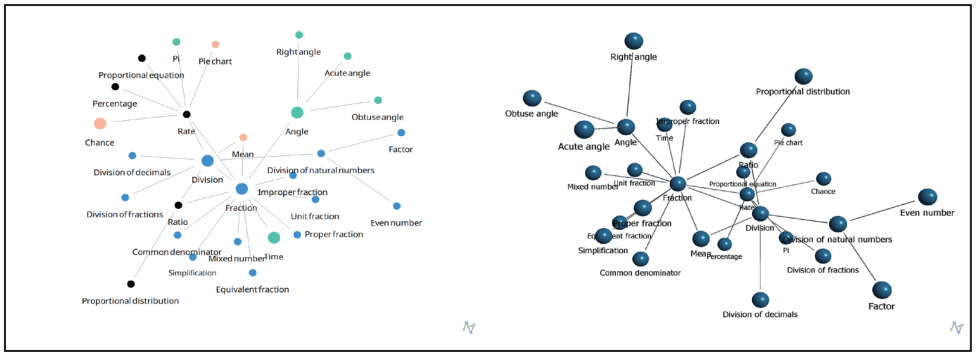


Figure 2. Structure of mathematical concepts based on transversal connection – equal partitioning

Conclusion

This study focused on visualizing the structure of mathematical concepts through transversal connections and deriving educational implications from this structure. Among the 88 concept nodes identified from the elementary mathematics curriculum, 28 were interconnected based on structural similarities, with fractions and division emerging as central hub concepts. In the knowledge structure centered on the idea of equal partitioning, fractions and division function as core hub concepts, providing a foundation for effectively implementing connectivity-based teaching and learning. These two concepts are directly linked to the partitioning structure through structural similarities and should be taught in an integrated manner alongside indirectly connected concepts such as ratio and rate. Therefore, the knowledge structure presented in this study serves to visualize positive transfer pathways that enable learners to organically connect, understand, and learn concepts centered on the meaning of equal partitioning.

Current curricula and textbooks often encourage fragmented learning by adhering strictly to domain boundaries. The transversal connections proposed in this study transcend such formal classifications, enabling learners to perceive mathematics as an integrated system rather than a collection of isolated concepts. Explicit incorporation of these connections into curricula and instructional practices may address the issue of fragmented learning in mathematics education.

Future research should explore how learners and teachers perceive and construct these conceptual connections in practice. Additionally, empirical studies on instructional designs centered on transversal connections could further validate their impact on learning outcomes and transferability.

References

Baroody, A. J., Cibulskis, M., Lai, M.-L., & Li, X. (2004). Comments on the use of learning trajectories in curriculum development and research. *Mathematical Thinking and Learning, 6*, 227-260.

Bruner, J. S. (1973). *The process of education* (H. Lee, Trans). Baeyoungsa. (Original work published 1960).

Carrillo-Yañez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P., Aguilar-González, Á., Ribeiro, M., & Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education, 20*(3), 236-253. https://doi.org/10.1080/14794802.2018.1479981

Gamboa, G., Caviedes

, S., & Badillo, E. (2022). Mathematical connections and the mathematics teacher's specialised knowledge. *Mathematics, 10*(21), 4010. https://doi.org/10.3390/math10214010

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*(pp. 65-97). New York: Macmillan.

Huisman, M. & Van Dujin, M. (2005). Software for social network analysis. In P. J. Carrington, J. Scott, & S. Wasserman (Eds.), *Models and Methods in Social Network Analysis*(pp. 270-316). Cambridge: Cambridge University Press.

Hurst, C. & Hurrell, D. (2014). Developing the big ideas of number. *International Journal of Educational Studies in Mathematics, 1*(2), 1-18. https://doi.org/10.17278/ijesim.2014.02.001

Lawshe, C. H. (1975). A quantitative approach to content validity. *Personnel Psychology, 28*(4), 563-575.

Lynch, M. (1988). The externalized retina: Selection and mathematization in the visual documentation of objects in the life sciences. *Human Studies, 11*(2), 201-234.

Marshall, S. P. (1995). *Schemas in problem solving: An integrated model of learning, memory, and instruction*. San Diego: Center for Research in Mathematics and Science Education, San Diego State University.

Montes, M. A., Aguilar, A., Carrillo, J., & Muñoz-Catalán, M. C. (2013). *MTSK: From common and horizon knowledge to knowledge of topics and structures*. In Proceedings of the CERME (Vol. 8). pp. 3185-3194. https://doi.org/10.13140/2.1.4338.0162

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Novick, L. R. & Hurley, S. M. (2001). To matrix, network or hierarchy: That is the question. *Cognitive Psychology, 42*, 158–216. https://doi.org/10.1006/cogp.2000.0746

Novick, L. R., Hurley, S. M., & Francis, M. (1999). Evidence for abstract, schematic knowledge of three spatial diagram representations. *Memory & Cognition, 27*(2), 288–308. https://doi.org/10.3758/bf03211413

Osborne, A. R. (1976). The mathematical and psychological foundations of measure. In R. A. Lesh & D. A. Bradbard (Eds.), *Number and measurement: Papers from a research workshop* (pp. 19–45). ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Rumelhart, D. E., & Norman, D. A. (1976). *Accretion, tuning and restructuring: Three modes of learning*. Center for Human Information Processing (Report No. 7602), San Diego: Department of Psychology, University of California. Retrieved from https://www.dsoergel.com/UBLIS571DS-06.1a-1Reading10RumelhartAccretionTuningAndRestructuring.pdf

Rumelhart, D. E., & Ortony, A. (1977). The Representation of Knowledge in Memory. In R.C. Anderson, R. J. Spiro, & W. E. Montague (Eds.), *Schooling and the acquisition of knowledge*(pp. 99-135). Hillsdale, NJ: Lawrence Erlbaum Associates.

Russell, S. (1999). Mathematical reasoning in the elementary grades. In L. V. Stiff and F. R. Curcio (Eds.), *Developing mathematical reasoning in grades K-12*(pp. 1-12). Reston, VA: NCTM.

Siemon, D. (2006). *Assessment for common misunderstandings*. Materials prepared for the Victorian Department of Education and Early Childhood Development. *Retrieved April 20, 2024, from* https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/misunderstandings.aspx

Siew, C. S. Q. (2020). Applications of network science to education research: Quantifying knowledge and the development of expertise through network analysis. *Education Sciences, 10*(4), 101. https://doi.org/10.3390/educsci10040101

Skemp, R. (1987). *The Psychology of Learning Mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Thorndike, E. L. (1922). *The psychology of arithmetic*. New York: Macmillan.

Yığ, K. G. (2022). Research trends in mathematics education: A quantitative content analysis of major journals 2017-2021. *Journal of Pedagogical Research, 6*(3), 137-153. https://doi.org/10.33902/JPR.202215529

Zhang, M. (2010). Social network analysis: History, concepts, and research. In B. Furht (Ed.), *Handbook of social network technologies and applications*(pp. 3-21). Springer.

TSG5-08

CRITICAL THINKING IN MATHEMATICS EDUCATION: A SYSTEMATIC REVIEW

Ersi Cha
East China Normal University, China, 51265500139@stu.ecnu.edu.cn

Shuhui Li
East China Normal University, China, shli@math.ecnu.edu.cn

This paper presents a systematic review of research on critical thinking in mathematics education (CT in ME) in the K-12 context. Thirteen studies, mainly journal articles and conference proceedings published since 2010, are identified and collected from four major databases. The results reveal that research on CT in K-12 ME is still limited despite an accelerated growth trend from 2020 to 2024. While early studies focused on fostering CT in ME, recent research has shifted towards mathematics-specific conceptualizations and measurements of CT. Even though the 13 studies cover diverse educational stages, high school students received more attention. Furthermore, the limited integration of digital tools for intervention and data analysis highlights a notable gap. Directions for future research are discussed in the end.

Keywords: critical thinking, mathematics education, systematic review

Introduction

Critical Thinking (CT), emphasized as one of the 21st-century competencies, has always been a hot topic among educational research and policies all over the world (e.g., MOE, 2023). A classic definition of CT is proposed by Ennis (1991), "reasonable and reflective thinking focused on deciding what to believe or do," which employs both dispositions and abilities. In mathematics education, Jablonka (2020) stated that explicit references to CT, as defined in educational psychology or philosophy, are not widespread; instead, general mathematical problem-solving and mathematical reasoning are more associated with CT. Another related concept, Critical Mathematics Education (CME), emphasizes "mathematics is formatting society" with critical education rather than focusing on critical thinking itself (Monteleone et al., 2023). As it does not align with the nature of CT in ME, studies falling within CME are excluded from this review.

Since the 2010s, there has been a growing trend of empirical research studies on CT in ME, however, a significant proportion of them focuses on higher education (Wang & Abdullah, 2024), with less attention given to K-12 students, which may produce valuable insights into the sustainable development of students' critical thinking and the preparation of mathematics teachers in the 21st-century. Therefore, our research question is: What is the recent development of research on critical thinking in K-12 mathematics education since 2010?

Methods

As shown in Figure 1, we conducted a comprehensive literature survey of studies published up to 2024 from four databases: Web of Science, SpringerLink, ScienceDirect, and Taylor & Francis. Using the keywords "critical thinking" AND "mathematics education", a total of 1,408 records under "article" or "dissertation thesis" were initially retrieved. Then, we filtered the 1,408 works per four criteria:

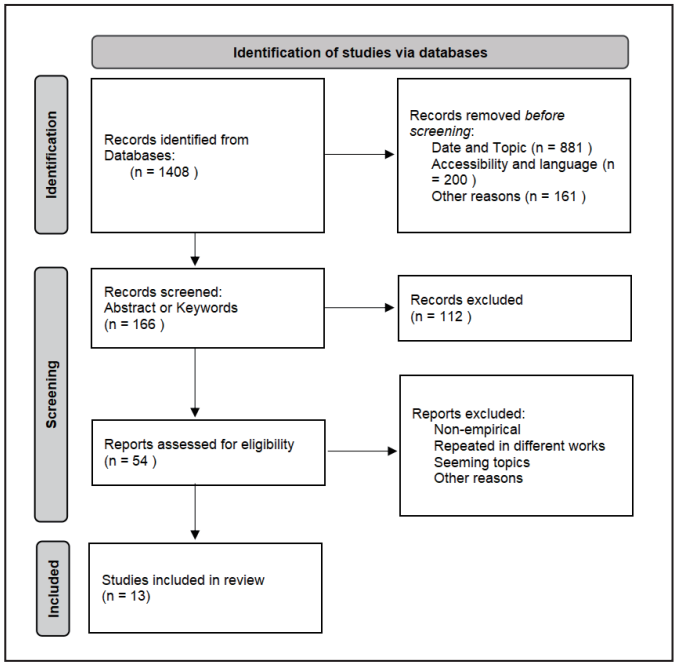


Figure 1. Review flowchart

(1) published since 2010, (2) full-text accessible, (3) written in English, and (4) not about higher education, resulting in 166 articles. Next, we screened 166 papers and excluded (1) non-empirical articles, (2) repeated works, (3) studies involving seeming topics (e.g., CME), and (4) articles that did not address K-12 students. In the end, we identified 13 studies, mostly (85%) journal papers. It suggests that current research on CT in ME is still limited. More than half of the studies (54%) were published from 2020 to 2024, indicating an accelerated growth trend.

Using a content analysis approach, considering the different roles of CT, we classified the 13 studies into four categories: (1) fostering CT in mathematics, (2) measuring and conceptualizing CT in mathematics, (3) developing CT as a by-product, and (4) others (see Table 1) and also coded their educational stage and sample size. Then, two independent coders coded the 13 articles. The percentage of agreement between the two researchers was 100%, which passed the reliability check.

Table 1. A coding framework

Category	Sub-category
Role of CT	Fostering CT in ME; Measuring and Conceptualizing CT in Mathematics; Developing CT as a by-product; Others
Educational Stage	Kindergarten; Primary; Middle School; High School
Sample Size	1-20; 21-80; Larger than 80; Not specified

Results

Overall, high school students (46%) were the most popular, probably due to their developed higher-order thinking. For sample size, studies utilizing no more than 20 participants (38%) collected and analyzed data from interviews; studies with a medium size (21-80) (31%) focused on interventions conducted by researchers; and studies with a large population (31%), either intervened or not, collected questionnaires and tests. Below is a detailed review in terms of different roles of CT.

Six studies (46%) belong to fostering CT in ME. Mathematical problem solving shares a great part with this category (4 in the 6 studies), which indicates a connection of CT, especially to the problem-solving cognitive process. For example, Dewolf et al. (2014) focused on developing CT through their interventions using illustrations that depicted the problem situation and/or warnings that alerted some nonstandard items. However, the results showed that neither the illustration nor the warning, or even the combination, had a positive impact. Later, Aizikovitsh-Udi and Cheng (2015) found the effectiveness of an infusion approach in fostering CT while teaching real-life related probability lessons. Dolapcioglu and Doğanay (2022) examined the development of CT skills of elementary students in mathematics classes, in which the authentic learning model was proved useful in developing five sub-skills of CT in mathematics understanding after six cycles.

Three studies (23%) fall into measuring and conceptualizing CT in mathematics. Researchers focused more on investigating the status quo of students' CT in mathematics rather than enhancing CT directly. Specifically, Bronkhorst et al. (2020) explored 16- and 17-year-old students' logical reasoning strategies in formal reasoning and everyday reasoning tasks, which is seen as a key aspect of critical thinking. Later, Sachdeva and Eggen (2021) examined learners' beliefs and self-reflection about their learning through questionnaires and interviews. However, the results showed that learners had limited or no experience in thinking critically about learning mathematics. Monteleone et al. (2023) established a critical mathematical thinking (CMT) framework for young students, where 5 themes of CMT (explaining, evaluating, creating, interpreting, and analyzing) with 14 sub-themes were extracted from literature and could be evidenced from young students.

Two studies (15%) are classified as developing CT as a by-product. They do not set developing CT as the main target but claim a significant improvement in CT through mathematics instruction. For example, Holmes and Hwang (2016) investigated the benefits of project-based learning on grade 8-9 students based on a two-year study, where CT served as a part of internal cognitive skills with positive results. Later, Luczak and Erwin (2023) used mathematical modelling in their case study, where CT was one of the abilities that they targeted to develop.

Two studies (15%) were sorted out as others. Aizikovitsh-Udi and Radakovic (2012) conducted an intervention using GeoGebra in the instruction of Bayes' Theorem, where high students may implement CT to support their understanding of probability. Zhou et al. (2024) designed and validated a Higher-Order Thinking Scale in mathematics, where CT was a sub-scale.

Concluding Remarks

In this survey, research studies on CT in ME in the K-12 context since 2010 were investigated. The main findings are listed as follows:

First, even with an accelerated growth trend in 2020-2024, research on critical thinking in K-12 mathematics education is still limited, compared to relatively more attention on CT through mathematics in higher education (Wang & Abdullah, 2024). It might be related to the traditional emphasis in K-12 mathematics curricula, which prioritizes mathematical knowledge (e.g., arithmetic, algebra, geometry) and high-stakes examination preparation over critical thinking, leaving less room for explicit CT integration. Furthermore, many national curriculum standards for mathematics just mention or acknowledge CT but lack clear implementation guidelines (e.g., MOE, 2022), and thus less attention is given to research on CT in K-12 mathematics education. As CT is emphasized as an essential competency for global citizens in the 21st century, further research in this direction is valuable to gain insights into measuring and fostering CT in K-12 mathematics education.

Second, high school students are the most popular participants, and the distribution of sample sizes (small, medium, large) is nearly equal. Furthermore, how to foster CT ability is always addressed when exploring kindergarten and primary school students; the disposition of CT is also raised in works covering middle and high school students, which is probably due to students' developed mentality, however, some of the desired dispositions such as internalization of CT skills may still not be achieved by students (Aizikovitsh-Udi & Cheng, 2015).

Finally, "fostering CT in ME" once dominated in the 2010s, with problem solving being its closest partner, but studies in recent years focused more on mathematics-oriented conceptualization and measurement of CT, reflecting a deepening understanding of CT from an independent subject's view. Moreover, the limited integration of digital tools for intervention and data analysis highlights a notable gap. In the age of digital transformation, especially students nowadays are surrounded with Generative AI, introducing new technologies can be a promising direction for future research.

References

Aizikovitsh-Udi, E. & Radakovic, N. (2012). Teaching probability by using geogebra dynamic tool and implemanting critical thinking skills. *Procedia - Social and Behavioral Sciences, 46*, 4943-4947.

Aizikovitsh-Udi, E. & Cheng, D. (2015). Developing critical thinking skills from dispositions to abilities: Mathematics education from early childhood to high school. *Creative Education, 6*, 455-462.

Bronkhorst, H., Roorda, G., Suhre, C., & Goedhart, M. (2020). Logical reasoning in formal and everyday reasoning tasks. *International Journal of Science and Mathematics Education, 18*(8), 1673-1694.

Dewolf, T., Van Dooren, W., Ev Cimen, E., & Verschaffel, L. (2014). The impact of illustrations and warnings on solving mathematical word problems realistically. *Journal of Experimental Education, 82*(1), 103-120.

Dolapcioglu, S. & Doğanay, A. (2022). Development of critical thinking in mathematics classes via authentic learning: An action research. *International Journal of Mathematical Education in Science and Technology, 53*(6), 1363-1386.

Ennis, R. H. (1991). Critical thinking: A streamlined conception. *Teaching Philosophy, 41*(1), 5-25.

Holmes, V. L. & Hwang, Y. (2016). Exploring the effects of project-based learning in secondary mathematics education. *Journal of Educational Research, 109*(5), 449-463.

Jablonka, E. (2020). Critical thinking in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 159-163). Springer.

Luczak, R. & Erwin, R. (2023). Mathematical modeling: A study of multidisciplinary benefits in the math classroom. *Teaching Mathematics and Its Applications, 42*(4), 325-342.

Ministry of Education of the People's Republic of China. (2022). *The mathematics curriculum standards for compulsory education*. Beijing Normal University Publishing Group.

Ministry of Education of the People's Republic of China. (2023). *Notice on the recommendation of experimental districts and experimental schools for compulsory education teaching reforms.* http://www.moe.gov.cn/srcsite/A06/s3321/202401/t20240102_1097465.html

Monteleone, C., Miller, J., & Warren, E. (2023). Conceptualising critical mathematical thinking in young students. *Mathematics Education Research Journal, 35*(2), 339-359.

Sachdeva, S. & Eggen, P. O. (2021). Learners' critical thinking about learning mathematics. *International Electronic Journal of Mathematics Education, 16*(3), em0644.

Wang, Q., & Abdullah, A. H. (2024). Enhancing students' critical thinking through mathematics in higher education: A Systemic Review. *SAGE Publications, 14*(3), 1-15.

Zhou, Y., Ning, Y. M., Chen, J. H., Zhang, W. J., & Wijaya, T. T. (2024). Development and validation of mathematical higher-order thinking scale for high school students. *Psychology in the Schools, 61*(8), 3160-3192.

TSG5-09

IS THERE A CONNECTION BETWEEN REASONING OF MATHEMATICS AND SCIENCE: A DOCUMENT ANALYSIS OF TIMSS ASSESSMENT FRAMEWORK

Kyong Mi Choi
University of Virginia, USA, kc9dx@virginia.edu

Jin Mun
Seoul National University, Republic of Korea, munjin0901@snu.ac.kr

This study investigates whether reasoning in mathematics and science is domain-specific or domain-general by analyzing the TIMSS assessment frameworks. While TIMSS presents reasoning separately for each discipline, the study explores potential cognitive connections, particularly through the roles of Mathematics Applying (MA) and Science Applying (SA) as mediating factors. Using transfer learning models, the research examines the relationships among Mathematics Reasoning (MR), Science Reasoning (SR), MA, and SA. Findings suggest that while MR and SR operate within distinct epistemological frameworks, MA and SA facilitate cross-disciplinary connections. The study highlights the importance of integrating reasoning and applying components in STEM education to enhance students' cognitive flexibility and problem-solving skills.

Keywords: reasoning, applying, cross-disciplinary, epistemological frameworks

Introduction

This study addresses part of this inquiry by analyzing an existing document—the TIMSS assessment frameworks. Through document analysis, we examine whether TIMSS frameworks treat reasoning as domain-specific or domain-general. While TIMSS presents reasoning descriptions separately for each discipline, potentially supporting domain-specificity, we hypothesize that our findings may reveal connections between reasoning in mathematics and science, suggesting a domain-general approach or transferable reasoning across domains. By the end of the study, we aim to clarify the theoretical connections (or distinctions) between reasoning in mathematics and science. By developing a framework that integrates skills across disciplines with a focus on shared cognitive resources like reasoning, we hope to enrich learners' intellectual capacity—what we term "intellectual resources" (Choi et al., 2020). Our qualitative approach involves analyzing the descriptions of Mathematics Reasoning (MR), Science Reasoning (SR), Mathematics Applying (MA), and Science Applying (SA) components to explore these interconnections. Drawing on Bailin's (2018) notion of reasoning as practice and Hammer et al.'s (2007) concept of utilization, we examine whether procedural knowledge (represented in the Applying domains) mediates the development of reasoning within and across disciplines.

Methodology

To guide this investigation, the following research questions are addressed:

1. How do Science Reasoning (SR) and Mathematics Reasoning (MR) connect when Mathematics Applying (MA) is incorporated into their relationship?
2. How do Science Reasoning (SR) and Mathematics Reasoning (MR) connect when Science Applying (SA) is incorporated into their relationship?
3. How is the relation between SR and MR when both Mathematics Applying (MA) and Science Applying (SA) are included?

Adapting Transfer Learning Models

To explore potential connections between cognitive elements in mathematics and science, we also adapted existing transfer learning frameworks. Pan and Yang's (2009) model of transfer learning was used to assess how cognitive elements might transfer from one domain to another. They emphasized that transfer is more likely when domains share common features or when learning in one domain reduces the cognitive load required to learn in another. Additionally, Varshney et al.'s (2022) framework of task-based transfer was applied to examine how cognitive tasks in one domain influence performance in another.

Analysis Procedure

The analysis aimed to identify relationships between components from the MR, MA, SR, and SA domains and to visualize these relationships through connecting lines between components from different cognitive domains. Using qualitative document analysis, we compare descriptions of Mathematics Reasoning (MR), Science Reasoning (SR), Mathematics Applying (MA), and Science Applying (SA) in the TIMSS frameworks. Document analysis was particularly useful for identifying patterns, commonalities, and changes in the descriptions of cognitive components (Bowen, 2009). The following steps were followed: Identifying Relationships, Commonality Identification, Subset Identification, Cross-Checking, Establishing Relationships

Findings

Though each overall connection between two cognitive domains (MR, MA, SA, SR) was evaluated based on every connection of all components in the two domains, it is difficult to explain all connections which are needed to understand the entire connection due to the length of the paper. Instead, every relationship is represented through the figures below and we excerpt some of the descriptions on every connection between two components of two domains.

Relationship Between Science Reasoning (SR) and Mathematics Reasoning (MR)

When comparing the components of SR and MR, we found that although some components share the same or similar names, their meanings differ significantly due to the nature of each discipline. This finding aligns with previous research (Hwang et al., 2020).

Mathematics Applying (MA) Mediates the Relationship between MR and SR

We present the relationship between Mathematics Reasoning (MR) and Science Reasoning (SR), with Mathematics Applying (MA) as a mediating factor. Based on our analysis of the components

in each of these domains, we identified the connections. To clarify the SR-MA-MR relationship, we analyzed it in two parts: SR-MA and MA-MR.

Due to the nature of their respective disciplines, Mathematics Reasoning (MR) and Science Reasoning (SR) are not directly related. However, these two types of reasoning are connected through the mediating factor of Mathematics Applying (MA). For example, MA's Determine→Formulate and MR's Evaluate are related through their emphasis on evaluating strategies and solutions. The ability to "evaluate alternative problem-solving strategies and solutions" (Mullis et al., 2015, p.27) serves as a theoretical basis for Determine→Formulate, as selecting efficient approaches requires evaluation. This connection is based on the subset criterion, as is the relationship between Determine→Formulate and MR's Justify, where Justify acts as a precursor to Determine→Formulate by requiring a logical basis for selecting strategies.

Science Applying (SA) Mediates the Relationship between MR and SR

Explanation of the MR-SA Connection

We present the findings from the comparisons of Science Applying (SA) and Mathematics Reasoning (MR) components to clarify their relationships. Likewise, SA plays a key mediating role in connecting SR and MR, establishing the MR-SA-SR pathway.

Explanation of the SR-SA-MA-MR Connection

In this section, we dive deeper into the interactions between MA and SA to provide a more comprehensive understanding of the overall SR-SA-MA-MR relationship. Contrary to our initial expectations of a strong connection, we observed minimal relevance between the components of the two domains, SA and MA. Except in few cases such as MA's Select and SA's Find solutions, most components between SA and MA do not show meaningful connections. For instance, SA's *Compare–Contrast–Classify*, which involves "identifying similarities and differences between groups" (Mullis et al., 2011, p.83), contrasts with MA's *Implement*, which focuses on executing "a set of mathematical instructions" (Mullis et al., 2011, p.44).

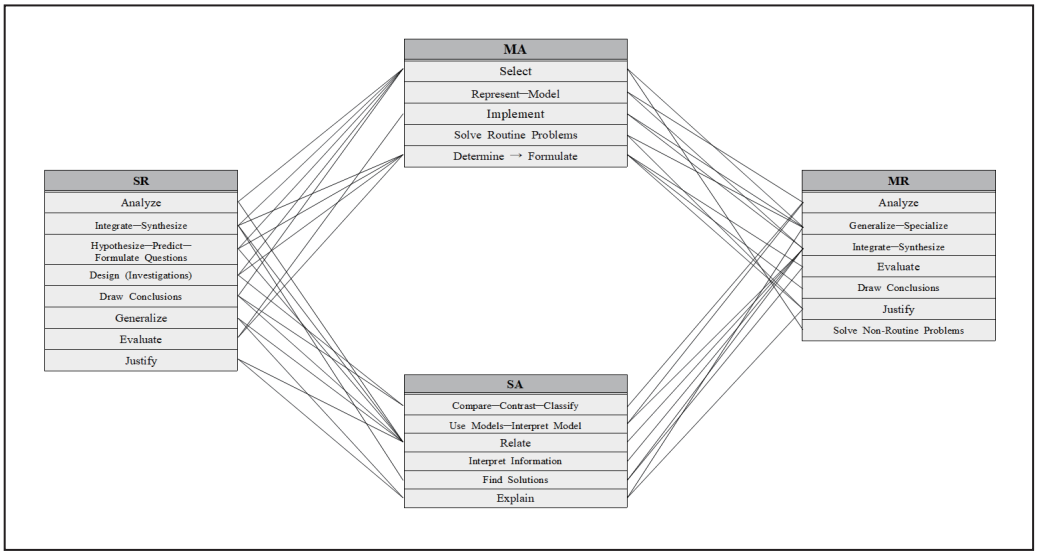


Figure 1. SR, SA, MA, MR relationship

Conclusion

SA and MA serve as two distinct mediators between SR and MR, each forming its own pathway, as illustrated in Figure 1. This model reflects the independent roles of SA and MA in connecting reasoning processes across science and mathematics. By acknowledging other potential influences, such as learning environments and individual differences, we embrace the complexity of these relationships, leading to a more nuanced understanding of how SR and MR are connected in educational contexts.

Discussions

Only a few direct connections were observed between MR and SR or between MA and SA, largely due to the fundamentally distinct cognitive and epistemological frameworks of mathematics and science. Mathematics emphasizes abstract reasoning, symbolic representation, and structured problem-solving, while science relies on empirical observation, experimental procedures, and interpreting natural phenomena (Klahr, 2000; Kuhn & Dean, 2005; Van de Sande et al., 2019; Zimmerman, 2007).

Despite this disciplinary separation, Applying and Reasoning components connect both within and across disciplines, as shown in the study, therefore, MA and SA serve as effective mediators when positioned between MR and SR, respectively. In this framework, applying components represent cognitive actions that students engage in within a problem-solving environment. In contrast, Reasoning components reflect epistemic actions—higher-order cognitive processes that shape how knowledge is constructed and understood. Reasoning actions require students to engage in reflective thinking, synthesis, and justification, moving beyond procedural application toward genuine understanding. These reasoning skills, once developed, become intellectual resources that students can draw upon across contexts, supporting knowledge generation and adaptability even when applied across different disciplines. The cognitive actions students engage in while learning and problem-solving (Applying components) underpin and enhance epistemic actions (Reasoning components), promoting a pathway to robust knowledge generation and cognitive advancement across domains.

Implications

This study provides valuable insights for enhancing everyday teaching practices, teacher education, curriculum design, and assessment in STEM education. Emphasizing cognitive connections, specifically through applying (A) and reasoning (R) components, can help shift educational focus from isolated content delivery to integrated problem-solving and critical thinking skills across mathematics and science. Encouraging students to use A skills as tools for developing R skills bridges disciplinary divides, supporting cognitive flexibility essential for real-world applications. In terms of follow-up study, empirical research involving student data, expanded component analysis, and exploration of contextual factors would contribute to a more nuanced understanding of how these cognitive connections manifest in diverse educational settings.

References

Assessment Frameworks for the TIMSS 2023. (n.d.). Timssandpirls.bc.edu. https://timssandpirls.bc.edu/timss2023/index.html

Bailin, S. (2002). Critical thinking and science education. *Science & education, 11*, 361-375.

Choi, K. M., Seo, K., Hand, B., & Hwang, J. (2020). A theoretical model for the utilization of intellectual resources between science and mathematics: An empirical study. *The Mathematical Education, 59*(4), 405-420.

Hwang, J., Choi, K. M., & Hand, B. (2020). Examining domain-general use of reasoning across science and mathematics through performance on standardized assessments. *Canadian Journal of Science, Mathematics and Technology Education, 20*, 521-537.

Klahr, D. (2000). Exploring science: The cognition and development of discovery processes. Cambridge, MA: MIT Press.

Kuhn, D., & Dean, D. (2005). Is developing scientific thinking all about learning to control variables? Psychological Science, 16(11), 866–870.

Mullis, I. V. S., Martin, M. O., Ruddock, G. J., O'Sullivan, C. Y. & Preuschoff, C. (2009). *TIMSS 2011 Assessment Frameworks.* MA: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.

Mullis, I. V. S., & Martin, M. O. (2013). *TIMSS 2015 Assessment Frameworks.* MA: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.

Mullis, I. V., & Martin, M. O. (2017). *TIMSS 2019 Assessment Frameworks.* International Association for the Evaluation of Educational Achievement. Herengracht 487, Amsterdam, 1017 BT, The Netherlands.

Mullis, I. V., Martin, M. O., & von Davier, M. (2021). TIMSS 2023 Assessment Frameworks. *International Association for the Evaluation of Educational Achievement.*

Pan, S. J., & Yang, Q. (2009). A survey on transfer learning. *IEEE Transactions on knowledge and data engineering*, 22(10), 1345-1359.

TIMSS Trends in International Mathematics and Science Study. (n.d.). Timssandpirls.bc.edu. https://timssandpirls.bc.edu/timss-landing.html

Van de Sande, E., Kleemans, T., Verhoeven, L., & Segers, E. (2019). The linguistic nature of children's scientific reasoning. *Learning and Instruction*, *62*, 20-26.

Varshney, N., Bakariya, B., & Kushwaha, A. K. S. (2022). Human activity recognition using deep transfer learning of cross position sensor based on vertical distribution of data. *Multimedia Tools and Applications, 81*(16), 22307-22322.

Zimmerman, C. (2007). The development of scientific thinking skills in elementary and middle school. Developmental Review, 27, 172–223.

TSG5-10

CONTEXT-BASED LEARNING FOR ELEMENTARY STUDENTS: CONSTRUCTING REPRESENTATIVENESS IN ARITHMETIC MEAN USING AIC THEORY

Youngin Kim
Seoul National University Graduate School, Republic of Korea, kyi9312@snu.ac.kr

Chanhee Park
Seoul National University Graduate School, Republic of Korea, savinamath@snu.ac.kr

Kyeong-Hwa Lee
Seoul National University, Republic of Korea, khmath@snu.ac.kr

This study utilized the Abstraction in Context (AiC) theory to analyze the learning process through which elementary school students develop an understanding of the concept of representativeness in the arithmetic mean via context-based tasks. The findings reveal that students progressed from an initial perception of the arithmetic mean as a simple arithmetic computation to recognizing the arithmetic mean as a representative measure of data, influenced by extreme values, and further to constructing scenarios where the arithmetic mean could represent diverse datasets. These results highlight that the integration of context-based pedagogical strategies in teaching elementary statistics promotes a deeper comprehension of the representativeness in arithmetic mean. This study underscores the importance of incorporating context-based teaching methodologies into the elementary statistics education curriculum to enhance students' conceptual understanding.

Keywords: Arithmetic mean, Representativeness, Context-based task, Abstraction in Context(AiC), Elementary statistics education

Introduction

The concept of the arithmetic mean encompasses both a mathematical interpretation as a division algorithm and a statistical role as a representative value that distributes or balances data evenly (Bremigan, 2003). In South Korea, elementary students encounter the arithmetic mean as their initial statistical measure to summarize data (Ministry of Education, 2015). However, prior studies in South Korea (Joo et al., 2010; Oh & Lee, 2008; Kim & Shin, 2019) have criticized the curriculum for focusing predominantly on algorithmic proficiency, often overlooking its fundamental role as a representative value, that is representativeness[1)]. The arithmetic mean may not always serve as an appropriate representative value due to its sensitivity to extreme values (Watson, 2006). This

1) In statistics education, the term 'representativeness' is commonly understood as the property of a sample representing its population. However, this study defines representativeness as the property of the arithmetic mean representing a dataset (Mokros & Russell, 1995; Watson, 2007; Makar & McPhee, 2009). This conceptualization is critical for understanding the distinctive characteristics of the arithmetic mean compared to other measures of central tendency, particularly when informally exploring the representational nature of different summary statistics.

highlights the necessity for a context-based pedagogical approach that allows the arithmetic mean to function as a meaningful summary value by accurately reflecting the central tendency of a dataset (Watson, 2006). Consequently, a pedagogical shift is required to transcend algorithmic computation and foster comprehensive understanding (Cai et al., 1999).

Dreyfus & Kidron (2014) proposed the AiC (Abstraction in Contexts) theory as an effective framework for describing the abstraction of mathematical concepts within contextual learning environments. This theory elucidates how students extract mathematical ideas from context and synthesize them into comprehending the arithmetic mean. This study aims to address the limitations of algorithm-centered approaches in teaching the arithmetic mean to elementary students, underscoring the importance of representativeness in learning through context-based tasks. It seeks to offer new insights into how elementary students develop the concept of the arithmetic mean and explore alternative pedagogical methods for teaching it.

Theorical Background

Construction of the Concept of Representativeness in Arithmetic Mean

Watson & Moritz (2000) emphasized the importance of recognizing the contexts in which the arithmetic mean functions as a measure, and studies by Mokros & Russell (1995) highlighted the role of understanding representativeness in forming a deeper comprehension of the arithmetic mean concept. Strauss & Bichler (1988) identified seven properties of the arithmetic mean, categorizing them into statistical and abstract aspects. The arithmetic mean possesses statistical properties such as being located between extreme values and having a sum of deviations from the mean equal to zero. Concurrently, it encompasses abstract properties that may not directly correspond to physical reality. Notably, the property of the arithmetic mean as a representative value encapsulates its essential meaning, underscoring the need to provide students with learning opportunities to deeply understand the representativeness in arithmetic mean.

Analysis of the Construction Process of the Concept of Representativeness in Arithmetic Mean Based on the AiC Theory

The AiC theory provides a theoretical framework for explaining how students construct abstract mathematical knowledge within various contexts, including data, learning, historical, social, and task contexts (Hershkowitz et al., 2001). The pedagogical model proposed by the AiC theory consists of three stages: ‘need,’ ‘emergence,’ and ‘consolidation’ (Dreyfus et al., 2015). This AiC perspective allows for a micro-level analysis of the process by which students construct mathematical concepts when presented with a sequence of tasks (Dreyfus & Kidron, 2014).

Meanwhile, the arithmetic mean does not exist independently of context as mathematical entities but are embedded within the distribution and relationships of the dataset (Moon et al., 2019; Yim, 2022). Mokros & Russell (1995) pointed out that measures of central tendency serve as central metrics of a dataset and that such measures cannot hold meaning unless the dataset itself is recognized as a mathematical entity. Based on the properties of the arithmetic mean outlined by Strauss & Bichler (1988), the structural knowledge elements required to form the concept of representativeness in the arithmetic mean were organized, as illustrated in [Figure 1].

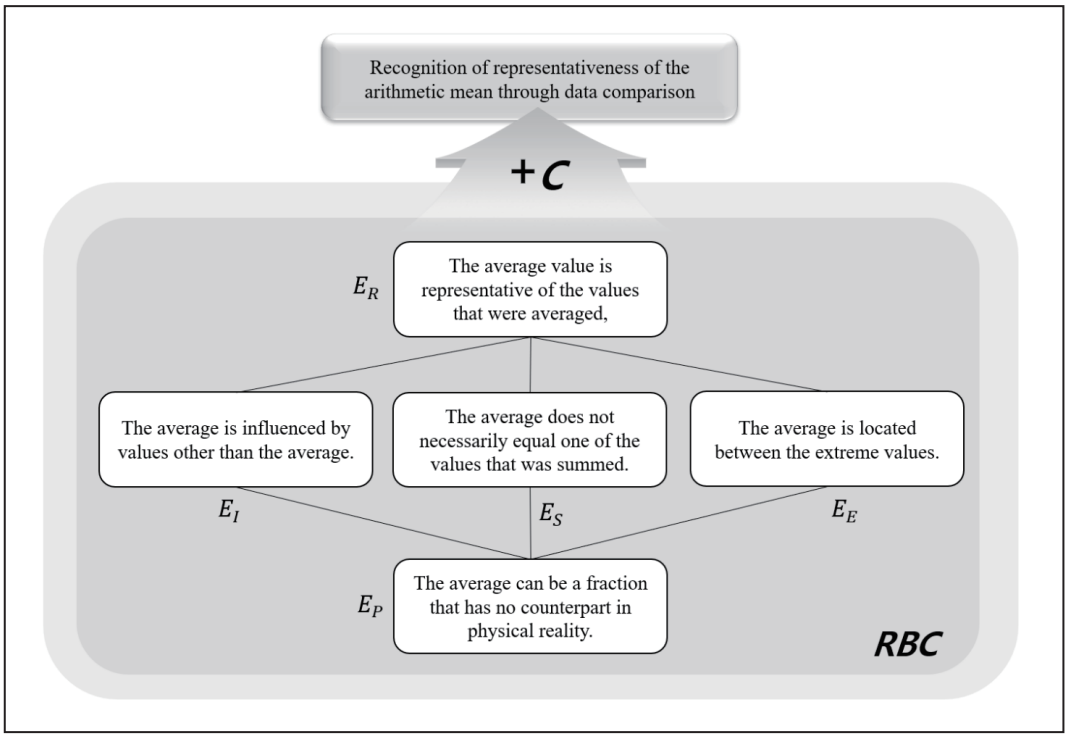


Figure 1. The structure of knowledge elements that define a concept of representativeness in arithmetic mean in this study

Research Design

The study involved 18 fifth and sixth-grade elementary school students who consented to participate. These students were general participants gathered for after-school classes from various elementary schools. Students work in pairs on the tasks collaboratively. The study extracted and presented several cases in which the students' constructed concepts of representativeness in arithmetic mean were clearly evident during three instructional sessions. To analyze the data, classroom scenes were recorded and transcribed, and the students' dialogues were analyzed from the perspective of AiC. The coding for recognition included discourse involving foundational knowledge elements, basic common sense, and interdisciplinary content connected to new knowledge elements, though it did not necessarily require prior knowledge.

Result Analysis

The Process of Constructing a Concept of Representativeness in Arithmetic Mean

Constructing E_P

Students demonstrated a notable focus on the fact that the given number in the third session's task was not an integer. This observation suggests that students may still struggle to differentiate between everyday numbers and arithmetic mean. This response highlights how students, through contextual learning opportunities connected to prior experiences, began to understand that the arithmetic mean does not necessarily correspond to a physical quantity. Such insights demonstrate the

potential for tasks like these to foster a deeper comprehension of the abstract nature of the arithmetic mean as a representative measure.

Constructing E_I, E_S, E_E

The researcher asked whether the number '0.778' could represent a real quantity. Student R elaborated, saying, "A house might not have any children, but in many cases, there might be 1, 2, 3, or even 4 children..." This response suggests that the idea of the arithmetic mean representing various datasets was beginning to form at an early stage. Responses in following indicate that the students were beginning to generate ideas about the presence of extreme values while simultaneously considering that most data points cluster around the arithmetic mean. Student T said, "Although there are families with 4 children, there are far more families that don't have any children at all, which brings the number down." This statement reflects the recurring emergence of the idea that the arithmetic mean can be considered a representative value, reinforcing their developing understanding of the concept.

Constructing E_R

Also Student R responded, "Yes, teacher, but I think they used this because there are families with no children. However, no one would think that families with 4 or 5 children don't exist, right? There are some, after all." This response demonstrates that Student R comprehends the arithmetic mean as more than just a numerical value, recognizing its susceptibility to extreme values and its application in representing diverse datasets.

Consolidation for Concept of Representativeness in Arithmetic Mean

Following group discussions, students who initially struggled began to integrate these elements. They discussed that the arithmetic mean better summarizes data when it is "concentrated" or "clustered" rather than "dispersed." This reflection allowed students to clearly recognize the critical role of the arithmetic mean in summarizing and representing data. Students also deliberated on the limitations of the arithmetic mean, explaining that its representativeness weakens when "data values are widely scattered" or when "extremely large or small values" are present. Such discussions reveal that students gained a deeper understanding of the scope of the arithmetic mean's applicability and recognized the need to consider alternative measures in specific situations.

Discussion

The findings reveal that students initially recognized the arithmetic mean as a rudimentary or procedural concept but later constructed the idea of the arithmetic mean as one of the representative values. Students progressed to an integrative understanding of representativeness, recognizing the arithmetic mean as a useful tool for summarizing data while also developing critical thinking about situations where the arithmetic mean may not adequately represent all data. These results align with the properties of the arithmetic mean proposed by Strauss & Bichler (1988) and support the effectiveness of Watson's (2006) context-based learning approach.

References

Bremigan, E. G. (2003). Developing a Meaningful Understanding of the Mean. *Mathematics Teaching in the Middle School, 9*(1), 22–26.

Cai, J., Moyer, J. C., & Grochowski, N. J. (1999). Making the Mean Meaningful; An Instructional Study. *Research in Middle Level Education Quarterly, 22*(4), 1–24.

Dreyfus, T., & Kidrun, I. (2014). *Introduction to abstraction in context.*In A. Bikner-Ahsbahs & S. Prediger. (Eds.), Networking theories group, networking of theories as a research practice in mathematics education. Advanced in mathematics education series (pp. 85-95). New York: Springer.

Dreyfus, T., Hershkowitz, R., & Schwarz, B. (2015). *The nested epistemic actions model for abstraction in context: Theory as methodological tool and methodological tool as theory.* Approaches to qualitative research in mathematics education: Examples of methodology and methods, 185-217.

Hershkowitz, R., Schwarz, B. B., & Dreyfus, T. (2001). Abstraction in context: Epistemic actions. *Journal for Research in Mathematics Education, 32,*195-222.

Joo, H. Y., Kim, K. M. & Hwang, W. H. (2010). Pre-service Teachers' Conceptualization of Arithmetic Mean. *The Mathematical Education, 49*(2), 199-221.

Kim, Y. J. & Shin, H. G. (2019). A Study of Development of Teaching and Learning Program on Representative Value in Elementary School. *The Journal of Korea Elementary Education, 30*(2), 37-52.

Makar, K., & McPhee, D. (2009, July). *Young children's explorations of average in an inquiry classroom.* In Crossing divides: Proceedings of the 32nd Annual Conference of the Mathemafics Educafion Research Group of Australasia (Vol. 1, pp. 347-354).

Ministry of Eduction (2015). *Elementary school mathematics teacher guidebook 5-2.* Seoul: Chunjae.

Mokros, J., & Russell, S. J. (1995). Children's concepts of average and representativeness. Journal for Research in Mathematics Education, 26(1), 20-39

Mokros, J., & Russell, S. J. (1996). What do children understand about average. *Teaching Children Mathematics, 2*(6), 360-364.

Moon, S. J., Noh, J. W., Ro, Y. S. & Lee, K. H. (2019). Professional development of an experienced teacher through research community activities: focusing on task modification and implementation to facilitate mathematical creativity. *The Mathematical Education, 58*(4), 545-566.

Oh, Y. Y. & Lee, M. Y. (2008). Exploring Directions for Improving Elementary Statistics Education. *The Journal of Korea Elementary Education, 19*(1), 1-13.

Strauss, S., & Bichler, E. (1988). The development of children's concepts of the arithmetic average. *Journal for Research in Mathematics Education, 19*(1), 64-80.

Watson, J. M. (2006). *Stastical Literacy at School: Growth and Goals.* Routledge.

Watson, J. M. (2007). The Role of Cognitive Conflict in Developing Students' Understanding of Average. *Educational Studies in Mathematics, 65*(1), 21–47.

Watson, J. M., & Moritz, J. B. (2000). The longitudinal development of understanding of average. *Mathematical Thinking and Learning, 2*(1-2), 11-50.

Yim, J. H. (2022). Various Meanings of Average and Implications for Teaching in Elementary School. *Journal of Elementary Mathematics Education in Korea, 26*(3), 155-173.

Topic Study Groups 5
(Day 3)

TSG5-11

A STUDY ON SPATIAL SKILLS TO ENHANCE MENTAL OPERATIONS IN ELEMENTARY SCHOOL STUDENTS

Satoshi Watanabe
Jissen Women's University, Japan, watanabe-satoshi@jissen.ac.jp

This study explores how primary school children mentally operate with nets of cubes and rectangular prisms. Findings indicate that children find it more challenging to mentally manipulate nets of rectangular prisms compared to cubes. Based on these findings, we designed age-appropriate interventions to enhance spatial skills and evaluated their effectiveness through practical classroom lessons. Observations revealed that spatial skills developed for assembling cubes were transferable to constructing more complex rectangular nets. Additionally, children who benefited from these spatial skills started applying them more proactively to similar challenges, indicating improved spatial abilities.

Keywords: spatial reasoning, spatial skills, mental operation

Background

Recent research highlights a strong correlation between spatial ability and mathematical performance (Mix & Cheng, 2012). Spatial ability not only correlates with mathematics achievement but also serves as a predictor of future performance, often demonstrating stronger predictive power than general mathematics scores (Mix & Chen, 2012). Spatial ability is multidimensional, encompassing various related skills. The psychometric tradition describes spatial ability as a general term that includes skills such as spatial visualization, mental rotation, and spatial orientation, as assessed by psychometric tests (Lowrie & Logan, 2017). Spatial visualization is defined as the ability or skill drawn upon to mentally transform or manipulate an object's spatial properties (Lowrie, 2019) and is a central component of spatial reasoning (Jansen et al., 2013).

A meta-analysis by Uttal et al. (2013) revealed that spatial reasoning training interventions significantly improve skills across all ages. More recently, classroom-based learning interventions have been explored. For instance, Lowrie (2019) examined a spatial visualization intervention aimed at enhancing both spatial reasoning and mathematics performance. This three-week program significantly improved students' spatial visualization and orientation skills by encouraging them to visualize and transform spatial information into schematic or pictorial forms to solve mathematical problems. However, further research is needed to identify the most effective spatial skills or strategies that students find useful, and to understand how these can be applied to solve spatial problems in classroom lessons.

Spatial visualization remains an underdeveloped area within the Japanese mathematics curriculum. In the fourth grade, Japanese students are introduced to the concept of transitioning between 2-D patterns, such as squares and rectangles, and 3-D forms like cubes and rectangular prisms. They are tasked with visualizing how nets assemble into 3-D shapes and with accurately constructing these shapes mentally. Teachers have observed that many students find it challenging to mentally con-

struct these nets. Key mental operations, including rotation, folding, bending, and scaling, are essential components of spatial skills (Uttal et al., 2013). Despite the importance of these skills, there is limited research on age-appropriate elementary school mathematics lessons designed to enhance students' spatial visualization. To address this gap, we investigated the difficulties that children face when mentally constructing rectangular and cubic nets. This study involved the administration of paper-pencil tests to 700 children in grades 1- 6. The students were asked to assemble five types of nets for cubes and rectangles and determine whether they could be correctly constructed. The result showed that the rate of correct answers is generally higher for cubes than for rectangular prisms. For instance, four out of five nets for cubes had a correct answer rate of over 90% by the second grade. In contrast, three out of five nets for rectangular prisms did not reach an 80% correct answer rate until the sixth grade. Based on this study, the following research questions were posed: First, how can age-appropriate interventions be designed to improve children's spatial abilities to mentally construct a net of cuboids? Second, what spatial abilities are most effective in helping children bend and manipulate surfaces in their minds?

Methodology

We designed learning activities for the intervention programs. In the first three lessons, I planned activities where children could explore the transformation of 2D and 3D shapes through hands-on manipulation of planes. These experiences were intended to develop children's ability to mentally manipulate surfaces. This study focused on the fourth and fifth lessons to examine how children acquire spatial skills necessary for mentally manipulating surfaces. The sequence of this intervention followed the order presented in the Japanese textbook. We implemented this instructional intervention in Class A, while for Class B, the lessons on the fourth and fifth days were reversed. This approach allowed us to determine which sequence of learning was more effective in developing children's spatial skills of mental operations.

A total of 64 fourth-grade students (ages 9-10) from two classes at a primary school in Tokyo participated in the intervention. The Design-Based Research method (Bakker & Van Eerde, 2015) was employed to closely examine the educational environment and teacher guidance that contributed to the development of individual spatial skills through retrospective analysis. Each lesson lasted 40 minutes. The author conducted four lessons (two for each class). This lesson plan was approved by the Ministry of Primary Education. The purpose of these lessons was to communicate to the parents in advance, and only the children whose parents gave consent participated. Four lessons were video recorded to understand the stages of each child's development in spatial skills of mental manipulation, and these recordings were planned to be used for future lessons. First, we conducted two interventions for Class A. The first intervention involved finding the two edges of a rectangular prism net that overlaps when assembled. The initial task was a basic exercise found in the textbook, followed by a more challenging task designed to encourage students to apply the spatial skills they developed in the previous task. The second intervention required the students to identify the missing face on a cube net that would allow it to be correctly assembled. Students used spatial skills to find missing faces and applied these skills to subsequent tasks. Next, we conducted lessons in reverse order for Class B. The first lesson involved finding the missing face on a cube net that would allow it to be assembled correctly. The second lesson focused on identifying the two edges of a rectangular prism net that would overlap when assembled. We observed the types of spatial skills the children used in these two lesson sequences. We also conducted a post-hoc survey to investigate which lesson sequence developed children's spatial skills more effectively. The survey included six

questions on finding overlapping edges in the nets of a rectangular prism and a cube and two questions on finding the position to attach the remaining face to the nets of the rectangular prism and cube with one face missing.

Results

The first lesson of class A focused on identifying the two edges of a flat rectangular pattern that would overlap when assembled. Twenty-one children used mental operations to find overlapping edges in question 1, whereas eight children connected the same-length edges with pencil lines to identify them. After sharing these spatial skills, the children proceeded to Question 2. In this question, 12 children used mental operations to find overlapping edges, whereas 17 connected same-length edges with pencil lines. Notably, nine of the twenty-one children who initially used mental operations switched to the pencil-line strategy in Question 2. This suggests that the children found it easier to identify overlapping edges by visually connecting edges of the same length rather than relying solely on mental operations. In the next lesson, they were tasked with finding a missing face on a cube net that could be assembled correctly. Because all sides of the cube are the same length, the children were unable to use the strategy they had learned from the previous lesson.

The students in Class B followed the lessons in the reverse order compared to Class A. In the first lesson, they were tasked with finding a missing face on a cube net that could be assembled correctly. Twenty children used the spatial skill of mental rotation, and three of them fixed the bottom face and manipulated the four sides to determine the position of the missing face. This strategy involved first establishing the bottom face and then mentally arranging the remaining four sides to determine the correct position of the top face. After sharing these spatial skills, the students proceeded to the next question, where 20 children employed the same mental strategy of fixing the bottom face and mentally positioning the four sides to identify missing faces. In the next lesson, the children were asked to imagine which two edges of the rectangular net would overlap when assembled. Twenty-three children applied the spatial skills they had learned in the previous lesson: setting the bottom face first and mentally manipulating the remaining four sides to ensure that the sides would overlap when the net was assembled into a rectangular prism. After using this skill, the children connected the edges of the same length to a pencil line to verify their answers. Subsequently, they addressed the next question. Fifteen children applied the skill of fixing the bottom face and mentally bending the four side faces while checking their answers by connecting edges of the same length with a pencil line.

After completing the two lessons, we conducted a post-hoc survey to investigate which sequence of lessons helped the children acquire spatial skills more effectively. A comparison of the children's learning in the two classes revealed the following. Class B children who learned to manipulate nets using cubes to fix the base, which are easy to manipulate in their minds, applied those skills to more difficult rectangular prisms. However, Class A children who learned the difficult mental operations of rectangular prisms were unable to use these skills and instead focused on the length of the edges to solve problems. The post-test survey included two types of tests: one to identify the two edges of the nets of a rectangle and a cube that would overlap when assembled, and the other to find the missing face of the nets that would allow it to be assembled correctly. In Class B, in addition to the two previously studied questions, the correct answer rate for the unlearned question of finding the missing face in a rectangular net increased from 60.4% to 90.7%. This suggests that the children in Class B applied the same spatial skills they used to mentally manipulate cube nets into rectangular

ones. Conversely, for the children in Class A, while the percentage of correct answers for questions they had studied increased, the percentage of correct answers to questions they had not studied increased only slightly. Furthermore, upon examining the post-test results of children whose correct answer rate was less than 50% on the pre-test, eight students in Class B improved their rates across all four types of questions. In contrast, the seven students in Class A who initially scored less than 50% on the pre-test improved their answer rates for only one type of question they learned. This suggests that even children who are not good at using spatial skills can improve their spatial skills by providing opportunities to understand and utilize the skills of their friends.

Table.1 Correct answer average of Pre and Post-test of spatial tasks

	Class A		Class B	
	pre	post	pre	post
Cube: look for a missing face	46.4	67.7	61.2	86.6
Rectangular prism: look for a missing face	46.4	54.4	60.4	90.7
Cube: Find the overlapping edges	46.4	59.1	61.2	73.6
Rectangular prism: Find the overlapping edges				

Discussion

Through the cube-to-rectangular prism intervention, the children acquired spatial skills for fixing the bottom face o0f the nets, and then applied this skill to rectangular nets, which are more challenging to manipulate mentally. By fixing the bottom face, the number of faces that needed to be manipulated in space was reduced, allowing for more precise spatial manipulation. The results observed in this study are consistent with previous research, which shows that children gradually learn to use multiple spatial skills through spatial learning experiences (Kageyama, 2000). Additionally, children used other spatial skills to check the accuracy of their ideas. Providing learning opportunities that encourage the sharing and use of multiple spatial skills in class will further develop children's spatial abilities. Future research should focus on developing curricula for learning spatial skills tailored to children's developmental stages.

References

Bakker, A. & Van Eerde, D. (2015). An introduction to design-based research education with an example from statistics education. Doing qualitative research: *Methodology and methods in mathematics education*.185-217. Springer.

Jansen, P., Schmelter, A., Quaiser-Pohl, C., Neuburger, S., & Heil, M. (2013). Mental rotation performance in primary school age children: Are there gender differences in chronometric tests? *Cognitive Development*, 28, 51-62.

Kageyama, K. (2000). Study on the Hierarchy of Spatial Thinking -Global Characterizing and

Concretization. *Japan Academic Society of Mathematics Education,* 6, 163-173.

Lowrie, T., Logan, T., & Ramful, A. (2017) Visuospatial training improves elementary students' mathematics performance. *British Journal of Education Psychology*, 87, 170-18.

Lowrie, T., Logan, T., & Hegarty, M. (2019) The Influence of spatial visualization training on students' spatial reasoning and mathematics performance. *Journal of Cognition and Development, 20*(5), 729-751.

Mix, K. S., & Cheng, Y. L. (2012). The relation between space and math: developmental and education implications. In J.B. Benson (Ed.), *Advances in child development and behavior*, 42, 197-243. San Diego, CA: Academic Press.

Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N.S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin, 139*(2), 352-402.

TSG5-12

EARLY SPATIAL REASONING: THE CASE OF PATH FINDING AND CONSTRUCTION

Kazuya Kageyama
Hiroshima University, Japan, kkageya@hiroshima-u.ac.jp

This article aims to investigate lower elementary school students' spatial reasoning working in the context of spatial task solving. Spatial reasoning is a complex ability with scales and types of space. The collaborative teacher and the author designed and conducted a series of mathematics lessons for third graders to engage in the "3D maze task" of path finding and constructing. Analysis of observed lesson data implies that third graders tend to use sensory-based reasoning while using cross-sectional diagrams effectively. It is not easy for them to imagine the overall structure of the object and reason about the invisible parts of the object. However, they could engage in the spatial task to work their multimodal senses, including sounds and touch sensations.

Keywords: spatial reasoning, path finding and construction task, elementary school students

Introduction

For humans living in three-dimensional physical space, spatial abilities are important, and they have been examined in various research fields and mathematics education (Foreman & Gillett, 1997). These abilities are often referred to as spatial reasoning or spatial imagination, and it has been argued that they are one of the basic cognitive abilities of humans, as well as the cognitive foundation of mathematics learning and that they are a form of literacy as a way of understanding the world (Sinclair, 2024). In recent years, a relationship between non-verbal spatial reasoning and mathematical achievement has been researched (e.g., Hawes, Gilligan-Lee, & Mix, 2023). However, it is still implicit in what context spatial reasoning is activated and what tools learners create and use. In fact, with a background in complexity science, Davis et al. (2015) suggests that spatial reasoning is a situational, complex ability and that multiple skills work in combination. Based on this argument, it is assumed that the developmental process of spatial reasoning is also similarly complex. Therefore, the development of spatial reasoning may improve the effectiveness of intertwining multiple problem-solving skills rather than the growth of a single skill, as has been argued by traditional experimental psychology. This article aims to investigate the emergence of complex spatial reasoning in young elementary school students during spatial tasks. After defining the concept of spatial reasoning, the article approaches how students create and use practical tools to solve spatial tasks during mathematics lessons.

Theoretical Background

Foreman and Gillett (1997) reviewed spatial reasoning research in the twentieth century in the context of psychological research and identified basic cognitive skills such as imagining, orienting, and mental rotation. In addition to these individual cognitive characteristics of spatial reasoning, its role in numeracy (Resnick & Lowrie, 2023) and measurement-based elementary school mathematics in-

struction and design (Lehrer & Schauble, 2023) has been asserted in recent years. Moreover, spatial reasoning also has an expansive one. For example, Freitas et al. (2021) argue that spatial reasoning contributes to cultural understanding by creating and reading diagrams.

Concerning the cognitive, functional, and expansive properties described above, Davis et al. (2015) proposed a combination of three core skills (*map-making, designing, projecting*) and six sub-skills (*constructing*, *interpreting*, *sensating*, *situating*, *moving*, *altering*) to represent the complexity of spatial reasoning, using the background of the purpose-oriented *understanding* and *transformative*. Concerning these trends, considering the complexity of spatial reasoning that works through mathematical activities, this research defines *spatial reasoning as the ability to reason systematically, rationally, and creatively in each situation to be effective in problem-solving.* Furthermore, this research includes the scale and type of space, assuming the place where spatial reasoning is activated. The *scale* of space refers to the size of the area that is possible for human sensory perception. It is composed of the micro-scale (such as the manipulatives for spatial figures in the classroom), which can be touched and wrapped with hands, and the meso scale (such as classrooms and schools), which is close to the same size as the body and can understand the overall structure through direct perception, and the macro scale (such as regions and the earth), which envelops the body but it is not easy to directly perceive the various parts. The *type* of space is related to Tall's three-world theory (Tall, 2004) and is set according to how spatial reasoning is activated (moving, transforming and describing objects with symbols, etc.): actual, possible, theoretical. This research assessed the background of students' behavior by using the scale and type of space and identified students' spatial reasoning by applying a combinatory spatial reasoning model initiated by Davis et al. (2015).

Methodology

The experimental lessons aimed to investigate the students' spatial reasoning when they engaged in a spatial task. The lesson was conducted with a class of 31 third graders from an elementary school attached to the national university. The author and the collaborative teacher designed the learning unit of three lessons. The designed task was a '3D maze' task. This task consisted of a path finding task, in which the students had to find a path inside a 3D object made up of multiple unit cubes that had an invisible path inside, and a path construction task, in which the students had to construct the 3D object in a way that would realize the path they had imagined beforehand. In constructing the 3D objects, we used Math Link Cubes made by Dream Blossom (Figure 1). These are plastic cubes with cut-outs on each side to connect them, allowing students to assemble and disassemble them freely.

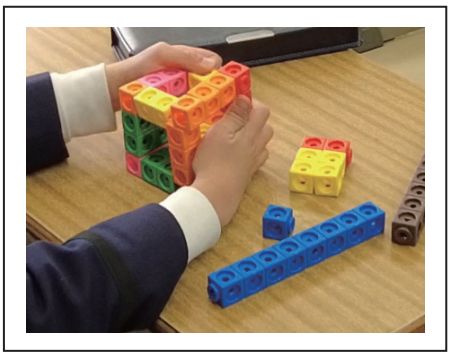


Figure 1. Manipulating the blocks

In the first lesson, the students used Cuisenairaire Rods to construct a path on a 2D plane. The activity aimed to understand the meaning of a path in this task. In the following lessons, the students combined unit cube blocks to discover paths inside 3×4×6 blocks, 3×3×3 blocks, and 4×4×4 blocks and to construct blocks with specific paths.

Through preliminary communication with several sixth graders before conducting this lesson, it was recognized that the students tended to use physical actions using their fingers, exploring the internal path, using marbles to explore by sound, and using descriptions of practical tools (i.e., cross-sectional diagrams that show the solid object in layers). Therefore, the teacher encouraged the third graders to use cross-sectional diagrams as practical tools. The sixth graders also suggested that a 4×4×4 block could hold the inner space of eight cubes, and a 5×5×5 block could hold the inner space of 27 cubes. The sixth graders understood that the key to this task was how to construct a path within this *possible micro* space besides *actual micro* space. This research assumed it was one of the developmental indicators of spatial reasoning related to the transition of space types.

Two cameras were installed at the back of the classroom to capture the whole blackboard from different angles, and the author operated one individual camera to record the movements of individual students. The lesson data analyzed included these videos and transcripts of the student's comments, worksheets completed by the students, field notes made by the observer (the author), and post-lesson discussions between the teacher and the author. This article focuses on the group of students' approach to the task and the subsequent three-dimensional construction process by one student who succeeded in finding a path in a 5×5×5 block. In order to analyze the multiple types of data obtained, the multimodal inference process, which consists of the student's speech, physical actions, and changes in the three-dimensional objects, was described.

Discussions and Implications

After understanding the meaning of the term 'path' in this task, in the second lesson, through manipulation of the 3×4×6 blocks, it was agreed that marbles would be used to search for paths, that the results would be shown in a diagram (cross-sectional view), and that there would be some restrictions on the search (for example, that the blocks would not be taken apart, and that the outside of the cross-sectional view would not be considered a path). At the end of the lesson, some students seemed confused about how to verify the paths. For example, when the marble placed inside the block stops moving, it is not clear to determine the direction of the path. At this point, the students are using sensory verification through the vibration and sound of the marble within the *actual, micro* space and are not using the shape of the block to reason the *possible, micro* space inside, as the sixth-grade students did.

Following this lesson, the students worked on a task to guess the path of a 4×4×4 block. The teacher asked them to draw the paths on each layer when looking at the block from above. Figure 2 shows the results of one student's reasoning, described through classroom discussion. The central idea was to connect the four cross-sections with paths by functioning constructing skill to understand the task situation. It was important to consider whether the shape of the partial *possible* path formed by the junction was reasonable rather than the position of the contact point between the cross-sectional diagrams.

Following the above path-finding task, the students moved on to the path-constructing task. The teacher asked each student to imagine a path that could be made using 4×4×4 blocks and construct it by combining unit cubes. The student, mainly analyzed in this article, had already succeeded in

finding a path using 5×5×5 blocks, which the teacher gave him individually, and he used the combination of partial paths as his basic strategy. First, this student constructed a 4×4 square array and then constructed pillars with a length of 4 from the four corners. To make the path more concrete, he constructed a face by arranging unit cubes in order between those four pillars and constructed a 4×4×4 block without the top and inside (Figure 1). Through these activities, he looked inside the 4×4×4 blocks from the outside, saying, 'I should not have done it. First, I cannot see it,' and measured the length of the path under construction by putting their fingers on it. He repeated the processes of taking it apart, modifying it, and reconstructing it to build the route while looking in from the outside at the blocks (Figure 1). The series of these behaviors seemed explorative but coherent in terms of spatial reasoning such as designing as a core spatial skill to transform the task situation.

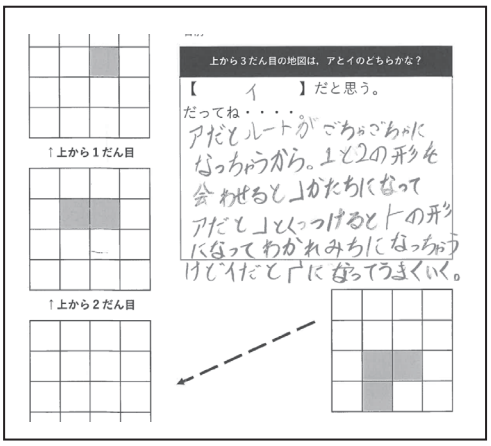


Figure 2. Student's description of the path

The following is a way for constructing a path used by another student during the summary phase of the lesson: 'When I first made it, I made it so that it would fall like a staircase, but the goal is supposed to be like this and like this (showing the direction of the path with gestures), (…) so I took it once and put in those blocks so that the path would go through properly.' In summary, spatial reasoning by third-graders has two functions. One is to work when searching for and identifying paths, and it promotes a shift from sensory-based search ways to ones that connect specific parts of cross-sectional diagrams. This shift suggests that students extend to *possible* space type beyond the sensations, rather than staying within the *actual* space type according to the intentional task-solving. The other is to work when constructing paths, leading to appropriate arrangements to realize paths as gaps in unit cubes in three dimensions. In this way, each cube is both a component of the block as a whole and a framework for *meso* scale spatial formation. Furthermore, they are viewed separately according to the problem situation, such as generating or alienating the path. Spatial reasoning encourages this kind of bricolage.

References

Davis, B. & Spatial Reasoning Study Group. (2015). *Spatial Reasoning in the Early Years*, Springer.

Foreman, N. & Gillett, R. (1997). *Handbook of Spatial Research Paradigms and Methodologies*, Psychology Press.

Freitas, L., et al. (2021). New Spatial Imaginaries for International Curriculum Projects: Creative Diagrams, Mapping Experiments, and Critical Cartography, *Qualitative Inquiry*, *28*(5), 507-521.

Hawes, Z. C. K., Gilligan-Lee, K. A., & Mix, K. S. (2023). Infusing spatial thinking into elementary and middle school mathematics: What, Why, and How? In K. M. Robinson, A. K. Dube, & D. Kotsopoulos (Eds.), *Mathematical Cognition and Understanding: Perspective on Mathematical Minds in the Elementary and Middle School Years* (pp. 13–33).

Lehrer, R., & Schauble, L. (2023). *Measuring and Visualizing Space in Elementary Mathematics Learning*, Routledge.

Resnick, I., & Lowrie, T. (2023). Spatial Reasoning Supports Preschool Numeracy: Findings From a Large-Scale Nationally Representative Randomized Control Trial. *Journal for research in mathematics education, 54*(5), 295-316.

Tall, D. (2004). Building theories: the three worlds of mathematics. *For the Learning of Mathematics, 23*(3), 29-32.

Acknowledgment

This work was supported by JSPS KAKENHI Grant Number 22H01010.

TSG5-13

A DESIGN-BASED STUDY ON INTEGRATING CREATIVE AND CRITICAL THINKING INTO MATHEMATICS-GROUNDING ACTIVITIES: PROPORTIONAL REASONING

Kai-Lin Yang
National Taiwan Normal University, Chinese Taipei, kailin@ntnu.edu.tw

Bo-Ting Liu
National Taiwan Normal University, Chinese Taipei, carrotonwater@gmail.com

Shu-Ting Yu
New Taipei Municipal DanFeng High School, Chinese Taipei, shutingyuu@gmail.com

Creative and critical thinking are essential components of mathematical literacy and crucial 21st-century skills. Mathematics-grounding activities, which involve behavioral, cognitive, and emotional engagement, provide an opportunity to cultivate these skills. This study explored how creative and critical thinking can be integrated into mathematics-grounding activities to enhance students' proportional reasoning abilities with the focus on two key questions: (1) What constitutes creative and critical thinking-oriented mathematics-grounding activities? (2) In the context of small-group teaching experiments, what are students' learning processes? This study followed three steps: exploration and development, analysis and thought experiments, and small-group trials, to evaluate feasibility and effectiveness. It was found that mathematics-grounding activities can be enriched by integrating worksheets with creative and critical thinking questions, and that students' learning can be facilitated through peer interactions. Further refinements to mathematics-grounding activities are provided to optimize the development of proportional reasoning.

Keywords: mathematics-grounding activities, creative thinking, critical thinking, proportional reasoning

Introduction

The Trends in International Mathematics and Science Study (TIMSS) reveals that Taiwanese students demonstrate high achievements in mathematics but exhibit low interest, confidence, and perceived value in learning mathematics compared to other countries (Mullis et al., 2020). The Programme for International Student Assessment (PISA) (Organisation for Economic Cooperation and Development [OECD], 2023) survey reveals that, although Taiwanese students perform excel in mathematical literacy, their attitudes toward learning mathematics are less positive, and their creative thinking skills remain inadequate (OECD, 2024).

Mathematics-grounding activities (MGAs) were developed to address these issues of high achievement but low interest and confidence, and have been shown to improve students' engagement in learning mathematics (Lin et al., 2018). Based on the enactivist learning perspective, Yang et al. (2021) proposed the theoretical principles and practical strategies for designing mathematics-grounding activities, highlighting three key processes: metaphorizing, scaffolding, and gamifying. Recognizing the importance of creative and critical thinking as essential 21st-century skills,

this study investigates how to integrate these thinking skills into mathematics-grounding activities which merge action, reflection, and emotional engagement. Focusing on proportional reasoning, the research questions are: (1) What constitutes creative and critical thinking-oriented mathematics-grounding activities? (2) In the context of small-group teaching experiments, what are the learning processes and outcomes for case-study students?

Method

This study adopted a design-based approach, encompassing the phases of preparation, implementation, and preliminary evaluation. The preparation phase involved literature review, activity development, self-analysis of tasks and thought experiments. During the implementation phase, small-group trials were conducted to evaluate feasibility and effectiveness. Eight students who had recently completed sixth grade (four from each of two schools in Taichung and Kaohsiung) were selected through convenience sampling. The implementation procedure is outlined in Figure 1. The preliminary evaluation analyzed students' cognitive construction and adjustment during the activities, providing a basis for refining and modifying the activities.

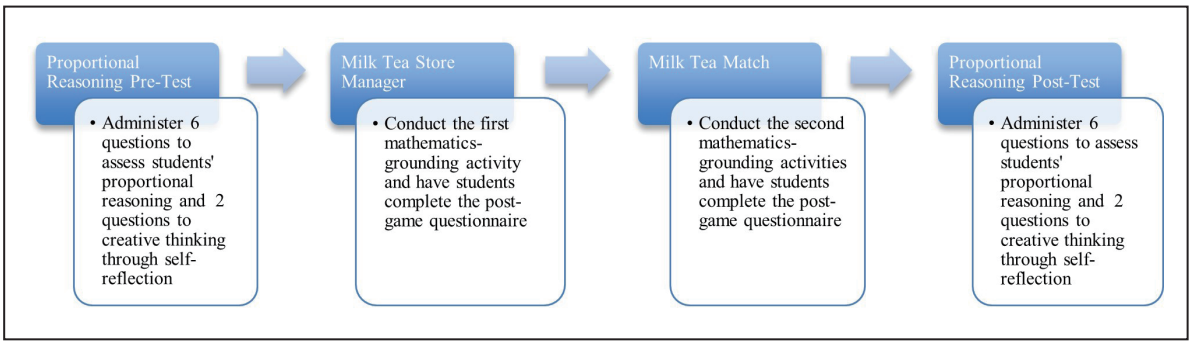


Figure 1. Implementation procedure

Results and Discussion

For developing learners' proportional reasoning, two MGAs were designed — **Milk Tea Store Manager** and **Milk Tea Match** — based on three design principles: metaphorizing, scaffolding, and gamifying(Yang et al., 2021). For developing creative and critical thinking skills, questions in worksheets were designed based on the three features of creativity — fluency, flexibility, and originality (Leikin & Lev, 2012)—and the three elements of critical thinking skills — perceiving possible invalidity of an argument, explaining for refutation, and evaluating the validity of inferences (Yang et al., 2023).

The "Milk Tea Store Manager" game features two types of cards: Milk Cards and Tea Cards, each displaying a specific value. Learners will be tasked with identifying Milk Cards with Tea Cards according to the recipe announced for each round to make milk tea with the specified flavor, a type of missing-value problems. If another student makes an incorrect match, they can call out "Catch!" to initiate an error check. For metaphorizing, the target domain focuses on proportional reasoning with non-integer multiples, while the source domain involves making milk tea according to a given recipe to produce the specified flavor. The repeated process of adjusting milk and tea amounts strengthens their proportional reasoning to recognize the underlying numerical structures beyond integer scaling. Through this experience, learners not only identify and construct the mathematical

structure underlying proportional reasoning by metaphorical inference from finding the amount of the milk and tea but also apply it back to the milk tea-making process. This iterative interaction allows them to embody proportional reasoning, as they refine their strategies for achieving the correct flavor, reinforcing the bidirectional connection between the source and target domains. For scaffolding, in addition to numerical representations of milk and tea quantities, some cards also provide visual representations of the number of cups used for milk or tea. This allows learners to observe the graphical representations and analyze proportional relationships. The error-checking mechanism also helps learners clarify concepts by considering the reasoning behind detected errors and analyzing the correct pairings to develop effective strategies. For gamifying, the game increases learners' motivation by awarding points for correct matches, with the player earning the highest score emerging as the winner.

During the initial implementation of the activity, several misconceptions and reasoning patterns were observed among students. At the beginning of the game, some students exhibited misconceptions such as confusing the red tea and milk cards (i.e., misinterpreting the ratio), believing that a larger amount of milk necessarily results in a stronger milk flavor, or assuming that milk teas with the same difference between milk and red tea quantities have the same flavor intensity. Moreover, while implementing the game, it was found that students with initial proportional reasoning skills (e.g., solving 72∶□ =18∶30，□= ?) did not immediately apply them. Through observing task-provided tables, recording accurate proportions, and identifying errors, students developed unit rate strategies to determine equivalent ratios (e.g., calculating the milk-to-tea ratio per unit).

To support students' development of critical and creative thinking, specific tasks and peer interactions were strategically embedded in the activity. Through the peer error-detection mechanism and the worksheet task designed to ask whether two milk teas with the same difference in milk and red tea amounts have the same milk flavor, students were able to actively identify contradictions and develop critical thinking skills. Additionally, another worksheet task prompted students to propose a less commonly considered yet more general method for creating milk teas with identical flavors. This encouraged them to reflect on the gameplay process and fostered the development of creative thinking through exploring alternative strategies for proportional reasoning.

The "Milk Tea Match" game features cards that provide information about both milk and tea quantities. However, each card contains only one type of information—either the actual amounts of milk and tea or the ratio between them. The difference from the "Milk Tea Store Manager" game is that each card in "Milk Tea Match" represents a specific milk tea flavor, and learners must find and match cards with the same flavor, a type of comparison problems. For metaphorizing, the target domain focuses on using a ratio as a unit to solve proportion problems, while the source domain is to compare multiple representations of milk and tea quantities with either the same or different flavors. For scaffolding, the cards feature various representations, including pure numerical values, the number of servings corresponding to given milk and tea amounts, numerical values with measuring cup markings, and comparisons indicating how many times the milk amount correspond to the tea amount. Additionally, the game includes an error-checking mechanism as learners can check and correct errors by calling out. For gamifying, players compete by making successful matches to earn points, with the highest scorer becoming the winner.

To examine the evolution of students' proportional reasoning, we first observed their reliance on previously learned strategies during gameplay. At the beginning, some students relied primarily on previous known strategy to verify if cards had equivalent ratios, for example the "inner ratio" strategy—first simplifying the ratio of milk to black tea into the smallest integer form, and then

using that ratio to infer the required amounts of milk and tea for other quantities in order to determine whether the flavors matched. Through analyzing cup numbers or scales, comparing multiple representations of proportions, and noticing the strategies used by other students, they developed a deeper understanding of equivalent ratios based on proportional reasoning. With the support of a worksheet and peer discussion, students were able to convert between representations to identify equivalent ratios.

Building upon students' gameplay experiences and worksheet engagement, additional evidence of the development of critical and creative thinking skills emerged. Through the error-detection mechanism in the game, students had the opportunity to evaluate the correctness of their peers' card pairings and articulate their reasoning, thereby developing critical thinking skills. Moreover, a worksheet prompt that asked students to propose a less common yet more general strategy for identifying milk teas with the same flavor encouraged them to approach the problem from different perspectives, fostering creative thinking and naturally leading to the use of unit rate strategies.

Conclusion and Suggestions

When developing a MGA integrated with creative and critical thinking, the MGA can be designed first, followed by the integration of worksheets containing questions that stimulate creative and critical thinking. This sequential approach ensures that the activity establishes a strong conceptual foundation, while the worksheets enhance students' ability to think critically and creatively, encouraging deeper analysis and understanding of mathematical concepts. When implementing the activity, it was found that students with stronger prior knowledge actively developed multiple strategies for proportional reasoning and demonstrated critical thinking through peer debugging. In contrast, those with weaker prior knowledge gradually became aware of and made sense of different strategies by interacting with peers and regulating their understanding as the evolving task.

The analysis revealed that students employed and developed various strategies (e.g., inner ratio, outer ratio, unit rate, and representational transformation) while engaging in "Milk Tea Store Manager" and "Milk Tea Match". Although the tasks targeted different learning goals, students' cognitive processes were similar. For the efficiency and feasibility of teaching in real classroom settings, the researcher plans to adjust and combine the two tasks into a single activity for greater effectiveness, renaming the game as *"Milk Tea Pair-Pair"* . Considering that the original worksheet questions (e.g., "How can you come up with more different methods?") were less effective in stimulating students' thinking, the questions will be revised to be more specific (e.g., "Come up with another method, different from the previous ones, to create milk tea with the same flavor"). By combining and refining these tasks, the learning mechanism for identifying proportional relationships evolves from initially identifying equivalent ratios beyond simple integer multiples to analyzing both the inner ratio and the outer ratio. Further implementation and teaching experiments will be conducted.

References

Mullis, I. V., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020, December). TIMSS 2019 international results in mathematics and science.

OECD. (2023). PISA 2022 Insights and Interpretations. OECD Publishing.

OECD (2024), PISA 2022 Results (Volume III): Creative Minds, Creative Schools, PISA, OECD Publishing, Paris, https://doi.org/10.1787/765ee8c2-en.

Leikin, R., & Lev, M. (2012). Mathematical creativity in generally gifted and mathematically excelling adolescents: What makes the difference? *Mathematics Education, 45*(183-), 179-183.

Lin, F.-L., Wang, T.-Y., & Yang, K.-L. (2018). Description and evaluation of a largescale project to facilitate student engagement in learning mathematics. *Studies in Educational Evaluation, 58*, 178–186.

Yang, K. L., Chen, C.H., Wu, W.R., Chen, Y. Z. (2023, August). Assessing Critical Thinking of Mathematical Literacy [Poster presentation]. The 20th Biennial European Association for Research on Learning and Instruction Conference, Thessaloniki, Greece.

Yang, K. L., Lin, F. L., & Tso, T. Y. (2022). An approach to enactivist perspective on learning: Mathematics-grounding activities. *The Asia-Pacific Education Researcher, 31*(6), 657-666.

TSG5-14

SURVEY ON THE FORMATION OF THE CONCEPT OF SPEED

Masahide Tsuda
Soka University, Japan, mtsuda@soka.ac.jp

This study investigates how learners intuitively recognize the concept of speed, focusing on both average and instantaneous speed. In elementary mathematics, speed is typically introduced through formulas and unit conversions, often with an emphasis on uniform motion. Prior studies have noted that such instruction may lead to a rigid, procedural understanding. To clarify learners' conceptual difficulties, this study analyzed their responses to tasks involving variable motion. The results revealed that learners tend to interpret instantaneous speed as constant and to rely heavily on average-speed-based reasoning. These findings suggest that many learners struggle to perceive speed as a dynamic and momentary quantity. To support deeper understanding, instructional approaches should explicitly distinguish between types of speed and incorporate the fundamental properties of quantities, including additivity, commutativity, and conservation. This study lays the groundwork for developing teaching strategies that foster more flexible and meaningful conceptions of speed.

Keywords: speed, quantity recognition, instantaneous speed, conceptual understanding, recognition survey

Introduction

The concept of "speed" is familiar in daily life—for example, when referring to the pace of a runner or the movement of a vehicle. However, in mathematics education, speed remains a difficult concept for learners due to the conceptual reasoning required to grasp both its meaning and its properties. In this study, "meaning and properties" refer not only to the numerical expression of speed as distance divided by time, but also to its mathematical characteristics such as additivity, commutativity, and conservation. The intuitive way in which learners perceive these aspects is referred to as their "recognition of speed."

Prior studies have identified several difficulties in how speed is taught and understood. In elementary education, speed is typically introduced as average speed and taught through formulas and unit conversions. Yoshizawa (2019) noted that this approach may lead to procedural learning without conceptual understanding. Tsuda (2024a) pointed out that textbook problems often assume uniform motion, failing to represent the complexity of real-world motion. In contrast, instantaneous speed varies moment by moment and cannot be derived from simple calculations. Nakao (2015) emphasized the instructional need to differentiate average and instantaneous speed. Although the latter is formally introduced at the junior high school level, it is necessary to examine how younger learners perceive it intuitively.

Another major challenge lies in understanding speed as a mathematical quantity. Unlike single-unit quantities such as length or time, which exhibit additivity (e.g., 20 m + 30 m = 50 m) and commutativity (e.g., 20 m + 30 m = 30 m + 20 m), speed—expressed with compound units like km/h—does not follow these basic properties in the same way. Kuroda (2010) argues for the importance of instruction that captures the characteristics of the unit of speed in order to clarify the difference between single-unit quantities and the compound-units quantity of "speed".

Furthermore, beyond understanding additivity and commutativity, the concept of conservation plays an important role in understanding speed. Conservation, in this context, refers to the idea that a quantity exists independently and is not merely the result of a calculation. For example, certain quantities such as instantaneous speed or temperature can be directly observed or measured, rather than computed from other values. In the case of speed, particularly when measured instantaneously with an instrument, the quantity can be recognized as already existing—its value is not dependent on prior calculations. Whether learners can grasp this concept of conservation, alongside the properties of additivity and commutativity, remains an open question. These issues highlight the need to investigate how learners perceive speed and to develop instructional approaches that foster deeper conceptual understanding.

Purpose

The aim of this study is to investigate how learners intuitively recognize the concept of speed, focusing on both its meaning—expressed as the relationship between distance and time—and its mathematical properties, such as additivity, commutativity, and conservation. By analyzing learners' difficulties in distinguishing between average and instantaneous speed, and by identifying patterns in their recognition of speed as a quantity, this study seeks to clarify key cognitive characteristics related to this concept. The findings are expected to provide a theoretical foundation for developing instructional approaches that foster a deeper and more flexible understanding of speed in mathematics education.

Analysis

Previous research on speed instruction has primarily focused on formula-based teaching and algebraic manipulation under the assumption of constant speed. Kumode et al. (2013, 2024) examined instructional practices in sixth grade that emphasized transforming speed-related formulas such as "speed = distance ÷ time," "distance = speed × time," and "time = distance ÷ speed." While these methods helped students understand formulaic relationships, many struggled to apply similar reasoning to other compound quantities like population density. This indicates that understanding the calculation mechanism alone is insufficient; learners also need experiences that reveal common properties across quantities.

Kawasaki et al. (2017, 2018) conducted surveys among fourth- to sixth-grade students to investigate their subjective difficulty with speed calculations. Results showed that learners across all grade levels found division-based problems particularly challenging. Moreover, many students who answered correctly were unable to explain the meanings of the quantities involved, suggesting a disconnect between procedural fluency and conceptual understanding. Shindo and Magara (2014) highlighted that learners often perceive formulas as fixed tools for numerical substitution rather than as expressions of relationships, a rigidity that presents a major obstacle to developing flexible understanding.

This rigidity may also explain learners' difficulties in recognizing instantaneous speed. Tsuda (2024b) conducted a survey involving 59 fifth-grade students, focusing on their understanding of the motion of a swing. The study found that many learners incorrectly believed the speed remained constant throughout the motion. In one task, students were asked to identify the point of maximum speed (correct answer: ②); in another, to select the diagram showing non-uniform movement from a sequence of images (correct answer: C). Many selected stationary points or diagrams with equal

intervals, reflecting a tendency to interpret motion based on average speed rather than instantaneous change. These findings suggest that a significant number of learners rely on formulaic interpretations and struggle to conceptualize speed as a dynamic and momentary phenomenon.

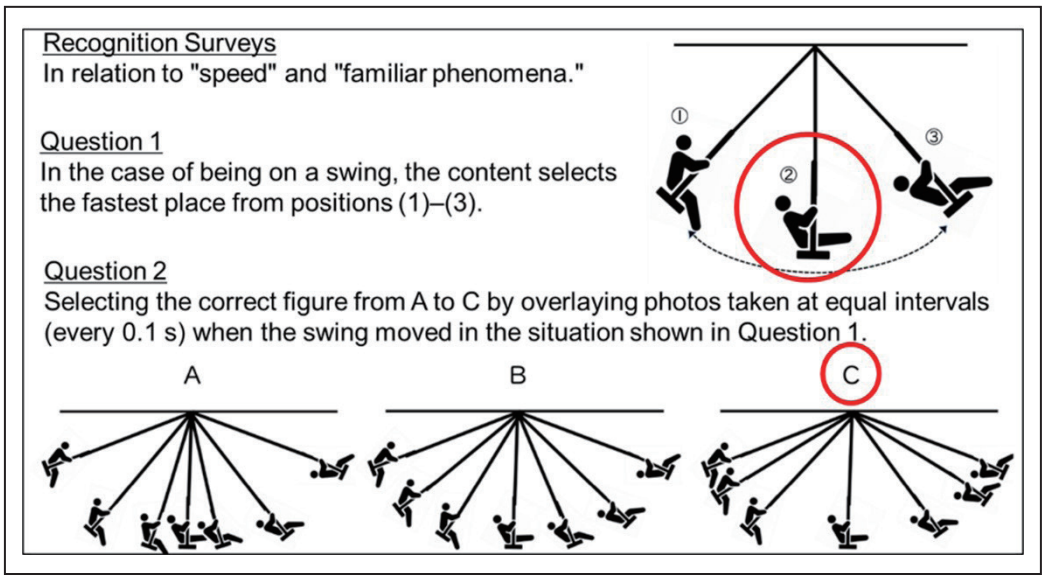


Figure 1: Overview of recognition survey questions and examples of correct answers

Discussion

As discussed in the previous section, many learners interpret speed in rigid, formula-based terms, often failing to recognize its dynamic and momentary nature. One theoretical perspective that helps clarify this issue is the concept of conservation. For example, when a car passes by, the value shown on the speedometer represents an inherent, momentary quantity possessed by the moving object. This exemplifies the conservation property of instantaneous speed—unlike average speed, which is derived from a calculation, instantaneous speed is perceived as existing directly and independently of measurement processes.

Magara (1992) emphasized that quantities for which conservation holds, including speed, should be understood as inherently existing rather than as constructs resulting from arithmetic operations. However, it remains unclear whether learners acquire this notion of conservation in the same developmental manner as they do with properties like additivity or commutativity. In this regard, psychological studies provide useful insights. Matsuda (2002) proposed that learners gradually construct the concept of speed through the integration of length and time concepts, along with the relational structures that emerge from them. This suggests that speed recognition involves a developmental process of coordinating multiple quantity concepts, rather than learning a single formula.

Recent mathematics education research also supports this view. A recognition survey by Kuroda, Nishida, and Tsuda (2024) found grade-level differences in how elementary students understand time and time-point calculations—fundamental knowledge for reasoning about speed. These results indicate that learners' naive perception of speed may be strongly influenced by their understanding of the quantity they have learned. To improve instruction, it is necessary to design learning experiences that explicitly reveal the properties of speed, connect it with previously learned quantities,

and differentiate between average and instantaneous speed. Clarifying these cognitive aspects can serve as a basis for more effective and meaningful mathematics instruction.

Conclusion and Future Directions

This study investigated the difficulties learners face in understanding the concept of speed from two perspectives. First, it was found that instruction emphasizing formulas and unit conversions often leads to a rigid understanding of speed, hindering learners' ability to recognize instantaneous speed as a dynamic, momentary quantity. Second, it was shown that many learners struggle to grasp speed as a mathematical quantity with properties that differ from those of simpler quantities, such as length or time.

Based on the analysis of learner responses and previous studies, the findings suggest that instruction should explicitly distinguish between average and instantaneous speed, while also addressing foundational properties such as additivity, commutativity, and conservation. To further develop effective instructional approaches, future research will involve systematic recognition surveys across different grade levels, with a focus on how learners' conceptual understanding of speed transforms through targeted instructional activities.

References

Kawasaki, M., & Endo, A. (2017). An analysis of the degree of understanding and the subjective difficulty of intensive quantity in upper grades of elementary school. *Japan Jornal of Mathematics Education and Related Fields, 58(3-4)*, 87-95.

Kawasaki, M., & Fukuma, T. (2018). An analysis of the understanding of quantities expressed by numbers given in problems among 4th, 5th, and 6th grade students. *Japan Jornal of Mathematics Education and Related Fields, 59(3-4)*, 17-25.

Kumode, M., Nisawa, Y., & Watanabe, N. (2013). A basic study on teaching 'speed' based on RTMaC lesson study in elementary school (1): Aspects of children's recognition of speed. *Japan Jornal of Mathematics Education and Related Fields, 54(3-4)*, 71-86.

Kumode, M., Watanabe, N., & Nisawa, Y. (2014). A basic study on teaching 'speed' based on RTMaC lesson study in elementary school (2): Educational practice on subject transformation of the speed formula. *Japan Jornal of Mathematics Education and Related Fields, 55(1-2)*, 21-32.

Kuroda, Y. (Ed.). (2010). Methods of teaching arithmetic in elementary school. *Kyoto: Minerva Shobō*.

Kuroda, Y., Nishida, J., & Tsuda, M. (2024). Grade-Specific Trends in Time and Time Calculations. *Collection of Papers Presented Mathematics Educational Society of Japan Summer Institute (Kanto Area)*, 50-53.

Magara, K. (1992). Children's Fundamental Misconception on Intensive Quantity Concepts and its Remolding. *Japanese Journal of Educational Psychology, 40(1)*, 20-28.

Matsuda, A. (2002). Development of relational concepts: Acquisition of time, distance, and speed concepts, and improvement of mathematics lessons on speed. In Development of relational concepts. *Kyoto: Kitaōji Shobō*.

Nakao, T. (2015). A study on the guidelines of speeds and means in the domain of Quantities and Measurements. *Departmental Bulletin Paper(Kwansei Gakuin University) Journal of Pedagogical Studies, 7*, 101-104.

Shindo, T., & Magara, K. (2014). Understanding “the Principle of Exchanging Variables” in a Formula of Intensive Quantity . *Japanese Journal of Psychology in Teaching and Learning, 10(1)*, 12-24.

Tsuda, M. (2024a). Reexamination of Difficulties in Understanding Speed: Focusing on Various Quantities in Arithmetic Instruction. *Collection of Papers Presented Mathematics Educational Society of Japan Summer Institute (Kanto Area)*, 24-27.

Tsuda, M. (2024b). Perception Survey Analysis of “Instantaneous Speed” Among Upper Elementary School Students -Focusing on the Relationship Between Misconceptions of Speed and Understanding of Formulas-, *Japan Jornal of Mathematics Education and Related Fields, 65(1-2)*, 15-24.

Yoshizawa, M. (2019). College students who don’t understand “%”: A fatal flaw in Japanese mathematics education. *Tokyo: Kobunsha Shinsho*, 18-22.

TSG5-15

EXPLORING THE ABSTRACTION PROCESS DURING ALGEBRA PROBLEM-SOLVING IN A PROGRAMMING-ENHANCED ENVIRONMENT

Haoyi Wang
University of Pennsylvania, USA, haoyiw@seas.upenn.edu

Biyao Liang
University of Hong Kong, China, biyao@hku.hk

Comparing multiple strategies for solving mathematics problems has been shown to enhance students' conceptual ad procedural knowledge. Drawing on Jean Piaget's notion of abstraction, this study extends previous research by modeling student thinking as they draw comparisons across isomorphic problems and multiple methods in a programming-enhanced mathematical environment. Data was collected using task-based clinical interviews. Results indicate an interconnected pattern of empirical and pseudo-empirical abstraction. Empirical abstraction was evident in output-level or program-level comparisons, often relying on figurative materials. Pseudo-empirical abstraction emerged when the student compared program-output pairs or output pairs under mental actions, indicating deeper engagement with the material. The student demonstrated a non-linear progression of abstraction levels, moving cyclically among levels rather than in a ladder-like pattern. We conclude by discussing how the characterization of these mental processes contributes to task design and future research.

Keywords: abstraction; problem-solving; scratch; algebra learning

Background

Comparison is a fundamental tool for learning, particularly in mathematics education, where comparing and reflecting on multiple strategies enhances students' mathematical learning (Silver et al., 2005). Expert teachers frequently use comparison as an effective instructional approach (e.g., Ball, 1993; Lampert, 1990; NCTM, 2014). With technological advancements, programming provides a natural context for comparison; the programming interface, for example, could allow for the simultaneous display of multiple programs, simplifying side-by-side comparison of command lines. However, few studies have modeled student thinking during such comparisons. Our study addresses this gap by employing Piaget's notion of abstraction. We aim to elucidate how drawing comparisons in a programming-enhanced environment can elicit abstraction processes in students, deepening their understanding of the concepts embedded. We ask two research questions:

1. What abstraction processes do students engage in when they compare problems and methods at different levels in programming-enhanced mathematical problem-solving?
2. How does each level of comparison provide affordances for student mathematical learning?

Theoretical Framework

Abstraction emphasizes a process from contextual experiences to abstract concepts. Empirical abstraction concerns surface-level similarities of object properties, limited to sensorimotor materials. Piaget (1970; 1976) described empirical abstraction as an inductive process that extracts the inherent properties of physical objects or the material characteristics of actions, as perceived by the subject. In the context of programming, empirical abstraction may occur when students compare outputs or programs, focusing solely on the figurative aspects of these observables.

Reflective abstraction involves coordination of one's actions and is no longer confined to the use of individual action or sensorimotor materials. Reflective abstraction is further differentiated into three types: pseudo-empirical abstraction, reflecting abstraction, and reflected abstraction. Piaget (1980) characterized the initial stage of reflective abstraction as pseudo-empirical. This abstraction is dependent on sensorimotor material and pertains to the properties of objects, but it also operates on the actions associated with these objects (von Glasersfeld, 1995). In the context of programming, students are likely to engage in this abstraction when they compare program-output pairs, as they often refer to the input blocks to explain the associated programming outputs.

Method

This study, part of a larger project on programming-based mathematical learning, investigates the abstraction processes of students in eastern China using Scratch, a visual programming interface. The research focuses on interviews (for a total of three hours) with one student Hayden (pseudonym) on the Flying Task. This task (as summarized in Table 1) is designed to develop students' ability to recognize linear relationships, encourage creative problem-solving, and reinforce algebraic understanding through visual and dynamic representation.

Data was collected using the task-based clinical interview approach (Ginsburg, 1997; Hunting, 1997), where students interact with the interviewer and a carefully designed task environment (Goldin, 2000). The tasks were constructed to elicit estimates of participants' existing and developing knowledge and mathematical reasoning (Maher, 1998). Students were asked to think aloud during problem-solving tasks (Clement, 2000).

All student activities were videotaped for detailed analysis, accompanied by transcripts and students' work on Scratch projects. Data from the task-based interviews were coded in MAXQDA and analyzed using conceptual analysis (von Glasersfeld, 1995; Thompson, 2008), guided by Piaget's abstraction framework. The study aimed to develop predictive cognitive models of student thinking. The authors collectively compared and reconciled codes, identifying students' pathways of abstraction in each task.

Results

In this task, students' actions indicated an interconnected pattern of empirical and pseudo-empirical abstraction. Initially, students scrutinized the similarities or disparities in programming outputs to justify solutions, dominated by figurative materials. This scenario transformed when reflection and abstract thinking came into play.

Empirical Abstraction

In the initial four-point task, Hayden employed the small staircase method (Fig. 1), constructing nine staircases of side length 30 among four points on the line $y = x$. He then used the diagonal method (Fig. 2) to guide the sprite along the line $y = x$. In his third attempt, he introduced a variation of the staircase method (Fig. 3), involving fewer but larger staircases. After explaining each method, Hayden compared the third method with the first. Hayden established empirical abstraction by comparing the figurative materials of two methods at the output level. He noted the physical size feature of the outputs, describing the last set of staircases as "large" and the former set as "small". This comparison implied his attention to relative figurative features.

In Excerpt 1, Hayden depicted the new set of points on a diagonal line of a rectangle as opposed to a square, indicating his focus on figurative properties. When questioned about increasing the step size of X and Y to 9 and 11 respectively, Hayden relied on the physical image of a potential programming output, anticipating that the diagonal line of a rectangle of size 9 by 11 would not align with that of size 18 by 22. This was an inaccurate interpretation as he did not realize the slope remained the same for both movements. His comparison was restricted to the resulting objects' figurative properties, indicating he had not abstracted the relationships underlying the observables.

Pseudo-Empirical Abstraction

As Hayden elaborated on the size difference of the drawn stairs in Excerpt 2, pseudo-empirical abstraction became evident. He referred to the computer program that generated the third output, shifting between the program and the output on the interface. He associated actions and results, describing pairs of code controlling coordinate movements and their effect on the sprite. This pairing was evident when he noted that increases on the x-axis and y-axis were interchangeable due to their identical results of flying across all four designated points with three staircases. His discussion and behaviors were still primarily dependent on the figurative features of the codes and outcomes.

For the interviewer's problem on a constant slope of 2, Hayden determined the value of Y, given X is 3. He summarized the multiplicative patterns into a rectangle-flying algorithm from mentally constructed actions of the sprite flying through four points. His algorithm indicated that X and Y increase at a geometric proportion, so given any value of X, he could find a corresponding Y by multiplying X with 2. This projection from multiple pairs of X and Y into an algorithm had its foundational steps as a pseudo-empirical abstraction. He paired the same output image with two series of actions: flying upward for two steps then right-hand for four steps, and flying upwards for one step then right-hand for two steps twice. This program-output-level comparison happened almost synchronously as Hayden compared operative actions at the program level. He constantly referred to the resulting graph while mentioning each program step, emphasizing objects rather than constructing mental actions at the re-presentational level, aligning with pseudo-empirical abstraction.

Hayden was tasked with programming a sprite to fly through five points on an inverted V shape. The interviewer introduced multiple isomorphic problems by adding points at different locations on the inverted V. Hayden's responses in Excerpt 4 demonstrate his process of pseudo-empirical abstraction as he adjusted the sprite's flight path to accommodate the new points. He constantly referred back to the figurative points on the Scratch interface, with his mouse cursor frequently pointing at both the programs and the outputs. Each action corresponded to a resulting figure of the sprite's location on the inverted V shape, a key characteristic of pseudo-empirical abstraction. Hayden further coordinated his actions to form an algorithm, adjusting each turn of flying distance to be one-third of the difference between the endpoints, repeating this turn three times to cross

points at one-third of the line. A similar approach was applied for crossing points at one-fourth of the line, repeating one-fourth flying distances four times.

Discussion

This study aimed to model student mathematical thinking during comparisons across isomorphic problems and various methods in a programming context. Our analysis of the student's implementations of Scratch programming tasks revealed that empirical abstraction was evident in output-level or program-level comparisons, often relying on figurative materials. Pseudo-empirical abstraction emerged when the student compared program-output pairs or output pairs under mental actions, indicating deeper engagement with the material. While empirical abstraction sometimes led to misconceptions due to overlooked operations, it can serve as a solid foundation for more advanced reflective abstraction and later constructions of mathematical concepts. In both sub-tasks, the student demonstrated a non-linear progression of abstraction levels, moving cyclically among levels rather than in a ladder-like pattern.

Ultimately, this study suggests that the level of abstraction involved in the comparison process can influence the degree of affordances for student mathematical learning. By understanding and leveraging these abstraction levels, educators can better support students in developing a deeper understanding of mathematical concepts through programming tasks. To further strengthen the educational implications, future studies could explore a broader range of programming tasks and settings to better understand the dynamics of abstraction in different contexts. This would provide a more comprehensive understanding of the potential of programming in mathematics education.

Limitations

This study has several limitations that should be noted. First, the small sample size limits the generalizability of the findings to a broader population. Second, the controlled study environment may not capture the complexities and dynamics of real-world classroom settings. Third, the study focused on a specific programming environment, which may not be representative of other programming tools or platforms. Future research could address these limitations by involving larger, more diverse samples and examining the impact of programming on abstraction in varied educational settings.

References

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal, 93*(4), 373–397. http://www.jstor.org/stable/1002018

Clement, J. (2000). Analysis of clinical interviews: Foundation and model viability. In R. Lesh & A. E. Kelly (Eds.), *Research design in mathematics and science education* (pp. 547–589). Erlbaum.

Ginsburg, H. P. (1997). *Entering the child's mind: The clinical interview in psychological research and practice.* Cambridge University Press.

Goldin, G. (2000). A scientific perspective on structures, task-based interviews in mathematics education research. In R. Lesh & A. E. Kelly (Eds.), *Research design in mathematics and science education* (pp. 517–545). Erlbaum.

Hunting, R. P. (1997). Clinical interview methods in mathematics education research and practice. *The Journal of Mathematical Behavior*. https://doi.org/10.1016/S0732-3123(97)90023-7

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*, 29-63. https://doi.org/10.2307/1163068

Maher, C. A. (1998). Constructivism and constructivist teaching – Can they co-exist? In O. Bjorkqvist (Ed.), *Mathematics teaching from a constructivist point of view* (pp. 29–42). Abo Akademi.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* National Council of Teachers of Mathematics.

Piaget, J. (1970). *Genetic epistemology* (E. Duckworth, Trans.). Norton.

Piaget, J. (1976). *The grasp of consciousness: Action and concept in the young child* (S. Wedgewood, Trans.). Harvard University Press.

Piaget, J. (1980). *Adaptation and intelligence: Organic selection and phenocopy* (Original work published 1974). University of Chicago Press.

Piaget, J. (2001). *Studies in reflecting abstraction.* Psychology Press.

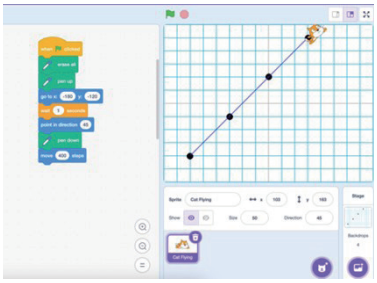
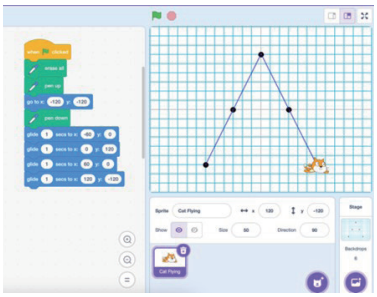
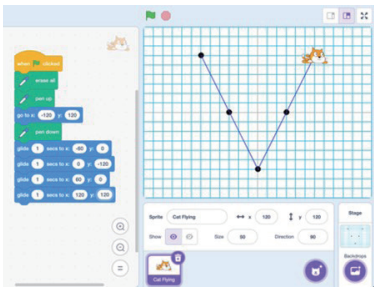
Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., & Strawhum, B. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *Journal of Mathematical Behavior, 24*, 287-301. https://doi.org/10.1016/j.jmathb.2005.09.009

Thompson, P. W. (2008). Conceptual analysis of mathematical ideas: Some spadework at the foundations of mathematics education. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Proceedings of the Annual Meeting of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 31–49). PME.

von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning.* Falmer Press.

Appendix A. Table

Table 1. Description of the task.

Name	Task	Normative Solution	Related Concept
Flying Task Sub-task 1	Fly through four points on .		Proportion; slope
Flying Task Sub-task 2	Fly through five points on the V-shaped lines.	 	Proportion; slope; pair-wise function

Appendix B. Figures

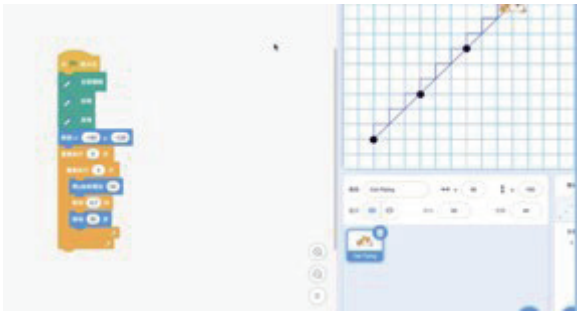


Fig. 1. *Small staircase method for flying through a line.*

Fig. 2. *Diagonal method for flying through a line.*

Fig. 3. *Large staircase method for flying through a line.*

Fig. 4. *Fly through a different line.*

Appendix C. (Translated) Excerpts

Note. "Int." stands for "Interviewer".

Excerpt 1

1 2	Hayden	It's not on this line, it's on another line. It's also a straight line. It's equivalent to the diagonal of a rectangle. (These points) are still connected.
3 4	Int.	So if this cat, first moves 9 steps, then moves 11 steps, and then the next one, it moves for 18 steps, for 22 steps. Are the points it lands on in a straight line?
5 6 7	Hayden	No, because the orientation of the diagonal of their squares has changed. (…) Because from this point to this point is slightly off, and from this point to this point is also slightly off, so they are not on the same straight line [pointing to Fig. 4].

Excerpt 2

1 2 3 4 5 6 7	Hayden	This is a large staircase, and the previous one is a small staircase. The large staircase is made up of three squares, drawing the two sides of a nine-square grid. Each square is 30, so I let the coordinate increase by 90, which means moving up by 90. Then the coordinate increases by 90, moving right by 90. Actually, these two can be swapped [pointing to codes for increasing and], and after swapping, it still draws a staircase here, passing through four points. Then repeat three times to draw three large staircases like this, until this point. [moving the mouse between the programming area and the stage] (…)

Excerpt 3

1 2	Int.	How about reducing the steps a bit? At first, it moves by 1 step, by 2 steps, then the cat moves by 2 steps, by 4 steps. If I continue with by 3 steps, would be moved by?
3 4 5 6 7	Hayden	by 6 steps. That is on a straight line because it increases by multiples. It goes from 1 to 2, then 2 to 4, then 3 to 6. It is still drawing a side. Moving up by two and then right by four is equivalent to moving up one square and right two squares, then up one square and right two squares again, so it is still on a straight line. (…) step increases by 90, moves up by 100, then increases by 90, and still increases by 100, so they are on a straight line.

Excerpt 4

1 2	Int.	Back to the inverted V situation, for example, we now add a point at one-third. If I want my cat to still pass through this newly added point, how would you change your program?
3 4 5 6 7	Hayden	Because I need to reach this point, I look at how to move one width and one length of this rectangle. So first, increase the coordinate by 20, then move up by 40, then 20, 40. Moving from this point to this point needs to be repeated three times. Moving from this point to this point also needs to be repeated three times. So in this direction, I need to repeat six times, then reverse and repeat six times again.
8	Int.	What about one-fourth?
9 10 12 13 14	Hayden	For one-fourth, I change the repetition to eight times, letting it move one-fourth each time. This point and the original point's rectangle have one width and one length. (…) should be 15, and should be 30. Because I need to move four times to this point, and the coordinate difference is 120. Each time I need to move 120 divided by 4 equals 30. The horizontal coordinate difference is 60, so 60 divided by 4 is 15.

TSG5-16

STRUCTURE SENSE AND FLEXIBILITY IN ALGEBRAIC PROBLEM-SOLVING: AN EXPLORATORY STUDY

Qiuyu Chen
University of Michigan, USA, qiuyuc@umich.edu

Haoyi Wang
University of Pennsylvania, USA, haoyiw@seas.upenn.edu

Jon Star
Harvard University, USA, jon_star@gse.harvard.edu

This exploratory study investigates the relationship between structure sense and potential flexibility in algebraic problem-solving. Through task-based interviews with three Grade 10 students, we analyzed how students identified and manipulated algebraic structures while solving a problem with multiple solution paths. Our findings suggest that students with stronger structure sense—who recognized and leveraged structural relationships rather than relying on surface features—demonstrated greater potential flexibility in generating alternative solutions. In contrast, students with weaker structure sense exhibited a formula-driven approach, limiting their problem-solving flexibility. These results highlight the role of structure sense in fostering mathematical flexibility and suggest the need for instructional approaches that explicitly develop structure sense. Further research should involve a larger sample and extend to other mathematical domains to explore the causal relationship between structure sense and problem-solving flexibility.

Keywords: flexibility, structure sense, problem-solving, algebra

Introduction

Previous research has identified structural understanding as essential to students' knowledge acquisition and performance in mathematical problem-solving (Richland et al., 2012). For example, students' difficulties in algebra often stem from a disconnect between the structural understanding of the algebraic system and the number system (Pittalis, 2023). Empirical evidence suggests that students may arrive at solutions or conclusions based on the surface structure of expressions rather than their underlying mathematical properties; for instance, they may incorrectly assume that "926 + 167 - 167" is not equal to "926 - 167 + 167" simply because notations of addition and subtraction appear in different positions (Linchevski & Livneh, 1999). Additionally, research has shown that students tend to rely on memorized procedures rather than viewing mathematics as a system and recognizing structural connections between problems or representations (Givvin, Stigler, & Thompson, 2011), which may lead to rigid problem-solving process.

Building on this body of research, Linchevski and Livneh (1999) introduced the concept of structure sense—the ability to "flexibly and creatively use equivalent structures of an expression" and actively engage in "the search for decomposition and recomposition of expressions" (p. 175). Structure sense offers a more specific lens for understanding students' structural reasoning in algebra. It

goes beyond formula memorization to involve recognizing underlying patterns and relationships within expressions. Such recognition demands versatility of thought, requiring learners to shift from an analytical approach to a holistic, global perspective (Tall & Thomas, 1991). In problem-solving contexts, it enables students to view solutions not merely as sequential steps but as interconnected, structurally rich processes—an approach aligned with deep procedural knowledge (Star, 2005). Despite its potential significance, the relationship between students' structure sense and their problem-solving flexibility has not been previously explored. Given this gap, it is reasonable to hypothesize that a more developed structure sense may be associated with greater flexibility. This study therefore takes an exploratory approach to investigate the potential connection between students' structure sense and their flexibility.

Theoretical Framework

Hickendorff's (2022) integrated conceptual model characterizes potential flexibility as the awareness of multiple strategies in mathematical problem-solving. In line with this model, we define students as exhibiting potential flexibility if they recognize multiple solution strategies for a given problem, even if they do not generate all correct solutions.

To examine the relationship between structure sense and potential flexibility, we adopt Novotná and Hoch's (2008) framework of structure sense (SS) in high school algebra, which includes the ability to: SS1: Recognize a familiar structure in its simplest form. SS2: Treat a compound term as a single entity and, through appropriate substitution, recognize a familiar structure in a more complex form. SS3: Choose appropriate manipulations to make the best use of a structure (p. 95). Among these, SS1 and SS2 are particularly relevant when students analyze algebraic expressions. For example, consider the expression $x^3 + \frac{1}{x^3}$. A student may interpret this in two structurally distinct ways: *Structure 1:* As a sum of a variable and its reciprocal. The expression can be rewritten as $a + \frac{1}{a}$ (SS1), where x^3 is treated as the compound term of the variable a (SS2). *Structure 2:* As a sum of two cubes. The expression follows the form $a^3 + b^3$ (SS1), where x and $\frac{1}{x}$ serve as the compound terms of the variable a and b respectively (SS2). A student's ability to recognize and manipulate fluidly between these different structures reflects their structure sense, while their capacity to apply multiple solution strategies indicates their potential flexibility.

Methods

This study involved three Grade 10 students—Clare, Dannie, and Emma—from a high school with average academic performance in Chengdu, China. Each student participated in a 30-minute individual task-based interview conducted by the second author. During the interview, students were presented with two algebra tasks and were asked to think aloud as they analyzed the problems and recorded their solutions. This think-aloud method was used to externalize their internal thinking processes, allowing us to explore how students construct mental representations of the problems and how these representations relate to their mathematical thinking process (Ericsson & Simon, 1980).

The problems presented to students were designed to allow for more than one valid solution strategy. After arriving at an initial solution, students were prompted to offer an alternative approach. If they experienced difficulty, the interviewer provided a consistent prompt to encourage further ex-

ploration. In this study, we used the number of distinct solutions a student generated as a proxy for their potential flexibility, as defined in Hickendorff's (2022) model. Specifically, if a student identified multiple strategies, even with prompts, they were considered to demonstrate potential flexibility; students who produced only a single approach or believed there was only one possible solution were regarded as exhibiting no flexibility. By analyzing students' problem analysis processes, we examined how students' structural interpretations of algebraic expressions may have supported or constrained their flexibility in problem-solving.

Results

In this study, we analyzed the interview section where the student was asked to solve the problem "Given that $x + \frac{1}{x} = 2$, find $x^3 + \frac{1}{x^3}$, find ." This is a typical problem which students will encounter after learning the sum of cubes formula in Grade 10 in China. It can be solved by a standard solution ("from the problem to the given condition"): converting the problem $x^3 + \frac{1}{x^3}$ to the expression of condition $x + \frac{1}{x}$ by using the sum of cubes formula $(x^3 + \frac{1}{x^3} = (x + \frac{1}{x})(x^2 - 1 + \frac{1}{x^2}))$ and the sum of squares formula $(x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2)$. Students in our study have learned both formulas before the interview. Alternatively, the question can also be solved with the idea "from the given condition to the problem": cubing the condition first and then converting it into the format of the problem.

Table 1 (see Appendix A) presents three excerpts of problem analysis process of three students before they begin writing their solutions. All three initially solved the problem using the standard problem-to-condition approach, yet their verbal articulations revealed distinct differences in their structure sense. As shown in Table 1, Clare identified the problem as involving a sum of cubes based on its structural features. While she recognized the expression as a sum, she specifically emphasized the presence of cube terms, which appeared to cue her toward applying the sum of cubes formula. This can be interpreted from her direct transition from identifying the problem structure to the usage of the sum of cubes formula without additional reasoning. After successfully giving the first solution, the interviewer asked Clare if there were any other strategies for solving the problem, Clare immediately responded "No" without hesitation.

Similarly, Dannie stated that she saw the element "cube" in the problem, which prompted her recall of the sum of cubes formula. This indicates that she focused more on the problem's surface structure (i.e. the number of power, 3) rather than its structural relationships. Furthermore, her reference to the sum of cubes formula as the "cubic formula" reinforces this interpretation, as it suggests that she associated the formula's application primarily with the presence of a cube rather than recognizing broader structural connections. Like Clare, when asked if multiple solutions existed, Dannie initially answered "No." Since neither Clare nor Dannie identified an alternative approach, we classify them as having no potential flexibility. Their explanations demonstrated their relatively low structure sense, as they primarily matched the problem's surface features to a known formula rather than engaging in deeper structural analysis. This formula-driven perspective even limited their expectation that alternative solutions might exist.

In contrast, while Emma also applied the standard formula initially, her approach to problem analysis was notably different. She first recognized the structure of the given condition as a sum and then treated the problem's overall structure in the same way. She explicitly connected the two by

stating that the sum in the problem was "a sum of powers, specifically the sum of cubes." Later, she reframed the problem in terms of its "sum and product" structure, referencing a concept previously introduced by her teacher. Unlike Clare and Dannie, who primarily focused on surface features, Emma analyzed the problem's structural properties, identifying both differences and commonalities between the given condition and the overall expression. Rather than merely recognizing the sum of cubes, she chose an appropriate representation that emphasized a broader structural framework ("sum and product"), demonstrating a more abstract and flexible understanding. When recalling the sum of cubes formula, she treated "cube" as a special case within a general power relationship, suggesting that her reasoning was not just procedural but structurally grounded. This indicates that Emma exhibited a stronger structure sense, engaging in deeper reasoning and demonstrating greater flexibility in problem-solving.

In sum, we found that students who represent the problem less structurally, focusing on its surface characteristics, are associated with the struggle to see the connection between the problem and the given condition. These students rely heavily on memorized procedures or formulas to solve the problem, which allows them to arrive at a correct solution but limits their ability to generate alternative approaches or even the belief that multiple solutions exist. In contrast, students with a stronger structure sense represent the problem more holistically, emphasizing its conceptual and structural characteristics over surface features. Recognizing the shared structure between the problem and the given condition enables them to interconvert expressions, enabling them to approach the problem from multiple perspectives and demonstrate flexibility spontaneously.

Conclusion

In this exploratory study, we observed a positive connection between students' structure sense and their potential flexibility. Students who perceive the underlying structure of a problem holistically are more likely to have deep procedural knowledge and demonstrate potential flexibility. Therefore, one implication of this study is the necessity of targeted interventions for students with weaker structure sense. For instance, curriculum designers could include progressive tasks that explicitly require students to identify and manipulate algebraic structures, which encourage flexibility. Given the small sample and non-experimental design, it remains unclear whether structure sense drives potential flexibility or vice versa. Future large-scale experimental studies should clarify this relationship and explore how structure sense manifests across different areas of mathematics beyond algebra.

References

Ericsson, K. A., & Simon, H. A. (1980). Verbal reports as data. *Psychological Review, 87*(3), 215-251.

Givvin, K. B., Stigler, J. W., & Thompson, B. J. (2011). What community college developmental mathematics students understand about mathematics, Part II: The interviews. *MathAMATYC Educator, 2*(3), 4-18.

Hickendorff, M. (2022). Flexibility and adaptivity in arithmetic strategy use: What children know and what they show. *Journal of Numerical Cognition, 8*(3), 367–381.

Linchevski, L., & Livneh, D. (1999). Structure sense: The relationship between algebraic and numerical contexts. *Educational Studies in Mathematics, 40*(2), 173-196.

Novotná, J., & Hoch, M. (2008). How structure sense for algebraic expressions or equations is related to structure sense for abstract algebra. *Mathematics Education Research Journal, 20*(2), 93-104.

Pittalis, M. (2023). Young students' arithmetic-algebraic structure sense: An empirical model and profiles of students. *International Journal of Science and Mathematics Education, 21*(6), 1865-1887.

Richland, L. E., Stigler, J. W., & Holyoak, K. J. (2012). Teaching the conceptual structure of mathematics. *Educational psychologist, 47*(3), 189-203.

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education, 36*(5), 404-411.

Tall, D., & Thomas, M. (1991). Encouraging versatile thinking in algebra using the computer. *Educational Studies in Mathematics, 22*(2), 125-147.

Appendix A

Table 1 Students' analysis of the problem

Clare	You can see that what it [the problem] is looking for is a sum of cubes, and then we can substitute it [pointing at the problem] into the sum of cubes formula and transform it.
Dannie	So I see that there is a cube here, which makes me think of the cubic formula. That is, is equal to [using the sum of cubes formula].
Emma	First, the problem gives , which is a sum. What we need to find later is also a sum, but it's a sum of powers, specifically the sum of cubes. Our teacher previously mentioned that whenever we see a sum of powers, we should think about the sum and product form. So I want to start by reviewing this target [refer to the problem] expression since the formula of the difference and sum of cubes provide some useful information.

TSG5-17

A COMPARATIVE STUDY OF INVERSE FUNCTIONS AMONG KOREAN AND CHINESE HIGH SCHOOL STUDENTS

Liu Wenting
Kaili University, China, 844503575@qq.com

Cheong-Soo Cho
Yeungnam University, Korea, chocs@yu.ac.kr

This study compared and analyzed the understanding and errors in finding inverse function equations and drawing inverse function graphs among 209 second-year high school students in Korea and China. Data was collected by extracting six types of functions from high school mathematics textbooks of both countries and designing questionnaire items based on them. The study compared and analyzed the results from six perspectives and, based on these findings, provided implications for teaching inverse functions in mathematics classrooms.

Keywords: understanding of inverse function, Korean and Chinese high school students, comparative study

Introduction

The concept of inverse functions is a fundamental idea in mathematics and plays a crucial role in mathematical thinking across all areas of the subject (Ellis, Ozgur, Kulow, Dogan, & Amidon, 2016; Ramful, 2014). It involves reversible reasoning, an essential mathematical cognitive process, yet students often struggle to fully grasp this concept (e.g., Paoletti, Stevens, Hobson, Moore, & LaForest., 2018; Zazkis & Kontorovich, 2016). The primary reason for this difficulty is that students tend to perceive inverse functions merely as an algebraic manipulation of swapping the variables x and y (Breen, Larson, O'Shea, & Pettersson, 2015), rather than understanding them as an actual process of reversal.

This study considers a comparative analysis with Chinese high school students, whose education system is similar to that of Korea, to be meaningful in identifying the causes of these results. China has achieved top scores multiple times in the PISA assessment and has demonstrated strong capabilities in mathematics and science research. Therefore, this study aims to compare and analyze the level of understanding and types of errors related to the concept of inverse functions among high school students in Korea and China. Through this comparison, the study seeks to provide insights into the teaching and learning of inverse functions.

Theoretical Background

The concept of inverse functions can be categorized into three perspectives: inverse as algebra, inverse as geometry, and inverse as the reverse of a process (Charlson & Oehrtman, 2005). The algebraic inverse refers to solving for y by swapping x and y, viewing functions as equations manipulated through algebraic operations. The geometric inverse is understood as the symmetry of a graph

with respect to the line $y = x$. The inverse as the reverse of a process considers the inverse function f^{-1} as the reversal of the process defined by the function f. These conceptualizations of inverse functions have been studied in teaching and learning, particularly in terms of their formal meaning and the undoing meaning as a reversal process (Stevens, Ko, Paoletti, Boileau, & Herbst, 2020).

Many researchers have analyzed students' and teachers' understanding of inverse functions, focusing on key elements such as the cancellation equations $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$ (Brown & Reynolds, 2007; Vidakovic, 1996), as well as the concept of injectivity (one-to-one correspondence) (Marmur & Zazkis, 2018; Wasserman, 2017). However, research findings indicate that most students and teachers do not fully comprehend these concepts.

Meanwhile, Paoletti et al. (2018) pointed out that students and teachers tend to have a limited understanding of inverse functions, focusing only on specific operations (e.g., swapping x and y, or graph symmetry). In particular, pre-service teachers showed a tendency to rely on the "conversion" technique. When drawing the graph of an inverse function, students often fail to consider symmetry with respect to the line $y = x$ or misinterpret the process as a transformation of the graph.

Other researchers (e.g., Oehrtman, Carlson, & Thompson, 2008; Teuscher, Palsky, & Palfreyman, 2018) have suggested that teaching inverse functions as a process of reversing the original function's operation is an effective approach. Research findings indicate that students who learn inverse functions through this reversal process perform better in solving inverse function tasks related to both decontextualization and contextualization.

Summarizing previous studies, the major difficulties students face in learning the concept of inverse functions can be categorized as follows: (1) symbolic misconceptions, (2) operational errors, (3) lack of recognition of symmetry, and (4) insufficient conceptual understanding due to differences in teaching methods.

Data Collection Methods and Procedures

Subjects

A total of 209 second-year high school students (162 from South Korea, 47 from China) participated in this study. All three high schools were private institutions with academic achievement levels at the mid-range within their respective regions.

Data Collection

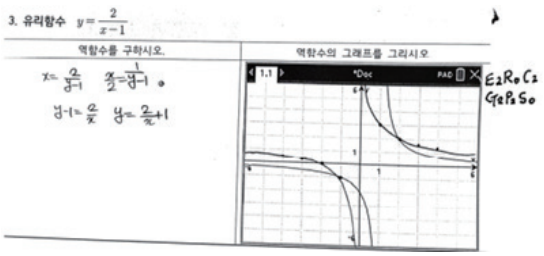
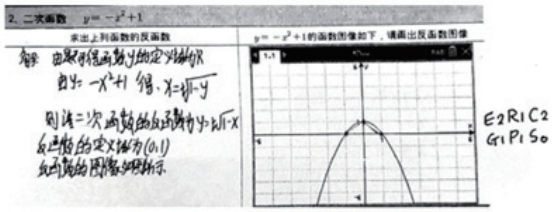
This study aims to compare the process of finding the inverse functions and graphing the inverse of six different functions: linear function $y = 3x - 2$, quadratic function $y = -x^2 + 1$, rational function $y = \frac{2}{x-1}$, radical function $y = \sqrt{x+1} - 2$, exponential function $y = 2^{-x+1} + 1$, logarithmic function $y = log_2 (x + 2)$. The questionnaire focused on deriving inverse function equations and graphing inverse functions. While domain, codomain, and range were not directly questioned, they were considered in the analysis process.

Data analysis

The data analysis was conducted based on six categories related to finding and graphing inverse functions: finding the inverse function equation (Expressions, E), considering the domain (Range, R), performing calculations for the inverse function (Calculations, C), graphing the inverse function

(Graphing, G), marking coordinates on the graph (Point, P), ensuring symmetry along the line $y = x$ (Symmetry, S). Student responses were coded as 0 (no response), 1 (incorrect answer), and 2 (correct answer). Three mathematics experts independently coded the responses, undergoing training until they achieved 90% inter-rater agreement. [Figure 1] presents an example of the coding results for the responses from Korean and Chinese students.

Figure 1. Example of coding for students' responses

Korean students' responses	Chinese students' responses
	

Findings

Comparison of Students' Results on Finding the Inverse Function Equation

The non-response rate among Korean students for finding the inverse function equations across the six functions ranged from 26.5% to 60.5%. In contrast, Chinese students had no non-responses for any function, and more than 85% correctly derived the inverse function equations for both the rational and logarithmic functions.

Comparison of Student's Results on Considering the Domain

Korean students exhibited a strong habit of disregarding the domain in the inverse process, leading to both a high error rate and a low accuracy rate. This is presumed to be due to a lack of emphasis on domain considerations when teaching the concept of inverse functions. Notably, when determining the inverse function of a rational function, Korean students did not consider the domain at all, whereas Chinese students accurately expressed the given function's domain and reflected it correctly when determining the inverse function's domain.

Comparison of Students' Results on Calculating Inverse Function Equations

Korean students showed a high non-response rate (30.9%–64.8%) in calculating inverse function equations. In contrast, Chinese students recorded a 0% non-response rate for all questions, demonstrating a thorough execution of the calculation process. Both countries showed the highest error rate in calculating the inverse function of the quadratic function $y = -x^2 + 1$, primarily due to not considering the sign (±) of the inverse function.

Comparison of Students' Results on Drawing Inverse Function Graphs

Korean students had the highest error rate in quadratic functions, while Chinese students recorded a 72.4% error rate in radical functions. Both Korean and Chinese students showed the lowest accuracy in drawing the inverse function graph of a radical function, primarily due to sketching the entire graph without considering the domain.

Comparison of Students' Results on Plotting Coordinates

Among Korean students, more than 80% drew inverse function graphs without plotting coordinates in exponential and logarithmic functions. On the other hand, Chinese students had a lower accuracy rate in correctly marking coordinates for exponential and logarithmic functions compared to other problems.

Comparison of Students' Results on Drawing Graphs Symmetric to the Line $y = x$

Most students of two countries did not utilize $y = x$ symmetry, with only a few Korean students attempting to apply it. Notably, in the case of logarithmic functions, Korean students represented the graph inaccurately, whereas Chinese students used coordinate symmetry to draw a more precise graph.

Conclusion

Across six aspects of inverse-function competence, Korean students showed markedly higher non-response and error rates—especially in handling domains and executing algebraic steps—whereas Chinese students, though stronger overall, struggled with sign conventions in quadratic inverses and with domain restrictions when graphing radicals. Both groups rarely linked an inverse equation to its graph or used the $y = x$ symmetry, signaling a shared conceptual gap that procedural skill alone cannot bridge.

To address these weaknesses, instruction should foreground *undoing* a one-to-one mapping: verify injectivity, reverse operations while tracking domains, recast the rule as a new function, plot key points, and confirm symmetry. Emphasizing domain–range exchange for Korean learners and coordinated algebraic–graphical reasoning for Chinese learners can directly target the observed error patterns, while future studies should test domain-focused and symmetry-based tasks across varied classrooms.

References

Bayazit, I., & Gray, E. (2004). *Understanding inverse functions: The relationship between teaching practice and student learning* (ED489691). International Group for the Psychology of Mathematics Education, 28th, Bergen, Norway.

Breen, S., Larson, N., O'Shea, A., & Pettersson, K. (2015). Students' concept images of inverse functions. In K. Krainer & N. Vondrová (Eds.), *Proceedings of the 9th congress of the European society for research in mathematics education* (pp. 2228-2234). Prague: Charles University and ERME.

Brown, C., & Reynolds, B. (2007). Delineating four conceptions of function: A case of composition and inverse. In Lamberg, T., & Wiest, L. R. (Eds.), *Proceedings of the 29th psychology of mathematics education* (pp. 190-193). Lake Tahoe, NV.

Carlson, M., & Oehrtman, M. (2005). Key aspects of knowing and learning the concept of function. *Educational Studies in Mathematics, 23*(3), 247-285.

Ellis, A. B., Ozgur, Z., Kulow, T., Dogan, M. F., & Amidon, J. (2016). An exponential growth learning trajectory: Students' emerging understanding of exponential growth through covariation. *Mathematical Thinking and Learning, 18*(3), 151-181.

Marmur, O., & Zazkis, R. (2018). Space of fuzziness: Avoidance of deterministic decisions in the case of the inverse function. *Educational Studies in Mathematics, 99*, 261-275.

Oehrtman, M., Carlson, M., & Thompson, P. W. (2008). Foundational reasoning abilities that promote coherence in students' function understanding. *Research and Teaching in Undergraduate Mathematics Education, 27*, 42.

Paoletti, T., Stevens, I. E., Hobson, N. L., Moore, K. C., & LaForest, K. R. (2018). Inverse function: Pre-service teachers' techniques and meanings. *Educational Studies in Mathematics, 97*, 93-109.

Ramful, A. (2014). Reversible reasoning in fractional situations: Theorems-in-action and constraints. *Journal of Mathematical Behavior, 33*, 119-130.

Stevens, I. E., Ko, I., Paoletti, T., Boileau, N., & Herbst, P. (2020). Introducing inverse function to high school students: Relating convention and reasoning. In *Mathematics education across cultures: Proceedings of the 42nd Meeting of the North American chapter of the international group for the psychology of mathematics education* (pp. 227-235). Mexico: Cinvestav

Teuscher, D., Palsky, K., & Palfreyman, C. Y. (2018). Inverse functions: Why switch the variable? *The Mathematics Teacher, 111*(5), 374-381.

Vidakovic, D. (1996). Learning the concept of inverse function. *Journal of Computers in Mathematics and Science Teaching, 15*(3), 295-318.

Wasserman, N. H. (2017). Making sense of abstract algebra: Exploring secondary teachers' understandings of inverse functions in relation to its group structure. *Mathematical Thinking and Learning, 19*(3), 181-201.

Wilson, F. C., Adamson, S., Cox, T., & O'Bryan, A. (2011). Inverse Functions: What our teachers didn't tell us. *The Mathematics Teacher, 104*(7), 500-507.

Zazkis, R., & Kontorovich, I. (2016). A curious case of superscript (-1): Prospective secondary mathematics teachers explain. *Journal of Mathematics Behavior, 43*, 98-110.

TSG5-18

GRADUATE STUDENTS IN EDUCATION' UNDERSTANDING OF ALGORITHMIC THINKING IN MATHEMATICS CLASSROOM THROUGH A COLORING BOOK WORKSHOP

Wiramon Kanlayarat
Khon Kaen University, Thailand, wiramon.k@kkumail.com

Narumon Changsri
Khon Kaen University, Thailand, changsri_crme@kku.ac.th

Maitree Inprasitha
Khon Kaen University, Thailand, inprasitha_crme@kku.ac.th

Algorithmic Thinking (AT) is an important skill for logical reasoning and problem-solving in mathematics education. This study explores how graduate students understand AT through a workshop that uses a coloring book activity based on the Open Approach. Four graduate students from different educational backgrounds participated in activities designed to develop AT's four key components, namely Search, Decomposing Problems, Linking Commands, and Recursion. Data were collected through pre- and post-workshop questionnaires, which included Likert scale items and open-ended questions, as well as observations. The findings suggest that participants improved their ability to break down complex problems, organize logical sequences, and apply recursive thinking. Participants also showed increased confidence in structuring problem-solving steps and recognizing patterns.

Keywords: algorithmic thinking, coloring book, graduate student, open approach, understanding

Introduction

In the era of rapid digital transformation, computer science has become a cornerstone of modern innovation, influencing many aspects of daily life. As technology continues to advance, the younger generation must develop essential skills to adapt and thrive (Stukalenko et al., 2016). Beyond foundational computational knowledge, logical thinking, structured reasoning, and problem-solving empower students to contribute meaningfully to this evolving world (Nahdi et al., 2024). Mathematics classrooms, once focused on numerical computation, are now seen as spaces for fostering higher-order cognitive skills, bridging mathematics and computer science education (Inprasitha, 2022).

Algorithmic Thinking (AT) has emerged as a key concept that links mathematics and computer science. Wing (2006) emphasized its pivotal role, while Kadijevich (2023) noted that AT equips students with the ability to solve problems logically and systematically, which are essential skills for success in computer science and related fields. Therefore, teaching AT in mathematics classrooms is a crucial step toward integrating computational thinking into education. One promising approach is the use of a Coloring Book within the Open Approach, as this method encourages students to work collaboratively, analyze problems critically, and develop creative reasoning (Changsri, 2024;

Changsri et al., 2024). Tasks that are structured yet open-ended allow students to practice breaking down problems, identifying patterns, and constructing logical sequences, which are all considered key components of AT.

Despite growing emphasis on AT, there is still a gap in applying it effectively in mathematics classrooms, especially through tools like the Coloring Book (Araya et al., 2021; Somsaman et al., 2021). Teachers often lack strategies for fostering AT, particularly with the Open Approach. This study addresses the gap through a workshop with graduate-level education students, investigating their understanding of AT before and after participation, and exploring how innovative practices can bridge mathematics and computational thinking.

Theoretical Framework

In mathematics education, the goal goes beyond mastering computations; it emphasizes logical reasoning and systematic problem-solving. Inprasitha (2022) highlighted that mathematical learning should cultivate skills like representing problems, justifying solutions, and reflecting on thinking. This aligns with Wing (2006), who described algorithmic thinking (AT) as central to computational thinking and closely linked to mathematical reasoning.

Somsaman et al. (2024) proposed a framework that conceptualizes AT into four core components: search (identifying specific elements under given conditions), decomposing problems (breaking down complex tasks into simpler parts), linking commands (building logical sequences or cycles), and recursion (reusing processes in repetitive or nested contexts). These components align with fundamental aspects of mathematical problem-solving, such as pattern recognition, ordering, abstraction, and generalization.

Changsri et al. (2024) showed that using Coloring Books within the Open Approach helps students engage with problems visually, reason collaboratively, and build structured thinking. This approach supports AT while enriching mathematical understanding. Thus, this study builds on the theoretical connection between AT and mathematics education.

Methodology

This study employed an experimental research design to examine graduate students' understanding of promoting Algorithmic Thinking (AT) using a Coloring Book in mathematics classrooms taught with the Open Approach. The participants were four master's degree students in education.

The research instrument was a questionnaire consisting of two parts. Part 1 was a 5-point Likert scale measuring understanding of four key components of AT, which are Search (locating elements under conditions), Decomposing Problems (breaking down complex tasks), Linking Commands (constructing sequences or cycles), and Recursion (reusing processes). Part 2 included open-ended questions to collect qualitative reflections.

Data were collected before and after a hands-on workshop where participants engaged with Coloring Book tasks following the Open Approach methodology. Quantitative data were analyzed descriptively, while qualitative data were analyzed thematically based on the four AT components.

Results

Quantitative Data Analysis

The table below summarizes the average scores and standard deviations of the participants' responses to the Likert scale questionnaire before and after the workshop. The data highlight the four key components of Algorithmic Thinking, including Search, Decomposing Problems, Linking Commands, and Recursion.

Table 1. Analysis of participants' likert scale responses before and after the workshop

AT Component	Pre-Workshop (Mean ± SD)	Post-Workshop (Mean ± SD)	Difference
Search	3.25 ± 0.50	4.50 ± 0.58	± 1.25
Decomposing Problems	3.00 ± 0.58	4.50 ± 0.50	± 1.50
Linking Commands	2.75 ± 0.65	4.25 ± 0.65	± 1.50
Recursion	2.50 ± 0.58	3.75 ± 0.50	± 1.25

The results show clear improvements in all four components of Algorithmic Thinking after the workshop. Search and Recursion saw moderate increases of +1.25, reflecting better identification of specific elements and iterative processes. The largest improvements, at +1.50 each, were observed in Decomposing Problems and Linking Commands, as participants demonstrated enhanced abilities to break down tasks and construct logical sequences. These results highlight the effectiveness of the workshop in promoting core Algorithmic Thinking skills.

Qualitative Data Analysis

Observation data were analyzed based on the four components of Algorithmic Thinking, revealing the following themes

1. Search: Participants found that identifying specific patterns or elements helped them complete tasks more efficiently. One remarked, "I learned to systematically analyze what to look for first." This was further supported by protocol data from the activity, as shown below.

   Item 22 - T: And why did you color green all the dress?

   Item 25 - D: I have a problem with this thing (Point to the belt.) I think this is a decoration, but it is still a part of the dress. So, I painted it green.

   Item 34 - T: Why did you color like this? (Point to blue balls in each box)

   Item 35 - D: Because of at least 2 balls, so 2, 3, 4, or 5 is still okay and I think it is interesting.

Participants engaged in an active search process, especially when interpreting ambiguous visual elements (e.g., the belt) or understanding constraints (e.g., "at least two blue balls"). The participant (D) demonstrated critical thinking in identifying elements beyond the explicit instruction, reflecting autonomous decision-making in the search process.

2. Decomposing Problems: Breaking down tasks into smaller steps was seen as a key strategy. One participant shared, "Dividing tasks into simpler steps allowed me to plan my work more effectively." This was further supported by protocol data from the activity, as shown below.

Item 46 - B: First of all, I painted 2 blue balls each … like … one, two, three, four like this.

Item 54 - B: I painted from this side. Left to right, first. And second, I take second place this like… Below…

Item 62 - A: First, I painted the dress green and same with Participant B, I didn't paint the belt.

Item 66 - A: And I painted the balls from left to right.

Participants demonstrated an ordered, sequential approach to handling the coloring task, carefully breaking it down into smaller sub-tasks (e.g., coloring the dress first, then the balls). They strategically prioritized which element to color first, following a stepwise approach that mirrors problem decomposition in computational thinking.

3. Linking Commands: Participants emphasized the value of seeing how steps connect. One noted, "The activity helped me see how one step leads to another, making problem-solving easier." This was also supported by protocol data from the activity, as shown below.

Item 47 - T: Why did you choose to color like this?

Item 48 - B: Yeah! This is the point. So, after that, I painted 2 balls each and I found that I wanted to paint the number like… this is I painted two (pointed to the ball she chose to color, viewing the balls horizontally.) painted two, two, two.

Item 52 - B: Firstly, I just painted this two two two two, and after that I checked the number of the paint, and I wanted to … horizontal (laughing).

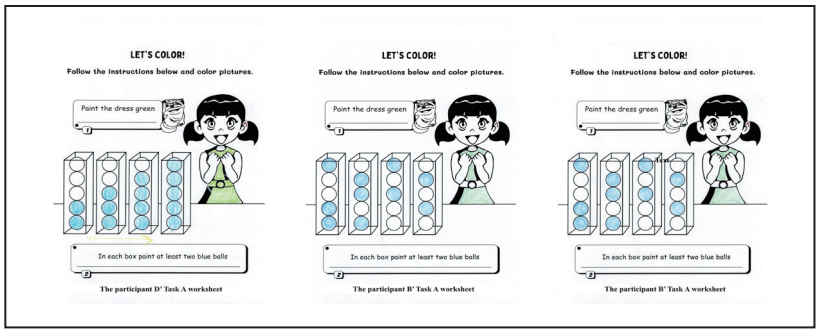


Figure. 1 The participants' task a worksheet

4. Recursion: Though initially challenging, recursion became clearer with practice. One participant noted, "Using repetitive steps was difficult at first but became clearer as I worked through examples." This was further supported by protocol data from the activity, as shown below.

Item 124 - B: I think about previous examples. So… some people think about the dress without thinking about the belt part, so why I picked two colors. Maybe some people painted the dress like upper part and lower part or just thought about this as one dress and another color in the belt part.

Item 127 - T: Participant D, why did you color balls like this?

Item 128 - D: I think the bottom balls are… umm.. in each box, the bottom one.

Item 130 - D: So, I painted the last one in each box and two balls in warm color and two balls in cool color. (Color by follow participant B' instructions)

Item 146 - All participants: Same.

Item 147 - T: Okay. Like when we have to write some coding, if we don't want the result to have errors, we have to fix it, right?

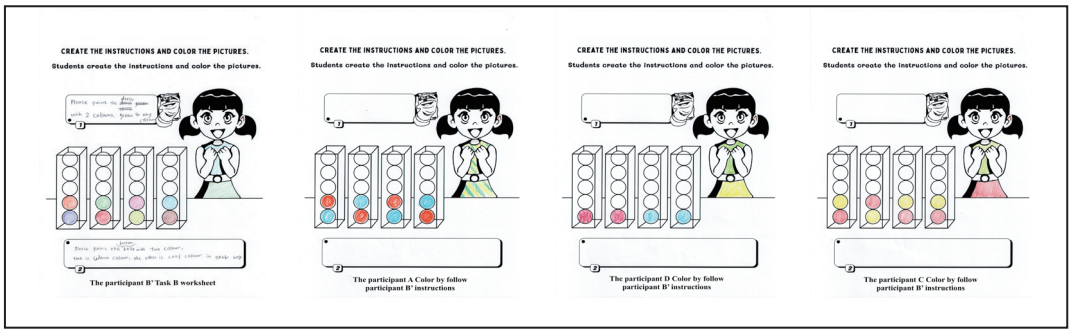


Figure 2. Instructions created by Participant B and responses from all participants in Task B

Conclusion

This study found that incorporating a Coloring Book within the Open Approach framework can effectively support the development of Algorithmic Thinking (AT) in graduate students in education. Both the quantitative and qualitative results indicated clear improvement in all four components of AT: Search, Decomposing Problems, Linking Commands, and Recursion, following the workshop. Notably, the components of Decomposing Problems and Linking Commands showed outstanding development. This suggests that step-by-step, hands-on activities can strongly enhance students' ability to think in structured and systematic ways. The qualitative insights further supported these findings by revealing participants' ability to reason critically, make independent decisions, and handle ambiguous tasks through repeated strategies. Overall, the results highlight the value of using open-ended mathematical activities together with unplugged tools to enhance learners' conceptual and algorithmic thinking. This approach may offer practical implications for designing classroom experiences that integrate AT more meaningfully into mathematics education.

References

Araya, R. (2019) *Colorea Ideas Coloring Book.* AutoMind. https://www.conectastem.cl/colorea-ideas/

Araya, R., & Isoda, M. (2021). Recommendations for curriculum reform on computational thinking. In M. Isoda, R. Araya, & M. Inprasitha (Eds.), *Developing computational thinking on AI and Big Data era for digital society: Recommendations from APEC InMside I Project* (pp. 9–39). APEC.

Araya, R., Isoda, M., & van der Molen Moris, J. (2021). Developing computational thinking teaching strategies to model pandemics and containment measures. *International Journal of Environmental Research and Public Health, 18*(23), 12520.

Changsri, N. (2024). *Pseudo coding with coloring (Mathematics classroom).* In The 17th National Open Class. Khon Kaen, Thailand: Institute for Research and Development in Teaching Profession for ASEAN.

Changsri, N., Inprasitha, M., Araya, R., & Isoda, M. (2024). Enhancing students' ideas of algorithm with coloring book in mathematics classroom. In T. Evans, O. Marmur, J. Hunter, G. Leach, & J. Jhagroo (Eds.), *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 1–117). PME.

Inprasitha, M. (2022). Lesson study and open approach development in Thailand: A longitudinal study. *International Journal for Lesson & Learning Studies, 11*(5), 1-15.

Kadijevich, D. M. (2023). *Computational/algorithmic thinking in school mathematics.* EMS Press. https://doi.org/10.4171/8ECM/40

Nahdi, D. S., Cahyaningsih, U., Jatisunda, M. G., & Rasyid, A. (2024). Mathematics Interest and Reading Comprehension as Correlates of Elementary Students' Mathematics Problem-Solving Skills. *Edukasiana: Jurnal Inovasi Pendidikan*, *3*(1), 115-127.

Somsaman, K., Isoda, M., & Araya, R. (2024). *Guidebook for unplugged computational thinking*. The Southeast Asian Ministers of Education Organization Regional Centre for STEM Education (SEAMEO STEM-ED).

Stukalenko, N. M., Zhakhina, B. B., Kukubaeva, A. K., Smagulova, N. K., & Kazhibaeva, G. K. (2016). Studying innovation technologies in modern education. *International Journal of Environmental and Science Education*, *11*(15), 7297-7308.

Wing, J. M. (2006). *Computational thinking. Communications of the ACM, 49*(3), 33- 35. https://doi.org/10.1145/1118178.1118215

Acknowledgement

This research was supported by the Center for Research in Mathematics Education (CRME), Faculty of Education, Khon Kaen University, Thailand. The authors sincerely appreciate CRME's support and the participants' valuable contributions

TSG5-19

EXAMINATION OF MATHEMATICAL DISCOURSE FOR INDIRECT PROOF OF THE CONVERSE OF THE INSCRIBED ANGLE THEOREM

Keiko Watanabe
Shiga University, Japan, mukai@edu.shiga-u.ac.jp

Masakazu Okazaki
Okayama University, Japan, masakazu@okayama-u.ac.jp

In this paper, based on Sfard's (2008) "Four Elements of Mathematical Discourse," we will examine the characteristics of the "mathematical nature" of students' discourse of a geometric indirect proof and its qualitative differences. Therefore, we consider the mathematical nature of discourse in relation to the objectivity and subjectivity of mathematics and create the framework of Examining of mathematical Discourse for the Indirect Proof.

Keywords: Indirect proof, mathematical discourse, objective perspective, subjective perspective

Introduction

Proof reflects the process of recognizing a mathematical proposition (Boero, 2007). In geometry proofs, the recognition of geometric phenomena must be consistent with the logic of the form using propositions. However, an indirect proof of geometry is not generated with the benefit of intuitive awareness of geometric phenomena, since geometrically "improbable" propositions are used in the proof. Therefore, indirect proofs of geometry require a high level of awareness of formal logic. We believe that there is a moment when the provers change their perspective on a mathematical proposition from a subjective one to an objective one, beyond the moment when the provers can construct an indirect proof. Therefore, a change in the provers' perspective is the key to developing an awareness of formal logic.

The purpose of this paper is to capture the characteristics of the discourses as students generate indirect proofs in a geometry class where the first-year high school students attempted to prove the converse of the inscribed angle theorem (CIAT).

A Framework for Examining Mathematical Discourse for Indirect Proofs

Sfard (2008) states that "mathematics begins where the tangible real-life objects end and where reflection on our own discourse about these objects begins" (p.216). The learner's discourse is expressed to recognize the existence of the objects of mathematics as an autopoietic system and to retell about them. Such discourse thus reflects the mathematical nature of the learner.

The characteristic of indirect proofs is that the proposition must be seen as a sentence with the structure "If A, then B". There must be a moment when the prover can shift the focus from the mathematical phenomenon to the truth or falsity of the proposition itself, when he or she begins to construct an indirect proof. We believe that it is at this moment that we can grasp the "qualitative

difference in mathematical discourse" in terms of the two perspectives of objective and subjective on mathematical propositions. Dawkins and Weber (2017) point to the differences in the values associated with mathematical knowledge, including proof, between mathematicians and learners of mathematics by identifying four values commonly shared in mathematicians community: (1) Mathematical knowledge is justified by a priori arguments, (2) mathematical knowledge and justifications should be a-contextual and specifically be independent of time and author, (3) mathematicians desire to increase their understanding of mathematics, and (4) mathematicians desire a set of consistent proof standards (Dawkins and Weber, 2017, p.128). The mathematicians' community strives to share a priori discourses, and these discourses value the objectification of mathematical knowledge. As high school students begin to develop values similar to those of mathematicians, there is a mixture of students who prioritize objective values and those who prioritize subjective values in mathematics classes. In this paper, we will reinterpret Sfard's (2008) "four elements of mathematical discourse: *Word Use, Visual Mediations, Narratives* and *Routines*" from the two perspectives (objective/subjective) and examine the differences in students' proofs.

Word use reflects "what the users are able to say about (and thus to see in) the world" (Sfard, 2008, p. 221). In *normative* "*word use*", mathematical objects are described from an *objective perspective* as things with unknown authors. It is a story about a fixed object outside the learner's own experience. It is neither a direct, empirical account of a mathematical object, nor a description of a personal matter. On the other hand, the process-oriented, personal "word use" tells us what "I" did with a mathematical object. It is a *descriptive* story (narrative) about one's own actions from a *subjective perspective*.

Visual mediators are visible objects that are manipulated as part of the communication process (Sfard, 2008). When learners subjectively describe the process of proof, they often use *diagrams* to explain how they understood the mathematical phenomenon. On the other hand, to generate indirect proofs ourselves, we need to *write out the propositions* to capture the if-then structure of a single proposition or/and the structure of multiple propositions that show a certain mathematical phenomenon.

Narrative is any sequence of utterances framed as a description of objects, that is subject to endorsement or rejection with the help of discourse-specific substantiation procedures (Sfard, 2008).

> "Mathematical discourse is conceived as one that should be impervious to any considerations other than *purely deductive relations* between narratives (Sfard, 2008, p.222)". However, many students construct subjective and *humanistic narratives*, and the criteria for approval vary across discourses because they incorporate each student's values and experiences. A *routine* is "a set of metarules that describe a repetitive discursive action." (Sfard, 2008, p.332).

Routines mathematize discourse, and they appear as a repeated activity in the process of proving. Mathematics develops by incorporating its own discourse mathematized (Sfard, 2008). The rules of a mathematics class evolve through the joint creation of routines by the teacher and students. The typical subjective meta-rule that governs proof activities is the way of constructing discourse that "directly" explains a proposition using concrete evidence and diagrams that are designed to *convey a sense of reality*. On the other hand, objective meta-rules are a way of constructing discourse based on *only logical rules*, looking at propositions from a bird's-eye view.

Table 1. A framework for capturing mathematical discourse from an objective/subjective perspective

Elements that capture mathematical discourse (Sfard, 2008)	Objective	Subjective
Word use	Normative	Descriptive
Visual mediations	Written proposition(s)	Diagram(s) of mathematical phenomena
Narratives	Purely Deductive Relations	Humanistic narratives
Routines	Constructing discourse based only on logical rules	Constructing discourse using expressions that convey a sense of reality

Mathematical Discourses in the Experimental Lesson

The CIAT is then formulated as "When two points C and P are on the same side of the line AB, if ∠APB = ∠ACB, then the four points ABCP lie on one circumference". In Japanese mathematics textbooks, the content of Figure 1 about the inscribed angle theorem is first shown, and the textbook states that the following proposition holds because the content shown in Figure 1 is valid. In the lesson of this paper, the teacher presented this description to the students and guided them to employ Figure 1 to prove the validity of the CIAT.

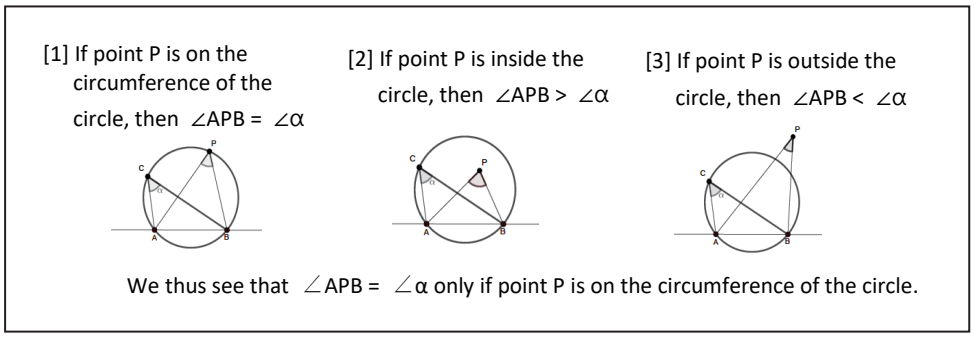


Figure 1 Textbook description (explanation A) (Okazaki & Watanabe, 2024, p.267)

The teacher asked the students to answer this question in groups of two or three. We pointed out a group of three students (Eri, Wari, and Kayu). Eri and Wari were constantly drawing diagrams and trying to prove that there are points C and P on the circumference. They agreed that the intersection of AP and BC is a point O (Figure 2) and that △AOC and △BOP are similar and ∠CAO = ∠PBO. Eri argued that there was no need to continue this proof, since it was accepted as something "obvious".

On the other hand, Wari regarded the description as a proof of what she "obviously" noticed by drawing the diagram Here, Wari added three sentences: "For arc CP, point A and point B are on the same circumference," "For arc AB, point C and point P are on the same circumference," and "So they correspond to each other's arcs." This allows her to conclude that "the arcs correspond to each other".

Despite being in the same group as Eri and Wari, Kayu did not engage in conversation with them, but

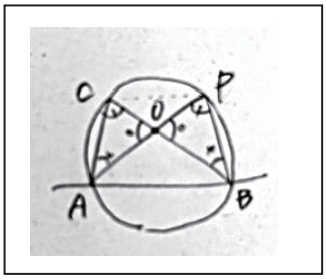


Figure 2. Wari's diagram for proving the CIAT

first stated, "Let A, B, and C be three points on a circumference, and if we take point P on the same side of the line AB as point C, then one of the following three cases is true: [I] Point P is inside the circle [II] Point P is outside the circle [III] Point P is on the circumference", next wrote "Suppose ∠APB = ∠ACB," and then muttered "Can we apply the contradiction method to this proposition?" Here, their teacher told them in a loud voice to consider the proof by making effective use of the "contradiction". Thus, Kayu proved that [I] above contradicts the inscribed angle theorem. After Kayu thought about his proof for a moment, he spoke to Eri.

Kayu: Isn't that it? Using "to be a contradiction".

Eri: What do you mean?

Kayu: First, the important thing is that you can use this, "∠APB=∠ACB because it is an assumption.

For example, in this case (above [I]), the angle APB is smaller than the angle AQB. So, using the

inscribed angle theorem, ∠AQB=∠ACB, so ∠APB is smaller than ∠ACB (using of explanation A in Figure 1). This contradicts the assumption.

Eri: What? What do you mean? Isn't ∠APB larger (than ∠ACB)? From the beginning. How else can it be shown in the diagram?

Kayu: No, that's not it. If you draw a diagram, you won't be able to see the contradiction.

Discussions

The provers who develop their discourse subjectively, like Eri and Wari, always use diagrams to explain the visible "reality", or what they have done, in a descriptive way. The proving activity carried out from a subjective perspective by Wari and Eri appeared to be progressing smoothly at first glance. However, the structure of the propositions was not discussed, and it was observed as

a discourse in which deductive relations were not always considered. Wari and Eri always used diagrams to explain the visible "reality". Just as Eri was unable to understand Kayu's proposition, "whether it can be expressed in a diagram or not" was Eri's routine for judging the truth or falsity of mathematical discourse.

When an indirect proof was generated, as in Kayu's case, the proposition was described and "what could be the assumption" was clearly stated, and the proof began by structuring the proposition. Within this structuring, it is possible to set up an "impossible situation" and consider purely deductive relationships. The mathematical nature of discourse for proving might be influenced by prover's different perspectives when confronted with mathematics.

References

Boero, P. (2007). *Theorems in school: From history, epistemology and cognition to classroom practice*. Rotterdam: Sense Publishers.

Dawkins, P.C. & Weber, K. (2017) Values and norms of proof for mathematicians and students. *Educational Studies in Mathematics*, *95*. 123-142.

Okazaki, M. & Watanabe, K. (2024) Prospective Teachers' Understanding of the Indirect Proof of Converse of the Inscribed Angle Theorem. In T. Evans, O. Marmur, J. Hunter, & G. Leach (Eds.). *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 265-272). PME.

Sfard, A. (2008) *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing*. Cambridge University Press.

TSG5-20

ENGAGING THROUGH REASONING: A CULTURAL-HISTORICAL ACTIVITY THEORY (CHAT) ANALYSIS OF STUDENTS' ENGAGEMENT IN ACCOUNTABLE TALK IN A HYFLEX MATHEMATICS CLASSROOM

Edrian Peter B. Villanueva
De La Salle University, Philippines, edrian_villanueva@dlsu.edu.ph

Frederick T. Talaue
De La Salle University, Philippines, frederick.talaue@dlsu.edu.ph

With digital transformation reshaping education, understanding how adaptive reasoning mediates student engagement in mathematical discourse is critical, particularly in HyFlex learning environments. This paper, grounded in Cultural-Historical Activity Theory (CHAT), examines how adaptive reasoning influences students' participation in Accountable Talk (AT) within a HyFlex mathematics classroom. Using a mixed-methods approach, we analyzed students' engagement across two mathematics topics. Quantitative findings show a strong correlation between adaptive reasoning and teacher-rated participation, while self-reported engagement inconsistencies suggest students may misperceive their participation. Qualitative discourse analysis reveals that students with strong reasoning skills actively engage in AT by demonstrating accountability to rigorous thinking, accurate knowledge, and the learning community. CHAT analysis highlights how adaptive reasoning mediates participation differently depending on content and learning mode. This study underscores the importance of structured discourse in fostering mathematical engagement, suggesting the need for scaffolded strategies to support reasoning in HyFlex mathematics instruction.

Keywords: HyFlex, Adaptive Reasoning, Accountable Talk, Mathematics Discourse, Cultural Historical Activity Theory

Introduction

Engaging in mathematical reasoning is fundamental to students' participation in meaningful discourse, particularly in hybrid learning environments. In the HyFlex model, which allows both in-person and online engagement, students navigate complex reasoning processes while adapting to varied instructional formats. Adaptive reasoning, a key component of mathematical proficiency, enables students to justify solutions, analyze arguments, and make connections across different mathematical contexts (Jacobs et al., 2022; Kilpatrick et al., 2001). However, the extent to which reasoning supports student engagement in Accountable Talk (AT)—a structured discourse approach that emphasizes justification and collaboration—remains an open question.

This study examines how adaptive reasoning mediates students' engagement in AT within a HyFlex mathematics classroom, where participation is shaped by social, cognitive, and technological factors. While prior research highlights the role of discourse-based strategies in fostering reasoning and engagement (Michaels et al., 2008; Resnick et al., 2018), the HyFlex setting presents unique challenges in sustaining productive discussions (Beatty, 2019; Raes et al., 2019). Using Cultural-Historical Activity Theory (CHAT) (Engeström, 2001), this study analyzes how students' reasoning abilities, AT norms, and hybrid learning structures interact to shape participation. The findings

contribute to a deeper understanding of how structured reasoning processes enable engagement in hybrid mathematics instruction.

Methods

This study is part of a larger research project examining the role of adaptive reasoning in mathematics learning within a HyFlex environment. A concurrent mixed-methods approach was employed to investigate how 34 college-level students' adaptive reasoning influences their participation in Accountable Talk (AT) within a HyFlex mathematics course. The study focused on two topics from the Mathematics in the Modern World curriculum: Mathematical Logic and Data Management. All participants completed the Mathematical Logic module, while only 25 students completed the Data Management module, resulting in a 26.5% attrition rate. To ensure consistency, only complete responses from the first topic were used in correlation analyses. Students ranged in age from 18 to 26, with self-reported genders including male, female, and prefer not to say.

To assess the relationship between adaptive reasoning and engagement, the study employed four quantitative measures. Adaptive reasoning scores were derived from custom pre- and post-tests, which included multiple-choice and explanation items. Open-ended responses were scored using a rubric that measured key indicators of reasoning, such as prediction, justification, pattern recognition, and conclusion drawing, on a 0–4 scale. Rubric reliability was established using Krippendorff's alpha (overall $\alpha = 0.80$). Teacher-Rated Participation (TRP) was based on structured rubrics reflecting AT norms, scored by the teacher after each topic. Self-Reported Participation (SRP) was gathered through a post-session Likert-scale questionnaire. Peer Evaluations were calculated by averaging the ratings given by each student's groupmates. Learning Management System (LMS) interaction data were collected from the digital platform and included logins, discussion contributions, and content access. These quantitative measures allowed for an objective assessment of how adaptive reasoning aligns with various forms of participation.

In addition to quantitative analysis, qualitative methods were employed to gain deeper insights into how students demonstrate adaptive reasoning in discourse. Classroom discussions were transcribed and analyzed using thematic coding, focusing on three key aspects of Accountable Talk: accountability to accurate knowledge, accountability to rigorous thinking, and accountability to the learning community. These dimensions were used to evaluate the extent to which students justified their reasoning, challenged ideas, and engaged collaboratively. Discourse excerpts were analyzed to identify specific reasoning strategies, such as providing justifications, questioning assumptions, and synthesizing multiple perspectives.

To provide a structured interpretation of the findings, Cultural Historical Activity Theory (CHAT) was utilized as a theoretical lens. CHAT was applied to examine the interplay between students' reasoning abilities, the tools used for discourse (including digital platforms), and the broader social dynamics influencing engagement in AT sessions. By integrating both quantitative and qualitative data, this study offers a comprehensive understanding of the role of adaptive reasoning in shaping students' participation in Accountable Talk within a HyFlex mathematics learning environment.

Results and Discussion

Using Cultural Historical Activity Theory (CHAT), this study examines how adaptive reasoning mediates student engagement in Accountable Talk (AT) within a HyFlex mathematics classroom. CHAT provides a framework for understanding how subjects (students), tools (AT and social language), rules (discussion norms), and community interact to shape participation.

Adaptive Reasoning as a Mediator in the Activity System

The subjects (students) engaged with AT as a mediating tool in mathematical discussions. Quantitative findings reveal a moderate to strong correlation between adaptive reasoning and teacher-rated participation (TRP) in Mathematical Logic ($\tau = .37$, $p < .05$) and Data Management ($\tau = .28$, $p < .05$). This suggests that students with higher reasoning skills engage more actively and align with AT norms. However, self-reported participation (SRP) correlations were weak ($\tau = .04$, $p = .77$ for Mathematical Logic; $\tau = .22$, $p = .13$ for Data Management), revealing a contradiction between student perceptions and observed engagement.

Interaction Between Tools, Rules, and Division of Labor

The division of labor within AT discussions varied based on students' reasoning abilities. Students with higher adaptive reasoning often assumed leadership roles, guiding discussions and synthesizing multiple perspectives. In contrast, students with lower adaptive reasoning engaged passively, relying on prompts from the teacher or peers. Qualitative discourse analysis shows that strong reasoners consistently demonstrated accountability to rigorous thinking by using logical connectors and justifications. For example, in Data Management discussions, students with strong reasoning skills referenced prior knowledge to analyze outliers in statistical data, strengthening their engagement. Conversely, students with weaker reasoning sought validation rather than initiating problem-solving.

Contradictions in Engagement Across HyFlex Modalities

The HyFlex model introduced contradictions in participation patterns, as in-person students engaged more actively than online students, who faced technological barriers and delayed feedback. CHAT analysis suggests that these constraints impacted students' ability to engage in AT and mediate reasoning through social interactions. Additionally, students with higher reasoning skills in virtual settings used written discourse more effectively to structure arguments, while those with weaker reasoning struggled to contribute meaningfully.

CHAT and the Misalignment of Self-Perceived Engagement

A key contradiction emerges in self-reported participation scores, which did not align with teacher-rated engagement or discourse analysis. Students with strong adaptive reasoning were highly engaged in AT, yet their self-assessments did not reflect their contributions. This suggests that metacognitive awareness gaps may lead students to underestimate their engagement, reinforcing the need for multiple assessment methods.

Implications

This study has key pedagogical and theoretical implications for mathematics education in HyFlex settings. Pedagogically, the strong link between adaptive reasoning and teacher-rated participation highlights the need for structured mathematical discourse to support engagement. Teachers should scaffold discussions to help students justify reasoning and collaborate effectively, particularly for those with lower adaptive reasoning. The discrepancy between self-reported and observed engagement suggests that alternative assessments such as peer evaluations and teacher observations are needed for accurate participation measurement.

Theoretically, this study extends CHAT by demonstrating how adaptive reasoning interacts with discourse norms, engagement structures, and digital mediation in HyFlex mathematics instruction.

The tools (AT and social language), rules (AT norms), and division of labor collectively shape how students engage in reasoning-centered discussions. Contradictions in engagement patterns—such as the discrepancy between observed and self-reported participation—underscore the complexity of reasoning as a socially mediated process rather than a purely cognitive skill.

The HyFlex model further complicates these interactions, as students in online settings often engage differently than those in face-to-face discussions. CHAT analysis reveals that digital mediation tools (e.g. LMS forums) function as alternative scaffolds for reasoning development, but they may not fully compensate for real-time verbal discourse that facilitates adaptive reasoning in in-person AT discussions. Future research should explore how digital discourse tools can better simulate the interactive nature of in-person AT, ensuring that students across HyFlex modalities experience equitable opportunities for reasoning development.

Conclusion

This study demonstrates that adaptive reasoning mediates student engagement in Accountable Talk (AT) within a HyFlex mathematics environment. Using Cultural Historical Activity Theory (CHAT), findings reveal that reasoning shapes participation roles, discourse quality, and collaborative interactions, with students exhibiting varied engagement patterns based on their reasoning abilities. Scaffolded discourse strategies and digital tools are essential for sustaining reasoning-driven engagement across learning modalities.

Key References

Beatty, B. J. (2019). *Hybrid-flexible course design: Implementing student-directed hybrid classes* (1st ed.). EdTech Books. https://doi.org/10.59668/33

Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work*, *14*(1), 133–156. https://doi.org/10.1080/13639080020028747

Jacobs, J., Scornavacco, K., Harty, C., Suresh, A., Lai, V., & Sumner, T. (2022). Promoting rich discussions in mathematics classrooms: Using personalized, automated feedback to support reflection and instructional change. *Teaching and Teacher Education*, *112*, 103631. https://doi.org/10.1016/j.tate.2022.103631

Kilpatrick, J., Swafford, J., Findell, B., & National Research Council (U.S.) (Eds.). (2001). *Adding it up: Helping children learn mathematics*. National Academy Press.

Michaels, S., O'Connor, C., & Resnick, L. B. (2008). Deliberative discourse idealized and realized: Accountable talk in the classroom and in civic life. *Studies in Philosophy and Education*, *27*(4), 283–297. https://doi.org/10.1007/s11217-007-9071-1

Raes, A., Detienne, L., Windey, I., & Depaepe, F. (2019). A systematic literature review on synchronous hybrid learning: Gaps identified. *Learning Environments Research*, *23*(3), 269–290. https://doi.org/10.1007/s10984-019-09303-z

Resnick, L. B., Asterhan, C. S. C., & Clarke, S. N. (2018). Accountable Talk: Instructional dialogue that builds the mind. In *Educational Practices Series*. The International Academy of Education.

Topic Study Groups 5
(Day 4)

TSG5-21

THE COHERENT AND STRATEGIC USES OF PIAGET'S NOTION OF PERTURBATION IN MATHEMATICS (TEACHER) EDUCATION

Biyao Liang
The University of Hong Kong, Hong Kong SAR, biyao@hku.hk

Piaget's learning theory, specifically his view of learning as a result of accommodation and reconciling perturbations, has been influential in research on students' mathematical learning. In this paper, the author discusses how the construct of perturbation is used in mathematics education research. Illustrating with empirical examples in extant studies, I first discuss how the notion of perturbation guides researchers' task design during constructivist teaching experiments to engender meaningful learning in students. Then, I discuss how researchers expand the use of this construct to the field to teacher education to promote teachers' development in mathematical knowledge for teaching.

Keywords: Piaget, learning theory, student thinking, teacher knowledge

Background and Motivation

Perturbation and accommodation are central constructs of Piaget's learning theory. According to Piaget, perturbation is a cognitive state occurring "when a person encounters an object or an event that he is unable to assimilate due to the inadequacy of cognitive structures" (Ginsburg & Opper, 1988, p. 227). Von Glasersfeld (1995) stated that being able to reconcile a perturbation through cognitive construction or reorganization (i.e., accommodation) is a hallmark of learning has occurred. Harel (2013) thus used the notion of the *intellectual need* to refer to a form of perturbation that affords learning, that is, "a perturbational state resulting from an individual's encounter with a situation that is incompatible with, or presents a problem that is unsolvable by, his or her current knowledge" (p. 122). Constructivist researchers have used the construct of perturbation explicitly and implicitly in their work, but there is no holistic discussion about how the construct is used in different ways and how it is explanatory to students' and teachers' learning activities in different contexts. In this paper, I discuss how the notion of perturbation has been operationalized in mathematics education research, including research on student learning and teachers' knowledge development.

Supporting and Accessing Student Learning

Constructivist research on student cognition has used perturbation as a critical tool to advance and access student thinking (Hackenberg, 2010; Steffe, 2004; Tzur, 1999). This is most salient within the constructivist teaching experiment methodology (Steffe & Thompson, 2000), in which researchers interact with students to systematically induce perturbations and support students in neutralizing these perturbations. Steffe and Thompson (2000) stated that a researcher's use of systematic approaches to induce perturbations and students' reconciliation of these perturbations provide the researcher with an opportunity to develop models of student learning that are more viable than those developed in instances in which students do not experience perturbations.

To illustrate, I draw on one example from my ongoing work about students' mathematical learning in a programming-based mathematical environment. In Liang et al. (under review), we report on two students' engagement in the Polygon Task. In these tasks, we asked the students to use the programming tool Scratch to first create a computer program that could draw a square, then an equilateral triangle, and finally a regular polygon with any number of sides. Figure 1 shows a breakdown of the behaviors associated with the program for drawing an equilateral triangle.

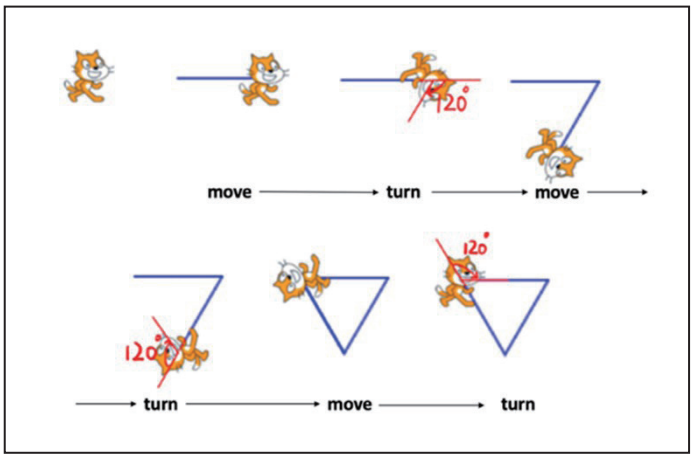


Figure 1. A simulation of the program behavior when drawing an equilateral triangle

One major challenge of the task is to identify the appropriate turning angle to construct the target polygon. Students often experience perturbations when they input a value of the interior angle into the coding block "Turn↷ [] degrees" and are unable to obtain the intended output. In our task design, we intentionally sequence the tasks so that students start with drawing a square where the interior angle and exterior angle are the same. Therefore, even if they fail to realize that the turning angle should be the exterior angle, they may still be able to obtain a satisfactory output by inputting 90 degrees. When transitioning to drawing a triangle, we expect (and observe) some students to experience perturbations after inputting 60 degrees. This allows us to gain insights into the students' debugging processes and how they accommodate their understanding. Finally, we ask students to generalize a way of calculating the turning angle to draw any regular polygon. Students may encounter further perturbations if they have focused on specific cases in the previous two tasks but have not yet generalized a method for determining the respective turning angle of any polygon. This challenge provides the intellectual need for students to construct the quantitative referent of the input angle value—namely the exterior angle—and identify a general way to calculate it of any polygon.

Promoting and Explaining Teachers' Knowledge Development

Recent research has extended Piaget's learning theory to teacher education, working to promote and explain teachers' constructions of mathematical knowledge for teaching (Liang, 2025; Silverman & Thompson, 2008; Tallman, 2023; Thompson, 2016). These researchers argued that perturbation, as an element fundamental to learning, is also essential to teachers' professional learning.

Competing Meaning as a Source of Perturbation

Moore et al. (2024) conceptualized a competing meanings perspective that identifies a form of learning that involves an individual: (1) experiencing a problematized extant meaning; (2) enacting an alternative meaning; and (3) comparing extant and alternative meanings. This perspective has important implications in advancing pre- and in-service teachers' mathematical content knowledge.

For example, Moore's team designed the Graphing Sine/Arcsine Task (Figure 2a) to support pre-service teachers in developing and reorganizing meanings for the arcsine relationship and graph. Extant literature has revealed that students tend to conceive of an inverse function as representing a different underlying relationship as the function of which it is an inverse, and thus it must be represented by a different-looking graph than its original function (Moore et al., 2014, 2019; Paoletti, 2020). With respect to the third prompt of this task, such a meaning would involve the students maintaining that Graph 1 *must* be rotated or reflected to obtain an arcsine graph. An alternative meaning grounded in covariational reasoning would involve students reasoning with amounts of change of two quantities to justify that a graph and its inverse graph do represent the *same* relationship. That is, any covariational relationship identified in Graph 1 can also be identified in Graph 2, and hence both graphs represent both relationships. The second question "Does this graph convey the same relationship as the second graph" can potentially generate perturbations in students because it invites their further thought on their work and draws the two meanings into competition with each other. Their former meaning will be problematized when the alternative meaning is available and when the students are positioned to compare the two meanings against each other.

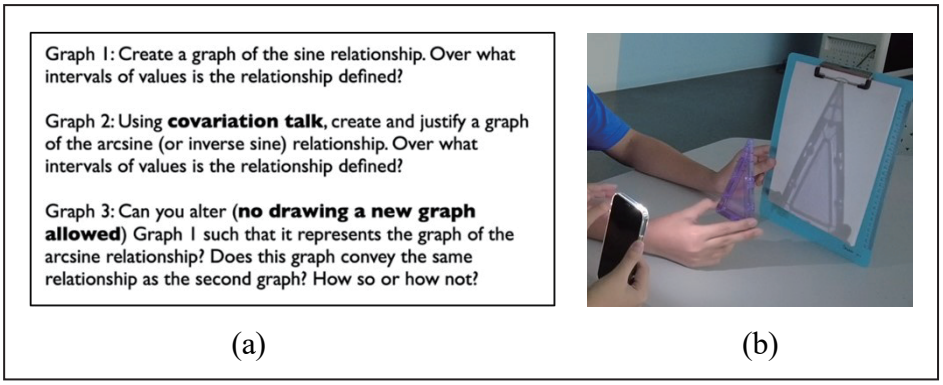


Figure 2. (a) The *graphing sine/arcsine task and (b) the triangle shadow situation*

Student Thinking as a Source of Perturbation

Another source of perturbation to teachers is student thinking. Constructivist teachers and researchers often find themselves experiencing perturbations when encountering student thinking. When attempting to understand student thinking, they inquire, "What might [these students] be thinking so that their actions make sense from [our] perspective?" They need to think within the constraints of the students, not only expecting but also actively seeking out novel ways of understanding that will allow them to be perturbed by their students (Schifter, 1998). In my work on teachers' learning stemming from student thinking, I identified evidence of teachers not only learning about what students are thinking but also enriching their personal mathematical knowledge as they reconcile the experienced perturbations arising from student thinking.

In Liang and Moore (under review), we described the teacher case of Ms. N who observed a student generating a solution that was entirely novel but led to a valid mathematical result. Ms. N experienced perturbation because she was unsure how the student's way of understanding was connected to the conventional way of understanding. In reconciling this perturbation, Ms. N worked to identify the underlying coherence between the two ways of understanding so that she could reinterpret the student's understanding through the lens of the conventional understanding, and alternatively, she could project the student's understanding onto the conventional understanding. I view this to be a minor accommodation of Ms. N because both understandings were available to her, and she reorganized them into a coherent structure.

In another case, a teacher's perturbation could lead to a major accommodation in her mathematical knowledge. Liang et al. (in preparation) described a teacher Ms. F who was working with a student to explore the relationship between the dimension of a triangle and that of its shadow (Figure 2b). At one point, the student made a hypothesis that when the triangle moves away from the light for equal amounts, the side length of its shadow increases by the same amount. Then, using a dynamic applet that simulates the situation, he explored how the side length of the shadow covaries with the distance between the triangle and the light. Eventually, he concluded that the side length of the shadow increases by decreasing amounts which contradicted his hypothesis. Ms. F was perturbed by the student's discovery because she ultimately thought the relationship was linear. This instance of student thinking motivated Ms. F to further work through the behind mathematics, and eventually, she realized that the relationship between the light-triangle distance and the side length of the triangle shadow is inverse proportional. Compared to the case of Ms. N, Ms. F's accommodation is major because she had constructed a new mathematical understanding that was not previously available.

Closing Remarks

To summarize, I discuss three uses of perturbation in research: (1) engendering students' intellectual needs to support and access student learning, (2) engendering competing meanings to promote teachers' development in mathematical knowledge, and (3) using student thinking as a source of perturbation to promote teachers' development in mathematical knowledge. The notion of perturbation both serves as a critical tool that guides educators' work with students and a useful analytical lens in research. On one hand, these uses can be applied purposefully in the contexts of task design and social interactions to influence students and teachers' learning. On the other hand, perturbation offers a theoretical lens through which researchers characterize and explain teachers' and students' learning. Future research can continue to draw attention to the critical perturbing moments in learning and investigate the learning opportunities they afford.

References

Ginsburg, H., & Opper, S. (1988). *Piaget's theory of intellectual development*. Prentice-Hall.

Hackenberg, A. J. (2010). Mathematical caring relations in action. *Journal for Research in Mathematics Education*, *41*(3), 236–273. https://doi.org/10.5951/jresematheduc.41.3.0236

Harel, G. (2013). Intellectual need. In K. R. Leatham (Ed.), *Vital directions for research in mathematics education* (pp. 119–151). Springer.

Liang, B. (2025). Mental processes underlying a mathematics teacher's learning from student thinking. *Journal of Mathematics Teacher Education*, *28*(1), 7–32. https://doi.org/10.1007/s10857-023-09601-7

Liang, B., Chen, Q., Ying, Z., & Ng, O. L. (under review). *Characterizing programming-based mathematics learning: An integration of psychological, mathematical and computational perspectives*.

Liang, B., Liu, R., Wu, Y., Huang, Y., & Liu, R. (in preparation). *Learning to decenter and decentering to Learn: A mathematics teacher's journey to exploring student meanings for geometric dilation*.

Liang, B., & Moore, K. C. (under review). *A framework for teacher's productive struggle with student thinking*.

Moore, K. C., Silverman, J., Paoletti, T., & LaForest, K. (2014). Breaking conventions to support quantitative reasoning. *Mathematics Teacher Educator*, *2*(2), 141–157. https://doi.org/10.5951/mathteaceduc.2.2.0141

Moore, K. C., Silverman, J., Paoletti, T., Liss, D., & Musgrave, S. (2019). Conventions, habits, and U.S. teachers' meanings for graphs. *Journal of Mathematical Behavior*, *53*, 179–195. https://doi.org/10.1016/j.jmathb.2018.08.002

Moore, K. C., Tasova, H. I., Bernardino, C.-S., Stevens, I. E., & Liang, B. (2024). Competing meanings, perturbation, and engendering shifts in (prospective) teacher meanings. In K. W. Kosko, J. Caniglia, S. Courtney, M. Zolfaghari, & G. A. Mooris (Eds.), *Proceedings of the forty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1145–1155). Kent State University.

Paoletti, T. (2020). Reasoning about relationships between quantities to reorganize inverse function meanings: The case of Arya. *The Journal of Mathematical Behavior*, *57*, 100741. https://doi.org/10.1016/j.jmathb.2019.100741

Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education*, *11*, 499–511. https://doi.org/10.1007/s10857-008-9089-5

Steffe, L. P. (2004). On the construction of learning trajectories of children: The case of commensurate fractions. *Mathematical Thinking and Learning*, *6*(2), 129–162.

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In R. Lesh & A. E. Kelly (Eds.), *Handbook of research design in mathematics and science education* (pp. 267–307). Lawrence Erlbaum Associates.

Tallman, M. (2023). What Makes Pedagogical Content Knowledge "Pedagogical"? Reconnecting PCK to Its Deweyan Foundations. *The Mathematics Educator*, *31*(1), 100–128.

Thompson, P. W. (2016). Researching mathematical meanings for teaching. In L. English & D. Kirshner (Eds.), *Handbook of International Research in Mathematics Education* (pp. 435–461). Taylor and Francis.

Tzur, R. (1999). An integrated study of children's construction of improper fractions and the teacher's role in promoting that learning. *Journal for Research in Mathematics Education*, *30*(4), 390–416. https://doi.org/10.2307/749707

von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning*. Falmer Press.

TSG5-22

USING ACE TEACHING CYCLE IN DEVELOPING A PRELIMINARY GENETIC DECOMPOSITION: A PROPOSED FRAMEWORK

Christian S. Abasta
Ateneo de Manila University, Philippines, christian.abasta@student.ateneo.edu

Lester Hao
Ateneo de Manila University, Philippines, lhao@ateneo.edu

APOS Theory is a theory that explains how students learn mathematical concepts by constructing their own knowledge. A genetic decomposition (GD) is a theoretical model that describes the development of students' understanding based on their knowledge constructions, whereas a preliminary GD (PGD) is a model that has not been empirically tested. APOS theory is conventionally used as a research framework that involves the development of a PGD, which informs the design and implementation of APOS-based instruction, called the ACE Teaching Cycle. Findings of the instruction are then used to refine the PGD. Traditionally, developing a PGD for a mathematical concept is anchored on a researcher's knowledge of APOS Theory, teaching experiences, and existing literature about the mathematical concept, which tends to focus on teacher-researcher's perspective. In response, the present study proposes a framework that considers the actual learning experiences of students through the ACE Teaching Cycle, in which the results are subsequently used to develop a PGD. Such a manner of development is seen as more student-centered and more reflective to student's cognitive processes.

Keywords: APOS Theory, ACE Teaching Cycle, Preliminary Genetic Decomposition

Introduction

APOS theory is a mathematics education theory that focuses on individual understanding of a mathematical concept, which is developed through an individual's construction and reconstruction of the mental structures, namely, the *actions, processes, objects, schemas* (Dubinsky, 2020; Dubinsky & Mcdonald, 2002). These mental structures are formed through mental mechanisms, namely: *interiorization, encapsulation, de-encapsulation, coordination,* and *reversal.* Arnon et al. (2014) illustrated the interrelatedness of the mental structures and mechanisms as follows: First, a concept in mathematics is conceived as an *action*, where students transform physical or mental objects through a series of instructions in which they perform a transformation either explicitly or from memory. Second, when students repeat and reflect on the *action*, then it turns into a *process* (i.e., interiorized *action*) through a mental mechanism called *interiorization.* This internal mental construct is manifested when students can perform similar *actions* without a need for an outside influence (e.g., teacher, classmate). Third, when a student displays awareness of the entire *process* and realizes that transformations can work on the *process* itself, then the *process* is *encapsulated* into a mental *object.* If necessary, an *object* can be *de-encapsulated* back into the *process,* indicating that acquiring mathematical knowledge under APOS Theory is nonlinear but hierarchical. Moreover, two or more existing *processes* can be *coordinated* to form a new *process.* Likewise, an existing *process* may be *reversed* to form another *process.* Finally, when these mental structures are organized into a coherent framework, *schema* of the mathematical concept is then constructed.

In APOS Theory, a genetic decomposition (GD) is a theoretical model which explains the construction of mental structures of an individual through mental mechanisms to understand a mathematical concept (Arnon et al., 2014; Asiala et al., 1996). Subsequently, the model is referred to as a preliminary GD (PGD) if it has not been experimentally tested or undergone refinement. Asiala et al. (1996) discussed a research framework that involves the development of a PGD as well as its refinement in Figure 1, which is briefly elaborated as follows: A PGD is conventionally developed through the researcher's knowledge of APOS theory, their teaching and learning experiences of and existing academic papers about a mathematical concept, or the concept's historical development (Arnon et al., 2014). Afterwards, the model informs the experimental instruction called the *ACE teaching cycle*, which is comprised of three phases: activity, classroom discussion, and exercise. Lastly, findings obtained from the instruction, possibly through observations and assessments, are then used to refine the PGD. This cycle can be repeated until the model satisfactorily describes the mental constructions of students in learning the mathematical concept (Arnon et al., 2014).

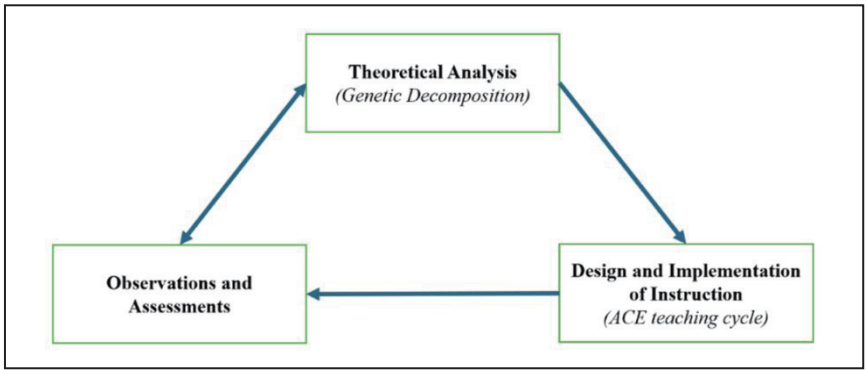


Figure 1. APOS Theory as a framework for research and development

In constructing a PGD, many studies (e.g., Asiala et al., 1996; Brijlall & Bansilal, 2010; Arnon et al., 2014; Duque-Marín et al., 2020; Tarr & Maharaj, 2021) tend to solely base their development on the perspective of researchers, i.e., researcher's teaching experiences and literature. While the researcher's teaching experiences and reflections are assumed to include students' input in the development of the PGDs, the cited studies may not have explicitly considered the actual experiences of students with the direct intention of constructing their respective PGDs. This is what the present research attempted to address by being more explicit in the development of a theoretical model and notably considering the actual learning experiences of students in the process of developing a PGD. By flipping the order of GD and ACE Teaching Cycle, the present study proposes the following conceptual framework in developing a PGD as shown in Figure 2.

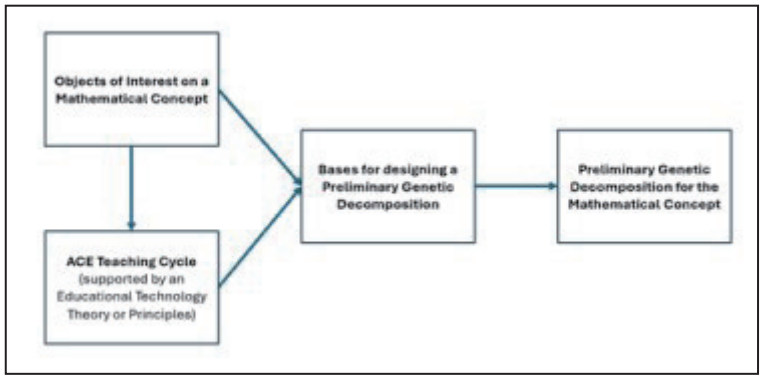


Figure 2. Conceptual framework of the present study

With the said modification, the present study argues that students' actual experiences are more directly characterized in the research process and eventually in the resulting theoretical model.

In light of the aforementioned, the present study aims to respond to the question: What insights were derived from the development of a PGD using the proposed framework?

Revisiting the Larger Study

The larger study that this paper is part of was an exploratory research that focused on constructing a PGD for graphing rational functions. Anchoring on the framework shown in Figure 2, the study employed systematic literature review and reflexivity as methods to determine the *objects of interest* (i.e., concepts, flow, and procedures students must grasp to learn a mathematical topic) on graphing rational functions, such as textbooks, learning modules, and researcher's teaching and learning experiences. Data obtained were subsequently used to design the ACE Teaching Cycle, from which classroom observations were utilized to reflect on students' class participation and their written works in learning graphing rational functions. Seven Grade 11 students underwent four teaching sessions of ACE Teaching Cycle inside a computer laboratory which were facilitated by the lead author. In addition, the participants were classified as high (HP), mid (MP), and low-performing (LP) students, and three heterogeneous groups were formed based on the said classifications. Data collected from the ACE Teaching Cycle were then analyzed through content analysis to determine specific *mental structures* and *mechanisms*, and served as bases in developing a PGD for graphing rational functions.

Discussion

By directly incorporating the actual learning experiences of students in developing a PGD, it is more likely for the resulting theoretical model to correctly predict the cognitive processes and challenges students encounter when learning a certain mathematical concept. For example, through the ACE Teaching Cycle, the study found that LP students encountered more challenges, such as difficulty in operating real numbers. Thus, it was found that some students, especially the LP ones, struggled in performing certain *actions* to determine corresponding characteristics of a rational function. This informed the researchers to include *prerequisite knowledge* in the PGD, in addition to *knowledge proper* (i.e., immediate knowledge in graphing rational functions). The *prerequisite knowledge* functions as a scaffolding for students in constructing certain *action* or *process* on a rational function, which will eventually lead to sketching its graph. The aforementioned was con-

sidered in the development of the PGD, unlike that of Tarr and Maharaj (2021) who assumed in their PGD for indefinite integrals that students had adequate knowledge about functions. They later found that majority of their participants were not equipped with prior knowledge of functions.

Additionally, the larger study incorporated technology through the use of Desmos Classroom; Mayer's (2009) principles of multimedia learning were adopted in ACE Teaching Cycle. It was observed that three MP and HP students remembered some activities inside the computer laboratory that helped them graph rational functions. For example, in graphing $h(x) = (x - 4) / (x^2 + x - 6)$, one student asked why they found two values of x in finding the vertical asymptotes of the function. Eventually, they realized that a rational function may have more than one vertical asymptote based on their Desmos Classroom activity. Moreover, after finding that the x-intercept is (4, 0), some students were hesitant to draw the graph of the function that passes through (4, 0) because they knew that $y = 0$ is the horizontal asymptote of the function. Later, the three groups of students realized that a horizontal asymptote may intersect the graph of a rational function; this was presented as an example in one of their Desmos Classroom activities as well. These observations show the *interiorization* of certain *actions* to the *process* understanding of asymptotes of a rational function.

Furthermore, the proposed framework adds focus on students' learning experiences, in which students take an active role in their knowledge construction. This aligns with the ideals of student-centered learning (Keiler, 2018). Herein, the study's ACE Teaching Cycle features cooperative learning (Arnon et al., 2014) as a student-centered approach (Spooner, 2015). In the larger study, it was observed that the LP students in their respective groups struggled in some parts of the graphing process. Their groupmates, especially the HP ones, helped them in accomplishing the task. For example, one LP student was having difficulty finding the zeroes of $h(x) = (x - 4) / (x^2 + x - 6)$, the HP groupmate provided directions on how to factor the quadratic trinomial. In the end, the LP student successfully factorized the trinomial and found the domain of the function. However, it was noted that despite accomplishing several tasks with support from their groupmates, the LP students still struggled with graphing rational functions in general. Nevertheless, cooperative learning was, at least, shown to contribute to the construction of *action* among LP students.

The aforementioned show how conducting the ACE Teaching Cycle could potentially inform the construction of the PGD. Prior knowledge and existing challenges could possibly be identified when the actual learning experiences of the students are considered through the ACE Teaching Cycle. Moreover, incorporating pedagogical tools and approaches, such as technology and cooperative learning, could also shed light on cognitive pathways in learning a certain mathematical concept. Thus, this paper argues that the proposed framework fosters the relationship between theory and practice, as the resulting PGD is grounded in the realities of actual classroom scenarios.

Conclusion

Developing a PGD for a mathematical concept is traditionally based on researcher's teaching experiences, reflections, and literature as well as knowledge about APOS Theory. This framework tends to allow researchers to assume students' knowledge about the mathematical concept concerned. While this method is theoretically sound and efficient, it may not fully grasp the intricacies of actual learning experiences, especially when students come from different performance levels. While the study's framework is time-consuming and more complex compared to the traditional framework, it argues that considering the actual learning experiences of students through the ACE Teaching Cycle allows for a more student-centered approach in developing a PGD that further reflects the cognitive pathways of students and the challenges that they encounter in learning mathematics.

References

Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Roa Fuentes, S., Trigueros, M., & Weller, K. (2014). *APOS Theory: A Framework for Research and Curriculum Development in Mathematics Education*. Springer New York. https://doi.org/10.1007/978-1-4614-7966-6

Asiala, M., Brown, A., DeVries, D., Dubinsky, E., Mathews, D., & Thomas, K. (1996). A framework for research and curriculum development in undergraduate mathematics education. In J. Kaput, A. Schoenfeld, E. Dubinsky, & T. Dick (Eds.), *CBMS Issues in Mathematics Education* (Vol. 6, pp. 1–32). American Mathematical Society. https://doi.org/10.1090/cbmath/006/01

Brijlall, D., & Bansilal, S. (2010). A genetic decomposition of the Riemann Sum by student teachers. *Proceedings of the Eighteenth Annual Meeting of the South African Association for Research in Mathematics, Science and Technology Education*, 131–138.

Dubinsky, E. (2020). Actions, Processes, Objects, Schemas (APOS) in Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 16–19). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_3

Dubinsky, E., & Mcdonald, M. A. (2002). APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research. In D. Holton, M. Artigue, U. Kirchgräber, J. Hillel, M. Niss, & A. Schoenfeld (Eds.), *The Teaching and Learning of Mathematics at University Level* (Vol. 7, pp. 275–282). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47231-7_25

Duque-Marín, E., Ramirez-Carrasco, C., & Altamirano-Espinoza, M. (2020). Construction of a genetic decomposition: Theoretical analysis of the concept of the value theorem medium. *Journal of Physics: Conference Series*, *1702*(1), 012025. https://doi.org/10.1088/1742-6596/1702/1/012025

Keiler, L. S. (2018). Teachers' roles and identities in student-centered classrooms. *International Journal of STEM Education*, *5*(1), 34. https://doi.org/10.1186/s40594-018-0131-6

Mayer, R. E. (2009). *Multimedia learning* (2nd ed). Cambridge University Press.

Spooner, E. (2015). *Interactive student centered learning: A cooperative approach to learning*. Rowman & Littlefield.

Tarr, H. L., & Maharaj, A. (2021). A preliminary genetic decomposition for conceptual understanding of the indefinite integral. *The Journal of Mathematical Behavior*, *63*, 100891. https://doi.org/10.1016/j.jmathb.2021.100891

TSG5-23

THE EFFECTS OF DIFFERENT INSTRUCTIONAL REPRESENTATIONS ON THE STUDENT ENGAGEMENT IN ELEMENTARY MATHEMATICS REMEDIAL TEACHING

Lan-Hsin Zhang
National Taipei University of Education, Taiwan, clanhsin@gmail.com

Chia-Jui Hsieh
National Taipei University of Education, Taiwan, paris@tea.ntue.edu.tw

This study investigates the impact of different teaching representations on student engagement in remedial mathematics education for primary school students. Utilizing a natural inquiry design, the research incorporates content analysis and a questionnaire survey to assess how teachers' use of various mathematical representations influences students' cognitive, emotional, and behavioral engagement. Data were collected from a sample of nine fourth-grade students and their teacher over a semester. The Mathematics Engagement Scale developed by Li and Gong (2019) was employed as the measurement tool, and K-means cluster analysis was used to differentiate teachers' representation styles. The findings indicate that (1) The teachers can exhibit two primary styles: favoring concrete representations and favoring abstract representations. (2) the abstract representations enhance cognitive and emotional engagement, while concrete representations foster a sense of belonging but reduce behavioral engagement. This study provides insights into effective strategies for remedial mathematics teaching.

Keywords: mathematics remedial teaching, instructional representations, student engagement

Background and Motivation

In mathematics education, representations serve as essential tools for both communication and the internalization of abstract concepts. The effective integration of diverse instructional representations can significantly enhance students' understanding and engagement. However, with the implementation of new competency-based curricula: such as the Taiwan 12-year national curriculum: traditional remedial teaching strategies face challenges in adapting both course content and instructional methods. Given that mathematics is inherently abstract, teachers must carefully select appropriate representations to aid student comprehension. In this context, the present study aims to investigate how teachers' flexible use and selection of different instructional representations in primary school remedial mathematics classes affect student engagement across cognitive, affective, and behavioral dimensions, ultimately providing concrete recommendations for practitioners and curriculum designers.

Research Purpose and Questions:

1. How do different mathematical teaching representations influence students' willingness and level of engagement in remedial mathematics?

2. What is the impact of teaching representations on the classroom practices and learning outcomes of remedial mathematics?

Literature Review

Teaching Representations

Mathematical concepts are inherently abstract, which necessitates the use of diverse representations in teaching, such as images, examples, drawings, and analogies, …, etc. The National Council of Teachers of Mathematics (NCTM, 2000) underscores that representations are a core process standard in mathematics instruction. Research (e.g., Hsieh, 2011; Song, 2008) has shown that when teachers appropriately select and integrate representations (by primarily employing concrete forms and supplementing them with mediating and abstract representations) students' conceptual understanding and affective engagement are significantly enhanced. However, it is important to note that overuse or inappropriate use of representations can negatively impact learning, especially for students with lower problem-solving accuracy (He & Lin, 1994).

Student Engagement

Student engagement is a multifaceted construct that encompasses cognitive, affective (emotional), and behavioral dimensions (Christenson et al., 2012). Cognitive engagement involves the use of learning strategies, self-regulation, and self-monitoring (Pintrich & Schrauben, 1992), while affective engagement relates to motivation, interest, and a sense of belonging (Kong, Wong, & Lam, 2003). Behavioral engagement reflects the actual level of participation in the classroom, including attention, diligence, and time spent on tasks (Patrick et al., 1993). In this study, the Mathematics Engagement Scale developed by Li and Gong (2019) is adopted, which further subdivides these dimensions into nine sub-dimensions to provide a comprehensive framework for analysis.

Research Methods

Research Design

In this study, a natural inquiry approach was adopted that combined content analysis with questionnaire surveys. The research employed the student engagement scale validated by Li and Kung (2019), which was adjusted to suit the specific course context. Additionally, observations were conducted during 15 remedial teaching sessions, where teachers used ten distinct types of representations: including examples, drawings, analogies, real-life instances, numerical examples, proofs, definitions, formulas, stories, and abstract symbols. Drawing on Song (2008), these representations were subsequently classified into three categories: concrete, mediating, and abstract.

Participants

The study involved one in-service remedial mathematics teacher and nine fourth-grade students (six boys, three girls) from a public elementary school in Taiwan. Due to incomplete participation, data from seven students were analyzed.

Instruments and Data Collection

Teaching Representation Records: Documented the types and frequency of representations used in each lesson.

Pre- and Post-Engagement Questionnaires: The study utilized a 41-item Mathematics Engagement Scale developed by Li and Gong (2019) as both a pre-test and a post-session feedback questionnaire. Due to time constraints during the post-session assessment, the post-test was divided into three versions (A, B, and C) and later merged for descriptive statistical analysis.

Data Analysis

K-means cluster analysis was applied to the data from the 15 teaching sessions to categorize the instructional representation usage. Pre- and post-test questionnaire scores measuring cognitive, affective, and behavioral engagement were then compared. Descriptive statistics were used to compare pre- and post-intervention engagement scores.

Results and Discussion

Types of Teaching Representations

This study conducted a K-means cluster analysis on the participating teachers' use of three types of representations: concrete, intermediate and abstract representations in every 15 lessons of the remedial teaching course. The results found that the participating teachers' use of teaching representations can be divided into two groups based on their Rasch logit scores, which were named *Predominantly Concrete Representation Style* (PCRS) and *Predominantly Abstract Representation Style* (PARS). A summary of the scores of the two groups in each type of representation is shown in Table 1.

PCRS: Characterized by higher scores in both concrete and mediating representations with minimal use of abstract representations.

PARS: Marked by a primary use of abstract representations with significantly lower use of concrete representations.

Table 1. A summary of the scores of the two groups

	Group 1(n=8)	Group2 (n=7)
Cluster name	PCRS	PARS
Concrete representations	12.50	2.50
Mediating representations	6.67	0.00
Abstract representations	0.00	6.67

Student Engagement

Pre-test data showed that students in classes taught with abstract representations had higher cognitive (3.92) and affective (4.00) engagement compared to those taught with concrete representations (3.54 and 3.64, respectively), although the behavioral engagement for the abstract group (3.55 pre-test and 3.19 post-test) declined. In the post-session feedback, students in the concrete representation group generally recorded slightly lower scores across most sub-dimensions, with the exception of a higher sense of belonging in affective engagement and increased time spent in behavioral engagement. In contrast, the abstract representation group exhibited improvements in cognitive and affective strategies but a general decrease in behavioral engagement. These results suggest that

while the use of abstract representations may promote deeper cognitive and affective engagement, the increased cognitive load might reduce students' active participation; conversely, concrete representations help establish a friendly classroom environment, though they may not fully enhance overall engagement.

Table 2. The summary scores at each level and the difference between the two types of teaching

Type	PCRS			PARS		
Mean	Pre-test	post-test	difference	Pre-test	post-test	difference
Cognitive Engagement	3.54	3.31	-0.23	3.92	3.99	0.06
Emotional Engagement	3.64	3.53	-0.11	4.00	4.13	0.13
Behavioral Engagement	2.90	2.87	-0.02	3.55	3.19	-0.36

Correlation Between Teaching Representations and Student Engagement

A significant correlation was found between the type of teaching representation and the level of student engagement. Abstract representations were positively correlated with cognitive and emotional engagement, suggesting that these methods promote deeper intellectual involvement and emotional connection to the subject matter. Conversely, concrete representations were more effective in fostering behavioral engagement, as students felt more comfortable and motivated to participate in classroom activities.

The study found that under instruction emphasizing concrete representations, students' affective engagement slightly decreased. However, students perceived that the teacher treated all students equally, with respect, and in a friendly manner during mathematics classes. In other words, when teachers employed more concrete representation-based teaching, students continued to perceive a supportive classroom environment, although they did not experience an increase in achievement-related joy. In contrast, students who received instruction with a greater emphasis on abstract representations showed improvements in both surface-level learning strategies and deep-level self-regulated learning strategies within cognitive engagement after the course. Their affective engagement, specifically in terms of interest and sense of belonging, also improved, while their achievement orientation scores declined compared to the pre-test. Regarding behavioral engagement, students' focus, diligence, and time investment were lower than in the pre-test. These results suggest that when students are exposed to more abstract instructional methods, the time they spend on learning mathematics tends to decrease.

Conclusion and Recommendations

This study confirms that different instructional representations in elementary mathematics remedial teaching significantly affect student engagement. Specifically:

- **Abstract representations** enhance students' cognitive and affective engagement, potentially leading to better conceptual understanding and intrinsic motivation.
- **Concrete representations** foster a friendly learning environment and a strong sense of belonging, though they may have limited impact on promoting active behavioral participation.

It is recommended that teachers adopt a flexible approach in remedial teaching (combining various representations based on lesson content and student needs) and provide appropriate guidance to achieve balanced development across all three engagement dimensions. Given the limited sample size in this study, future research should involve a larger sample to explore additional influencing factors and further validate these findings.

References

Christenson, S. L., Reschly, A. L., & Wylie, C. (2012). *Handbook of research on student engagement*. Springer.

Ho, Y. C., & Lin, C. C. (1994). The Effects of Representation Strategy Teaching on the Problem Solving. *Bulletin of Educational Psychology, 27*, 259~279.

Hsieh, C. J. (2011). *Concept Images for Mathematics Teaching of Mathematics Intern Teachers: The Aspect of Student Engagement.* National Taiwan Normal University.

Hsieh, F. J. (2012). Indicators of Professional Mathematics Teaching Competence at The Secondary School Level. *Secondary Education, 63*(3), 30–47.

Kong, Q. P., Wong, N. Y., & Lam, C. C. (2003). Student engagement in mathematics: Development of instrument and validation of construct. *Mathematics Education Research Journal, 15*(1), 4–21.

Lee, C. Y., & Kung, H. Y. (2019). The Development of the Inventory of Mathematics Academic Engagement for Junior High School Students. *Psychological Testing, 66*(1), 83–116.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Patrick, B. C., Skinner, E. A., & Connell, J. P. (1993). What motivates children's behavior and emotion? *Journal of Personality and Social Psychology, 65*(4), 781–791.

Pintrich, P. R., & Schrauben, B. (1992). Students' motivational beliefs and their cognitive engagement in classroom academic tasks. *Student Perceptions in the Classroom*, *7*, 149-183.

TSG5-24

THE TEACHER AS A MEDIATOR IN A PROPORTIONAL REASONING TASK

Elaine Yu Ling Cai
National Institute of Education, Nanyang Technological University, Singapore, elaine.cai@nie.edu.sg

Conducting a lesson on teaching through problem-solving by building on students' ideas is a demanding task. This paper explores the implementation of Teaching through Problem-solving (TTP) in a mathematics classroom, focusing on how a teacher navigated the pedagogical challenges of supporting student learning without overtly providing solutions. Drawing on Sato's (2017) perspective, the study highlights the use of "jumping tasks"—rich, accessible problems with multiple entry points and solution paths—as a means to scaffold students' independent mathematical thinking.

Keywords: problem-solving, proportional reasoning

Introduction

The Singapore Primary Mathematics Syllabus aims to enable all students to acquire mathematical concepts and skills for everyday use and continuous learning in mathematics; develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving; and build confidence and foster interest in mathematics (CPDD, 2023).

Literature Review

Helping students develop the above thinking skills requires highly developed pedagogical strategies which is dependent on a teacher's expertise. Sugiyama (2008) suggests that a Level 1 teacher lectures essential mathematics such as facts, concepts, procedures and practices, a Level 2 teacher lectures the meaning behind the mathematics and a Level 3 teacher guides students to become independent learners who can discover essential mathematics. Level 3 embodies the Teaching through Problem-solving (TTP) approach to leading a classroom by teachers who must be able to support students when they are struggling while not explaining the reasoning outright. Sato (2017) views that even for students who are beginning mathematics problem-solvers, the use of a jumping task as a springboard for learning as part of teaching through problem-solving bridges between what students are able to do independently and what they ought to learn. Moreover, such a task affords both multiple solution pathways as well as multiple entry points so that the task is accessible to all students.

However, teaching mathematics through problem solving is pedagogically demanding in three aspects: designing the problem task, anticipating students' thinking, and orchestrating discussions or *Neriage* (Takahashi, 2021). Therefore, this paper will be framed around the key research question: How did a mathematics teacher overcome challenges in TTP, mediating mathematics learning and students' development of cognition for a proportional reasoning task?

The recent emphasis on *Big Ideas* in the 2021 primary syllabus in Singapore (CPDD, 2023) listed proportionality as one of the six clusters of ideas. Here, proportionality is defined as a relationship

between two quantities that allows one quantity to be computed from the other based on multiplicative reasoning. Underlying the concept of proportionality are two quantities that vary in such a way that the ratio between them remains the same (p.16). Proportional reasoning in the Grade 5 curriculum is positioned under the topic of rate. Under this topic, students are required to work out rate as the amount of the quantity per unit of another quantity and find rate, total amount, or number of units, given the other two quantities (CPDD, 2023).

Methods

The vignettes described in this paper were developed from data collected as part of a study which focused on teaching through problem solving. In this paper, I shall present snippets of Austin's initial attempt using the TTP approach with a class of grade 5 students around a typical mathematics question on the comparison of the pricing of donuts from two bakeries. The task is presented in Figure 1.

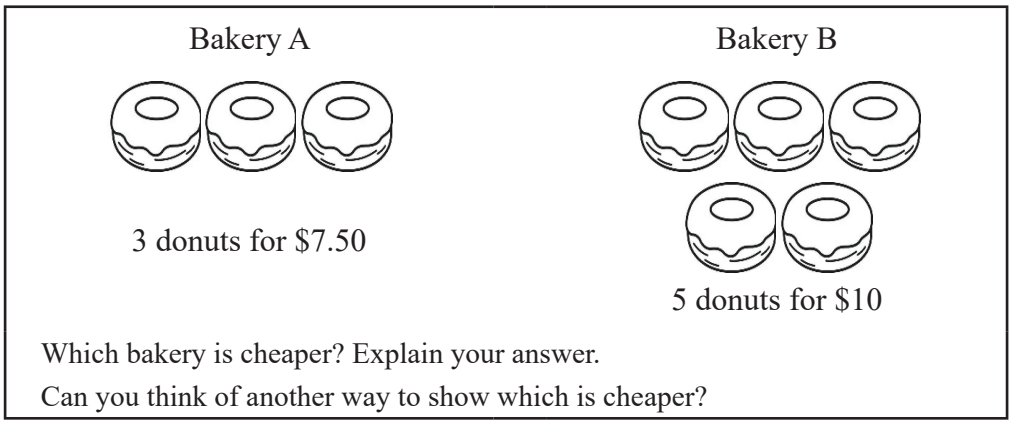


Figure 1: Task on proportional reasoning.

Data were generated from video and voice recordings of Austin and his students during the lesson, scanned students' class work and instructional materials such as slides used during the lesson. Pedagogically significant moments related to Austin's instruction were identified and examined, so that suggestions could be made on how his teaching through problem-solving approaches could be refined to support his learners.

Findings and Discussion

Overview of the Lesson

This problem which is based on the concept of proportional reasoning is a challenge for many students based on their verbal and written responses. The task required students to initiate mathematical steps which were not explicitly stated in the question. There are a few approaches to derive the answer underlying common principles. A common approach would be to make equivalent the number of donuts from both bakeries, compare the prices of the donuts and conclude that the cheaper bakery would be the one with the lower price. A second approach would be to make the prices of the donuts from both bakeries equivalent and the bakery which allowed more donuts to be purchased would be the cheaper bakery.

Designing the Problem Task

Austin's design of the task was a suitable TTP task as it afforded opportunities for students to engage in thinking with and about mathematical ideas highlighted by Anthony and Walshaw (2009). Austin modelled thinking aloud to his students, illustrating his thought processes in an authentic scenario. This episode created an impetus for students to know how to solve such questions so that when faced with similar situations, they would be able to apply similar thought process to make economical decisions.

> Austin: We have to decide where we want to buy donuts. The prices from the two bakeries are not the same. You have Bakery A. If we buy 3 donuts, it will cost us $7.50. Bakery B sells 5 donuts for $10. Now you see, the number and the cost are all not the same. So it would be difficult for her to compare. She wants to buy the cheaper one, no one wants to buy the more expensive one.

In Austin's presentation, he narrated that the price of the donuts and the number of donuts were different, hence, adding to the difficulty of comparing the price. He also added that the choice of the bakery would be the one which has a cheaper price as a criteria. The task also required students to justify their choice of the bakery and solve the question using multiple approaches. Austin's modelling thinking aloud helped to engage learners in thinking with and about mathematical ideas. Furthermore, the teacher used a suitable starting point, showing her understanding of her students' difficulties and common misconceptions. This showed that she had considered how best to connect students' current conceptions to the task, in order to provide platforms for students to generate new ideas or come up with strategies to the problem and learn from their mistakes or sub-optimal solutions Kapur (2014).

Anticipating Students' Thinking

> Student A: 3 donuts for $7.50 and 5 donuts for $10. So $7.50 times 2 is $15 and 3 times 2 is 6 donuts. So $15 for 6 donuts and 5 donuts for $10. So 5 donuts is cheaper.
>
> Austin: But this one is not the same. This is 6 and this is 5, this is $15 and this is $10. It is not the same.

Austin gave the student feedback that her step had led to different number of donuts and different prices for Bakery A and Bakery B. The student probably did not know how to or that she had to continue listing the number of donuts and their corresponding prices in their multiples beyond the second multiple. Here, Austin showed that he had drawn on his mathematical knowledge for teaching as the depth of the teacher's understanding of the task is a major determinant of his choice of examples, explanations, exercises and follow-up to his students' responses (Ball, 2000). Therefore, the wordings, numbers and representations in the task design were carefully considered, whereby the students had to initiate making the number of donuts or the cost of donuts in both bakeries equivalent. This could not be executed in a single step as the number or cost of donuts in one bakery was not a multiple of the other.

Orchestrating Discussions

> Austin: How do you get 5 to become 15?
> Student: Times 3
> Austin: What did you do? 3 is $7.50, right? Then why did you times 2?
> Student: Because I want to find 5 donuts how much.
>
> Austin: What should you do next?

Austin orchestrated the discussion with his student not by conducting an entirely free-reigned improvisation in the classroom, but instead, he took on a more listening and interactive stance to teaching as evidenced by anticipating, interpreting and responding appropriately to her students' ideas in class. Furthermore, she had connected her students' solutions to the mathematical point of the lesson, which lies at the centre of a successful *Neriage*.

Taken together, the above vignettes showed instances of how a teacher at Level 3 would enact a TTPS lesson and in the process, support students when they are struggling while not explaining the reasoning outright. It is an approach which takes time for the teacher to adopt and develop while planning the lesson and for students to get used to in a mathematics lesson for the benefit of the development of cognition in a proportional reasoning task.

References

Ball, D. L. (2000). Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education*, *51*(3), 241–247.

Curriculum Planning and Development Division (2023). *Primary mathematics teaching and learning syllabus*. Ministry of Education, Singapore.

Kapur, M. (2014). Productive failure in learning math. *Cogn Sci*, *38*(5), 1008-1022. https://doi.org/10.1111/cogs.12107

Sato, M. (2017, October 21). *Democratizing Schools as Learning Community in Asian Context* [Powerpoint Slides]. http://school-lc.com/wp-content/uploads/SATOKeynote.pdf

Sugiyama, Y. (2008). *Introduction to Elementary Mathematics Education*. Tokyo: Toyokan Publishing Co.

Takahashi, A. (2021). *Teaching mathematics through problem solving: A pedagogical approach from Japan*. Routledge.

Acknowledgement

This paper refers to data from the research project 'Teaching Mathematics through problem-solving: Developing a pedagogical approach for low progress students in Singapore'' (OER21/22C-BH), funded by the Office of Educational Research (OER), National Institute of Education (NIE), Nanyang Technological University (NTU), Singapore as part of the NIE Education Research Funding Programme (ERFP). The views expressed in this paper are the author's and do not necessarily represent the views of NIE.

TSG5-25

ANALYZING INTERDEPENDENCY OF COGNITIVE ATTRIBUTES IN MATHEMATICS AND SCIENCE

Jihyun Hwang
Korea National University of Education, South Korea, jihyun-hwang@knue.ac.kr

Jeong Su Ahn
Sejong Academy of Science and Arts, South Korea, integer@sasa.hs.kr

Kyong Mi Choi
University of Virginia, USA, kc9dx@virginia.edu

Brian Hand
University of Iowa, USA, brian-hand@uiowa.edu

This study explores the emerging trend in mathematics and science education, shifting the focus from traditional content-driven curricula to practice-based approaches as outlined by the CCSSM and NGSS frameworks. It emphasizes the utilization of cognitive attributes within these subjects and investigates how these attributes interconnect both within and across mathematics and science. To conduct this investigation, researchers analyzed standardized test responses from a cohort of 909 students spanning grades 5 to 7. In the first phase of the study, the Generalized DINA model was applied to estimate the mastery levels of various cognitive attributes. Following this, association rule analysis was used to examine the interdependencies between these attributes. The results revealed significant differences in the patterns of cognitive attributes between mathematics and science. Specifically, the interdependencies among science attributes were found to be more vulnerable to disruptions during school transitions, although a partial recovery was observed by the seventh grade. In contrast, mathematics exhibited a strengthening of attribute interdependencies after the transition, particularly among inductive cognitive skills. Additionally, both subjects showed robust interdependent relationships in deductive cognitive attributes in the fifth grade, which tended to weaken by the sixth grade. These findings underscore the importance of developing educational strategies that foster interconnected cognitive skills and advocate for an integrated approach in the design and implementation of STEM curricula.

Keywords: Interdependency, Cognitive Attributes, Association Rule Analysis, STEM

Introduction

This study examines the interdependencies among cognitive attributes in mathematics and science education. The analysis of students' responses to standardized assessments is employed to identify patterns in cognitive attribute development and their interactions over time within each discipline.

Recent shifts in educational paradigms have witnessed an emphasis on cognitive attributes over rote content memorization, as evidenced by prominent curricular frameworks such as the Principles and Standards for School Mathematics (NCTM, 2000) and the Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices & Council of

Chief State School Officers, 2010). These frameworks promote practice standards that underscore the importance of problem-solving, reasoning, proof, communication, and presentation in mathematics learning. Similarly, the Next Generation Science Standards (NGSS; NGSS Lead States, 2013) introduce practice-based standards aimed at enhancing students' cognitive skills in science education. These developments mark a shift toward integrating cognitive attributes as fundamental elements of STEM education, acknowledging their significance alongside content knowledge.

The effective implementation of these practice standards, therefore, necessitates the leveraging of cognitive attributes such as pattern recognition and problem-solving (Hwang et al., 2020). The recognition of these interdependencies enables educators to create more cohesive and engaging learning experiences, ultimately equipping students with the cognitive skills necessary for solving complex real-world problems.

Research Questions

Building on this emerging perspective on cognitive practices, we seek to explore the interdependencies between mathematics and science cognitive attributes students used to answer standardized test items. To examine the interdependencies, we used the association rule to analyze the probabilities to master cognitive attributes.

The questions guiding this study are: (1) what does intra-discipline interdependency among cognitive attributes in mathematics and science look like? (2) How does the interdependency among cognitive attributes change as the grade level of participating students increases? Particularly, are there negative effects of school transition on interdependency among the cognitive practices?

Participants

Data were collected from a cohort of 909 fifth-grade students from school districts in Iowa. According to demographic information, 92% of the students were white, and 31.1% received free or reduced lunch in grade 5 when the SWH approach was introduced.

All students participated in the Iowa Assessments over three consecutive years (grades 5–7). Students were required to solve 66, 65, and 70 mathematics test items in grades 5, 6, and 7, respectively, within a one-hour time limit. For data analysis, their responses were recorded in binary form, with 0 for incorrect answers and 1 for correct answers.

Data Analysis

This study employs a two-phase data analysis approach. In the first phase, the GDINA model is applied to estimate students' mastery of cognitive attributes. This dataset serves as the basis for the second phase, where association rule analysis is used to examine interdependencies between cognitive attributes. Interdependency strength is measured through confidence, which assesses the likelihood of mastering one attribute given another, and lift, which evaluates the impact of these relationships on cognitive behavior.

A lift value greater than 1 indicates a positive correlation, suggesting that mastering one attribute increases the likelihood of mastering another. A lift value below 1 implies a negative correlation, where acquiring one attribute reduces the likelihood of acquiring the other. A lift value of exactly 1 signals independence, meaning one attribute does not influence the occurrence of the other.

The ranges of support and confidence values (0 to 1) reflect the strength of association. To ensure significance, a minimum support threshold is established, and patterns with high confidence and lift values are examined further. This process allows researchers to identify meaningful interdependencies, providing insights that can inform curriculum development and teaching strategies by recognizing critical cognitive relationships in learning.

Results

Figure 1 and 2 shows inductive cognitive attributes were strongly interrelated across grades 5, 6, and 7 in Mathematics. For instance, students identified as mastering 'Select' were very likely to also master 'Analyze', and vice versa. The attribute 'Represent', typically considered as deductive, was also found to be strongly related to both 'Select' and 'Analyze'. This indicates that strong interdependency can exist between cognitive attributes used in different types of reasoning, inductive and deductive inferences within mathematics.

Among the science cognitive attributes, 'Find Solution' and 'Explain' demonstrated a strong relationship at grades 5 and 7. However, it is important to consider that such interdependency may stem from students' learning experiences, where they often use these attributes together, rather than from the types of reasoning themselves. Consequently, the strength of these interdependency is likely influenced by the learning environments in which students are engaged.

A significant observation is that both science and mathematics showed interdependent relationships in deductive cognitive attributes in 5th grade, which then disappeared in 6th grade. However, while mathematics regained these interdependent relationships as the grade level increased, science did not exhibit such a recovery.

We applied association rule analysis, a data mining technique, to determine the probabilities of mastering each cognitive attribute and to observe the strength of their interdependency. By calculating support, confidence, and lift values, we were able to discern how these relationships function between areas in science and mathematics. Therefore, we recommend future research to bridge the gap between students' cognitive attributes found in classroom contexts and the probability of mastering these attributes. While the list of cognitive attributes in this study is not exhaustive, future research can expand on these findings, potentially uncovering additional cognitive attributes used in inductive and deductive inferences.

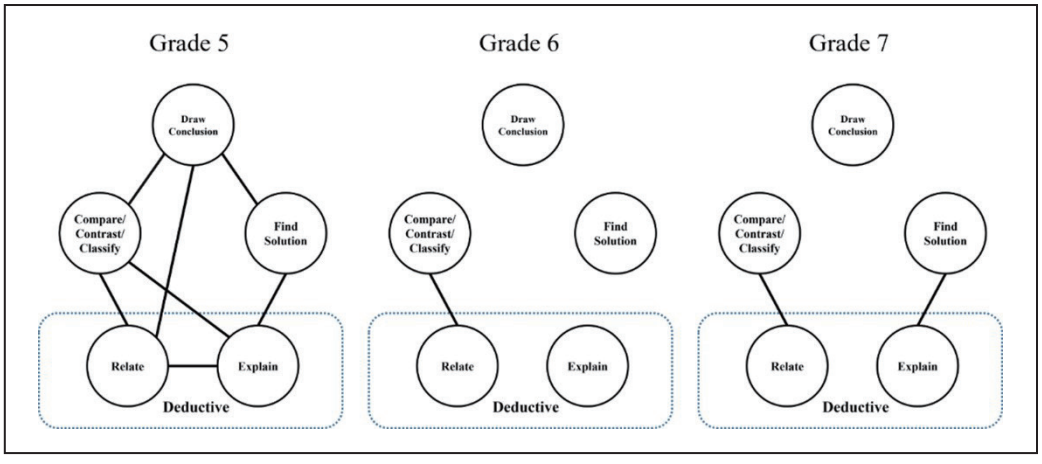


Figure 1. Interdependence between the science cognitive attributes

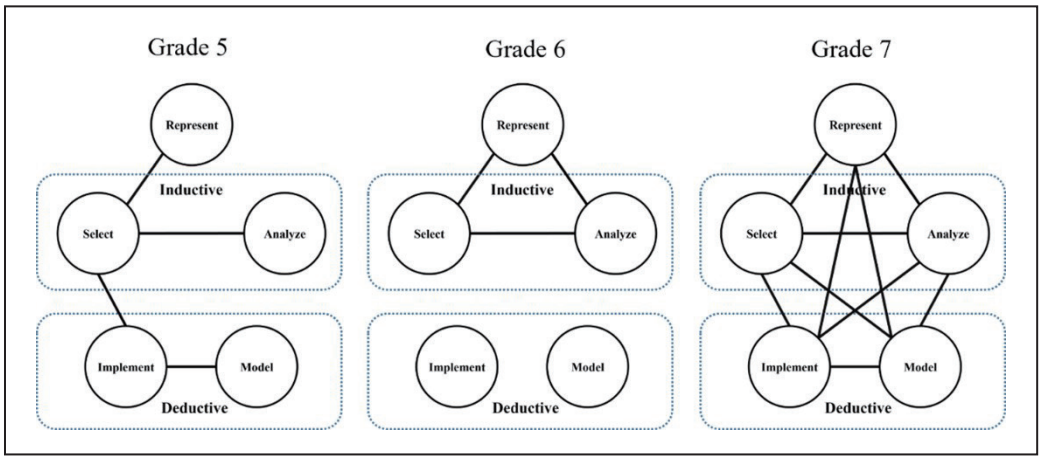


Figure 2. Interdependence between the mathematics cognitive attributes

References

Hwang, J., Choi, K. M., & Hand, B. (2020). Examining domain-general use of reasoning across science and mathematics through performance on standardized assessments. *Canadian Journal of Science, Mathematics and Technology Education, 20*, 521–537. https://doi.org/10.1007/s42330-020-00108-4.

Ma, W., de la Torre, J., Sorrel, M., & Jiang, Z. (2019). gdina: The generalized DINA model framework.R package version 2.7.3.

Mislevy, R. J. (1994). *Probability-based inference in cognitive diagnosis*. Educational Testing Service.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*. NCTM.

National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.*

NGSS Lead States. (2013). *Next generation science standards: For states, by states.* The National Academies Press.

TSG5-26

CHILDREN'S MATHEMATICAL NOTICING IN AN ONLINE PEER-GENERATED STRATEGY VIDEO

Ho-Chieh Lin
Qing Shui Elementary School, Taiwan, linhochieh@gmail.com

Theodore Chao
California State University, Fullerton, U.S.A., thchao@fullerton.edu

Julie M. Amador
University of Idaho, U.S.A., jamador@uidaho.edu

Noticing skills are vital for students to engage with peers' mathematical ideas, yet research on K-12 students' noticing remains limited. This study examines how four elementary students noticed a peer's mathematical thinking through an online strategy video. The participants first solved a fraction comparison problem individually and then watched a peer-generated strategy video. Finally, they used structured prompts to provide their noticing of the mathematical ideas presented in the video. Guided by an adapted noticing framework, the analysis reveals that the participants tended to focus on solution paths that differed most from their own. Moreover, they attended to certain erroneous details and struggled to understand how their peer arrived at their conclusions. This study highlights the need for instructional practices—such as structured video noticing tasks—that encourage students to move beyond error-spotting and toward genuine sense-making of peer mathematical ideas.

Keywords: student noticing, peer engagement, online video, elementary school education

Introduction

Empirical studies show that students rarely question or critique the ideas their peers propose during mathematical discussions (Hähkiöniemi, 2022; Rumsey et al., 2022). One explanation is that students do not fully engage with or try to make sense of their peers' ideas (Conner, 2022). Effective engagement requires robust noticing skills, defined as "selecting, interpreting, and working with particular mathematical features or regularities when multiple sources of information compete for one's attention" (Lobato et al., 2013, p. 809). Thus, understanding students' noticing could illuminate why they seldom challenge their peers' ideas.

While extensive research has examined teacher noticing, little is known about how K–12 students engage in noticing (Hohensee, 2016; Jacobs & Spangler, 2017; Lobato et al., 2013). Prior work by Lobato and colleagues (2013) demonstrates that the mathematical patterns middle school students noticed during lessons influenced their subsequent reasoning, underscoring the importance of noticing skills in mathematical learning. Yet, within the limited literature on student noticing, how K-12 students attend to and interpret their peers' mathematical thinking remains an open question.

To address this gap, this study explores how elementary children notice a peer's solution to a fraction comparison problem through a structured online video-based activity, which was developed

in the first author's prior work (Lin, 2022). Given the widespread use of video in studies of teacher noticing (e.g., van Es & Sherin, 2002, 2008), employing this medium to investigate student noticing is a logical extension. Ultimately, this study aims to offer insights into student noticing and inform innovative instructional approaches that enhance mathematical learning. The research question guiding this study is: *How do children notice a peer's mathematical thinking through viewing and commenting on their strategy video?*

Theoretical Framework

Conceptualization of student noticing is still emerging (Hohensee, 2016; Lobato et al., 2012, 2013). Lobato et al. (2013) adopted a macro perspective, defining student noticing as a phenomenon distributed across individual cognition, social interactions, material resources, and normed practices. In this study, the focus is on the individual aspect, and the framework of Jacobs, Lamb, and Philipp (2010) is adapted. Specifically, noticing is conceptualized as comprising two dimensions: (a) attending—identifying significant details in another's mathematical ideas; and (b) interpreting—using one's knowledge and experience to make sense of those observed details.

Methods

Four elementary students from a state in the midwestern region of the U.S.A. participated in this study: two fourth-grade boys (Haoxin and Cheng-Yang) and two fifth-grade girls (Mei-Ling and Pei-Chen), all pseudonyms. Three participants (Cheng-Yang, Mei-Ling, Pei-Chen) identified as Taiwanese American, and one (Haoxin) as Chinese American. Haoxin and Cheng-Yang were close friends, while Mei-Ling and Pei-Chen knew each other through shared extracurricular activities. All four reported high confidence in mathematics. Additionally, a sixth-grade Taiwanese American girl named Pin-Hsin (pseudonym) contributed her strategy video but was not interviewed.

Grounded in Goldin's (2000) task-based interview framework, all sessions were held on Zoom using a custom protocol designed for this study. During orientation, each participant learned to navigate the Flip platform (now retired) by creating a brief self-introduction video and viewing their partner's introductory video. In the main activity, each participant first solved the Pizza Parlor Problem—a fraction comparison task adapted from Lubienski (2000). This problem was chosen for its potential to elicit multiple solution methods. Once they had solved the problem, they explained their strategies to the interviewer. Next, participants individually watch Pin-Hsin's strategy video on Flip at least twice. They then completed a Google Docs–based video noticing form, using three sentence stems: "I notice…," "I wonder…," and "Also, I want to say…." These prompts were designed to guide their attention to the mathematical details and underlying reasoning in the video. Finally, participants talked about their typed responses during a one-on-one follow-up interview.

Each Zoom session lasted approximately 60–90 minutes and was video-recorded with participants' consent. Typed responses to the noticing form were collected through Google Docs, and participants' written work on the Pizza Parlor Problem was gathered for further analysis. Throughout the process, memos were written to document immediate observations and reflections.

The Pizza Parlor Problem

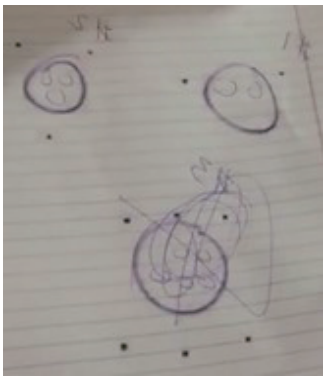
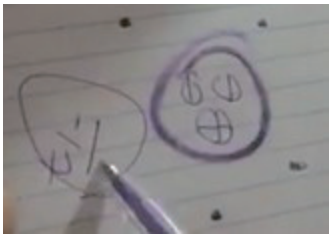
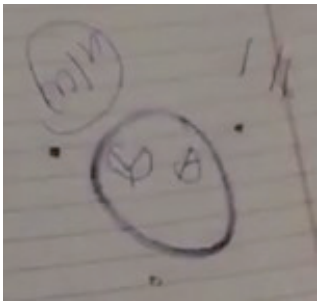
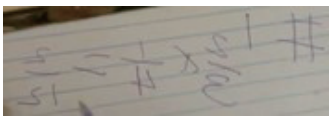
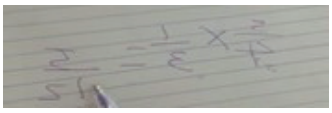
Each participant independently solved the following problem:

> Imagine that you go to a party. You see two, three, and five pizzas being served respectively at three tables of your friends: one table with two people, another table with three people, and the last table

with six people. Friends at each table call out to you to pull up a chair and want to equally share the pizza. You are hungry and want to join one of the tables where you can have the most pizza. Is there any helpful way that you can use mathematics to help with your decision-making? What are your first, second, and third choices? Explain your reasoning.

After solving the above problem, participants individually watched Pin-Hsin's strategy video, which can be divided into four parts (see Table 1).

Table 1. Pin-Hsin's strategies and noted mistakes

Part	Screenshots of *Pin-Hsin's* Strategy	*Strategies & Mistakes*
1		Table Three Strategy: Pin-Hsin drew circles and dots to represent the problem, then quickly eliminated Table Three by suggesting, "More people, you basically have less food." However, this informal reasoning was flawed because the share depends on both the number of people and the amount of pizza.
2		Table Two Strategy: Pin-Hsin used drawings to divide pizzas—splitting two pizzas into halves and one pizza into fourths—then incorrectly claimed each person would receive 1¼ pizzas. The mistake arose from misinterpreting the part-whole relationships, as the correct share should have been ¾ pizza per person.
3		Table One Strategy: Pin-Hsin divided two pizzas into thirds, arriving at a share of 2/3 per person, which was executed without error.
4	 	Common Denominator Strategy: Pin-Hsin attempted to compare 2/3 and 1¼ by converting them to equivalent fractions, but she applied the algorithm incorrectly. Rather than properly scaling the fractions, she multiplied denominators inappropriately (multiplying 2/3 by 1/4 and 1¼ by 1/3), resulting in 2/12 and 5/12. The proper method would have involved scaling 2/3 by 4/4 and 1¼ by 3/3.

Data Analysis

Data for this study came from multiple sources: (a) video recordings of research sessions and one-on-one interviews, (b) participants' typed responses in the video noticing forms, (c) participants' written work, and (d) the researcher's memos. Thematic analysis was conducted using Braun and Clarke's (2012) six-phase framework. Participants' typed responses were first organized in a spreadsheet and reviewed to generate initial codes. Video recordings were then examined to gather additional evidence that could clarify or enrich participants' noticing, and these insights were integrated into the spreadsheet. Through iterative review of all consolidated data and with continual reference to Pin-Hsin's strategy video for contextual grounding, emergent themes were identified and refined. This process ensured that the analysis remained closely tied to the participants' reasoning and their noticing of the mathematical ideas presented in the video.

Discussion and Implications

This study extends prior research on K–12 student noticing (e.g., Lobato et al., 2013) by focusing on how elementary children attended and interpreted a peer's mathematical thinking through a structured video noticing task. The findings reveal that participants selectively attended to certain parts of Pin-Hsin's strategies—most frequently the Table Three and Common Denominator approaches—while overlooking others, such as the Table One Strategy. One possible explanation is the cognitive load associated with the large amount of information presented in Pin-Hsin's video, prompting students to focus on strategies that diverged most from their own. For example, Haoxin, Cheng-Yang, and Mei-Ling all commented on Pin-Hsin's quick dismissal of Table Three, whereas they had begun their own problem-solving with Table One and approached the Table Three situation using very different reasoning.

Additionally, the fifth graders appeared to exhibit stronger noticing skills than the fourth graders, as they attended to and challenged specific numerical details like 1¼ or 5/12. However, none of the participants fully interpreted the broader rationale behind the details they identified, indicating that while emergent noticing skills are present, students still struggled to integrate isolated observations into a coherent understanding of their peers' overall reasoning. One possible explanation is that K–12 mathematics instruction, particularly in upper elementary grades, tends to emphasize correctness, speed, and procedural efficiency over deep comprehension of another person's thinking. As a result, students might have limited practice articulating and interpreting the reasoning behind someone else's solution. Instead, they tend to compare strategies, identify differences, and "grade" their peers' work rather than seek understanding.

This study is limited by its small sample size and focus on a single task. Nevertheless, the results underscore the importance of designing instructional activities that support students in genuinely making sense of their peers' mathematical ideas, rather than simply seeking out errors or differences. For instance, regularly engaging students in structured video noticing tasks may support the development of their attending and interpreting skills, guiding them beyond isolated details toward a more integrated grasp of underlying mathematical reasoning.

References

Braun, V., & Clarke, V. (2012). Thematic analysis. In H. Cooper, P. M. Camic, D. L. Long, A. T. Panter, D. Rindskopf, & K. J. Sher (Eds.), *APA handbook of research methods in psychology, Vol 2: Research designs: Quantitative, qualitative, neuropsychological, and biological.* (pp. 57–71). American Psychological Association.

Conner, A. (2022). Participation in Argumentation: Teacher and Student Roles Across the Grades. In K. N. Bieda, A. Conner, K. W. Kosko, & M. Staples (Eds.), *Conceptions and Consequences of Mathematical Argumentation, Justification, and Proof* (pp. 277–286). Springer International Publishing.

Goldin, G. A. (2000). A scientific perspective on structures, task-based interviews in mathematics education research. In *Research Design in Mathematics and Science Education*. L. Erlbaum.

Hähkiöniemi, M. (2022). Argumentation in the Context of High School Mathematics: Examining Dialogic Aspects of Argumentation. In K. N. Bieda, A. Conner, K. W. Kosko, & M. Staples (Eds.), *Conceptions and Consequences of Mathematical Argumentation, Justification, and Proof* (pp. 145–158). Springer International Publishing.

Hohensee, C. (2016). Student noticing in classroom settings: A process underlying influences on prior ways of reasoning. *The Journal of Mathematical Behavior*, *42*, 69–91.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional Noticing of Children's Mathematical Thinking. *Journal for Research in Mathematics Education*, *41*(2), 169–202.

Jacobs, V. R., & Spangler, D. A. (2017). Research on core practices in K-12 mathematics teaching. *Compendium for Research in Mathematics Education*, 766–792.

Lin, H.-C. (2022). Designing an online video-based environment for promoting mathematical argumentation. In Lischka, A. E., Dyer, E. B., Jones, R. S., Lovett, J. N., Strayer, J., & Drown, S. (2022), *Proceedings of the 44th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Middle Tennessee State University.

Lobato, J., Hohensee, C., & Rhodehamel, B. (2013). Students' Mathematical Noticing. *Journal for Research in Mathematics Education*, *44*(5), 809–850.

Lobato, J., Rhodehamel, B., & Hohensee, C. (2012). "Noticing" as an Alternative Transfer of Learning Process. *Journal of the Learning Sciences*, *21*(3), 433–482.

Lubienski, S. T. (2000). Problem Solving as a Means toward Mathematics for All: An Exploratory Look through a Class Lens. *Journal for Research in Mathematics Education*, *31*(4), 454–482.

Rumsey, C., Whitacre, I., Atabaş, Ş., & Smith, J. L. (2022). Argumentation in the Context of Elementary Grades: The Role of Participants, Tasks, and Tools. In K. N. Bieda, A. Conner, K. W. Kosko, & M. Staples (Eds.), *Conceptions and Consequences of Mathematical Argumentation, Justification, and Proof* (pp. 19–34). Springer International Publishing.

van Es, E. A., & Sherin, M. G. (2002). Learning to Notice: Scaffolding New Teachers' Interpretations of Classroom Interactions. *Journal of Technology and Teacher Education*, *10*(4), 571–596.

van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education*, *24*(2), 244–276.

van Es, E. A., & Sherin, M. G. (2021). Expanding on prior conceptualizations of teacher noticing. *ZDM – Mathematics Education*, *53*(1), 17–27.

Acknowledgements

This material is based upon work supported by a Taiwanese Overseas Pioneers (TOP) Grant from the National Science and Technology Council of Taiwan. Any opinions, findings, conclusions, or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the Council.

TSG5-27

READING COMPREHENSION OF REAL-WORLD PROBLEMS: AN ERP STUDY

Wei-Jung Lai
National Tsing Hua University, Taiwan, weilong881110@gmail.com

Yu Chen
National Tsing Hua University, Taiwan, dave900120@gmail.com

Hui-Yu Hsu
National Tsing Hua University, Taiwan, huiyuhsu@mx.nthu.edu.tw

Zai-Fu Yao
National Tsing Hua University, Taiwan, zfyao@mx.nthu.edu.tw

Chen-Yu Yao
National Tsing Hua University, Taiwan, john21997034@gmail.com

Tsu-Jen Ding
National Tsing Hua University, Taiwan, ding@mx.nthu.edu.tw

This study investigates cognitive processes in reading comprehension as students transform real-world problems into linear equations with two variables under bilingual (Chinese and English) conditions. Using event-related potentials (ERP), we examined how students allocate cognitive resources during problem-solving. Analyses focused on P300, N400, and P600 components to capture attention allocation, semantic processing, and integrative comprehension. The results revealed significant differences in working memory load and attention allocation between key and non-key information. Additionally, P600 amplitudes progressively increased, indicating greater engagement in integration and evaluation, while N400 amplitudes decreased, reflecting reduced cognitive load related to expectation processing. These findings provide insights into how students process linguistic information, interpret mathematical texts, and construct equations.

Keywords: Reading comprehension, Equation, ERP, Bilingual literacy

Introduction

Mathematics education aims to establish a strong connection between real-world contexts and abstract mathematical concepts, not only to deepen students' understanding of mathematical principles and their underlying meanings but also to enhance their ability to apply mathematics in solving a wide array of complex problems encountered in future careers (Porter et al., 2011). Proficiency in algebraic concepts provides the foundation for learning advanced mathematics and developing higher-order thinking skills (e.g., systematic reasoning). Acquiring algebraic competence is crucial for success across various disciplines, particularly in STEM fields (English, 2016). However, equa-

tions—an integral part of the algebra curriculum—have consistently been regarded as challenging for students to master (Kieran, 1992). These difficulties include the technical manipulation of equations, the interpretation of solutions, and the process of converting real-world contexts into mathematical equations (and vice versa).

This study focuses on transforming real-world contexts into linear equations with two variables and examines the role of language in shaping that transformation under bilingual education policies. Due to economic globalization, bilingual education has become a growing trend in many countries. Within this context, language serves not only as a medium of communication but also as a critical factor that influences the effectiveness of mathematics learning. Our study explores how students transform real-world context presented in both Chinese and English into mathematical equations, providing insights into the cognitive processes underpinning bilingual mathematical literacy. This transformation can be informed by linguistic research indicating that reading comprehension involves intertwined cognitive processes, such as decoding, meaning construction, and integrating prior knowledge with new information (Kintsch, 1998). Mathematics education researchers have long sought to understand how students process word problems; successful problem solvers tend to excel at extracting mathematical relationships embedded in text. By utilizing ERP technique, we aim to explore this transformation across different language formats from a reading comprehension perspective.

Literature Review

Reading comprehension involves decoding, constructing meaning, and integrating prior knowledge with new information. According to Kintsch (1998), the construction-integration model highlights two main phases: constructing a mental representation based on textual information and integrating this representation with existing knowledge. This model underscores the dynamic and interactive nature of reading comprehension. Similarly, Perfetti and Stafura (2014) examined how lexical quality affects reading comprehension, asserting that strong vocabulary knowledge and efficient word recognition are essential for understanding text. Students with more advanced linguistic skills can process complex syntactic structures and derive meaning from context more effectively.

Keywords play a vital role in guiding students' comprehension by signaling key ideas and relationships within a text. Verschaffel (2000) noted that in word problems, keywords such as "total," "per," and "difference" indicate the operations or relationships required to solve the problem. Powell et al. (2022) reported that students often use a "keyword strategy" without fully comprehending text's underlying meaning; they distinguish between cases where the keyword is tied to a specific operation and those where it is not. Students' ability to focus on crucial information varies with their reading proficiency and prior knowledge. Research by McNamara et al. (1996) demonstrates that skilled readers are better at identifying and prioritizing critical details, often guided by explicit cues or organizational structures in the text. In contrast, less-skilled readers tend to struggle with discerning relevant points from unimportant details, leading to weaker comprehension. Although numerous studies have explored reading comprehension in various fields, few have used neuroscience to investigate reading comprehension within mathematical problem-solving context.

Methodology

Instrument

We developed a test to examine how real-world context problems in Chinese and English affect students' ability to transform contexts into mathematical equations. To maintain consistency, we used identical problems and contexts in both languages. Each problem was structured into nine structured stages (S1–S9), to maintain a direct correspondence between the two language versions. Key information—such as coefficients, variables, central relationships, and total quantities—was presented in S2, S3, S5, S6, S7, and S8, while non-key information appeared in S1 and S4. Stage S9 was designated for final equation judgment. This structured approach minimized linguistic variations and enabled reliable comparisons. In S9, participants were asked to evaluate whether a given equation accurately represented the realistic context. Below is an example in both languages.

Table 1. E-prime model for test design (chinese and english versions)

	S1	S2	S3	S4	S5	S6	S7	S8	S9
Chinese	小李買了	x 張椅子	每張 250 元，	以及	y 張桌子	每張 350 元。	他總共花了	850 元。	250 x + 350 y = 850
English	Leo bought	x chairs	at 250 dollars each,	and	y tables	at 350 dollars each.	He spent a total of	850 dollars.	250 x + 350 y = 850

Subjects

Forty Taiwanese undergraduate students (23 males and 17 females) participated in the study, all of whom were right-handed. Their English proficiency was assessed using the Common European Framework of Reference for Languages (CEFR), revealing that 2 students were at the C1 (advanced) level, 21 at B2 (upper-intermediate), 15 at B1 (intermediate), and 2 at A2 (basic).

Data Collection and Analysis Procedure

The test was conducted using E-Prime software. Before the main test, participants completed several practice trials to familiarize themselves with the testing procedure. Each trial began with a 500 ms fixation period, followed by stages S1–S8, each displayed for 1200 ms, and concluded with stage S9, which was displayed for 2000 ms. An inter-stimulus interval (ISI) between each stage was 200 ms. Each trial ended with a 1000 ms resting period before the next fixation. A total of 168 trials were administered, randomized to minimize learning transfer between Chinese and English versions. Electrophysiological data were recorded continuously using a 64-electrode Neuroscan Quik-Cap at a sampling rate of 1000 Hz. Electrode impedance was kept below 5 kΩ, ensuring minimal noise interference. Event-related potentials (ERPs) were processed using BrainVision Analyzer 3 software. Zero-phase filtering was applied with a band-pass range of 0.53 to 30 Hz to remove slow drifts and high-frequency noise. The data were re-referenced to the average of all electrodes, and trials with amplitude fluctuations exceeding ±80 μV were excluded to limit artifacts related to eye movements or muscle activity. Ocular artifacts were corrected using the method proposed by Gratton et al. (1983), and channels with more than 10% artifact contamination were interpolated via spherical spline methods. EEG data were segmented into epochs ranging from -200 ms to 1000 ms relative to stimulus onset, with a 200 ms pre-stimulus baseline applied. Cleaned data were averaged to obtain ERPs for each stage and electrode site.

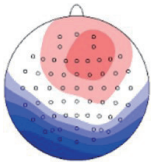
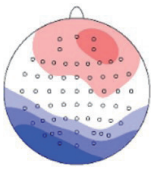
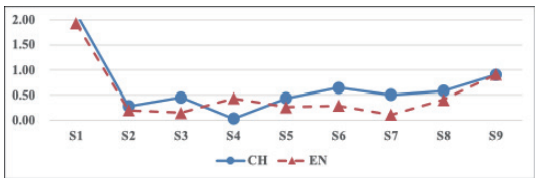
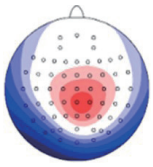
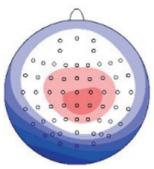
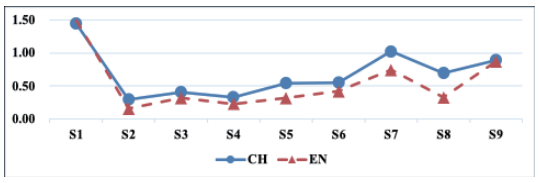
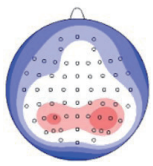
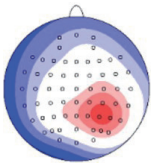
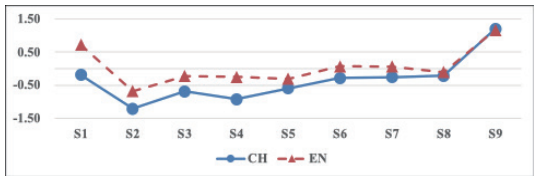
Our Analysis of Reading Comprehension Focused on Three ERP Components

P300, P600, and N400. P300 is associated with working memory and attention allocation (Donchin, 1981; Polich, 2007), indicating how students process the encoding and interpretation of problems presented in different language. Cain et al. (2004) also noted that working memory is central to reading comprehension. The P600 component represents the evaluation phase in reading comprehension; according to Monitoring Theory (Vissers et al., 2008), P600 reflects sentence reanalysis, while Brouwer et al. (2012) further described it as reflecting integration and information updating processes. N400 is involved in error detection and expectation during reading comprehension, particularly for identifying semantic incongruities and violations in real-world context problems (Kutas & Hillyard, 1980). We evaluated both accuracy and reaction times for correct responses, along with electrophysiological measures (P300, N400, and P600), across stages S1–S9 using appropriate statistical methods.

Findings

Table 2. Topographies of ERP Components (P300, N400, and P600)

Stage: S7 ; Channel: Cz

Time Window	CH	EN	Plotted figures
P300 (200 ms - 350 ms)			
P600 (600 ms-1000 ms)			
N400 (350 ms - 600 ms)			

We examined P300 amplitudes by using different stages (S1 to S9) as baselines and calculating amplitude differences across stages. The results indicated that students allocate greater neural resources to key words. Specifically, there were no significant differences in P300 amplitudes among key stages (S2, S3, S5, S6, S7, and S8). However, when comparing key-information stages with non-key stages (S1 and S4), significant differences emerged. For instance, in Chinese problems, comparisons between S2 and S5 ($P = 0.312$), S3 and S5 ($P = 0.905$), S6 and S5 ($P = 0.195$), and S7 and S5 ($P = 0.660$) showed no significant differences. Similarly, in English problems, comparisons between S2 and S5 ($P = 0.694$), S3 and S5 ($P = 0.467$), S6 and S5 ($P = 0.863$), and S7 and S5 ($P = 0.336$) were also not significant. Notably, S1 had the highest P300 amplitude across both languages (mean for Chinese = 2.131, mean for English = 1.935), differing significantly from all other stages. This finding underscores the unique processing role of S1 (see Table 2).

Integration and monitoring are metacognitive processes associated with higher-level reading comprehension and are linked to the P600 component. The analysis of P600 amplitudes revealed a pattern complementary to that of N400. Although Stage 4 showed a slight dip in amplitude, P600 amplitudes gradually increased in the later stages (see Table 2). Stages 3, 5, 6, and 7 exhibited marked increases, with Stage 7 displaying the highest amplitude (apart from Stage 1), regardless of whether the problems were presented in Chinese or English. For example, in Chinese, the mean P600 amplitudes at Stages 3 through 9 were 0.408, 0.324, 0.544, 0.551, 1.025, 0.696, and 0.894, respectively. In English, these values were 0.314, 0.227, 0.318, 0.417, 0.743, 0.325, and 0.872. These findings highlight the importance of Stage 7 as a critical juncture, where students evaluate whether the information provided aligns with the equation.

We analyzed N400 to explore processes related to expectation and error detection. Stage 2 exhibited the largest N400 amplitude (see Table 2), with particularly high values in Chinese (mean = -1.215), followed by a gradual decline in subsequent stages. This suggests that introducing variables at Stage 2 triggered robust error-detection and expectation mechanisms, regardless of whether the context was presented in Chinese or English. In contrast, amplitudes from Stages 4 through 8 steadily decreased in both languages. For instance, in Chinese, the mean amplitudes at Stage 4, 5, 6, 7, and 8 were -0.923, -0.598, -0.283, -0.255, and -0.212, respectively. These findings imply that once the variables were introduced, the additional information incrementally reduced participants' expectation-driven cognitive load. When tasks were presented in Chinese, overall amplitudes for P300, N400, and P600 were higher than for the same tasks in English.

Discussion

This study investigated reading comprehension in the context of transforming real-world problems into mathematical equations. Analyses of P300 component revealed that students devoted significantly more attention to key information relevant to forming equations, irrespective of whether problems were presented in Chinese or English. However, we found no significant differences among the key-information stages themselves, suggesting that students distinguish between key and non-key information and allocate comparable attention to each key element. Additionally, the working memory load remained consistent across these key stages, indicating steady engagement during decoding and meaning construction.

Higher-level comprehension processes were examined through the P600 component, which showed that Stage 2 elicited the smallest amplitude, while Stages 3, 5, 6, and 7 exhibited pronounced increases, culminating in the highest amplitude at Stage 7. In language-learning context, P600 has been tied to syntactic integration, suggesting that students required additional effort to integrate the equation's key components—an effect more pronounced in Chinese than in English. Although N400 is also tied to semantic processing—particularly in detecting expectations and violations—its pattern proved complementary to that of P600, regardless of the problem's language. Stage 4 showed a minor dip in P600 amplitude, followed by a steady increase in later stages, whereas N400 amplitude was initially high (reflecting error detection) but gradually tapered off, indicating decreasing cognitive load in semantic expectation as the problem unfolded. This trajectory may reflect a "second-look" or reprocessing mechanism, whereby readers review contextual meaning. In line with these findings, S2, the initial key stage, triggered strong error detection, which subsided as comprehension progressed. Overall, the interplay between N400 and P600 suggests that detection and integration processes allocate cognitive resources in tandem yet maintain some degree of independence.

When tasks were presented in Chinese, overall amplitudes for P300, N400, and P600 were higher than for the same tasks in English, indicating that language familiarity strongly influences cognitive processing during reading comprehension. Nevertheless, students in both language conditions concentrated on the critical information for forming equations, underscoring the role of key words in comprehending real-world context problems. Taken together, these findings extend beyond a simple comparison of mathematical word problems in two languages. They provide deeper neurophysiological insights into bilingual mathematical literacy, shedding light on how language proficiency can shape the cognitive processes that underlie reading comprehension in mathematics.

References

Brouwer, H., Fitz, H., & Hoeks, J. (2012). Getting real about semantic illusions: rethinking the functional role of the P600 in language comprehension. *Brain Research*, *1446*, 127-143.

Cain, K., Oakhill, J., & Bryant, P. (2004). Children's reading comprehension ability: Concurrent prediction by working memory, verbal ability, and component skills. *Journal of Educational Psychology*, *96*(1), 31.

Donchin, E. (1981). Surprise!… surprise? *Psychophysiology*, *18*(5), 493-513.

English, L. D. (2016). STEM education K-12: Perspectives on integration. *International Journal of STEM education*, *3*, 1-8.

Gratton, G., Coles, M. G., & Donchin, E. (1983). A new method for off-line removal of ocular artifact. *Electroencephalography and Clinical Neurophysiology*, *55*(4), 468-484.

Kieran, C. (1992). The learning and teaching of school algebra. I DA Grouws (red.) Handbook of research on mathematics teaching and learning, pp. 390-419. In: New York: Macmillan.

Kintsch, W. (1998). The representation of knowledge in minds and machines. *International Journal of Psychology*, *33*(6), 411-420.

Kutas, M., & Hillyard, S. A. (1980). Reading Senseless Sentences: Brain Potentials Reflect Semantic Incongruity. *Science*, *207*(4427), 203-205. https://doi.org/doi:10.1126/science.7350657

McNamara, D. S., Kintsch, E., Songer, N. B., & Kintsch, W. (1996). Are good texts always better? Interactions of text coherence, background knowledge, and levels of understanding in learning from text. *Cognition and Instruction*, *14*(1), 1-43.

Perfetti, C., & Stafura, J. (2014). Word knowledge in a theory of reading comprehension. *Scientific Studies of Reading*, *18*(1), 22-37.

Polich, J. (2007). Updating P300: an integrative theory of P3a and P3b. *Clinical Meurophysiology*, *118*(10), 2128-2148.

Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Common core standards: The new US intended curriculum. *Educational Researcher*, *40*(3), 103-116.

Powell, S. R., Namkung, J. M., & Lin, X. (2022). An Investigation of Using Keywords to Solve Word Problems. *The Elementary School Journal*, *122*(3), 452-473. https://doi.org/10.1086/717888

Verschaffel, L. (2000). Making sense of word problems. In: Swets & Zeitlinger.

Vissers, C. T. W., Kolk, H. H., Van de Meerendonk, N., & Chwilla, D. J. (2008). Monitoring in language perception: evidence from ERPs in a picture–sentence matching task. *Neuropsychologia*, *46*(4), 967-982.

TSG5-28

PHONE FREQUENCY DISTRIBUTIONS IN MATHEMATICAL LANGUAGE: A STATISTICAL ANALYSIS OF MANDARIN

Pu Yu
National Chengchi University, Taiwan, acadyupu@gmail.com

I-Ping Wan
National Chengchi University, Taiwan, 111555004@g.nccu.edu.tw

This study investigates phone frequency distributions in mathematical discourse, comparing them to general spoken Mandarin to explore whether mathematical language follows established linguistic models. Using data from 600 elementary school math word problems (CMATH) and general Taiwan Mandarin speech, the study examines how phone frequency distributions align with models such as Zipf's Law, the Yule function, and the Cocho/Beta function. Results indicate that both mathematical discourse and general Mandarin speech follow the Yule function, with minimal divergence between the two datasets (Jensen-Shannon divergence = 0.008780). This suggests that mathematical language does not impose distinct phonological processing demands, supporting the idea that it shares similar phonological structures with general spoken Mandarin. These findings contribute to understanding phonological processing in mathematical learning and inform curriculum and bilingual education strategies.

Keywords: phonological processing, phone frequency, mathematical discourse, Mandarin, numerical cognition

Introduction

The triple code model (TCM; Dehaene, 1992) proposes that numerical cognition involves verbal, visual, and magnitude-based representations. Verbal processing is essential for arithmetic tasks, while other aspects rely on visual and magnitude-based representations. Phonological processing (PP), which supports verbal numerical tasks, plays a crucial role in mathematical learning (Peng et al., 2018; Li et al., 2023). While previous research has primarily focused on phonological processing in arithmetic fact retrieval, it is also important to consider how phonemic patterns in mathematical discourse influence verbal processing. Edwards et al. (2015) demonstrated that phoneme frequency impacts the order of phoneme acquisition, with more frequent phonemes being acquired earlier. This suggests that phoneme frequency plays a key role in phonological processing, which may affect mathematical verbal processing, especially in early learning. If mathematical discourse exhibits distinct phoneme distributions, it may impose additional phonological processing demands. This study examines phone[1] frequency in both mathematical and general spoken Mandarin, with the aim of understanding how these phone distributions may influence phonological processing in mathematical discourse.

1 A phone is a speech sound without phonemic contrast. The classification of Mandarin phonemes is debatable due to phonotactic constraints (e.g., [tɕ], [tɕʰ], [ɕ] only before [i], [y]). Thus, this study uses "phone" instead of "phoneme" for Mandarin.

Phoneme Frequency in Mathematical Learning

Phonological processing consists of phonemic awareness, phonological memory, and rapid automatized naming (RAN), with varying contributions to mathematical development (Espinas & Fuchs, 2022). These skills affect number identification, arithmetic computation, and problem-solving (Yang et al., 2022), with deficits linked to mathematical difficulties (Peng et al., 2018).

Among these components, phonemic awareness is the strongest predictor of mathematical fact retrieval and problem-solving (Matejko et al., 2023; Li et al., 2023). Phonological memory has a weaker influence and often loses significance when controlling for other variables (Amland et al., 2025; Matejko et al., 2023; Li et al., 2023). RAN shows a moderate correlation with mathematical fluency (Amland et al., 2025), with increasing relevance in age but limited impact on accuracy-based tasks (Yang et al., 2022; Chen et al., 2025).

Phoneme distribution varies across languages, influencing phonological processing in mathematical discourse. Tambovtsev & Martindale (2007) found that the Yule function best describes phoneme distributions across languages. However, research suggested that Mandarin Chinese follows the Cocho/Beta function (Deng, 2016). Wan et al. (2024) found that Taiwan Mandarin aligns better with the Yule function than Zipf's Law, suggesting regional variations in phone distributions.

Despite these findings, it remains unclear whether mathematical language follows known phone distributions or exhibits unique patterns. This study addresses:

1. Does mathematical discourse follow known linguistic models (e.g., Zipf's Law, Yule function, Cocho/Beta function)?
2. How does its phone frequency compare to general spoken Mandarin?
3. Do these patterns suggest additional phonological processing demands?

Methodology

This study examines phone frequency distributions in mathematical discourse by comparing 600 elementary school math word problems (CMATH; Wei et al., 2023) with general Taiwan Mandarin speech (Wan et al., 2024). The goal is to determine whether mathematical language follows established linguistic models or exhibits unique phone patterns.

The methodology involves converting numeric digits to Chinese characters, transcribing text into IPA phonetic symbols, and calculating phone frequency distributions. These distributions are fitted to Zipf's Law, the Yule function, and the Cocho/Beta function, with model performance evaluated using R^2, AIC, and BIC. Jensen-Shannon (JS) divergence measures the similarity between phone frequency distributions in mathematical discourse and general spoken Mandarin. This framework assesses whether mathematical discourse aligns with general linguistic trends or demonstrates unique patterns.

Results and Discussion

The analysis of general Mandarin speech and mathematical language datasets revealed highly similar phone frequency distributions, with all three models performing comparably (Table 1 & Table 2). The Yule model consistently showed the best fit, achieving the highest R^2 values (0.964 and 0.984, respectively) and the lowest AIC and BIC scores, confirming its superior explanatory power. Table 1 and Table 2 show that the Yule model consistently outperforms Zipf's Law and the Cocho/Beta model in both datasets. These results indicate that mathematical discourse follows the same statistical distribution as general spoken Mandarin, aligning more closely with the Yule function than with Zipf's Law or the Cocho/Beta model.

Table 1. Model performance metrics for general Mandarin speech

Model	R^2	AIC	BIC
Zipf-law	0.862	819.0821.9	819.0821.9
Yule	0.964	771.5775.6	771.5775.6
Cocho/Beta	0.954	780.7784.8	780.7784.8

Table 2. Model performance metrics for mathematical language

Model	R^2	AIC	BIC
Zipf-law	0.894	460.3	463.2
Yule	0.984	392.9	397.1
Cocho/Beta	0.971	415.3	419.4

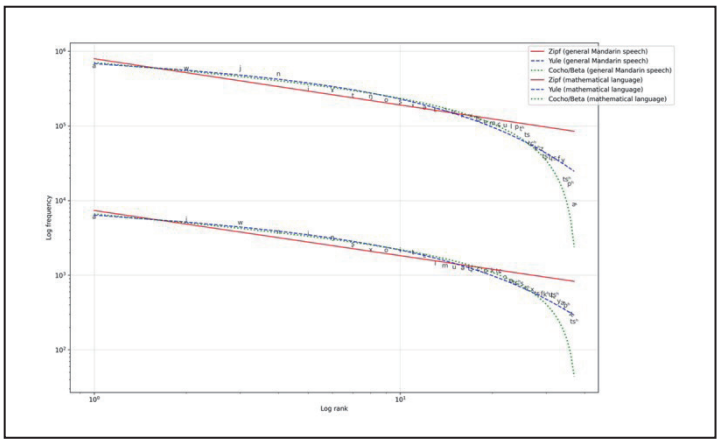


Figure 1. Phone frequency distribution on log-log scale

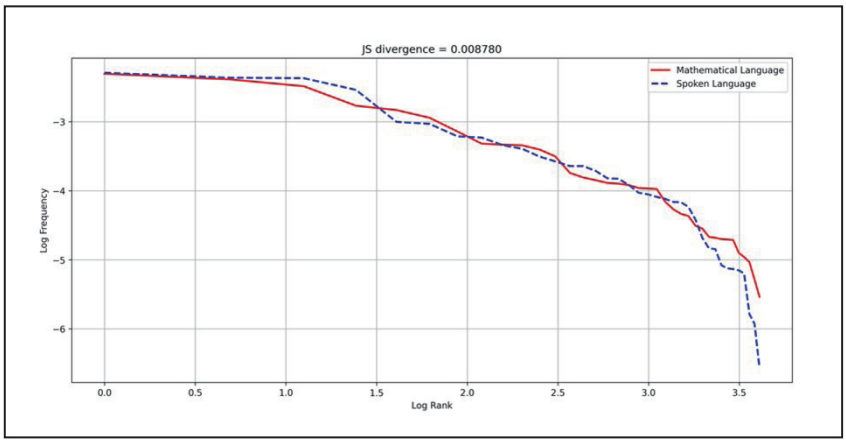


Figure 2. Jensen-Shannon divergence analysis of aligned phone frequency distributions

As shown in Figure 1, the log-log scale comparison confirms that phone frequency distributions are nearly identical across both datasets. Figure 2 further supports this, revealing a low Jensen-Shannon (JS) divergence of 0.008780, calculated after aligning phone frequencies. Despite minor rank shifts, the overall distributions remain highly similar, suggesting that mathematical language does not impose distinct phonological processing demands compared to general spoken Mandarin.

Conclusion

This study examines phone occurrence patterns in mathematical language, comparing them to known linguistic models and general spoken Mandarin. Results indicate that the Yule function best describes both datasets, suggesting that mathematical discourse follows general phone frequency distributions rather than forming a distinct statistical structure (RQ1). The low Jensen-Shannon (JS) divergence of 0.008780 further confirms minimal divergence between the two, implying similar phonological processing demands (RQ2, RQ3). Future research should explore spoken instructional language to determine whether teacher discourse follows similar patterns. Additionally, analyzing bilingual math instruction may reveal how linguistic structure affects mathematical cognition. These findings inform curriculum design, phonological training, and bilingual education strategies.

References

Amland, T., Grande, G., Scherer, R., Lervåg, A., & Melby-Lervåg, M. (2025). Cognitive factors underlying mathematical skills: A systematic review and meta-analysis. *Psychological Bulletin, 151*(1), 88-129.

Chen, X., Ma, C., Yang, Z., & Yang, X. (2025). Bidirectional relationships between phonological processing and basic number knowledge in kindergarten children: A longitudinal study. *BMC Psychology, 13*(49).

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*(1-2), 1-42.

Deng, Y. (2016). Some statistical properties of phonemes in Standard Chinese. *Journal of Quantitative Linguistics, 23*(1), 30-48.

Edwards, J., Beckman, M. E., & Munson, B. (2015). Frequency effects in phonological acquisition. *Journal of Child Language*, *42*(2), 306-311.

Espinas, D. R., & Fuchs, L. S. (2022). The effects of language instruction on math development. *Child Development Perspectives, 16*(2), 69-75.

Li, L., Zhang, H., Qi, Y., Lei, X., Yu, X., & Liu, H. (2023). More than visual-spatial skills: The important role of phonological awareness in mathematical abilities among Chinese primary school children. *Current Psychology, 42*, 31810-31823.

Matejko, A. A., Lozano, M., Schlosberg, N., McKay, C., Core, L., Revsine, C., Davis, S. N., & Eden, G. F. (2023). The relationship between phonological processing and arithmetic in children with learning disabilities. *Developmental Science, 26*(2), e13294.

Peng, P., Wang, C., & Namkung, J. (2018). Understanding the cognition related to mathematics difficulties: A meta-analysis on the cognitive deficit profiles and the bottleneck theory. *Review of Educational Research, 88*(3), 434-476.

Tambovtsev, Y., & Martindale, C. (2007). Phoneme frequencies follow a Yule distribution. *SKASE Journal of Theoretical Linguistics, 4*(2), 1-11.

Wan, I.-P., Allassonnière-Tang, M., & Yu, P. (2024). Early segmental production in Thai preschool children learning Mandarin. *International Journal of Asian Language Processing, 34*(2), 2450005-1-2450005-22.

Wei, T., Luan, J., Liu, W., Dong, S., & Wang, B. (2023). Cmath: Can your language model pass chinese elementary school math test?. *arXiv preprint arXiv:2306.16636*.

Yang, X., Yan, M., Ruan, Y., Ku, S. Y. Y., Lo, J. C. M., Peng, P., & McBride, C. (2022). Relations among phonological processing skills and mathematics in children: A meta-analysis. *Journal of Educational Psychology*, *114*(2), 289-307.

TSG5-29

A NEUROCOGNITIVE PERSPECTIVE ON THE DEVELOPMENT OF FRACTIONAL UNDERSTANDING BASED ON APOS THEORY

Doyeon Ahn
Korea National University of Education, Republic of Korea, ahndy91@gmail.com

Kwang-Ho Lee
Korea National University of Education, Republic of Korea, paransol@knue.ac.kr

This theoretical study explores the development of fractional understanding through the lens of APOS theory (Action–Process–Object–Schema) and connects each stage to findings from contemporary neuroscience. Focusing on the cognitive shift from the partitive unit fraction scheme (PUFS) to the partitive fraction scheme (PFS), the study reviews recent research using fNIRS, EEG, and fMRI to identify how brain networks support conceptual transitions. Evidence suggests that each APOS stage engages distinct but overlapping neural systems, including fronto-parietal, prefrontal-temporal, and basal ganglia-limbic networks. These patterns reflect increasing abstraction and integration in learners' mathematical cognition. The findings offer a framework for future empirical research using neuroimaging to investigate how students at various stages of fractional reasoning exhibit differences in brain activation. This synthesis aims to support the design of neuroscience-informed instructional interventions in mathematics education.

Keywords:APOS Theory, Brain networks fNIRS EEG

Introduction

Fractional understanding is a cornerstone of mathematical cognition, yet many learners struggle to grasp fractions, especially during the transition from simple unit fractions to general fractions. Research shows that mastery of fractions is critical for later algebra success. In mathematics education, the development of fractional knowledge has been described via cognitive schemes. In particular, children first form a partitive unit fraction scheme (PUFS), understanding fractions like 1/n as one part of an equally partitioned whole, and later develop a more advanced partitive fraction scheme (PFS) for general fractions m/n. This article adopts APOS theory (Action–Process–Object–Schema) as a theoretical lens to analyze this transition from PUFS to PFS and connects each APOS stage with findings from neuroscience. We integrate up-to-date literature (2019 onward) from fNIRS, EEG, and fMRI studies on mathematical cognition and fractional reasoning to propose a theoretical model of how brain networks support each stage of fractional understanding. This literature-based analysis is intended to inform the design of future fNIRS-based empirical research, where brain activation patterns of students at different APOS stages of fraction understanding can be observed and compared.

Theoretical Background

APOS theory is a constructivist framework for learning that describes how mathematical concepts are constructed in the mind. According to APOS, an individual's understanding of a concept

evolves through four types of mental structures: APOS theory is a constructivist framework that describes how mathematical understanding develops through four stages. In the Action stage, learners perform procedures step by step with external support. In the Process stage, they begin to mentally simulate these procedures. In the Object stage, the processes become encapsulated as unified conceptual entities. Finally, the Schema stage involves integrating these objects into a flexible and interconnected structure that supports broader reasoning.

In fraction learning, the partitive unit fraction scheme (PUFS) involves constructing 1/n by partitioning a whole into n equal parts and taking one part. The partitive fraction scheme (PFS) builds on PUFS by generating m/n as m iterations of 1/n. This progression aligns with the development through APOS stages: learners move from concrete actions to mental processes, then to conceptual objects, and finally to integrated schemas.

Literature Review and Analysis

The following table summarizes key findings from neuroscience literature about each APOS stage, the cognitive characteristics involved in fraction learning, and the corresponding brain networks activated:

Table 1. Summary of cognitive characteristics and brain networks across APOS stages

APOS Stage	Cognitive Characteristics	Activated Brain Network	Studies
Action	Physical partitioning, iterating 1/n	Visual-parietal, motor areas, bilateral IPS	Dehaene (1999); Menon (2014)
Process	Mental simulation of partitioning and iteration	Fronto-parietal network (DLPFC, IPS)	Grabner (2007); Baker et al. (2020)
Object	Treating m/n as a conceptual object	IPS specialization, prefrontal-temporal regions	Wortha et al. (2020); Ischebeck et al. (2009)
Schema	Flexible reasoning with integrated fraction concepts	Frontal-parietal-temporal integration; basal ganglia	Fonseca et al. (2024); Rivera et al. (2005)

At the Action stage, learners construct fractions like 1/n through direct manipulation. Neuroscientific evidence shows that this stage engages visual and spatial processing networks, particularly in the bilateral intraparietal sulcus (IPS), motor cortex, and visual cortices.

As learners begin to internalize at the Process stage, their working memory and visual-spatial reasoning become more active. fNIRS studies report increased activation in the dorsolateral prefrontal cortex and posterior parietal cortex during mental estimation and proportional reasoning.

The Object stage reflects a learner's ability to encapsulate fractional processes into conceptual objects. Research shows that with training, learners begin to activate IPS regions for fractions similarly to how they do for whole numbers. This indicates efficiency and automaticity in processing m/n as a magnitude.

At the Schema level, students apply fractional reasoning flexibly across various contexts. Neural activity becomes more integrated across the prefrontal, parietal, and temporal lobes. Studies link schema-level understanding with coordinated network activation and high connectivity, supporting complex problem-solving and transfer.

Discussion

This review highlights several key implications. First, each APOS stage in the development of fractional understanding corresponds to distinctive cognitive characteristics and is associated with specific—but interactive—patterns of brain activation. For example, during the Action stage, learners perform hands-on partitioning and iteration, which engages not only visual and motor areas but also language and numerical regions involved in labeling and comparing quantities (Dehaene, 1999; Menon, 2014).

Second, in the Process and Object stages, working memory, mental imagery, and numerical abstraction become increasingly important. These are supported by fronto-parietal coordination, particularly involving the dorsolateral prefrontal cortex and intraparietal sulcus (Grabner, 2007; Baker et al., 2020). In the Object stage, numerical representations become more automatic and abstract, and learners begin to treat fractions like m/n as coherent magnitudes. This shift is reflected in more efficient IPS activation and greater involvement of prefrontal-temporal networks (Wortha et al., 2020; Ischebeck et al., 2009).

Third, the Schema stage is marked by flexible and integrated fractional reasoning. Learners at this stage activate widespread, interconnected neural systems that support conceptual transfer and problem-solving. The basal ganglia and limbic system may support strategy selection and cognitive control, working in conjunction with the frontal, parietal, and temporal lobes (Fonseca et al., 2024; Rivera et al., 2005).

Finally, this theoretical synthesis suggests new directions for empirical research using fNIRS. By analyzing differences in brain activation patterns across students operating at different APOS stages, future studies could identify neural indicators of conceptual growth. Such insights would enable more refined diagnostic and instructional tools, grounded in both cognitive theory and neuroscience.

Conclusion

This study synthesized APOS theory with recent neuroscience literature to theorize how brain networks support each stage in the development of fractional understanding. Rather than being localized to discrete brain regions, neural processing of fractions reflects interactive and coordinated activity across multiple networks. These findings provide a theoretical foundation for future empirical studies using fNIRS or EEG, to identify neural signatures associated with different levels of fractional reasoning in learners. The integration of APOS theory and neuroscience offers a promising direction for educational research, potentially leading to more targeted interventions for students at different developmental stages of mathematical understanding.

References.

Arnon, I., Cottrill, J., Dubinsky, E., et al. (2014). *APOS theory: A framework for research and curriculum development in mathematics education.* Springer.

Ashcraft, M. H. (2002). Math anxiety: Personal, educational, and cognitive consequences. *Current Directions in Psychological Science, 11*(5), 181–185.

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry, 46*(1), 3–18.

Butterworth, B. (2018). *The mathematical brain.* Macmillan.

Dehaene, S. (1999). *The number sense: How the mind creates mathematics.* Oxford University Press.

Dubinsky, E. (1991). Reflective abstraction in advanced mathematical thinking. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 95–123). Springer.

Dubinsky, E., & McDonald, M. A. (2001). APOS: A constructivist theory of learning in undergraduate mathematics education research. In D. Holton (Ed.), *The teaching and learning of mathematics at university level* (pp. 275–282). Springer.

Grabner, R. H. (2007). Individual differences in mathematical competence predict parietal brain activation during mental calculation. *NeuroImage, 38*(2), 346–356.

Knops, A., Thirion, B., Hubbard, E. M., Michel, V., & Dehaene, S. (2009). Recruitment of an area involved in eye movements during mental arithmetic. *Science, 324*(5934), 1583–1585.

Menon, V. (2014). Arithmetic in the child and adult brain: An fMRI study. *Nature Reviews Neuroscience, 15*(4), 277–292.

Rivera, S. M., Reiss, A. L., Eckert, M. A., & Menon, V. (2005). Developmental changes in mental arithmetic: Evidence for increased functional specialization in the left inferior parietal cortex. *NeuroImage, 28*(4), 1220–1235.

Baker, J. M., Bruno, J. L., Gundran, A., & Souza, M. (2020). fNIRS evidence of age-related changes in mental magnitude estimation. *NeuroImage*, *213*, 116731.

Dehaene, S. (1999). *The number sense: How the mind creates mathematics*. Oxford University Press.

Fonseca, P., Garcia, A., & Lopez, S. (2024). Fraction problem-solving and fronto-parietal network synchronization in children: A preprint study. *bioRxiv*.

Grabner, R. H. (2007). Individual differences in mathematical competence predict parietal brain activation during mental calculation. *NeuroImage, 38*(2), 346–356

Ischebeck, A., Zamarian, L., Siedentopf, C. M., Koppelstätter, F., Benke, T., & Delazer, M. (2009). How specifically do we learn? A functional magnetic resonance imaging study of learning fractions. *NeuroImage, 45*(3), 1055–1065.

Menon, V. (2014). Arithmetic in the child and adult brain: An fMRI study. *Nature Reviews Neuroscience*, *15*(4), 277–292.

Rivera, S. M., Reiss, A. L., Eckert, M. A., & Menon, V. (2005). Developmental changes in mental arithmetic: Evidence for increased functional specialization in the left inferior parietal cortex. *NeuroImage*, *28*(4), 1220–1235.

Wortha, R. P., Kumar, R., & Menon, V. (2020). Learning fractions through number-line estimation: Neural correlates of symbolic and spatial representation. *Developmental Cognitive Neuroscience, 41*, 100740.

TSG5-30

FALLACY OF COMPOSITION AND COIN TOSS SEQUENCES: COMPARING SEQUENCES WITH THE SAME HEADS-TO-TAILS RATIOS

Ippo Ishibashi
Okayama University, Japan, iishibashi@okayama-u.ac.jp

This paper discusses the relationship between the fallacy of composition and the 50–50 approach in a question that requires a comparison of sequences with the same ratios of heads to tails based on findings from previous studies. According to the analysis, in a comparison between sequences with the same heads-to-tails ratios, more respondents consider "sequences that are more random" or "sequences that have patterns" in combination with the application of the fallacy of composition compared with respondents who may have applied the fallacy of composition alone. Moreover, the 50–50 approach observed in a comparison of sequences may be the same as the fallacy of composition observed in a comparison of events.

Keywords: heuristics, biases, logical fallacies, 50–50 approach, probability

Introduction

According to Tversky and Kahneman (1974), people often believe that in coin tosses, the sequence HTHTTH is more likely to occur than the sequence HHHTTT. Chernoff et al. (2018) claims that this error has been attributed to an application of the "representativeness heuristic" (e.g., Kahneman & Tversky, 1972), the "fallacy of composition" (e.g., Chernoff et al., 2018), and so on.

The purpose of this paper is to update the way learners' beliefs are viewed in probability education research by proposing a new perspective on these two beliefs, using literature interpretation as a research method.

Representative Heuristic and Fallacy of Composition

Representative Heuristic

Suppose a person determines that HHHHT is less likely to occur than HHTHT (hereinafter, "error α"). If this belief is informed by an application of the representativeness heuristic, then this person must have determined, based on the law of large numbers, that the ratio of heads to tails would approach 1:1 (Kahneman & Tversky, 1972; Tversky & Kahneman, 1974).

At first glance, error α may appear to be an application of the representativeness heuristic. However, Chernoff et al. (2018) state that the following line of reasoning is always raised whenever this case is presented at a mathematics education conference:

> Yes, your results (and analysis) demonstrate that the participants deem [using H for head and T for tail] HHHHT as less likely than HHTHT, but they're *probably* comparing the event [i.e., a set of outcomes] four-heads-and-one-tail to the event three-heads-and-two-tails, in which case, then, 'HHHHT' *is* less likely than 'HHTHT'. This means that, although their answer is incorrect for the question you

> gave them, they're not necessarily incorrect if they're comparing events instead of sequences. (Chernoff et al., 2018, p. 278) (Italics by the original authors)

Kahneman and Tversky (1972) and Tversky and Kahneman (1974) are unable to respond to this line of reasoning (hereafter "line of reasoning A"). Thus, a fresh survey needed to be conducted (Chernoff et al., 2018).

Fallacy of composition

In crafting the survey, Chernoff et al. (2018) focus on the comparison of the likelihood of the occurrence of events. This approach is different from that of previous surveys, which focus on sequences. One of the survey questions designed is shown in Figure 1.

Which of the following is the least likely result of six flips of a fair coin? Please justify your response

a. Five heads and one tail
b. Four heads and two tails
c. Both results (a and b) are equally likely to occur

Figure 1. Survey question by chernoff et al. (2018, p. 284)

For the question in Figure 1, the correct answer is "a." If survey participants can compare events correctly (as line of reasoning A asserts), then many of them would be able to provide the correct answer. However, in the survey conducted by Chernoff et al. (2018) targeting 106 prospective mathematics teachers, 23 participants (about 22%) chose "a," 10 (about 9%) chose "b," and 73 (about 69%) chose "c." Thus, Chernoff et al. (2018) refute line of reasoning A on the grounds that the survey participants failed to compare events correctly. Moreover, 14 of the 73 participants who chose "c" responded that they chose this answer because "equiprobability is true of the events (the whole) from the fact that is true of the coin (the part)" (p. 285). Chernoff et al. (2018) regard this reasoning as an application of the fallacy of composition.

Chernoff et al. (2018) further state that error α is an application of the fallacy of composition as well. This is because, although earlier evidence points to the application of the law of large numbers to small samples as the fundamental cause of the representativeness heuristic (Kahneman & Tversky, 1972; Tversky & Kahneman, 1974), subsequent studies (e.g., Stohl, 2005) reveal that participants do not necessarily have the understanding of the law of large numbers as assumed by researchers. If error α is indeed an application of the fallacy of composition, as Chernoff et al. (2018) argue, it can be attributed to an inference that the ratio of heads to tails would be 1:1 for a sequence of coin tosses. Such an inference is based on the equiprobability of individual coin tosses and the fact that the sequence consists of fair coin tosses (Chernoff et al., 2018). According to this explanation, the survey participants inferred that HHHHT was less likely to occur than HHTHT because the ratio of heads to tails is 4:1 for HHHHT, which is farther from 1:1 compared with the 3:2 for HHTHT. This explanation, according to which error α results from the fallacy of composition, does not assume that participants have an understanding of the law of large numbers. Thus, it avoids the problem of the representativeness heuristic, which assumes such an understanding (Chernoff et al., 2018).

Research Question

The explanation of error α by Chernoff et al. (2018), which assumes that it results from an application of the fallacy of composition, covers two sequences (HHHHT and HHTHT) whose ratios of heads to tails differ. However, this explanation does not cover sequences with the same heads-to-tails ratios.

Suppose that the fallacy of composition is applied and that the ratios of heads to tails of two sequences are the same. Participants may determine that the occurrence of these two sequences is equally likely. Thus, this study establishes the following research question.

> RQ: Can any inference believed to be an application of the fallacy of composition occur in a question that compares sequences that have the same heads-to-tails ratios?

Fallacy of Composition and Coin Toss Sequence

Survey Question and Results of Rubel (2007)

To answer the research question above, this study is based on the findings of Rubel (2007), who investigated student thinking on tasks involving compound events and independent trials of coin tosses. Rubel (2007) presented the question in Figure 2 to 173 students in grades 5, 7, 9, and 11. The present study analyzes Rubel's (2007) findings with a new perspective and objective.

Suppose you toss a fair coin six times, recording the result of each toss. For instance, if you toss one heads and then five tails in a row, you would write HTTTTT. Which is the most likely result?

	Toss 1	Toss 2	Toss 3	Toss 4	Toss 5	Toss 6
a.	H	T	H	T	H	T
b.	H	H	T	H	T	T
c.	H	H	H	T	T	T
d.	T	T	T	H	T	T

e. All are equally likely

Explain.

Figure 2. Survey question from rubel (2007, p. 537)

Analysis

For the question in Figure 2, the correct answer is "e." If the fallacy of composition is applied to Figure 2, then any one of a, b, or c, which have a 1:1 heads-to-tails ratio, is most likely to occur. This is because, after referring to the equiprobability of individual coin tosses and the fact that the sequence consists of fair coin tosses, one can infer the ratio of 1:1 for the sequence of coin tosses as well.

Of the students, 116 (67%) chose the correct answer "e," while 18 wrote that any of the sequences with three heads and three tails would be the most likely. They represent about 10% of the total and about 32% of the 57 students who chose wrong answers. Meanwhile, to 33 students choose one out of a, b, and c. They represent 19% of the total and about 58% of the 57 students who chose wrong answers.

Discussion and Conclusion

Of the students who selected wrong answers to Figure 2, about 32% are believed to have applied the fallacy of composition. The results indicate that what is believed to be an application of the fallacy of composition does occur even in a question that compares sequences with the same heads-to-tails ratios. Meanwhile, about 58% of the students answered that even if the ratio of heads to tails is 1:1 for all cases, the likelihood of occurrence would differ among these sequences. These are inferences that cannot be explained by the fallacy of composition alone. According to Rubel (2007), students who answered that HHTHTT was the most likely to occur believed that the results of coin tosses were random and that the number of heads and tails would be the same. Those who responded that HTHTHT and HHHTTT were the most likely to occur believed that there are patterns to the sequences of coin-toss results and that the numbers of heads and tails should be the same. This indicates that, when comparing sequences with the same heads-to-tails ratios, students may also consider "sequences that are more random" or "sequences that have patterns" in addition to experiencing the fallacy of composition.

Among students who responded that all were equally likely (option e), some may have applied the fallacy of composition. These were students who used the "50–50 approach." Under this approach, a term coined by Rubel (2007), "one takes the probability of a specific outcome from the case of a single event, which in this case is 50%, or 1/2, and extends this ratio to any general compound case" (p. 542). Regarding Figure 2, Rubel (2007) cites the following as examples of the reasons this approach is used by some students: "all coins have a 50% chance of heads or tails" (p. 546) and "there are two sides and a 50–50 chance of getting any pattern" (p. 547). These reasons seem to be essentially the same as the reason some participants chose "c" in Figure 1, as explained by Chernoff et al. (2018): "equiprobability is true of the events (the whole) from the fact that is true of the coin (the part)" (p. 285). Thus, in Figure 2, which compares sequences, the same response is observed as that observed in Figure 1, which compares events rather than sequences. This may be because the students who applied the 50–50 approach did not consider the sequences, and thus, they showed the same response as that obtained in the study by Chernoff et al. (2018), which compared events.

Note that the students in the two surveys are different; whether the students who gave reasons for what is considered a synthesis error for one survey question would give the same reasons for the other survey question is unknown. This is a limitation of this paper.

References

Chernoff, E. J., Vashchyshyn, I., & Neufeld, H. (2018). Comparing the relative probabilities of events. In C. Batanero, & E. J. Chernoff (Eds.), *Teaching and Learning Stochastics* (pp. 277–291). Springer. https://doi.org/10.1007/978-3-319-72871-1_16

Kahneman, D., & Tversky, A. (1972). Subjective probability: A judgement of representativeness. *Cognitive Psychology*, *3*(3), 430–454. https://doi.org/10.1016/0010-0285(72)90016-3

Rubel, L. H. (2007). Middle school and high school students' probabilistic reasoning on coin tasks. *Journal for Research in Mathematics Education*, *38*(5), 531–556. https://www.jstor.org/stable/30034964

Stohl, H. (2005). Probability in teacher education and development. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 345–366). Springer. https://doi.org/10.1007/0-387-24530-8_15

Tversky, A., & Kahneman, D. (1974). Judgement under uncertainty: Heuristics and biases. *Science*, *185*(4157), 1124–1131. https://doi.org/10.1126/science.185.4157.1124

MEMO

MEMO

KOREA 2025

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 6:

The Essence of Mathematics Education in the Use of Digital Technology

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 6
(Day 2)

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

TSG6-1-1

LEARNING TEACHING MATHEMATICS WITH INSTRUCTIONAL TECHNOLOGY: A PRAXEOLOGICAL ANALYSIS OF A GHANAIAN MATHEMATICS TEACHER EDUCATION COURSE

Farouq Sessah Mensah
Stockholm University, Sweden, farouq-sessah.mensah@su.se

This study explores how a mathematics teacher educator prepares pre-service teachers to integrate instructional technology into mathematics teaching, using the Anthropological Theory of the Didactic (ATD) as a framework. A qualitative case study at a Ghanaian university discusses the MTE's practices through video observations and a semi-structured interview. Findings reveal that representation of practice through modelling provided PSTs with practical examples of instructional integration, while decomposition allowed for refining lesson plans through structured analysis. However, the absence of approximations of practice due to time constraints limited experiential learning opportunities. The study emphasises the need for explicitly articulating meta-didactical logos and simulated teaching environments to bridge theory and practice.

Keywords: Instructional technology, anthropological theory of the didactic, mathematics teacher education, pedagogies of practice

Introduction

The demands of modern educational environments require high-quality mathematics instruction, of which instructional technology plays an integral part. I refer to instructional technologies (IT) as the intentional integration of various digital technologies into the classroom to increase the efficiency with which teachers and students can accomplish their tasks or the depth with which teachers and students can acquire new knowledge. There is a growing expectation that integrating IT in mathematics teaching and learning can contribute to meaningful learning (e.g., Hillmayr et al., 2020). However, integrating IT in teaching and learning does not always produce the desired learning outcomes (e.g., Player-Koro, 2013). The literature highlights the transformative power of IT when used meaningfully in teaching and learning (e.g., Hoyles, 2018). While it could provide meaningful learning opportunities for students, integrating IT into teaching practices also demands a lot of teacher effort. Some researchers have acknowledged the complexities of teaching mathematics with IT (e.g., Rakes et al., 2022). The challenges of integrating IT into teachers' daily routines remain unresolved despite many years of research and development. From the Ghanaian context, for example, studies on technology-enhanced learning (e.g., Buabeng-Andoh, 2019) have reported low levels of IT integration into core teaching practices. This low level of IT integration raises the question of how pre-service mathematics teachers (PSTs) are prepared for the complex work of teaching mathematics integrating IT.

Theoretical Consideration and Research Question

The study is grounded in the Anthropological Theory of the Didactic (ATD) (Chevallard, 1992), linked to the processes of teaching and learning mathematics. ATD considers doing, teaching, diffusing, creating, and transposing mathematics, as well as any other kind of knowledge, to be human activities taking place in institutional settings. ATD holds that any human activity can be understood as a praxeology. The ATD frames human activity as praxeology, comprising praxis and logos. Praxis involves tasks and the techniques used to solve them, while logos encompasses the theoretical discourse that explains and justifies these techniques. The ATD distinguishes between mathematical praxeologies (MP) and didactic praxeologies (DP) (Chevallard & Bosch, 2020). MPs involve types of tasks, techniques, technologies, and inherently mathematical theories. In contrast, DPs refer to praxeologies that support the teaching or learning of MPs, typically reflecting the activities of school teachers. In teacher education (TE), however, the focus shifts: the DP itself becomes the knowledge to be taught as PSTs learn how to teach. Arzarello et al. (2014) extend this by introducing meta-didactical praxeologies, which encompass the tasks, techniques, and justifying discourses related to teaching how to teach mathematics. In this context, mathematics education becomes the object of study. To avoid confusion, I will use the term meta-didactical praxeology when referring to praxeologies where didactical knowledge rather than mathematics is the central content to be taught and learned. From a praxeological perspective, the following research question guided the study: *How are meta-didactical praxeologies privileged when a mathematics teacher educator describes their teaching of IT integration in mathematics teaching?*

Method

The study is designed as a qualitative single-case study. It serves as a preliminary analysis, offering a nuanced, context-specific, and empirically rich account that lays the foundation for a broader, multi-case investigation. The design enables an initial critical reflection, analysis, interpretation, and deeper understanding of the research problem.

Case Context

The study was conducted in a teacher preparation course at a public university in Ghana. The course was designed to allow PSTs to develop skills in designing instructional materials, identifying various teaching and learning resources (TLRs), using manipulatives and using technology applications to meet students' diverse learning needs. The study focused on an MTE, referred to by the pseudonym Benjamin, as the principal participant, along with his 204 PSTs enrolled in the course.

Data Collection and Analysis of the Case

Data was collected through video observations and a semi-structured interview with Benjamin. Selected teaching episodes were drawn from the classroom observations, and the interview was audio-recorded and transcribed for analysis. The data was analysed by drawing on a reference model.

Reference Model

A reference model (RM) is a framework used by researchers to examine and respond to didactical situations within specific institutional contexts (see Wijayanti & Winslow, 2017). The RM served as the study's analytical framework. To answer the research question, I drew on Grossman et al. (2009) pedagogies of the practice of representation, decomposition, and approximation as reference models to analyse the meta-didactic praxis. The meta-didactic logos was determined inductively

by looking for substantiations of the identified praxis. In the course, representation occurred when PSTs engaged with IT as part of their learning process or observed the MTE modelling its use. Decomposition involved PSTs critically analysing the use of various IT tools across different mathematical task situations. Approximation refers to PSTs actively enacting mathematics teaching with IT and reflecting on their experiences to develop their instructional practices.

Results and Discussions

Representation of Practice

The observation data showed how Benjamin, through the modelling of lessons, privileged different pedagogical and didactical strategies that integrate IT, providing PSTs with concrete examples of meaningful classroom application. This modelling showcased diverse pedagogical and didactical praxis of representing mathematical concepts. Benjamin indicated in the interview:

> Sometimes, I select three or four topics, such as solid mensuration, quadratic equations, and algebraic expressions. I then demonstrate the teaching process by enacting lessons based on these topics, supporting them with worksheets containing related activities for the students to engage.

The meta-didactic praxis of modelling teaching by integrating IT and hands-on activities suggests opportunities for PSTs to connect more deeply with the subject matter. This praxis reflects ATD's emphasis on immersing PSTs in real classroom dynamics where theoretical knowledge is contextualised and practised. Research also highlights the value of such modelling in enabling PSTs to internalise methods for making abstract concepts accessible and engaging (e.g., Ball & Cohen, 1999). Although implicit, the meta-didactic logos of representation reflected the need to foster active engagement and improve teaching efficiency through modelling.

Decomposition of Practice

Benjamin exemplified decomposition by guiding PSTs through designing and evaluating lesson plans. In dissecting the lesson planning process into its fundamental elements - objectives, instructional strategies, materials, and assessments, Benjamin facilitated a structured reflection that enabled PSTs to refine each component. Benjamin explained:

> So when they design their lesson plans, I check whether what they have designed can support the SHS students in learning the concept they want to teach through discussions.

The meta-didactical praxis of designing and evaluating lesson plans aligns with research in mathematics teacher education. It emphasises the importance of breaking down practice into core elements to support PSTs in developing targeted instructional skills (Ball & Cohen, 1999). The logos for the meta-didactical praxis of decomposition of practice was not evident from the data. While the decomposition of practice is critical for developing pedagogical content knowledge, it often remains implicit within instructional settings (Ball & Cohen, 1999). The absence of explicit logos for the meta-didactical praxis of decomposition highlights a broader challenge to IT integration in mathematics teacher education. The non-articulation of logos might hinder PSTs from fully grasping the nuanced decision-making processes involved in teaching with IT, which is essential for their development as reflective practitioners.

Approximation of Practice

The findings highlight a gap in implementing approximations of practice within the teacher education course. Although Benjamin recognised the value of having PSTs enact lessons they designed, time constraints prevented this crucial step from being integrated into the course. Benjamin acknowledged this limitation, noting:

> Yes, that is the unfortunate part we couldn't do... But it would have been very productive. Having them design and enact. Because designing a lesson is different from enacting it.

This statement highlights the distinction between lesson planning and the actual enactment of teaching, a crucial component of approximations of practice. In the context of mathematics teacher education, previous research has emphasised the importance of providing PSTs with opportunities to engage in approximations of practice. This approach allows PSTs to bridge the gap between theory and practice by actively participating in teaching scenarios that resemble real classroom situations (Grossman et al., 2009). The omission of this step, as highlighted in the interview, suggests a missed opportunity for PSTs to develop their pedagogical and didactical skills more deeply.

Concluding Remarks

This study highlights the role of representation, decomposition, and approximation of practice in preparing PSTs to teach mathematics using IT. Through modelling, Benjamin demonstrated IT integration with pedagogical and didactical strategies, providing PSTs with concrete examples of effective classroom practices. Decomposition of practice offered opportunities for PSTs to analyse and refine lesson planning. The meta-didactic logos underline the mata-didactic praxis of representation, and decomposition was not articulated. Approximations of practice were absent due to time constraints, reflecting a missed opportunity to bridge the gap between theoretical knowledge and practical application. The study suggests that MTEs explicitly articulate their meta-didactic logos and include approximations of practice through simulated teaching environments.

References

Arzarello, F., Robutti, O., Sabena, C., Cusi, A., Garuti, R., Malara, N., & Martignone, F. (2014). Meta-didactical transposition: A theoretical model for teacher education programmes. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The mathematics teacher in the digital era: An international perspective on technology focused professional development* (pp. 347–372). Springer Netherlands. https://doi.org/10.1007/978-94-007-4638-1_15

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3–32). Jossey-Bass.

Buabeng-Andoh, C. (2019). Factors that influence teachers' pedagogical use of ICT in secondary schools: A case of Ghana. *Contemporary Educational Technology, 10*(3), 272–288. https://doi.org/10.30935/cet.590099

Chevallard. (1992). Fundamental concepts in didactics: Perspectives provided by an anthropological approach. In Research in *didactique of mathematics: Selected papers* (pp. 131–168). La pense´e sauvage.

Chevallard, Y., & Bosch, M. (2020). Anthropological theory of the didactic (ATD). I S. Lerman (Red.). *In Encyclopedia of Mathematics Education* (pp. 53–61). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0

Grossman, P., Hammerness, K., & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers and Teaching, 15(2)*, 273–289. https://doi.org/10.1080/13540600902875340

Hillmayr, D., Ziernwald, L., Reinhold, F., Hofer, S. I., & Reiss, K. M. (2020). The potential of digital tools to enhance mathematics and science learning in secondary schools: A context-specific meta-analysis. *Computers & Education, 153*, 1-25. https://doi.org/10.1016/j.compedu.2020.103897

Hoyles, C. (2018). Transforming the mathematical practices of learners and teachers through digital technology. *Research in Mathematics Education, 20*(3), 209–228. https://doi.org/10.1080/14794802.2018.1484799

Player-Koro, C. (2013). Hype, hope and ICT in teacher education: A Bernsteinian perspective. *Learning, Media and Technology, 38*(1), 26–40. https://doi.org/10.1080/17439884.2011.637503

Rakes, C. R., Stites, M. L., Ronau, R. N., Bush, S. B., Fisher, M. H., Safi, F., Desai, S., Schmidt, A., Andreasen, J. B., Saderholm, J., Amick, L., Mohr-Schroeder, M. J., & Viera, J. (2022). Teaching mathematics with technology: TPACK and effective teaching practices. *Education Sciences, 12*(2), 133. https://doi.org/10.3390/educsci12020133

Wijayanti, D., & Winslow, C. (2017). Mathematical practice in textbooks analysis: Praxeological reference models, the case of proportion. *REDIMAT, 6*(3), 307–330. https://doi.org/10.17583/redimat.2017.2078

Acknowledgement

I acknowledge the REMATH research school, funded by the Swedish Research Council (2021-00534) and Helge Ax: son Johnsons stiftelse (Nr F23-0012).

TSG6-1-2

EXPLORING PRESERVICE TEACHERS' INTEGRATION OF AI TOOL THROUGH AI-TPACK AND SAMR MODEL

YuJin Lee
Heungryong Elementary School, Korea, kjyjl4231@naver.com

This study examines how PSTs integrate AI tools, specifically 'TocToc-Math', into their lesson plans and the relationship between SAMR levels and their AI-TPACK. The results show that a significant number of PSTs planned lessons at the augmentation level of the SAMR model, with few reaching the redefinition level. While PSTs recognized the advantages of 'TocToc-Math', they predominantly used it to enhance existing activities rather than modifying or creating new ones. This suggests that although PSTs are aware of the potential of AI tools, they struggle to apply them in transformative ways. Furthermore, while higher SAMR levels were generally associated with higher AI-TPACK scores, statistical analysis showed no significant differences across the SAMR categories. These findings suggest that enhancing AI-TPACK alone is insufficient to elevate AI integration. It emphasizes the importance of teacher education in helping PSTs move beyond simple usage of AI tools and shift towards transformative integration.

Keywords: elementary preservice teachers, AI-TPACK, SAMR model, TocToc-Math, AI tool

Introduction

Recent advancements in Artificial Intelligence (AI) have led to significant transformations across various fields, including education. AI tools have enabled the design of personalized learning pathways by analyzing student data and providing optimized educational content tailored to individual needs (Chen et al., 2020; Zawacki-Richter et al., 2019). In particular, in Korea, there has been a growing effort to incorporate AI into public education, exemplified by AI-driven educational tools such as AI digital textbooks and 'TocToc-Math'. However, while AI holds great promise, it cannot replace the essential role of the teacher in education (Gentile et al., 2023; Selwyn, 2019). The effective integration of AI tools into education, therefore, requires the active engagement of teachers, emphasizing the importance of teachers' knowledge in utilizing AI for enhancing pedagogy.

Given this background, this study aims to examine how preservice teachers (PSTs) integrate AI tools into their lesson plan and how their AI-TPACK competencies relate to this integration. Accordingly, the research questions guiding this study are as follows: (a) How do PSTs integrate AI tools (TocToc-Math) into their lesson plans? (b) What is the relationship between the SAMR levels observed in PSTs' lesson plans and their AI-TPACK competencies?

Literature Review

TPACK is a concept that describes the knowledge framework teachers must possess to effectively integrate technology into their teaching (Mishra & Koehler, 2006), and it has been recognized as a flexible framework for both pedagogical approaches and technological tools (Mishra et al., 2010; Valtonen et al., 2017). In particular, AI-TPACK expands the existing TPACK framework

by emphasizing the integration of AI technologies with education. Among the studies that address AI-TPACK, Celik (2023) argued that to effectively utilize AI-based tools in education, teachers must possess not only technological and pedagogical knowledge, but also ethical knowledge. He extended the concept of TPACK by introducing Intelligent-TPACK, which incorporates the ethical considerations needed when using AI tools. Additionally, research focusing on generative AI has introduced the concept of contextual knowledge (XK) (Mishra et al., 2023), which similarly emphasizes the need for teachers to account for the instability and opacity of generative AI, akin to Celik's ethical considerations.

While AI-TPACK emphasizes the knowledge teachers need to effectively integrate AI tools into their teaching, the SAMR model provides a framework for understanding how technology can be integrated into the classroom, particularly in mathematical tasks and activities. Various studies have explored how to integrate technological tools into teaching. Among these, the SAMR model is particularly suitable for integrating technological tools into mathematical tasks and activities. The SAMR model consists of four levels: Substitution, Augmentation, Modification, and Redefinition (Hamilton, Rosenberg, & Akcaoglu, 2016). The first two levels (Substitution and Augmentation) are categorized as enhancement, while the latter two (Modification and Redefinition) are considered transformation. The Substitution level represents a direct replacement of traditional methods with technology, without functional change. At the Augmentation level, technology still substitutes for traditional methods, but it adds enhancements that improve the learning experience. The Modification level involves a significant redesign of the learning task, and the highest level, Redefinition, allows for the creation of new tasks that were previously unimaginable without technology.

Methods

This study was conducted with 50 PSTs enrolled in their second year at a national university of education in Korea. These PSTs participated in lectures on the principles and methods of teaching and learning mathematics, during which they were also introduced to the functionalities and usage of 'TocToc-Math', an AI-based elementary mathematics classroom support system developed for Korean public education. To address the first research question, the PSTs were asked to select a lesson from the 6th-grade second-semester unit "Space and Solid Figures" and create a lesson plan using 'TocToc-Math'. The lesson plan included descriptions of the lesson content, learning objectives, and strategies for integrating 'TocToc-Math'. The collected lesson plans were analyzed using the SAMR model to access how the PSTs integrated 'TocToc-Math' into their teaching. Examples of the lesson plans analyzed using the SAMR model are shown in Table 1. To address the second research question, the PSTs' AI-TPACK was measured, and their AI-TPACK was analyzed according to the SAMR level. Specifically, the differences in AI-TPACK scores between the groups were analyzed. To measure the PSTs' AI-TPACK, the study used a survey tool adapted from Lee (2024), based on the Intelligent-TPACK framework by Celik (2023), specifically tailored to the context of mathematics education in Korea. Participants responded to the AI-TPACK survey via Google Forms, using a 5-point Likert scale. Based on the collected data, the Mann-Whitney U test was used in this study to analyze the differences in the mean AI-TPACK scores between the Enhancement group and the Transformation group within the SAMR model.

Table 1. Example activities based on SAMR levels

SAMR levels	Example
Substitution	Substitute the traditional building blocks with the building blocks tool in 'TocToc-Math' or replace the mathematics workbook or assessment with the 'TocToc-Math' lesson assessment.
Augmentation	Understand the advantages of the 'TocToc-Math' building blocks tool over the traditional one and plan activities that leverage its enhanced features.
Modification	Use the 'TocToc-Math' building blocks tool to modify textbook activities and plan different tasks. Incorporate the advantages of 'TocToc-Math' assessment for real-time feedback and group activities.
Redefinition	Create activities where students observe real buildings from multiple angles using programs like Google Earth, and present adaptive questions based on student levels through 'TocToc-Math' assessment activities.

Findings and Discussion

Integration of AI Tools in Lesson Plans

An analysis of PSTs' lesson plans, categorized by the SAMR levels, revealed the following distribution: 4% in the Substitution level, 58% in the Augmentation level, 32% in the Modification level, and 6% in the Redefinition level. Most PSTs augmented or modified AI tools to enhance or restructure their lesson activities, focusing on functional improvements over traditional tools. Specifically, 86% of PSTs utilized the 'TocToc-Math' building block tools, and 64% used them for assessments. Additionally, 8% of PSTs incorporated other AI tools alongside.

Looking at the usage of 'TocToc-Math', PSTs took advantage of the building block tools, such as the ability to easily observe shapes from different perspectives (top, front, and side views). This was more challenging with physical blocks due to limited angles and the need for movement. This allowed them to restructure the activities. For assessment in 'TocToc-Math', PSTs highlighted the immediate feedback and its ability to present level-specific questions based on the students' proficiency.

In the Augmentation level, most PSTs suggested using the advantages of the building block tools to enhance existing activities. In the Modification level, as shown in Figure 1, PSTs utilized 'TocToc-Math' to create group quizzes where students could freely stack blocks and compare the shapes from different perspectives. This activity would have been difficult to execute using traditional blocks, as the shapes would need to be drawn from multiple angles. By restructuring the activity with building block tools, PSTs were able to provide richer examples and deeper exploration than traditional textbook activities. At the Redefinition level, PSTs planned activities that involved using Google Earth or online maps to observe architectural structures from various directions. Additionally, they encouraged students to upload images of buildings or shapes created with 'TocToc-Math' from different angles to collaborative platforms such as Naver Whale Space, Padlet, or Google Board. This activity allowed students to discuss and share their observations. This level of inte-

gration represents a complete transformation of traditional tasks, introducing new possibilities for learning and collaboration.

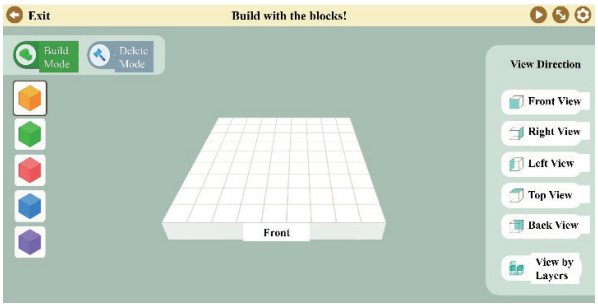
TocToc-Math building block tools	Example activities
	In the group, each participant uses the 'TocToc-Math' building blocks tool to build the shape they want. Then, they capture the shape from the front, back, and side views. The other members look at the photos captured and try to recreate the shape. Afterward, they check if the shape they built matches the original one.

Figure 1. Example activities at the modification level

Relationship Between SAMR Levels and PSTs' AI-TPACK

The distribution of AI-TPACK components across SAMR levels is shown in Table 2. This table presents the average values of AI-TK, AI-TPK, AI-TCK, AI-TPACK, and Ethics for each SAMR level. To examine the relationship between SAMR levels and AI-TPACK, a Mann-Whitney U test was conducted. The results indicated no significant differences between the Enhancement group (Substitution and Augmentation levels) and the Transformation group (Modification and Redefinition levels), suggesting that the AI-TPACK did not differ significantly across the SAMR levels.

However, despite the lack of statistical significance, a general trend was observed where higher SAMR levels tended to be associated with higher AI-TPACK levels. Specifically, when comparing the Augmentation and Modification levels, excluding the smaller sample sizes in the Substitution and Redefinition levels, this trend was observed across all AI-TPACK components. Although these differences were not statistically significant, this trend suggests that PSTs at the Modification level exhibited higher levels of AI-TK, AI-TPK, AI-TCK, AI-TPACK, and ethical knowledge compared to those at the Augmentation level.

Table 2. AI-TPACK based on SAMR levels

SAMR levels		N	AI-TK	AI-TPK	AI-TCK	AI-TPACK	Ethics
Enhancement	Substitution	2(4%)	2.67	3.13	2.87	2.90	3.10
	Augmentation	29(58%)	2.80	3.07	2.81	2.93	3.06
Transformation	Modification	16(32%)	3.04	3.25	3.11	3.13	3.22
	Redefinition	3(6%)	2.80	4.00	3.15	3.05	4.05
Mann-Whitney		z	-1.69	-1.54	-1.80	-0.94	-1.07
		p	0.12	0.20	0.72	0.35	0.29

Conclusions

This study analyzed how PSTs integrate AI tools into their lesson plans and the relationship between SAMR levels and their AI-TPACK. Based on the findings, the following implications for teacher education and AI tool integration in lessons are presented.

First, the results of this study show that a significant number of PSTs tended to plan lessons at the Augmentation level of the SAMR model when integrating AI tools. Notably, instances of lesson designs that reached the Redefinition level were rare. While the PSTs seemed to be aware of the advantages of 'TocToc-Math', they predominantly utilized these tools at the level of enhancing existing activities, rather than modifying or creating new ones. These findings are similar to the findings in Shin (2022), where prospective teachers recognize the potential of ITSs, but often position them as servants. This suggests that while PSTs recognize the potential of AI tools, they struggle to apply them in transformative ways that could fundamentally alter the structure of their lessons. Therefore, teacher education should focus on helping modify activities and design new ones using AI tools, shifting from simple usage to transformative integration.

Second, although a general trend emerged showing that higher SAMR levels were associated with higher AI-TPACK scores, the statistical analysis revealed no significant differences across the SAMR categories. This suggests that enhancing AI-TPACK alone is insufficient to elevate AI integration. Additional factors beyond AI-TPACK development are needed, particularly knowledge of diverse AI tools. The results showed that PSTs' AI-TK was relatively low, and they rarely incorporated other AI tools into their lesson plans. Moreover, when using 'TocToc-Math', it was evident that PSTs primarily focused on the tool's manipulative functions, such as building blocks, rather than its AI features, such as AI-based assessments. This suggests a low level of engagement with the AI features of the tool, which likely contributed to the weaker correlation between AI-TPACK and SAMR levels observed in the lesson plans. Therefore, comprehensive training is needed to enhance PSTs' knowledge of diverse AI tools and encourage their deeper integration.

References

Celik, I. (2023). Towards Intelligent-TPACK: An empirical study on teachers' professional knowledge to ethically integrate artificial intelligence (AI)-based tools into education. *Computers in Human Behavior, 138*, 107468.

Chen, L., Chen, P., & Lin, Z. (2020). Artificial intelligence in education: A review. *IEEE Access, 8*, 75264-75278.

Gentile, M., Città, G., Perna, S., & Allegra, M. (2023). Do we still need teachers? Navigating the paradigm shift of the teacher's role in the AI era. *Frontiers in Education, 8*, 1161777.

Hamilton, E. R., Rosenberg, J. M., & Akcaoglu, M. (2016). The substitution augmentation modification redefinition (SAMR) model: A critical review and suggestions for its use. *TechTrends*, *60*, 433-441.

Lee, Y. (2024). Analysis of elementary teachers' AI-TPACK in mathematics education. *Education of Primary School Mathematics*, *27*(4), 463-479.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017-1054.

Mishra, P., Koehler, M. J., & Henriksen, D. (2010). The 7 transdisciplinary habits of mind: Extending the TPACK framework towards 21st century learning. *Educational Technology, 51*(2), 22-28.

Mishra, P., Warr, M., & Islam, R. (2023). TPACK in the age of ChatGPT and Generative AI. *Journal of Digital Learning in Teacher Education, 39*(4), 235-251.

Selwyn, N. (2019). *Should robots replace teachers?: AI and the future of education.* John Wiley & Sons.

Shin, D. (2022). Teaching mathematics integrating intelligent tutoring systems: Investigating prospective teachers' concerns and TPACK. *International Journal of Science and Mathematics Education*, *20*(8), 1659-1676.

Valtonen, T., Sointu, E., Kukkonen, J., Kontkanen, S., Lambert, M. C., & Mäkitalo-Siegl, K. (2017). TPACK updated to measure pre-service teachers' twenty-first century skills. *Australasian Journal of Educational Technology, 33*(3), 491-502.

Zawacki-Richter, O., Marín, V. I., Bond, M., & Gouverneur, F. (2019). Systematic review of research on artificial intelligence applications in higher education—where are the educators? *International Journal of Educational Technology in Higher Education, 16*, 1-27.

TSG6-1-3

PRE-SERVICE TEACHERS' SELF-EFFICACY OF USING GENERATIVE ARTIFICIAL INTELLIGENCE IN MATHEMATICS INSTRUCTION

Norbert Noster
University of Würzburg, Germany, norbert.noster@uni-wuerzburg.de

Hans-Stefan Siller
University of Würzburg, Germany, hans-stefan.siller@uni-wuerzburg.de

Nils Buchholtz
University of Hamburg, Germany, nils.buchholtz@uni-hamburg.de

Artificial intelligence, and especially generative variants, receive increasing attention in mathematics education research. This includes their prevalence in educational contexts as well as the need to educate learners about the challenges evolving around this topic. In order for the latter to happen in school, mathematics teachers require specific competencies for using applications of generative artificial intelligence and guiding learners while making use of the technology. Therefore, this study focuses on generative artificial intelligence related self-efficacy, which has been empirically studied as a predictor for the use and implementation of technology in the Technology Acceptance Model. To gain insight into pre-service teachers' self-efficacy regarding this technology, a quantitative study was carried out with 192 pre-service teachers from the University of Hamburg and Würzburg. A scale to measure generative artificial intelligence related self-efficacy that has produced reliable results is presented in this study. The data suggests that future teachers feel partially confident in making use of generative artificial intelligence in their future mathematics lessons.

Keywords: Artificial Intelligence, Teacher Education, Self-Efficacy, Mathematics Education

Introduction

With the availability of Large Language Models and other generative tools, the topic of artificial intelligence has received increasing attention in recent months. In the field of education, claims were made promoting the potential of generative AI for teaching and learning (Sabzalieva & Valentini, 2023). While different studies started discussing them, artificial intelligence slowly but surely finds its way into school education. This includes the use by teachers for preparing instruction (Buchholtz & Huget, 2024; Gurl et al., 2024) and fostering learning processes from a teaching perspective (Noster et al., 2024b). Central to this is the teachers' competence in using tools of artificial intelligence for domain-specific purposes, which was shown to not be sufficiently developed (Noster et al., 2024a). Therefore, the focus of this study lies on teachers and teacher education. One model that studies the use of technology in professional settings is the technology acceptance model (TAM), which has been empirically studied for more than 30 years (Davis & Granić, 2024). For the purpose of this study, it is used in the context of pre-service teachers intending to make use of artificial intelligence as technology in their future instruction. The focus is put on pre-service teachers in order to gain insights into their educational needs in teacher training to help them teach students mathematics with the help of artificial intelligence.

Theoretical Background

The TAM model's goal is to test for users' acceptance of technology, for which perceived usefulness and perceived ease of use were identified to be core determinants to influence the intention of using and actually using a technology (Davis & Granić, 2024). These two determinants in itself are affected by external variables. Self-efficacy was proven to be a reliable predictor for perceived ease of use in the context of learners' usage of e-portfolios (Abdullah et al., 2016), as well as pre-service teachers' intention to use mobile devices in their future instruction (Sánchez-Prieto et al., 2017). Furthermore, Jin (2014) found that self-efficacy also affects perceived usefulness in addition to perceived ease of use. Therefore, the goal is to examine pre-service teachers' self-efficacy regarding the implementation of generative artificial intelligence in mathematics education as a first step towards acceptance of the technology for professional purposes in teaching and learning.

The construct of self-efficacy is defined by Bandura (1997) as "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (p.3). It is noteworthy that this construct takes on a subjective perspective that is said to influence behavior, similar to claims made with the TAM model. Regarding the use of technology, this notion has been adapted for computer self-efficacy as "a judgment of one's capability to use a computer" (Compeau & Higgins, 1995, p.192). In a similar manner, we operationalize *generative artificial intelligence related self-efficacy* as a judgment of one's capability to use tools that are based on generative artificial intelligence. This relates to pre-service teachers' self-efficacy in using this technology. Taking up Bandura's notion of a given attainment the relation to using technology with regard to mathematics instruction is being added. This is done to include the pre-service teachers' later professional environments in which they should put generative artificial intelligence to use.

Methodology

Data was collected in the summer of 2024 at the universities of Würzburg (n=96) and Hamburg (n=96) using an online survey tool. The questionnaire developed by Buchholtz et al. (2024) was distributed via mail and included a brief explanation of the term artificial intelligence. Furthermore, it was specified that the questionnaire specifically evolves around generative artificial intelligence. Participation was voluntary. In total, 192 pre-service teachers were recruited as participants for the study, with an age range from 19 to 45 years and an average age of 23,3 years (*sd*=4,66). These include students who become teachers at grammar schools (n=89), elementary schools (n=52), middle schools (n=13), vocational schools (n=9), as well as schools for children with special needs (primary n=14; secondary n=14).

The survey contained demographic questions and a total of 87 items. Each item consists of a statement, which the participants were asked to rate on a five-point scale (fully agree, tend to agree, partly agree and disagree, tend to disagree, fully disagree). The addressed topics are possible applications and benefits of artificial intelligence in mathematics lessons for pupils, challenges and risks in the use of artificial intelligence technologies with regard to pupils, possible applications of artificial intelligence in mathematics lessons for teachers, challenges and risks of using artificial intelligence technologies with regard to teachers, and attitudes towards self-reported artificial intelligence competencies of teachers.

The items regarding attitudes towards self-reported artificial intelligence competencies contain self-efficacy-related statements, which this study focuses on. Generative artificial intelligence related self-efficacy in mathematics education is operationalized using the following statements: "*I feel*

well prepared to integrate AI technologies into my math lessons" (SE1); "*I feel overwhelmed by the topic of "artificial intelligence*" implying a low judgment of one's capability (SE2); "*I feel able to cope with the technical challenges of using AI technologies in mathematics lessons*" (SE3); "*I have technical difficulties in dealing with generative AI Tools.*" implying a low judgment of one's capability (SE4), "*I feel able to interpret the data and decisions provided by AI technologies to specifically improve my teaching*" (SE5); "*I feel able to assess the opportunities and limitations of using AI technologies in the classroom*" (SE6); "*I feel able to assess the ethical implications of using AI technologies in the classroom*" (SE7). The statements were chosen through discussion among the authors. Additionally, two researchers from mathematics education confirmed face validity of these items regarding self-efficacy. Confirmatory factor analysis (CFA) is carried out to test validity using diagonally weighted least squares method due to the ordinal nature of the data and is considered acceptable with $CFI > 0.95$ and $SRMR < 0.09$ (Hu & Bentler, 1999).

For the purpose of interpretation and analysis, the answers were assigned whole numbers from -2 (fully disagree) to 2 (fully agree). The assigned numbers for items SE2 and SE4 were multiplied by -1 to account for the fact that they are formulated negatively.

Results

In every item, at least one participant fully agreed with the statement. This is also true for fully disagreeing with a statement. The number of missing answers ranges from 30 to 33 per item related to self-efficacy. Validity is confirmed through CFA ($CFI \approx 0.995$, $SRMR \approx 0.051$). With $\alpha = .79$, the reliability of the scale can be considered acceptable and almost high. The results of the descriptive analysis are presented in Table 1.

Table 1. Descriptive statistics of the self-efficacy items

	SE1	SE2	SE3	SE4	SE5	SE6	SE7
Median	-1	0	1	1	0	0	1
Mean	-0.5556	0.2531	0.4037	0.7516	0.4088	0.3395	0.02484
SD	1.153	1.155	1.086	1.019	1.069	0.973	0.974

Note. SD refers to standard deviation.

Item SE1 stands out as it has the lowest mean, the sole negative median. The highest mean corresponds to item SE4 with a median of 1. The statements of SE3 and SE5 were similarly rated and have the second-highest mean but differ in the median. Item SE6 has a lower mean, followed by SE2. The mean of SE7 stands out as it is close to zero. The standard deviations range from 0.973 to 1.15 and take on rather similar values across the scale.

Discussion & Conclusion

The answers to the items regarding self-efficacy show that in no cases do the pre-service teachers appear to be neither overly confident nor unconfident, as the averages are dispersing around zero. It is also noteworthy that the pre-service teachers feel rather unprepared for using the technology in their lessons. This is despite the fact that they seem to have less trouble using generative artificial intelligence tools. This points towards the need to address the usage of such tools for mathematics education in pre-service teacher education rather than addressing technical aspects. However, the

latter cannot be completely omitted as the participants felt to be partially overwhelmed by the topic of artificial intelligence. Self-efficacy regarding opportunities and limitations for instruction is leaning towards the positive side while remaining rather low on average. Furthermore, the pre-service teachers feel somewhat confident in being able to interpret the results of generative artificial intelligence to improve teaching. Another point that should be taken up is the discussion of ethical implications regarding the topic of artificial intelligence, since pre-service teachers only partially appear to be able to assess these with regard to their instruction.

Looking at these results with a rather neutral level of self-efficacy through the lens of the TAM model allows for further implications. Since the items can be considered external factors influencing perceived ease of use and perceived usefulness, it appears likely that the pre-service teachers also have mixed feelings about these two variables with regard to generative artificial intelligence. Keeping in mind that these two factors influence the intent of using a certain technology for professional purposes, it seems not too likely that the pre-service teachers would use generative artificial intelligence in mathematics instruction. This was not examined in this current analysis, which is why this implication should be taken with caution. Further insights regarding the individual variable of the TAM model are necessary.

Although the sample size is moderate, with almost 200 participants, it is noteworthy that the participating pre-service teachers are not necessarily representative of other future teachers. Given the novelty of the technology and the little research that has been done yet, there is little reason to believe that the studied sample differs greatly from other pre-service teachers.

Despite the limitations, this study provides valuable insights into pre-service teachers' self-efficacy regarding the usage of generative artificial intelligence in mathematics instruction. These show that further action is required in teacher education in order to increase the future teachers' intention of using generative artificial intelligence and unlocking their potential in their future classrooms.

References

Abdullah, F., Ward, R., & Ahmed, E. (2016). Investigating the influence of the most commonly used external variables of TAM on students' Perceived Ease of Use (PEOU) and Perceived Usefulness (PU) of e-portfolios. *Computers in Human Behavior*, *63*, 75–90. https://doi.org/10.1016/j.chb.2016.05.014

Bandura, A. (1997). *Self-efficacy: The exercise of control*. Freeman.

Buchholtz, N., & Huget, J. (2024). ChatGPT as a reflection tool to promote the lesson planning competencies of pre-service teachers. *Proceedings of the 17th ERME Topic Conference MEDA 4 – University of Bari Aldo Moro, Italy, 3-6 September 2024*, 129–136.

Buchholtz, N., Schorcht, S., Bata, K., Baumanns, L., Decker, M., Dilling, F., Knoth, N., Laupichler, M. C., Noster, N., Mah, D.-K., Peters, F., Pinski, M., Siller, H.-S., Strohmaier, A., & Witzke, I. (2024). *Fragebogen zu Überzeugungen und Bewertungen angehender und praktizierender Mathematiklehrkräfte im Bezug auf den Einsatz von Künstlicher Intelligenz (KI) im Kontext von Lehr-Lernprozessen*. Unpublished document.

Compeau, D. R., & Higgins, C. A. (1995). Computer Self-Efficacy: Development of a Measure and Initial Test. *MIS Quarterly*, *19*(2), 189–211. https://doi.org/10.2307/249688

Davis, F. D., & Granić, A. (2024). *The Technology Acceptance Model: 30 Years of TAM*. Springer International Publishing. https://doi.org/10.1007/978-3-030-45274-2

Gurl, T. J., Markinson, M. P., & Artzt, A. F. (2024). Using ChatGPT as a Lesson Planning Assistant with Preservice Secondary Mathematics Teachers. *Digital Experiences in Mathematics Education*, *11*, 114–139. https://doi.org/10.1007/s40751-024-00162-9

Jin, C.-H. (2014). Adoption of e-book among college students: The perspective of an integrated TAM. *Computers in Human Behavior*, *41*, 471–477. https://doi.org/10.1016/j.chb.2014.09.056

Noster, N., Gerber, S., & Siller, H.-S. (2024a). Pre-Service Teachers' Approaches in Solving Mathematics Tasks with ChatGPT. *Digital Experiences in Mathematics Education*, *10*(3), 543–567. https://doi.org/10.1007/s40751-024-00155-8

Noster, N., Gerber, S., & Siller, H.-S. (2024b). Tasks incorporating the use of ChatGPT in mathematics education designed by pre-service teachers. *Proceedings of the 17th ERME Topic Conference MEDA4*, 303–310.

Sabzalieva, E., & Valentini, A. (2023). *ChatGPT and artificial intelligence in higher education: Quick start guide.* UNESCO. https://unesdoc.unesco.org/ark:/48223/pf0000385146

Sánchez-Prieto, J. C., Olmos-Migueláñez, S., & García-Peñalvo, F. J. (2017). MLearning and pre-service teachers: An assessment of the behavioral intention using an expanded TAM model. *Computers in Human Behavior*, *72*, 644–654. https://doi.org/10.1016/j.chb.2016.09.061

TSG6-1-4

STUDENTS' PERCEPTIONS OF GENERATIVE ARTIFICIAL INTELLIGENCE AND UNDERSTANDING OF MATHEMATICAL PROOFS

Yujin Lee
Kangwon National University, Korea, ylee@kangwon.ac.kr

Kyungwon Lee
Seoul National University, Korea, kyungwon.lee.snu@gmail.com

Oh Nam Kwon
Seoul National University, Korea, onkwon@snu.ac.kr

As generative AI tools become increasingly integrated into education, their impact on students' mathematical reasoning and perceptions of proof remains an important area of study. This research explores how students use ChatGPT in the process of mathematical proving and how their perceptions of ChatGPT and mathematical proof evolve through interaction with the tool. The study includes a case analysis of a student who leveraged ChatGPT to refine his proof, illustrating how AI-assisted reasoning influenced his approach. Additionally, pre- and post-test results revealed a statistically significant improvement in students' perceptions of ChatGPT after engaging with it in proving. Beyond individual learning outcomes, this study provides broader implications for the use of generative AI tools in higher education, particularly in the context of mathematical reasoning and proof construction. It highlights ChatGPT's potential as a supportive tool for mathematical proving, while also raising questions about students' critical engagement with AI-generated proofs.

Keywords: Generative AI, ChatGPT, Perceptions, Proofs

Introduction

This study explores how undergraduate students use artificial intelligence (genAI) to prove mathematical statements and examine changes in their perceptions throughout the process. As genAI continues to be integrated into education, discussions on its role in mathematical learning have also progressed. Yoon et al. (2024), in an exploratory study involving three undergraduate students, proposed the Students' Interactive Proving Experience with AI (SIPE-AI) framework to explain how students use genAI in proof construction. Their analysis demonstrated that students' interactions with genAI in proving could be associated with various factors. While Yoon et al. (2024) provided valuable insights, its small sample size highlights the need for further research with a larger participant group. Building on these findings, this study seeks to deepen our understanding of how undergraduate students engage with genAI in proof construction and how their perceptions shift before and after proving. Through this analysis, we aim to refine the SIPE-AI framework and demonstrate practical applications for undergraduate teaching and learning in proving with genAI. This analysis will provide pedagogical implications by demonstrating practices of teaching and learning proofs using generative artificial intelligence, while also offering methodological implications for mathematics learning with the use of generative AI. The research question is as follows: *How do under-*

graduate students learn mathematical proofs using genAI, and how do their conceptions change during this process?

Literature Review

GenAI can support student learning by providing personalized learning environments tailored to individual ability levels and offering immediate feedback through interactive engagement (Baidoo-Anu & Ansah, 2023). However, there are significant limitations of genAI that require careful consideration. GenAI may provide insufficiently detailed explanations or inaccurate information (Su & Yang, 2023). Recent studies indicate that students have experienced challenges such as concerns over the reliability of information provided by genAI, difficulties in phrasing questions to elicit desired responses, and confusion caused by ambiguous answers generated by genAI (Ngo, 2023). As discussed above, genAI offers unique advantages to learning environments while simultaneously presenting potential challenges. To address these challenges, it is essential to focus on students' perceptions of genAI in use. Such insights can help guide students toward critical and thoughtful use, avoiding overreliance on technology. Yoon et al. (2024) identified three factors influencing students' use of genAI in proving through task-based clinical interviews: conceptions of mathematical proofs, conceptions of genAI in use, and ethical considerations with genAI. This study aims to investigate how students' perceptions of genAI change as they use it for proving. By focusing on students' perceptions, this study aims to build on and extend the findings of Yoon et al. (2024).

Methodology

The participants in this study consisted of 33 undergraduate students enrolled in a mathematics education technology course. ChatGPT, developed by OpenAI, is the most widely used and commonly recognized tool in Korea. Therefore, this study utilized ChatGPT as one of the genAI tools. Among the participants, 26 students had prior experience using ChatGPT, while seven had not. Additionally, 15 students had previously used ChatGPT for proving mathematical statements, whereas 18 students had not. We asked the participants to use ChatGPT to prove mathematical statements. To examine shifts in their perceptions, we conducted a survey before and after they completed their proofs using ChatGPT. We conducted a two-week intervention involving mathematical proof activities. The activities included using ChatGPT to assist in understanding how to prove mathematical statements. The intervention involved two main activities. First, students used ChatGPT to explore potential solutions before attempting to prove the theorem on their own. Second, students independently developed their own solutions and then used ChatGPT to enhance or refine their proofs. After completing both activities separately, students were also asked to prove similar theorems independently to reinforce and deepen their understanding of the proof process. To investigate whether students were willing to follow ChatGPT-generated proofs, we examined the extent to which their approaches to proving theorems aligned with ChatGPT's solutions. This analysis considered the timing of ChatGPT adoption—before or after proving—to determine its influence on students' reasoning. We adopted a framework incorporating three key factors: the set of accepted statements, modes of argumentation, and modes of argument representation (Stylianides, 2007). To illustrate our findings, we present an example from a selected participant. To explore the participants' perceptions of ChatGPT, ethics, and mathematical proofs, pre- and post-tests were administered immediately before and after the intervention. The study utilized 22 items related to ChatGPT use, 11 items concerning ethics in ChatGPT, 7 items focused on mathematical proofs, and 5 items addressing mathematical proofs using ChatGPT, based on previous research (Farhi et al., 2023; Malmstrom et

al., 2023; Shoufan, 2023; Welding, 2023). A 5-point Likert scale was used, ranging from 1 (Strongly Disagree) to 5 (Strongly Agree). To analyze the collected data, paired t-tests were implemented.

Results

Similarity of Proofs

Student A began with the 'prove first and ChatGPT later' activity, in which he initially attempted to prove the theorem independently before using ChatGPT to refine and finalize his proof. In this process, he treated ChatGPT's results as a reference to developing his own proof rather than relying on them entirely. Notably, he revised the modes of argument representation, adapting how the proof was structured and communicated. However, when engaging in the 'ChatGPT first and prove later' activity, his approach shifted. Instead of constructing the proof independently, he was more inclined to directly adopt ChatGPT's results with minimal personal modification. This suggests that the order in which ChatGPT was introduced influenced his level of engagement in the proving process, with prior independent proving encouraging deeper mathematical reasoning and later exposure to ChatGPT leading to a more passive acceptance of the AI-generated proof.

Students' Perceptions

Table 1 presents the results of the paired t-tests, comparing pre- and post-test data. For each category, the table includes means, standard deviations (SD), t-values, and p-values, followed by an analysis of the findings related to the research question.

Table 1. Mean, SD, t, and p in students' perceptions by categories

Category	test	Mean	SD	t	p
ChatGPT Perceptions	Pre	68.97	8.656	-6.167	<0.001
	Post	77.60	10.278		
Ethical Considerations of ChatGPT	Pre	35.90	4.270	-1.927	0.064
	Post	37.57	4.423		
Proof Perceptions	Pre	28.80	3.652	1.008	0.322
	Post	27.97	4.612		
Perceptions of proof using ChatGPT	Pre	17.73	2.559	-1.227	0.230
	Post	18.40	2.415		

Regarding ChatGPT perceptions, the mean score for the post-test was higher than that of the pre-test, indicating an improvement in students' perceptions of ChatGPT following the intervention. The t-value for ChatGPT use was -6.167, and it was statistically significant ($p < 0.001$). This suggests that the students' perceptions of ChatGPT increased significantly after engaging in the intervention.

In terms of ethical considerations of ChatGPT and perceptions of proof using ChatGPT, the mean scores for the post-tests were also higher than the pre-test mean scores. However, the differences were not statistically significant, as indicated by p-values greater than 0.05. This implies that while there was an improvement in students' ethical considerations of ChatGPT and the use of ChatGPT in proofs, the change was not strong enough to be considered statistically meaningful. For mathe-

matical proofs, the mean score of the post-test was lower than the pre-test mean score. However, this difference was not statistically significant, with a p-value of 0.005.

Although students' performance or understanding of mathematical proofs appeared to decrease slightly after the intervention, this change was not statistically significant, suggesting that the intervention did not have a major negative impact on students' proof abilities in this context. Figure 1 visualizes the mean differences between pre- and post-test results.

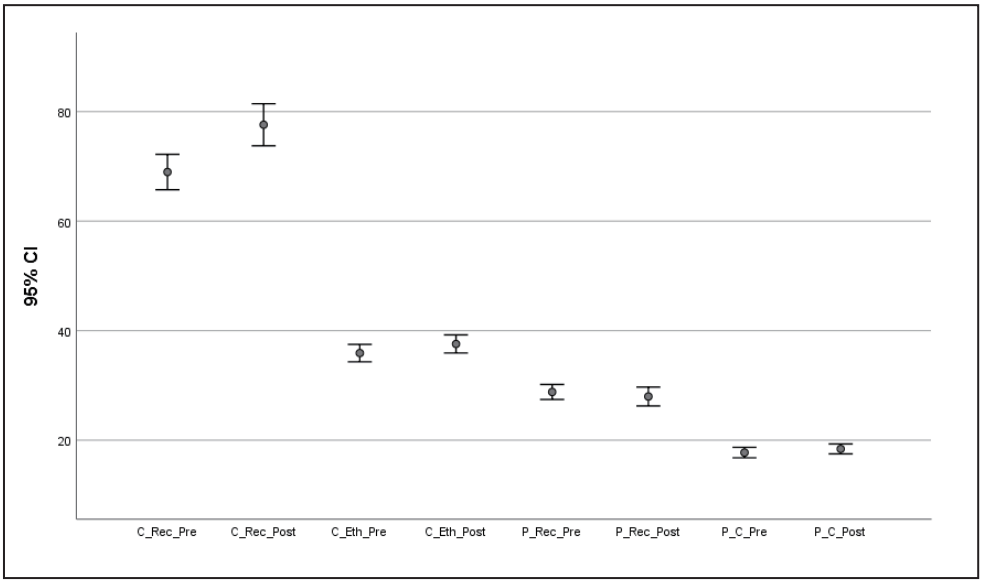


Figure 1. Pre- and post-test results

Conclusion

This study examined the changes in students' perceptions of ChatGPT during their proving process. The pre- and post-test results revealed a statistically significant improvement in students' perceptions of ChatGPT after engaging with the tool for proving. This suggests that students' views evolve during proof construction, which indicates that educators could play a role in guiding students to use ChatGPT productively. Furthermore, this study provides insights into the implications of genAI tools in higher education, particularly in shaping students' understanding of AI-assisted learning and its role in mathematical reasoning. It highlights the potential of ChatGPT as a supportive tool for mathematical proving, while also addressing concerns regarding students' critical engagement with AI-generated proofs. By analyzing students' actual patterns of use and their evolving perceptions, this study offers practical considerations for integrating genAI tools in undergraduate mathematics education.

References

Baidoo-Anu, D., & Ansah, L. O. (2023). Education in the era of generative artificial intelligence (AI): Understanding the potential benefits of ChatGPT in promoting teaching and learning. *Journal of AI*, *7*(1), 52-62.

Farhi, F., Jeljeli, R., Aburezeq, I., Dweikat, F. F., Al-shami, S. A., & Slamene, R. (2023). Analyzing the students' views, concerns, and perceived ethics about chat GPT usage. *Computers and Education: Artificial Intelligence*, 100180.

Isoda, M. (2010). Lesson study: Problem solving approaches in mathematics education as a Japanese experience. *Procedia-Social and Behavioral Sciences, 8*, 17-27.

Malmström, H., Stöhr, C., & Ou, W. (2023). *Chatbots and other AI for learning: A survey of use and views among university students in Sweden [Application/pdf]. Chalmers University of Technology.*

Ngo, T. T. A. (2023). The Perception by University Students of the Use of ChatGPT in Education. *International Journal of Emerging Technologies in Learning (Online)*, *18*(17), 4-19.

Shoufan, A. (2023). Exploring students' perceptions of ChatGPT: Thematic analysis and follow-up survey. *IEEE Access*, *11*, 38805-38818.

Stylianides, A. L. (2007). Proof and proving in school mathematics. *Journal for Research in Mathematics Education*, *38*(3), 289-321.

Su, J., & Yang, W. (2023). Unlocking the power of ChatGPT: A framework for applying generative AI in education. *ECNU Review of Education*, 1-12.

Watanabe, T., Lo, J.-J., & Son, J.-W. (2017). Intended treatment of fractions and fraction operations in mathematics curricula from Japan, Korea, and Taiwan. In J.-W. Son, T. Watanabe, & J.-J. Lo (Eds.), *What Matters? Research Trends in International Comparative Studies in Mathematics Education* (pp. 33-61). Springer.

Welding, L. (2023). Half of college students say using AI is cheating. BestColleges.

Vygotsky, L. (1978). Mind in society: *The development of higher psychological processes*. Cambridge. Harvard University Press.

Yoon, H., Hwang, J., Lee, K., Roh, K. H., & Kwon, O. N. (2024). Students' use of generative artificial intelligence for proving mathematical statements. *ZDM–Mathematics Education*, *56*(7), 1531–1551.

TSG6-1-5

SIMULATED PRACTICE-BASED TEACHING FOR PRESERVICE TEACHERS USING AN AI CHATBOT

Taekwon Son
Bongmyong Elementary School, Korea, sontaekwon7@gmail.com

Dabae Lee
Kennesaw State University, USA, dlee159@kennesaw.edu

Sheunghyun Yeo
Daegu National University of Education, Korea,shyeo@dnue.ac.kr

Corey Webel
University of Missouri, USA, webelcm@missouri.edu

This study examines the use of an AI-based chatbot to enhance preservice teachers' (PSTs) responsive teaching skills through simulated interactions. The chatbot features a virtual student with misconceptions and a virtual mentor providing feedback on questioning strategies. Using an exploratory design, 148 PSTs from two universities practiced eliciting and responding to student thinking in fraction concepts. Latent profile analysis identified four questioning profiles, revealing that high-leverage teaching moves with high responsiveness led to the most improvement. Findings suggest AI-driven simulations can effectively support PSTs in developing questioning strategies and responsive teaching skills.

Keywords: Responsive teaching, AI chatbot, preservice teacher education, simulated teaching, professional noticing, questioning strategies, mathematics education

Introduction

Preservice teachers (PSTs) have limited opportunities to develop responsive teaching skills before engaging in real classroom field experiences. Additionally, the absence of teacher educators during field placements may hinder the provision of immediate, constructive feedback on the effectiveness of PSTs' teaching strategies. AI chatbots offer a promising solution by providing personalized learning environments where PSTs can approximate teaching practices, refine their responsive teaching skills, and receive real-time, personalized feedback. However, research on the use of AI chatbots for such purposes remains scarce. To address this gap, we designed an AI chatbot system featuring two virtual agents: a virtual student exhibiting misconceptions and a virtual mentor programmed to provide feedback on PSTs' questioning strategies. This study aims to examine how PSTs engage with the AI chatbot and how their teaching moves and responsiveness evolve through simulated interactions. The study addresses the following research questions: RQ1. What profiles of PSTs' questioning emerge in terms of their teaching moves and responsiveness? RQ2. How do PSTs' questioning patterns change after receiving feedback from the virtual mentor?

Theoretical Framework

Responsive teaching emphasizes attending to students' ideas and adapting instruction to support their mathematical sense-making (Dyer & Sherin, 2016; Pierson, 2008). Effective responsive teaching involves continual adjustment to students' mathematical thinking through instructional decision-making (Jacobs & Empson, 2016). Research has demonstrated that responsive teaching fosters deeper mathematical understanding (Carpenter et al., 2014; Pierson, 2008), encourages student engagement (Flood et al., 2020), and promotes equitable learning environments by valuing diverse perspectives (Robertson et al., 2016). Despite prior research on responsive teaching, empirical studies exploring specific teaching moves and their relationship to responsiveness remain limited. Given the various ways responsive teaching can manifest, further research is needed to analyze specific instructional moves (Jacobs & Empson, 2016) and degrees of responsiveness (Kavanagh et al., 2020) to better understand the nuances of effective responsive teaching.

Traditional university coursework primarily focuses on developing pedagogical knowledge, while instructional practice is often limited to field placements. In response, practice-based teacher education has emerged as an approach emphasizing the identification and development of core teaching practices (Grossman et al., 2009; Ball & Forzani, 2009). Grossman et al. (2009) delineate three phases of practice-based teacher education: Representation of practice through videos and instructional artifacts, Decomposition of practice by breaking down complex instructional techniques into their components, and Approximation of practice through simulated environments that enable novices to practice discrete instructional skills. Approximation of practice, in particular, allows PSTs to develop and refine their questioning techniques in a controlled, low-risk environment (Santagata & Yeh, 2014). In this study, PSTs engaged in simulated questioning interactions with a virtual student, modeled on common fraction-related misconceptions, as a way to approximate real classroom discourse.

Methodology

This study employed an exploratory research design, which serves as a preliminary investigation into a novel issue and lays the groundwork for future research (Singh, 2007). Participants included 148 PSTs enrolled in elementary mathematics methods courses at two South Korean teacher education institutions: University A (n=53, 18 males, 35 females) and University B (n=95, 41 males, 54 females). These PSTs were in the second year of a four-year elementary teacher preparation program, where they studied mathematical content knowledge (e.g., fraction concepts) and pedagogical strategies for eliciting and responding to student thinking during problem-solving activities.

Two types of data were collected. Conversation data is interaction logs between the PSTs and the AI chatbot, stored in JSON format and analyzed using a custom-built program that converted data into an interpretable format. Post-survey data is reflections from PSTs collected through Google Forms following their chatbot interactions.

We developed an analytical framework informed by literature to categorize. First, teaching moves include High-Leverage Teaching Moves (HLTM) that elicit or build on student thinking and Low-Leverage Teaching Moves (LLTM) that do not support student understanding. Second, responsiveness include High, Low, or Unclassified responsiveness based on whether questions built upon the student's previous responses. Latent profile analysis (LPA) was conducted using the *mclust* package in R (Scrucca et al., 2017), with teaching moves and responsiveness as profile variables. Using the profiles identified in RQ1, we examined how PSTs' questioning strategies evolved

after receiving feedback from the virtual mentor. Questions were coded into four categories: HH (High-Leverage Teaching Moves & High Responsiveness), HL (High-Leverage Teaching Moves & Low Responsiveness), LH (Low-Leverage Teaching Moves & High Responsiveness), and LL (Low-Leverage Teaching Moves & Low Responsiveness).

Results

Model fit indices (Table 1) supported a four-profile solution. The identified profiles (Table 4) included: Profile 1: HLTM & High Responsiveness, Profile 2: LLTM & High Responsiveness, Profile 3: LLTM & Low Responsiveness, and Profile 4: Medium-Leverage Teaching Moves & Medium Responsiveness.

Table 1. Fit indices for different models

Model	AIC	CAIC	BIC	SABIC	Entropy	BLRT (p)	No. of profiles < 5%
2-profile	25.8129	54.43237	45.93929	23.79897	0.72414	> .05	0
3-profile	7.81949	46.57146	36.57146	4.942443	0.806857	< .001	1
4-profile	**4.054806**	**52.93929**	**41.43237**	**7.314645**	**0.840772**	**< .001**	**0**
5-profile	4.60658	66.60974	50.60974	12.0033	0.725941	> .05	1

Note. AIC = Akaike's Information Criterion; CAIC = the Constant Akaike's Information Criterion; BIC = Bayesian Information Criterion; SABIC = sample size adjusted Bayesian Information Criterion; BLRT(p) = p-Value for the bootstrap likelihood ratio test

PSTs' questioning patterns were analyzed before and after receiving feedback from the virtual mentor. Among the profiles, HLTM & HR (23 out of 27) received mostly positive feedback, while LLTM & HR (22 out of 31) and MLTM & MR (23 out of 35) received a mix of positive and constructive feedback. LLTM & HR received slightly more positive feedback, whereas MLTM & MR received more constructive feedback. LLTM & LR (20 out of 38) predominantly received constructive feedback. The following sections provide a detailed analysis of question quality changes for each profile.

Discussions

Findings highlight the importance of both teaching moves and responsiveness in effective questioning. PSTs who engaged in high-leverage questioning with high responsiveness demonstrated the most growth. The AI-based chatbot provided an effective simulation for approximating teaching practice and fostering pedagogical reflection. Compared to traditional digital tools that offer limited response choices (Estapa et al., 2018), the chatbot enabled open-ended questioning and real-time feedback. Future studies should explore how AI chatbots can further support responsive teaching beyond fraction concepts.

References

Ball, D., & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 60*(5), 497-511.

Carpenter, T., Fennema, E., Franke, M. L. L., Levi, L., & Empson, S. (2014). *Children's mathematics: Cognitively guided instruction*. Heinemann.

Dyer, E. B., & Sherin, M. G. (2016). Instructional reasoning about interpretations of student thinking that supports responsive teaching in secondary mathematics. *ZDM the Mathematics, 48*, 69-82.

Estapa, A. T., Amador, J., Kosko, K. W., Weston, T., de Araujo, Z., & Aming-Attai, R. (2018). Preservice teachers' articulated noticing through pedagogies of practice. *Journal of Mathematics Teacher Education, 21*, 387-415.

Flood, V. J., Shvarts, A., & Abrahamson, D. (2020). Teaching with embodied learning technologies for mathematics: Responsive teaching for embodied learning. *ZDM the Mathematics, 52*(7), 1307-1331.

Grossman, P., Hammerness, K., & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers and Teaching, 15*(2), 273-289

Jacobs, V. R., & Empson, S. B. (2016). Responding to children's mathematical thinking in the moment: An emerging framework of teaching moves. *ZDM the Mathematics, 48*, 185-197.

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169-202.

Kavanagh, S. S., Metz, M., Hauser, M., Fogo, B., Taylor, M. W., & Carlson, J. (2020). Practicing responsiveness: Using approximations of teaching to develop teachers' responsiveness to students' ideas. *Journal of Teacher Education*, *71*(1), 94-107.

Pierson, J. L. (2008). *The relationship between patterns of classroom discourse and mathematics learning*. The University of Texas at Austin.

Robertson, A. D., Scherr, R. E., & Hammer, D. (Eds.). (2016). *Responsive teaching in science and mathematics.* Routledge.

Santagata, R., & Yeh, C. (2014). Learning to teach mathematics and to analyze teaching effectiveness: Evidence from a video-and practice-based approach. *Journal of Mathematics Teacher Education, 17*, 491-514.

Scrucca, L., Fop, M., Murphy, T. B., & Raftery, A. E. (2017). mclust 5: Clustering, classification and density estimation using Gaussian finite mixture models. *The R Journal, 8*(1), 205-233.

Sherin, M., Jacobs, V., & Philipp, R. (2011). *Mathematics teacher noticing: Seeing through teachers' eyes*. Taylor & Francis.

Singh, K. (2007). *Quantitative social research methods.* SAGE Publications.

Webel, C., & Conner, K. A. (2017). Using simulated teaching experiences to perturb preservice teachers' mathematics questioning practices. *Mathematics Teacher Educator, 6*(1), 9-26.

TSG6-1-6

HOW SECONDARY MATHEMATICS TEACHERS LEVERAGE AI-POWERED TOOLS TO ENHANCE TEACHING EFFICIENCY

Wing Kin Cheng
Hong Kong Metropolitan University, Hong Kong, wikcheng@hkmu.edu.hk

On Ping Chung
Hong Kong Chinese Women's Club College, Hong Kong, ansonchung12@gmail.com

Kwan Ngok Ko
HKFYG Lee Shau Kee College, Hong Kong, matt19017@gmail.com

Ming Yan Tsui
United Christian College, Hong Kong, tmy@ucc.edu.hk

This paper presents several scenarios on how secondary mathematics teachers choose and use AI-powered tools to handle some major tasks. Preliminary data reveals that secondary mathematics teachers expect that AI can (1) help them to prepare remedials and enrichment exercises; (2) help students to understand mathematics concepts; and (3) solve mathematics questions and problems with clear solutions. AI-powered tools have significant enhanced teachers' capacity in these areas. Three strategies, namely, 'Choosing AI-powered tools', 'building prompt library' and 'exploring the interplay between different AI tools' were found to have enhanced teachers' teaching efficiency.

Keywords: AI-powered tools, secondary mathematics teachers, teaching efficiency, prompt library, interplay

Background

The HKSAR Government (2023) has recognized the importance of AI in education, emphasizing its integration into the curriculum, personalized learning, and teacher professional development. The Education Bureau (EDB) promotes AI advancements through collaborative programs, including a new AI module for junior secondary students set to launch before the 2024-25 academic year. Research by Yau et al. (2023) highlight successful collaborations between university professors and secondary teachers to design an effective AI curriculum, fostering student AI literacy and confidence.

Effective AI integration in education relies on teacher readiness and training. Chiu & Li (2023) emphasize the need for professional support, while Ravi et al. (2023) recommend preparatory training before introducing AI curricula. Studies show that AI-assisted teaching positively impacts student engagement and learning outcomes (Mohamed et al., 2022). Research opportunities in Hong Kong remain to explore AI applications in mathematics education, enhancing teaching practices and addressing existing gaps in the curriculum.

Current Research

The research project entitled "Advancing AI in Teaching Secondary Mathematics in Hong Kong" was initiated in June 2024, aiming at gathering insights from secondary mathematics teachers regarding their views on using AI in instruction. The objective is to create innovative teaching methodologies that incorporate AI in the mathematics curriculum.

The first goal is to assess teachers' attitudes, opinions, and experiences with AI in education through both quantitative and qualitative approaches. A 5-point Likert scale questionnaire is utilized to evaluate teachers' perspectives on the integration of AI in mathematics instruction.

The second goal focuses on developing effective AI-based teaching resources and exploring innovative pedagogical strategies. This will involve a series of training sessions and collaborative meetings to equip teachers with the necessary skills and knowledge for AI implementation. Teachers will be organized into different groups to investigate various topics, including (1) creating teaching materials, (2) designing assessments, (3) monitoring student progress, (4) developing remedial and enrichment resources, (5) ethical AI usage, and (6) teacher adaptation to AI. They will experiment with different generative AI tools relevant to their chosen topics. Discussions during these meetings will be recorded for further analysis. Each group will contribute their adapted AI teaching materials, which the project team will compile at a later stage. Participants will also be interviewed about their experiences in integrating generative AI into their teaching practices.

Introduction

Preliminary Findings

This paper presents the preliminary findings of the second part of the study in which the major needs of teachers in using AI-powered tools have been elicited. The research team has subscribed some AI-powered tools including *Thetawise Pro*, *Wolfram Alpha Premium* and *Mathpix Pro*. These tools have been selected based on the suggestion made and trial exercise experimented by the research team and teachers. Different free AI tools have also been used such as *POE*, *Copilot*, *Magic School*, *websim ai*, and *Perplexity*. The tools have been made available for teachers to try out their ideas throughout the research.

Descriptions of functions and roles of the AI tools.

This part provides a brief description of the functions and roles of the AI tools used in this research.

AI tools	Functions of the AI tools	Roles of the AI tools in this research

Methodology

12 mathematics teachers were recruited on a voluntary basis. Each teacher was invited to join two working groups to explore how they can leverage AI-powered tools to teach some selected topics.

For example, in the group 'developing remedial and enrichment materials', teachers can choose AI-powered tools that help them prepare materials correspondingly. Several insightful scenarios will be presented to demonstrate how mathematics teachers leverage AI-powered tools in different tasks.

Choosing AI-powered Tools

Teacher A compared the capacity of the AI-powered tools, *Thetawise* (paid version), *Copilot* (free version), *POE* (free version) and *Wolfram Alpha* (paid version).

Identification of Mistakes

Teacher A tested different tools to explore whether they can facilitate identification of students' mistakes. *POE* and *Wolfram Alpha* failed to identify the mistakes, while *Thetawise* and *Copilot* did. The following shows the teacher's trial with *Copilot*. Teacher A used *Copilot WhatsApp* to snip the students' work and asked *Copilot* to identify students' mistaken step as shown in Figure 1. The output transcription generated by *Copilot* is also shown below.

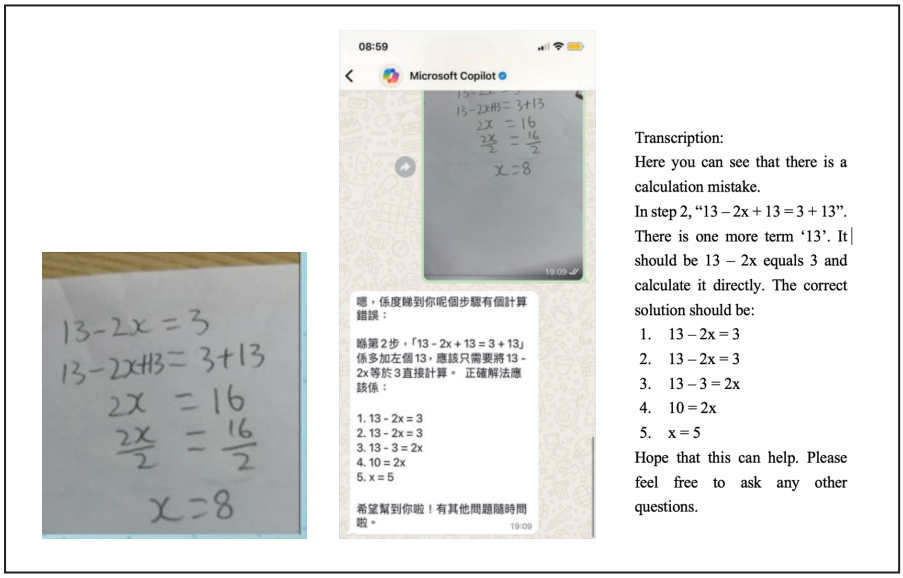
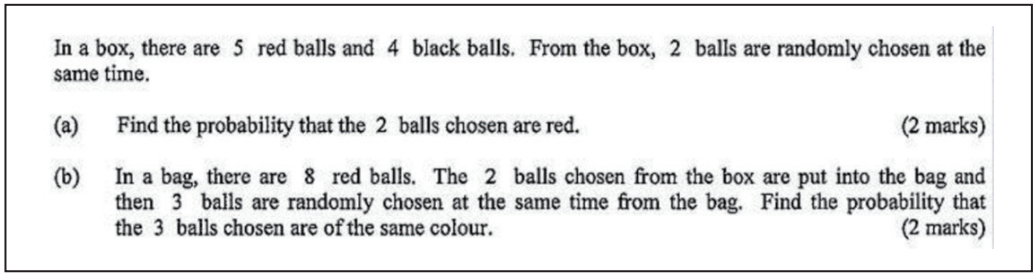


Figure 1. Students' working steps, copilot output and the transcription of the copilot output

Solving Problems

Teacher A wanted to test the problem-solving ability of different AI tools. Figure 2 shows the image that teacher A snipped to *POE*, *Wolfram Alpha*, *Thetawise* and *Copilot*. *POE* and *Wolfram Alpha* did not recognize the questions correctly. *Thetawise* showed the best solution. The following link shows the text format of the questions from snipping the questions and the results provided by *Thetawise*. https://thetawise.ai/shared?conversationId=d132a724-4648-49bd-9e65-5027a8af19e4



In a box, there are 5 red balls and 4 black balls. From the box, 2 balls are randomly chosen at the same time.

(a) Find the probability that the 2 balls chosen are red. (2 marks)

(b) In a bag, there are 8 red balls. The 2 balls chosen from the box are put into the bag and then 3 balls are randomly chosen at the same time from the bag. Find the probability that the 3 balls chosen are of the same colour. (2 marks)

Figure 2. The question used in testing problem-solving ability of AI tools

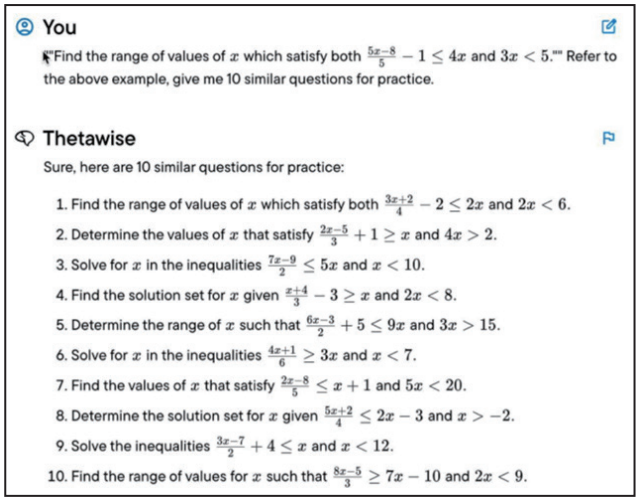
Teacher A concludes that he would choose *Thetawise* and *Copilot* over *POE* and *Wolfram Alpha* as *Thetawise* and *Copilot* can recognize images and present them as mathematics expressions. *Thetawise* can also provide step-by-step solutions, but the solutions provided by *Thetawise* is not always accurate.

Building Prompt Library for Mathematics Teachers

Teacher B tried free AI tools to prepare a set of consolidation exercises for students in a remedial class. He used *POE* to complete the tasks. He employed prompt engineering techniques; the dialogue can be found in the link https://poe.com/s/2t87uGL01y5I0Iqvj82a. Teacher B merged the prompts, as shown in https://poe.com/s/mFpQjW20pt24YU0aVHHk. It becomes a resource in his prompt library that he intended to establish – 'Solving quadratic equations with solutions'. When teacher fed it to the same AI engine, it would provide similar responses. It largely enhances teachers' efficiency as teachers do not have to go through the long dialogue with AI again.

Interplay Between Different AI Tools

Teacher C aimed to prepare remedial exercises for students who demonstrated a pre-determined level of difficulties. He used a paid AI tool *Thetawise* to generate the question as shown in Figure 3 and then used another paid AI tool *Mathpix* to convert the results from Thetawise into a *Microsoft Word* file so that the formula can be edited by Equation in *Word*. He reported that the task can be completed within 10 minutes. This greatly enhances teachers' efficiency in the preparation of teaching materials.



You

"Find the range of values of x which satisfy both $\frac{5x-8}{5} - 1 \leq 4x$ and $3x < 5$." Refer to the above example, give me 10 similar questions for practice.

Thetawise

Sure, here are 10 similar questions for practice:

1. Find the range of values of x which satisfy both $\frac{3x+2}{4} - 2 \leq 2x$ and $2x < 6$.
2. Determine the values of x that satisfy $\frac{2x-5}{3} + 1 \geq x$ and $4x > 2$.
3. Solve for x in the inequalities $\frac{7x-9}{2} \leq 5x$ and $x < 10$.
4. Find the solution set for x given $\frac{x+4}{3} - 3 \geq x$ and $2x < 8$.
5. Determine the range of x such that $\frac{6x-3}{2} + 5 \leq 9x$ and $3x > 15$.
6. Solve for x in the inequalities $\frac{4x+1}{6} \geq 3x$ and $x < 7$.
7. Find the values of x that satisfy $\frac{2x-8}{5} \leq x + 1$ and $5x < 20$.
8. Determine the solution set for x given $\frac{5x+2}{4} \leq 2x - 3$ and $x > -2$.
9. Solve the inequalities $\frac{3x-7}{2} + 4 \leq x$ and $x < 12$.
10. Find the range of values for x such that $\frac{8x-5}{3} \geq 7x - 10$ and $2x < 9$.

Figure 3. Results from thetawise on preparing exercises

Ongoing works

Teachers in this research are now testing how different AI tools can help them to enhance teaching and learning efficiency. Some teachers also explore how they can teach students using AI-powered tools to learn, such as identification of their own mistakes.

Research Opportunities

This research shows different examples on how AI-powered tools can enhance teachers' teaching and learning efficiency. With emerging AI technology, various AI-powered tools would further enhance teachers' teaching and learning efficiency. Building a prompt library and interplay between different AI tools are two areas that researchers can further investigate into.

References

Chiu, T.K., & Li, Y. (2023). How Can Emerging Technologies Impact STEM Education? *Journal for STEM Education Research, 6,* 375-384.

The HKSAR Government. (2023a, January 25). 'Bringing AI into Classrooms' Retrieved from https://www.news.gov.hk/eng/2023/01/20230116/20230116_110626_007.html

Mohamed, M.Z., Hidayat, R., Suhaizi, M., Mahmud, M.K.H., & Baharuddin, S.N. (2022). Artificial intelligence in mathematics education: A systematic literature review. *International Electronic Journal of Mathematics Education, 17*(3), em0694.

Ravi, P., Broski, A., Stump, G., Abelson, H., Klopfer, E., & Breazeal, C. (2023). Understanding Teacher Perspectives and Experiences after Deployment of AI Literacy Curriculum in Middle-school Classrooms. *arXiv preprint arXiv:2312.04839*.

Yau, K. W., Chai, C. S., Chiu, T. K., Meng, H., King, I., & Yam, Y. (2023). A phenomenographic approach on teacher conceptions of teaching Artificial Intelligence (AI) in K-12 schools. *Education and Information Technologies, 28*(1), 1041-1064.

TSG6-2-1

EQT-TECH: FRAMEWORK FOR INTEGRATING TECHNOLOGY TO ADVANCE MATH THINKING AND LEARNING

Jennifer Suh
George Mason University, USA, jsuh4@gmu.edu

Jennifer A. Wolfe
University of Arizona, USA, jwolfe628@arizona.edu

Kristine Ho
University of California, Los Angeles, USA, krho@gseis.ucla.edu

Equity-centered Transformative Technology (EqT-tech) Lesson Analysis Tool builds on the seminal work on technology, pedagogy, and content knowledge and considers how technology can support equitable teaching practices. The goal of the tool is to develop teachers' awareness and action to bring equity to the forefront in the use of technology in the mathematics classroom. This paper introduces the framework and lesson analysis tool and describes case studies from preservice courses and inservice professional development that used the tool to support teachers in implementing lessons with digital technology with equity at the forefront.

Keywords: Mathematical modeling, Equitable Teaching, Data Literacy and Placed-based learning

Introduction of the EqT-tech Framework

Digital equity is a multifaceted issue that extends beyond mere access to digital devices and the internet. It is crucial to differentiate between digital technologies that genuinely promote deep and meaningful math teaching and learning and those that do not. This differentiation is essential for providing professional development for both preservice and inservice teachers, enabling them to effectively integrate technology and expand educational opportunities for all students. The Equity-centered Transformative Technology (EQT-tech) framework (Suh et al., 2023) integrates research from the fields of equity, mathematics, and technology (Aguirre et al., 2013; Gutiérrez, 2009; McCulloch, et al., 2021; Mishra et al., 2006). It builds on existing frameworks and emphasizes the transformative potential of technology in education. The tool is designed to help educators evaluate technology tools and practices for their potential to support equitable mathematics teaching. The framework draws on key recommendations from the National Council of Teachers of Mathematics (NCTM, 2022) and the Association of Mathematics Teacher Educators (AMTE, 2017, 2022), which emphasize the importance of equitable technology integration in mathematics education.

The EQT-tech Lesson Analysis Tool was developed in response to the need for additional support in analyzing and evaluating technologies that support effective and equitable mathematics teaching practices. The authors synthesized existing equity frameworks in mathematics education and incorporated the concept of "transformative potential" from Jemal's (2017) work, which focuses on the levels of consciousness and action necessary to transform inequitable conditions. The framework

examines the affordances of technology and its potential to bring about necessary changes for a more equity-centered classroom. The development process involved a research synthesis to identify the intersection of equity, mathematics education, and technology. The Six Dimensions of the EqT-tech include: 1) Access to Inquiry-based Learning: Technology should support exploration, inquiry, and problem-solving, allowing students to craft their understanding through meaningful mathematical tasks; 2) Promote Math Identity through Authorship and Agency: Technology should enable students to create, represent, and share their mathematical thinking, fostering positive mathematical identities; 3) Formative Assessment and Differentiation: Technology should facilitate formative assessment and differentiation, providing educators with insights into student understanding and allowing for tailored instruction; 4) Empower Students through Collaboration: Technology should support collaboration and collective thinking, enabling students to work together on group-worthy tasks and share ideas; 5) Amplification of Mathematical and Cognitive Processes: Technology should enhance mathematical reasoning and problem-solving, providing tools that allow students to explore and generalize mathematical concepts; and 6) Insight into Social Justice Issues: Technology should help students understand and address social justice issues, using data and analysis to highlight inequities and promote action. The EqT-tech lesson analysis tool was designed to bring awareness to the potential technology has to promote equity while recognizing that it alone cannot ensure equitable learning or include all six dimensions. Therefore, it was critical for teachers to also develop deep technological pedagogical content knowledge (Mishra et al., 2006) to further create an equitable teaching and learning environment. In addition, *The 5 Practices for Orchestrating Productive Mathematics Discussions* by Smith and Stein (2018) enhances lesson planning by guiding teachers through anticipating student responses, monitoring their work, selecting specific responses to highlight, sequencing these responses to build a coherent narrative, and connecting them to key mathematical ideas. This structured approach ensures that discussions are purposeful and effective, promoting deeper understanding and equitable participation. By following these practices, teachers can create more intentional and impactful technology-enhanced mathematics lessons.

Methods

We present cases involving preservice teachers (PSTs) in a mathematics methods course and inservice teachers (ISTs) in professional development (PD) sessions, focusing on integrating technology to promote equity and social justice in mathematics education. Teachers in all the cases presented focused on learning about *The 5 Practices* (Smith & Stein, 2018). Data were collected through lesson reflections, and in class assignments. The exemplar cases illustrate ways in which EqT-tech dimensions emerged in each of the enactments.

Case of Elementary Teacher's Use of Digital Tools to Advance Equitable Teaching

In a mathematics methods course, the MTE assigned PSTs to evaluate technology using the EQT-Tech Lesson analysis tool so that they could detail the mathematics features, support features to identify affordances of using the dimensions of equitable technology use. This analysis helped PSTs anticipate students' responses and monitor a focal student on their mathematics understanding around multiplication. The use of the technology tool had the best mathematical and support features to design a sequence of learning activities. For example, Ms. Selena chose the Bunny Times app (NCTM) for her focal student to support her in developing ideas around unitizing and chunking to skip count. The fog feature obscured the number of carrots in the array so the student Sara could focus on developing skip counting as a strategy to build on multiplication fluency. In this example, Selena felt that the math action technology highlighted *Dimension 5: Amplifying Math*

Cognitive Processes with the concept of unitizing to extend student thinking and *Dimension 3: Formative Assessment and Differentiation* for her focal learner.

In an IST professional development, another case of use of the EQT-Tech Lesson analysis tool was with a 6th grade teacher who was planning a lesson around data standards. The class explored the issue of access to sports facilities and opportunities in their community. The lesson was designed to take students through the data cycle: formulating questions, collecting data, organizing and representing data, analyzing data, and taking action. They used a suite of Google tools including Google Maps-used to analyze access to sports facilities in different communities, highlighting disparities; Google Forms-created and disseminated surveys to collect data on students' sports participation and barriers; Google Sheets-organized and represented survey data, allowing students to create visualizations and analyze results; Google Slides: used to map out the layout and timing for a sports clinic, facilitating collaboration and planning. In doing so, they gained insight using technology on a community issue (Dimension 6); discovered the usefulness of analyzing data and using the mean as a central measure (Dimension 1); developed a solution through collective thinking (Dimension 4); and took ownership in their mathematical thinking (Dimension 2).

Case of using Desmos with Preservice Teachers To Highlight Formative Assessment and Differentiate to build Collective Learning and Math Identity

In this case, we share secondary mathematics PSTs engaged in a collaborative mathematics task using Desmos, designed to foster equity-centered transformative technology practices. This collaborative mathematics task was structured around the 5 Practices (Smith & Stein, 2018) framework. The PSTs had access to dynamic tools within Desmos, supporting inquiry-based learning and deep mathematical sense-making (Dimension 1). The task provided equitable structures for participation, affirming math identity through authorship and agency (Dimension 2). As PSTs worked through the task, the course instructor used the teacher dashboard to provide real-time formative feedback and scaffolding in the form of assessing and advancing questions (Dimension 3). The collaborative nature of the task, facilitated in and through Desmos, encouraged collective thinking, critical analysis, and distributed authority by honoring all PSTs' ideas (Dimension 4).

The collaborative Desmos task was intentionally designed to amplify the mathematical and cognitive processes (Dimension 5). The PSTs engaged in rich mathematics discussions, leveraging the technology-mediated features of Desmos to enhance their mathematical understanding, communication of mathematical ideas, and considerations for the pedagogical practices they would enact when analyzing high school students' work on the task. Throughout the task, the PSTs were prompted to reflect on their experiences and to consider how they might use Desmos as a tool for engaging in the 5 Practices in their future classroom to promote equity and student access.

Discussion and Implication for Mathematics Education

The different cases illustrate two different approaches to integrating equity-centered transformative practices mediated by technology where one case focuses on providing learning supports for individual students to extend their learning whereas the other focuses responding to social inequities centered on a community issue. The nature of the task afforded opportunities to use technology in a unique way as highlighted in Dimension 6, using Google Map as a Geographic Information System and Data Analysis through Google Sheets to gain insights into a social issue.

The case of secondary PSTs underscores the potential of using the 5 Practices (Smith & Stein,

2018) framework with equity-centered transformative technology in preservice mathematics teacher education. By engaging secondary PSTs in a collaborative task using Desmos, this particular case illustrates how technology can be leveraged to support inquiry-based learning, affirmation of teacher's mathematics and professional identities, provide assessment and differentiation, foster collective thinking, and amplify mathematical and cognitive processes. The format of the collaborative task allowed secondary PSTs to experience the 5 Practices in action while simultaneously reflecting on their application in their future classroom. This approach aligns with Jansen's (2020) emphasis on revising to learn, as preservice secondary teachers had the opportunity to refine their understanding and application of the 5 Practices through a collaborative task. The use of Desmos as a platform for the task highlights the importance of selecting technology tools that support equity-centered transformative practices. By providing access to dynamic tools and promoting equitable participation, Desmos served as an effective medium for engaging preservice teachers in the 5 Practices while addressing multiple dimensions of equity-centered transformative technology.

These case studies we have shared contribute to the growing body of research on the equity-centered, critically reflective professional development and preparation of K-12 inservice and preservice teachers. These cases offer valuable insight into the successful integration of the 5 Practices framework with equity-centered transformative technology in mathematics teacher education. The findings and implications presented here should inform future research and practice in the field, contributing to the development of critically reflective, equity-centered mathematics teachers.

References

Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics: Rethinking equity-based practices*. National Council of Teachers of Mathematics.

Association of Mathematics Teacher Educators. (2017). *Standards for Preparing Teachers of Mathematics.* Available online at amte.net/standards.

Association of Mathematics Teacher Educators. (2022). *Technology Position Statement for Mathematics Educators*. Available online at https://amte.net/news/2022/06/press-release-amte-statement-technology

Jansen, A. (2020). *Rough draft math: Revising to Learn.* Stenhouse.

Jemal, A. (2017) Critical consciousness: A critique and critical analysis of the literature. *The Urban Review*, *49*(4), 602-626.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, *108*(6), 1017-1054. https://dx.doi.org/10.1111/j.1467-9620.2006.00684.x

McCulloch, A., Leatham, K., & Bailey, N. (2021). Theoretically framing the pedagogy of learning to teach mathematics with technology. *Contemporary Issues in Technology & Teacher Education*, *21*(2), 325–359.

National Council of Teachers of Mathematics (2022). Position Statement for Technology in Mathematics. NCTM.

Smith, M. S., & Stein, M. K. (2018). *The 5 Practices for orchestrating productive mathematics discussions* (2nd ed.). Cowin.

Suh, J., & Roscioli, K. (2023). Learning trajectory-based formative assessment & sequenced digital learning activities in math class. In C. L. Webb & A. L. Lindner (Eds.), *Preparing preservice teachers to integrate technology in K-12 classrooms: Standards and best practices* (pp. 271-294). IGI Global.

TSG6-2-2

EVOLVING TRENDS IN MATHEMATICS EDUCATION RESEARCH ON DIGITAL TECHNOLOGY: A DYNAMIC TOPIC MODELING ANALYSIS

Kyeong-Hwa Lee
Seoul National University, Korea, khmath@snu.ac.kr

Yeongjun Kim
Seoul National University, Korea, astre0422@snu.ac.kr

JinHyeong Park
Gyeongin National University of Education, Korea, demxas@ginue.ac.kr

Dongjo Shin
Pusan National University, Korea, djshin@pusan.ac.kr

Song Hee Lee
Seoul National University, Korea, thd2432@gmail.com

This study explores evolving trends in mathematics education research focusing on the integration of digital technology, analyzing publications from 2001 to 2024 in six SSCI-indexed journals. Using Dynamic Topic Modeling (DTM), 30 key topics were identified, spanning mathematics education themes such as problem-solving, mathematical modeling, and calculus, alongside technology-focused areas like dynamic geometry, computer algebra systems, and online learning. Results highlight a growing emphasis on teacher expertise, feedback, and online learning, while traditional technologies like calculators and CAS show declining interest. The findings underscore the influence of technological advancements and global shifts, such as the COVID-19 pandemic, in reshaping educational practices. This study offers a nuanced perspective on how research topics evolve over time and provides valuable insights for guiding future research and practice in technology-integrated mathematics education.

Keywords: digital technology, Dynamic Topic Modeling, trend

Introduction

The integration of technology into mathematics education has been a longstanding topic of debate and interest (Bartolini Bussi & Borba, 2010; Olsher et al., 2023). In recent decades, the rapid development of digital technology has led researchers to explore various approaches to integrating it into mathematics teaching and learning (Bray & Tangney, 2017; Drijvers & Sinclair, 2023; Herfort et al., 2023; Hoyles, 2018; Lagrange et al., 2003; Luz & Yerushalmy, 2023; Weigand, Trgalova, & Tabach, 2024; Yerushalmy, Nagari-Haddif & Olsher, 2017).

This ongoing expansion and rapid evolution of research necessitates a systematic review of these studies to better understand the evolving focus of the research community after 2001 to the present. However, previous research has noted that traditional reviews involving manual coding and analy-

sis face challenges, such as the difficulty of ensuring fully transparent coding and, particularly with the surge in research on the integration of digital technology in mathematics education, the overwhelming workload required for large datasets. To address these challenges, there has been a growing focus on exploring and understanding these shifts through the application of methodologies such as topic modeling (Foster & Inglis, 2019; Herfort et al., 2023; Hwang, Flavin & Lee, 2023; Inglis & Foster, 2018; Marks et al., 2020).

In this context, the present study aims to explore trends in technology-integrated mathematics education research through topic modeling, focusing on major mathematics education journals. In this study, Dynamic Topic Modeling (DTM), which is recognized as an effective methodology for identifying temporal changes in topics based on Latent Dirichlet Allocation (LDA), is utilized (Blei & Lafferty, 2006). From the preceding discussions, the following research questions have been formulated for this study.

RQ 1: What are the distinct topics in articles from prominent mathematics education journals that focus on the utilization of technology?

RQ 2: How have these topics evolved from 2001 to the present?

Topic Modeling in Mathematics Education Research

Topic modeling, which is an unsupervised method of content analysis using statistical models to identify topics in large sets of textual data (Boyd-Graber et al., 2017), has been increasingly used in recent studies for automatic literature reviews (Herfort et al., 2023).

Studies using topic modeling to analyze general trends in mathematics education research have identified only a single topic related to technology (see Foster & Inglis, 2019; Inglis & Foster, 2018; Marks et al., 2020). Among studies focusing exclusively on technology-related research, Herfort et al. (2023) highlighted topics such as *embodied perspectives* and s*emiotics*, while also identifying *the instrumental approach* as a notable addition. Furthermore, themes like *CAS, dynamic geometry, programming, e-blended learning, spreadsheets,* and *game-based learning* emerged as unique to technology-focused studies and were absent from broader studies on mathematics education trends. In contrast, Hwang et al. (2023), with a smaller set of topics, identified *ICT (information and communication technology)* as a prominent theme but observed minimal distinct technology-specific topics.

Building on this review, the present study applies topic modeling to mathematics education research incorporating technology from 2001 onward, focusing on identifying recent trends and changes. By narrowing the vertical scope to research conducted since 2001 and broadening the horizontal scope to include major journals rather than specific journals or conferences, this study aims to capture the collective trends in the use of technology within the mathematics education research community.

Method

We collected research articles published between 2001 and 2024 from six SSCI-indexed mathematics education journals (ESM, JRME, ZDM, MTL, IJSME, JMTE). Using the Web of Science, Scopus, and ERIC databases, we searched for studies with the following keywords: “technology,” “online,” “digital,” “software,” “computer,” and “calculator”. These keywords were selected from among several candidates based on the criterion that each term, when searched independently, retrieved a sufficient number of publications. To ensure relevance, we excluded studies that did not

focus on the use of technology in mathematics teaching, learning, or assessment. Specifically, we filtered out papers that (1) were unrelated to mathematics education, (2) did not involve technology-integrated education, (3) used technology solely as a data collection or analysis tool, (4) were limited to teacher education contexts, or (5) were review studies that did not specifically address technology use. Based on these criteria, a total of 521 articles were selected.

To analyze topic evolution over time, we employed DTM, which identifies temporal variations in topics while assuming continuity in word distributions across consecutive time intervals (Blei & Lafferty, 2006). For this analysis, we divided the collected documents into eight intervals, each spanning three years from 2001 onward. The DTM was implemented using the *ldaseqmodel* module in the *Gensim* library. We adjusted hyperparameters, including the number of topics, and selected the most appropriate model based on topic coherence scores. The DTM produced a set of 20 key terms for each topic in each of the eight time intervals, enabling a detailed analysis of topic evolution. Finally, we examined the generated word distributions of each topic in conjunction with the original documents to assign meaningful labels to the identified topics.

The results were analyzed following the procedure, consisting of four key steps (Shen & Mariano, 2024). First, we identified "hot topics" by measuring the proportion of key topic-related words within the corpus at specific time intervals. Second, we conducted a trend analysis of topic heat evolution to examine fluctuations in topic prominence over time. Third, we evaluated topic similarity across different intervals to trace the evolutionary paths of topics throughout the study period. Finally, we performed a detailed analysis of the evolution of topic-specific keywords using outputs of DTM, allowing us to observe how focal points within each topic shifted over time.

Results and Discussion

A total of 30 topics related to the use of technology in mathematics education were identified through the DTM. These topics encompassed a wide range of research areas, including mathematics education themes such as *problem-solving, mathematical modeling, statistical reasoning, geometric proof, algebra,* and *calculus.* Additionally, theoretical perspectives such as *the instrumental approach, semiotics,* and *embodied cognition* emerged. Technology-focused topics included *computer algebra systems (CAS), dynamic geometry, calculators, online learning, game-based learning.* Furthermore, broader educational themes such as teacher resource design, teacher professional development, and assessment were also identified.

Among the research topics, *problem-solving, dynamic geometry, teaching resource design* have consistently held the highest proportion over an extended period. Additionally, *teacher expertise, feedback,* and *online learning* have shown a significant upward trend, while topics such as *calculators, CAS, problem-solving, and calculus* have exhibited a declining trend. Some of these trends can be interpreted as being influenced by external factors. For instance, research attention shifting from traditional tools like calculators or CAS to more accessible, educationally potent alternatives such as GeoGebra or Desmos (Engelbrecht & Borba, 2023; Bray & Tangney, 2017). Notably, the rising trend in online learning has emerged as a new research direction, largely driven by the impact of the COVID-19 pandemic.

The rising topic *teacher expertise* exhibited strong interconnections with other areas, absorbing related themes such as pedagogy in specific content domains, task design, and assessment over time. This trend results from the expanded, multidimensional role teachers face in integrating technology, which is reflected in evolving teacher education standards and drives the increasing demand for

focused teacher professional development (Dockendorff & Gomez Zaccarelli, 2024). In contrast, the declining topic *problem-solving* has fragmented into topics like *modeling, statistical reasoning,* and *geometric proof,* suggesting a trend toward specialization. The influence of international assessments such as PISA and national educational reforms has increased the emphasis in educational policies on specific mathematical competencies, including mathematical modeling and statistical reasoning. Additionally, advancements in digital tools may have accelerated this subject specialization by facilitating particular types of problem-solving associated with these competencies (Santos-Trigo, 2024).

Shifts in research focus are observed not only in topic evolution but also in changes to key terms within the same topic. For instance, while *computer* was once a prominent keyword across multiple topics, its significance has declined in recent years. Conversely, the increasing importance of *environment* suggests a shift in perspective—viewing technology not merely as a medium for learning but as an integral part of the learning environment. Keywords such as "formative", "feedback", and "competency" have emerged as trend terms, appearing across various topics rather than being confined to a single domain. This reflects a fundamental shift in education within digital environments—from merely transmitting knowledge, performing procedures, and assessing outcomes, to supporting learning processes and fostering competencies essential for future society—an emphasis now reflected across diverse research domains (Drijvers & Sinclair, 2023).

The findings of this study largely align with previous research on technology use trends in mathematics education (Herfort et al., 2023; Hwang, Flavin & Lee, 2023). However, this study also identifies new trends emerging in the 2020s that were not covered in earlier analyses. Unlike prior studies that primarily examined topic proportion trends, this research employs DTM to trace how topics evolve over time and how keyword shifts within the same topic reflect changes in focus.

References

Bartolini Bussi, M. G. & Borba, M. C. (2010). The role of resources and technology in mathematics education. *ZDM, 42*, 1-4.

Blei, D., & Lafferty, J. (2006). Correlated topic models. *Advances in Neural Information Processing Systems, 18*, 147.

Boyd-Graber, J., Hu, Y., & Mimno, D. (2017). Applications of topic models. *Foundations and Trends ® in Information Retrieval, 11*(2–3), 143–296. https://doi.org/10.1561/1500000030

Bray, A. & Tangney, B. (2017). Technology usage in mathematics education research - A systematic review of recent trends. *Computers & Education, 114*, 255-273.

Dockendorff, M., & Zaccarelli, F. G. (2024). Successfully preparing future mathematics teachers for digital technology integration: A literature review. *International Journal of Mathematical Education in Science and Technology*.

Drijvers, P. & Sinclair, N. (2023). The role of digital technologies in mathematics education: purposes and perspectives. ZDM. https://doi.org/10.1007/s11858-023-01535-x.

Engelbrecht, J., & Borba, M. C. (2023). Recent developments in using digital technology in mathematics education. *ZDM – Mathematics Education*. https://doi.org/10.1007/s11858-023-01530-2

Foster, C. & Inglis, M. (2019). Mathematics Teacher Professional Journals: What Topics Appear and How Has This Changed over Time? *International Journal of Science and Mathematics Education, 17,* 1627–1648.

Herfort, J.D.; Tamborg, A.L.; Meier, F.; Allsopp, B.B.; Misfeldt, M. (2023). Twenty years of research on technology in mathematics education at CERME: A literature review based on a data science approach. *Educational Studies in Mathematics, 112*, 309–336.

Hoyles, C. (2018). Transforming the mathematical practices of learners and teachers through digital technology. *Research in Mathematics Education, 20*(3), 209–228. https://doi.org/10.1080/14794802.2018.1484799

Hwang, S., FLavin, E. & Lee, J.-E. (2023). Exploring research trends of technology use in mathematics education: A scoping review using topic modeling. *Education and Information Technologies, 28*, 10753–10780.

Inglis, M. & Foster, C. (2018). Five decades of mathematics education research, *Journal for Research in Mathematics Education, 49*(4), 462-500.

Lagrange, J.-B., Artigue, M., Laborde, C., & Trouche, L. (2003). Technology and mathematics education: A multidimensional study of the evolution of research and innovation. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), S*econd international handbook of mathematics education* (pp. 237–269). Springer Netherlands. https:// doi. org/ 10. 1007/978- 94- 010- 0273-8_9

Luz, Y. & Yerushalmy, M. (2023). Computer-supported assessment of geometric exploration using variation theory. *Journal for Research in Mathematics Education , 54* (2). 141–174. https://doi.org/10.5951/jresematheduc-2020-0260

Marks, R., Foster, C., Barclay, N., Barnes, A., & Treacy, P. (2020). A comparative synthesis of UK mathematics education research: What are we talking about and do we align with international discourse? *Research in Mathematics Education, 23*(1), 39–62. https://doi.org/10.1080/14794802.2020.1725612

Olsher, S., Abrahamson, D., Arcavi, A., Arzarello, F., Chazan, D., Clark-Wilson, A., Leikin, R., Sinclair, N., & Yerushalmy, M. (2023). Problem solving with technology: Multiple perspectives on mathematical conjecturing. In Proceedings of the International Group for the Psychology of Mathematics Education (Vol. 46, PP. 134–158).

Santos-Trigo, M. (2024). Problem solving in mathematics education: Tracing its foundations and current research-practice trends. *ZDM – Mathematics Education*, *56*(2), 211–222. https://doi.org/10.1007/s11858-024-01578-8

Shen, J., & Mariano, V. Y. (2024). Analyzing Alzheimer's Disease Research Trends: Insights From Improved Dynamic Topic Modeling. *IEEE Access*, *12*, 106121–106132. https://doi.org/10.1109/ACCESS.2024.3437656

Weigand, H. G., Trgalova, J., & Tabach, M. (2024). Mathematics teaching, learning, and assessment in the digital age. ZDM Mathematics Education, 56, 525–541. https:// doi. org/ 10. 1007/ s11858-024-01612-9

Yerushalmy, M., Nagari-Haddif, G., & Olsher, S. (2017). Design of tasks for online assessment that supports understanding of students' conceptions. ZDM Mathematics Education. doi:10.1007/ s11858-017-0871-7.

TSG6-2-3

INVESTIGATING PRESERVICE TEACHERS' USE OF A DYNAMIC DIGITAL TOOL TO SOLVE A FRACTION MULTIPLICATION PROBLEM

Mi Yeon Lee
Arizona State University, USA, mlee115@asu.edu

Sheunghyun Yeo
Daegu National University of Education, Korea, shyeo@dnue.ac.kr

Ji Nam Hwang
Yongjeong Elementary School, Korea, whiyoung10@naver.com

This paper examines how preservice teachers (PSTs) engage with a dynamic digital tool while solving a fraction multiplication problem. Sixty participants from South Korea were introduced to the tool, a technological environment designed to support fraction comprehension. They then utilized the tool to solve a fraction multiplication problem, justifying their answers through their problem-solving process. The findings suggest that most PSTs were able to correctly calculate fraction multiplication procedurally, but used the dynamic digital tool in three different ways to demonstrate the quantitative meanings of fraction multiplication. Implications are discussed for the improvement of PSTs' mathematical knowledge for teaching fraction multiplication.

Keywords: Preservice teachers, fraction multiplication, dynamic digital tool,

Introduction

The appropriate use of multiple representations is a key indicator of mathematical understanding and a central feature of effective problem solvers (NCTM, 2014). In the digital era, understanding how preservice teachers (PSTs) use digital tools to represent important mathematical concepts in problem-solving is essential for preparing them to teach effectively in their future classrooms. Research indicates that teachers often rely primarily on symbolic notation in their instruction (Izak, 2008; Lee et al, 2011. Even when incorporating visual representations, they tend to use them mainly to demonstrate answers or procedures rather than to deepen students' conceptual understanding (Izsák, 2008; Lee et al., 2011; Lee, 2017; Lo & Luo, 2012). This limited use of representations may stem from teachers' own lack of understanding of representations or insufficient knowledge to guide students in using them to learn mathematical concepts (Lee et al., 2011; Lee, 2021; Lee & Lee, 2019, 2021).

To address these challenges in teacher education, this study explores how elementary PSTs use a dynamic digital tool to solve a fraction multiplication problem. In this study, the dynamic digital tool, FracTopia (www.steamup.academy/en/fractopia), is specifically designed for deeper exploration of fraction concepts and operations. It includes six fraction *models*[1] that help bridge the gap

1 Representations refer to the various ways mathematical concepts can be visualized, while models serve as structured tools for representing mathematical relationships and ideas. Thus, while all models are representations, not all representations function as models. However, in this study, we specifically focus on visual representations related to fractions, which are embodied by FracTopia, so the terms models and representations are used interchangeably.

between abstract mathematical concepts and visual representations. This study aims to contribute to the growing body of research on the role of technology in enhancing mathematical understanding. The research questions guiding this study were: (1) How do PSTs solve a fraction multiplication problem using FracTopia? and (2) What types of errors PSTs demonstrate in solving a fraction multiplication problem using FracTopia?

Theoretical Underpinning

To teach fractions effectively, educators need both procedural knowledge (content knowledge, CK) and a deep understanding of why these procedures work, along with insight into potential student misconceptions (pedagogical content knowledge, PCK). However, research shows that even teachers fluent in fraction computation often lack understanding of its underlying concepts (Lee, 2017; Lee & Lee, 2020, 2023; Rosli et al., 2013). To effectively use visual representations for fractions, PSTs must have a clear understanding of the units that the quantities in those representations refer to (Jacobson & Izsák, 2015). To represent fraction multiplication quantitatively using visual representations, teachers' flexibility with these referent units is critical (Izsák, 2008; Lee 2917; Lee et al., 2011).

Digital tools not only create traditional symbols but also serve as dynamic mediators that aid students in understanding mathematical relationships and properties through visual representations (Lee & Yeo, 2024; Yeo & Webel, 2024). Existing knowledge influences the use of tools, while the limitations and advantages of the tools, in turn, impact human knowledge and thinking (Bartolini & Mariotti, 2008; Vygotsky, 1978). By observing how FracTopia tool was utilized, the study aimed to understand how PSTs use FracTopia to articulate the underlying mathematical concepts and procedures involved in fraction multiplication by visualizing the quantitative meaning of the operation.

Methods

In this study, 60 PSTs were recruited from a teacher preparation program in South Korea and enrolled at an elementary mathematics methods course in the second year of each program. PSTs were introduced to FracTopia, a virtual manipulative tool featuring models of (1) Fraction Circles, (2) Fraction Rectangles, (3) Fraction Bars, (4) Number Line, (5) Fraction Sets, and (6) Fraction Rulers. Afterward, they completed a questionnaire including four fraction problems covering addition, subtraction, multiplication, and division. As part of a larger investigation, this study specifically focuses on examining how PSTs solve the fraction multiplication problem, 2/3 x 3/8 using FracTopia. Notably, in Korea, this expression is interpreted as '3/8 of 2/3,' whereas in the U.S., it is understood as '2/3 of 3/8 (Ministry of Education, 2020).

To address the first research question, PSTs' responses to the target task were categorized based on their correctness and the approach used to solve the problem. We then coded the data in terms of correct or incorrect solutions and representations. To address the second research question about the purpose of PSTs' use of FracTopia, a four-phase inductive content analysis was conducted: (1) reviewing PSTs' written responses; (2) drafting a coding scheme by referring to prior study (Lee, 2017) and finalizing the coding scheme (see Table 1) after applying it to our data sample; (3) coding all data; and (4) counting the frequencies and interpreting the result. For the reliability of the coding, we coded first ten responses from PSTs and discussing the discrepancy, resulting in 100% agreement on the coding of all the examples.

Table 1. Coding scheme of this study

Category	Sub-category	Explanation
Computational reasoning	Fraction Multiplication Algorithm-Based Solution	The multiplier, multiplicand, and result are simply represented in a model without reflecting the meaning of the operation or the relationship between the fractions – even when the given quantities are converted to a common denominator, the meaning of the operation does not emerge in the process.
Operational reasoning	O1: The use of algebraic property (e.g., commutative property, associative property)	Uses the commutative property to change the order of operations or the associative property to transform fractions while solving the problem using representations (e.g., Transforming 2/3 x 3/8 (i.e., 3/8 of 2/3 from the Korean perspective of fraction multiplication) to 3/8 x 2/3 (i.e., 2/3 of 3/8 from the Korean perspective using commutative property)
	O2: Partial use of algebraic knowledge (e.g., computing common denominators in mind)	Omitting the explanation of the process where the denominators are converted to 12 or 24 while representing 3/8 of 2/3 (i.e., the Korean perspective of 2/3 x 3/8)
Quantitative reasoning	Q1: Additive use of the models	Describing the process of representing 3/8 of 2/3 (i.e., the Korean perspective of 2/3 x 3/8) additively
	Q2: Multiplicative use of the models	Describing the process of representing 3/8 of 2/3 (i.e., the Korean perspective of 2/3 x 3/8) multiplicatively

Findings and Discussion

The Accuracy of PSTs' Responses to a Fraction Multiplication Problem

To address the first research question, we evaluated the correctness of PSTs' responses. A response was considered correct when both the answer and the process were accurate. Specifically, if PSTs provided the correct answer and followed the correct procedure, or used a fraction model correctly without errors, we deemed the response correct. On the other hand, if PSTs provided a correct answer but included errors in their process (e.g., improper use of the set model, mislabeling in the number line model, etc.), the response was considered incorrect.

Out of 60 PSTs, 20 (33%) provided correct responses across all six fraction models in FracTopia, while 40 PSTs (67%) gave at least one incorrect response (see Table 2). None of the PSTs provided incorrect answers. However, all 40 PSTs made some errors in using fraction models within FracTopia.

Table 2. Numbers of PSTs who provided correct or incorrect responses

Models	Correct (No. of PSTs)	Incorrect (No. of PSTs)
Fraction Circle	51	9
Fraction Square	60	0
Fraction Bar	49	11
Number Line	49	11
Fraction Set	37	23
Fraction Ruler	58	2

Note. The total frequency of incorrect responses is 56 because 15 out of 40 PSTs provided incorrect responses in two fractional models and 1 PST responded incorrectly in three fraction models. The total error rate is 16% (=56/360).

Fifty-six instances of errors in the use of fraction models were categorized into five types. The most common issue was the inappropriate use of a fraction model (n=23), followed by representing multiple fractions within the same whole (n=20). The third most frequent error involved using natural numbers to represent fractions on the number line model (n=10). Additionally, errors related to the concept of the whole (n=1) and a lack of understanding of referent units (n=2) were also observed.

This finding indicates that, although the overall error rate in the use of fraction models was relatively low at 16%, there is still a need to ensure PSTs are well-prepared to teach fractions effectively using these models, given that two-thirds of PSTs (67%) demonstrated some errors in their use of fraction models. In particular, PSTs appeared to struggle with understanding the features of set models, which is aligned with prior literature (Van de Walle et al., 2013). Notably, in this study, about one-third of PSTs tended to use the fraction set model in FracTopia similarly to an area or length model, treating it as though it had continuous rather than discrete features. Also, PSTs' many errors in using fraction models seem to be related to misunderstandings of referent units, unitizing, and the concept of a whole—key foundations for developing advanced fractional understanding (Steffe & Olive, 2010).

Types of PSTs' Use of FracTopia in Solving a Fraction Multiplication Problem Correctly

Building on the previous study (Lee, 2017), we further analyzed PSTs' correct responses by categorizing them into three groups based on how they used fraction models to connect their fractional reasoning with procedures or symbolic notations (refer to Table 1). Table 3 presents the distribution of PSTs' correct responses across all six fraction models provided in FracTopia.

Table 3. The way PSTs use FracTopia in solving a fraction multiplication problem correctly

Category	Fraction Circle	Fraction Square	Fraction Bar	Number Line	Fraction Set	Fraction Ruler	Total (%)[a]
C	11	3	11	9	5	8	47 (15%)
O1	7	3	2	3	6	8	29 (9.5%)
O2	14	6	15	15	18	19	87 (29%)
Q1	8	4	5	5	2	5	29 (9.5%)
Q2	11	44	16	17	6	18	112 (37%)
Total	51	60	49	49	37	58	

Note. The frequency was calculated based on 60 PSTs to determine how many participants provided correct responses in each category. [a] The percentage reported in this table are based on a total of 304 occurrences, which exceeds the number of participants.

In terms of total distribution, the fraction square model (n=60) had the highest frequency, followed by the fraction ruler (n=58) and fraction circle (n=51). Additionally, PSTs faced more difficulties with the fraction set model compared to the area and length models, such as the fraction circle, fraction square, fraction bar, number line, and fraction ruler. In terms of the types of representations used to solve the fraction multiplication problem with FracTopia, quantitative reasoning occurred most frequently (46.5%), followed by operational reasoning with partial use of algebraic knowledge (38.5%). Computational reasoning was used least frequently (15%). Interestingly, the largest number of PSTs (n=48) demonstrated quantitative reasoning when solving a fraction multiplication problem using the fraction square model, while the smallest number of PSTs (n=8) showed quantitative reasoning with the fraction set model. Length models, such as the fraction bar (n=21) and number line (n=22), also appeared to help PSTs address fractions quantitatively by subdividing the length of the fractional quantity. This finding indicates that all 60 PSTs correctly used the fraction square model, which may align with the recommendation to use area models based on rectangles when teaching the multiplication of a proper fraction by another proper fraction (van de Walle et al., 2013). We attribute the high frequency of quantitative reasoning in the fraction square model to the fact that its crossed area directly illustrates the product of fractions, reducing the likelihood of errors. However, it is also possibility that the design features of the fraction square model in FracTopia, which emphasize overlapped area representation, contribute to this high frequency of quantitative reasoning in this specific model. Additionally, given the challenges PSTs face in understanding and using set models, teacher educators should provide more opportunities for PSTs to engage with set models when learning and teaching fractions.

Conclusions

Compared to traditional hand-drawing methods, digital tools can help shift the focus toward conceptual thinking by reducing the cognitive load associated with manually creating representations. However, despite this advantage, PSTs still encounter challenges when using digital models. One possibility for such challenge is the insufficient time allocated in their methods course for engaging with this digital tool, which may hinder PSTs' deeper comprehension. Another potential factor is their incomplete understanding of fundamental fraction concepts, operations and the key attributes of corresponding models. Previous literature (Bartolini & Mariotti, 2008; Yeo & Webel, 2024) highlights the interaction between the tool and the user, demonstrating in this study how PSTs' prior knowledge influenced their use of the digital tool, while the tool's affordances and limitations shaped their thinking and reasoning. The findings from this study underscore the need for more targeted instruction on both the conceptual foundations of fractions and the effective use of digital fraction models in teacher education. Furthermore, this study suggests the importance of investigating PSTs' conceptual understanding when using digital tools.

References

Bartolini Bussi, M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective. In L. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (2nd ed., pp. 746–783). Routledge.

Izsák, A. (2008). Mathematical knowledge for teaching fraction multiplication. *Cognition and Instruction, 26*(1), 95–143.

Jacobson, E., & Izsák, A. (2015). Knowledge and motivation as mediators in mathematics teaching practice: The case of drawn models for fraction arithmetic. *Journal of Mathematics Teacher Education, 18*(5), 467–488.

Lee, J., & Lee, M. Y. (2020). Pre-service teachers' exploration of model breaking points. *International Journal of Science and Mathematics Education, 18*, 549–575.

Lee, J., & Lee, M. Y. (2023). How elementary prospective teachers use three fraction models: Their perceptions and difficulties. *Journal of Mathematics Teacher Education, 26*(4), 455–480.

Lee, M. Y. (2017). Pre-service teachers' flexibility with referent units in solving a fraction division problem. *Educational Studies in Mathematics, 96*(3), 327–348.

Lee, M. Y. (2021). Using a technology tool to help pre-service teachers notice students' reasoning and errors on a mathematics problem. *ZDM - Mathematics Education*, *53*(1), 135-149.

Lee, M. Y., & Lee, J. (2019). Pre-service teachers' perceptions of the use of representations and suggestions for students' incorrect Use. *Eurasia Journal of Mathematics, Science and Technology Education, 15*(9), 1-21.

Lee, M. Y., & Lee, J. (2021). Pre-service teachers' selection, interpretation, and sequence of fraction examples. *International Journal of Science and Mathematics Education, 19*(3), 539-558.

Lee, M. Y., & Yeo, S. (2024). Elementary preservice teachers' fraction-as-measure reasoning with a dynamic tool. *Journal of Mathematics Teacher Education*.

Lee, S. J., Brown, R. E., & Orrill, C. H. (2011). Mathematics teachers' reasoning about fractions and decimals using drawn representations. *Mathematical Thinking and Learning, 13*(3), 198–220.

Ministry of Education. (2020). *2020 Revised National Curriculum for Mathematics*. Ministry of Education.

National Council of Teachers of Mathematics [NCTM]. (2014). *Principles to actions: Ensuring mathematical success for all*. Author.

Rosli, R., Han, S., Capraro, R., & Capraro, M. (2013). Exploring preservice teachers' computational and representational knowledge of content and teaching fractions. *Journal of Korean Society of Mathematics Education, 17*(4), 221–241.

Steffe, L. P., & Olive, J. (2010). *Children's fractional knowledge*. Springer.

van de Walle, J., Karp, K.S., & Bay-Williams, J. M. (2013). *Elementary and middle school mathematics: Teaching developmentally* (8th ed.). Pearson

Vygotsky, L. S. (1978). *Mind in society*. Harvard University Press.

Yeo, S., & Webel, C. (2024). Elementary students' fraction reasoning: A measurement approach to fractions in a dynamic environment. *Mathematical Thinking and Learning, 26*(1), 20–46.

TSG6-2-4

USE OF GEOGEBRA IN ADDITION AND SUBTRACTION OF FRACTIONS WITH DIFFERENT DENOMINATORS IN MOZAMBIQUE

Fernando Macuacua Junior
Naruto University of Education, Japan, macuacuafj@yahoo.com

Abstract: This study examines the effectiveness of GeoGebra as a pedagogical tool for teaching addition and subtraction of fractions with different denominators to 7th-grade students in Mozambique. The research focuses on two key aspects: the impact of GeoGebra on students' conceptual understanding and their ability to visualize fraction operations, as well as their perceptions of the tool as a learning aid. A quasi-experimental design involved 102 secondary school students divided into control and experimental groups. The experimental group received instruction using GeoGebra, while the control group utilized traditional teaching methods. Quantitative data were collected through pre- and post-tests to measure performance gains, while qualitative data were gathered through questionnaires to explore student perceptions. Preliminary findings indicate that GeoGebra enhances students' comprehension and visualization of fraction operations, increasing their engagement and confidence in problem-solving. The results highlight its potential to overcome challenges in teaching fractions and offer insights into effective technology integration in mathematics education.

Keywords: GeoGebra, fractions, mathematics education, Mozambique, technology.

Introduction

Teaching fractions, especially the addition and subtraction of fractions with different denominators, poses significant challenges in mathematics education. Students often struggle to grasp the abstract nature of fractions and the procedural complexities involved in operations with them. This difficulty is further compounded in resource-limited educational contexts, such as Mozambique, where traditional teaching methods dominate and access to innovative instructional tools is limited.

GeoGebra, dynamic mathematics software, offers a promising solution by providing visual and interactive representations that simplify complex concepts. GeoGebra can bridge the gap between abstract mathematical ideas and practical understanding by enabling students to visualize and manipulate fraction operations. However, while extensive research highlights the benefits of educational technology in developed countries, its application and impact in developing nations remain underexplored.

This study investigates the effectiveness of GeoGebra as a teaching tool for fractions in Mozambican classrooms. Specifically, it examines how the software enhances students' comprehension and engagement when learning the addition and subtraction of fractions with different denominators. By comparing traditional instruction with a GeoGebra-supported approach, this research aims to provide insights into the potential of integrating technology into mathematics education in developing contexts.

Research Questions

Research Questions:

1. How does using GeoGebra impact students' understanding of adding and subtracting fractions with different denominators in secondary school?
2. What are secondary school students' perceptions of GeoGebra as a learning tool for the addition and subtraction of fractions with different denominators?

Literature Review

Teaching fractions, notably addition and subtraction with different denominators, is widely recognized as one of the most challenging topics in mathematics education. Students often struggle to understand fractions' abstract nature and develop conceptual and procedural knowledge. This challenge is especially prevalent in resource-constrained contexts, where traditional teaching methods dominate and interactive tools are rarely available (Edivaldo & Fernando, 2022).

Recent research has explored combining GeoGebra with hybrid teaching approaches to maximize its benefits in diverse educational settings. For instance, Mulat et al. (2021) demonstrated that integrating GeoGebra with strategies guided by the Action, Process, Object, and Schema Theory (APOS) significantly improved students' conceptual understanding in African contexts. These findings suggest that blending GeoGebra with pedagogical innovations can enhance learning outcomes, particularly in environments where resources are limited.

While GeoGebra's potential to transform mathematics education is well-documented, its adoption in developing contexts, such as Mozambique, faces significant barriers. These include limited access to technology, insufficient teacher training, and inadequate school infrastructure. Addressing these challenges is crucial to maximizing GeoGebra's impact on mathematics education in the region. Future research should focus on scalable strategies for integrating GeoGebra into low-resource settings and investigate its long-term effects on student engagement and achievement across various mathematical topics.

Method

The study employed a quasi-experimental design to evaluate the effectiveness of GeoGebra in teaching fraction operations to seventh-grade students in Mozambique. It involved 102 participants, divided into a control group using traditional teaching methods and an experimental group using GeoGebra. Over one week, both groups completed pre- and post-tests, while the experimental group also answered a questionnaire and participated in interviews. Data were analyzed using JASP and Jamovi for statistical tests, and Google Forms was used to process qualitative interview responses.

The analysis focused on students' performance, engagement, and conceptual understanding of fractions.

Results

This section presents the results of the data analysis, addressing two research questions. The first explores the impact of GeoGebra on students' understanding of adding and subtracting fractions

with different denominators. The second **explores** secondary school students' perceptions of GeoGebra as a learning tool for these operations.

Research Question 1: How does using GeoGebra impact students' understanding of the addition and subtraction of fractions with different denominators in secondary school?

Table 1. Descriptive statistics of the Pre-test

		N	Mean	SD
Pre - test	Control	52	8.458	2.301
	Experimental	50	8.536	2.442

Table 1 shows descriptive statistics for the Pre-test scores of the control and experimental groups. Both groups had complete data with no missing values. The control group had a mean score of 8.458 with a standard deviation of 2.301, while the experimental group had a slightly higher mean of 8.536 and a standard deviation of 2.442.

Table 2. Independent samples t-Test of Pre-test

Pre – test	t	d.f	p	Mean Dif.
	-0.167	100	0.868	0.470

The results in Table 2 show that the mean score difference between the groups was 0.470 with a t value of −0.167. However, the p-value was 0.868 ($p = 0.868 > 0.05$), indicating no statistically significant difference between the control and experimental groups. This suggests that both groups started with comparable levels of knowledge and understanding of the addition and subtraction of fractions with different denominators.

Table 3. Descriptive statistics of Post-test

		N	Mean	SD
Post - test	Control	49	10.5	2.75
	Experimental	49	12.3	3.25

Table 3 shows that, after the intervention, the experimental group demonstrated a significant improvement in their post-test scores compared to the control group. The mean post-test score for the experimental group was 12.3, while the control group had a mean of 10.5. Both groups had initially equal sample sizes (n = 49), but three students were absent from the control group and one student were absent from the experimental group. Additionally, the experimental group exhibited slightly more variability in their scores.

Table 4. Independent samples t-Test of Post-test

Post - test	t	d.f	p	Mean Dif.
	-3.05	96	0.003	-1.86

Table 4 shows that the mean score difference between the groups was -1.86, with a t-value of −3.05. Additionally, the p-value was low ($p = 0.003 < 0.05$), indicating a statistically significant difference between the groups.

This finding highlights that students in the experimental group reported higher levels of engagement and understanding when using GeoGebra than traditional methods.

Research Question 2: What are secondary school students' perceptions of GeoGebra as a learning tool for the addition and subtraction of fractions with different denominators?

Did GeoGebra help you to understand operations with fractions better?

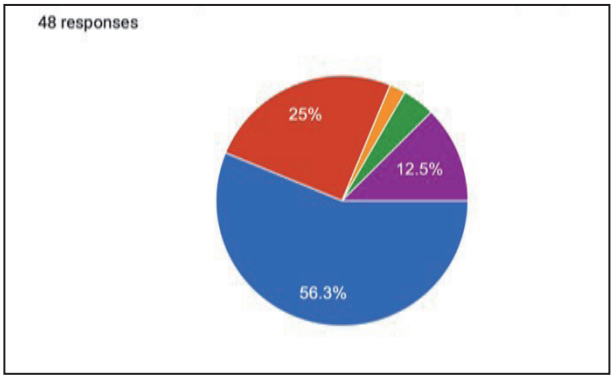


Figure 1. Students' perception of using GeoGebra

Figure 1 illustrates students' perceptions of how GeoGebra helped them visualize operations with fractions. A majority, 81.3% (56.3% "significantly" and 25% "to a certain extent"), felt that GeoGebra improved their understanding.

A smaller portion, 12.5%, was neutral, while very few found it unhelpful (12.5% combined in the "not really" and "not at all" categories). These results suggest that GeoGebra is an effective tool for helping students visualize and understand fraction operations.

Discussion

This study highlights the effectiveness of GeoGebra in teaching fractions, particularly in helping students understand common denominators and operations. Its visualizations made relationships between fractions clearer, contributing to improved performance, aligning with previous findings such as Zulnaidi and Zamri (2017).

In Mozambique, where traditional methods dominate, incorporating GeoGebra could transform mathematics education by offering more interactive learning experiences. The intervention was conducted directly by the researcher, ensuring consistent application of the software.

Some limitations emerged, such as two students finding GeoGebra less engaging, possibly due to the short intervention period. However, interest from other students suggests that greater exposure could enhance its benefits over time.

Conclusion

This study demonstrated that GeoGebra significantly improves students' performance in adding and subtracting fractions with different denominators. Despite these positive outcomes, limited resources in most Mozambican schools hinder the consistent integration of technology into classrooms. Additionally, the short duration of the intervention and the regional sample size represent limitations that may affect the generalizability of the findings.

Nevertheless, the results suggest that GeoGebra enhances student engagement and improves understanding of mathematical concepts. Future research should investigate the long-term impact of using GeoGebra and explore strategies to expand its application across various mathematical topics, while also addressing issues related to resource availability and teacher training in resource-limited settings.

Although this study focused on the addition and subtraction of fractions with different denominators, future investigations could examine the use of GeoGebra in other areas of mathematics, such as algebra, geometry, and statistics, to assess the broader effects of the tool on students' mathematical thinking development.

References

Edivaldo, M. & Fernando, S. (2022). Challenges in Teaching Fractions.

Hart, W. (1989). Fractions: *Equivalence and Addition in "Children's Mathematical Frameworks 8 -13"*. NFER-Nelson. England

Mulat, G., Mulugeta A., & Tadele E. W. (2021). Implementing GeoGebra integrated with multi-teaching approaches guided by the APOS theory to enhance students' conceptual understanding of limits in Ethiopian Universities. *Heliyon 7*(5). https://doi.org/10.1016/j.heliyon. 2021.e07012

Mulat, M., Zewde, A., & Beyene, G. (2021). Integrating GeoGebra with hybrid teaching approaches based on the APOS Theory to enhance students' conceptual understanding in African contexts. *Journal of Educational Technology & Society*, 24(3), 101-113.

Zulnaidi, H., & Zamri, H. (2017). The Impact of Technology on Learning Fractions.

TSG6-2-5

'GREEDY PIG' FOR A MATHEMATICAL INQUIRY WITH PROGRAMMING IN SECONDARY SCHOOL

Dai Kishimoto
Waseda University, Japan, ksks_dai0249@akane.waseda.jp

Takeshi Miyakawa
Waseda University, Japan, tmiyakawa@waseda.jp

'Greedy Pig' is a dice game that may be related to probability and statistics for finding a winning strategy. We believe that it has a potential as a theme of mathematical inquiry which is carried out through programming. In this study, we designed a question for inquiry and analyzed its possible itineraries to examine its potentials for inquiry-based learning. Specifically, using the framework of Study and Research Paths, we designed an initial question and conducted an a priori analysis using the Q-A map as an analytical tool. As a result, we elucidated the structure of inquiry activities and demonstrated the potential for the dialectic of mathematics and programming to emerge within the inquiry.

Keywords: mathematical inquiry, programming, Anthropological Theory of the Didactic, Study and Research Paths

Introduction and Theoretical Framework

Programming has gained significant attention across various educational fields, yet its integration into school curricula and policies varies from country to country. For example, Sweden incorporates programming into mathematics and complementary subjects such as crafts (Bocconi et al., 2018). In contrast, some countries treat programming as a distinct subject with a strong focus on informatics or computing (Vinnervik, 2023). This is the case in the Japanese secondary school curricula, where programming is taught only in informatics class, not at all in mathematics. Consequently, research on the use of programming for mathematical learning remains limited, and studies examining the relationship between mathematics and programming are particularly scarce in Japan (Kishimoto & Miyakawa, 2023). However, we argue that mathematics and programming share a strong conceptual relationship, and the inquiry rooted in both disciplines can foster deeper and complementary learning. Therefore, this study aims to explore the potential of mathematical inquiry with programming in the case of 'Greedy Pig'. Specifically, we design a question (or inquiry) related to this game and examine the characteristic features we could expect with this question in secondary school. Implementation of the lessons is beyond the scope of this paper and will be reported elsewhere.

In this research, mathematical inquiry is conceptualized within the framework of the *Anthropological Theory of the Didactic* (ATD) (Chevallard & Bosch, 2019), specifically by the notion of *Study and Research Paths* (SRP). SRP represents the structure and functioning of a researcher's inquiry, operating under the assumption that all necessary resources (e.g., PC, internet, books) are readily accessible and that requisite knowledge will be acquired as needed. SRP begins with an *initial question* (Q_0) that possesses *generative power*, and the progression of inquiry is characterized by

the notion of *dialectic*. This study employs two types of dialectics as perspectives to design and analyse the inquiry. The first is the *dialectic of questions and answers.* This perspective provides a fundamental mechanism of the inquiry that posits that the inquiry evolves not only through finding an answer to a question, but also through getting new questions (Chevallard & Bosch, 2019). The second perspective is the *dialectic of mathematics and programming*, which allows us to capture the dynamic interplay between mathematical reasoning and programming activities throughout the inquiry process (Kishimoto & Miyakawa, 2023). We investigate in this paper *these two dialectics that may take place in the SRP we designed with Greedy Pig.*

The rules of 'Greedy Pig' are as follows: the goal of this game is to be the first player to reach 100 points. On each turn, a player rolls a die repeatedly until either a 1 is rolled or they choose to hold, in which case they score the sum of their rolls (i.e., the turn total). At any point during their turn, the player must decide between two options: rolling or holding. If they roll a 1, they score nothing, and their turn ends, passing play to their opponent. If they roll any number other than 1, that number is added to their turn total, and their turn continues. Alternatively, if the player chooses to hold, their turn total is added to their overall score, and the turn passes to their opponent. Until now, this game has been studied by various mathematicians (e.g., Neller & Presser, 2010). The initial question we designed is Q_0*: What is the winning strategy for this game?*

Research Method

To examine the potential of the initial question given above, we carry out an *a priori analysis* (Bikner-Ahsbahs et al., 2023). This refers to an analysis typically conducted before the experiment or data collection. Originally proposed as part of the methodology of *didactic engineering* within the *Theory of Didactical Situations* (TDS) (Artigue, 2020). It is also adopted in the research with ATD. In this study, we make use of the concept of the *questions-answers map* (Q-A map) (Winsløw et al., 2013) as an analytical tool. This concept allows for the representation of the inquiry process structure using a tree diagram and also makes visible the dialectic of questions and answers that takes place within the inquiry. This analysis of the generating power of Q_0 ensures a viability of the inquiry process and also gives the teacher a first insight into the students' possible inquiries (Bosch, 2018). So, we extract the questions and answers that arise when initiating the inquiry from Q_0 and construct a Q-A map. Through this process, we clarify the overall structure of the inquiry and identify the characteristics of the two dialectics. From these results, we examine the potential of mathematical inquiry with programming.

In the Q-A map presented in this paper, questions and answers related to programming are highlighted in gray, while questions and answers related to mathematics are highlighted in light blue. Additionally, some elements in the Q-A map of this paper are marked with ♦ or ♥ next to A. A♦ represents an existing answer obtained through the use of *media* (e.g., the internet, books, papers, lectures), while A♥ represents the final answer.

Result

The inquiry using our Q_0 can be divided into two phases: the *mathematical or theoretical phase* and the *statistical phase*. We provide a detailed analysis of the structure of the inquiry and the dialectic that emerged within it, dividing it into two phases.

Phase 1: The Mathematical or Theoretical Phase

When an inquiry begins from Q_0, two approaches may be taken: practical and mathematical. The practical approach begins with developing strategies by playing a game using real dice. In this context, programming a system allowing simulations of the game (Code.1) will be effective in collecting data. On the other hand, the mathematical approach begins with calculating various probabilities and expected values that allow the player to decide how to play. Some strategies will be formulated based on the results of the calculations (e.g., "Stops rolling the dice when the score for the turn exceeds 20 points"). Subsequently, the question arises: "Q: Are the strategies based on these numbers effective?" To effectively evaluate the strength of the strategies, it will be expected to develop a program (Code.2) to automate the opponent's actions. At this point, the inquiry progresses from phase 1 to phase 2. Figure 1 is the Q-A map for phase 1.

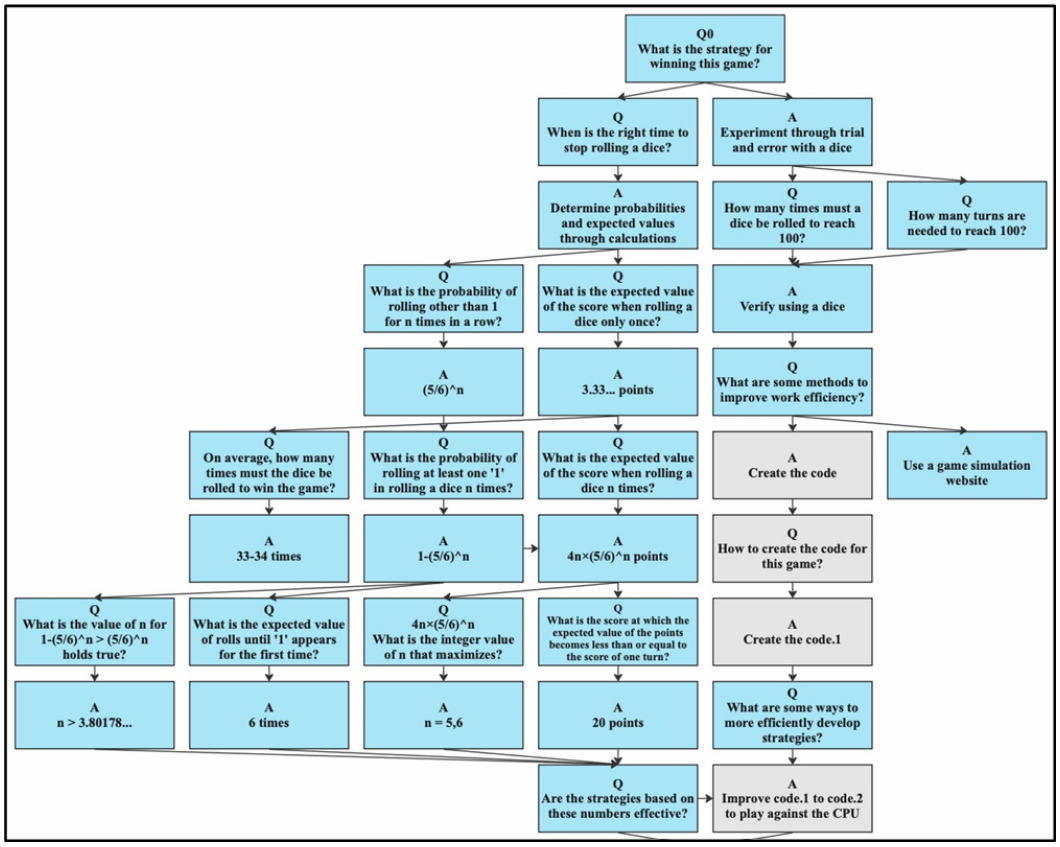


Figure 1. Q-A map for phase 1

Phase 2: The Statistical Phase

Phase 2 begins with evaluating the effectiveness of the strategies developed in phase 1. Then, Code.2 will be improved to Code.3, which enables multiple battles between CPUs, allowing for a more detailed assessment of the strategies. And, an activity will arise to record the number of victories and calculate the win rates of the CPUs in Code.3. For instance, if Strategy A and Strategy B compete 10,000 times, with Strategy A winning 5,100 times and Strategy B winning 4,900 times, the following question arises: "Q: What are the methods to evaluate strategies more mathematically?". This initiates the validation process for Strategy A through hypothesis testing. Finally, by integrating the results and ideas obtained through these activities, an original strategy

(A♥) will be developed, marking the completion of both phase 2 and the inquiry. Figure 2 is the Q-A map for phase 2.

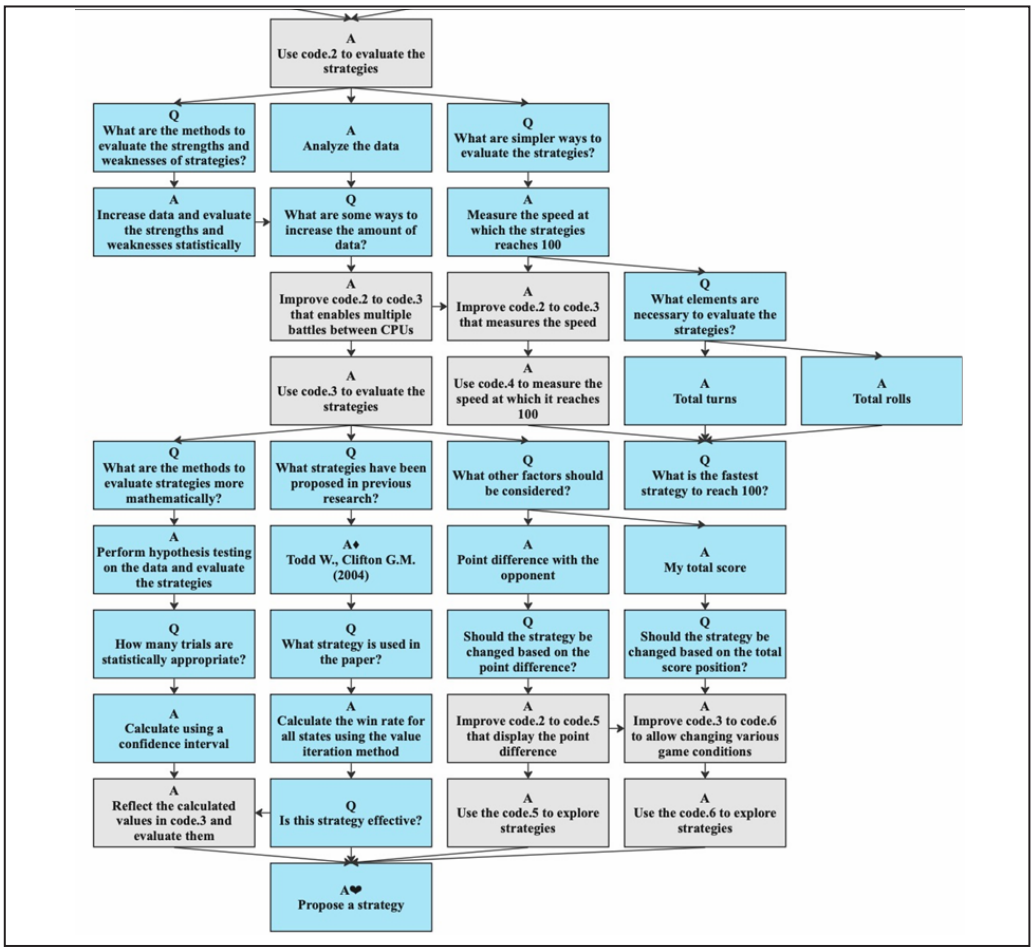


Figure 2. Q-A map for Phase 2

Discussion and Conclusions

A priori analysis demonstrated how the initial question using 'Greedy Pig' may be approached in the context of inquiry-based learning, leading students to inquire about the game through programming. We highlighted two distinct potentials of the inquiry with programming. First, a dialectic of mathematics and programming emerges on the one hand to improve the efficiency of rolling the dice in Phase 1 and on the other hand to evaluate the strategies in Phase 2. Therefore, this inquiry has the potential to generate various dialectic of mathematics and programming. In particular, when determining the number of trials using the confidence interval in Phase 2, there is a possibility that a more concrete dialectic will emerge, integrating mathematical knowledge into the code. Second, two types of activities arise in this inquiry: the mathematical or theoretical phase and the statistical phase. In other words, there is a possibility of learning about *theoretical probability* and *empirical probability* (Ishibashi, 2022) and also understanding their relationships within this inquiry.

Furthermore, to more effectively demonstrate the potential of this game, it is necessary to conduct verification tests. To this end, we are currently conducting a teaching experiment using Q_0 at a senior high school in Japan.

References

Artigue, M. (2020). Didactical engineering. In S. Lerman (Ed.), *Encyclopedia of mathematics education. Second Edition* (pp. 202–206). New York: Springer. https://doi.org/10.1007/978-3-030-15789-0_44

Bikner-Ahsbahs, A., Kidron, I., Shinno, Y., & Miyakawa, T. (2023). Guest Editorial: Rethinking the diversity of theories in mathematics education. *Contributions related to the Topic Study Group 57 of ICME 14*. *16*. 21-26. 10.24529/hjme.1602.

Bocconi, S., Chioccariello, A., & Earp, J. (2018). The nordic approach to introducing computational thinking and programming in compulsory Education. *Report Prepared for the Nordic@BETT2018 Steering Group, 42*.

Bosch, M. (2018). Study and Research Paths: a model for inquiry. *In Proceedings Of The International Congress Of Mathematicians 2018, 3*, 4001-4022.

Chevallard, Y., & Bosch, M. (2019). A short (and somewhat subjective) glossary of the ATD. In M. Bosch, Y. Chevallard, F. García, & J. Monaghan (Eds.), *Working with the Anthropological Theory of the Didactic in Mathematics Education: A Comprehensive Casebook* (1 ed., pp. xviii-xxxvii). London: Routledge.

Ishibashi, I. (2022). Analyzing experimental and theoretical probabilities in Japanese 7th and 8th grade textbooks. *International Electronic Journal of Mathematics Education, 17*(3), em0690. https://doi.org/10.29333/iejme/12061

Kishimoto, D., & Miyakawa, T. (2023). The Place of Programming in Mathematics Learning: through an experiment with the Collatz problem. *Journal of JASME Research in Mathematics Education, 29*(1), 41-53.

Neller, Todd W., & Clifton G.M. Presser. (2010). Practical Play of the Dice Game Pig. *The UMAP Journal, 31*, 1, 5-19.

Vinnervik, P. (2023). Programming in school technology education: the shaping of a new subject content. *Int J Technol Des Educ 33,* 1449–1470. https://doi.org/10.1007/s10798-022-09773-y

Winsløw, C., Matheron, Y., & Mercier, A. (2013). Study and research courses as an epistemological model for didactic. *Educational Studies in Mathematics, 83*(2), 267-284.

https://doi.org/10.1007/s10649-012-9453-3

TSG6-2-6

STUDENTS' REASONING OF QUADRATIC GROWTH IN A PROGRAMMING-BASED ENVIRONMENT

Xuan Su
The University of Hong Kong, China, xuansu24@hku.hk

Biyao Liang
The University of Hong Kong, China, biyao@hku.hk

Integrating programming into mathematics curricula can help students build mathematical knowledge. However, the use of programming in the teaching of functions remains underexplored. This study aims to fill this gap by investigating how programming can help or challenge students understand the second difference of quadratic functions. This study used two one-on-one semi-structured task-based interviews, introducing the context of growing squares to examine the reasoning process of an eighth-grade student. Data was analyzed by open-coding the interview transcripts. The results showed that programming enabled the student to visualize and refine his understanding of the second difference. This study provides mathematics teachers with practical examples and insights into how programming can support teaching abstract mathematical concepts.

Keywords: Programming, Mathematical Reasoning, Quadratic Function, Second Difference

Background

In response to the growing emphasis on Science, Technology, Engineering and Mathematics (STEM) education in the 21st century, many studies have advocated the integration of programming into mathematics curricula (Wang et al., 2022). Unlike the static process in paper-and-pencil environments, programming's interactivity and immediate feedback help students develop logical reasoning and problem-solving through creating, debugging, and modifying code (Kaufmann & Stenseth, 2021; Ng & Cui, 2021; Niemelä et al., 2017). In addition, programming has the potential to support students in communicating through multiple representations, such as words, symbols, and diagrams, making abstract concepts more concrete (DeJarnette, 2019).

Building on these general benefits, programming has been shown to enhance students' understanding of algebra (Bråting & Kilhamn, 2021). For example, programming algorithms based on repeated addition could help students grasp the concept of multiplication (Sherin, 2001). Besides, Gorman et al. (2014) found that students could master set operations and logical reasoning through programming tasks related to relational algebra. However, its potential for teaching function relationships, such as the second difference, remains underexplored.

The second difference reflects the "change of change" in a function and requires students to understand how quantities change at different levels. This layered way of thinking is closely related to computational thinking (CT), which emphasizes decomposing complex problems into smaller and manageable parts (Wing, 2006). Similarly, programming involves features such as loops, variables and conditional statements to systematically model these layered relationships in order to provide

a natural framework for supporting students' mathematical reasoning. Therefore, we aim to design a task that encourages students to explore the second difference of quadratic function through programming and provides practical insights for educators. The guiding research question is: *How can programming help or challenge students understand the concept of second difference?*

Methodology

Participant

The participant in this study was Bao (pseudonym), a Chinese eighth-grade student who had not yet learned function at school but was familiar with Scratch. Bao's active personality made it easier to capture his cognitive processes during the interviews. His average math grades could present both challenges and opportunities in the task.

Research Design

This study used two one-on-one semi-structured task-based interviews, each lasting 1.5 hours. In each interview, Bao was asked to complete specified mathematical tasks, and the researcher guided his thinking with questions like, "Why do you think so?" and "Why do you do this?" After the first interview, the task design was adjusted based on Bao's feedback. All the interviews were conducted after school to avoid disrupting Bao's regular school hours.

Task Design

This task placed the first and second differences of quadratic function in the context of "growing square" to explore changes in side lengths and areas. It consisted of two activities, one in a paper-and-pencil environment and one in a programming environment, corresponding to the two interviews.

To clarify the relevant mathematical concepts, we used the quadratic function $y = x^2$, where x represents the side length of a square and y represents the area, with a growing square as the figural pattern (*Figure 1a*). For this case, assuming the side length increases by 1 each time, the area would be 1, 4, 9, 16, 25... The first difference of the area would be 1, 3, 5, 7, 9..., and the second difference remains constant at 2. This framework guided Bao's exploration of the second difference.

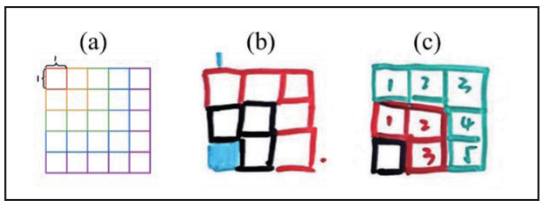


Figure 1. Figural pattern of the growing square

The first activity served as a foundation for exploring the second difference by introducing the first difference. Bao was asked to draw growth patterns of squares with increasing side lengths on grid paper. The researcher guided him in analyzing the first difference in side length and area.

The second activity introduced code as a new representation to help Bao explore the second difference. Bao was asked to select one pattern from the first activity and program an animation to

recreate it using the cloning feature in Scratch. The initial code provided contained a parent square with a side length of 30 (Figure 2), considered as 1 unit for convenience. After completing the animation, Bao was asked to explain the code's logical structure and identify the second difference in area for the animation.

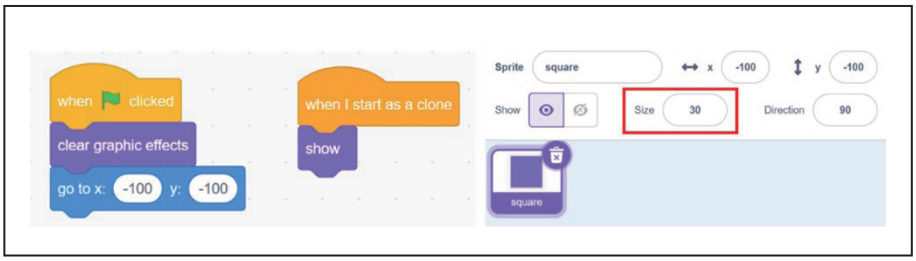


Figure 2. Initial code and parent square provided to the student

Data Collection and Analysis

Data collection included audio and video recordings, Scratch code, and written work. Open coding was conducted by the first author to analyze Bao's reasoning process, and memos were written to track cognitive shifts and summarize key insights. This preliminary analysis provides the basis for further refinement of codes and the development of Bao's cognitive model in future work.

To enhance coding validity, the first author regularly discussed with the second author to reflect on Bao's reasoning. The absence of a second coder is acknowledged as a limitation, and future research could include independent double coding to establish inter-coder consistency. In addition, although there was no strict time limit for the programming task, Bao completed it within one hour. This ensured that the duration of participation was reasonable and minimized potential effects from prolonged exposure to the programming environment.

Results

This section reports Bao's performance in the second activity, demonstrating how he developed abstract thinking and an understanding of the second difference through programming. In the first activity, Bao initially understood the first difference by drawing growth patterns of squares. Therefore, in the second activity, we moved straight into programming.

Bao began by coding the L-shaped growth pattern from his hand-drawn sketches (*Figure 1b*), explaining, "This pattern's side lengths increase by 1 at a time, so it involves fewer blocks, making the programming simpler." This shows how his initial understanding of the first difference informed his approach to coding. Before coding, Bao predicted, "These small squares will probably appear sequentially in a certain order," which reflected his shift from visual to programming-based thinking. He then established the sequence for generating and placing each clone (*Figure 1c*), which guided his coding logic.

Bao's initial strategy was to start by coding 2×2 squares, then 3×3 and 4×4. He said, "Because it's easier to find patterns that way." This indicates his awareness of the need to eventually abstract and simplify the code. Using a parent square, he moved clones stepwise (up, right, then down) to represent a 2×2 square (*Figure 3a*), adding "color effects" to distinguish squares of different side length.

When extending the code to 3×3 squares, Bao copied the 2×2 code, assuming the logic remained the same. However, when Bao ran the code, he noticed that the overall position of the third L-shape

was incorrect (*Figure 3b*). After reviewing the code, he realized, "The first clone of the third L is in the wrong position, affecting the others." To correct this, Bao identified that he had not reset the final clone's position from the 2×2 code. Adding "go to x: -100 y: -100" (*Figure 3c*) resolved the issue, allowing the correct generation of the 3×3 square.

For the 4×4 square, Bao still copied and modified the code for the 3 × 3 square (*Figure 3d*). He soon realized the repetitive commands "change x by 30" and "change y by -30" could be replaced with loops, stating, "These actions repeat three times, so I can simplify them using 'repeat 3'" (*Figure 3e*). He further optimized earlier codes by applying loops to the 2×2 and 3×3 patterns. Bao noticed that the y-position changes (30, 60, 90) at the beginning aligned with each L-shape's first clone, simplifying these steps using multiplication (1×30, 2×30, 3×30). He explained, "Loops aren't used here because the first clone jumps directly without leaving traces, which loops would create."

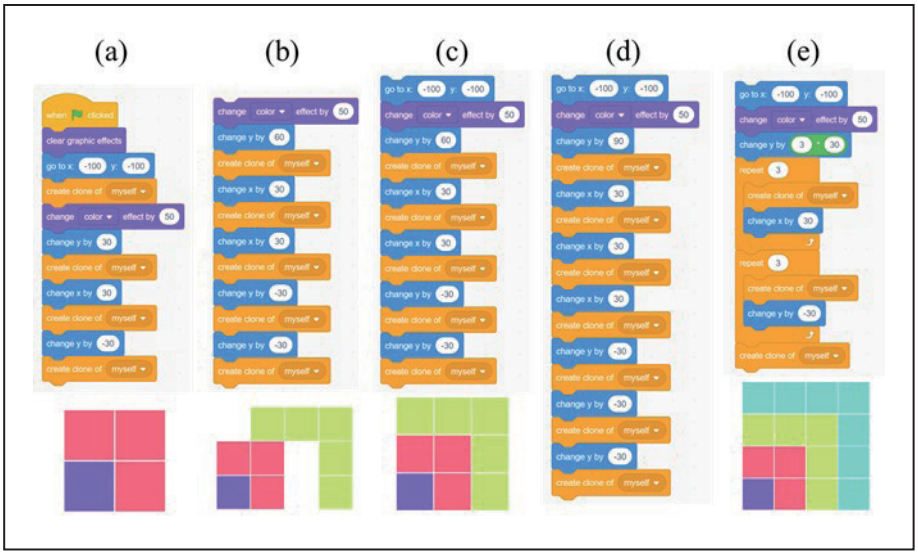


Figure 3. Code for drawing squares from 2×2 to 4×4

Eventually, Bao generalized the pattern, "For an n × n square, jump (n-1) steps, then move right (n-1), and finally down (n-1). So, both loops repeat (n-1) times." This marked his transition from concrete manipulation to abstract reasoning.

To make the program adaptable for any square size, Bao added an input function (*Figure 4a*), explaining, "If we enter a number, the program gives an 'answer' to simplify the code." He initially set all number of times to repeat "answer-1," but observed missing L-shapes when running the code (*Figure 4c*). Reviewing the code, Bao realized, "I need a variable to track changes." He created a "side length" variable, initialized at 1, and added a loop repeating "answer-1" times (*Figure 4b*). With each iteration, "change side length by 1" updated the pattern dynamically. He also changed the "answer-1" inside the loop to "side length-1". He distinguished between the static "answer" (defining the number of L-shapes) and the dynamic "side length" (controlling clone placement), concluding, "The answer is fixed, but side length changes with each loop to adjust the shape." This correction enabled the program to draw squares of any size (*Figure 4d*).

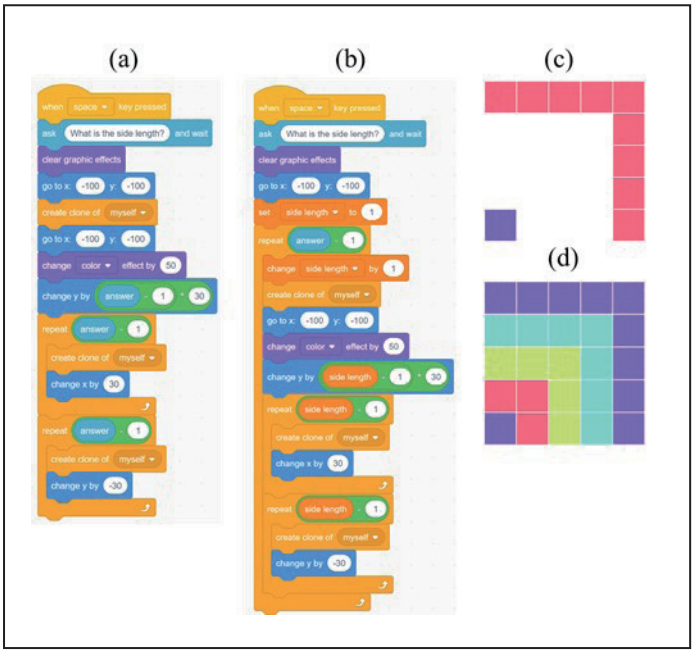


Figure 4. Generalized code for squares of varying side length

After completing the programming, the researcher tested Bao's understanding of the second difference. Bao observed, "The increment of the area increment is 2," explaining, "At each side of each L-shape, there is one more small square than the previous L-shape, and 1 plus 1 is 2." When asked to identify this in the code, he said, "In these two places where 'repeat side length - 1'. They represent the two sides of the L-shape. As the loop goes on, the side length increases by 1 each time, representing that each side will have one extra square than each side of the previous L-shape."

Discussion and Conclusion

Unlike traditional paper-and-pencil environments, programming provides opportunities for iterative debugging and logical optimization (Kaufmann & Stenseth, 2021). We support this view through Bao's learning process. In his final code, the outer loop controlled square growth (representing changes in side length), while the inner loop controlled the assembly of L-shaped blocks (representing changes in area). Through iterative debugging, he gradually grasped the corresponding second difference by understanding the nested structures in the code. This process allows the abstract concept to be visualized in programming operations, thus helping Bao to internalize it (DeJarnette, 2019).

Additionally, Bao encountered some challenges in resetting clone locations and distinguishing between static parameters (e.g., 'answer') and dynamic variables (e.g., 'side length'). These challenges exposed his conceptual misunderstandings and showed how programming reveals conceptual differences not easily seen in paper-and-pencil environments.

This study provides practical insights for educators on using programming to support students' understanding of abstract mathematical concepts, such as the second difference. Building on these findings, future research could explore how Scratch can be leveraged to introduce more complex algebraic topics, including but not limited to quadratic functions.

References

Bråting, K., & Kilhamn, C. (2021). Exploring the intersection of algebraic and computational thinking. *Mathematical Thinking and Learning*, *23*(2), 170–185. https://doi.org/10.1080/10986065.2020.1779012

DeJarnette, A. F. (2019). Students' challenges with symbols and diagrams when using a programming environment in mathematics. *Digital Experiences in Mathematics Education*, *5*, 36–58. https://doi.org/10.1007/S40751-018-0044-5

Gorman, J., Gsell, S., & Mayfield, C. (2014). Learning relational algebra by snapping blocks. *Proceedings of the 45th ACM Technical Symposium on Computer Science Education*, 73–78. https://doi.org/10.1145/2538862.2538961

Kaufmann, O. T., & Stenseth, B. (2021). Programming in mathematics education. *International Journal of Mathematical Education in Science and Technology*, *52*(7), 1029–1048. https://doi.org/10.1080/0020739X.2020.1736349

Ng, O. L., & Cui, Z. (2021). Examining primary students' mathematical problem-solving in a programming context: Towards computationally enhanced mathematics education. *ZDM – Mathematics Education*, *53*(4), 847–860. https://doi.org/10.1007/s11858-020-01200-7

Niemelä, P., Partanen, T., Harsu, M., Leppänen, L., & Ihantola, P. (2017). Computational thinking as an emergent learning trajectory of mathematics. *Proceedings of the 17th Koli Calling International Conference on Computing Education Research*, 70–79. https://doi.org/10.1145/3141880.3141885

Sherin, B. L. (2001). A Comparison of Programming Languages and Algebraic Notation as Expressive Languages for Physics. *International Journal of Computers for Mathematical Learning*, *6*(1), 1–61. https://doi.org/10.1023/A:1011434026437

Wang, C., Shen, J., & Chao, J. (2022). Integrating Computational Thinking in STEM Education: A Literature Review. *International Journal of Science and Mathematics Education*, *20*(8), 1949–1972. https://doi.org/10.1007/S10763-021-10227-5/METRICS

Wing, J. M. (2006). Computational thinking. *Communications of the ACM*, *49*(3), 33–35. https://doi.org/10.1145/1118178.1118215

TSG6-2-7

ENHANCING COMPUTATIONAL THINKING THROUGH MACHINE LEARNING IN DAILY WASTE SORTING PROBLEM SITUATIONS

Anucha Koyata
Khon Kaen University, Thailand, anucha_koyata@kkumail.com

Narumon Changsri
Khon Kaen University, Thailand, changsri_crme@kku.ac.th

Maitree Inprasitha
Khon Kaen University, Thailand, inprasitha_crme@kku.ac.th

This study aims to enhance computational thinking skills through the integration of machine learning in daily waste sorting problem situations, using a TLSOA. Conducted with 12 seventh-grade students from a school in Khon Kaen, Thailand, data was collected through classroom observations and analysis of students' work. Qualitative analysis revealed that students developed skills in pattern recognition, data analysis, and algorithmic thinking while interacting with machine learning. These activities fostered systematic problem-solving, collaboration, and data-driven decision-making. Students gained insight into the complexities of waste sorting and the necessity of careful categorization. Training models sparked discussions on waste classification and bin allocation, deepening their understanding of waste management challenges. Additionally, students acknowledged the limitations of machine learning, emphasizing the importance of quality training data. They also demonstrated critical judgment in evaluating machine-generated outputs, acknowledging that machine-learning models can err and require responsible usage.

Keywords: Computational Thinking, Machine Learning, Transformative Lesson Study incorporated with Open Approach (TLSOA), Middle School Education

Introduction

Computational thinking has emerged as a fundamental skill in modern education, enabling students to approach problem-solving with systematic precision and efficiency (Wing, 2006). As technology continues to reshape educational landscapes, the integration of machine learning (ML) into classrooms presents compelling opportunities to enhance students' problem-solving capabilities while developing essential computational skills (Barr & Stephenson, 2011; Goodfellow, Bengio, & Courville, 2016).

Waste sorting represents an ideal context for developing computational thinking, as it requires pattern recognition, data analysis, and systematic decision-making skills that align naturally with ML applications (Ng, Su, & Chu, 2024; Yadav, Hong, & Stephenson, 2016). By engaging students in ML-based waste classification projects, this study seeks to strengthen computational thinking while fostering environmental awareness and creating meaningful connections between technology and real-world challenges (Beers, 2011).

The research employs a Transformative Lesson Study incorporated with Open Approach (TLSOA), which emphasizes active learning and collaborative problem-solving. This methodological framework provides a structured yet flexible environment for students to develop thinking skills and adapt to complex problem-solving scenarios. The TLSOA framework allows students to refine their understanding through practical problem-solving and reflection (Inprasitha, 2006, 2010, 2022).

This study addresses these research gaps by examining the synergistic relationship between ML implementation and the TLSOA model. The findings aim to provide educators and policymakers with evidence-based strategies for enhancing students' computational thinking and problem-solving capabilities through advanced technology integration through the TLSOA framework, this research contributes to our understanding of how emerging technologies can be effectively utilized to develop essential 21st-century skills while addressing pressing real-world challenges. The study's outcomes offer valuable insights into the design and implementation of technology-enhanced learning experiences that promote both computational thinking and environmental stewardship.

Methodology

This study aimed to enhance computational thinking skills by integrating machine learning into waste-sorting problem-solving scenarios, using the TLSOA framework (Inprasitha, 2006, 2010, 2022). The research was conducted with 12 seventh-grade students in Khon Kaen, Thailand. A qualitative approach was employed to explore teaching processes and student engagement in machine learning-based activities.

Research Design

A qualitative approach was employed to explore teaching processes and student engagement in machine learning-based waste sorting activities. The study utilized the TLSOA framework, which emphasizes collaborative plan, do, and see to foster critical thinking and enrich learning experiences. The methodology followed the three core phases of the TLSOA framework:

Step 1 Collaboratively: Plan The first step involved the lesson study (LS) team and participating teachers collaboratively designing lessons that align with the principles of computational thinking and machine learning in the context of waste sorting. The planning process focused on creating open-ended problems that would stimulate students' critical thinking, creativity, and collaboration. The research team formulated open-ended problems such as "How do students determine which colored bin to dispose of their waste in? How can students train a machine learning model to sort waste into the correct colored bins?" These problems were designed to have multiple solutions, encouraging students to explore diverse approaches. The problems also incorporated real-world data to make the activities authentic and relevant. During the planning phase, the LS team iteratively refined the open-ended problem to ensure it was challenging yet accessible for 7th-grade students. This iterative process ensured the problem aligned with the student's cognitive level and fostered meaningful engagement.

Step 2 Collaboratively Do: The second step involved implementing the planned lessons in the classroom. The focus was on observing students' thinking processes rather than evaluating the teacher's teaching ability. Teachers acted as facilitators, guiding students through the four phases of OA while documenting their interactions and learning progress.

1. posing open-ended problems: Teachers introduced the waste sorting problem and introduced basic ML concepts using a beginner-friendly ML platform called Machine Learning for Kids to students and encouraged them to brainstorm potential solutions. For example, students were tasked with training a simple machine-learning model to classify waste items based on labeled datasets. This activity required students to engage in pattern recognition, data analysis, and algorithmic thinking.
2. students' self-learning: During the self-learning phase, students worked in small groups to explore the problem using machine-learning tools. They collected and labeled waste data, trained their models, and tested their accuracy. Teachers observed how students approached the task, noting their problem-solving strategies and areas where they encountered difficulties.
3. whole-class discussion and comparison: After the self-learning phase, students presented their findings to the class. Discussions centered on comparing different approaches to waste classification, analyzing the strengths and weaknesses of each group's model, and reflecting on the challenges of accurate categorization. Teachers encouraged students to critically evaluate their models and consider ways to improve them.
4. summarizing by connecting students' ideas: At the end of the lesson, teachers summarized the key takeaways, emphasizing the importance of computational thinking skills such as pattern recognition, abstraction, and algorithmic design. They also highlighted the ethical implications of using machine learning in waste management, such as the need for quality training data and responsible usage.

Step 3 Collaboratively See: The final step involved reflecting on the implemented lessons to identify areas for improvement. Observers, including teachers and researchers, met to reflect on the learning activities. They discussed students' engagement, the effectiveness of the open-ended problems, and the extent to which the lessons achieved the desired learning outcomes. These reflections provided valuable insights into students' thinking processes and helped identify areas where additional support was needed.

Data Collection and Analysis

The data presented in this study were collected during the 18th National Open Class, an annual nationwide initiative organized under the framework of the Professional Learning Community (PLC). Which represents PLC level 5, an annual cycle within the teacher education institution network. This event is part of a broader effort to enhance teaching practices and foster collaboration among educators across Thailand. Data was collected through 1) Classroom Observations: Documenting students' interactions, problem-solving strategies, and engagement during the lessons. 2) Analysis of Students' Work: Examining students' ML models, written reflections, and group presentations to assess their development of computational thinking skills.

Qualitative data was thematic analysis was conducted using predefined codes based on computational thinking framework. Patterns of development in key areas—such as abstraction, data representation, and debugging—were identified. Interactions were also examined using rubrics that assessed collaborative problem-solving skills and computational processes.

Ethical Considerations

The study adhered to all ethical standards, obtaining consent from all participants. Personal information was kept confidential to ensure that students and teachers participated voluntarily and safely.

Results and Discussion

The findings indicate that engaging students in ML-based waste sorting activities promoted significant development in computational thinking. Students demonstrated increased proficiency in: 1) Pattern recognition – They identified visual features to categorize waste (e.g., color, shape, material). 2) Algorithmic thinking – Student groups iteratively modified input data and adjusted model structures to improve classification outcomes. 3) Data decomposition and abstraction – Students broke down waste into subcategories (e.g., recyclable vs. non-recyclable) and abstracted essential features for machine learning model training

Figure 1. Student-trained and tested ML model and result recording sheet.

The ML models trained by students also supported these observations. For instance, groups that used a higher number of training samples per class achieved higher model accuracies, whereas groups with fewer training samples had lower accuracy. This highlights the relationship between data volume and model performance—a relationship that students themselves recognized and articulated in their reflections. Moreover, students developed a growing awareness of ML's inherent limitations, leading them to apply critical judgment when evaluating ML-generated classifications and understanding the ethical implications of ML decision-making. Through these experiences, students also acknowledged the subjectivity involved in waste sorting, realizing that different criteria might be applied in different contexts. This realization reinforced their ability to make informed decisions when categorizing waste and strengthened their computational thinking in tandem with their environmental consciousness.

Discussion and Conclusion

The findings of this study align with previous research on the benefits of integrating computational tools into education (Grover & Pea, 2013). By incorporating machine learning into waste sorting activities, students not only developed computational thinking skills but also gained valuable insights into environmental management. The use of TLSOA created a dynamic learning environment that promoted active participation and reflection, according to Inprasitha (2022).

This study demonstrates the potential of integrating machine learning into classroom activities to enhance computational thinking skills and raise awareness of environmental issues. By using the TLSOA framework, educators can create engaging and reflective learning experiences that prepare students for the challenges of the modern world. The findings suggest that ML-based activities can be a valuable addition to middle school curricula, fostering both technological proficiency and environmental stewardship.

Acknowledgment

The Research on Enhancing Computational Thinking Through Machine Learning in Daily Waste Sorting Problem Situations has received funding support from the Fundamental Fund of Khon Kaen University from The National Science, Research and Innovation Fund or NSRF, Thailand.

References

Beers, S. Z. (2011). *21st-century skills: Preparing students for their future.* National Education Technology Plan: Students at the Center. U.S. Department of Education.

Barr, V., & Stephenson, C. (2011). Bringing computational thinking to K-12: What is involved and what is the role of the computer science education community? *ACM Inroads, 2*(1), 48-54. https://doi.org/10.1145/1929887.1929905

Goodfellow, I., Bengio, Y., & Courville, A. (2016). *Deep learning.* MIT Press.

Grover, S., & Pea, R. (2013). Computational thinking in K–12: A review of the state of the field. *Educational Researcher, 42*(1), 38-43. https://doi.org/10.3102/0013189X12463051

Inprasitha, M. (2006). Open-ended approach and teacher education. *Tsukuba Journal of Educational Study in Mathematics, 25*, 169-177.

Inprasitha, M. (2010). One feature of adaptive Lesson Study in Thailand – designing learning unit. *Proceedings of the 45th Korean National Meeting of Mathematics Education, Korean Society of Mathematics Education*, Seoul, Korea, pp. 193-206.

Inprasitha, M. (2022). Lesson study and open approach development in Thailand: a longitudinal study. *International Journal for Lesson and Learning Studies, 11*(5), 1-15. https://doi.org/10.1108/IJLLS-04-2021-0029

Ng, D.T.K., Su, J. & Chu, S.K.W. (2024). Fostering Secondary School Students' AI Literacy through Making AI-Driven Recycling Bins. *Education and Information Technologies, 29(*8):1-32 https://doi.org/10.1007/s10639-023-12183-9

Wing, J. M. (2006). Computational thinking. *Communications of the ACM, 49*(3), 33-35. https://doi.org/10.1145/1118178.1118215

Yadav, A., Hong, H. & Stephenson, C. (2016). Computational Thinking for All: Pedagogical Approaches to Embedding 21st Century Problem Solving in K-12 Classrooms. *TechTrends, 60*, 565–568. https://doi.org/10.1007/s11528-016-0087-7

Topic Study Groups 6
(Day 3)

TSG6-1-7

CRITICAL REFLECTIONS ON ALGORITHMIC LEARNING DESIGN AND HUMAN EXISTENCE THROUGH HEIDEGGERIAN ONTOLOGY

Jiyoung, Kim
Seoul Dungyang Elementary School, South Korea, 2250@daum.net

Mangoo, Park
Seoul National Univ. of Education, South Korea, mpark29@snue.ac.kr

With the rise of AI-driven educational systems, personalized learning algorithms have gained significant attention for their potential to optimize student learning. However, these systems may also limit learners' cognitive flexibility by reinforcing existing thought patterns and restricting exposure to diverse perspectives. This study critically examines AI-based learning design through Heidegger's existential ontology, focusing on the implications of technology as both a revealing (Aletheia) and concealing (Verbergung) force. By analyzing AI-driven learning through the concepts of Zeug (equipment), Bestand (enframing), and Eigentlichkeit (authentic existence), this study explores whether AI learning design supports learners' self-directed inquiry or confines them within algorithmic constraints. The findings suggest that while AI systems enhance efficiency, they risk reducing learners to data-driven entities, potentially undermining the openness necessary for authentic intellectual development. To address this, the study proposes alternative AI learning design principles that promote cognitive exploration and learner autonomy.

Keywords: AI-based learning, Heidegger, existential ontology, cognitive framing, algorithmic constraints

Introduction

With the advent of the Fourth Industrial Revolution, learning design using artificial intelligence (AI) and big data is rapidly spreading in the education sector (McNamara et al., 2020; Holmes et al., 2019). AI-driven learning design aims to provide personalized learning by analyzing learners' individual characteristics and learning data, which offers technological possibilities to improve learning efficiency and maximize learner engagement. These systems work by analyzing learners' learning patterns in real-time, assessing their performance, and suggesting the most effective learning paths (Knox, 2020).

These capabilities offered by AI-based learning systems are viewed positively as they allow for a shift away from the traditional one-size-fits-all approach to education and allow for instruction that is tailored to the individual learner (Popenici & Kerr, 2017). In particular, features such as automated learning content recommendations, feedback systems, and learning path optimization using AI algorithms are playing an important role in improving learners' learning experience and providing individualized learning opportunities (Williamson, 2017). However, it has been raised that these technological advancements do not guarantee learners' expanded thinking and autonomy, but rather may act in ways that reinforce and entrench certain thinking frames (Selwyn, 2019).

In particular, when AI-driven instructional design recommends optimal learning paths based on a learner's past learning history and patterns, this has the potential to make it difficult for learners to

break out of their existing ways of thinking (Biesta, 2019). While algorithms can identify learners' learning tendencies based on data analysis and recommend content accordingly, there is a risk that they operate in a way that reinforces filter bubbles and confirmation bias (Pariser, 2011; Sunstein, 2017), meaning that learners are likely to learn in a way that reinforces existing ways of thinking rather than exploring new ways of thinking or critical perspectives (Greller & Drachsler, 2012).

These issues require serious philosophical discussion, as AI-driven learning design can operate in ways that do not expand learners' thinking possibilities, but rather frame their thinking. Martin Heidegger (1927/1996) pointed out that technology is not just a tool, but can have a fundamental impact on the way we exist (Dasein). He warned that technology can limit the possibilities of human existence in a certain way by "revealing" the world in a certain way (Aletheia) (Heidegger, 1977). In other words, technology can define the relationship between human beings and the world in a certain way, which in turn shapes human thought and practice in a certain way (Dreyfus, 1992). These Heideggerian ontological concepts can provide an important theoretical framework for analyzing the impact of AI-based learning designs on learners' ways of thinking and being.

The study addressed three primary research questions.

1. How can we interpret AI-driven learning design from a Heideggerian ontological perspective?
2. Does AI-driven learning design expand the possibilities of learners' existence, or does it limit them by defining them in certain ways?
3. What are alternative design principles for ontologically reframing AI-based learning design?

Theoretical Framework

Along with the Fourth Industrial Revolution, learning design using artificial intelligence (AI) is rapidly spreading in the education sector. In particular, algorithm-based systems are evolving to provide personalized learning paths that reflect learners' individual characteristics (McNamara et al., 2020; Popenici & Kerr, 2017). These systems aim to analyze learning data in real-time and recommend content appropriate for each learner, thereby enhancing efficiency (Holmes, 2019). However, such personalization has also raised concerns about reinforcing existing thinking patterns and narrowing cognitive diversity (Selwyn, 2019; Williamson, 2017).

Algorithms in AI-based learning design may reinforce filter bubbles and confirmation bias, thereby restricting the learner's exposure to new perspectives (Pariser, 2011; Sunstein, 2017). This concern goes beyond pedagogy and suggests an ontological issue regarding the shaping of human thought (Biesta, 2019). From a Heideggerian perspective, technology is not merely instrumental but has the potential to configure human ways of being. Heidegger warned that modern technology risks reducing humans to "resources" (Bestand), limiting them to roles defined by technological systems rather than enabling existential exploration (Heidegger, 1977; Dreyfus, 1992).

In *Being and Time* (1927/1996), Heidegger introduced the concept of *Zeug* (instrumentality), emphasizing that tools are not neutral objects but are meaningful within a practical context. This insight can be applied to AI systems, which can act as tools for learning. However, when AI-based systems start categorizing and optimizing learners according to predefined data patterns, they may function less as instruments and more as ontological structures that delimit possibilities of thought and being (Heidegger, 1977; Harman, 2011).

Therefore, this study draws on Heidegger's ontological concepts to analyze whether AI-based learning design expands or limits learners' existential possibilities. It explores whether such systems promote *Eigentlichkeit* (authentic existence) by enabling learners to navigate learning paths independently, or lead to *Uneigentlichkeit* (inauthentic existence) by shaping them within algorithmically preconfigured paths (Wrathall, 2010; Biesta, 2020). The study ultimately calls for alternative learning design principles that align technological affordances with the broader aim of supporting self-directed and critical modes of thinking.

Research Method

This study employs a **critical literature review** as its research methodology, aligning with the framework proposed by Boell and Cecez-Kecmanovic (2014) and Snyder (2019). This approach is particularly suitable for theoretical inquiries such as this one, which aim to interrogate existing assumptions and introduce new conceptual lenses rather than merely summarizing previous work. The critical review method enables a nuanced exploration of how AI-based learning designs intersect with Heideggerian ontology, while identifying conceptual gaps in both educational technology and philosophy of education literature.

The research proceeded through the following three steps:

1. **Ontological Grounding**: We conducted an in-depth conceptual analysis of Heidegger's ontological framework, focusing on key concepts such as *Zeug* (equipment), *Bestand* (standing-reserve), *Aletheia* (unconcealment), and *Eigentlichkeit* (authentic existence). This philosophical foundation was necessary to analyze the ontological implications of AI-based learning environments.
2. **Pedagogical Literature Review**: We systematically reviewed empirical and conceptual studies on AI-based learning design, with particular attention to critiques concerning personalization, algorithmic recommendation, learner autonomy, and confirmation bias (e.g., Knox, 2020; Selwyn, 2019; Williamson, 2017). This step identified how AI systems function in current educational settings and how their design influences learners' thinking processes.
3. **Conceptual Synthesis and Reframing**: We critically synthesized findings from steps 1 and 2 to develop a Heideggerian critique of AI-based learning design. This included an evaluation of whether current AI applications in education promote or inhibit authentic intellectual development (*Eigentlichkeit*). Based on this, we proposed alternative design principles that foreground learner autonomy and ontological openness.

Validity and Theoretical Rigor: To ensure theoretical robustness, each stage of the review was guided by principles of conceptual saturation and consistency with Heidegger's original texts (1927/1996; 1977), supplemented by secondary analyses (Dreyfus, 1992; Wrathall, 2010). The coherence of arguments was cross-validated through dialogic engagement with interdisciplinary sources in education, philosophy, and technology studies. Although empirical generalizability is not the aim of this study, the conceptual validity is ensured through analytic depth and philosophical fidelity.

This methodologically structured engagement provides a solid foundation for interpreting AI learning environments through existential ontology and lays the groundwork for future empirical inquiry.

Discussion

Based on Heidegger's ontological concepts, this study analyzed the impact of AI-based learning design on learners' ways of thinking and being. We found that AI-based learning design tends to reflect learners' individual characteristics while reinforcing existing thinking patterns, limiting diversity of thought (Selwyn, 2021; Knox, 2020). This suggests that AI works to reinforce learners' existing thinking frameworks in the process of recommending personalized learning content based on their past performance and learning patterns, which may limit their ability to experience new thinking opportunities (Greller & Drachsler, 2012; Selwyn, 2019). This tendency is similar to the problem of filter bubbles and confirmation bias, which are common in algorithmic recommendation systems, and imply structural limitations that encourage learners to think within a particular learning path (Pariser, 2011; Sunstein, 2018)

Heidegger pointed out that technology can serve to "reveal" (Aletheia) the world in a certain way, while simultaneously concealing (Verbergung) other possibilities of existence (Heidegger, 1977). Applying these concepts to the design of AI-powered learning, we can see that as AI systems suggest optimal learning paths based on a learner's training data, they have the potential to structure the learning experience in certain ways and block out new possibilities. For example, if an AI repeatedly recommends only certain types of learning materials based on a learner's past performance, the learner may lose the opportunity to discover new learning paths or expand their existing frame of mind (Biesta, 2019).

This structure connects to what Heidegger warned about as the "danger of technological existence. Heidegger believed that when humans are forced by technology to understand the world in a certain way, there is a danger that their way of being is determined by the possibilities offered by technology (Heidegger, 1927/1996). This can be equally applicable to AI-based learning design, and when AI operates in a way that predetermines the learner's learning path, learners are more likely to think only within the boundaries of what the algorithm allows, rather than actively exploring the possibilities of their own existence (Wrathall, 2010).

This can have the effect of limiting learners' self-direction and creative thinking. Existing research also suggests that if AI-based learning designs are overly data-driven, learners experience less active thinking and risk becoming mere consumers of information (Peters, 2017; Selwyn, 2019). Therefore, it is important to be wary of the possibility that AI-based learning systems may operate in ways that guide and constrain learners' thinking in a particular direction, rather than helping them expand their thinking.

Heidegger believed that human existence (Dasein) can oscillate between "original existence" (Eigentlichkeit) and "unoriginal existence" (Uneigentlichkeit) in the process of opening up its own possibilities (Heidegger, 1927/1996). Authentic existence refers to a state of being in which human beings are autonomously exploring their own possibilities, while non-authentic existence refers to a state in which their way of being is defined by their environment or technological systems (Wrathall, 2010). When AI-driven learning design operates in a way that prescribes a learner's thinking in a particular way, there is a risk that the learner will be pushed out of their natural way of being and into a non-natural way of being.

For example, if learners passively follow the learning paths suggested by AI systems without critically examining them, their thought processes are increasingly framed by algorithms (Biesta, 2020). This means that learners can operate in ways that exist only within the possibilities set by the technology, rather than self-directed learning paths. Therefore, if AI-driven learning design is to work

towards expanding the possibilities of learners' existence, it must put in place structures that enable learners to actively navigate their own learning paths.

Conclusion

1. Design to Break Algorithmic Self-repetition

Current AI-based learning designs work by analyzing learners' past learning patterns to deliver optimal learning content. However, this approach runs the risk of forcing learners to repeat their thinking within a certain range and blocking the possibility of new thinking (Selwyn, 2019). Therefore, AI should be designed to provide unexpected learning opportunities instead of following learners' existing patterns. For example, this could include the ability to suggest new topics that are unrelated to the learner's learning patterns or content that can encourage critical thinking (Peters, 2017).

2. Allow Learners to Design Their Own Learning paths

Rather than forcing learners to make choices, AI systems should present them with a variety of learning options and allow them to explore them on their own. In other words, AI should be designed in such a way that it does not steer learners' thinking in a particular direction, but rather offers different learning paths that allow learners to discover new possibilities (Biesta, 2020). To this end, it is necessary to include features that allow AI to guide learners to set their own learning goals and directions.

3. Build Learning Environments Where AI is an Assistant, not a Master

Current AI-powered learning designs rely on AI to design learning paths and guide learners. However, this approach can limit the learner's experience of active thinking and increases the risk of learners uncritically following the AI's recommendations. Therefore, AI should function as an assistant to support learners' thinking, not as the main agent of learning.

This study analyzed the impact of AI-based learning design on learners' ways of being and thought processes, and identified the possibility that AI can operate in a way that prescribes thinking in a particular direction rather than expanding learners' thinking. To address this, this study suggests that AI learning designs should be designed to expand learners' possibilities of being, and that they should be designed to support learners' self-direction and thought expansion. Future research will need to empirically examine these theoretical discussions and analyze the impact of actual AI-based learning systems on learners' thought expansion.

References

Biesta, G. (2019). *Reclaiming Teaching for Teacher Education*. The Future of Teaching, 10.

Biesta, G. (2020). *Educational research: An unorthodox introduction*. Bloomsbury Publishing.

Boell, S. K., & Cecez-Kecmanovic, D. (2014). A hermeneutic approach for conducting literature reviews. *Communications of the Association for Information Systems, 34*(1), 12.

Dreyfus, H. L. (1992). *What Computers Still Can't Do: A Critique of Artificial Reason*. MIT Press.

Greller, W., & Drachsler, H. (2012). Translating learning into numbers. *Educational Technology & Society, 15*(3), 42–57.

Harman, G. (2011). *Heidegger explained: From phenomenon to thing*. Open Court.

Heidegger, M. (1927/1996). *Being and Time* (J. Stambaugh, Trans.). SUNY Press.

Heidegger, M. (1977). *The question concerning technology*. Garland.

Holmes, W., Bialik, M., & Fadel, C. (2019). *Artificial intelligence in education promises and implications for teaching and learning*. Center for Curriculum Redesign.

Knox, J. (2020). Artificial intelligence and education in China. *Learning, Media and Technology, 45*(3), 298–311.

McNamara, D. S., Graesser, A. C., McCarthy, P. M., & Cai, Z. (2014). *Automated evaluation of text and discourse with Coh-Metrix*. Cambridge University Press.

Pariser, E. (2011). *The filter bubble: What the Internet is hiding from you*. Penguin.

Peters, M. A. (2017). *Heidegger, phenomenology, education*. Routledge.

Popenici, S. A., & Kerr, S. (2017). Exploring the impact of AI on higher education. *Research and Practice in Technology Enhanced Learning, 12*(1), 22.

Selwyn, N. (2019). *Should robots replace teachers?: AI and the future of education*. Wiley.

Selwyn, N. (2021). *Education and technology: Key issues and debates* (2nd ed.). Bloomsbury.

Snyder, H. (2019). Literature review as a research methodology. *Journal of Business Research, 104*, 333–339.

Sunstein, C. (2017). *#Republic: Divided democracy in the age of social media*. Princeton University Press.

Williamson, B. (2018). *Big data in education: The digital future of learning, policy and practice*. SAGE.

Wrathall, M. A. (2010). *Heidegger and unconcealment: Truth, language, and history*. Cambridge University Press.

TSG6-1-8

EXPLORING THE LONG-TERM LEGACY OF COVID DISRUPTION TO MATHEMATICS TEACHING IN AUSTRALIAN PRIMARY SCHOOLS

Catherine Attard
Western Sydney University, Australia, c.attard@westernsydney.edu.au

Kathryn Holmes
Western Sydney University, Australia, k.holmes@westernsydneyedu.au

Prior to the COVID-19 pandemic, the use of digital technologies in mathematics education was inconsistent. The lockdowns that occurred during 2020 and 2021 forced teachers to shift from face-to-face to online teaching with little preparation or professional development. In this paper we report on a longitudinal study exploring the impacts of the pandemic on primary school classroom teachers and their mathematics teaching practices. Data drawn from eight teachers interviewed in 2020 and in 2024 highlight inconsistencies that remain in relation to digital technology use in Australian primary mathematics classrooms. The data also sheds light on some of the legacies of the pandemic in relation to increased collaboration and communication, and changes in mathematics pedagogy that have been retained.

Keywords: primary mathematics, digital resources, COVID-19, pedagogy

Mathematics Education Disrupted

It is a long-held belief that digital technology can and should be regarded as a disruptive pedagogy (Hedberg, 2011). However, prior to the onset of the COVID19 pandemic, its use in mathematics classrooms was inconsistent in quality, quantity, and effectiveness (OECD, 2016), and according to Drijvers and Sinclair (2024) the integration of digital technologies into classroom teaching and learning remains inconsistent. While the pandemic caused a major disruption to all aspects of education (Courtney et al., 2022), it remains unclear if the dramatic forced shift from face-to-face to online teaching and learning of mathematics has had any long-term impacts in mathematics classrooms. In this paper we report on data collected from a longitudinal study seeking to explore the impacts and influence of the pandemic on classroom teachers and their mathematics teaching practices in Australia. We draw on data from eight primary school teachers who were interviewed in 2020 following the first set of COVID-19 lockdowns, and again in 2024.

A Framework for Technology use in Mathematics Classrooms

The Technology Integration Pyramid (Mathematics) (TIP(M)) (Attard & Holmes, 2020) is a framework that emerged from existing frameworks (e.g. TPACK, SAMR) and the findings of a study investigating the technology-infused practices of 10 teachers of mathematics from preschool to senior secondary settings across Australia. While previously existing models are useful, limitations were noted and addressed within the TIP(M) framework. TIP(M) is conceptualised as a three-dimensional model (a square pyramid) to illustrate the connections and inter-related elements for con-

sideration when planning for the use of any technology, regardless of device, software, access and school context. Of particular relevance to this paper, is the base of the TIP(M) that acknowledges the complexities and challenges from within and beyond a school that influence how teachers and students use digital technologies in mathematics classrooms. The model identifies four interrelated influences (the triangular faces of the pyramid): community, commitment, culture, and context. These influences, along with the four classroom considerations: mathematics, engagement, pedagogy, and tools, are used in this paper to explore the long-term legacy of the disruptions caused by the pandemic.

Methods

A mixed methods approach was utilised in the larger study from which this paper is drawn, to explore the short- and long-term impacts of the COVID-19 pandemic on technology-related classroom mathematics practices. The study involved a survey, and in-depth interviews conducted with primary and secondary teachers across Australia in late 2020. Interview participants were invited to participate again in 2024. For this paper, we draw on interview data collected from eight primary teachers.

Participants

Survey respondents were asked to indicate their willingness to participate in an interview and a follow-up study was conducted in 2024. Twenty-three teachers participated in the 2020 interview, and 12 agreed to take part in 2024. Participants' experience teaching ranged from nine to 39 years, six taught in government schools, and two in catholic systemic schools.

Data Collection and Analysis

Each participant undertook two interviews conducted via the *Zoom* platform. Interview questions were based on the four elements on the base of the TIP(M), and the four considerations for teaching as described above. In the second interview participants were asked the same questions and were reminded of their responses from 2020 and prompted to discuss any long-term impacts. Each interview took one hour and was recorded and transcribed verbatim. Data analysis was conducted in alignment with the structure of the TIP(M).

Findings

The participants worked in diverse school settings including a remote school with 14 students (Renee) and a large metropolitan school (up to 1600 students) with a high proportion of students from non-English speaking backgrounds (Lisa). Four of the teachers (Caroline, Renee, Mandy, Frank) had moved schools/roles in between the 2020 and 2024 interviews. The school *contexts* also varied considerably with respect to the availability of digital devices and supporting infrastructure.

A school *culture* that supported and encouraged teacher collaboration was found to be critical for effective teaching of mathematics with technology. Teacher collaboration took many forms including the sharing of resources and teaching strategies via *Google Classroom*, the production of digital resources such as videos and programming. As Harry stated: "We basically collaborate on every single thing that we do. So we share programs, we share lessons, we share assessments, we discuss results and direction that we're going in." (2024)

There was only one school where collaboration was not integral to teacher practice beyond compliance with the same high level program documents. Renee explained: "It's not planned. It's not valued by the principal" (Renee, 2020). When interviewed in 2024, Renee reported that the collaboration at her school had not improved.

By necessity, during the pandemic schools had to devise new ways of communicating with the parent *community* as they were more reliant on them as partners for ensuring that the students were learning. For many schools this need prompted the introduction of new digital tools such as *Google Classroom* or the *See-Saw* app. As a result of their introduction during the pandemic, these tools are now commonplace in schools, and beyond having the capacity to facilitate parent communication, they also play a key role in supporting teacher collaboration and sharing of practice.

The teachers reported varying levels of *commitment* from school leadership to support the use of new digital technologies. For some schools, in the years following the pandemic, the need to engage with system-level initiatives has reduced professional learning (PL) opportunities related to technology, as opposed to the necessity for this type of PL during the pandemic. It was common for teachers to be teaching each other rather than seeking external PL opportunities.

Classroom Considerations (Mathematics, Pedagogy, Tools and Engagement)

The teachers generally expressed the view that they were confident with understanding the *mathematics* content they were teaching, but were not always confident that they could appropriately teach this content to learners of different levels. During the pandemic, the teachers had to make decisions to reduce the content to be covered, as it wasn't always possible to teach at the same pace as they would in a face-to-face classroom. In some cases this resulted in a greater focus on foundational concepts rather than attempting to cover the whole syllabus.

In 2020 all teachers indicated significant *pedagogical* changes were necessary to facilitate learning. While technology enabled many of these changes, the teachers also developed new non-technology-based approaches to maximise learning at home, and many of these changes were retained. Relying on digital tools such as shared *Google Docs*, forced the teachers to be more structured in terms of the pedagogical decisions they were making. More purposeful decisions were made when designing tasks so that they were accessible for all students. The digital tools also provided automated feedback to teachers and students about their knowledge, allowing the teachers to target their pedagogical decisions in response to student progress.

While the digital *tools* were essential during the pandemic, post-COVID many teachers were more critically evaluating their use back in the classroom. In 2024, the teachers are more tech-savvy and integrate technology seamlessly into their work, both when planning and teaching. Students are regularly documenting their work using photos attached to assigned activities, allowing teachers to have real-time insights into their thinking.

All teachers agreed that their students were generally more *engaged* in learning when using technology, however, they were cautious about always using technology and felt that they needed to supplement its use with "hands-on, non-tech" activities. The heavy reliance on technology during the pandemic did, over time, result in a drop in student engagement, requiring further pedagogical changes to be made.

Discussion

For the primary school teachers in this study, there is no doubt that the pandemic provided a sudden impetus for change in their teaching practices. New technological tools became essential for communicating with students and parents and changed the way students engaged with mathematical content. The increase in available technologies required teachers to become more critical in making decisions about its use, and in the long-term, to view its benefits and challenges more holistically, often reverting to non-technology-based pedagogies to improve student engagement and learning. The step-change in technology use also facilitated new ways for teachers to share their practice and become more collaborative when planning to teach. Learning management systems are now commonplace in primary schools and provide the basis for more structured approaches to teaching that can be developed and shared collaboratively. These systems have also improved communication with the parent community.

Although this study is limited and the findings are not able to be generalised more broadly, it does highlight that for some, the COVID-19 pandemic has left a positive legacy in terms of technology-infused mathematics pedagogy, teacher collaboration, and communication within and beyond the school. However, it is clear from this small study that there are inconsistencies in the use of digital resources that should be addressed at system and school leadership levels. The study also highlighted the usefulness of the TIP(M) as a tool to identify the complex relationships between system and school contexts and their influence on teachers' decision making in mathematics classrooms.

References

Attard, C., & Holmes, K. (2020). *Technology-enabled Mathematics Education: Optimising Student Engagement*. Routledge.

Courtney, S. A., Miller, M. E. S., & Gisondo, M. J. (2022). The Impact of COVID-19 on Teachers' Integration of Digital Technology. *Contemporary Educational Technology*, *14*(4), ep387.

Drijvers, P., & Sinclair, N. (2024). The role of digital technologies in mathematics education: Purposes and perspectives. *ZDM – Mathematics Education*, *56*(2), 239–248.

Hedberg, J. G. (2011). Towards a disruptive pedagogy: Changing classroom practice with technologies and digital content. *Educational Media International*, *48*(1), 1–16.

OECD. (2016). *Students, Computers and Learning: Making the Connection, PISA*.

TSG6-1-9

KOREAN TEACHERS' PERCEPTION ON AI-EMPOWERED DIGITAL TEXTBOOKS IN MATHEMATICS

Jukyung Park
Seoul Oryun Elementary School, Korea, bluelihd@gmail.com

Hoyun Cho
Capital University, U.S.A., hcho1416@capital.edu

Jung Y. Colen
Bellarmine University, U.S.A., jcolen@bellarmine.edu

Nayoung Kwon
Inha University, Korea, rykwon@inha.ac.kr

Seunghyun Yeo
Daegu National University of Education, Korea, shyeo@dnue.ac.kr

Jinho Kim
Daegu National University of Education, Korea, jk478kim@dnue.ac.kr

Byungeun Pak
C, U.S.A., b.pak@utahtech.edu

This study examines the role of digital and AI-empowered mathematics textbooks in modern education, focusing on the influence of teachers' background characteristics. As digital transformation and artificial intelligence reshape educational methodologies, understanding how educators' school levels, experiences, and prior exposure to resources such as e-books impact their perceptions and utilization of these tools is crucial. Through a comprehensive analysis, this research identifies key factors affecting the adoption and effectiveness of AI-empowered mathematics education. The results indicate that teachers' familiarity with digital tools plays a significant role in their willingness to integrate AI-based resources into their teaching practices. Furthermore, this study highlights the need for targeted professional development programs to bridge the gap in digital literacy and enhance the effective implementation of AI-empowered learning environments. The findings provide insights into optimizing AI-empowered learning environments and offer practical implications for curriculum development and teacher training programs.

Keywords: digital textbook, AI digital textbooks, school level, teachers' career experience, prior usage experience

Introduction

The rapid advancement of digital transformation and artificial intelligence technology has fundamentally reshaped various sectors of society, with education being no exception (Holmes et al.,

2019). The integration of AI technology into educational practices has become a critical necessity, especially in mathematics education, where personalized learning approaches are crucial for meeting diverse student needs (Sharma, 2024). Mathematics, as a subject, poses unique challenges in accommodating individual learning paces and comprehension levels, making it an ideal candidate for AI-enhanced educational tools (Holmes et al., 2022). With the mandatory adoption of AI Digital Textbooks (AIDT) for the upcoming academic year in South Korea, this study provides a comprehensive analysis of teachers' perceptions, with a particular focus on the challenges educators face in their implementation. By examining current digital textbooks usage patterns and attitudes toward AIDT, this research aims to provide empirical evidence to inform the implementation strategies for next-generation mathematics textbooks. This study explores the following research questions:

What factors influence teachers' perceptions and adoption intentions of AIDT in K-12 education settings?

- Does the intention to use AIDT significantly vary according to school level, teachers' career experience, and prior usage experience?
- How do school level, teachers' career experience, and prior usage experience influence their perception of AIDT's effectiveness?

Theoretical Underpinning

Digital textbooks are electronically formatted instructional materials that integrate diverse multimedia elements—such as text, images, video, animations, and interactive simulations—to enhance the visualization and understanding of abstract mathematical concepts (Pepin et al., 2017). The emergence of digital textbooks is driven by rapid advancements in information and communication technologies, increased internet accessibility, and the proliferation of mobile devices, which together have fostered a shift from static printed materials to dynamic, engaging learning environments. Building on this foundation, AIDT incorporates advanced artificial intelligence techniques, including machine learning, natural language processing, and data analytics, to provide personalized learning experiences that adapt content and feedback to individual learners' needs, thereby addressing the limitations of traditional educational resources and promoting a more efficient, customized, and equitable instructional framework in mathematics education (Kim et al., 2024). Kim and her colleagues (2024) surveyed 132 elementary school Korean teachers regarding the implementation of math AIDT and found that while a majority exhibited a low perception of the necessity for these textbooks, a subset recognized their potential for personalized learning and enhanced teaching support.

Methodology

This research conducted a survey to explore teachers' perceptions of mathematics digital textbooks and the upcoming implementation of AIDT march 2025 in Korea. The survey was administered over six weeks, from December 13, 2024, to January 27, 2025. This timeframe was chosen as it aligned with the initial exposure of teachers to AIDT in mathematics who had successfully passed the textbook evaluation process, thereby providing a relevant context for assessing teachers' insights and attitudes.

The survey questionnaire consists of 11 background questions regarding the teachers' school level, experience, and prior usage experience with digital textbooks, as well as 13 questions assessing

their overall thoughts on the utilization of digital textbooks and AIDT. The specific survey questions for the latter are shown in the table below

Table 1. Content of survey questions

Category		Description
Digital math text -books (e.g., e-book)	Need	Is there a need for digital textbooks in math, such as e-books?
	Utilization, Ease of use, Difficulties (experienced)	How much do you use digital math textbooks, such as e-books, in your classroom?
		Do you feel convenience/difficulties using digital math textbooks in your classroom?
	Supporting students' learning	Do you think using digital math textbooks in your classroom helps students learn, and why?
	Supporting teachers' instruction	Do you think digital math textbooks support teachers' instruction, and why?
Math AIDT	Expectations for Utilization	If math AIDT is introduced, to what extent will you use it in your lessons?
	Supporting students' learning	Do you think using math AIDT in your classroom will benefit students' learn, and why?
	Supporting teachers' instruction	Do you think using math AIDT in your classroom will benefit teachers' instruction,, and why?

The survey was sent to K-12 math teachers across the country, with a total of 204 respondents. The distribution of teachers surveyed was 133 (65.2%) from elementary schools, 33 (16.2%) from middle schools, and 38 (18.6%) from high schools. According to their teaching experience, 26 (12.7%) had 0-5 years of experience, 35 (17.2%) had 5-10 years of experience, 43 (21.1%) had 10-15 years of experience, 45 (22.1%) had 15-20 years of experience, 44 (21.6%) had 20-25 years of experience, 8 (3.9%) had 25-30 years of experience, and 3 (1.5%) had more than 30 years of experience. There were 98 (48.0%) teachers with experience using digital math textbooks and 106 (52.0%) teachers without experience.

Findings

First, at the school level, the perceived need for mathematics digital textbooks in e-book format and the extent to which they would be helpful to students were highest in middle schools, and the extent to which they would be helpful to teachers was highest in elementary schools. Expectations for classroom use of math AIDTs and the extent to which they would support students and teachers were highest in elementary schools.

Second, in terms of years of teaching experience, perceptions of the need for digital mathematics textbooks in e-book format were highest among teachers with 5-10 years of experience and lowest among teachers with less than 5 years of experience. Perceptions of the extent to which digital mathematics textbooks support student learning and teacher instruction were positive among teachers with 5-10 years of experience and 15-20 years of experience. Expectations for the use of math AIDTs in the classroom and the extent to which they support student learning were higher among teachers with 15-20 years of experience, and expectations for instructional support were highest among teachers with less than 5 years of experience.

Third, perceptions of the need for digital mathematics textbooks and the extent to which they support student and teacher learning were higher among teachers with prior usage experience compared to teachers without experience. Teachers with prior usage experience with mathematics digital textbooks reported higher usability and ease of use and fewer difficulties in using them. Teachers with prior usage experience also had higher expectations of math AIDTs than teachers without prior usage experience. Teachers who had never used digital math textbooks in their classrooms had lower perceptions of the usefulness of math AIDTs and their support for student learning than the average of teachers who had.

Discussion

The findings of this study offer insights into teachers' perceptions of AIDT in mathematics education in South Korea, which is facing its mandatory adoption of AIDT in the upcoming academic year. The analysis reveals nuanced variations in AIDT's perceived necessity, effectiveness, and implementation feasibility across different school levels and teaching experiences.

The results indicate that middle school teachers exhibit the highest perceived need for digital textbooks, while elementary school teachers have the most optimistic expectations for AIDT in supporting student learning and instructional practices. This disparity suggests that middle school teachers recognize the challenges associated with traditional instructional methods and view digital textbooks as necessary tools to enhance engagement and comprehension. Conversely, elementary school teachers' higher expectations for AIDT may stem from the adaptability of AI-empowered resources in addressing diverse learning needs at early developmental stages. However, high school teachers showed comparatively lower enthusiasm.

When examining based on their experiences, teachers with 5–10 years of experience demonstrated the highest perceived need for digital textbooks. This suggests that teachers in this career stage are more open to technological innovations than their less experienced or veteran counterparts. This group also exhibited strong support for AIDT in facilitating student learning and instructional practices. In contrast, teachers with over 30 years of experience reported the lowest levels of interest in AIDT, reflecting potential resistance to adopting new technologies or a preference for traditional teaching methods.

We also found that teachers' previous experience with digital math textbooks strongly influenced their perceptions of their utilization, ease of use, and overall effectiveness: teachers with previous experience using these resources tended to find them more conducive to student learning and teaching, while those without experience were less confident in their usefulness. This suggests that familiarity with digital textbooks plays an important role in shaping attitudes toward their adoption and integration into the classroom.

These findings underscore the importance of targeted professional development and training programs to address concerns and build digital competency across all grade and experience levels. By addressing teachers' concerns and promoting strategic professional development initiatives, stakeholders can enhance the adoption and efficacy of AIDT, ultimately fostering more personalized and effective mathematical learning environments for students.

References

Holmes, W., Bialik, M., & Fadel, C. (2019). *Artificial intelligence in education: Promises and implications for teaching and learning*. Center for Curriculum Redesign.

Holmes, W., Bialik, M., & Fadel, C. (2022). Artificial intelligence in mathematics education: The good, the bad, and the ugly. *International Journal of Educational Research*.

Sharma, P. (2024). Revolutionizing Math Education: The Power of Personalized Learning. *International Journal for Multidisciplinary Research*.

Kim, S., Lee, G., & Kim, H. J. (2024). Introduction of AI digital textbooks in mathematics: Elementary school teachers' perceptions, needs, and challenges. *Education of Primary School Mathematics*, *27*(3), 199-226.

Pepin, B., Choppin, J., Ruthven, K., & Sinclair, N. (2017). Digital curriculum resources in mathematics education: foundations for change. *ZDM*, *49*, 645-661.

TSG6-1-10

HARNESSING AI IN SINGAPORE PRIMARY MATHEMATICS CLASSROOM - REDEFINING THE ROLE OF A TEACHER

Suo Hui Chang
Academy of Singapore Teachers, Singapore, chang_suo_hui@moe.gov.sg

Guat Bee Loh
Academy of Singapore Teachers, Singapore, loh_guat_bee@moe.gov.sg

Ming Kwang Choon
Academy of Singapore Teachers, Singapore, choon_ming_kwang@moe.gov.sg

Wan Teh
Academy of Singapore Teachers, Singapore, teh_wan@moe.gov.sg

Two Artificial Intelligent (AI)-enabled features, Adaptive Learning System (ALS) and Feedback Assistant Mathematics (FA-Math) were rolled out recently within the Singapore Student Learning Space (SLS) to revolutionise primary mathematics education with the promise of more personalised learning. In supporting teachers' adoption of these AI features in the classroom, it became evident that the utilization of AI goes beyond simply a plug-and-play requiring no adjustments or proactivity of teachers or students. This paper seeks to illustrate through two cases, one on the use of ALS in a Primary 5 classroom and another on the use of FA-Math in a Primary 3 classroom, how teaching and learning with AI looks like. The illustrations focus on the interactions of teachers, students and the AI-features to illuminate the need for teachers to redefine and make shifts in current practices to optimize student learning in the classroom integrated with AI.

Keywords: Artificial Intelligent, Primary Mathematics, Teacher

Introduction

Singapore mathematics classroom comprised, on average, 30 to 40 diverse students with different learning needs and learning progression. Traditional teaching approaches that often "pitch to the middle" with "one-size-fits-all" learning experience can barely provide equal access to learning for everyone. However, attempts by teachers to differentiate instructions are often met with challenges particularly the large class size that impedes the collection of assessment information to feed instruction forward (Heng, 2023). Two newly launched features in the Singapore SLS, ALS and FA-Math, powered by artificial intelligence have the potential to support teachers in differentiating instruction in a large class. ALS affords self-directed learning of students by providing each a personalised learning pathway to learn at their own pace, recommending learning resources and practice questions and offering immediate feedback based on student readiness, strengths and needs (Singapore Student Learning Space, 2025). There are four learning modes in ALS: Guided Learning, Mastery Check, Revision and Test Myself. FA-Math offers step-by-step feedback to students on their answers (Singapore Student Learning Space, 2025). The line-by-line feedback generated by

FA-Math during the solution process sensitises students to their errors and misconceptions. In this paper, we examine the impact of ALS and FA-math in teachers' practice in Singapore classrooms focusing on the necessary shifts to be made in complementing the AI-features to optimise student learning.

Literature Review

AI "is a technology that builds systems to think and act like humans with the ability to achieve goals" (Akgun & Greenhow, 2021, p433). The use of AI in education broadly includes (i) Intelligent Tutoring System (ITS) where a pedagogical agent serves as a virtual teacher to customise instruction and to provide immediate feedback to students, (ii) Adaptive Learning System (ALS) where lesson-based tasks or game-based tasks are sequenced against fine-grained conceptual or knowledge pieces that builds on one another leading students towards constructing a complete schema of mastered concepts, and (iii) educational robotics with their manipulatable capabilities supports student exploration and creativity.

Proponents of AI in education highlight the power of AI in evaluating student learning by interpreting patterns of collected information, making autonomous adjustments and providing personlised feedback and instruction customised to student pace to maximise outcomes. Empowered by machine learning capabilities, AI can "learn by itself" from large data sets through mining and analytics, making it more responsive to the needs of diverse students (Mohamed et al, 2022; Hwang, 2022; Voskoglou & Salem, 2020). In addition, AI helps students find answer faster and easier without the presence of a teacher and supports their development of mathematical and cognitive skills (Chan, 2024). Yet, Cope et al. (2020) indicate that AI will never replace the teacher. While AI supports teacher in time management as they take over part of teachers' workload for grading, record keeping, teachers can turn their attention to other things such as focusing on the more human side of teacher-student relations (Felix, 2021) and checking student learning progress and devise teaching strategies to better support student learning (Chan, 2024; Hwang, 2022). Tan et al. (2024) in their study on FA-Math use in Singapore classroom highlighted the importance of "teacher-in-the-loop" where the teacher remains actively monitoring student learning and providing support whenever needed.

The capabilities and affordances of AI in education offers much potential and benefits in the future of teaching and learning. To navigate amidst the change, it is necessary to investigate and redefine the current practices particularly teachers' role in the proper use of AI in and out of the classroom. The next part of this paper aims to explicate the shifts in teachers' role observed from the use of ALS and FA-Math in SLS through two separate cases.

The case of ALS in a Primary 5 Classroom

Teacher A is a Master Teacher[1)] with more than 20 years who is prototyping the use of ALS as part of the exploration of new pedagogies using AI. He began the lesson to a mixed-gender class of 40 students, aged 11 years old, on Primary 5 concept of expressing a part of a whole as a percentage. He posed a problem on comparing the number of people who are obese in two different countries and engaged the students in a class discussion to find out which of the two countries had a more

1) Master Teachers are national Teacher Leaders who provides pedagogical leadership and mentorship to foster a teacher-led culture of professional excellence in the teaching fraternity.

serious issue with obesity. During the discussion, he taught the concept of percentage as a part of a whole highlighting the importance of interpreting the provided data based on its respective whole. After ascertaining the level of acquisition of all students, he found that 47% of the students achieved desirable level of acquisition. He then instructed them to log in to SLS to select the Guided Learning mode in ALS for independent learning to reinforce the learned concept. In this mode, students' learning was supported by instructional videos, scaffolds such as hints for solving questions, and feedback, including step-by-step explanations and model drawings, allowing them to learn from their errors (see Figure 1).

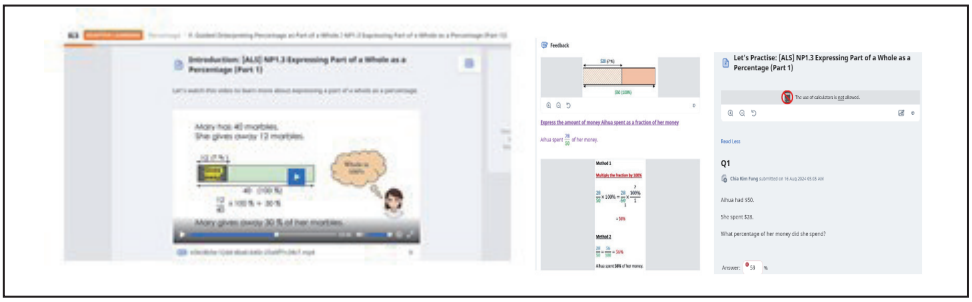


Figure 1: ALS guided learning mode

Teacher A continued to support the remaining 53% of the students in the class who required direct teacher attention in learning the concept. While doing so, he periodically checked the ALS learning progression dashboard (see Figure 2) to monitor the learning of the 43% of students assigned to learn with the AI feature. One student was identified by ALS as "needing attention". After a few minutes, no student appeared under "needing attention". That student managed to utilise the hints and teaching videos to correct the mistakes and progressed on to answer the remaining questions correctly.

The AI-featured ALS, served as a capacity multiplier to support the teacher in engaging a group of students who progressed faster within the class to reinforce their learning after the whole-class introduction of the new concept. This enabled the teacher to attend to those in need of direct teacher intervention to learn the required concept. The affordance of a learning progress dashboard in ALS supported teacher in monitoring the independent learning of the more readied group. The consolidated data and analysis streamlined teacher's work thus achieving greater efficiency and effectiveness as the teacher sought to differentiate instruction according to students' learning pace.

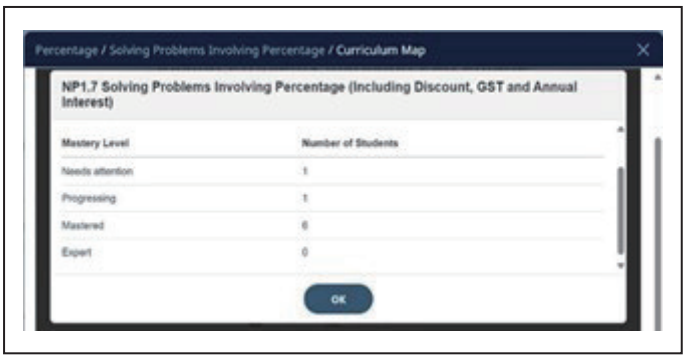


Figure 2: Learning progress dashboard in ALS

The case of FA-Math in a Primary 3 Classroom

Teacher B, an experience teacher of more than 15 years, assigned 30 students, aged 9 years old in a mixed-gender class, to complete a lesson package for Primary 3 Addition of like and unlike Fractions that harnessed FA-Math. It was the first time Teacher B conducted a lesson with FA-Math. Prior to this, she underwent briefings focusing on some pedagogical considerations and a hands-on session with FA-Math with the team of authors of the paper. Refer to Figure 3 for the screenshot of one student's interface that included the interactions with FA-Math and her experience.

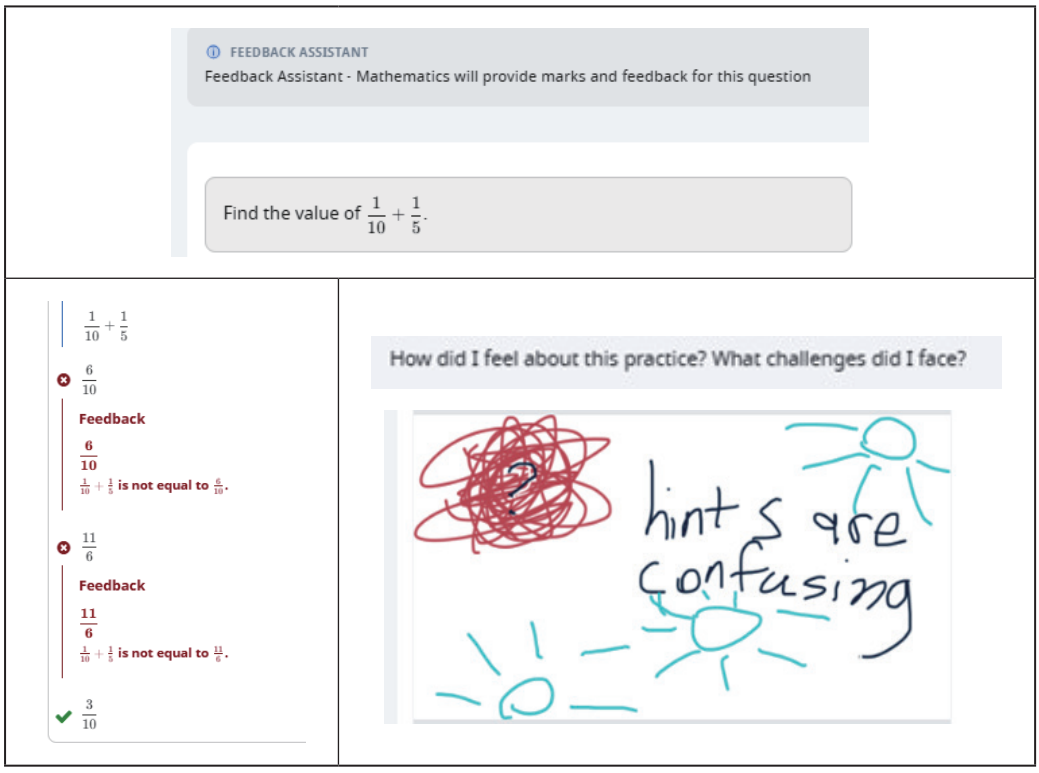


Figure 3: FA-Math lesson package for P3 addition of like and unlike Fractions

This student was observed unable to solve the question despite the provision of the line-by-line feedback from FA-Math which she found confusing. Her case was picked up by the teacher who then guided her in-person to solve the question. This showed that the AI-powered FA-Math requires students to be equipped with digital literacy to understand the line-by-line hints and feedback pushed out to feed learning forward. Teachers are to factor in instructions to familiarise students with the technological skills needed to navigate the user interface and solution input of the system. Both digital literacy and technological skills of the students thus become a necessary instructional consideration for teachers when designing an AI-powered learning environment. The active monitoring of the teacher remains important to provide targeted support to students who are unable to move their learning forward with technology. Rather than the usual walk-around in the class, monitoring can be achieved in the digital space via the device and incorporated dashboards and systems.

Discussion

From the two cases illustrated above, the AI-powered ALS and FA-Math supported teachers in differentiating instruction. Students who are able to progress independently with AI continued to be engaged and learn. The teacher is freed-up to attend to students who need greater support. Teacher can also provide more targeted support to students by studying their interactions with AI and analyse their struggles. Teachers have traditionally been the sole knowledge dispenser, orchestrator of knowledge construction and facilitator of discourse in the classroom of diverse learners. AI-powered ALS and FA-Math abdicated the role of teacher as a better candidate for knowledge dispenser with the value proposition of customising to the immediate needs of individual student. Therefore to achieve a complementary role between human and machine, teachers are to redefine the teaching role to focus on equipping students in the area of digital literacy, technological skills, feedback literacy for self-directed and independent learning with AI. With the affordance of AI as a capacity multiplier, teachers can now provide targeted support to students with specific needs and expand the role of facilitating and orchestrating learning opportunities for students to think critically, adaptively and creatively; a role that AI cannot achieve as they are created and programmed by humans and thus could not supersede the quality of human reasoning (Voskoglou & Salem, 2020). Most importantly, learning is a social activity. The physical presence of a teacher exuding enthusiasm, disposition and empathy establishes a profound affiliation and trust that in turn, encourages and motivates students to learn (Felix, 2020).

Conclusion

This exploratory study set in an authentic primary math classroom setting, surfaced some necessary shifts in teacher's role, with the use of AI. Some shifts highlighted includes monitoring via AI data analytics rather than via whole-class walk around, a shared teacher role with AI where the physical presence of the teacher can be placed more strategically to support students with higher needs. Future direction can include more case studies involving teachers with different technological and pedagogical experience to provide more insights into the different shifts needed. This serves to inform professional learning directions and foci for both pre-service and in-service teachers.

References

Akgun, S., & Greenhow, C. (2021). Artificial intelligence in education: Addressing ethical challenges in K-12 settings. *AI Ethics*, 2(3), p431-440. https://doi.org/10.1007/s43681-021-00096-7

Chan, G. (2024, July 29). askST: What are MOE's newest artificial intelligence tools, and how are schools using them? *The Straits Times*. https://www.straitstimes.com/singapore/askst-what-are-moe-s-newest-artificial-intelligence-tools-and-how-are-schools-using-them

Cope, B., Kalantzis, M., & Searsmith, D. (2020). Artificial intelligence for education: Knowledge and its assessment in AI-enabled learning ecologies. *Educational Philosophy and Theory*, 53(12), p1229-1245. https://doi.org/10.1080/00131857.2020.1728732

Felix, C. V. (2020). The role of the teacher and AI in education. In *Innovations in higher education teaching and learning* (pp. 33–48). https://doi.org/10.1108/s2055-364120200000033003

Heng, T. T. (2023). Lessons on educational borrowing and change: teachers' implementation of differentiated instruction in Singapore, *Pedagogy, Culture & Society*, https://doi.org/10.1080/14681366.2023.2166094

Hwang, S. (2022). Examining the effects of artificial intelligence on elementary students' mathematics achievement: A meta-analysis. *Sustainability*, *14*(20), 13185.

Mohamed, M. Z. b., Hidayat, R., Suhaizi, N. N. b., Sabri, N. b. M., Mahmud, M. K. H. b., & Baharuddin, S. N. b. (2022). Artificial Intelligence in mathematics education: A systematic literature review. *International Electronic Journal of Mathematics Education*, 17(3), em0694. https://doi.org/10.29333/iejme/12132

Singapore Student Learning Space (2025). About AI in SLS. https://www.learning.moe.edu.sg/ai-in-sls/about-ai-in-sls/

Tan, L. S., Lee, M. L. S., Loh, G. B., & Chang, S. H., (2024, July 7-14) *Artificial intelligence-powered feedback assistant for mathematics: Making metacognition accessible to all students* [Extended paper]. The 15th International Congress on Mathematical Education, Sydney, Australia.

Voskoglou, M. G., & Salem, A. M. (2020). Benefits and limitations of the artificial with respect to the traditional learning of mathematics. *Mathematics*, *8*(4), 611. https://doi.org/10.3390/math8040611

TSG6-1-11

ANALYSIS OF THE STRUCTURING FEATURES OF CLASSROOM PRACTICE IN A TRIGONOMETRIC RATIOS LESSON FOR ACUTE ANGLES USING GENERATIVE AI

Jaeeun, Joo
Mudung Middle School, South Korea, mumaroma@naver.com

Bomi, Shin
Chonnam National University, South Korea, bomi0210@chonnam.ac.kr

This study explores instructional strategies for integrating generative AI into mathematics lessons to establish a learner-centered learning environment. Specifically, a self-study analysis was conducted on the teaching of trigonometric ratios for acute angles, based on the curriculum script component of Ruthven's (2009, 2014) framework of structuring features of classroom practice (SFCP). The findings indicate that the teacher modified and adjusted the curriculum script in response to unexpected student reactions. Additionally, while the teacher provided opportunities for reflective reasoning by evaluating and correcting AI-generated errors, difficulties in clearly identifying the causes of these errors were also observed.

Key words: generative AI, structuring features of classroom practice, curriculum script, self-study, trigonometric ratios

Introduction

The advancement of digital tools is transforming pedagogical approaches in mathematics education, and the recent emergence of generative AI, such as ChatGPT, is accelerating the shift toward learner-centered instructional models (Wardat et al., 2023). In particular, generative AI supports students in exploring mathematical concepts and developing problem-solving strategies, and its potential applications in mathematics education are gaining increasing attention (Kim et al., 2023). Accordingly, this study aims to analyze the alternative pathways and challenges that emerge in the teaching and learning process when generative AI is integrated into classroom lessons. Based on this analysis, the study seeks to propose practical strategies for implementing generative AI-based instruction. To this end, generative AI-based instructional materials on trigonometric ratios for acute angles were implemented, and the curriculum script component of Ruthven's (2009, 2014) framework of Structuring Features of Classroom Practice (SFCP) was employed to examine the structural features of classroom enactment.

Theoretical Background

Ruthven's (2009, 2014) SFCP framework is an analytical tool for examining classroom implementation involving technology. It consists of five key components: working environment, resource system, activity structure, curriculum script, and time economy. In this study, due to space limitations, the focus is primarily on the curriculum script.

The curriculum script outlines the instructional plan for teaching curriculum content, including goals, resources, actions, expectations, challenges, and alternative pathways. When integrating new technologies, teachers need to revise and adjust this script to guide instructional design and implementation. This includes selecting tasks for technology-enhanced lessons, anticipating and managing student response patterns, and recognizing how technology may support or hinder learning objectives in order to respond appropriately during the lesson.

Research Methodology

This study applied a **self-study methodology** to analyze the implementation of a generative AI-based trigonometric ratio lesson using self-study of teaching practice analysis (Park & Kim, 2017). Accordingly, the research focused on the teacher conducting the lesson and the lesson itself. While the **curriculum script** component of the **SFCP framework** was used to analyze the lesson, students' activities were indirectly included in the process.

The teacher who conducted the lesson is one of the researchers of this study, a mathematics teacher who has primarily worked in middle schools. This teacher has regularly implemented technology-based lessons and has conducted various instructional experiments using generative AI tools. The lesson was carried out in a third-year class (25 students) at Y Middle School, located in an urban area. The students had previously learned the definitions of trigonometric ratios over two prior lessons and were now encountering trigonometric ratios for acute angles for the first time. The generative AI tool used was AskUp (October 2023 version), which is based on the ChatGPT API. Students accessed the AI easily using their personal smartphones in the classroom. Throughout the study, lesson recordings and transcripts, student activity materials, and a self-reflection journal were collected and analyzed.

Results

Teacher's Response to Student Prompting and Response Patterns in Generative AI Tasks

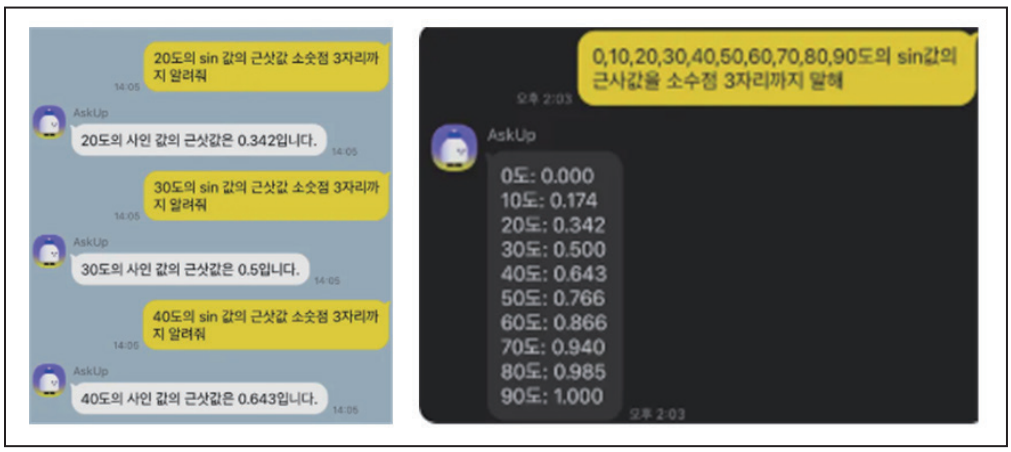


Figure 1. Examples of student-entered prompts for generating trigonometric ratio values

Students were assigned a task to input prompts into generative AI to observe changes in the approximate values of trigonometric ratios for angles ranging from 0° to 90°. Each group formulated prompts, entered them into the AI, recorded the generated values, and analyzed the observed patterns in the trigonometric ratio changes. The goal was for students to inductively infer the properties of trigonometric ratios for acute angles.The teacher initially expected students to input angles one by one to obtain corresponding values, as shown on the left side of Figure 1. However, student 11 structured their prompts differently, as illustrated on the right side of Figure 1, by requesting multiple values at once with a prompt such as, "Provide the approximate sine values for 0°, 10°, …, 90° rounded to three decimal places."

When this type of prompt appeared, the teacher shared it with the whole class and commented, "This is also a good way to enter the prompt," providing both rephrasing feedback and positive reinforcement. Encouraged by this, other students revised their own prompts and began to focus more on analyzing the patterns of change. This process illustrates how the curriculum script was adjusted to an alternative pathway in response to students' actions.

Opportunities for Reflective Reasoning and Challenges in Analyzing Errors in Incorrectly Generated Trigonometric Values

The teacher shared an output error case with student 9, where the tangent value of 80° was displayed as a negative number, as shown in Figure 2. Rather than dismissing this as a limitation of generative AI, the teacher used it as an opportunity to reinforce students' understanding of the geometric definition of trigonometric ratios. By posing questions such as, "As you know, the tangent value is the ratio of lengths. Can it be negative?", the teacher encouraged students to critically reflect on the concept. This demonstrated how the curriculum script was adjusted to anticipate potential difficulties in the teaching and learning process involving generative AI.

However, difficulties were observed in clearly articulating the cause of the AI-generated error. For instance, in the teacher's statement, "So, the AI might have misinterpreted the angle and recognized it as a length, such as 80 cm.", the explanation remained ambigu-
ous. This highlights the challenges in precisely diagnosing AI-generated errors and suggests the need for more structured approaches to support students in critically evaluating AI outputs.

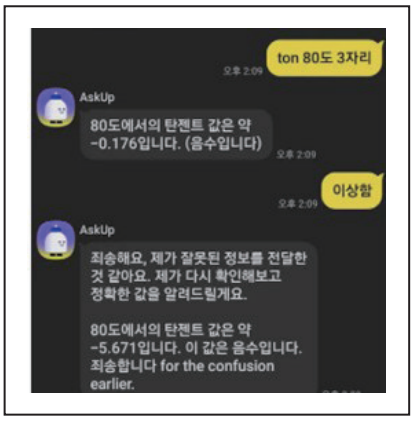


Figure 2. Example of an error in trigonometric ratio output generated by AI (Student 9)

Identifying the Role of Generative AI in Trigonometric Calculations and Comparing It with Trigonometric Tables

The teacher first asked students to locate the value of sin 41° in the trigonometric table provided in the textbook, and then engaged them in the following dialogue:

Teacher: 0.6561 came out, right? … Remember the example I gave earlier, 41.5 degrees? Can you find this one using the table?
Students: No~
Teacher: No, you can't. Then what should you do in this case?
Student 7: AI.
Teacher: Right. You would have to ask AI or use a calculator.

Through this discussion, students recognized the limitations of trigonometric tables and realized that tools such as calculators or AI are necessary for finding values that are not explicitly listed. This discussion process served as an opportunity to explore the educational role of generative AI, shifting the perception of AI from a potential distraction to a useful learning tool. This aligns with Busuttil and Calleja (2025), who emphasize the need to transition generative AI from being seen as an obstacle to being recognized as a valuable educational resource.

Conclusion

This study analyzed the alternative pathways and challenges in the teaching and learning process in AI-integrated lessons. While this study focused on the application of AI in teaching trigonometric ratios, future research should explore how AI can be utilized across various mathematical concepts. Continuous practical research is necessary to ensure that AI fosters critical thinking and creative problem-solving skills in students.

References

Busuttil, L., & Calleja, J. (2025). Teachers' beliefs and practices about the potential of ChatGPT in teaching Mathematics in secondary schools. *Digital Experiences in Mathematics Education,* 1-27.

Kim, J., Kwon, M., & Pang, J. (2023). Elementary School Teachers' Perceptions of Using Artificial Intelligence in Mathematics Education. *Education of Primary School Mathematics, 26*(4), 299–316.

Park, C., & Kim, Y. (2017). A Methodological Inquiry of Self-study. *The Journal of Elementary Education, 30*(4), 75-104.

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. *Éducation et didactique,* (3-1), 131-159.

Ruthven, K. (2014). Frameworks for analysing the expertise that underpins successful integration of digital technologies into everyday teaching practice. *The mathematics teacher in the digital era: An international perspective on technology focused professional development,* 373-393.

Simsek, A., Bretscher, N., Clark-Wilson, A., & Hoyles, C. (2023). Characterising Features of Secondary Teachers' Curriculum Scripts for Geometric Similarity with Dynamic Mathematical Technology. In *The Mathematics Teacher in the Digital Era: International Research on Professional Learning and Practice* (pp. 263-288). Cham: Springer International Publishing.

Wardat, Y., Tashtoush, M. A., AlAli, R., & Jarrah, A. M. (2023). ChatGPT: A revolutionary tool for teaching and learning mathematics. *Eurasia Journal of Mathematics, Science and Technology Education, 19*(7), em2286.

TSG6-1-12

EXPLORING ACTOR-NETWORKS IN MATHEMATICS CLASSES USING GENERATIVE AI

Jungeun Yoon
Seoul National University, Korea, yoonhoho1004@snu.ac.kr

Oh Nam Kwon
Seoul National University, Korea, onkwon@snu.ac.kr

Recently, a new materialist perspective has emerged, expanding the scope of mathematics teaching and learning to include both human and non-human actors through the recognition of technology's agency. Latour's Actor-Network Theory (ANT) offers an alternative framework by assigning technology equal ontological status to human actors in an era of the increasing integration of advanced technologies into educational settings. This study aims to explore the actor-networks in a high school mathematics class that incorporates generative AI, using ANT as a theoretical framework. Mathematics classes incorporating generative AI were examined through the lens of the translation process, with a focus on the formation and transformation of actor-networks and generative AI's dynamic agency. Through this, the contribution of ANT interpreting AI-based mathematics teaching and learning was discussed.

Keywords: generative AI, mathematics class, actor, agency, network, translation, Actor-Network Theory (ANT)

Introduction

Generative AI is gaining attention in education due to its ability to engage in natural conversations with humans on various topics. Teachers can use generative AI to establish teaching and learning plans or receive materials, and students can use generative AI as an assistant teacher to engage in individualized and proactive learning without time and space constraints (Lv, 2023). Generative AI differs from traditional AI in that it facilitates a higher volume of communication between teachers and students with improved quality, making it a valuable partner for collaborative tasks rather than merely a tool used by teachers and students (Yoon & Kwon, 2024).

In mathematics classes using generative AI where human and non-human actors coexist, a theory or perspective that views both humans and non-humans as subjects of agency is required to deeply understand the learning process and interrelationships between actors (Latour, 2005). Actor Network Theory (hereinafter referred to as ANT) represents an effort to move beyond traditional human-centered perspectives by reconceptualizing human actions through the formation and dissolution of socio-material networks (Kim, 2016). While previous studies have explored AI in education, few have examined AI as an active actor in classroom networks. Accordingly, this study analyzed the interactions and networks among human and non-human actors in mathematics classes using generative AI within the framework of ANT. Through this, the formation and transformation of networks among various actors, including teachers, students, generative AI, prompts, classrooms, laptops, and mathematical concepts were explored. The research questions of this study are as follows.

1. How were design and implementation of mathematics classes using generative AI translated?
2. How were actor-networks formed and transformed in mathematics classes using generative AI?

Theoretical Perspective: Actor-Network Theory (ANT)

ANT is a theoretical and methodological framework that emerged in the late 20th century within the field of Science and Technology Studies (STS), which reflects posthumanism and offers a sociological perspective on science and technology. According to ANT, humans address various problems through interactions with other humans, but in doing so, they encounter additional challenges that necessitate the involvement of non-human actors. In ANT, humans and non-humans cannot be separated, and what we call society is a human-non-human complex (collective) (Latour, 1999). Actors are human and non-human elements that have agency and create changes in the world. Here, human actors are not the core or node that forms the center of the network, but the network they are in, which means that the actor is a network in itself (Fenwick & Edwards, 2010). Actor-networks are built, transformed, and stabilized through translation, which is a process of mobilizing and arranging actors to form a network of relationships, and is accompanied by the formation of power and order. The translation consists of four stages: problematization, interessement, enrollment, and mobilization, which occur cyclically rather than linearly and can be interrupted by something or repeated (Callon, 1986). Table 1 outlines these stages in detail.

Table 1. Translation process of network by Callon (1986)

Stage	Description of stage
Problematization	The stage where an actor checks the entire network and then identifies or disrupts the situation
Interessement	The stage where a new actor appears in an existing network and attracts attention.
Enrollment	The stage of assigning a new role to a newly emerged actor
Mobilization	The stage where actors in new roles form a new network to complete the translation

Methods

The participants were 24 students from one class of first-year students at a Korean public high school. They were 12 boys and 12 girls, and their math scores varied. Mathematics classes using generative AI were designed and implemented over a total of 6 sessions from August 2024 to November 2024, and most students faithfully participated in them. The researcher participated in all classes, observing and recording interactions among human and non-human actors within the networks. After classes, individual student interviews were conducted to deeply analyze the network. In addition, interviews with generative AI were also conducted according to heuristic of 'Interviewing Objects' (Adams & Thompson, 2016). Students' activity data, teacher reflection logs, participant observation logs, record of interviews with students or object were collected to conduct continuous cross-analysis from various angles. The researcher circularly analyzed the insider's perspective (emic codes) and outsider's perspective (etic codes) and continuously compared these, and constructed open coding, axial coding, and selective coding (Strauss, 1987). Analysis of data was conducted continuously and cyclically until the end of the study.

In this study, Latour's ANT theory (Latour, 1999; 2005) and translation process (Callon, 1986) was adopted as the theoretical framework for mathematics classes using generative AI for the following three reasons. First, it is adapted to analyze networks of human and non-human actors (e.g., teachers, students, AI) in mathematics class using generative AI. Second, it can capture changes in the interactions and roles of humans and generative AI through the network. Third, the process of knowledge formation of ANT theory is useful in exploring meaning of knowledge and the nature of learning in mathematics teaching and learning in class with technology.

Results

Translation of Actor-Networks in Mathematics Class Using Generative AI

Mathematics class using generative AI has been designed and implemented. The translation process was repeated each time a new activity such as drawing graphs with generative AI, solving mathematics problems with generative AI, and prompt engineering. Although there were various actors in mathematics classes using generative AI, this study mainly explored the agency of the representative actors: Teachers, Students, Generative AI.

First, in <Problematization> stage, preparations were made for overall lesson design and implementation. Teacher Y, a mathematics teacher, established a learning environment and prepared orientation materials, activity sheets of error analysis of mathematics problem and mathematical prompt engineering. Students signed up for each generative AI and online class through Google login, and learned how to use generative AI through orientation. Also, mathematics classes using generative AI were set as an 'obligatory passage point (OPP)' for teachers and students.

Second, in <Interessement> stage, it was necessary to build an attention-getting device or strategy to block the actors and other entities that wanted to define each actor's identity differently. Teachers were interested in the 'practice of digital mathematics education' and introduced AI-based error analysis tasks to encourage students to view AI as a problem-solving. Students recognized generative AI as a tool to help them learn and began taking interests in new activities. Through the teacher's attention strategy, students were able to attend classes using generative AI.

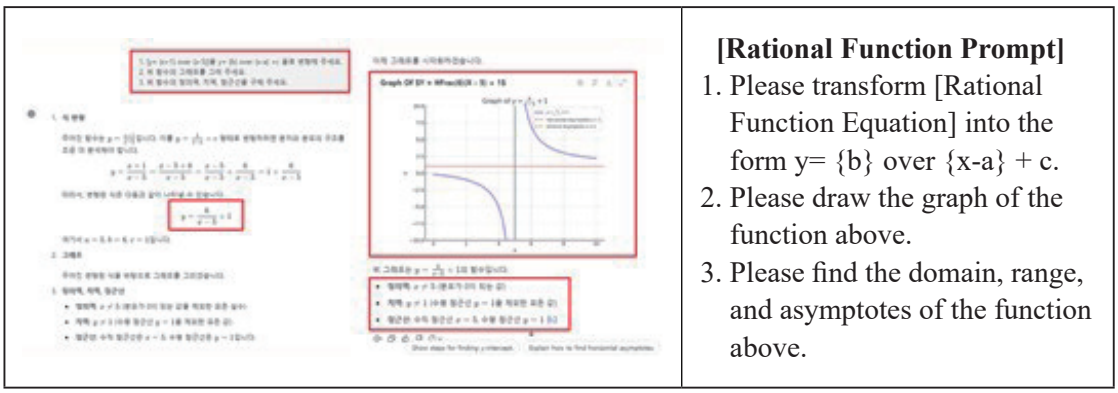
Third, in <Enrollment> stage, the role of each actor was determined. The teacher's role was to successfully complete a mathematics class using generative AI while continuously resolving various conflict situations and obstacles during the class (e.g., unable to log in, unable to use the internet, lack of time for the class, etc.). The students' role was to successfully complete the tasks, such as exploring math concepts using generative AI and analyzing errors in generative AI answers. During group activities, they were given roles within the group (leader, helper, recorder, AI expert) as in Figure 1. The role of generative AI is to be frequently utilized and to sustain its presence among teachers and students in the school education environment.

Figure 1. Students' roles in groups (leader, AI expert, recorder)

Fourth, <Mobilization> is the process in which a representative actor is created and the actor is incorporated into a new network or expanded to other areas. A new actor 'generative AI' secured representation and authority as the center of a stabilized translation in the network alliance and dominates and reproduces actor-network through recording and prescription.

Formation and Transformation of Actor-Networks in Mathematics Classes Using Generative AI

As the classes progressed, the networks between generative AI and students, as well as between generative AI and mathematical learning processes, became more robust. In the early stages, students either stuck to their existing learning methods or used generative AI to check their own learning even though the teacher suggested the use of generative AI. However, students gradually used generative AI to understand mathematics concepts and graphs, and they also conducted in-depth learning by comparing answers of generative AI with their own answers. For example, students actively used generative AI to create step-by-step prompts as shown in Figure 2 to check the graph, domain, range, and asymptotes of rational functions. Generative AI promoted students' individualized and self-directed learning.



[Rational Function Prompt]

1. Please transform [Rational Function Equation] into the form y= {b} over {x-a} + c.
2. Please draw the graph of the function above.
3. Please find the domain, range, and asymptotes of the function above.

Figure 2. Collaborative learning with generative AI

In class, the network from generative AI became gradually strengthened. Initially perceived as an unfamiliar tool, generative AI gradually became central to the classroom as students recognized its value as both an assistant teacher and a learning tool. Through this process, students developed proficiency in utilizing generative AI for concept exploration and problem-solving, fostering a collaborative learning environment with AI. Also, the generative AI-student network was gradually strengthened as the class continues. When the network was temporarily weakened due to the generative AI exceeding its capacity, another generative AI or program entered as a new actor such as ChatGPT, Claude, Gemini, and Perplexity, and a new network was created. Students compared answers from various generative AI tools and also used graphing programs such as Geogebra and Mathway as auxiliary tools. Also, errors in generative AI answers to math problems were critically analyzed and categorized as condition recognition error, expression error, and calculation error. Students were able to learn in-depth by collaborating with generative AI, and these experiences were extended to classes of other subjects or future career.

Conclusion & Discussion

This study explored the networks among human and non-human actors in mathematics classes using generative AI within the framework of ANT. ANT expands existing actors to include a new non-human actors, and explored actor-networks and a new form of "materiality" (Braidotti, 2019) in mathematics classes. Materiality was generated among various actors, including teachers, students, generative AI, prompts, classrooms, laptops, and mathematical concepts for each student and fostered networks and interactions. This study is significant in that it moves away from the perspective of considering learning as an individual's competency and views it as the formation and transformation of networks between multiple actors in AI era. In class, generative AI was not just a tool, but an active agent that increasingly combined with multiple actors to create and generate mathematics learning. Future research could further investigate how digital tools influence the formation of actor-networks in various educational contexts.

References

Adams, C., & Thompson, T. L. (2016). *Researching a posthuman world: Interviews with digital objects.* Springer.

Braidotti, R. (2019). A theoretical framework for the critical posthumanities. *Theory, Culture & Society, 36*(6), 31–61.

Callon, M. (1986). Some elements of a sociology of translation : Domestication of the scallops and the fishermen of St. Brieuc Bay'. In J. Law (ed.). *Power, Action and Belief: A new sociology of knowledge?* (pp. 196-233). London: Routledge.

Fenwick, T., & Edwards, R. (2010). *Actor-network theory in education.* Routledge.

Kim, H. (2016). Technoscience and Democracy from the Perspective of ANT. *Journal of Korean Social Trend and Perspective, 112,* 209-231.

Latour, B. (1999). On recalling ANT. *The sociological review, 47*(sup1), 15-25.

Latour, B. (2005). *Reassembling the Social: An Introduction to Actor-Network Theory*. Oxford: Oxford University Press.

Lv, H. Z. (2023). Innovative music education: Using an AI-based flipped classroom. *Education and Information Technologies, 28*(11), 15301-15316.

Yoon, J. & Kwon, O. N. (2024). Systematic literature review on AI-based mathematics teaching and learning: Focusing on the role of AI and teachers. *The Mathematics Education, 63*(3), 573-591.

TSG6-1-13

UTILIZATION OF CHEATING-RESISTANT QUESTIONS USING STACK IN THE COURSE OF LINEAR ALGEBRA

Kentaro Yoshitomi
Osaka Metropolitan University, Japan, yositomi@omu.ac.jp

STACK is a question plugin that works on Moodle and is an effective tool for e-learning in mathematics. Its core functions are to provide questions with random numbers, automatically mark the answers, and provide feedback corresponding to the students' answers. The feedback function is particularly effective for students' learning, and the author has developed various linear algebra questions using STACK, and has used them as preparation questions, in-class exercises, and review questions. When these exercises are used as part of a grading system to motivate students to complete them, as with online quizzes, there is a problem with cheating using computing software, and more recently, AI. Until now, I have developed questions with the intention of making them resistant to such cheating, but in this era of AI coexistence, it is necessary to verify whether they are still resistant. In addition, we will examine how to make the best use of questions that are and are not resistant based on past usage results and suggest ways to do so. We would like to consider how to use them in "Action" of APOS theory.

Keywords: STACK, Mathematics e-Learning, Cheating resistance, APOS theory

Overview of the Target Class and How to Use the Online Questions

This research is about the implementation and effectiveness verification of online questions for formative and grading assessment in a linear algebra course for first-year engineering students. This is an individual effort, not an institutional initiative. Usually, there are 60 to 80 students in each class, and this includes students who are repeating the course. The syllabus is the same for all classes, but only a small number of teachers use online questions.

In the first semester, topics covered include equations of planes in space, numerical vectors and matrices, Gaussian elimination, and determinants. The second semester introduces abstract vector spaces, including the concepts of basis and dimension, as well as linear maps, dimension formulas, representation matrices, inner products, orthogonalization, eigenspaces, and diagonalization.

At our university, Moodle is used as an LMS. On Moodle, the STACK question plugin, which uses Maxima, one of the free computer algebra systems (CAS), in the background, is available for use once it has been installed. STACK can generate random questions and give partial scores and instant feedback in response to user input. Thus, the system can be used for learning and test evaluation purposes. There is a STACK question bank project called Abacus, mainly in North Europe, and it is actively used. The fact that the system is becoming popular worldwide can be seen from the fact that the number of inquiries to the developers is increasing rapidly. On the contrary, as far as the author knows, there have been few attempts to utilize STACK organizationally in Japan.

The author started to develop videos for flipped learning since 2014 and started to develop STACK questions since 2015. The questions developed abundantly during Covid-19 have been used in face-to-face classes to the present day. In this paper, the development and utilization of the questions,

especially from the viewpoint of cheating resistance, will be discussed.

Why Cheating-Resistant Questions are Required?

The primary purpose of online quizzes is to provide learning exercises to deepen understanding. However, to encourage active student engagement, the author informs students that the completion status will be incorporated into the grade evaluation to the extent of around 10%. Also, the quizzes that we used to give in class used to take up about 1/5 to 1/4 of the class time, and it was also time-consuming to mark them. Therefore, another purpose of the online quizzes is to have students take the quizzes for assessment outside of class online.

When considering the use of online questions from the above perspective, since they are carried out outside of class, the problem of cheating using calculation apps etc. has become an issue. Therefore, this is why it is necessary to develop cheating-resistant questions.

What Questions have Cheating-Resistance Property?

The author thinks that there are two main types of questions that are resistant to cheating.

One is a question that is quite easy to compute, and it takes much less time for the student to think about the answer themselves than it does for them to cheat and copy it. This could be a question that could be used for Action in APOS theory, for example. As a concrete example, consider the following question about a representation matrix. Given the size (type) of the representation matrix of a linear map from the polynomial space P_n to P_m, the question is to find n and m. Although any AI, such as chatGPT or Gemini, can easily give the correct answer to this question, if you were to ask 10 questions of various patterns (including cases where one of the domains or ranges is a numerical vector space), the AI would not answer all of them, but would only explain the first one, so students would look at the AI's answer and try to do the rest themselves. In this way, although it is a simple task, it can be said that the "Action" has been successfully executed, which is expected to lead to a conceptual understanding of the representation matrix based on the "Object" of understanding that the mapping defined by the $n \times m$ matrix is a mapping from the m-dimensional numerical vector space to the n-dimensional numerical vector space.

Another type of cheating-resistant question is a complicated question that cannot be answered using just computing software. These are questions that require logical reasoning and cannot be answered without properly understanding the question text, or questions that cannot be answered correctly unless one understands the definitions. For example, questions that require finding the basis and dimension of the kernel of a linear map. A while ago, ChatGPT made the mistake of stating that the number of vectors that constitute the basis was different from the dimension, a mistake that is often made even by actual students. In the current situation where AIs that are capable of logical thinking have appeared, questions like this may no longer be considered cheating resistant. However, when considering the use of computing software alone, the author believes that it was sufficiently cheating resistant. However, unfortunately, questions like this, which are a little difficult, can also be a cause of students' frustration. Even in the 2024 class after Covid-19, when the author used online questions like this, many students (about 10%) stopped coming to class because they couldn't understand them. In the future, it is necessary to change the policy to use AI to explain the questions, and to use the questions as a first pattern that allows students to repeat the same questions by themselves while understanding them. In other words, it is thought that a lesson design that prevents students from having problems and steadily improves their understanding is required by making use of AI.

Correlation Analysis of Online Exercises and Final Examinations (2021 case)

Figure 1 presents a scatter plot illustrating the relationship between the achievement (completion rate) of online exercises and the final exam performance of the class of 2021. Generally, students who complete more online exercises tend to achieve higher grades. However, a notable concern is the presence of a few students with high completion rates but low final exam scores, which could indicate potential issues such as cheating and warrants further data analysis. Additionally, some students who did not engage with the online exercises still attained a moderate final grade, though their performance remained lower than those who actively completed the exercises. This suggests that attempting the online exercises may have contributed to better conceptual understanding and retention among students.

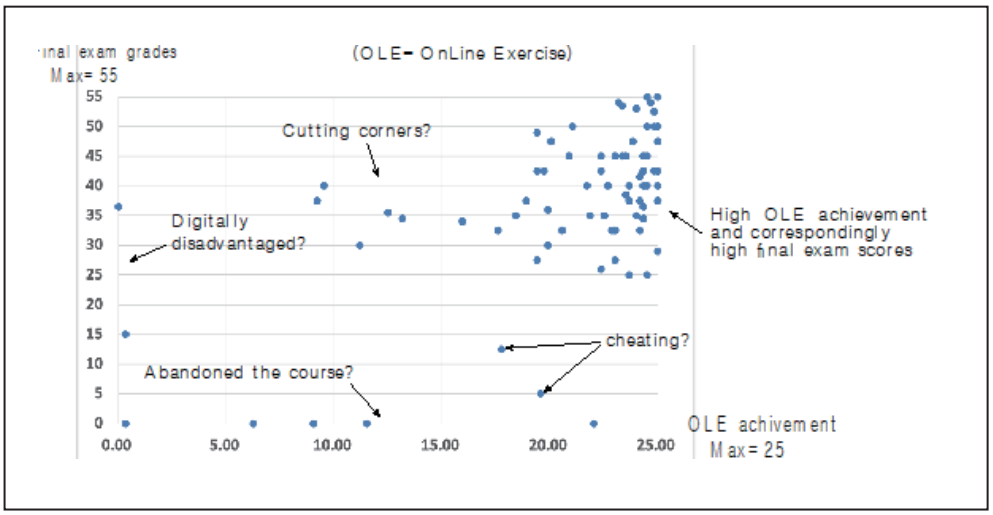


Figure 1. Relationship between OLE achievement and final exam grades

Examples of Questions

Figure 2 is an example of a multiple choice question (hereafter referred to as MCQ). The author believes that MCQs are effective for checking definitions and basic properties. For example, in order to make students aware of the regularity of matrices, it is necessary to make them firmly aware from an early stage that matrices with the same rows or columns, or zero rows or zero columns, are not regular. Therefore, in questions that ask students to choose a matrix that is not regular, the author deliberately gives many such matrices. By repeating this, we hope that when it comes to diagonalization, even in cases where diagonalization is not possible, no students will answer that it is possible to diagonalize using a non-regular diagonalization matrix. Of course, questions like this can be called the first type of cheating-resistant question. Note that, for the question on the right of Figure 2, both ChatGPT 4-o and o3-mini fail to recognize the matrix correctly and answer 1 and 3 as the correct answers. However, the explanation of the concept is correct, so one can see that the AI serves the role of a hint.

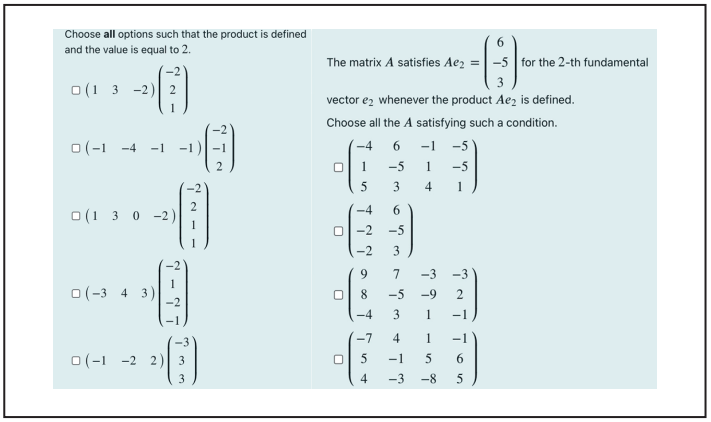


Figure 2. Examples (MCQ)

Conclusion and Discussion based on APOS Theory

The author has been aiming to use online teaching materials with cheating-resistant questions to deepen students' understanding. The author believes that there has been a certain level of effectiveness. However, when considering from the perspective of individual students, difficult questions can cause issues for students. Also, with the advent of AI, it has become an era where even questions that require reasoning can be cheated. From the standpoint of making questions available that students need to think about for themselves, I think it is necessary to evolve them into online teaching materials that students can progress through without difficulty, with the support of AI. And the author thinks it would be better to be able to carry out assessments effectively in the classroom preventing cheating by using Safe Exam Browser and other methods.

APOS theory provides a useful framework to describe and enhance mathematical learning by identifying cognitive stages: Action, Process, Object, and Schema. Within this framework, STACK's randomized and feedback-oriented exercises play a pivotal role, especially in reinforcing the Action stage, where students engage directly with mathematical operations and procedures.

From an educational perspective, integrating STACK within APOS theory-driven instruction allows educators to scaffold learning effectively. Initial exercises aimed at the Action stage can gradually lead to more sophisticated tasks that require conceptual reasoning and understanding at the Process and Object stages. By designing online questions aligned with this framework, educators can facilitate a smooth cognitive transition, assisting students in building a solid Schema of linear algebra concepts. In the future, the author plans to explore more objective methods to evaluate the establishment students' Schema.

References

Yoshitomi, K. (2022). Multiple-choice questions using STACK with partial score and feedbacks, ATCM 2022 Proceedings (pp. 333–342).

Sangwin, C. J. (2014). Computer Aided Assessment of Mathematics. Oxford University Press, Oxford, UK.

Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Fuentes, S.-R., Trigueros, M., & Weller, K. (2013). APOS Theory, Springer New York, NY.

Jörg Härterich (2019). Using randomized quizzes in undergraduate linear algebra and multivariable calculus. In: Contributions to the 1st International STACK conference 2018.

Acknowledgement

This work was supported by JSPS KAKENHI Grant Number JP24H001681

TSG6-1-14

STUDENT INTERACTIONS WITH CHATGPT: ANALYZING PROBLEM-POSING PROCESSES IN A NUMBER THEORY COURSE

Ah Ra Cho
Seoul National University, Korea, ahra604@snu.ac.kr

Oh Nam Kwon
Seoul National University, Korea, onkwon@snu.ac.kr

This study explores how undergraduate students in a number theory course collaborated with ChatGPT to pose, review, and solve problems. Using the ***Students' Interactive Proving Experience with AI (SIPE-AI)*** *framework, the research analyzed processes such as prompt engineering, problem refinement, and iterative revisions. The findings reveal that while ChatGPT enhanced students' mathematical understanding and creativity, its limitations such as difficulties in addressing deep conceptual reasoning required students to refine prompts and critically evaluate its outputs. Through this iterative process, students improved their problem-posing skills, deepened their understanding of challenging mathematical concepts, and developed proficiency in generative AI tools. This study highlights the educational value of incorporating generative AI into mathematics education, emphasizing the importance of structured guidance in prompt engineering and self-directed learning strategies. The insights offer practical recommendations for educators integrating generative AI into teaching practices, fostering more dynamic and interactive learning environments.*

Keywords: ChatGPT, Number theory, Generative AI, Higher education

Introduction

ChatGPT has shown stronger performance in tasks requiring natural language skills and general knowledge than in those demanding specialized knowledge and logical reasoning, such as mathematics (Lo, 2023). Current research on generative AI in mathematics education focuses on problem-solving performance (Kwon et al., 2023; Oh, 2023; Kang, 2024) or teacher-led applications (Van Long et al., 2023; Zong & Krishnamachari, 2023), with limited exploration of learner-driven interactions. Yoon et al. (2024) proposed the Students' Interactive Proving Experience with AI (SIPE-AI) framework, which forms the theoretical basis for this study, examining generative AI's role in proving activities. Problem-posing plays a critical role in fostering problem-solving strategies and creativity in mathematics education (Polya, 1945); however, students often face challenges in posing and refining problems due to time constraints and insufficient feedback. Collaborating with ChatGPT in problem-posing activities offers a potential solution, enabling deeper conceptual understanding and serving as an effective pedagogical tool. This study investigates how students in a number theory course used ChatGPT to pose and review problems through four assignments, focusing on their interactions and outcomes.

Theoretical Framework

This study utilized the Students' Interactive Proving Experience with AI (SIPE-AI) framework (Yoon et al., 2024) to analyze students' interactions with ChatGPT during the problem-posing assignment. The framework comprises two dimensions: (1) processes, referring to students' actions and behaviors when using generative AI, and (2) factors, including conceptions of mathematical proofs, perceptions of generative AI, and ethical considerations. The SIPE-AI framework, originally designed to analyze students' interactions with generative AI during the proof process, was deemed sufficiently applicable for analyzing student-led learning interactions with ChatGPT in problem-posing activities. Figure 1, which illustrates students' generative AI usage patterns, supports this applicability. However, this study focuses solely on the processes involved in using ChatGPT to pose number theory problems, excluding the analysis of factors.

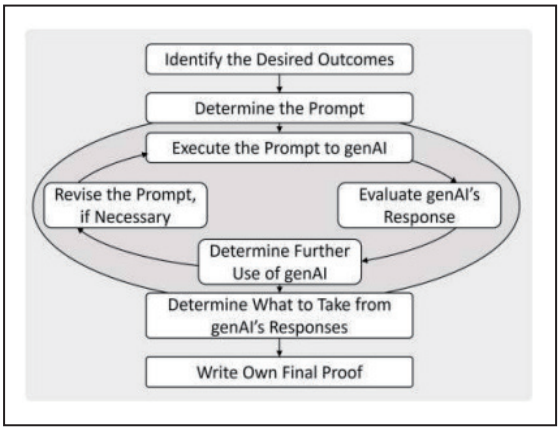


Figure 1. Students' process patterns of using gen AI in proving

Method

This study was conducted in a number theory course from September to December 2024, with 18 students from the Department of Mathematics Education at the College of Education in Seoul. Over four assignments, students collaborated with ChatGPT to pose, solve, and revise problems based on monthly course content. To enhance the quality of generative AI outputs, students were guided in various prompt engineering techniques, including role-playing (setting specific roles for ChatGPT) and few-shot prompting (providing example problems). The analysis focused on the processes dimension of the SIPE-AI framework, excluding the "Write own final proof" stage as the assignments centered on problem posing and solving rather than proof construction.

Findings

Identify the Desired Outcomes

Students began the 'Collaborating with ChatGPT to Pose Number Theory Problems' assignment by specifying their preferred outcomes, often focusing on challenging concepts. For example, student P (Assignment 2) expressed interest in "posing a problem about solving systems of congruences related to the determinant of the coefficient matrix." One student (student H, Assignment 2) stated,

> "I still lack a full understanding of the Fundamental Theorem of Arithmetic, so I chose it as my topic to enhance my learning. I expect that completing the series of processes, including discussions and revisions with GPT, will deepen my understanding."

As students became more familiar with the tasks in later assignments, they often skipped this initial step and proceeded directly to prompting.

Determine the Prompt & Execute the Prompt to Generative AI

Initially, students used zero-shot prompts but gradually adopted role-playing and few-shot techniques, assigning ChatGPT roles such as a university professor to pose problems aligned with class content. Over time, students crafted more detailed prompts and employed Chain-of-Thought (CoT) prompting, guiding ChatGPT step-by-step to pose problems tailored to their desired learning outcomes.

For instance, student K (Assignment 2) wrote:

1. Briefly explain the Chinese Remainder Theorem.
2. Rewrite the Chinese Remainder Theorem, limiting it to two congruences, and explain how to find the unique integer x that satisfies the conditions of the two congruences.
3. Prove that the method for finding the unique integer x in two congruences also holds for an arbitrary k number of congruences.
4. Based on the above discussion, pose problems for undergraduate students learning number theory.

Before prompting, students organized their preliminary learning materials and set explicit guidelines for using ChatGPT. Rather than requesting a single question, they asked ChatGPT to generate multiple options, showcasing a proactive approach. For example, student S (Assignment 2) provided the following guidelines:

1. Use theorems covered in the lecture but avoid directly turning them into problems.
2. Pose questions suitable for undergraduate number theory students.
3. Do not overly rely on ChatGPT.
4. Avoid problems requiring overly complex numerical calculations.

Evaluate Generative AI's Response

After presenting prompts to ChatGPT, students evaluated whether the responses aligned with their intended concepts and difficulty levels. However, ideal problems were rarely achieved on the first attempt. Students often found the outputs lacking, as ChatGPT either failed to reflect the intended content, repeated class material or produced overly simplistic problems. For example, in Assignment 3, student H remarked, "Hmm, that's already covered in lecture notes on page 10, theorem 4.4 (3)." Similarly, student S (Assignment 1) noted:

> "No specific errors were found, but despite asking for the exact relationship between the greatest common divisor and least common multiple of multiple numbers, it only provided the fact that the product of the greatest common divisor and least common multiple is less than the product of all the numbers."

Determine Further Use of Generative AI & Revise the Prompt, if Necessary

Throughout the assignment 'Collaborating with ChatGPT to Pose Problems,' students consistently revised their prompts to elicit better responses. When ChatGPT's outputs did not meet their expectations—either being overly simplistic or diverging from their intended concepts—they refined their prompts or shifted to different topics. For example, in Assignment 3, student H observed, "It's now familiar that ChatGPT suggests existing propositions." Similarly, in Assignment 4, student D critiqued ChatGPT's simplicity, stating, "The applied problem seems too simple. Pose a more advanced problem and provide a solution." Even when appropriate problems were posed, errors often appeared in the solutions. Students carefully reviewed ChatGPT's responses, identified inaccuracies, and requested corrections. Also, student J (Assignment 1) pointed out:

> "(ChatGPT's response) 'Since c must be a multiple of 72, the values of c, which are multiples of 72 such as 72, 144, and 216, must be excluded.' If c must be a multiple of 72, why should 72, 144, and 216 be excluded? Does the definition of a multiple need to be restricted to natural numbers?"

Determine What to Take from Generative AI's Response

Students asked ChatGPT to pose multiple problems, selected the one most aligned with their preferences, and refined it further to complete their assignments. They thoroughly reviewed ChatGPT's problems and solutions, correcting errors to ensure accuracy before submission. Finally, students compiled their prompts, processes, and reflections as their final submissions.

For instance, student H noted,

> "I found the problem of determining whether a specific polynomial is a quadratic residue interesting, so I asked about the proof process. I like the approach of dividing into cases and verifying each one. Thanks! I'll submit this problem as my assignment."

Conclusion & Discussion

This study explored how students collaborated with ChatGPT to pose and review problems in a number theory course, focusing on the process dimension of the SIPE-AI framework. By applying the SIPE-AI framework to problem-posing activities, this study demonstrates its versatility beyond proving contexts. Problem-posing, like proving, involves iterative processes of reasoning, refining, and validating mathematical ideas. This work not only broadens the applicability of SIPE-AI but also highlights its potential as a robust framework for analyzing diverse mathematical learning activities. The findings confirm ChatGPT's potential to enhance mathematical abilities and foundational knowledge, while also revealing limitations, such as difficulties in deep conceptual understanding and the generation of simplistic or flawed outputs. These challenges required students to revise prompts iteratively, which in turn reinforced their understanding of key mathematical concepts and improved their problem-posing skills. The results emphasize the importance of regular practice in prompt engineering and maintaining originality to foster self-directed learning. Furthermore, educators play a critical role in guiding students to effectively use generative AI, ensuring that its integration supports both critical thinking and creative problem-solving. However, this study does not provide direct evidence of improvements in students' mathematical achievement or creativity, which remains an important direction for future research. By providing strategies for integrating generative AI into mathematics education, this study offers practical insights for educators and lays the groundwork for future research on enhancing student interactions with AI tools.

References

Kang, Y. J. (2024). Exploring pedagogical strategies for utilizing ChatGPT in solving mathematical word problems. *Journal of Mathematics Education, 38*(1), 49–67.

Kwon, O. N., Oh, S. J., Yoon, J. E., Lee, K. W., Shin, B. C., & Jeong, W. (2023). Analysis of ChatGPT's mathematical performance: Focusing on solving problems from national academic achievement tests and the College Scholastic Ability Test in mathematics. *Journal of Mathematics Education, 37*(2), 233–256.

Lo, C. K. (2023). What is the impact of ChatGPT on education? A rapid review of the literature. *Education Sciences*, *13*(4), 410.

Oh, S. J. (2023). Exploring effective ChatGPT prompts in solving mathematical problems: Focusing on quadratic equations and quadratic functions. *Journal of Mathematics Education, 37*(3), 945–967.

Polya, G. (1945). *How to Solve It: A New Aspect of Mathematical Method*. Princeton University Press.

Van Long, P. P., Vu, D. A., Hoang, N. M., Do, X. L., & Luu, A. T. (2023). ChatGPT as a math questioner? Evaluating ChatGPT on generating pre-university math questions. *arXiv preprint*, arXiv:2312.01661. https://arxiv.org/abs/2312.01661

Yoon, H., Hwang, J., Lee, K., Roh, K. H., & Kwon, O. N. (2024). Students' use of generative artificial intelligence for proving mathematical statements. *ZDM–Mathematics Education, 56*, 1531-1551.

Zong, M., & Krishnamachari, B. (2023, June). Solving math word problems concerning systems of equations with gpt-3. In *Proceedings of the AAAI Conference on Artificial Intelligence* (Vol. 37, No. 13, pp. 15972-15979).

TSG6-1-15

THE DEVELOPMENT OF PRESERVICE MATHEMATICS TEACHERS' SPATIAL REASONING: STRENGTHS AND CHALLENGES

Lizhen Chen
Western Washington University, USA, chenl24@wwu.edu

Spatial reasoning predicts mathematics performance. However, spatial reasoning has been a less developed area compared to other topics like integer operations in American mathematics curriculum and relevant research on preservice teachers' (PSTs) spatial reasoning is even less in USA. PSTs who do not develop sound spatial reasoning often continue struggling to teach geometry in their future K-12 classrooms. This study aims to investigate 1) how PSTs navigate open-ended geometric block building tasks using an online tool and 2) what strengths and challenges they experience in developing spatial reasoning.

Keywords: Geometry and Spatial Reasoning, Preservice Teacher Education, Problem Solving

Spatial reasoning has long been a difficult area for mathematics learners (Bjork & Bjork, 2011). It strongly predicts mathematics performance (Fias & Fischer, 2005; Hawes et al., 2022; Kinach, 2012). However, spatial reasoning has been a less developed area compared to other topics like integer operations in American mathematics curriculum (Bruce & Hawes, 2015) and relevant research on preservice teachers' (PSTs) spatial reasoning is even less in USA. PSTs who do not develop sound spatial reasoning often continue struggling to teach geometry in their future K-12 classrooms. Therefore, it becomes especially important to delve into PSTs' development of spatial reasoning and reveal the strengths and challenges they experience, by doing which it is possible to create effective curriculum and instruction in preservice teacher education.

Theoretic Framework

According to Uttal et al. (2013), spatial reasoning includes the static versus dynamic skills and intrinsic versus extrinsic skills (i.e., attributes within an object versus relations among multiple objects). Based on these two dimensions, there are four spatial reasoning categories: intrinsic-static skills, intrinsic-dynamic skills, extrinsic-static skills, and extrinsic-dynamic skills. The extrinsic-dynamic skills may be most challenging for students to develop for two possible reasons: first, multiple moving objects are involved; second, students have to consider the relative positions among multiple objects instead of only viewing objects from their own perspective (Davis & the Spatial Reasoning Study Group, 2015).

Learning mathematics with understanding necessitates some struggles and is likely to deepen learners' understanding of spatial concepts and relationships (Hiebert & Wearne, 2003). Piaget (1960) described learners' struggles as a process of resolving disequilibrium and developing new understanding. Tasks containing "desirable difficulties" can "optimize long-term retention and transfer" (Bjork & Bjork, 2011, p. 57). When students inquire into diverse problem-solving strategies, they are actually practicing their spatial reasoning skills and reasoning about feasible versus problemat-

ics solutions. This study aims to investigate 1) how PSTs navigate open-ended geometric block building tasks using an online tool and 2) what strengths and challenges they experience in developing spatial reasoning.

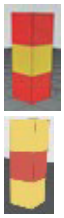
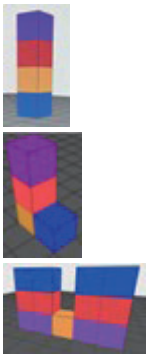
Methods

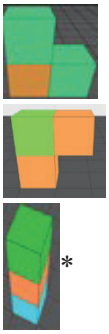
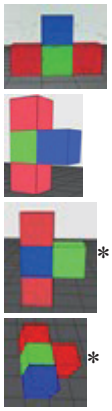
Participants and Data Collection

Twenty-two PSTs participated in this study. They were enrolled in teacher preparation programs in an American public university in 2024: 13 PSTs in early childhood education, seven PSTs in elementary education, and two PSTs in inclusive education. At the time of data collection, PSTs were taking either the last course in a three-course sequence for PK-3 mathematics or the last course in a three-course sequence for K-8 mathematics. All of these courses integrate content and methods. One important topic is geometric thinking and spatial reasoning. Before data collection, PSTs had been engaged in defining different 2D and 3D shapes (e.g., rhombuses, parallelograms, trapezoids, cylinders, pyramids etc.), reasoning about the properties (e.g., sides, angles, vertices, faces, etc.) of different polygonal and 3D shapes, and exploring geometric transformations using GeoGebra, manipulatives, and bodily movements.

PSTs recorded their thinking processes when they worked on five 3D block building tasks (See three-block tasks and four-block tasks in Table 1). The 3D block building tasks provided clues in three-four sentences. PSTs need to build a 3D structure using those clues and make sure all blocks were connected. All tasks were purposefully designed to allow for multiple solutions, of which PSTs were well informed. Table 1 presents some correct and incorrect 3D structures as examples. Most PSTs used an online tool, i.e., 3D Block Builder, and a few students used multi-link cubes or both to make 3D structures. Their screenshots of their structures and written thinking processes were collected.

Table 1. 3D Block building tasks

	Three-block Tasks	Exemplar Structures		Four-Block Tasks	Exemplar Structures
Task #1	This is a three-block tower. The highest block is on the third level. There is a red block directly below a yellow block. There is a red block directly on top of a yellow block.		Task #4	This is a tower of four blocks. The red block is between the blue one and the purple one. The blue and the purple blocks are not between any other blocks. The orange block shares face with two other blocks.	 *

Task #2	This is a three-block tower. The highest block is on the second level. There is green block directly above an orange block. All the green and orange blocks of the same color do not share faces.	 *	Task #5	There are four blocks in all. One of the blocks is blue. The green block shares one face with each of the other three blocks. Two red blocks do not touch each other.	 * *
Task #3	All three blocks are connected. The purple and the blue blocks are below the green one. The green and purple blocks only share face with another block.	----		----	

* Note. These are examples of incorrect structures. The green-orange-blue structure was made by the PST F5. The 13-block structure in Task #4 and the structure made of red-blue-red blocks stacked vertically and the green block on side in Task #5 were made by W9. The last four-block structure in Task #5 was made by F9

Data Analysis

The grounded coding theory (Glaser & Strauss, 1967) was used to analyze PSTs' responses to the 3D block building tasks. Four themes are found: vocabulary understanding, initial partial structures, the use of only one versus multiple structures, and unnoticed information.

Results

On average, 78% of the PSTs created only one 3D block building structure for each task whereas the rest 22% provided more than one structure (see Table 2). Half of those PSTs believed that there was only one structure after their trials. For example, in Task #1, all 22 PSTs provided only one structure as their solutions. Actually, the number of red and yellow blocks remains open-ended. When the relative positions of the blocks are considered, there could be two red blocks and one yellow block OR two yellow blocks and one red block. One PST (W2) commented, "I believe this is the only solution to this task (Task #1), as there are only… two reds and one yellow… there are no other possibilities." The number of PSTs who created red-yellow-red structures (15 PSTs) in Task #1 is twice as many as that of PSTs who created yellow-red-yellow structures (seven PSTs).

Table 2. Percentage of PSTs who created only one versus multiple structures (N=22)

	PSTs Providing Only One Structure	PSTs providing Multiple Structures
Task #1	100% (22/22)	----
Task #2	68% (15/22)	32% (7/22)
Task #3	82% (18/22)	18% (4/22)
Task #4	59% (13/22)	41% (9/22)
Task #5	82% (18/22)	18% (4/22)
Average	78%	22%

PSTs' understanding of geometric concepts affects PSTs' success in creating feasible 3D structures. Some PSTs had difficulty in interpreting the meaning of two blocks sharing faces. The PST, W12, thought when the green and purple blocks only share face with another block in Task #3, it means "the green and purple cannot touch so purple had to go on the bottom making the blue in the middle." The PST, F4, mistakenly thought the green and purple blocks share faces with each other. In fact, Task #3 allows both possibilities in terms of the meanings of *sharing faces*: a) the green and purple blocks do not touch each other at all, nor sharing faces, edges, or vertices; b) the green and purple blocks only touch each other along an edge, still not sharing faces with each other (see the second structure in Task #3 in Table 1). In Tasks #2 & #5, about one fourth of PSTs thought two blocks not touching as them not sharing faces and thought it was ok to share corners or edges. Besides, quite a few PSTs struggled to understand the meanings of "between" in geometry. The PST, W9, mistakenly thought "between" means touching. When she read "The red block is between the blue one and the purple one" in Task #4, she stacked them together on top of each other, which led her to place the last orange block by the purple block and introduce more than four blocks in her final structure (see the 13-block structure in Task #4 in Table 1). Another misleading conception that many PSTs held is that "between" means all blocks are placed in a linear manner. A bit more than half of the PSTs created a vertical tower structure in Task #4 where all four blocks stack directly above each other. Only four PSTs thought of alternative structures either in an L shape or in a T shape. The PST, W4, saw the red block between the purple and blue blocks from a side orientation although these three blocks were not in the same column (see the second exemplar structure in Task #4 in Table 1).

The initial partial structures that PSTs built for their overall 3D structures play an important role in their success. Those who came up with multiple structures saw more than one possibility when they built their initial partial structures for Task #3. The PST, F2, reasoned, "Since purple and blue blocks must be under the green block then there must be space for at least one to two blocks under the green block." By contrast, those who created incorrect initial structures had a high potential of creating mismatching 3D structures. For example, the PST, W9, explained her initial red-blue-red structure, "I knew that there are two reds because it says two red blocks that do not touch each other. So I placed red, blue and red at the bottom leaving that space." This initial structure left the PST feeling confused about where to place the green block to make sure the green one shares one face with each of the other three blocks. In the end, she left her confusion unsolved.

PSTs encountered an impasse when their initial ideas conflicted with given information; they tended to stick with their initial ideas without revisions when they applied their reasoning to build 3D block structures. For example, F5 made a three-block-high structure for Task #2 and later realized that the highest block was on the second level. She renamed her structure levels: Level 2 - green, Level 1 - orange, Level 0 – blue, which do not match the purpose of the task. Another situation is that some PSTs likely ignored some key information in the task when their attention was focused on the placement of individual blocks. For instance, in Task #5, F9 seemed to explore different positions for those two red blocks. After finding one feasible structure where two red blocks were placed on the opposite faces of the green block, she thought it was feasible as well to place the red blocks on adjacent faces of the green block, thus explaining, "the two red blocks are on adjacent sides of the green block with the blue connecting to an open face of the green block" (see the last structure in Task #5 in Table 1). However, F9 forgot the clue that red blocks do not touch.

Discussion and Implications

Although the 3D block builder provides PSTs with a convenience of rotating and viewing their structures from any perspectives, results still show that it is a challenge for PSTs to develop a sound understanding of geometric positions and orientations when multiple blocks are involved. Teacher educators could design meaningful contexts to reinforce PSTs' vocabulary understanding such as between, sharing faces, sharing edges, below, above, and touching. For example, embodied activities could help PSTs use their body movements to experience the changes of locations. Moreover, many PSTs found it challenging to coordinate the relevant positions of multiple blocks and likely let some essential information go unnoticed even when they double checked their structures. Future studies could provide scaffolds that alleviate PSTs' short-term memory load and help them focus on the relationships between two shapes at a time. Using worked examples to scaffold PSTs' building of initial structures could be beneficial. Most important of all, PSTs should gain more experience of practicing their extrinsic-dynamic skills with technology to enhance their spatial reasoning skills.

References

Bjork, E. L., & Bjork, R. A. (2011). Making things hard on yourself, but in a good way: Creating desirable difficulties to enhance learning. In M. A. Gernsbacher, R. W. Pew, L. M. Hough, & J. R. Pomerantz (Eds.), *Psychology and the real world: Essays illustrating fundamental contributions to society* (pp. 56–64). New York: Worth Publishers.

Bruce, C. D., & Hawes, Z. (2015). The role of 2D and 3D mental rotation in mathematics for young children: what is it? Why does it matter? And what can we do about it?.*ZDM, 47*, 331-343. DOI 10.1007/s11858-014-0637-4.

Davis, B., & the Spatial Reasoning Study Group (2015). *Spatial reasoning in the early years: Principles, assertions, and speculations*. Routledge: New York.

Fias, W., & Fischer, M. H. (2005). Spatial representation of numbers. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 43–54). Psychological Press.

Glaser, B., & Strauss, A. (1967). *The discovery of grounded theory*. Aldine Publishing Company, Hawthorne, NY.

Hawes, Z., Gilligan-Lee, K. A., & Mix, K. S. (2022). Effects of spatial training on mathematics performance: A meta-analysis. *Developmental Psychology, 58*(1), 112–137. https://doi.org/10.1037/dev00 01281

Hiebert, J., & Wearne, D. (2003). Developing understanding through problem solving. In H. L. Schoen & R. I. Charles (Eds.), *Teaching mathematics through problem solving: Grades* (pp. 6–12). Reston, VA: NCTM.

Kinach, B. M. (2012). Fostering spatial vs. metric understanding in geometry. *The Mathematics Teacher, 105*(7), 534–540.

Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological bulletin, 139*(2), 352. doi: 10.1037/a0028446

TSG6-1-16

DESIGNING AND REVISING DYNAMIC GEOMETRY TASKS BY PRE-SERVICE ELEMENTARY TEACHERS: THEORETICAL NETWORKING OF NOTICING AND TPACK

Nayoung Ku
Gyeonggi Science High School, South Korea, guri39@gmail.com

This study investigated noticing characteristics of pre-service elementary teachers during task design using dynamic geometry software (DGS), simulated teaching, and task revision with feedback. The study analyzed cases from five groups (11 participants) who attended lectures on using AlgeoMath. An analytical framework was developed through networking of noticing and technological pedagogical content knowledge (TPACK) theories. Results revealed that pre-service teachers demonstrated noticing based on TCK and TPACK during the task design process, with TPACK-based noticing having the most significant influence during task revision. These findings suggest the need for pre-service teachers to develop the ability to effectively use technological tools as authoring tools and the necessity of support to enhance TPK-based noticing. This study contributes by exploring noticing in the integration of technological resources and proposing a synthesized understanding of noticing and TPACK theories.

Keywords: noticing, TPACK, DGS, AlgeoMath, task design

Introduction

Teachers require professional vision to recognize complex classroom situations and provide learning opportunities (Choy & Dindyal, 2021). Pre-service teachers, often limited in teaching experience, may notice various aspects but lack depth in their analyses (Amador, 2016). Therefore, research is needed to investigate pre-service teachers' noticing throughout the entire process of designing, enacting, and reflecting on lessons.

AlgeoMath, a Korean DGS (Kim et al., 2020), has been integrated into textbooks and AI digital textbooks. Although noticing is known to be knowledge-based reasoning (Sherin & van Es, 2009), research focusing on teachers' knowledge remains limited. Previous studies (Hwang et al., 2020; Na, 2021) explored elementary pre-service teachers' Pedagogical Content Knowledge (PCK)-based noticing but were limited to observing others' lessons rather than designing their own. Moreover, these studies did not address knowledge related to technological tools. By networking TPACK and noticing theories, this study investigated elementary pre-service teachers' noticing during task design and revision using AlgeoMath.

Literature Review

Teacher's noticing consists of two main subprocesses: selective attention and knowledge-based reasoning (Sherin & van Es, 2009). According to Shulman (1986), teachers' knowledge encompasses Content Knowledge (CK), General Pedagogical Knowledge (PK), and their fusion into Pedagogical Content Knowledge (PCK). Teachers reason about information based on their knowledge in specific situations, thereby concretizing their understanding of learners and pedagogical decision-making (Ding & Dominguez, 2016; Hwang et al. 2020; Na, 2021).

To integrate technological resource into mathematics teaching practices, teachers need knowledge of content, pedagogy, and technological resources. The TPACK framework (Koehler & Mishra, 2009) highlights the interplay among CK, PK, Technological Knowledge (TK), as well as their intersections such as PCK, Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), and TPACK. This framework provides a comprehensive lens for examining the skills and knowledge teachers need to orchestrate instruction utilizing available digital resources.

Method

Design of the Study

This research employed a case study methodology involving 11 pre-service elementary teachers enrolled in the course "Elementary Mathematics Education Using Technological Resources" within the Mathematics Education Department at a metropolitan University of Education. The course covered AlgeoMath practice with geometric tools (weeks 3-5) and block coding (weeks 7-9).

Five groups (G1-G5), each consisting of 2-3 pre-service teachers, collaborated on lesson design using AlgeoMath. In week 12, participants selected their teaching content. During weeks 13 and 14, each group conducted simulated lesson and received feedback from both the professor and peers. In week 15, groups revised their initial tasks.

Data Collection and Analysis

The study collected both written and video data, including task design reports and lesson plans for initial and revised tasks. An analytical framework was developed by networking noticing and TPACK theories through Prediger et al.'s (2008) coordinating strategy.

Drawing on previous research (Choy & Dindyal, 2021; Na, 2021), units of analysis were identified as text segments expressing new ideas about AlgeoMath-based tasks from reports, interview sheets, and lesson transcripts. Given the study's focus on designing and revising tasks using DGS, the analytical framework focused on TCK, TPK, and TPACK. Hansen et al.'s (2016) TPACK analysis method for technology-based mathematics instruction was also referenced.

The analytical framework developed for networking noticing and TPACK is presented in Table 1. TCK-based noticing addresses how teachers connect mathematical content with technological tools. TPK-based noticing focuses on non-mathematical aspects such as device management, usage instructions, and student organization. TPACK-based noticing encompasses how technological tools support students' mathematical understanding and anticipates potential learning difficulties

Table 1. Analytical framework for networking between noticing and TPACK

Category	Description	Example
TCK-based noticing	Noticing grounded in knowledge that connects mathematical content with technological resources	(G1) The distance between parallel lines is defined by the length of the shortest segment connecting them. Through AlgeoMath, students can verify that this shortest segment is perpendicular to both lines. (…) This approach helps students understand that the perpendicular property of the distance applies to both parallel lines.
TPK-based noticing	Noticing based on knowledge that is not directly related to mathematical content, but involves general pedagogical aspects in technology use (e.g., time management, seating arrangements, device management)	(G4) After using AlgeoMath, I realized the importance of completely turning off digital devices and store them in a desk drawer when not in use, helping students maintain focus on the lesson content.
TPACK-based noticing	Noticing founded on knowledge that combines technological resources with mathematical content to support student learning and address content-specific needs	(G1) Students were guided to use inductive reasoning to determine which of the connecting segments between two parallel lines should be defined as the distance. Among the various segments, they discovered on their own that selecting the shortest segment yields a unique, constant value, which makes it more convenient.

Results & Discussion

During initial task design, pre-service teachers exhibited TCK- and TPACK-based noticing, while TPK-based noticing was not observed. Although they were introduced to various AlgeoMath tools, they primarily used ‘length’, ‘area’, and ‘angle’ functions. This aligns with Ku’s (2023) finding that pre-service teachers tend to prefer ‘measurement’ and ‘transformation’ tools, indicating a need for support in exploring a wider range of tool applications. Moreover, pre-service teachers exhibiting TCK-based noticing developed lessons that extended beyond textbook presentations to deepen students’ conceptual understanding.

TPACK-based noticing significantly influenced task revisions. After the simulated lesson, the pre-service teachers received TCK-, TPK-, and TPACK-based feedback. While maintaining their initial design goals, they revied tasks to strengthen conceptual understanding (G1, G3) and to anticipate student challenges (G2, G4). Despite attempts to incorporate TCK-based feedback, limited familiarity with AlgeoMath tools and technical constraints hindered the full implementation of suggested revisions.

When designing their second tasks, the pre-service teachers demonstrated more sophisticated and knowledge-based noticing than in their initial designs. For instance, G1, G2, and G4 provided more

detailed and nuanced rationales for their task revisions, explicitly connecting their decisions to students' mathematical thinking or the functionalities of AlgeoMath. Compared to the first task, the grounds for their noticing became more comprehensive and well-supported. These findings support Hansen et al. (2016), who reported that the components of TPACK tend to manifest in increasingly integrated ways as teachers engage with technological tools.

This study networks noticing and TPACK theories through qualitative analysis, moving beyond traditional TPACK research that relies on quantitative measurement instruments. This qualitative approach facilitates a more in-depth analysis of TPACK-based noticing. Recent research (Kadluba et al., 2025) emphasizes the necessity for a more focused analysis of TPACK components, especially those involving CK. This study fills this gap by examining concrete instances of TPACK implementation.

Moreover, this study suggests an instructional approach to develop pre-service teachers' skills in using technological tools like AlgeoMath as authoring tools. Through examining how experts transform textbook activities into digital content and analyzing specific commands and functions, pre-service teachers can enhance their lesson design capabilities. This practical approach improves their capacity to integrate technology effectively into mathematics instruction.

The findings from this study seek to encourage further research on technology-based lesson design by both in-service and pre-service teachers.

References

Amador, J. (2016). Professional noticing practices of novice mathematics teacher educators. *International Journal of Science and Mathematics Education, 14*(1), 217-241.

Choy, B. H., & Dindyal, J. (2021). Productive teacher noticing and affordances of typical problems. *ZDM–Mathematics Education, 53*(1), 195-213.

Ding, L., & Domínguez, H. (2016). Opportunities to notice: Chinese prospective teachers noticing students' ideas in a distance formula lesson. *Journal of Mathematics Teacher Education, 19*(4), 325-347.

Hansen, A., Mavrikis, M., & Geraniou, E. (2016). Supporting teachers' technological pedagogical content knowledge of fractions through co-designing a virtual manipulative. *Journal of Mathematics Teacher Education, 19*, 205-226.

Hwang, S., Son, T., & Yeo, S. H. (2020). Examining the characteristics of novice elementary mathematics teacher' noticing. *School Mathematics, 22*(4), 811-831.

Kadluba et al. (2025). How much C is in TPACK? A systematic review on the assessment of TPACK in mathematics. *Educational Studies in Mathematics, 118*(2), 169-199.

Kim, H. K. et al. (2020). *AlgeoMath 3rd year development research report*. Research Report BD20020009, Korea Foundation for the Advancement of Science and Creativity.

Koehler, M., & Mishra, P. (2009). What is technological pedagogical content knowledge (TPACK)?. *Contemporary issues in technology and teacher education, 9*(1), 60-70.

Ku, N. (2023). Analyzing preservice elementary teachers' use of AlgeoMath in the design of tasks. *School Mathematics, 25*(4), 569-593.

Ku, N. (2025). Designing and revising dynamic geometry tasks by pre-service elementary teachers: Theoretical networking of noticing and TPACK. *Journal of Educational Research in Mathematics, 35*(1), 1-30.

Na. G-S. (2021). A Study on the Comparison of Elementary Preservice Teachers' Noticing to a Mathematics Lesson. *School Mathematics, 23*(2), 271-290.

Prediger, S., Bikner-Ahsbahs, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches: First steps towards a conceptual framework. *ZDM-Mathematics Education, 40*(2), 165-178.

Sherin, M. G., & van Es, E. A. (2009). Effects of video club participation on teachers' professional vision. *Journal of Teacher Education, 60*(1), 20-37.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

TSG6-2-8

META-ANALYSIS OF THE EFFECTS OF EDUCATIONAL ROBOTICS ON K-12 STUDENTS' MATHEMATICS LEARNING

Ziyi Xiao
School of Mathematical Sciences, East China Normal University, China, 51275500106@stu.ecnu.edu.cn

Di Liu
School of Mathematical Sciences, East China Normal University, China, dliu@spe.ecnu.edu.cn

This systematic review and meta-analysis endeavors to evaluate the impact of educational robotics on enhancing mathematics learning. Based on 43 data sets sourced from 18 articles, we identified an overall effect size of 0.590 using a random-effects model. Key variables that emerged as significant include sample size, educational level, intervention duration and mathematical content. Educational robotics was particularly effective in the following contexts: sample sizes ranging from 41 to 60, kindergarten and high school students, and intervention durations lasting from 1 week to 1 month. It was especially beneficial in improving attitudes towards mathematics learning. Additionally, this study aims to demonstrate how to effectively utilize educational robotics to improve student learning outcomes in mathematics.

Keywords: Educational robotics, Effectiveness, Mathematics learning, Meta-Analysis

Introduction

With the innovation and development of technology, educational robotics, which incorporates digital techniques such as artificial intelligence, mechanical manufacturing and electronic sensors (Ouyang & Xu, 2024), has been transforming education and supporting students in multiple learning contexts (Evripidou et al., 2020). Educational robotics, aims to achieve pedagogical purposes by conceptualizing, creating and implementing robotic prototypes (Lopez-Caudana et al., 2020) or by interacting with students (Pivetti et al., 2020), and it can be categorized into robotics kits, construction kits and social robots (Virnes, 2014). The use of educational robotics has attracted the attention especially in mathematics education (Benitti, 2012), which can not only eliminate the abstraction of mathematics and help students establish a connection between mathematics and the real world by studying the practical applications of technological concepts (Adams & Cook, 2017), but also alleviate the problem of students' lack of interest (Kim & Lee, 2016).

Contemporary applications of educational robotics enhanced children's mathematical performance in various domains, including computational thinking (Chiazzese et al., 2019), spatial ability (González-Calero et al., 2019), logical mathematical skills (Muñoz et al., 2020), geometry knowledge and statistics knowledge (Rico-Bautista et al., 2019), arithmetical operation (Konijn & Hoorn, 2020), and problem-solving skills (Nemiro, 2020; Popa, 2020). Beyond cognitive gains, educational robotics has also been utilized to foster non-cognitive skills essential for mathematics learning, including interest (Kim & Lee, 2016) and motivation (Lopez-Caudana et al., 2020).

The studies that investigate the impact of educational robotics include Ouyang and Xu (2024), who only regard mathematics as a subject within STEM education, without independently quantifying the impact of educational robotics in mathematics education. Zhong and Xia (2018) conducted a systematic review of the application of robots in mathematics learning, but did not accurately calculate the effect size and further examine the variables regarded as moderators.

To fill these gaps, this study aims to comprehensively explore the role of educational robotics in mathematics education from both quantitative and qualitative perspectives, and to take into account factors that might influence the role. Therefore, our current systematic review and meta-analysis seeks to answer the following research questions: (1) What is the overall effectiveness of educational robotics on mathematics learning outcomes? (2)Which variables moderate the overall effectiveness of educational robotics on mathematics learning?

Methods

Study Inclusion Criteria

In order to be eligible for inclusion in this meta-analysis, studies must satisfy the following six criteria: (1) studies were empirical studies investigating the application of physical educational robotics in mathematics education; (2) studies were conducted in regular K-12 classroom settings; (3) studies employed an experimental or quasi-experimental design with a control group using traditional instruction methods; (4) studies provided sufficient quantitative data for effect size calculations (e.g., means, standard deviations, sample sizes, or t, F values); (5) studies were peer-reviewed journal articles published between January 2000 and January 2025; (6) studies were written in English with full text accessible.

Study Search

We employed the snowballing technique (Wohlin, 2014) to search for literature in the Web of Science Core Collection, Scopus, EBSCO Academic Search Complete and ScienceDirect, using two keyword strings combinations: combination 1 ((robot*) AND (“mathematics education” OR “mathematics learning” OR “mathematics teaching” OR “mathematics achievement” OR “mathematical area”)) and combination 2 (“educational robotics” OR “educational robots” OR “robotic education” OR “teaching robotics”) AND (“math*” OR “algebra” OR “calculus” OR “statistics” OR “probability” OR “geometry” OR “trigonometry” OR “arithmetic” OR “number theory” OR “number sense”). The first author screened all the articles, while a second researcher reviewed them again independently. Any disagreements were resolved through discussion until consensus was reached. Finally, we included 43 samples from 18 articles for systematic review and meta-analysis.

Data Extraction and Coding

Four variables were included in the coding scheme: sample size, educational level, intervention duration, mathematical content. Two coders with backgrounds in mathematics education independently coded all studies. Ultimately, the inter-rater agreement of the two researchers’ coding results was 95%, showing a high degree of consistency.

Effect Size Calculation and Data Analysis

The Comprehensive Meta-Analysis software (version 3) was employed to conduct data analysis. Hedges’ g was used as the measure of effect size to compare outcomes between groups using educational robotics and traditional instruction.

Results

Overall Effect Size of Educational robotics on Mathematics Learning

Heterogeneity analysis indicated significant variability among the studies, as evidenced by a significant Q statistic ($Q = 351.143$, $df = 42$, $p < 0.001$) and a moderately high I^2 value ($I^2 = 88.039$). Given the observed heterogeneity, a random-effects model was deemed appropriate for calculating the pooled effect size. Under the random-effects model, the pooled effect size Hedges' g was found to be 0.590 ($SE = 0.093$, 95% CI: 0.408–0.772, $Z = 6.352$, $p < 0.001$). Among the 43 unadjusted overall effect sizes, 35 indicated a positive influence, while 8 were in a negative direction. The results suggest a positive and medium effect of educational robotics on mathematics learning.

Moderator Analysis

The results of this analysis are detailed in Table 1 below.

Table 1. Results for moderator analyses

Variable	Level	k	g	95% CI	Q_B	p
Sample size	5-20	7	0.303*	[0.002, 0.603]	9.480	0.050
	21-40	19	0.750**	[0.334, 1.165]		
	41-60	5	0.966**	[0.577, 1.356]		
	61-80	4	0.533	[-0.338, 1.404]		
	> 80	8	0.386**	[0.155, 0.617]		
Educational level	Kindergarten	2	1.087	[-0.183, 2.356]	4.908	0.179
	Elementary	25	0.495**	[0.284, 0.706]		
	Middle	11	0.573**	[0.060, 1.086]		
	High	5	0.966**	[0.577, 1.356]		
Intervention duration	< 1 day	4	0.189	[-0.082, 0.460]	9.790	0.044
	1 day-1 week	7	0.334	[-0.011, 0.679]		
	1 week-1 month	8	0.699**	[0.400, 0.998]		
	1 month-2 months	9	0.737**	[0.333, 1.141]		
	> 2 months	15	0.675**	[0.303, 1.047]		
Mathematical content type	Knowledge	17	0.736**	[0.446, 1.026]	5.051	0.080
	Skill	18	0.351**	[0.094, 0.609]		
	Attitude	8	0.858**	[0.305, 1.412]		

Sample size ($Q_B = 9.480$, $p = 0.050$) and intervention duration ($Q_B = 9.790$, $p < 0.05$) significantly moderate the results, indicating that the effects of educational robotics on mathematics learning vary by these variables. Studies with a sample size of 41–60 had the largest effect size ($g = 0.966$, $p < 0.01$). Interventions lasting 1 month-2 months showed a promising effect size ($g = 0.737$, $p < 0.01$), while no significant effect was observed in studies with the shortest intervention duration (1 day-1 week).

Educational level (Q_B = 4.908, p = 0.179 > 0.05) and mathematical content type (Q_B = 5.051, p = 0.080 > 0.05) were not identified as moderators. Except for kindergarten, statistically significant effects were observed in elementary (g = 0.495, p < 0.01), middle (g = 0.573, p < 0.01) and high school stages (g = 0.966, p < 0.01), with the effect size increasing with educational level. Mathematical content type had a significant effect on knowledge (g = 0.736, p < 0.01), skill (g = 0.351, p < 0.01), and attitude (g = 0.858, p < 0.01), the effects on attitude and knowledge were much larger than those on skill.

Discussion

This systematic review and meta-analysis of 43 samples from 18 articles showed that the use of educational robotics had a significant and positive impact on mathematics learning among K-12 students, with a medium effect size of 0.590 under the random-effects model. Therefore, students who use educational robotics for mathematics learning perform better than those who do not.

In moderator analyses, sample size and intervention duration significantly influenced the effectiveness of educational robotics. Our study revealed that a sample size of 41-60 is most effective in improving mathematics learning with robots, suggesting it may be the optimal size for robot-assisted classrooms. Regarding intervention duration, different from Zhang and Zhu (2022), this study uniquely identifies one month to two months as the potential optimal intervention period, with the effect diminishing when the duration exceeded two months. In addition to being limited by unfamiliarity with the operation, short-term intervention may fail to achieve the transition from surface memory to transfer due to the need for repeated practice and feedback in mastering mathematics concepts. On the other hand, the repetition brought about by longer intervention periods may reduce students' interest, thus resulting in a diminishing effect. Nevertheless, among the included studies, the number of studies with an intervention duration of more than two months is the largest, which may represent a trend.

In addition, educational level and mathematical content type were not serve as a key moderator. However, it was notable that educational robotics had the largest impact high school students' mathematics learning, which was consistent with Wang et al. (2023). With higher hands-on skills, metacognitive abilities and more extensive experience, high school students can master the principles and structures of robots rapidly and focus more on the function of robots rather than appearance. For example, Chang & Chen (2020) used sailboat testing and design to provide real-world problem situations, assisting students in understanding and applying trigonometric functions when calculating the forces on sails. For mathematical content type, specifically, the effect sizes for knowledge and attitudes were larger than those for skills. This may not only be due to the complexity of mathematical skills hindering the development of short-term interventions, but also because of the differences between the educational robotics environment and the testing context, which impede the transfer of mathematical skills to testing. Future research should focus on using larger samples and continue to explore the impact of intervention duration on effectiveness and ways to enhance mathematical skill.

Reference

Adams, K. D., & Cook, A. M. (2017). Performing mathematics activities with non-standard units of measurement using robots controlled via speech-generating devices: three case studies. *Disability and rehabilitation. Assistive technology*, *12*(5), 491–503. https://doi.org/10.3109/17483107.2016.1151954

Aslan, D., Dağaynası, S., & Ceylan, M. (2024). Technology and geometry: Fostering young children's geometrical concepts through a research-based robotic coding program. *Education and Information Technologies*, *29*(17), 22699–22721. https://doi.org/10.1007/s10639-024-12747-3

Barker, B. S., Nugent, G., & Grandgenett, N. F. (2008). Examining 4-H Robotics and Geospatial Technologies in the Learning of Science, Technology, Engineering, and Mathematics Topics. *The Journal of Extension*, *46*(3), 1-8. https://digitalcommons.unomaha.edu/tedfacpub/37

Benitti, F.B. (2012). Exploring the educational potential of robotics in schools: A systematic review. *Computers & Education, 58*(3), 978-988. https://doi.org/10.1016/j.compedu.2011.10.006

Chang, C. C., & Chen, Y. (2020). Cognition, Attitude, and Interest in Cross-Disciplinary i-STEM Robotics Curriculum Developed by Thematic Integration Approaches of Webbed and Threaded Models: a Concurrent Embedded Mixed Methods Study. *Journal of Science Education and Technology*, *29*(5), 622–634. https://doi.org/10.1007/s10956-020-09841-9

Ceylan, M., & Aslan, D. (2024). The effect of learning trajectories-based coding education program on preschoolers' mathematical measurement skills. *Education and Information Technologies*, *29*, 7737–7757. https://doi.org/10.1007/s10639-023-12107-7

Chiazzese, G., Arrigo, M., Chifari, A., Lonati, V., & Tosto, C. (2019). Educational Chang, C. C., & Chen, Y. (2020). Cognition, Attitude, and Interest in Cross-Disciplinary i-STEM Robotics Curriculum Developed by Thematic Integration Approaches of Webbed and Threaded Models: a Concurrent Embedded Mixed Methods Study. *Journal of Science Education and Technology*, *29*(5), 622–634. https://doi.org/10.1007/s10956-020-09841-9

Ching, Y. H., Yang, D., Wang, S., Baek, Y., Swanson, S., & Chittoori, B. (2019). Elementary School Student Development of STEM Attitudes and Perceived Learning in a STEM Integrated Robotics Curriculum. *TechTrends*, *63*(3), 590-601. https://doi.org/10.1007/s11528-019-00388-0

Evripidou, S., Georgiou, K., Doitsidis, L., Amanatiadis, A.A., Zinonos, Z., & Chatzichristofis, S.A. (2020). Educational Robotics: Platforms, Competitions and Expected Learning Outcomes. *IEEE Access, 8*, 219534-219562. https://doi.org/10.1109/access.2020.3042555

Fanchamps, N. L. J. A., Slangen, L., Hennissen, P., & Specht, M. (2021). The influence of SRA programming on algorithmic thinking and self-efficacy using Lego robotics in two types of instruction. *International Journal of Technology and Design Education*, *31*, 203–222. https://doi.org/10.1007/s10798-019-09559-9

González-Calero, J. A., Cózar-Gutiérrez, R., Villena, R., & Merino, J. M. (2019). The development of mental rotation abilities through robotics-based instruction: An experience mediated by gender. *British Journal of Educational Technology*, *50*(6), 3198-3213. https://doi.org/10.1111/bjet.12726

Jeno, L.M., Vandvik, V., Eliassen, S., & Grytnes, J. (2019). Testing the novelty effect of an m-learning tool on internalization and achievement: A Self-Determination Theory approach. *Computers & Education*, *128*, 398-413. https://doi.org/10.1016/j.compedu.2018.10.008

Julià, C., & Antolí, J. Ò. (2016). Spatial ability learning through educational robotics. *International Journal of Technology and Design Education*, *26*(2), 185–203. https://doi.org/10.1007/s10798-015-9307-2

Kim, S., & Lee, C. (2016). Effects of robot for teaching geometry to fourth graders. *International Journal of Innovation in Science and Mathematics Education, 24*(2), 52-70. https://api.semanticscholar.org/CorpusID:56309827

Konijn, E. A., & Hoorn, J. F. (2020). Robot tutor and pupils' educational ability: Teaching the times tables. *Computers & Education, 157,* Article 103970. https://doi.org/10.1016/j.compedu.2020.103970

La Paglia, F., Rizzo, R., & La Barbera, D. (2011). Use of robotics kits for the enhancement of metacognitive skills of mathematics: a possible approach. *Studies in health technology and informatics*, *167*(1), 26–30. https://www.researchgate.net/publication/51231099

Laughlin, S. R. (2013). Robotics: Assessing its role in improving mathematics skills for grades 4 to 5 (Publication No. 3557417) [Doctoral dissertation, Capella University]. ProQuest LLC.

Lindh, J., & Holgersson, T. (2007). Does lego training stimulate pupils' ability to solve logical problems. *Computers & Education*, *49*(4), 1097-1111. https://doi.org/10.1016/j.compedu.2005.12.008

Lopez-Caudana, E., Ramirez-Montoya, M. S., Martínez-Pérez, S., & Rodríguez-Abitia, G. (2020). Using Robotics to Enhance Active Learning in Mathematics: A Multi-Scenario Study. *Mathematics*, *8*(12), 2163. https://doi.org/10.3390/math8122163

Mishra, D., Lugo, R.G., Parish, K., & Tilden, S. (2023). Metacognitive Processes Involved in Human Robot Interaction in the School Learning Environment. In *Proceedings of the 25th Human-Computer Interaction International Conference* (pp. 85–100). https://doi.org/10.1007/978-3-031-35599-8_6

Muñoz, L., Villarreal, V., Morales, I., Gonzalez, J., & Nielsen, M. (2020). Developing an Interactive Environment Through the Teaching of Mathematics with Small Robots. *Sensors (Basel, Switzerland)*, *20*(7), 1935. https://doi.org/10.3390/s20071935

Nemiro, J. E. (2020). Developing collaborative behaviours in elementary school students: a comparison of robotics versus maths teams. *Educational Studies*, *47*(5), 521–537. https://doi.org/10.1080/03055698.2020.1716209

Ortiz, A. (2011). *Fifth Grade Students' Understanding of Ratio and Proportion in an Engineering Robotics Program* (Publication No. 3422310) [Doctoral dissertation, Tufts University]. ProQuest LLC.

Ouyang, F., & Xu, W. (2024). The effects of educational robotics in STEM education: a multilevel meta-analysis. *International Journal of STEM Education, 11*(1), 7. https://doi.org/10.1186/s40594-024-00469-4

Pivetti, M., Di Battista, S., Agatolio, F., Simaku, B., Moro, M., & Menegatti, E. (2020). Educational Robotics for children with neurodevelopmental disorders: A systematic review. *Heliyon*, *6*(10), e05160. https://doi.org/10.1016/j.heliyon.2020.e05160

Popa, A. M. (2020). Development of the Children's Abilities in School. A comparative study between the efficiency of the Robotics vs Applied Mathematics in Movement Transmission. *Journal of Educational Sciences, 41*(1), 47-61. https://doi.org/10.35923/JES.2020.1.04

Rico-Bautista, N. A., Rico-Bautista, D. W., & Medina-Cárdenas, Y. C. (2019). Collaborative work as a learning strategy to teach mathematics incorporating robotics using led godt education system and fischertechnik in seventh graders at the school Isidro Caballero Delgado in Floridablanca Santander Colombia. *Journal of Physics: Conference Series, 1386*(1), Article 012146. https://doi.org/10.1088/1742-6596/1386/1/012146

Sanders, T. S. (2019). *A Causal-Comparative Study of the Relationship Between Fifth-grade Girls' Mathematics Achievement and Participation in a Code-Based Robotics Program* (Publication No. 27739092) [Doctoral dissertation, Northcentral University]. ProQuest LLC.

Simley, T., Mack, N. A., Pittman, T., Cook, C., Cummings, R., Moon, D., & Gosha, K. (2020). Assessing the Efficacy of Integrating Computer Science, Math, and Science in a Middle School Sphero Robotics Summer Program. *2020 Research on Equity and Sustained Participation in Engineering, Computing, and Technology (RESPECT)*, *1*,1-8. https://doi.org/10.1109/RESPECT49803.2020.9272479

Varaman, P., Kumar, J. A., Rabu, S. N. A., & Osman, S. (2024). The effect of educational robots on primary schools' mathematics learning achievement, interest, and attitude. *Journal of educators online*, *21*(2), 17. https://doi.org/10.9743/JEO.2024.21.2.17

Virnes, M. (2014). *Four Seasons of Educational Robotics: Substantive Theory on the Encounters between Educational Robotics and Children in the Dimensions of Access and Ownership*. The University of Eastern Finland: Kuopio, Finland.

Wang, K., Sang, G.-Y., Huang, L.-Z., Li, S.-H., & Guo, J.-W. (2023). The Effectiveness of Educational Robots in Improving Learning Outcomes: A Meta-Analysis. *Sustainability*, *15*(5), 4637. https://doi.org/10.3390/su15054637

Wohlin, C. (2014). Guidelines for snowballing in systematic literature studies and a replication in software engineering. In *Proceedings of the 18th international conference on evaluation and assessment in software engineering* (pp. 1–10). https://doi.org/10.1145/2601248.2601268

Zhang, Y., & Zhu, Y. (2022). Effects of educational robotics on the creativity and problem-solving skills of K-12 students: a meta-analysis. *Educational Studies*, *50*(6), 1539–1557. https://doi.org/10.1080/03055698.2022.2107873

Zhong, B., Xia, L. (2020). A Systematic Review on Exploring the Potential of Educational Robotics in Mathematics Education. *International Journal of Science and Mathematics Education*, *18*, 79–101. https://doi.org/10.1007/s10763-018-09939-y

TSG6-2-9

EXPLORING MATHEMATICAL NARRATIVES AND CITIZENS' COMMUNICATION IN KOREAN YOUTUBE SHORTS

Kyungwon Lee
Seoul National University, Korea, kyungwon.lee.snu@gmail.com

Chaereen Han
Jeonju National University of Education, Korea, chaereen@jnue.kr

Yujin Lee
Kangwon National University, Korea, ylee@kangwon.ac.kr

Oh Nam Kwon
Seoul National University, Korea, onkwon@snu.ac.kr

This study explores mathematical narratives and citizens' communication in Korean YouTube Shorts. Social media serves as a platform for this discussion but also as a public forum for broader communication about mathematics. YouTube Shorts, a feature introduced in 2021, offers an accessible platform for citizens to engage with a diverse range of topics, including mathematical content, all within one minute. Following the analysis of 95 Korean YouTube Shorts videos, four key strands were identified: Mathematics introductions, arithmetic methods, creative mathematics challenges, and controversial mathematical topics. The comments on YouTube Shorts served as a platform for discussing mathematics. The findings of this study illuminate the types of mathematical narratives and aspects of citizens' communication about mathematics in YouTube Shorts.

Keywords: YouTube Shorts, Numeracy, Korea

Introduction

This study aims to investigate key strands of mathematical narratives and aspects of citizens' communication in Korean YouTube Shorts based on Lee et al. (2025). With the start of YouTube in 2005, the creation and sharing of content in video formats was activated. The exploration of better formats for sharing resources has continued to evolve through social networking services. Recently, short video formats like YouTube Shorts have provided alternative and more accessible ways for users to create and share information. However, limited information exists on the types of mathematical content and citizens' communication featured in this short video format, which provides insights into their understanding of mathematics. Specifically, this study focuses on understanding key strands of mathematical narratives in Korean YouTube Shorts and grasping the wide range of citizens' discussions about mathematics on social media in Korea. The research question is as follows: *What kind of mathematical narratives are conveyed in Korean YouTube Shorts, and how do Korean citizens communicate about them?*

Related Literature

Korea is among the countries with high accessibility to social media, to the extent that it has developed and popularized social media platforms. Korean citizens are thus naturally exposed to the use of globally prevalent social media. The development of social media in Korea is closely related to the development of Korean culture (K-culture) such as music, dramas, and other forms of entertainment. Research focusing on social media in mathematics education is still in its infancy. Bini et al. (2022) conceptualized mathematical memes as hybrid representations that blend mathematical statements with humor and popular culture, facilitated by the Internet and social media. Based on this, Bini et al. (2023) explored the creative process behind mathematical internet memes in three social media (Facebook, Instagram, and Reddit), focusing on how mathematical concepts are merged with digital culture to create educational content. Exploring the mathematical content discussed on social media will be an increasingly significant topic in mathematics education research. The content offered on social media serves as a new learning medium for mathematics, engaging not only students and teachers but also citizens.

Traditionally rooted in arithmetic, one of the fundamental 3Rs (Reading, wRiting, and aRithmetic) in school education, numeracy derived from arithmetic is an essential skill and foundational competence for learning, alongside reading and writing. As society advances, the increasing demand for numeracy skills in various aspects of adult life has elevated numeracy to a crucial skill for success in contemporary society (Condeli et al., 2006). However, several challenges hinder the conventional approach to adult numeracy development (Carpentieri et al., 2010). Social media offers an alternative platform to address these challenges, providing interactive and engaging formats that enhance numeracy skills and broaden access to mathematical resources.

Methods

YouTube Shorts, launched in 2021 as a competitor to TikTok and Instagram Reels, is a short-form content, presented within one minute, developed by YouTube. This study targeted YouTube Shorts videos that recorded over 500,000 views by February 2024. The threshold of 500,000 views was selected to represent approximately 1% of the Korean population, which is about 51.75 million. Initially, keyword search results were obtained using NetMiner, a software valued for its effectiveness in social network analysis. Videos were reviewed to determine whether they were general videos or YouTube Shorts that might be included in the analysis. Subsequently, a search was conducted on YouTube in Korean for 'mathematics (translated from Korean '수학' into English)' to focus on YouTube Shorts videos, resulting in the selection of 184 videos. From these, videos that do not contain mathematical content (e.g., promotion for private education, humor, and grading mathematics scores) were excluded from the analysis, leaving 95 videos as the final subject. The selection of the 95 videos was based on mutual agreement among the authors. The content of the 95 videos and the comments added to the videos were reviewed, especially those comments that received the most Likes and the responses to those comments.

Findings

95 videos were categorized into four key strands of mathematical narratives in YouTube Shorts (See Figure 1). First, Mathematics Introductions focused on general mathematical topics, including brief explanations of school and university-level mathematics, as well as content beyond the classroom, such as the history of mathematics (n = 31, 32.6%). Second, Arithmetic Methods covered topics

like multiplication tables, methods for calculating exponents, or real-life money calculations (n = 28, 29.5%). Third, Creative Mathematics Challenges aimed to stimulate creative thinking with content like one-stroke drawing or problem-solving puzzles (n = 20, 21.1%). Lastly, Controversial Mathematical Topics presented problems with multiple interpretations, addressing controversial mathematical questions (n = 16, 16.8%).

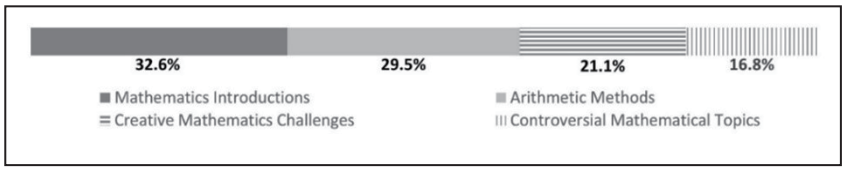


Figure 1. The distribution of mathematical narratives in Korean YouTube Shorts

Mathematics Introductions

A YouTube Shorts video, with 0.5M views, deals with the convergence of integrals (See Figure 2 (left-above)). It suggests that the integral of an odd function $f(x)$ might be interpreted as resulting in 0. However, since $f(x)$ is not continuous across the entire set of real numbers, this assumption is incorrect. The results of the two integrals diverge, so their sum cannot be considered zero. Viewers suggested their opinions through the comments on the videos and expressed their agreement using the 'Like' button. Viewers noted the importance of drawing function graphs in calculus and implicitly explained the significance of considering points of discontinuity.

Arithmetic Methods

In a video titled "A Problem Only Koreans Can Easily Solve", which recorded 10M views, the influencer asks if one can calculate 3+3×3−3+3 within 5 seconds (See Figure 2 (right-above)). The video mentions that the correct answer is not provided and that there is controversy over the correct answer internationally. The comment mentioning the correct answer as 'B.12' received the most Likes (34K). The most liked comment in response to this was asking why 12 is the correct answer and how some got 18.

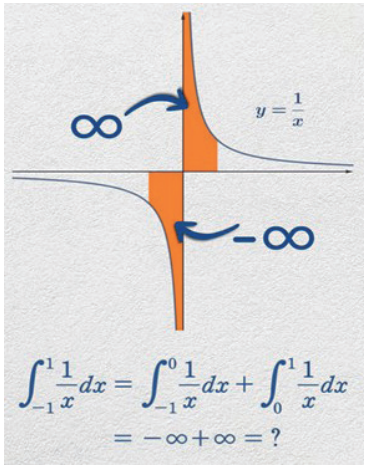
Creative Mathematics Challenges

A YouTube Shorts video with 2.7 million views features a logic-based number puzzle that requires identifying the pattern in the equations: 2▣3=9, 5▣4=26, 6▣2=13, 7▣4=? (See Figure 2 (left-below)). To solve it, one must deduce that the operation involves addition. The sum of three consecutive natural numbers starting from 2 equals 9. Similarly, the sum of four consecutive natural numbers starting from 5 is 26, and the sum of two consecutive natural numbers starting from 6 is 13. Thus, the sum of four consecutive natural numbers starting from 7 equals 34. The number before ▣ indicates the starting point of the consecutive natural numbers, and the number after ▣ indicates the count of consecutive numbers to be summed. The equals sign yields the result of this summation. One viewer commented that 2▣4 can be calculated as 14, which provides an example of applying the content of the video using different numbers.

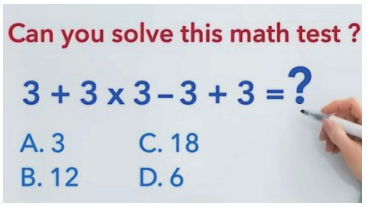
Controversial Mathematical Topics

A YouTube Shorts video with 5M views proposed a mathematics quiz (See Figure 2 (right-below)). While '6' might be the expected answer, the video suggests that '6' is not the answer without presenting the correct one. In the comments, the most Likes (8K) were for a comment stating '6' is the

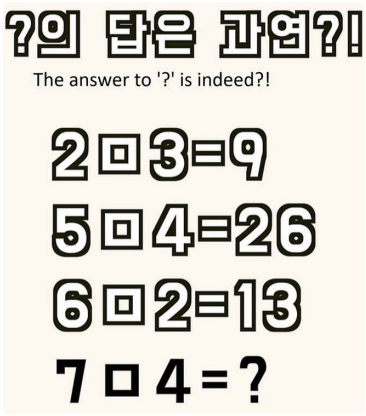
answer. Some comments mentioned that since the video said ‘6’ is not the answer but did not say ‘6’ is not, ‘6’ can be considered the correct answer. The answer can vary depending on the viewer’s interpretation of the problem..



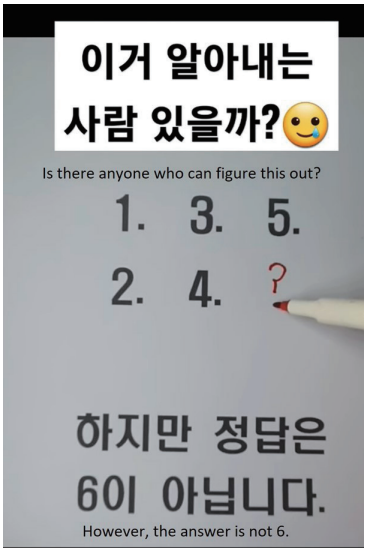
Example of *Mathematical Introductions*

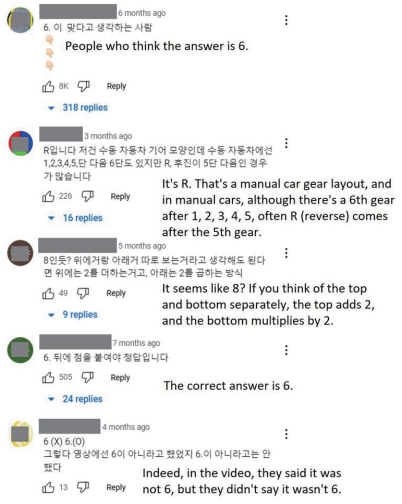


Examples of *Arithmetic Methods* and citizens’ communication



Example of *Creative Mathematics Challenges*





Examples of *Controversial Mathematical Topics* and citizens’ communication

Figure 2. Examples of mathematical narratives and citizens’ communication in YouTube Shorts

Conclusion

In this study, we identified four key strands concerning mathematics narratives in Korean YouTube Shorts. Additionally, we analyzed communication and prevailing opinions expressed by citizens, shedding light on their understanding and perspectives regarding mathematics. We found that comments on YouTube Shorts related to mathematics serve as a platform for citizens’ communication about mathematics. By presenting explanations to viewers in unique ways, commenters seek val-

idation through Likes and comments, fostering a community-driven approach to finding the most compelling explanations. The examples presented in this study suggest that social media can have a significant influence on people's everyday lives and an effect on their perceptions and ideas of mathematics.

References

Bini, G., Bikner-Ahsbahs, A., & Robutti, O. (2023). "How to meme it": reverse engineering the creative process of mathematical Internet memes. *Educational Studies in Mathematics, 112*(1), 141-174.

Bini, G., Robutti, O., & Bikner-Ahsbahs, A. (2022). Maths in the time of social media: Conceptualizing the Internet phenomenon of mathematical memes. *International Journal of Mathematical Education in Science and Technology, 53*(6), 1257-1296.

Carpentieri, J. D., Litster, J., & Frumkin, L. (2010). *Adult numeracy. A review of research: research report commissioned by BBC Learning and conducted by NRDC*. National Research and Development Centre for Adult Literacy and Numeracy, London.

Condelli, L., Safford-Ramus, K., Sherman, R., Coben, D., Gal, I., & Hector-Mason, A. (2006). *A review of the literature in adult numeracy: Research and conceptual issues*. Prepared by American Institutes for Research for the Adult Numeracy Initiative of the US Department of Education's Office of Vocational and Adult Education. https://files.eric.ed.gov/fulltext/ED495456.pdf.

Lee, K., Han, C., Lee, C., & Kwon, O. N. (2025). Exploring mathematical narratives and citizens' communication in Korean YouTube Shorts. In Engelbrecht, J., Oates, C., & Borba, M. C. (Eds.), *Social Media in the Changing Mathematics Classroom* (pp. 311-331). Springer.

TSG6-2-10

DESIGNING AUGMENTED PAPER-BASED MATHEMATICS LEARNING ENVIRONMENT FROM AN EMBODIED COGNITION PERSPECTIVE: APPLICATION TO THE TRANSLATION OF QUADRATIC FUNCTIONS

Hyowon, Wang
Sookmyung Women's University, South Korea, eduwang1010.dev@gmail.com

Yunjoo, Yoo
Seoul National University, South Korea, yyoo@snu.ac.kr

This study explores the design and implementation of an augmented paper-based mathematics learning environment (APMLE) for learning translation of quadratic functions from an embodied cognition perspective. The APMLE integrates augmented reality (AR) tools with paper-based worksheets, allowing learners to explore mathematical concepts through sensorimotor experiences, self-directed inquiry, multimodal representation, and incorporation of AR tools. The study applied the APMLE in a classroom with two ninth-grade students and analyzed their learning processes through video recordings. Findings indicate that AR tools effectively support concept formation, formalization, and problem-solving by enhancing embodied experiences. The results highlight the potential of APMLEs in mathematics education, suggesting the need for further research on their application to various mathematical concepts and larger student groups.

Keywords: embodied cognition, augmented paper, augmented reality, learning environment design

Introduction

In recent years, there has been a growing interest in learning environments that utilize augmented reality (AR) tools, which overlay virtual images onto real-world scenes in real time (Wei et al., 2021; Mohammadhossein et al., 2024). While many studies have explored the motivational effects of AR or its use for video playback (Romadon & Kartiak, 2024; Sholikhah & Cahyono, 2021; Bakri, Sumardani et al., 2019; Bakri, Pratiwi et al.), systematic research on AR environments that support mathematical concept formation through interaction remains limited (Masmuzidin et al., 2022).

From the perspective of embodied cognition, learners form concepts through physical movement and sensorimotor experiences (Nagataki & Hirose, 2007). AR tools can play a crucial role in linking such embodied experiences to mathematical exploration. In particular, AR tools can enable learners to directly manipulate graphs, visually explore changes, and progressively develop mathematical concepts (Bujak et al., 2013). In this context, this study focuses on designing an augmented paper-based mathematics learning environment utilizing augmented paper technology, a type of AR technology.

Paper-based media continue to play a significant role in educational settings, providing the advantage of allowing learners to write and manipulate content directly (Ackerman & Goldsmith, 2011).

Augmented paper technology extends the learning experience by overlaying digital resources from AR tools onto paper-based content, thereby connecting the information presented in both media (Han et al., 2021). Such an augmented paper-based learning environment maintains the benefits of traditional paper while enabling learners to explore mathematical concepts through physical interactions supported by AR tools (Wang, 2025).

Therefore, this study aims to design an augmented paper-based mathematics learning environment (APMLE) for learning translation of quadratic functions from an embodied cognition perspective. It will be applied in an actual classroom to analyze how students engage in the learning process. Through this investigation, the study seeks to explore the role of APMLEs in concept formation and problem-solving processes, ultimately contributing to discussions on their potential educational applications.

Theoretical Background

Augmented Paper

As technology has advanced, the resources available through digital media have become much more abundant, yet paper media continues to be used in many settings, including classrooms (Prieto et al., 2014; Sellen & Harper, 2003). This is because paper and digital media differ in how they present information and induce learners' responses—making it difficult to fully substitute one for the other (Noyes & Garland, 2008)—and because paper media offers several advantages over digital media. Compared to digital media, paper media has been found to be superior in metacognitive aspects, such as regulating the learning process (Ackerman & Goldsmith, 2011), and multiple studies continue to report that paper media could bring better learning outcomes (Mangen et al., 2013; Chen et al., 2014; Delgado et al., 2018).

Paper media has advantages in terms of tangibility, as users can physically touch it; accessibility, as it is cost-effective and readily available; and flexibility, as users can freely modify its form to suit their needs (Han et al., 2021). These advantages enable intuitive interactions in paper-based environments, allowing users to freely annotate, rearrange, and manipulate the layout of paper for interaction (Liu, 2005; O'Hara & Sellen, 1997; Johnson et al., 1993; Prieto et al., 2014). Therefore, to leverage the advantages of paper media along with the capabilities of digital media (e.g. multimedia sources, virtual 3d models), augmented paper technology (Mackay & Fayard, 1999) has been proposed. Augmented paper technology refers to the use of physical paper as an interface to access digital resources provided by computing devices. It allows users to simultaneously perceive and interact with both paper and digital media, enabling a blended sensory and interactive experience.

Augmented Paper-based Mathematics Learning Environments from an Embodied Cognition Perspective

From the embodied cognition perspective, human cognition can be understood as a functional and dynamic system that evolves and adapts through continuous interaction between the body and the environment. When artifacts are appropriately integrated, the cognitive system can be extended (Shvarts & Abrahamson, 2023; Lockman, 2000). In this context, augmented reality mathematics worksheets (ARMW), a type of augmented paper technology, combine physical paper with digital resources, enabling learners to perceive multiple representations simultaneously and actively establish conceptual connections. Thus ARMW contribute to the extension of learners' cognitive systems in APMLE, leading to the following characteristics in the learning process.

First, ARMW strengthen learners' representational associations and promote meaning-making by linking paper-based learning environments with digital media. The integrated learning environment, which combines physical paper with virtual objects provided by AR tools, allows learners to maintain the spatiotemporal continuity of information while actively translating between different representations. This process plays a crucial role in fostering deep understanding (Ainsworth, 1999). Second, learners can actively form embodied experiences through physical interactions facilitated by AR tools and reflect on these experiences to connect them to mathematical concepts (Flood et al., 2022). AR tools enable learners to manipulate virtual images that are difficult to implement in reality, thereby encouraging richer physical activities and fostering the formation of virtual perceptual structures (Ladel & Kortenkamp, 2016). Lastly, ARMW promote mathematical discourse through collaborative interactions. Learners engage in a process of reflecting on their perceptual-motor experiences, verbalizing their understanding, and coordinating with others to construct meaning (Shvarts & Abrahamson, 2023). Through the harmonious integration of physical experiences and digital information, an APMLE minimizes learners' cognitive load while providing an in-depth learning experience. In this study, based on the design principles for developing an APMLE from an embodied cognition perspective (Wang, 2025), we designed and developed an APMLE for exploring translation of quadratic functions and applied to the classroom.

Result

Instructional Design

In this study, an APMLE was developed and applied in a lesson to support the learning translation of quadratic functions. The lesson was conducted as a single session with two ninth-grade students who had low academic achievement. The instructional design emphasized the process of concept formation based on learners' physical experiences, followed by formalization and incorporation. The lesson was structured into three stages: the introduction stage, the concept learning stage, and the application stage.

In the introduction stage, students first sketched basic quadratic function graphs by hand and then used an AR tool to compare their graphs on the ARMW with the given reference graphs. Through this process, students directly observed changes in the graphs, forming foundational experiences for learning the concept of quadratic function translation. In the concept learning stage, students visually explored the horizontal and vertical translations of quadratic function graphs using the AR tool. They manipulated a slider on the screen to move the graph and observed the resulting changes, which they recorded on their worksheets. Following this activity, a discussion was held in which students analyzed, based on their sensorimotor experiences, the elements that changed and remained unchanged during the translation process. Through this reflection, they formalized their understanding of the concept. In the application stage, students completed tasks that required them to apply the concept of quadratic function translations without using the AR tool. They either sketched translated graphs by hand or identified translated graphs among given options. These activities helped students apply their newly acquired knowledge and incorporate the AR tool.

Findings

This study analyzed how students' learning processes unfolded within the APMLE during an actual classroom lesson. To achieve this, video recordings were made of the students' interactions and their tablet PC screens during the lesson. These recordings were analyzed in conjunction with the

design principles used in developing the APMLE, as well as the gesture categorization framework proposed by Moon (2020). Moon (2020) categorized the gestures of teachers and students related to mathematical meaning-making into two types: gestures of actualization of the virtual and gestures of realization of the possible. The actualization of the virtual refers to the process by which new mathematical meanings are formed by breaking away from existing meaning systems, habits, or rules. This includes moments when learners notice new objects or generate novel ideas. The realization of the possible refers to the formation of mathematical meaning in accordance with previously followed meaning systems, habits, or rules. It encompasses moments of refining logic or symbols through inductive or deductive reasoning (de Freitas & Sinclair, 2014).

The analysis revealed that students' learning progressed through the following stages: exploring concepts through sensorimotor experiences, constructing concepts through self-directed inquiry, formalizing concepts through multimodal representation, and applying concepts through the incorporation of the AR tool. Furthermore, gestures of actualization of the virtual were prominently observed during the stages in which students engaged with the augmented reality tools, while gestures of realization of the possible appeared in the process of formalizing the embodied experiences formed through the use of these tools.

Students actively explored mathematical concepts by sketching and comparing graphs using the AR tool. They autonomously manipulated sliders to observe changes in quadratic function translations, gradually structuring their understanding. Additionally, they formalized their conceptual understanding through physical gestures and verbal explanations. When solving problems without the AR tool, they demonstrated an ability to mentally reenact their prior experiences with the tool's functions. These findings suggest that an AR-based learning environment effectively supports concept learning by utilizing students' embodied experiences while also reinforcing conceptual formalization and incorporation.

Conclusion

This study designed and developed an APMLE for learning translations of quadratic functions, applied it in an actual classroom, and analyzed students' learning processes. The analysis of students' learning revealed that they explored mathematical concepts through sensorimotor experiences using the AR tool and proceeded through stages of conceptual formalization and incorporation. These findings indicate that the APMLE facilitate mathematics learning by leveraging students' embodied experiences. The AR-based environment supported concept formation, multimodal formalization, and successful incorporation of the tool.

However, this study has a limitation in that it was conducted with a small number of students in order to observe the process of mathematical meaning-making. In addition, the activity of perceiving and exploring mathematical concepts by using an AR tool in conjunction with a paper-based environment can impose cognitive load on students who are not yet familiar with the tool, and it was observed that sufficient time is required for them to become accustomed to such activities. Future research should consider the environmental conditions that allow students to use AR tools proficiently, develop APMLEs for a wider range of mathematical concepts, and conduct studies with larger groups of learners to more specifically examine how the experiences provided by APMLEs contribute to students' conceptual understanding.

References

Ackerman, R., & Goldsmith, M. (2011). Metacognitive regulation of text learning: on screen versus on paper. *Journal of experimental psychology: Applied*, *17*(1), 18.

Ainsworth, S. (1999). The functions of multiple representations. *Computers & education*, *33*(2-3), 131-152.

Bakri, F., Sumardani, D., & Muliyati, D. (2019). Integrating augmented reality into worksheets: Unveil learning to support higher-order thinking skills. In F. Bakri, et al. (Eds.), *AIP Conference Proceedings* (Vol. 2169, No. 1), 020012. AIP Publishing.

Bakri, F., Pratiwi, S., & Muliyati, D. (2019). Video-enriched worksheet based on augmented reality technology: The heat experiment is easier. In F. Bakri, et al. (Eds.), *AIP Conference Proceedings* (Vol. 2169, No. 1), 020010. AIP Publishing.

Bujak, K. R., Radu, I., Catrambone, R., MacIntyre, B., Zheng, R., & Golubski, G. (2013). A psychological perspective on augmented reality in the mathematics classroom. *Computers & Education*, *68*, 536-544.

Chen, G., Cheng, W., Chang, T. W., Zheng, X., & Huang, R. (2014). A comparison of reading comprehension across paper, computer screens, and tablets: Does tablet familiarity matter?. *Journal of computers in education*, *1*, 213-225.

de Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. Cambridge University Press.

Delgado, P., Vargas, C., Ackerman, R., & Salmerón, L. (2018). Don't throw away your printed books: A meta-analysis on the effects of reading media on reading comprehension. *Educational research review*, *25*, 23-38.

Flood, V. J., Shvarts, A., & Abrahamson, D. (2022). Responsive Teaching for Embodied Learning with Technology. In S. L. Macrine & J. M. Fugate. (Eds.), *Movement Matters: How Embodied Cognition Informs Teaching and Learning* (pp. 179-195). Cambridge, UK: Cambridge University Press.

Han, F., Cheng, Y., Strachan, M., & Ma, X. (2021). Hybrid paper-digital interfaces: A systematic literature review. In W. Ju, et al. (Eds.), *Proceedings of the 2021 ACM Designing Interactive Systems Conference* (pp. 1087-1100).

Johnson, W., Jellinek, H., Klotz Jr, L., Rao, R., & Card, S. K. (1993). Bridging the paper and electronic worlds: the paper user interface. In B. Arnold, et al. (Eds.), *Proceedings of the INTERACT'93 and CHI'93 Conference on Human Factors in Computing Systems* (pp. 507-512). Association for Computing Machinery.

Ladel, S., & Kortenkamp, U. (2016). Artifact-Centric Activity Theory—A framework for the analysis of the design and use of virtual manipulatives. In P. Moyer-Packenham. (Ed.) *International perspectives on teaching and learning mathematics with virtual manipulatives* (Vol. 7, pp. 25-40). Springer, Cham.

Liu, Z. (2005). Reading behavior in the digital environment: Changes in reading behavior over the past ten years. *Journal of documentation*, *61*(6), 700-712.

Lockman, J. J. (2000). A perception–action perspective on tool use development. *Child development*, *71*(1), 137-144.

Mackay, W. E., & Fayard, A. L. (1999). Designing interactive paper: lessons from three augmented reality projects. In *Proceedings of IWAR* (Vol. 98, pp. 81-90).

Mangen, A., Walgermo, B. R., & Brønnick, K. (2013). Reading linear texts on paper versus computer screen: Effects on reading comprehension. *International journal of educational research*, *58*, 61-68.

Masmuzidin, M. Z., Aziz, N. A. A., & Suhaimi, S. (2022). A Systematic Review of Augmented Reality Design for Young Children. *International Journal of Interactive Mobile Technologies*, *16*(17), 60-74.

Mohammadhossein, N., Richter, A., & Lukosch, S. (2024). Augmented reality in learning settings: a systematic analysis of its benefits and avenues for future studies. *Communications of the Association for Information Systems*, *54*(1), 29-49.

Moon, S. J. (2020). Categorization of gestures in mathematics education. *The Journal of Learner-Centered Curriculum and Instruction*, *20*(17), 859-878.

Nagataki, S., & Hirose, S. (2007). Phenomenology and the third generation of cognitive science: Towards a cognitive phenomenology of the body. *Human Studies*, *30*, 219-232.

Noyes, J. M., & Garland, K. J. (2008). Computer-vs. paper-based tasks: Are they equivalent?. *Ergonomics*, *51*(9), 1352-1375.

O'hara, K., & Sellen, A. (1997). A comparison of reading paper and on-line documents. In R. Grinter, et al. (Eds.), *Proceedings of the ACM SIGCHI Conference on Human factors in computing systems* (pp. 335-342). Association for Computing Machinery.

Prieto, L. P., Wen, Y., Caballero, D., & Dillenbourg, P. (2014). Review of augmented paper systems in education: an orchestration perspective. *Educational Technology & Society*, *17*(4), 169-186.

Romadon, A. G., & Kartika, E. D. (2024). Development of ethnomathematics student worksheets based on augmented reality on geometric material. *International Journal Of Multidisciplinary Research And Analysis*, *7*(3), 1043-1049.

Sellen, A. J., & Harper, R. H. (2003). *The myth of the paperless office*. MIT press.

Sholikhah, B. U., & Cahyono, A. N. (2021). Augmented reality student worksheets for learning mathematics during the COVID-19 pandemic. In *Journal of Physics: Conference Series* (Vol. 1918, No. 4, p. 042063). IOP Publishing.

Shvarts, A., & Abrahamson, D. (2023). Coordination dynamics of semiotic mediation: A functional dynamic systems perspective on mathematics teaching/learning. *Constructivist Foundations*, *18*(2), 220-234.

Wang, H. (2025). Development of Design Principles for Embodied Cognition-Based Augmented Reality Mathematics Learning Environments: Focusing on Augmented Paper Technology. [Doctoral dissertation, Seoul National University].

Wei, C. Y., Kuah, Y. C., Ng, C. P., & Lau, W. K. (2021). Augmented Reality (AR) as an enhancement teaching tool: Are educators ready for it?. *Contemporary Educational Technology*, *13*(3), ep303.

TSG6-2-11

COLLABORATIVE PRACTICES IN VIRTUAL GROUP WORK ON DYNAMIC GEOMETRY TASKS

Younggon Bae
The University of Texas Rio Grande Valley, USA, younggon.bae@utrgv.edu

V. Rani Satyam
Virginia Commonwealth University, USA, vrsatyam@vcu.edu

Zareen G. Aga
James Madison University, USA, agazg@jmu.edu

The goal of this study is to explore productive ways to engage students in group work using dynamic geometry tasks in online synchronous classroom environments. We aim to understand the social, mathematical, and technological aspects of student collaboration in virtual spaces. We analyzed how three online groups of students collaboratively worked on dynamic geometry tasks of exploring interactive kaleidoscope applets in Desmos and producing visual representations and written descriptions of geometric transformations used in the applets. We identified three emerging practices of the students collaboratively working on dynamic geometry tasks in online synchronous environments: (a) drawing in response, (b) co-construction, and (c) writing in real time. We also discuss inequity in students' participation in collaborative practices in an online environment and possible ways to ensure equitable learning opportunities for online students.

Keywords: Dynamic geometry, Task design, Collaborative practice, Online synchronous learning environment

Introduction

We investigated productive ways to engage students in group work on dynamic geometry tasks in online synchronous classroom environments. With the increasing adoption of online learning platforms, educators face challenges in fostering meaningful collaboration among students, particularly in STEM fields where tasks often require deep engagement with conceptual understanding and problem-solving while collaborating with others (e.g., Tomaszewski, 2023). The study focuses on how students' social, mathematical, and technological interactions are mediated in virtual spaces and seeks to identify core practices that can enhance collaborative learning in geometry. It also addresses equity issues in virtual collaboration, recognizing that not all students participate or benefit equally from such activities. In this study, we conceptualized *virtual collaboration* as students' social interactions in online synchronous group work settings, enabling meaningful engagement with mathematical processes. Based on this framework, our research question is: How do students engage in virtual collaboration for learning geometry in an online synchronous environment? In particular, we focused on identifying students' emergent practices of virtual collaboration that demonstrate social, technological, and mathematical aspects.

Framework

The study is grounded in a framework that integrates three core aspects of virtual collaboration: social interaction, technological mediation, and mathematical engagement. Prior studies have provided insights into understanding three different aspects of collaborative mathematical practices in online synchronous environments (e.g., Cohen, 1990; Johnson & Johnson, 1999; Sampson & Clark, 2009; Webb, 2013). Effective online collaborative learning requires equitable participation, group norms, and a safe space for students to share ideas and engage in collective knowledge construction (e.g., Barron, 2000; Blatchford et al., 2003; Patterson, 2019; Sandoval, 2003). Technology (e.g., dynamic geometry environments) mediates students' interactions and supports collective sense-making, communication, and knowledge-building in collaborative mathematical tasks (e.g., Alqahtani & Powell, 2016; 2017; Edson et al., 2018; Medina & Stahl, 2021). Collaborative group work in technology-integrated environments fosters mathematical inquiry and disciplinary engagement by enabling students to explore, represent, and refine their understanding of mathematical concepts (e.g., Bieda et al., 2020; Edson & Phillips, 2021; Stahl, 2016). A framework that incorporates all three aspects of collaboration can be useful for researchers who face challenges in understanding how students work with others (i.e., social) on challenging mathematical tasks (i.e., mathematical) using technological tools in an online environment (i.e., technological). It also provides insights into productive ways to foster students' meaningful collaboration in such environments regarding the connections among the aspects as well as practical guidelines for educators to enhance their instructional practices such as task design, technology selection, group norm settings, and evaluation.

Methods

Participants were pre-service mathematics teachers (PSTs) enrolled in online mathematical content and methods courses at three US universities. Each of the three authors taught one of the courses independently and implemented a Desmos task in online synchronous settings via Zoom. Tasks were designed to be open-ended and high cognitive demand tasks (Stein & Lane, 1996), requiring students to generate group outcomes with an emphasis on the collaborative aspects of task completion, including screen sharing, drawing, and discussing findings. Students first individually explored Desmos applets representing geometric transformations (e.g., translation, reflection, rotation, and dilation), then collaborated in groups to create visual representations and written descriptions of each transformation. Participants shared their Desmos screens in Zoom and used Google Slides for collaborative work. We analyzed video recordings of group discussions, screen shares from Zoom sessions, and artifacts such as students' Desmos drawings and written descriptions in Google Slides. From each of the three courses, we selected the group that showed the most active interaction with each other, to select episodes with strong virtual collaboration. We conducted the first-level coding to describe student interactions along one or more components of virtual collaboration (social, technological, and mathematical) using the deductive coding method (Miles et al., 2014). After identifying episodes with one or more of the above codes, we conducted second-level coding of each episode using the constant comparative method (Glaser & Strauss, 1967) to find emergent themes as potential practices.

Findings

We identified three key practices that facilitated collaboration in online synchronous environments.

Drawing-in-Response

This practice was identified when a student explained mathematical ideas involving geometric objects, and another student drew figures in the shared screen of Desmos. This practice emerged as a way for students to be responsive to each other's thinking and supported clearer communication and mathematical reasoning. Without a direct request, the drawer often drew figures on the shared screen while making sense of the conversation, to demonstrate their interpretation, or to respond to the speaker non-verbally. For example, Val described to the group how he drew triangles, and then Tom drew them on his shared Desmos screen, of his own accord. Val then described more objects he had drawn - in particular lines in the middle of the screen - and his reasoning while pointing at the screen:

> Tom: Did anyone have a specific approach to this?
> Val: Well, I drew triangles first, just to see how it worked
> Tom: So like you started at a point, and then drew a triangle?
> Val: Yeah. And then I also just drew a line in the middle and then up…just to see how it would affect [the kaleidoscope] when you go all the way up and [the line] touches.

Drawing-in-response supports students socially interacting with each other in a collaborative environment by allowing multi-modal communication about geometric ideas, improving the quality of their discussion for collaborative tasks involving spatial and geometric figures that are difficult to convey in verbal explanation. Drawing-in-response enables students to engage in mathematical processes as they create, observe, and interpret geometric diagrams. When drawing figures following others' verbal explanations, students are expected to make sense of mathematical ideas and to demonstrate geometric construction. Drawing-in-response provides opportunities to learn how to utilize technological environments for online synchronous group work in a shared virtual space for remote students.

Co-construction

This practice occurred when multiple students were involved in decision-making collaboratively while constructing a figure. This takes the form of one person sharing their screen and taking control of drawing, while others suggest how to draw, add, or modify existing figures (e.g., "Why don't we try this?", "Can you draw vertical lines from that point?"). This practice emerged repeatedly across groups when students built off each other's contributions to investigate specific properties of the kaleidoscopes and to produce the best representation to submit for the task. For example, Tom (Group 3) drew similar hexagons on the rotational symmetry applet. Val suggested extending the sides of a hexagon on one side to illustrate the rotation precisely and distinguish it from reflection: "I think you should include the black hexagon though for having the rays come out of it". At the end, elements of all three students' individual work became a part of the group's drawing.

Writing in Real Time

This practice was identified when students collectively produced their written descriptions for the kaleidoscopes in a shared Google slide. This practice involved writing on the slide, suggesting changes, editing, proofreading, asking for help, and/or asking for reassurance of what was written. For example, students queried others about what to write (e.g., "Can you repeat what you just said?"). Students would add or modify existing descriptions written by another person. For example, Tom read their descriptions for the rotation applet out loud and tried to improve the last written sentence, "equal distance points from the origin as well as the points themselves".

Tom: Let's see. So for every point...I'd say, rather than "has a given distance," say "When creating points…the point you create has a given distance or a certain distance from the origin. The kaleidoscope then creates five more points with that same distance?
Val: Okay, I get what you mean, "in creating…" I'll say "when creating a point."

Writing in real time involved students' social engagement with each other including assigning different roles (e.g., writing, editing, proofreading, etc.) and discussing what content they want to include and how to express them. Technology provided students with a shared workspace where they could read, type, and edit text synchronously, so everyone in their groups could access the space and make changes in the writing in real time. Students attended to precision in their mathematical language by looking for accurate vocabulary and formal expressions to deliver their ideas.

Equitable Distribution of Opportunities to Collaborate

The study also revealed inequities in students' participation during group work. Some students dominated discussions or tool usage, while others struggled to contribute. For example, students who lacked confidence or technical proficiency were less likely to take leadership roles in drawing or writing. This allowed for missed opportunities for communication of ideas, hindering the process of collaborative meaning-making. This indicates the need for instructors to establish group norms and role assignments (e.g., drawer, writer, etc.). We also sound that students used their drawing and writing as a form of support to exercise their agency and share their ideas.

Discussion and Conclusion

The study suggests a framework to design and implement collaborative tasks for fostering meaningful student interactions and engagement in virtual settings. Three collaborative practices—drawing-in-response, co-construction, and writing in real-time—manifested the usefulness of the framework and offered insights into students social, mathematical, and technological engagements. The analysis showed that DGEs played a key role in facilitating students' meaningful discussions and collaboration by representing geometric ideas. The study also highlights the need for instructors to address equity by establishing norms for collaboration in online settings that ensure all students have opportunities to participate and contribute. Future research is needed to explore the generalizability of these practices across diverse contexts and student populations. Furthermore, refining task designs and technological tools could enhance their capacity to support equitable and productive group work.

References

Alqahtani, M. M., & Powell, A. B. (2016). Instrumental appropriation of a collaborative, dynamic-geometry environment and geometrical understanding. *International Journal of Education in Mathematics, Science and Technology, 4*(2), 72.

Alqahtani, M. M., & Powell, A. B. (2017). Mediational activities in a dynamic geometry environment and teachers' specialized content knowledge. *Journal of Mathematical Behavior, 48*, 77-94. https://doi.org/10.1016/j.jmathb.2017.08.004

Barron, B. (2000). Achieving coordination in collaborative problem-solving groups. *Journal of the Learning Sciences, 9*(4), 403-436. https://doi.org/10.1207/S15327809JLS0904_2

Bieda, K. N., Going, T., Kursav, M., & Edson, A.J. (2020, April 17-21). Entailments of productive disciplinary engagement in technology-enhanced mathematics classrooms [Conference paper]. *The 2020 American Education Research Association Annual Meeting*, San Francisco.

Blatchford, P., Kutnick, P., Baines, E., & Galton, M. (2003). Toward a social pedagogy of classroom group work. *International Journal of Educational Research, 39*(1-2), 153-172. https://doi.org/10.1016/S0883-0355(03)00078-8

Cohen, E. G. (1990). Continuing to cooperate: Prerequisites for persistence. *The Phi Delta Kappan, 72*(2), 134-138.

Edson, A. J., Kursav, M. N., & Sharma, A. (2018). Promoting collaboration and mathematical engagement in a digital learning environment. In T. E. Hodges, G. J. Roy, & A. M. Tyminski (Eds.), *Proceedings of the 40th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1243-1246). University of South Carolina & Clemson University.

Edson, A. J., & Phillips, E.D. (2021) Connecting a teacher dashboard to a student digital collaborative environment: supporting teacher enactment of problem-based mathematics curriculum. *ZDM Mathematics Education, 53*, 1285-1298. https://doi.org/10.1007/s11858-021-01310-w

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Hawthorne, NY: Aldine.

Johnson, D. W., & Johnson, R. T. (1999). Making cooperative learning work. *Theory into Practice, 38*(2), 67-73. https://doi.org/10.1080/00405849909543834

Medina, R., & Stahl, G. (2021). Analysis of group practices. In U. Cress, C. Rose, A. Wise, & J. Oshima (Eds.), *International handbook of computer-supported collaborative learning* (pp. 199-218). Springer. https://doi.org/10.1007/978-3-030-65291-3_11

Miles, M. B., Huberman, A. M., & Saldana J. (2014). *Qualitative data analysis: A methods sourcebook*. SAGEPublication. Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017-1054. https://doi.org/10.1111/j.1467-9620.2006.00684.x

Patterson, A. D. (2019). Equity in groupwork: The social process of creating justice in a science classroom. *Cultural Studies of Science Education, 14*, 361-381. https://doi.org/10.1007/s11422-019-09918-x

Sampson, V., & Clark, D. (2009). The impact of collaboration on the outcomes of scientific argumentation. *Science Education, 93*(3), 448-484. https://doi.org/10.1002/sce.20306

Sandoval, W. A. (2003). Conceptual and epistemic aspects of students' scientific explanations. *Journal of the Learning Sciences, 12*(1), 5-51. https://doi.org/10.1207/S15327809JLS1201_2

Stahl, G. (2016). *Constructing dynamic triangles together: The development of mathematical group cognition*. Cambridge University Press

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*(1), 50-80. https://doi.org/10.1080/1380361960020103

Tomaszewski, S. (2023). Modifying the didactical tetrahedron to describe aspects of mathematical digital collaborative learning. In *Thirteenth Congress of the European Society for Research in Mathematics Education* (CERME13). Alfred Renyi Institute of Mathematics; Eotvos Lorand University of Budapest.

Webb, N. (2013). Information processing approaches to collaborative learning. In C. E. Hmelo-Silver (Ed.), *The international handbook of collaborative learning* (pp. 19-40). Routledge.

Acknowledgement

The research presented in this conference paper is based on a study that was previously published as Bae, Y., Aga, Z. G., & Satyam, V. R. (2024). Collaborative practices in virtual group work on dynamic geometry tasks. *Journal of the Korean Society of Mathematical Education Series D: Research in Mathematical Education 27*(3), 367-399.

TSG6-2-12

PROVING ACTS WITH DIGITAL TOOLS FOR INQUIRING INTO MATHEMATICAL PROPOSITIONS IN LEARNING GEOMETRY

Keiko Watanabe
Shiga University, Japan, mukai@edu.shiga-u.ac.jp

Masataka Koyama
Hiroshima University, Japan, mkoyama@hiroshima-u.ac.jp

The purpose of this paper is to consider which functions of digital tools are useful in proving acts for inquiring into mathematical propositions in learning geometry. To this end, we first discuss the need for reflective development of proving acts for inquiring into a proposition. Then, we develop a framework that captures the two levels that are essential to this developmental process. Finally, we identify three key functions (copying, pasting and modifying a part of the pasted proof description) of digital tools used for the proving acts. As an example, we show that the proof methods and the conclusion of Proposition (B) concerning triangles ABC can be used to prove Proposition (A) in an isosceles triangle ABC (AB=AC).

Keywords: digital tool, proving act, mathematical proposition, learning, geometry

Proving Act as an Activity for Inquiring into Mathematical Propositions

Knowing a proposition is related to constructing a proof of the proposition. In mathematics, a proposition is a principal expression that indicates a mathematical idea, and "mathematical ideas do not exist as plain facts but as statements which are accepted only once they have been proved explicitly" (Balacheff, 2010, p.117). In mathematical inquiry, *proving* is an essential activity, because it involves thinking deeply about the truth, value, and nature of mathematical phenomena, and then clarifying them by following a logical path. In this paper, the term *proving act* is used to refer to all mathematical acts performed to prove a mathematical proposition. The purpose of proving acts is always directed towards the inquiry of mathematical propositions. We will show one possible direction in which proving acts with digital tools are developed reflectively in learning geometry.

The process of inquiry into a proposition makes us construct a proof, and conversely, how we construct a proof makes us reflectively grasp the proposition (Okazaki & Watanabe, 2021). Many studies have focused on finding counterexamples to a mathematical proposition and the mutual relationship between a proof and its refutation based on Lakatos' description of proof and refutation. The main theme of these studies is how learners acquire mathematical theorems, by exploring how to modify the proposition and its proof with the aim of dealing with counterexamples (Komatsu & Jones, 2019). However, in actual school mathematics, there are not many opportunities to refute. In Japanese high schools, there are even fewer opportunities for students to encounter and refute incomplete propositions that allow them to discover counterexamples. Therefore, in this paper, we will clarify the role of digital tools in the process of re-understanding a mathematical proposition by re-proving it from a different perspective.

A Framework for Reflective Development of Proving Acts

Balacheff (2010) argued that in order to identify proof, it is necessary to clarify the nature of the relation between the subject's knowing and what is involved in the 'proof', by structuring the relationship between 'explanation', 'proof'' and 'mathematical proof'. On the other hand, this study argues that 'mathematical proof' can be further structured by learners' knowledge of the mathematical subject. Proving acts develop reflectively as learners grow in their learning about the subject. For example, let's say that a Proposition (A) about an isosceles triangle and its Proof1 (A) are given. In the same context, when Proposition (B) is given for a triangle, it is usual to generate a Proof (B) of Proposition (B). At this point, it becomes possible to actively reconsider Proposition (A) in a different way. Specifically, the proof of the "partially modified" version of Proof (B) should be a different proof of Proposition (A). In this paper, we will denote such a proof as Proof2 (A). In this way, students who have learned to perceive isosceles triangles conceptually through comparison with other triangles are able to construct a proof for isosceles triangles (Proof2 (A)) from the proof for triangles (Proof (B)), and through these proofs, students are able to conceptualize them. The proving act that makes it possible to construct Proof2 (A) is different from the proving act that simply constructs Proof1 (A) of Proposition (A). The difference between Proof1 (A) and Proof2 (A) is due to the difference in the "subject's knowing" levels. The shifting of levels from the former to the latter is prompted by the realization of Proposition (B) which completely contains Proposition (A) and its Proof (B). This is a clear shift from a lower-level to an upper-level in terms of the relationship between "subject's knowing" and "what is involved in a proof" (Figure 1).

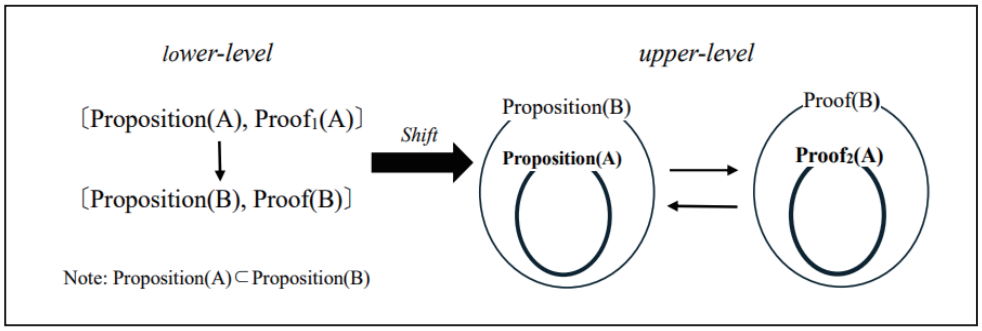


Figure 1. Two levels of mathematical proof through reflective development of proving acts

The Use of Digital Tools to Re-generate Mathematical Proofs

In order to develop the proving act in a reflective way, that is, to develop the proving act for the purpose of inquiry into propositions, two acts are necessary to achieve the level shift shown in Figure 1. One act is to *copy and paste* the description of the proof or the diagrams used to prove a proposition, either partially or in their entirety, as necessary. In the development of proving acts, the act of *copy and paste* basically means that the relationship between the copied subject (Proof (B) in Figure 1) and the pasted object (a different proof from $Proof_1$ (A) of Proposition (A) in Figure 1) is regarded as the same kind. The other act is to *modify* the copied proof descriptions and/or figures. This act highlights the *difference* between Proof (B) and Proof (A), and clarifies how Proof (A) can be generated as a *special* proof of Proof (B). In other words, the modifying act makes the boundary between Proof (B) and $Proof_2$ (A) more distinct as shown in Figure1.

Case Study with Digital Tools: Generating Two Levels of Mathematical Proof

The above mentioned can be demonstrated using the following proposition as an example, that is, Proposition (A); "Let D be the intersection of the bisector of ∠A and BC in an isosceles triangle ABC (AB = AC). Then, BD = DC." In Japan, students usually encounter the proving problem of Proposition (A) when they are in their second year of junior high school. At that time, the students had just learned the congruence conditions for triangles, so they intuitively looked for congruent triangles and constructed Proof$_1$ (A) of Proposition (A). After this, in the third year of junior high school in Japan, students encounter the Proposition (B);"Let D be the intersection of the bisector of ∠A and BC in a triangle ABC. Then, BD:DC = AB:AC." For example, the following kind of Proof (B) is constructed by the students; "Draw a line parallel to AD that passes through point C, and let E be the intersection with the extension of BA. Therefore, AE = AC. Also, from △BAD∽△BEC, BA:AE = BD:DC. That is, AB:AC = BD:DC." In this paper, we set out to develop a proving act that inquiries into Proposition (A), and in relation to the similarities between Proposition (A) and Proposition (B), we improve Proof (B) into a proof of Proposition (A) (but as a different proof from Proof $_1$(A)), and generate a special proof based on the structure of Proof (B), namely Proof $_2$ (A).

The acts that are carried out currently with digital tools are very basic: *copying* and *pasting*. That is, whole of the description of Proof (B) is copied and pasted to become the proof of Proposition (A). Inquiring into Proposition (A) is realized as a *partially modifying act* for Proof (B), and the result appears as Proof$_2$ (A) (Figure 2).

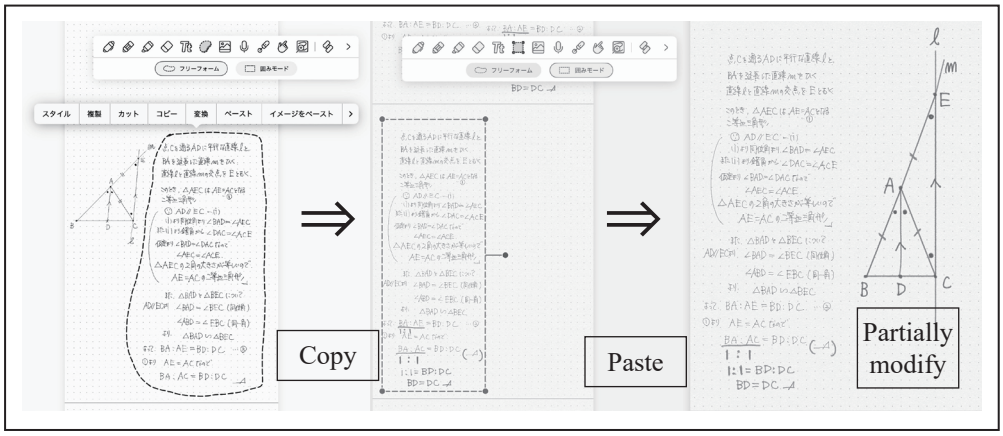


Figure 2. Acts with digital tools for developing the proving acts

Three Key Functions of Digital Tools for the Proving Acts

In the generation of a mathematical proof, the act of "copying the statement of the proof" is accomplished when a learner intuitively recognizes the similarity between an already-proven proposition and an upcoming proposition, and grasps the inclusion relationship between the objects of multiple propositions (for example, the isosceles triangle that is the object of Proposition (A) and the triangle that is the object of Proposition (B). All isosceles triangles are triangles.) in the aspect of "object". This aspect can be seen as an aspect of the recognition of propositions and proofs that we pass through in order to move away from the level that deals with a single combination of proposition and proof (*lower-level* in Figure 1).

However, it is not always the case that learners at *lower-level* are able to logically grasp the relationship between the propositions themselves. In order to reach an even higher level, it will be necessary to conduct inquiries in the aspect of "proof". This is achieved by the act of modifying a part of the copied proof. In the example given above, the conclusion of Proposition (B) concerning triangles (BA:AC = BD:DC*) can be used, if it is viewed as the conclusion of Proposition (A) in an isosceles triangle. In other words, in an isosceles triangle ABC, by expressing the fact that "BA = AC" as "BA:AC = 1:1" with a ratio, the above equation (*) becomes 1:1 = BD:DC. So, BD=DC (Figure 3).

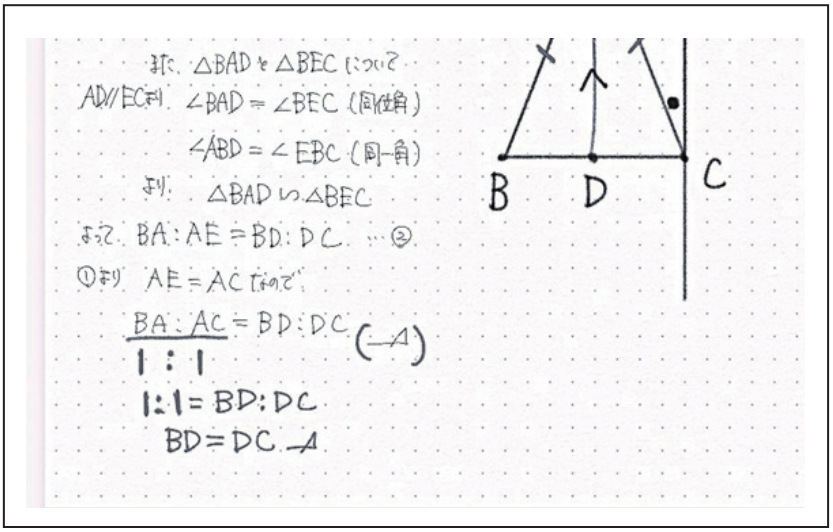


Figure 3. *Partial modification in the copied description of proof at upper-level*

In addition, in order to capture the proving acts for inquiring into mathematical propositions, it would be necessary to use functions that can record changes in the digital screen and also record the learner's voice. Moreover, investigating the basis for learners' decisions to "copy and paste" and to "modify the copied and pasted proof description" could become one way of clarifying how the inquiry into a mathematical proposition can be realized through proving the proposition.

References

Balacheff, N. (2010) Bridging knowing and proving in mathematics: An essay from a didactical perspective. In G. Hanna, H. N. Jahnke, & H. Pulte, (Eds.), *Explanation and Proof in Mathematics* (pp. 115-135). Heidelberg. Springer.

Komatsu, K. & Jones, K. (2019) Task design principles for heuristic refutation in dynamic geometry environments. *International Journal of Science and Mathematics Education*, *17*(4), 801-824.

Okazaki, M. & Watanabe, K. (2021). Enhancing the level of geometric thinking through learning the inscribed angle theorem. In M. Inprasitha, N. Changsri, & N. Boonsena, (Eds.), *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education*, *3* (pp. 420-430). Khon Kaen, Thailand: PME.

TSG6-2-13

CAPTURING COMPUTATIONAL THINKING IN EARLY MATHEMATICS EDUCATION: A CASE STUDY FROM A CHINESE KINDERGARTEN

Ying Zhang
University of Hong Kong, zhangy15@connect.hku.hk

Gary Wong
University of Hong Kong, wongkwg@hku.hk

Integrating computational thinking (CT) into mathematics education has been increasingly regarded as promising to transform mathematics education in the digital era, while also fostering essential problem-solving skills associated with CT. Although this integration has been explored at primary school and upper levels, the line of research in early childhood remains underdeveloped. As an initial step addressing this gap, this study investigates the CT competence embedded in existing mathematics classroom activities with varying levels of cognitive demand, through analysing semi-structured interviews with teachers in a Chinese kindergarten. By mapping mathematics activities onto CT elements identified, the findings contribute to the body of literature in not only enriching conceptual understanding of early CT but also informing future efforts to intentionally integrate CT into formal early childhood education through mathematics.

Keywords: Mathematics, Early Childhood Education, Computational Thinking, Integration, Classroom Activity

Background

To prepare younger generations for an AI-empowered future, there remains much to explore in transforming the traditional education paradigm—one that has been largely focused on disciplinary knowledge to train the workforce in response to the Industrial Revolution—towards the development of more generic skills upon the digital revolution in recent decades (Collins & Halverson, 2010). Against the backdrop of this educational shift, computational thinking (CT) was popularised by Wing (2006) as an essential set of thinking skills in problem-solving situations and is necessary for everyone to grasp from an early age. Since then, researchers (e.g., Barr & Stephenson, 2011; Lee et al., 2020) have advocated for integrating CT into K-12 curriculum through different subjects, including mathematics, to help students accommodate the increasingly computational world.

A recent systematic review synthesised 21 empirical studies on the interplay between CT and early mathematics education at the preK-2 level (Zhang & Wong, 2024), which has provided insights not only into the cognitive mechanisms linking computational and mathematical thinking (e.g., Gerosa et al., 2021) but also into the complex interactions between the learning of CT components and mathematics content. For instance, children engaged in numerical, spatial, and measurement learning while developing step-by-step algorithms and debugging errors encountered when programming a robot to move as intended (e.g., Berson et al., 2023; Welch et al., 2022). However, most of the reviewed studies were conducted in programming-oriented activities within early STE(A)M contexts and lacked a specific focus on integrating CT into the mathematics domain as the primary

instructional goal. Hence, to systematically integrate CT into early mathematics education by attending to their compatibility, this study takes an initial step to investigate the extent to which CT is manifested in existing early mathematics classroom activities in the Chinese context. The research question guiding this study is: What elements of CT can be identified, and how are they embedded in early mathematics classroom activities?

Method

Data was collected from a public kindergarten in Shenzhen, China, upon receiving signed consent forms from the principal and from eleven teachers who voluntarily participated. The teacher participants are all females and come from three grade levels (K1-K3) within the kindergarten. Their teaching experience ranges from 3 to 14 years, and all hold a bachelor's degree in early childhood education. None of them has prior experience integrating CT into their teaching. All names of the interviewed teachers are pseudonymised in this study to ensure confidentiality. The first author conducted semi-structured interviews with individual teachers, which lasted approximately 50 minutes and were held in a quiet room at the kindergarten during teachers' spare time. All interviews were audio-recorded and transcribed for data analysis. Sample questions included: "Can you describe a recent mathematics-related activity you conducted in your class?" and "What is the detailed teaching and learning process, and what materials do you use?" The qualitative thematic analysis method was employed to analyse the data (Braun & Clarke, 2006).

We first code the activities according to the mathematics topics mentioned in the Chinese official document of *Early Learning and Development Guideline for Children Aged 3-6 Years* (Ministry of Education of, 2012), including (1) number and arithmetic, (2) shape and space; (3) measurement; (4) data handling; and (5) patterning. Then, the framework of Stein et al. (1996) was utilised to label the level of cognitive demand (hereinafter referred to as CD) required in the mathematics activities, ranging from (1) Level 1: memorisation, (2) Level 2: the use of formulas, algorithms, or procedures *without* connection to concepts, understanding, or meaning, (3) Level 3: the use of formulas, algorithms, or procedures *with* connection to concepts, understanding, or meaning, to (4) Level 4: doing mathematics. Following that, we mapped the mathematics activities onto CT competence with the framework adapted from Ye et al. (2023) (see Table 1). Iterative refinement was performed through multiple rounds of data review to ensure coding validity.

Table 1. CT competence framework adopted from Ye et al. (2023)

Algorithmic thinking	Developing step-by-step solutions to be followed in sequence.
Decomposition	Breaking down a complex problem into more manageable parts.
Pattern recognition	Identifying similarities among and within problems.
Abstraction	Focusing on key information and ignoring irrelevant details.
Data practices	Collecting, analysing, and representing data.
Testing and debugging	Recognising and fixing errors to make solutions work as expected.

Findings

Mathematics Activities Supporting Algorithmic Thinking, Testing and Debugging

Sequencing activity (CD-Level 3): Asking children to sequence materials according to certain mathematical properties, such as number, size, or length, is a good way to foster algorithmic thinking as well as testing and debugging when they encounter unexpected outcomes. Ivy and Zia described scenarios where children were engaged in tasks to organise toys into a sequence ordered from smallest to largest, or from lowest to highest. Zia also introduced a cubic kit, each cube being printed with one of the 1-10 whole numbers, and children were encouraged to sequence the mixed-up cubes into an ordinal line from 1 to 10. These activities engage children in sorting algorithms as they arrange materials in a sequential order with mathematical knowledge (e.g., number and measurement) by systematically comparing and evaluating items. During the process, if the ordering of certain materials appears to conflict with the overall sequencing rule, children must identify the source of the error and determine how to correct it to make the sequence as intended, as illustrated in Kelly's "Finding the longest and shortest sticks" activity.

Mathematics Activities Supporting Decomposition

Number/Shape decomposition (CD-Level 3): In early childhood mathematical activities, children often decompose numbers and shapes, which can also be seen as a means for cultivating CT decomposition to recognise smaller parts comprising a larger whole. Alice and Jenny mentioned the activities, asking children to decompose the number 5, under the tasks of figuring out how many boys and girls were within today's unattended children, or exploring how 5 can be decomposed in different ways using blocks. Daisy also introduced the process of children building figures (e.g., flowers) with different shapes of tangram sets, during which children are provided with opportunities to explore not only the geometric attributes of shapes but also the CT decomposition by identifying the part-whole relationships.

Mathematics Activities Supporting Pattern Recognition

Shape classification (CD-Level 2): When teaching concepts of shapes, a common activity involves asking children to sort materials, which fosters pattern recognition as children identify similarities and differences among items with different shapes. For instance, Cindy described a shelf featuring a circle, a square, and a triangle in her classroom, with which children were tasked with matching materials to their corresponding shape slot.

Patterning activity (CD-Level 3): Patterning activities require children to identify the repeated rules that occur in a sequence (e.g., ABAB) according to certain characteristics (e.g., colour, size), which lays a foundation for early algebraic thinking while also supporting pattern recognition in CT. For instance, Anna presented a line of flowers in the repetition of red, blue, red, blue, and invited children to observe whether there is any pattern embedded. Kelly not only encouraged children to identify patterns but also guided them to *extend* the patterns by providing additional manipulatives for children to choose from and to continue the pattern.

Mathematics Activities Supporting Abstraction and Data Practices

Making parking plots (CD-Level 4): Kelly described a measurement activity where young children created parking plots for their bikes in the kindergarten, which indicated their engagement in abstraction and data practices. Children began by measuring the length and width of all types of bikes (data collection). Then, they counted the number of each type (data analysis) and represented

the types on paper by abstracting key features and then annotated with measurement information (data representation and abstraction). After that, the children used crayons to draw the parking plots. It is noteworthy that decomposition and pattern recognition were also involved in this activity.

Everyday life survey (CD-Level 3): Alice and Jenny demonstrated the process of handling data from the "I am going to primary school" survey. Children were asked to fill out which primary school they planned to attend. After collecting all the surveys, the whole class reviewed them together while creating a table, listing the names of the primary schools and counting the corresponding number of students attending each. Then, children were guided to create a bar graph by abstracting key information into simplified visualisations and ignoring the irrelevant details.

Naturally vs. Implicitly: How CT is embedded in Early Mathematics Classroom Activities

We further categorise the mathematics activities into two forms through which CT is manifested: The first form is *naturally CT-embedded*, wherein CT components are inherently overlapped with the mathematical content itself. Examples include number/shape decomposition (CD-Level 3), shape classification (CD-Level 2) and patterning activity (CD-Level 3), and data handling activities (CD-Level 3/4), where CT components—namely decomposition, pattern recognition, and data practices—are conceptually in line with mathematical content. The second form is *implicitly CT-embedded*, in which CT and mathematics interact with each other more dynamically as children engage in problem-solving. For instance, in the sequencing activity (CD-Level 3), algorithmic thinking and testing and debugging are supported as children apply their knowledge of number ordinality or measurement knowledge to arrange manipulatives into a specific sequence.

Discussion and Conclusion

This study set out to explore CT competence embedded in the existing early mathematics classroom activities with varying levels of cognitive demand in a Chinese kindergarten. Identifying mathematics activities that unintentionally encompass CT not only deepens our conceptual understanding of how CT manifests within existing mathematics education from an interdisciplinary perspective but also informs future efforts to intentionally integrate CT—the essential human cognition—into early childhood education through mathematics (Lee et al., 2023). The example activities, labelled according to different levels of cognitive demand, can further assist teachers in recognising and adjusting instructional design based on students' learning needs. Finally, we acknowledge potential bias, as findings were derived from teachers' perspectives rather than direct field observation. The small sample size from a single Chinese kindergarten may also limit the generalisability of the results to broader contexts.

References

Barr, V., & Stephenson, C. (2011). Bringing computational thinking to K-12: What is Involved and what is the role of the computer science education community? *ACM Inroads*, *2*(1), 48–54. https://doi.org/10.1145/1929887.1929905

Berson, I., Berson, M., McKinnon, C., Aradhya, D., Alyaeesh, M., Luo, W., & Shapiro, B. (2023). An exploration of robot programming as a foundation for spatial reasoning and computational thinking in preschoolers' guided play. *EARLY CHILDHOOD RESEARCH QUARTERLY*, *65*, 57–67. https://doi.org/10.1016/j.ecresq.2023.05.015

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Collins, A., & Halverson, R. (2010). The second educational revolution: Rethinking education in the age of technology. *Journal of Computer Assisted Learning*, *26*(1), 18–27. https://doi.org/10.1111/j.1365-2729.2009.00339.x

Gerosa, A., Koleszar, V., Tejera, G., Gómez-Sena, L., & Carboni, A. (2021). Cognitive abilities and computational thinking at age 5: Evidence for associations to sequencing and symbolic number comparison. *COMPUTERS AND EDUCATION OPEN*, *2*. https://doi.org/10.1016/j.caeo.2021.100043

Lee, I., Grover, S., Martin, F., Pillai, S., & Malyn-Smith, J. (2020). Computational Thinking from a Disciplinary Perspective: Integrating Computational Thinking in K-12 Science, Technology, Engineering, and Mathematics Education. *Journal of Science Education and Technology, 29*(1), 1–8. https://doi.org/10.1007/s10956-019-09803-w

Lee, J., Joswick, C., & Pole, K. (2023). Classroom Play and Activities to Support Computational Thinking Development in Early Childhood. *EARLY CHILDHOOD EDUCATION JOURNAL, 51*(3), 457–468. https://doi.org/10.1007/s10643-022-01319-0

Ministry of Education of the People's Republic of China. (2012). *Early learning and development guidelines for children aged 3 to 6 years.* Beijing, P. R. China: Ministry of Education.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms. *American Educational Research Journal*, *33*(2), 455–488. https://doi.org/10.3102/00028312033002455

Welch, L., Shumway, J., Clarke-Midura, J., & Lee, V. (2022). Exploring Measurement through Coding: Children's Conceptions of a Dynamic Linear Unit with Robot Coding Toys. *EDUCATION SCIENCES*, *12*(2). https://doi.org/10.3390/educsci12020143

Wing, J. M. (2006). Computational thinking. *Communications of the ACM, 49*(3), 33–35.

Ye, H., Liang, B., Ng, O.-L., & Chai, C. S. (2023). Integration of computational thinking in K-12 mathematics education: A systematic review on CT-based mathematics instruction and student learning. *International Journal of STEM Education*, *10*(1), 3. https://doi.org/10.1186/s40594-023-00396-w

Zhang, Y., & Wong, G. K. (2024). Exploring the interplay of computational thinking and mathematics in early childhood education: A systematic review. In *Proceedings of the 2024 IEEE Global Engineering Education Conference (EDUCON)* (pp. 1-10). IEEE. https://doi.org/10.1109/EDUCON60312.2024.10578653

Acknowledgement

The first author gratefully acknowledges the support of the kindergarten principal and teacher participants during data collection and extends special thanks to Dr. Oi-Lam Ng for her valuable professional insights.

TSG6-2-14

INTERACTIVE HIGH SCHOOL APPS FOR BRIDGING MATHEMATICAL LEARNING: PERSPECTIVES FROM A HIGH SCHOOL IN QUEZON CITY, PHILIPPINES

Ma. Louise Antonette N. De Las Peñas
Ateneo de Manila University, Philippines, mdelaspenas@ateneo.edu

Maria Alva Aberin
Ateneo de Manila University, Philippines, maberin@ateneo.edu

Debbie Marie B. Verzosa
University of Southern Mindanao, Philippines, dmbverzosa@usm.edu.ph

Mark Anthony C. Tolentino
Ateneo de Manila University, Philippines, mtolentino@ateneo.edu

Juan Carlo Mallari
Ateneo de Manila University, Philippines, jmallari@ateneo.edu

Jumela F. Sarmiento
Ateneo de Manila University, Philippines

Agnes D. Garciano
Ateneo de Manila University, Philippines

This paper discusses the design and use of mathematical applications (apps) to support high school learners in attaining pre-identified Most Essential Learning Competencies (MELCs). In particular, we present apps on the topics of linear equations and inequalities, geometric theorems and trigonometric identities. Game design elements were integrated in the apps to facilitate a setting wherein students can visualize mathematical concepts and develop and strengthen their mathematical skills. The deployment of the apps to a public high school through a community LTE network is briefly discussed. Initial testing revealed significant improvements in student performance after using the apps.

Keywords: Math apps, Digital technology, Digital tools, Community LTE network

Introduction

The use of digital tools in teaching mathematics has grown rapidly the past five years. First, at the time of the Covid-19 pandemic, there was a drastic shift to distance education, and various technological approaches were explored by teachers and school administrators to facilitate student learning. Post-pandemic, there was the challenge to help students sustain mathematical learning. Various

strategies were carried out to address this concern, some of which employed the aid of digital tools to facilitate student-centered learning.

This paper reports a government funded project (De Las Peñas et al., 2023) entitled "Mathematical Resources for Distance Learning Using Community LTE Networks" during the period April 2021-May 2023 when the authors developed interactive mathematical applications (apps) designed to address the Most Essential Learning Competencies (MELCs) prescribed by the Philippine Department of Education (DepEd) (Department of Education [DepEd], 2020). The apps were intended to supplement available mathematical resources when DepEd prescribed distance learning due to the pandemic; and help bridge the learning gap when face-to-face instruction resumed in August 2022. These apps are interactive, engaging, easy to use with limited supervision and can be played using a mobile phone, tablet or a computer; and are also freely available via the Internet. Despite these multiple alternatives, many students still cannot access these apps in the Philippines because only about 17.7% of households have access to the Internet (National ICT Survey Results, 2019). To address the problem of providing access to quality education to schools whose children belonging to households with limited/no access to the Internet, the project explored alternative avenues for delivery of mathematical resources, such as local community LTE networks and digital datacasting frameworks, for particular schools in collaboration with DepEd School Division offices (DepEd SDOs). In this paper, we discuss the deployment of the mathematical apps via a community LTE network to one public high school in Quezon City within the Philippines' capital region.

Methodology

We first outline the steps considered in developing the mathematical apps. The first step was to consider the least learned competencies from school administrators and teachers, based on their experiences with the students. The next step was to align the apps with the Most Essential Learning Competencies (Department of Education [DepEd], 2020) prescribed by the DepEd Curriculum. Then the apps were designed incorporating game elements (see Figure 1(a)) (Shi &Shih, 2015). Finally, the appropriate scientific framework and pedagogies were embedded in the app design.

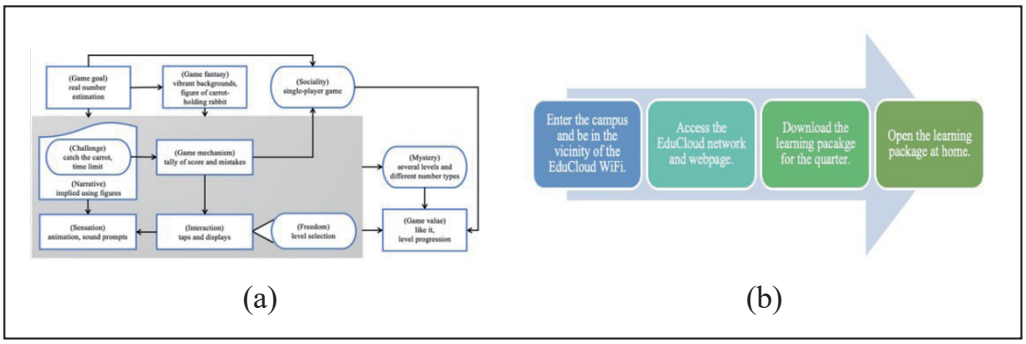


Figure 1.(a) Game design principles in developing a math App; (b) Steps in accessing educloud

The mobile cloud network architecture for the community LTE network used is called *EduCloud* developed by the Ateneo Innovation Center and allows for a resilient distribution of educational content for remote asynchronous learning. *EduCloud* "offers a low-cost, low technology requirement, easily deployable and wide range of compatibility systems for the distribution of open ed-

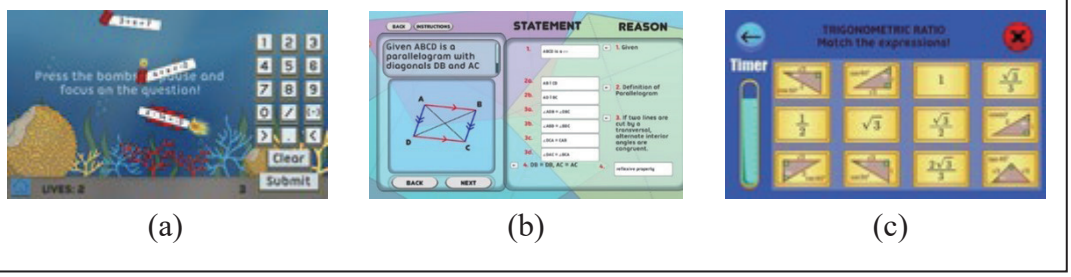
ucational resources" (Villanueva et al., 2021, p. 402). The collaboration of the authors with the *Educloud* team and the DepEd SDO of Quezon City led to the deployment of math apps supplemented with instructional materials and videos through *Educloud*, to a public high school in Quezon City with a population of about 5,000. The main factors considered for the choice of school were: i) a substantial percentage of the student population did not have ready access to the internet (about 95%); and ii) faculty members and students would be ready for the adoption of the use of technological tools in teaching and learning mathematics, respectively. The methodology for the dissemination of the math resources through *Educloud* included: i) planning sessions with the *Educloud* team, ii) close coordination with school administrators for the administration of a survey of learners' profile and deployment of lessons, iii) extensive training sessions with the administrators, teachers and parents on the downloading of math resources from *Educloud*; and iv) quarterly deployment of the math resources to coincide with the lessons of the students. *Educloud* was set up in a strategic point in the school where parents/students can access the device easily. Figure 1(b) shows a flowchart of how apps were accessed in the school through *Educloud* (De Las Peñas, et al., 2022).

Results and Discussion

In the course of the project, several apps for high school students were designed on topics such as solving linear equations and inequalities, proving geometric theorems, identifying classes of various real numbers, generating the equation of a line, and recognizing trigonometric identities. The apps can be accessed for free from [https://mathplusresources.wordpress.com/]. For purposes of this paper, we present three mathematical apps and describe briefly their underlying scientific framework.

Just Keep Solving!

This app allows students to practice solving linear equations and inequalities. The app features an underwater setting and students must correctly answer the questions to prevent the bomb from falling and threatening ocean corals (Figure 2(a)). The app, intended for Grade 7 students, incorporates the deliberate practice model. Drill-and-practice is often associated with low-level or procedural skills; deliberate practice, however, involves the learning of higher-order skills (Lehtinen et al., 2017). These include developing skills that have not yet been learned but are known by a knowledgeable other (such as a teacher) who can provide support, and the provision for feedback.



(a) (b) (c)

Figure 2. Screenshots from the apps a) Just Keep Solving; b) Two Column Proof and c) Trigomatch

Two Column Proof.

Using a two-column format, the app, intended for Grade 9 students, facilitates proving using two column proofs of: 1) Properties of parallelograms 2) Conditions that make a quadrilateral a parallelogram and 3) Theorems on trapezoids and kites. A student enters a response to a line of proof shown on the screen. To aid in visualization of statements or phrases, a right arrow button can be clicked, triggering an animation. The design of the Two Column Proof app was grounded on the use of visualization and imagery in mathematics, which has drawn the attention of experts in the field of psychology of mathematics education (Presmeg, 2006). Drawing on the body of work on mathematical visualization, Rivera (2011) presented a strong rationale for a visually oriented mathematics curriculum. Presenting a problem together with a diagram significantly enabled students with initial difficulties to solve problems successfully. Drawing on the idea of diagrams as effective visual scaffolds provided that students use them meaningfully, the app presents a diagram integrated into a feedback mechanism representing a target theorem. In proving for example "The diagonals of a parallelogram form two congruent triangles" in the "Reason" column the definition of a parallelogram, is given (see Figure 2(b)). The visualization component involves, relevant parts of the geometric figure are highlighted such as pairs of sides. Color flashes are provided as visual hints. To visualize the properties of a parallelogram the animation involves the first pair of sides AB and CD (repeated flashes in red); followed by the second pair of sides, AD and BC (repeated flashes in blue). Congruence of sides is hinted as well by highlighting the congruence symbols.

TrigoMatch.

A set of cards is shown, and the user must find a match between two equivalent representations or two equivalent trigonometric expressions (Figure 2(c)). The app is intended for Grade 9 and illustrates the six trigonometric ratios. The design of the app is anchored on the presentation of trigonometric representations through interactive computer-aided instruction. As stated in Jenkins (2022), when visual representations of trigonometry are presented using interactive technology, these are more accurate and allow the learners to proceed at their own pace.

In the time of distance learning, the teachers presented an app to the students via Google classroom, and students submitted screenshots of work in the app via messenger chatroom. During the resumption of face-to-face instruction, the apps were used in certain times of the week where math classes were carried out with technology, and in remediation classes outside regular class hours.

Conclusion and Future Direction

The implementation of our project to schools in collaboration with DepEd SDOs, to help address the issue of learning gaps through mathematical apps, is one step towards addressing this problem in the Philippines. In the near future, there is still a need to thoroughly assess the effectiveness of the math apps in student learning. Initial results, however, point to the potential of the apps in mathematical learning. For instance, in the school year 2022-23 (De Las Peñas et al; 2023), during the deployment to a high school in Quezon City, a study was conducted to assess the effectiveness of the Two Column Proof app. Two pairs of control-experimental Grade 9 classes were selected using purposive sampling so that the classes have similar mathematics grades. Informed consent was obtained from these classes; moreover, to ensure privacy, all collected data was anonymized, securely stored, and used solely for research purposes. These classes were then given pretests and posttests covering the following competencies: (i) proves theorems on the different kinds of a parallelogram (rectangle, rhombus, square), (ii) proves theorems on trapezoids and kites. In control classes, only

traditional teaching methods were applied and the app was not integrated at all; on the other hand, in experimental classes, the app was integrated in synchronous lessons and in homework. For the initial analysis, a simple comparison of the mean pretest and posttest scores was carried out. For the first pair of classes, the control class (38 students) recorded a mean gain score of 30.94% while the experimental class (40 students) recorded a mean gain score of 44.90%. For the second pair of classes, the gap between mean gain scores was wider: 25.73% for the control (25 students) and 79.29% for the experimental (27 students). These results indicate the potential of the Two Column Proof app to help students learn geometry, particularly in topics on proving.

References

Aberin, A., De Las Peñas, M.L.A.N., Garciano, A., Mallari, J.C., Sarmiento, J., Tolentino, M., & Verzosa, D. (2023). Enhancing Trigonometry Learning through a Mobile App. In: Shih, J.L. et al (Eds) *ICCE 2023 - 31st International Conference on Computers in Education Proceedings 2*, 405-410.

De Las Peñas, M. L. A. N., Aberin, M. A. Q., Garciano, A. D., Mallari, J. C. F., Sarmiento, J. F., Tolentino, M. A. C., & Verzosa, D. M. B. (2023). A Visualization App on Proving Geometric Concepts. In: Shih, J.L. et al (Eds) *ICCE 2023 - 31st International Conference on Computers in Education Proceedings 2,* 491-496.

De Las Penas, M. L. A. N., Aberin, M. A. Q., Garciano, A. D., Mallari, J. C. F., Sarmiento, J. F., Tolentino, M. A. C., & Versoza, D. M. B. (2022). Deployment of Mathematical Resources to a Philippine High School through a Community LTE Network. In W.C. Yang, M. Majewski, D. Meade, & W.K. Ho (Eds.) *Proceedings of the 27th Asian Technology Conference in Mathematics*, 126-140

De Las Peñas, M. L. A. N., Aberin, M. A. Q., Garciano, A. D., Sarmiento, J. F., Tolentino, M. A.C., Verzosa, D. M. B., & Mallari, J. C. F. (2023). *Terminal report (Department of Science and Technology Philippine Council for Industry, Energy and Emerging Technology Research and Development): Mathematical resources for distance learning utilizing community LTE networks and television frequencies.*

Department of Education (DepEd). (2020). *K to 12 Most Essential Learning Competencies with Corresponding CG Codes.* https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf

Department of Information and Communications Technology. (2019). *2019 National ICT Household Survey Results.* Retrieved 7/20/2022 from https://dict.gov.ph/ictstatistics/2019-national-ict-household-survey-results/

Garciano, A. D., Aberin, M. A. Q., De Las Peñas, M. L. A. N., Mallari, J. C. F., Sarmiento, J. F., Tolentino, M. A. C., & Verzosa, D. M. B. (2023). Practice through Play Using Mobile Technology. *International Conferences e-Society 2023 and Mobile Learning 2023 Proceedings,* 351-358.

Jenkins, R. (2022). *Impact of interactive computer-aided instruction in learning trigonometry in a high school precalculus course.* [Doctoral dissertation, Franklin University]. https://fuse.franklin.edu/ doc-pub/71

Lehtinen, E., et al. (2017). Cultivating mathematical skills: From Drill-and-Practice to Deliberate Practice. *ZDM Mathematics Education*, *49*, 625-636.

Rivera, F. (2011). *Towards a visually-oriented school mathematics curriculum: Research, theory, practice, and issues*. Dordrecht, the Netherlands: Springer

Presmeg, N. (2006). Research on visualization in learning and teaching mathematics. In A. Gutierrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 205-235). Rotterdam, the Netherlands: Sense Publishers.

Rivera, F. (2011). *Towards a visually-oriented school mathematics curriculum: Research, theory, practice, and issues*. Dordrecht, the Netherlands: Springer

Shi, Y.-R., & Shih, J.-L. (2015). Game factors and game-based learning design model. *International Journal of Computer Games Technology*, *2015*, 1-11.

Villanueva, J.C.P.M., Melendres, M.A.V., Lagunzad, C.G.B., and Libatique, N.J.C. (2021). Design and Deployment of a Mobile Learning Cloud Network to Facilitate Open Educational Resources for Asynchronous Learning. In Rodrigo, M. M. T. et al. (Eds.), *Proceedings of the 29th International Conference on Computers in Education*. 398-403.

Acknowledgments

This paper is one of the outputs of the Ateneo Mathplus Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors would like to thank the Department of Science and Technology- Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the project. We thank our app developers Mr. Victor Antonio M. Ortega, Mr. Nigel Benedict Cargo, Mr. Jose Teodoro Lacson and our artist Mr. Amiel Damian F. Justiniani.

TSG6-2-15

A PRELIMINARY STUDY ON 5- TO 6-YEAR-OLD CHILDREN'S SELECTION PREFERENCES AND SALIENT INFORMATION IN MATHEMATICAL SHORT VIDEOS.

Wan-Hua, Deng
National Pingtung University Department of Early Childhood Education, Taiwan, wanhua0830@gmail.com

Chia-Yen, Hsieh
National Pingtung University Department of Early Childhood Education, Taiwan,
banduna@mail.nptu.edu.tw

En-Qi, Chen
National Pingtung University Department of Early Childhood Education, Taiwan,
myday120612@gmail.com

This preliminary study explores 5- to 6-year-old children's preferences for salient information in mathematical short videos. As digital media and short videos become increasingly prevalent in early childhood mathematics education, understanding children's responses to different design elements is crucial for enhancing teaching effectiveness. Using a convenience sampling method, we recruited 30 kindergarten children from southern Taiwan. A questionnaire survey with a Likert five-point scale and open-ended questions was conducted to assess their preferences and selection tendencies through quantitative and qualitative analyses. Findings indicate that salient information significantly influences children's preferences, and prior experiences also shape their choices. These insights offer valuable guidance for educators and content creators in refining instructional design, increasing engagement, and improving learning outcomes.

Keywords: Early Childhood Education, Mathematical Short Videos, Salient Information, Learning Preferences

Motivation

In recent years, the rapid advancement of electronic devices has fueled the global surge in real-time video recording and short video sharing (Xu et al., 2023). The rise of short videos has not only transformed social media interactions but also introduced new opportunities in education. Teaching through short videos can enhance students' learning motivation, driven by their curiosity about video content and the incentive of gaining instructor recognition through engagement (Bravo et al., 2011).

In Taiwan's early childhood education system, mathematics is not a standalone subject but is integrated into the curriculum under the Cognitive Domain, as specified in the Curriculum Design for Early Childhood (Ministry of Education, 2017). Concepts such as quantity, counting, numbers, shapes, and spatial awareness are embedded within lesson planning. Game-based learning and diverse instructional strategies have been shown to significantly enhance children's mathematical learning outcomes (Yeh, 2003; Liao, 2021; Chen & Wang, 2021). Furthermore, the use of educa-

tional videos not only improves learning effectiveness but also serves as a reusable shared resource (Simo et al., 2010). Short videos provide a more cost-effective and flexible alternative to traditional materials. When properly designed in terms of content, information density, and duration, short videos can further enhance their educational value (Bravo et al., 2011). The influence of short videos extends beyond learning motivation; the salient information within videos may also impact children's cognitive and emotional responses. Salient information refers to elements with distinctive or prominent features (Tao, 2011). In this study, it specifically refers to auditory and visual elements within the videos, which are hypothesized to influence children's engagement with mathematical content. However, due to individual differences in perception, children may process the same stimuli differently, affecting their attention allocation and comprehension.

According to Information Processing Theory (Miller, 1956), human learning and thinking function similarly to computer-based information processing. Salient information in videos captures viewers' attention, directing their focus to specific content (Tao, 2011). Understanding the variations in children's responses to these stimuli is crucial for optimizing educational video content. Beyond video characteristics, children's prior experiences and content selection preferences also play a critical role. This aligns with the concept of learning transfer (Thorndike, 1901), which examines whether learners can apply previous knowledge to new contexts (Lin & Zheng , 2022). Children's selection of short videos is not solely driven by immediate content appeal but also influenced by prior exposure to mathematical concepts, familiarity with specific teaching styles, and personal interests. This raises a key question: Do children's prior experiences affect their preferences for different types of salient information in videos, ultimately influencing their engagement and learning outcomes?

Based on this background, the research objectives are as follows:

Research Questions

I. Investigate the Factors and Characteristics That Cause Significant Differences in Children's Responses to Salient Information in Short Mathematical Videos.

II. Analyze Whether Children's Prior Experiences Influence Their Selection Preferences for Mathematical Short Videos.

Research Instrument

This study primarily utilizes a researcher-developed assessment, the *Mathematical Short Video Preference Test*, which is designed based on the Cognitive Domain objectives outlined in the *Curriculum Design for Early Childhood* (Ministry of Education, 2017). Specifically, it aligns with the goal "Recognize-2-1: Organizing Mathematical Information in Daily Life" and focuses on six key mathematical concepts: 1. The shape and appearance of numbers.2. Number Sequences 3. Composite number 4. Integer factorization 5. Geometric figures 6. Long and short arrangement.

Mathematical short videos were selected ranging from 30 seconds to 1 minute 30 seconds, with two different presentation styles for each category, totaling 12 videos. When choosing the videos, we looked at things like how fast they move, how visually busy they are, and whether the cultural context makes sense for the kids. Some videos move at a slower pace and use gentle animation, so the scenes stay on screen a bit longer — that way, it's easier for young children to follow along and understand what's happening.

In terms of visuals, we paid attention to how much is going on in each scene — like how many things are on screen, how intense the animation is, and how colors are used. Some videos have a clean, simple look, while others are packed with decorative images. As for cultural context, some of the videos feature familiar characters like" *Qiaohu"*, a well-known figure among young children in Taiwan. The idea behind using characters that kids already recognize is to see whether their past media experiences might affect their motivation to watch and their preferences. By including a variety of video styles, the study creates a chance to explore how young children pay attention and process information differently depending on what they're watching.

To refine the salient information incorporated in these videos, discussions were held with preschool teachers to predict which elements children might find most engaging. These insights served as a reference for identifying and analyzing salient information within the videos.

Table 1. Video encoding method:

Category name	First video	Second video
1.The shape and appearance of numbers.	1-1	1-2
2.Sequence of digits	2-1	2-2
3. Composite number	3-1	3-2
4. Integer factorization	4-1	4-2
5. Geometric figures	5-1	5-2
6. Long and short arrangement	6-1	6-2

Study Object

This preliminary study aims to explore children's reactions and content preferences when watching mathematical short videos. To achieve this, convenience sampling was used to randomly select 30 kindergarteners from a preschool in southern Taiwan. Each participant completed two experimental tasks using a Likert five-point scale. (Evaluating their preference for mathematical short videos &explaining the reasons for liking or disliking specific elements in the videos.)

Data Analysis

This study employs both quantitative and qualitative data analysis methods, as outlined below:

I. Quantitative Analysis

As a preliminary study with a limited sample size of 30 participants, non-parametric statistical methods were used for data analysis. Specifically, we used the Mann-Whitney U test to analyze differences in children's preferences between the First Video and Second Video.

II. Qualitative Analysis

This study adopts the open coding method proposed by Khandkar (2009) to analyze children's responses. The responses were transcribed verbatim, systematically coded, and categorized to derive qualitative insights from the data.

Results

This chapter analyzes the research data obtained from questionnaires and individual interviews regarding children's experiences watching mathematical short videos. The analysis is divided into two sections:

I. Reasons and Characteristics of Significant Differences in Video Preferences:

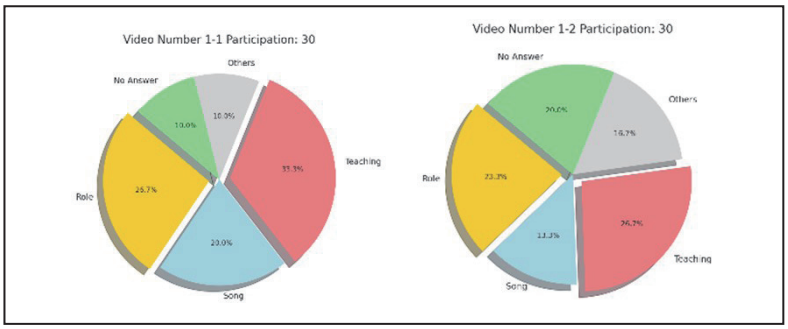


Figure 1. Category one analysis chart (shape and appearance of numbers)

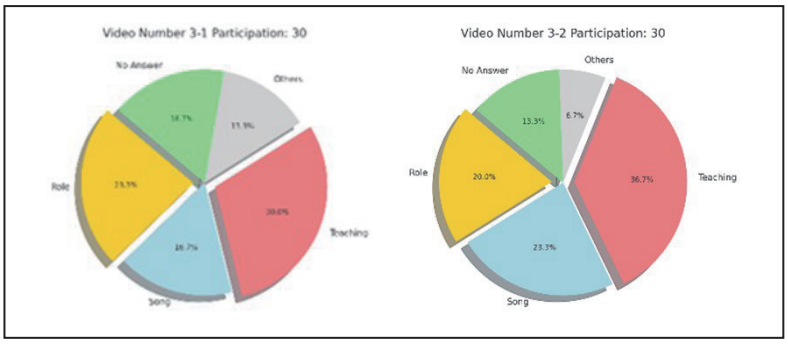


Figure 2. Category three analysis chart (Composite number)

Using SPSS analysis, significant differences were found in two categories among the 12 videos: Shape and Appearance of Numbers ($z = 2.808$, $p = .005$),Composite Numbers ($z = 2.310$, $p = .021$). In both cases, First Video > Second Video in preference. Therefore, data labeled as 1-1, 1-2, 3-1, and 3-2 were selected for detailed analysis. Through interviews and qualitative coding, children described the specific features they noticed in the videos and explained their reasons for liking or disliking them. The analysis, as illustrated in the charts, identifies five key categories of salient feedback elements in mathematical instructional videos that capture children's attention: Teaching Methods. Characters. Songs. Other and No Response. The following sections provide a detailed explanation of each category.

(I.) Teaching Methods

Among all videos, teaching methods were the most frequently noticed element by children. The comparison between Video 3-1 and Video 3-2 highlights differences in instructional approaches: Video 3-1 utilized a singing-based teaching method with a slower pace, which some children found

helpful. One child noted, "*The numbers stay on the screen longer, making them easier to remember.*" (T12 20241225) Video 3-2, which used a lecture-based teaching approach, received more negative feedback. A child commented, "*I don't think the teaching in Video 3-2 is good; I couldn't understand it.*" (T14 20241231) Although both Video 1-1 and Video 1-2 incorporated singing-based teaching, Video 1-1 also included movement and dancing, which children found engaging. One child remarked, "*This video has dancing, it looks really fun*!" (T6 20241224). Additionally, another child noted that incorporating movement made the learning experience more interactive, saying, "*I like following the actions, so the video is less boring*." (T22 20250102)

Compared to the lecture-based approach, singing-based teaching appears to enhance children's comprehension of the content. Furthermore, incorporating movement and physical actions within instructional videos can significantly increase children's engagement. This approach not only makes learning more enjoyable but also encourages active participation, allowing children to learn by doing rather than passively watching.

(II.) Characters

When the instructional video started playing, some children immediately recognized familiar characters. One child exclaimed, "*1-1 has Qiaohu*!" (T2 20241223). In Video 1-1, the presence of Qiaohu, a well-known character, made some children more eager to watch and express their preference for the video. However, others found certain characters less appealing, with one child commenting, "*Qiaohu is for babies.*" (T14 20241231). This suggests that both age appropriateness and novelty of characters influence children's acceptance of the video. In Videos 3-1 and 3-2, characters such as number blocks and animal figures successfully captured the attention of some children. Notably, animal characters in Video 3-2 received frequent positive feedback, with children remarking that "the animals are cute." *However, not all character designs were well received. Some children commented that "the blocks look weird*." (T17 20250102) or "*I don't like the milk in the video.*" (T22 20250103).

Character design plays a crucial role in engagement and learning interest. Familiar characters may increase motivation, but if perceived as too childish, they might reduce engagement for older preschoolers. Additionally, while new and visually appealing characters (such as animals) can attract attention, those that fail to resonate with children's preferences may diminish learning interest.

Familiar characters can enhance viewing motivation, but if they are too commonly seen, some children may lose interest. In contrast, novels and visually appealing characters tend to better stimulate children's learning motivation. Therefore, character design in educational videos should strike a balance between familiarity and novelty to effectively engage a diverse group of young learners.

(III.) Songs

along. For example, one child remarked, "*The song in 3-1 sounds nice—one, two…*" (T25 20250103), while another commented, "*This song is better, I keep singing it.*" (T29 20250106). However, if the music is unappealing or does not connect well with the learning content, it may negatively impact the viewing experience. In Video 1-2, for instance, a child stated, "*This song doesn't sound good*." which diminished the overall attractiveness of the math video.

Appropriate songs and music can enhance learning motivation, but if the music does not align well with the video content, it may reduce interest and engagement. Therefore, future mathematical video designs should carefully consider song selection and the coherence between music and learning content to ensure that music effectively supports the learning

(IV.) Other

Beyond teaching methods, characters, and songs, elements such as visual design, animation effects, and video length also influenced children's viewing experience. In Video 1-2, the teacher used manipulatives as teaching aids, which drew particular attention from some children. Additionally, video length emerged as a salient factor. If a video was too short (less than one minute), some children felt it ended before they fully grasped the content. One child commented, "*This video is too short—I didn't finish learning before it ended.*" (T10 20241225). Similarly, another child provided feedback to the teacher, saying, "*I think the video should be longer; otherwise, I won't be able to learn it well.*" (T13 20241225).

(V.) No Response

Some children, however, did not provide any feedback after watching the video.

II. The Impact of Prior Experience on Children's Selection Preferences for Mathematical Short Videos

Through the analysis of children's verbatim transcripts, this study examines whether their prior experiences—such as previous exposure to similar videos, familiarity with related mathematical concepts, or recognition of specific media or characters—influence their preferences for mathematical short videos. The results indicate that children's prior experiences significantly impact their video selection and learning motivation. The key areas of influence include media familiarity, learning style, and existing mathematical knowledge. The discussion is divided into the following two aspects:

(I.) Media Familiarity

Children who have previously watched similar videos on TikTok or other short-video platforms demonstrated higher acceptance and learning motivation toward mathematical videos with similar styles. For example, one child remarked, "*I've seen this video at home, I think I watched it on TikTok, and it helped me remember numbers.*" (T9 20241225). Another child stated, "*I've watched this on my mom's phone before.*" (T26 20250103). When a character is both familiar and favored, it can enhance children's learning motivation, making them more engaged and willing to interact with the educational content.

(II.) Character Familiarity

Familiar characters evoke reactions based on previous exposure to similar figures. For example, one child mentioned, "*This is Qiaohu! My mom subscribed to this for me before—I like Qiaohu.*" (T22 20250102). Another child stated, "*I prefer 1-1 because it has Qiaohu.*" (T1 20241223).

A closer look shows that children's video preferences aren't just shaped by obvious things like teaching style, characters, or songs — they also seem closely tied to deeper features of the video's overall design. For example, kids' preference for Video 1-1 might have to do with its slow pace, clean and simple visuals, and the use of familiar characters like "Qiao Hu", which Taiwanese children already know and love. These elements likely help reduce the mental effort needed to process the video, making it easier for children to stay focused and understand what's going on. On the flip side, Video 3-2 features fast pacing, frequent animation changes, and new, unfamiliar characters that lack cultural relevance. For some kids, this might feel overwhelming or disconnected, leading to negative reactions or lower engagement. These findings reflect ideas from information processing theory, which suggests that our brains have limited cognitive resources — when there's too much

going on visually or aurally, it can reduce how much young children pay attention or understand what they're supposed to be learning. All in all, the pace, visual complexity, and cultural familiarity in a video may interact in complex ways with children's preferences and learning engagement — something future research should dig deeper into.

Conclusion

The results indicate that when watching mathematical short videos, children are most attentive to three key types of salient information: "teaching methods," "characters," and "songs." Among these factors, teaching methods play a key role in influencing children's learning engagement. Singing-based instruction is more effective than lecture-style teaching in enhancing learning interest. Furthermore, videos that incorporate movement or action design further increase children's involvement and participation. Regarding characters, novels and visually appealing characters are more effective in sustaining children's learning interest. However, characters that children were exposed to at a younger age can also capture their attention and engagement. The selection of songs plays a crucial role in engagement. Music that is easy to learn and well-connected to the content can enhance children's viewing duration and learning focus.

Additionally, prior experiences, such as previous media exposure and familiarity with specific mathematical concepts, also influence video selection preferences. Familiar video styles and characters can improve children's learning acceptance, whereas overly unfamiliar content may reduce their learning motivation.

Recommendation

For future selection and design of mathematical short learning videos, the following considerations are recommended: Prioritize singing and movement-based teaching to enhance learning engagement. Balance familiarity and novelty in character design to sustain children's interest. Incorporate songs that are easy to sing along with and well-aligned with the content to strengthen learning motivation. Adjust video length to ensure that it effectively delivers the learning content while remaining suitable for children's attention span. Future research could further explore the long-term impact of these video elements on children's mathematical learning outcomes, optimizing the educational application of short videos in early childhood mathematics education.

Also, when designing videos for young children, it's important to consider where they are in terms of cognitive development. The pacing of the video should be just right — not too fast or packed with information, so kids don't get overwhelmed or confused. Visually, the design should be clean and simple, avoiding distracting animations or too many on-screen elements that could pull attention away from the main learning points. As for cultural context, it's helpful to strike a balance between familiar characters and fresh, exciting content — this way, children can feel an emotional connection while still being curious and motivated to explore. These design strategies can really help make short-form videos more effective and engaging for early childhood math learning.

AI Self-Disclosure

During the preparation of this thesis, as English is not the researcher's first language, OpenAI's ChatGPT was utilized to assist with translating, refining sentence structure, and optimizing grammar for improved readability. After using this tool/service, the author carefully reviewed all content

and edited it as necessary to align with the thesis writing requirements. The author takes full responsibility for the final content of this publication.

References

Bravo, E., Amante, B., Simo, P., Enache, M., & Fernandez, V. (2011). Video as a new teaching tool to increase student motivation. In *2011 IEEE Global Engineering Education Conference (EDUCON)* (pp. 638–642). https://doi.org/10.1109/EDUCON.2011.5773205

Chen, Y.-H., & Wang, W.-C. (2021). The impact of tabletop games implemented in primary school mathematics curriculum on learning motivation and learning interest. *International Journal of Digital Media Design, 13*(1), 27–38. https://doi.org/10.29465/ijdmd.202106.13.0003

Khandkar, S. H. (2009). *Open coding*. University of Calgary.

Liao, Y.-J. (2021). *A study on the effects of game-based instruction on interest and performance in elementary school mathematics: A case of Grade 1 geometry* [Master's thesis, National Taichung University of Education]. National Digital Library of Theses and Dissertations in Taiwan. https://hdl.handle.net/11296/5tpf96

Lin, S.-H., & Cheng, M.-T. (2022). The influence of problem-based learning on learning transfer and learning-related activity emotions. *Chinese Journal of Science Education, 30*(3), 193–216. https://doi.org/10.6173/CJSE.202209_30(3).0001

Ministry of Education. (2017). *Curriculum outlines for preschool care-taking activities*. Author.

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review, 63*(2), 81–97. https://doi.org/10.1037/h0043158

Simo, P., Fernandez, V., Algaba, I., Salan, N., Enache, M., Albareda-Sambola, M., Bravo, E. R., Suñé, A., Garcia-Almiñana, D., Amante, B., & Rajadell, M. (2010). Video stream and teaching channels: Quantitative analysis of the use of low-cost educational videos on the web. *Procedia - Social and Behavioral Sciences, 2*(2), 2937–2941. https://doi.org/10.1016/j.sbspro.2010.03.444

Tao, C.-C. (2011). How do media stimuli receive attention: Salience or relevance? The perspective of cognitive approaches to media. *Mass Communication Research, 107*, 245–290. https://doi.org/10.30386/MCR.201104_(107).0007

Thorndike, E. L., & Woodworth, R. S. (1901). The influence of improvement in one mental function upon the efficiency of other functions. *Psychological Review, 8*, 247–261. https://doi.org/10.1037/h0074898

Xu, Z., Gao, X., Wei, J., Liu, H., & Zhang, Y. (2023). Adolescent user behaviors on short video application, cognitive functioning, and academic performance. *Computers & Education, 203*, 104865. https://doi.org/10.1016/j.compedu.2023.104865

Yeh, S.-C. (2003). *The effects of game-based mathematics instruction on students' learning* [Master's thesis, Taichung Teachers College]. National Digital Library of Theses and Dissertations in Taiwan. https://hdl.handle.net/11296/268g66

TSG6-2-16

AN ANALYSIS ON THE DIGITAL ARTIFACT AFFORDANCE THROUGH ANT AND INSTRUMENTAL APPROACH

Jiyoung, Kim
Seoul Dungyang Elementary School, South Korea, 2250@daum.net

Mangoo, Park
Seoul National Univ. of Education, South Korea, mpark29@snue.ac.kr

This study explores how digital artifacts influence instructional design and practices in elementary mathematics classrooms. Employing Actor-Network Theory (ANT) and the instrumental approach, two experienced teachers were analyzed through observations, interviews, and instructional materials. While both used digital tools, Teacher J integrated them to foster inquiry and collaboration, whereas Teacher K emphasized task management. The findings highlight that teachers' affordance awareness and orchestration strategies shape whether artifacts function as central or auxiliary actors. Affordance-aware design promotes creative and collaborative learning, underscoring the need for targeted teacher training.

Keywords: Digital Artifact, Affordance, ANT, Instrumental Approach

Introduction

This study aimed to comprehensively analyze the impact of digital artifacts on teachers' instructional design and implementation processes in elementary mathematics classrooms. The investigation specifically focused on how the affordances of digital artifacts and their status in instructional design influence teachers' usage patterns. The rapid development of digital technology has ushered in a paradigm shift in educational environments. Particularly in mathematics education, digital artifacts have transitioned from being auxiliary tools replicating traditional classroom practices to becoming central elements that reshape learning processes(Clark-Wilson et al., 2014; Hoyles & Noss, 2003). Digital artifacts now play a crucial role in reconstructing learning environments, mediating interactions between teachers and learners, and enhancing the overall quality of learning experiences(Passey, 2011).

The study addressed two primary research questions.

1. How do the affordances of digital artifacts manifest across the stages of Actor-Network Theory (ANT) in mathematics classrooms?
2. how do teachers' instrumental genesis processes and their utilization patterns of digital artifacts vary in relation to these affordances?

Theoretical Framework and Analytical Approach

This study employed a theoretical networking approach to integrate affordance theory, Actor-Network Theory (ANT), and the instrumental approach into a unified analytical framework. Actor-Net-

work Theory, developed by Latour, Callon, and Law, provided the foundation for examining the interactions between human and non-human actors—teachers, students, and digital artifacts—within the learning environment(Callon, 1986; Latour, 2005). ANT's stages—problematisation, interessement, enrollment, and mobilization—guided the analysis of how digital artifacts emerged, were perceived, implemented, and influenced educational practices.

The affordance theory was instrumental in identifying the potential and possibilities offered by digital artifacts. Specifically, it focused on the physical, virtual, and virtually mediated social affordances of these artifacts and how they were recognized and utilized by teachers and learners(Kennewell, 2001). The cyclical framework of affordances (emergence, recognition, implementation, and impact) enabled a dynamic analysis of digital artifacts' roles in shaping the learning environment(Ostern & Rosemann, 2021).

The instrumental approach, encompassing instrumental genesis and orchestration, served as the second pillar of the analytical framework(Guin et al., 2005; Trouche, 2004). Instrumental genesis examined how teachers internalized and adapted digital artifacts to align with their pedagogical goals. Instrumental orchestration analyzed how these artifacts were externalized and utilized within classroom interactions to achieve instructional objectives(Drijvers et al., 2010). Together, these frameworks provided a robust structure for exploring the multifaceted roles of digital artifacts in mathematics education.

Research Method

This study adopted a qualitative case study methodology to enable an in-depth and context-sensitive analysis of how two elementary school teachers utilized digital artifacts in mathematics instruction. The case study design was appropriate for capturing the complex, situated, and process-oriented nature of digital artifact use in real classroom contexts, consistent with recommendations from Yin (2018) and Stake (1995).

Participant Selection and Comparative Design

Two elementary school teachers, referred to as Teacher J and Teacher K, were purposefully selected based on the principle of maximum variation sampling. Both participants shared comparable teaching characteristics—including similar levels of TPACK (Technological Pedagogical Content Knowledge), years of teaching experience (19 years for J; 13 years for K), and frequency of digital artifact use in regular practice. However, they represented contrasting levels of **affordance awareness**, as indicated by pre-screening interviews and an affordance observation checklist, which served as the theoretical anchor for this contrastive case design.

Data Collection Procedures

Data were collected over a four-week period and included the following:

- **Classroom Observations**: Each teacher was observed across three mathematics lessons, resulting in six complete lesson records. Video recordings captured teacher-student-digital artifact interactions, and field notes documented contextual elements.
- **Pre- and Post-Lesson Interviews**: Each teacher participated in semi-structured interviews before and after each lesson. Pre-lesson interviews focused on instructional goals and digital design intentions, while post-lesson interviews elicited teacher reflections on student engagement, affordance realization, and tool effectiveness.

- **Instructional Design Materials**: Lesson plans, presentation slides, worksheets, and student-generated digital artifacts (e.g., Book Creator projects, Padlet boards) were collected and analyzed to trace the materialization of affordance-aware design.

All data sources were triangulated to ensure methodological rigor and to enhance the credibility and trustworthiness of findings (Lincoln & Guba, 1985).

Analytical Procedures

The analysis followed a multi-layered interpretive procedure structured by the integrated theoretical framework. First, a **cyclical affordance analysis** (emergence–recognition–implementation–impact) was conducted for each lesson episode to trace the trajectory of affordance realization. Second, classroom episodes were coded according to the **four phases of Actor-Network Theory** (problematization, interessement, enrollment, and mobilization) to examine the positioning of digital artifacts as central or auxiliary actors. Third, the **instrumental approach**—particularly instrumental genesis and orchestration—was applied to analyze teachers' evolving schemes and orchestrational strategies during tool use.

By employing theoretical triangulation (ANT, affordance theory, and instrumental approach) and using structured within-case and cross-case analyses, the study ensured analytical transparency and theoretical robustness.

Findings

ANT Analysis of Digital Artifact Affordances

The analysis revealed that digital artifacts functioned differently depending on the stages of Actor-Network Theory (ANT) and the teacher's instructional design. Teacher J's lessons demonstrated the dynamic role of digital artifacts as central actors facilitating inquiry, collaboration, and knowledge construction. For instance, J utilized Book Creator to enable students to co-construct visual explanations of mathematical concepts, leveraging both virtual and socially mediated affordances (Majchzak & Markus, 2012).

Conversely, Teacher K employed digital artifacts primarily as auxiliary actors. Kahoot was used for individual quiz-based interactions, where students selected predefined responses and received automated feedback. While such use highlighted physical affordances (e.g., selecting via touch interface), it limited the opportunity to activate deeper levels of engagement. Padlet, similarly, was used for task submission rather than collaborative ideation or dialogic exchange, restricting its affordance potential.

Importantly, this analysis does not suggest that any particular artifact is inherently superior. Rather, the findings emphasize that the **same digital artifact can function differently** depending on the teacher's perception of affordance, the epistemic framing of the tool, and the teacher's instrumental orchestration capabilities. For example, although Teacher J also used Padlet and QuizAnn, these tools were embedded in sequences designed to scaffold prior knowledge and promote peer sharing—an orchestration fundamentally distinct from Teacher K's usage. This reinforces the central claim that affordances are not fixed properties of tools but are co-constructed in practice (Kennewell, 2001; Ertmer & Ottenbreit-Leftwich, 2010).

To enhance the validity of these findings, multiple forms of data—including classroom video records, teacher interviews, and instructional materials—were triangulated. Coding was performed in

iterative cycles, with analytic categories derived from the theoretical framework (ANT and affordance theory), ensuring consistency across cases. Member checking with the participants was conducted to confirm the accuracy of interpretation, further enhancing the credibility of the findings.

Instrumental Approach to Affordance Implementation

The instrumental analysis underscored the decisive role of teachers' design capacities in determining the depth and quality of affordance implementation (Trouche, 2004). Teacher J exhibited a sophisticated orchestration of tools, combining physical, virtual, and social affordances to cultivate exploratory and interactive learning environments. AlgeoMath enabled real-time manipulation of geometric shapes, activating dynamic virtual affordances, while Book Creator fostered peer dialogue and multimodal expression.

In contrast, Teacher K's orchestration strategy reflected a more procedural and efficiency-oriented use of tools. Kahoot and Padlet were used in ways that prioritized classroom management and task completion, which, while effective for certain instructional goals, offered limited support for collaborative or generative learning.

The validity of these comparative findings rests on the **analytic coherence across multiple data sources** and the use of a theoretically grounded framework. The affordance cycles and ANT stages were applied systematically to both cases, revealing not only how tools were used, but how they became meaningful within particular pedagogical cultures. These interpretations were not made solely on observable behaviors but were triangulated with teachers' own accounts of intention and reflection, thereby reducing interpretive bias.

While this study is limited in scope to two cases, the robust alignment between theoretical constructs and empirical patterns suggests strong **theoretical generalizability** (Yin, 2018). Future research could extend this work by including more varied classroom contexts to further test the transferability of these findings.

Conclusion

Role of Digital Artifacts Across ANT Stages

Digital artifacts exhibit varying roles—central or auxiliary actors—depending on the teacher's instructional goals and design strategies. During the ANT stages, artifacts facilitated or limited interaction and knowledge construction. J's effective orchestration of artifacts demonstrated their potential as central actors, while K's selective use reflected their role as auxiliary aids.

Affordance Implementation and Learning Environments

When digital artifacts serve as central actors, they enable student-centered, creative, and collaborative learning environments. Conversely, when used as auxiliary aids, they support teacher-centered approaches, effectively motivating students and reinforcing prior learning but offering limited opportunities for deeper exploration(Clark-Wilson & Hoyles, 2017).

Contributions and Implications

This study contributes to the growing body of research on digital artifacts in education by empirically demonstrating the multifaceted impact of affordances on learning environments. It highlights the critical role of teachers' design capabilities in realizing the full potential of digital artifacts. By integrating affordance theory with ANT and the instrumental approach, the study provides a comprehensive framework for analyzing digital artifacts' roles in education.

Practical implications include the need for targeted professional development programs that equip teachers with the skills to design and implement affordance-rich learning environments. These programs should emphasize the strategic integration of digital artifacts as central actors in instructional design.

References

Callon, M. (1986). Some elements of a sociology of translation: Domestication of the scallops and the fishermen of St. Brieuc Bay. In J. Law (Ed.), *Power, action and belief: A new sociology of knowledge?*(pp. 196–233). London, England: Routledge.

Clark-Wilson, A., Hoyles, C., & Noss, R. (2014). Scaling a technology-based innovation: Windows on the evolution of mathematics teachers' practices. *ZDM Mathematics Education, 46*(5), 717–730.

Clark-Wilson, A., & Hoyles, C. (2017). *Dynamic digital technologies for dynamic mathematics: Implications for teachers' knowledge and practice*. UCL IOE Press.

Drijvers, P., Kieran, C., Mariotti, M. A., Ainley, J., Andresen, M., Chan, Y. C., ... & Meagher, M. (2010). Integrating technology into mathematics education: Theoretical perspectives. *Mathematics education and technology-rethinking the terrain: The 17th ICMI Study*, 89-132.

Ertmer, P. A., & Ottenbreit-Leftwich, A. T. (2010). Teacher technology change: How knowledge, confidence, beliefs, and culture intersect. *Journal of Research on Technology in Education, 42*(3), 255–284.

Guin, D., Ruthven, K., & Trouche, L. (2005). *The didactical challenge of symbolic calculators: Turning a computational device into a mathematical instrument*. Springer.

Guin, D., & Trouche, L. (1998). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning, 3*(3), 195–227.

Hoyles, C., & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second International Handbook of Mathematics Education*, 323–349.

Kennewell, S. (2001). Using affordances and constraints to evaluate the use of information and communications technology in teaching and learning. *Journal of Information Technology for Teacher Education, 10*(1–2), 101–116.

Latour, B. (2005*). Reassembling the social: An introduction to actor-network theory*. Oxford, England: Oxford University Press.

Lincoln, Y. S., & Guba, E. G. (1988, April). *Criteria for assessing naturalistic inquiries as reports*. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA.

Majchrzak, A., & Markus, M. L. (2012). Technology affordances and constraints in management information systems (MIS). *Encyclopedia of Management Theory,(Ed: E. Kessler), Sage Publications, Forthcoming*.

Ostern, N. K., & Rosemann, M. (2021). A Framework for Digital Affordances. In F. Rowe, R. E. Amrani, M. Limayem, S. Matook, C. Rosenkranz, E. A. Whitley, A. E. Quammah (Eds.), *Proceedings European Conference on Information Systems (ECIS)*, (pp. 1-16), Marrakech, Morocco.

Passey, D. (2011). Learning mathematics using digital resources: Impacts on learning and teaching for 11-to 14-year-old pupils. *Mathematics education with digital technology*, *4*, 6-60.

Prestridge, S. (2012). The beliefs behind the teacher that influences their ICT practices. *Computers & Education, 58*(1), 449–458.

Stake, R. E. (1995). *The art of case study research*. SAGE Publications.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning, 9*(3), 281–307.

Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). SAGE Publications.

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 6
(Day 4)

TSG6-1-17

TOWARDS A DIGITAL MATHEMATICS TEACHING ANALYTIC FRAMEWORK

Erlina Ronda
University of the Philippines, Philippines, erronda@up.edu.ph

Abigail Gonzales
University of the Philippines, Philippines, abgonzales5@up.edu.ph

In this paper, we offer a framework for characterizing lessons that use digital mathematics resources in teaching. The framework is derived from existing lenses on studying teaching practices and teacher knowledge involving use of technology. What differentiates the framework from existing ones is that it is not only descriptive of the various aspects of instruction but also evaluative of the extent through which the digital mathematics resource is transforming the learning and the quality of mathematics made available to learn in a lesson. We present the framework here and exemplify its use in one of the lessons from our project where we investigated how teachers appropriate GeoGebra-based resources in their settings after participating in our professional development course.

Keywords: GeoGebra, Digital Mathematics, Analytic Frameworks, Instruction

Introduction and Background

The programming and the digitalization of nearly all aspects of the mathematical system and the accessibility of these 'digital mathematics' are fast transforming the way mathematics is taught and learned in schools. Digital mathematics refers to mathematics accessed or engaged in its digital form such as those in mathematical software or applications that mimics or obeys the logic and rules of mathematics, such as computer algebra systems. The shifts brought about by these digital resources necessitate targeted support for teachers as well as new lenses to describe how the use of these is impacting instruction.

GeoGebra is a mathematics software where learners and teachers can engage interactively with various mathematical systems in digital form. This application is free yet only a few teachers in our country have knowledge of it; of those that do usually use it for drawing graphs and geometric figures. To support teachers in resourcing their teaching with GeoGebra, we developed a three-part professional development course for 42 secondary school mathematics teachers in the Philippines. The course was conducted within a year, with each part totaling about 30 contact hours. Part 1 focused on learning the GeoGebra tools and the mathematics behind the tools. Part 2 is on designing GeoGebra activities and lessons informed by the mathematics discourse in instruction framework (Adler & Ronda, 2015). Part 3 introduced them to the process of doing Lesson Study so that they can investigate and continue to learn with their peers about how they can effectively use GeoGebra in their teaching. To assess and evaluate the training and to further understand how teachers appropriate what they learned given their technology-limited classroom settings, we developed a lens through which we can analyze their lessons. Thus, the framework we present here.

The Digital Mathematics Teaching Analytic Framework (DMT)

The study adopted a sociocultural perspective to understand GeoGebra-resourced teaching and so we need a framework that will enable us to describe instruction in this perspective. In addition, we also want to be able to describe the quality of the mathematics instruction. So, the framework needs to be both descriptive and evaluative. To do this we draw on frameworks from the literature. We considered Ruthven's (2009) *Structuring Classroom Practice Framework (SCPF)*. The SCPF recognizes that teacher knowledge is descriptive of various elements that structure instructional practice, namely the *working environment*, *resource system*, *activity structure*, *curriculum script*, and *time economy*. While we agree that these are indeed critical elements in instruction, the descriptions of each element are too general for our purposes. So, we operationalized what they are in relation to our research setting and goal and rename some of them as we see fit. We incorporated indicators that would be evaluative of quality of instruction and of the mathematics made available to learn in the lesson. We call the resulting framework *Digital Mathematics Teaching Analytic Framework* and is constitutive of the following facets or aspects of instruction

Digital Mathematics Resource System

One of the components of the SCPF framework is the resource system. For our study, we only focused on the resources where the learners engaged with mathematics in digital form which are the GeoGebra-based activities. Since we are interested in describing teachers' appropriation of this digital resource in instruction, it was necessary to describe the extent to which the use of the resource is transforming the experiencing of mathematics. To do this, we were inspired by Puentedura's (2006) work, but we renamed and redefined them to serve as an evaluative framework. At the lowest end, the use of the resource is labelled as *integration* when the learning experience with or without the resource is the same. That is, the GeoGebra activities are integrated into teaching; however, they serve merely as a direct replacement for traditional tools or methods in the teaching and learning of mathematics, without necessarily altering the overall mathematical experience. At the other end of the spectrum is *transformation* where the use of the GeoGebra activities creates entirely new ways of learning and engaging with mathematics, enabling learners to explore mathematical concepts dynamically and interactively in ways that were previously impossible without technology. In between these are *enhancement* and *innovation*. The digital resource system is labelled innovation when the introduction of new GeoGebra functionalities significantly enhance the teaching and learning experience by transforming the way mathematical concepts are visualized, analyzed, or manipulated in digital form. The resource system is coded enhancement, just above integration, when the use of GeoGebra activities improve the presentation and understanding of mathematics through visual, dynamic, or interactive elements, although the underlying mathematical concepts remain the same.

Mathematics Discourse

Optimizing the use of the resource may not always indicate providing quality mathematics in teaching. We needed another lens through which we can characterize instruction so that it is descriptive and evaluative of the quality of the mathematics that is made available to learn in the GeoGebra-resourced lesson. We think that it would be more appropriate to call this facet as *mathematics discourse in lesson script*. Drawing from Sfard's (2012) notion of mathematics discourse, we define quality of mathematics in terms of the quality of mathematics discourse in the lesson. Here, we adapted the mathematics discourse in instruction framework (Adler & Ronda, 2015) which characterize the quality of mathematics discourse in the lesson in terms of the *tasks* through which learners engage with mathematical processes, through the *examples space* through which opportunity for

generalizing the mathematical objects or abstraction of mathematical entities are created, through *naming* or use of words to refer to the mathematics in the lessons, and through *substantiation* to describe the criteria to which claims and methods about the mathematics in the lesson are substantiated. Each of these elements have indicators of levels in 'formality' of mathematics engaged in the Vygotskian sense of scientific concepts.

Interaction with the Resource

Critical to describing instruction is the type of interaction of the teachers and the learners with the digital resource. Here, our interest is in characterizing the structure of the teacher and the learner interaction with the resources. To describe the nature of this interaction, we draw on Drijver et al's (2013) characterization of the whole class and individual interaction with resources. Drijvers et al proposed a total of nine categorizations, four of which can also be used to describe individual orchestration. Examples of these orchestrations we used in our data analysis include *sherpa at work*, *teach with screen*, etc. However, unlike with the resource system and the lesson script, these types of interaction cannot be considered evaluative but rather, descriptive. Nevertheless, we think that this element in the framework is crucial since it could be connected to the resource system and the quality of mathematics discourse and could help in explaining and making sense of the results of the analysis. Link to describing the type of interaction with the resource is the time spent interacting with them as it could impact on learning or may help explain it and so we included engagement time specifically for this purpose.

Learning Environment

The fourth and last element of the framework is the learning environment where the teaching and learning process takes place. It is expected that this environment could change because of the use of digital mathematics resources and thus could impact on the teaching and learning process. It could take place in a traditional classroom, in a computer laboratory, virtually, with individual or shared resources etc., and so it matters to also get them into focus in the analysis.

Exemplification

We present in Table 1 the result of our analysis of one lesson to exemplify the use of the framework. The analytic process involves reading the lesson transcript in its entirety to identify the main object of learning of the lesson and then chunking the lesson into episodes for detailed analysis using the DMT framework before summarizing them again in a table for a synthesis of the lesson transcript analysis. The main object of learning of the lesson was to determine the effects of the changing values of a, h and k in the equation $f(x) = a(x - h)^2 + k$ of a quadratic function on its graph. The team constructed two GeoGebra applets for this lesson. The first utilizes the slider tool to show the link between the parameters a, h and k and the behavior of graph of the function and the second is about the difference and connection between the graphs of a linear function, $f(x) = x$, and a quadratic function, $g(x) = x^2$. As a digital resource, the first applet is coded as innovation due to two key features: the slider for changing cases and the immediate visualization of changes through the dynamically updating graph. The second applet was only coded as enhancement because the slider for the graphs improved the presentation of mathematics, allowing users to easily display or hide objects without necessarily changing the experiencing of the mathematics. The accompanying mathematics discourse were of the highest level, except for the nature of the task where the highest level considered is problem solving. The example space aided learners to generalize the effects of the parameters to the graph. There was also appropriate use of math words, and the claims made were substan-

tiated mathematically. The interaction with the resource was mostly spot and show with Sherpa at work, meaning the teacher used learners' answers, recorded in the web-based GeoGebra activity, to discuss concepts and provide deeper insights. The learners presented their work and explained their answers as their work was projected on the screen hence the almost equal time with the teacher in terms of interacting with the digital resource.

Table 1. Analysis of one lesson based on DMT analytic framework

Learning environment	Digital resource system	Interaction with the resource	Mathematics Discourse
• Traditional classroom • Projector screen • Smart phones from learners; some shared with the group	• Applet 1: Innovation • App 2: Enhancement	• 49% Spot-and-show with Sherpa-at-work • 25% Explain the screen • 17% Work-and-walk-by • 9% Guide-and-explain Shared by teacher and learners: 60%; Teacher: 21%; Learners 19%	• T3 – Applying • E4 – Fusion • N4 – Mostly formal mathematical names • S4 – conjectures made were legitimated with math principles, definitions, properties

Discussion

The DMT can be used to assess the extent to which teachers are appropriating digital mathematics resources introduced to them in a professional development course, as we have done so in our project. While it is still a work-in-progress, we think it holds promise both in terms of characterizing digital-mathematics-resourced instruction and in understanding how the different components of instruction are related and support each other.

References

Adler, J., & Ronda, E. (2015). A framework for describing mathematics discourse in instruction and interpreting differences in teaching. *African Journal of Research in Mathematics, Science and Technology Education*, *19*(3), 237-254.

Drijvers, P., Tacoma, S., Besamusca, A., Doorman, M., & Boon, P. (2013). Digital resources inviting changes in mid-adopting teachers' practices and orchestrations. *ZDM*, *45*, 987-1001.

Puentedura, R. (2006). Transformation, technology, and education [Blog post]. Retrieved from http://hippasus.com/resources/tte/.

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. Éducation *et didactique, 3*(1), 131-159

TSG6-1-18

ELECTRONIC FEEDBACK IN FORMATIVE ASSESSMENT: ENHANCING STUDENT UNDERSTANDING IN MATHEMATICS

Huan Chin
School of Educational Studies, Universiti Sains Malaysia, Malaysia, chinhuan@usm.my

Cheng Meng Chew
School of Education, Humanities, and Social Sciences, Wawasan Open University, Malaysia. cmchew@wou.edu.my

With technological advancement, electronic feedback could be integrated into the formative assessment system to support mathematics learning. The existing studies mainly focus on examining the effectiveness of electronic feedback to support students' mathematics learning, but they often do not explain how students process and utilize this feedback to enhance their understanding. To fill in this research gap, the case study that involved 18 Grade Five students in National Primary School, National Type Chinese Primary School, and National Type Tamil Primary School was conducted to explore how the electronic feedback in formative assessment supports students' mathematics learning. The data was collected through semi-structured interviews and analyzed using thematic analysis, followed by cross-case analysis. The findings indicated that the electronic feedback generated immediately in the formative assessment could support students to learn from the mistakes made and construct schema. Thus, educators are encouraged to integrate electronic feedback into formative assessment systems to provide timely feedback that enhances students' understanding and engagement.

Keywords: case study, electronic feedback, formative assessment, mathematics understanding,

Introduction

Feedback is one of the fundamental facets of effective mathematics learning. It is the second part of the formative assessment process (Sadler, 1989). After being assessed formally or informally through formative assessment, feedback is given to the students so that they can monitor their strengths and weaknesses in their mathematics knowledge and skill acquisition. As such, they can regulate their learning by reinforcing the skills that they had mastered and improved the skills that they had not mastered (Sadler, 1989). Thus, giving feedback is one of the crucial formative assessment practices to support students' mastery of mathematical knowledge and skills. According to Kehrer et al. (2013), the corrective feedback should be given in a timely manner to avoid reinforcement of the mistakes made. Nonetheless, providing corrective feedback to each student could be challenging due to the large class sizes (Paris, 2022). With this practical constraint, corrective feedback is usually being delivered to the students after the teachers have graded the assignments or homework. In fact, the teachers could only pinpoint the mistakes made by majority of students due to the time constraints. With the advancement of technology, electronic feedback integrated in the formative assessment system emerges as an important educational tool that may support mathematics learning. Existing studies (i.e., Barana et al., 2021; Chin et al., 2021; Huang et al., 2024) examine the effectiveness of electronic feedback system in supporting students' mathematics learn-

ing, but they often do not explain how students process and utilize this feedback to enhance their understanding. In fact, this information is crucial to identify factors that influence effectiveness of electronic feedback and thereby refining the feedback strategies. Thus, this research is conducted to explore how electronic feedback in formative assessment supports students' mathematics learning.

Method

Research Design

The research was conducted by adopting multiple-case study research design to increase the robustness of the study (Yin, 2018). The research was conducted in one classroom each in the three types of primary school in Malaysia, namely National Primary School (NPS), National Type Chinese Primary School (NTCPS), and National Type Tamil Primary School (NTTPS) in Penang, Malaysia. The Grade 5 students attempted the digital formative assessment with 12 to 15 multiple choice items in the computer laboratory individually after they have learnt each subtopic. When once they submitted their answer for each item, they would receive corrective feedback written using respective medium of instruction practiced by each school type. Despite different languages being used, the content of the feedback is the same. The feedback consists of notification of correct or wrong, the correct answer, and the full solution as shown in Table 1. They are required to read and understand the feedback given in the formative assessment system.

Table 1. Component of feedback

Component of Feedback	Feedback for Correct Answer	Feedback for Incorrect Answer
Notification of correct or incorrect	Your answer is correct	Your answer is wrong
Correct answer	-	The correct answer is 'A'
Full solution	The correct solution: 1 year = 12 months 3 x 12 months = 36 months 36 months + 8 months = 44 months	The correct solution: 1 year = 12 months 3 x 12 months = 36 months 36 months + 8 months = 44 months

Research Participants

In this multiple case-study, the unit of analysis are the students in different school types who have read the feedback generated in the formative assessments. The research participants in each school type were selected using the maximum variation purposive sampling technique to ensure the representativeness of the research. The process began with categorizing students into three mathematics achievement categories based on their overall grade computed using their scores in the formative assessment. For example, if the student correctly answered all items correctly for Relationship of Time (14 items), Addition of Time (14 items), Subtraction of Time (12 items), Multiplication of Time (12 items), and Division of Time (12 items) and Problem Solving involving Time (15 items), their total score will be 100 percent (79 items / 79 items), and their grade is 'A' following the Grading System in Malaysian Primary School (MOE, 2016). The students obtained grade A (score range: 80-100) were categorized as "high achiever", the students obtained grade B (score range: 65-

79), C (score range: 50-64), and D (score range: 40-49) were categorized as "average achiever" and the students obtained grade E (score range: 0-39) were categorized as "low achievers". Then, for each school type, two students with different genders in each achievement category who have fulfilled the following criteria will be selected as the research participants: (i) the student has completed all digital formative assessments in the system developed; (ii) The student has read the feedback generated in the formative assessments system developed. Thus, this study involved 18 research participants from three types of elementary schools in Malaysia.

Data Collection and Analysis

The qualitative data was collected through semi-structured interviews conducted by the researchers in the schools. The interviews of the 18 research participants were audio recorded and transcribed verbatim by the research assistant for the data analysis. The data analysis was conducted in two stages in this study. At the first stage, thematic analysis was conducted following the framework proposed by Clarke and Braun (2013) to identify the themes which can reflect the qualitative data collected in the study. This was followed by cross-case analysis to determine the common themes. As such, transferability of the findings in different context could be enhanced.

Results

Electronic Feedback Supported Students to Learn from Mistakes

Majority of the participants from NTCPS (5 out of 6 participants) and NTTPS (4 out of 6 participants) did agree that the feedback given helps them to learn from mistake. This is because the full solution given in the feedback could support detection and correction of mistake(s) as stated by the participant ESJKTY5H1 from NTTPS pupil: "*[…] I'll correct my mistake based on the full solution. So, I don't repeat the mistake*". Similar responses were given by the participants from SJKC. The participant ESJKCY5A2 said "[Based on the feedback] *I will know what mistakes I've made. Following on the correct method* [given], *in the next question, I won't make the mistake again*." According to them, they would identify her mistakes by comparing their working with the full solution. After that, they would correct her mistakes and learned from her mistakes. Thus, the full solution given in the feedback could support the detection and correction of mistake(s). In short, the feedback is given to the pupils to support their learning. In this study, the participants make use of the information provided in the feedback to support the process of learning from error, after the internalisation process such as filtering and selecting the important information.

Electronic Feedback Supported Students in Schema Construction

In this study, majority of students from the three school types (NPS: 4/6; NTCPS: 5/6; NTTPS: 5/6) indicate that electronic feedback generated in formative assessment system could promoting schema construction and thereby support students' learning. According to the participant ESKY5A1, the full solution given in the feedback could serve as directive guide. The participant mentioned that "*I know how to do after reading the solution…*" (ESKY5A2_86). Similar responses were given by the participants from NTCPS and NTTPS. For instance, the participant ESJKCY5H1 from NTCPS stated that "… I will also know how to do, if the next questions are almost similar [with this question] ….". Likewise, participant ESJKTY5A2 from NTTPS also stated that "*I'll know why I'm wrong and how to do…I'll ask my teacher for that one...err….that full solution that I didn't understand".* Through reading the feedback, the participant was directed to answer the similar questions. This is because schema construction was initiated when she read the feedback. The new knowledge

was built upon the prior knowledge in the process of knowledge acquisition. Thus, she would be able to answer similar question by applying the same method once she had learned the method shown in the feedback.

Discussion and Conclusion

The findings of this study indicate that the electronic feedback generated immediately in the formative assessment could support students to learn from the mistake. This finding was supported by Chin and Lim (2018) who affirmed that the full solution served as the source of error detection and error correction. In this study, the participants identified their mistake through comparing their solution with the full solution given. After that, they correct their mistake based on the full solution. Although the feedback did not pinpoint the pupils' mistake, the feedback was still useful for error detection. This is supported by the claim made by Manson and Ayres (2021) in which the full solution could support indirect error flagging. The findings of this study also indicate that the electronic feedback generated support schema construction. The use of the full solution as directive guide to support pupils learning. Due to low level of prior knowledge, the pupils with low performance could hardly identify their mistakes even though the full solution is provided (Zamora et al, 2018). Thus, the full solution provided would serve as directive guide rather than the source of error detection and error correction. The pupils received full guidance in the feedback which was presented as a full worked solution. By studying the full solution, the pupils actively construct the schema that can be used to solve the related items (Zamora et al., 2018). In sum, the findings indicated that electronic feedback generated in formative assessment system could be useful for supporting students' mathematics learning. Thus, educators should consider integrating electronic feedback tools into their teaching practices to provide timely, personalized, and actionable feedback that enhances students' understanding and engagement. While the indirect nature of the data limits direct observation of how students use feedback, it still offers valuable insight into their reflective processes. Future research could address this limitation by employing think-aloud protocols to more closely examine feedback use in practice.

References

Barana, A., Marchisio, M., & Sacchet, M. (2021). Interactive feedback for learning mathematics in a digital learning environment. *Education Sciences*, *11*(6), Article 279. https://doi.org/10.3390/educsci11060279

Chin, H., Chew, C. M., & Lim, H. L. (2021). Incorporating feedback in online cognitive diagnostic assessment for enhancing grade five students' achievement in 'time'. *Journal of Computers in Education*, *8*(2), 183-212.

Chin, S. F., & Lim, H. L. (2018). The effect of computerized feedback on students' misconceptions in algebraic expression. *Pertanika Social Science & Humanities, 26*(3), 1387-1403.

Clarke, V., & Braun, V. (2013). Teaching thematic analysis: Overcoming challenges and developing strategies for effective learning. *The Psychologist, 26*(2), 120-123.

Huang, J., Cai, Y., Lv, Z., Huang, Y., & Zheng, X. L. (2024). Toward self-regulated learning: effects of different types of data-driven feedback on pupils' mathematics word problem-solving performance. *Frontiers in Psychology*, *15*, Article 1356852. https://doi.org/10.3389/fpsyg.2024.1356852

Kehrer, P., Kelly, K. & Heffernan, N. (2013). Does immediate feedback while doing homework improve learning. In C. Boonthum-Denecke & G. M. Youngblood (Eds.), *Proceedings of the Twenty-Sixth International Florida Artificial Intelligence Research Society Conference* (pp. 542-545). AAAI Press.

Manson, E., & Ayres, P. (2021). Investigating how errors should be flagged and worked examples structured when providing feedback to novice learners of mathematics. *Educational Psychology*, 1-19. Advance online publication. https://doi.org/10.1080/01443410.2019.1650895

Ministry of Education Malaysia [MOE] (2016a). *Gred sekolah rendah* [Primary school grade]. Retrieved from https://sapsnkra.moe.gov.my/dokumen/GRED.pdf

Paris, B. (2022). Instructors' perspectives of challenges and barriers to providing effective feedback. *Teaching and Learning Inquiry*, Article *10*. https://doi.org/10.20343/teachlearninqu.10.3

Sadler, D. R. (1989). Formative assessment and the design of instructional systems. *Instructional Science*, *18*(2), 119-144.

Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). Sage publications

Zamora, Á., Suárez, J. M., & Ardura, D. (2018). Error detection and self-assessment as mechanisms to promote self-regulation of learning among secondary education students. *The Journal of Educational Research*, *111*(2), 175-185.

TSG6-1-19

PERCEIVED IMPACT OF AI-GENERATED FEEDBACK ON UNDERGRADUATE MATHEMATICS LEARNING IN GEOMETRIC PROOF

Aldrich Agad
University of the Philippines Los Baños, Philippines, asagad@up.edu.ph

Joseph Ma. Steven Cabalo
Department of Education Schools Division of Lipa City, Ateneo de Manila University, Philippines, joseph.cabalo@deped.gov.ph, joseph.cabalo@student.ateneo.edu

Lady Angela Rocena
University of the Philippines Integrated School, Philippines, lmrocena@up.edu.ph

Christine Nicole Victorio
Ateneo de Manila University, National University Philippines, christine.victorio@student.ateneo.edu, cnvictorio@nu-moa.edu.ph

This study investigates undergraduate students' perceptions of ChatGPT-generated feedback on mathematical proofs in projective geometry. Six senior-year BS Mathematics students participated in a problem-solving task, receiving Artificial Intelligence (AI) generated feedback before revising their proofs. Thematic analysis of post-task interviews revealed that students found the feedback confusing, primarily due to ChatGPT's focus on structure rather than mathematical accuracy. Many perceived the feedback as redundant or misaligned with their professors' expectations, leading to low trust in the feedback generated. While some students acknowledged AI's usefulness in concept recall, most preferred traditional learning resources such as textbooks and instructor feedback. The findings highlight concerns regarding AI's effectiveness in mathematical proof evaluation, emphasizing the need for improvements in AI's ability to provide precise, pedagogically relevant feedback. This study underscores the importance of aligning AI-generated responses with academic expectations in mathematics education.

Keywords: ChatGPT, AI-generated feedback, student perceptions, mathematical proofs

Introduction

Artificial Intelligence (AI) has been rapidly expanding and integrating into various aspects of education. A survey by the Digital Education Council (2024) revealed that 86% of bachelor, masters, and doctorate students across 16 countries use AI in their studies, with 24% relying on it daily. As AI becomes more common among students, its role in mathematical problem-solving has also been explored, particularly with ChatGPT (generative pre-trained transformer) which has been shown to provide comprehensive instruction and assistance in geometry (Wardat, 2023).

Previous research highlights AI's potential in education. Alsaiari and co-researchers (2024) pointed out that AI feedback's effectiveness depends on how students engage with it, with emotionally enriched feedback improving perceptions but showing mixed results in engagement and work qual-

ity. Meanwhile, another study found that AI-generated formative feedback boosts motivation by enhancing confidence, promoting socio-emotional interaction, and increasing the perceived importance of mathematics (Zheng & Tse, 2023).

Large language models (LLMs) like ChatGPT are gaining interest for their impact on student learning (Yan et al., 2024). However, concerns remain. Maulida et al. (2024) warned of over-reliance on AI, emphasizing the need for activities that promote critical thinking. On the other hand, Dietvorst et al. (2015) described "Algorithm Aversion", where people distrust AI despite its accuracy, preferring human judgment after observing similar errors. Given these challenges, Maulida et al. (2024) stressed the need for careful AI integration in education, ensuring it complements rather than replaces traditional teaching. Similarly, Melchor et al. (2023) also called for further research to refine AI's role in mathematics education for Generation Alpha students.

This study explores students' perceptions of AI in mathematics, focusing on its use in assessments, trust in AI-generated feedback, and its benefits and limitations in mathematical problem-solving. Specifically, it aims to answer: (1) What are the undergraduate students' perceptions on AI-generated feedback on mathematical proofs specifically, about projective geometry? (2) To what extent do undergraduate students trust AI-generated feedback on mathematics assignments and activities?

Methodology

This study explored undergraduate students' perception of ChatGPT-generated feedback in mathematics through a proving exercise followed by a post-task interview. Thematic analysis identified patterns in the effectiveness, trustworthiness, and impact of AI feedback on their proving exercise.

The topic of the exercise focused on Projective Geometry, a core senior-year subject in the BS Mathematics curriculum. This study involved six students enrolled in AY 2024-2025 who had completed the prerequisite Euclidean Geometry course. This is important as their selection ensured a strong foundation in proof techniques while presenting them with a reasonably challenging problem to assess their proving skills. Participants provided informed consent in compliance with the Philippine's Republic Act No. 10173, otherwise known as the Data Privacy Act of 2012.

The study began with a problem-solving task where participants were given the axioms of an affine plane and asked to prove a specific theorem using only these axioms. This task ensures that all participants engage with the problem at the same level, eliminating external influences such as prior exposure to the proof. Once participants completed their proof, they submitted their solution to ChatGPT using a standardized templated prompt to ensure consistent responses. ChatGPT then generated feedback evaluating the correctness, clarity, and logical structure of the proof, which also provided suggestions for improvement. The prompt used by the researchers was modified from Nazaretsky et al. (2024) on their study where they asked OpenAI GPT interface to provide elaborated formative feedback:

You are a college professor teaching Projective Geometry to 4th year BS Mathematics students. I am one of your students. You gave me the following task:

Prove the statement, "An affine plane contains at least four points" using the following axioms.

Axiom A1: Given two distinct points P and Q, there is one and only one line that contains both P and Q.
Axiom A2: Given a line l and a point P not on l, there is one and only one line m that passes through P and is parallel to l.
Axiom A3: There exist three non-collinear points.

My Proof: [INSERT]

I uploaded the proof I wrote. I want you to provide me feedback in the form of comments that identify errors in my proof and guide me toward making my answer more accurate and well-structured. Your feedback should focus on pinpointing mistakes, explaining why they are incorrect, and suggesting improvements without directly providing the final answer. This is the only feedback that you can give me for this task, so write all comments and questions that you think are necessary for me to satisfactorily revise my draft proof.

Figure 1. Prompt template used in ChatGPT by the participants

After receiving the AI-generated feedback, participants had the opportunity to revise their proof based on the feedback they received. They were given the options to accept, modify, or reject the AI's suggestions, depending on their assessment of its relevance and accuracy. This step allowed researchers to analyze how students engage with AI feedback and whether it influences their problem-solving and revision strategies.

Following the revision phase, participants completed a qualitative post-task interview that explicitly probed their behavioral engagement with feedback, including questions such as "How did you use the feedback to revise and improve your initial answer?", "Would you consider using AI-generated feedback in future mathematics tasks? If so, how?". While these questions aimed to capture participants' reported actions and intentions during revision, the study relied solely on self-described behaviors rather than direct observation or analysis of their actual revisions. Thematic analysis of responses identified patterns in perceived usefulness, trustworthiness, and revision strategies, but findings remain limited to participants' recollective accounts, which may not fully align with their real-time decision-making processes.

Analysis and Discussion

This study examined students' perceptions of ChatGPT-generated feedback on their projective geometry proofs and their trust in its accuracy.

Results revealed that the students perceived the feedback provided by ChatGPT confusing because ChatGPT: provided feedback per line of the provided proof, and not as a whole; failed to assess the overall flow of the proof; gave contradicting feedback. For example, a student said:

> It was confusing especially [because] the response that was given by ChatGPT was per line, so it wasn't able to assess my proof overall and it wasn't able to tell me if the flow of my proof was right or wrong. It simply assessed my proof per line, which was, again, confusing, since I don't know if the feedback that ChatGPT gave was supporting each other, since some feedback it gave was contradicting.

Other students perceive ChatGPT's feedback to their proofs unhelpful because they were only reiterations or elaborations of their own arguments, making their proofs complicated and not as straightforward as their initial proof. For instance, a student noted:

> Well technically, I did not improve my proof because as per ChatGPT it was already correct; it just needs more elaboration on some things but as but personally, I don't want that, because it would just make my proof more complicated. I want it to be as straightforward as possible.

Moreover, some feedback focused on grammar and sentence structure which the students do not find helpful because their professors prefer a specific way of writing proofs.

Meanwhile, some students find ChatGPT feedback helpful as it helped them structure their proofs with better transition from one argument to another. In one case, a student said:

> …I improved my decision from the proof that I made. Like, for the first, how it connects, how the … first part of my proving connects to the second part and the conclusion. And it also construct [sic] my wordings like some ... more correct or more ... not confusing.

When asked about the students' trust in AI-generated feedback, the majority expressed that they do not trust it and do not see themselves using it in their future proving tasks. Moreover, the students highlighted that there are other references that can be used such as YouTube videos, mathematics textbooks, and professors who can provide more helpful and reliable feedback. Still, some noted that it can be trusted in recalling and learning mathematical concepts or in instances where they need guidance on how to start their proofs.

Conclusions and Recommendations

This study explored undergraduate students' perceptions of AI-generated feedback in mathematical proof-writing, with a focus on projective geometry. While ChatGPT delivered structured feedback, participants predominantly reported confusion and limited utility, favoring conventional resources such as textbooks and instructor-led guidance. A subset of students recognized AI's supplementary benefits, including concept reinforcement and transitional phrasing, yet the majority expressed skepticism toward its capacity to enhance core proving competencies. Criticisms centered on fragmented line-by-line evaluations, internal contradictions, and deviations from educational standards emphasizing concision and logical rigor. These findings underscore significant challenges in deploying AI tools within proof-based mathematics education.

To advance AI's educational utility, future iterations must prioritize more systematic and direct feedback. Subsequent research should: (1) conduct comparative analyses of AI and human feedback efficacy; (2) examine how users' perceptions and specific situations affect their trust in AI systems; (3) optimize prompt engineering to emphasize logical coherence; (4) develop domain-specific LLMs tailored to advanced mathematical discourse; and (5) design AI literacy curricula to foster critical engagement with automated feedback.

References

Alsaiari, O., Baghaei, N., Lahza, H., Lodge, J., Boden, M., & Khosravi, H. (2024). Emotionally Enriched Feedback via Generative AI. *arXiv preprint arXiv:2410.15077.*

Dietvorst, B. J., Simmons, J. P., & Massey, C. (2015). Algorithm aversion: people erroneously avoid algorithms after seeing them err. *Journal of experimental psychology: General, 144*(1), 114. https://papers.ssrn.com/sol3/Delivery.cfm?abstractid=2466040

Maulida, L., Nurossobah, P., Aura, B. A., Nengsih, E. D., & Rasilah, R. (2024). Improving The Effectiveness of Mathematics Learning Through Artificial Intelligence: Literature Review. *Journal of General Education and Humanities, 3*(4), 323-338. https://journal-gehu.com/index.php/gehu/article/download/267/282

Melchor, P. J. M., Lomibao, L. S., & Parcutilo, J. O. (2023). Exploring the potential of AI integration in mathematics education for Generation Alpha—Approaches, challenges, and readiness of Philippine tertiary classrooms: A literature review. *Journal of Innovations in Teaching and Learning, 3(*1), 39-44.

Nazaretsky, T., Mejia-Domenzain, P., Swamy, V., Frej, J., & Käser, T. (2024, September). AI or human? Evaluating student feedback perceptions in higher education. In *European Conference on Technology Enhanced Learning* (pp. 284-298). Cham: Springer Nature Switzerland. https://osf.io/6zm83/download

Wardat, Y., Tashtoush, M. A., AlAli, R., & Jarrah, A. M. (2023). ChatGPT: A revolutionary tool for teaching and learning mathematics. *Eurasia Journal of Mathematics, Science and Technology Education, 19*(7), em2286. https://www.ejmste.com/download/chatgpt-a-revolutionary-tool-for-teaching-and-learning-mathematics-13272.pdf

Yan, L., Sha, L., Zhao, L., Li, Y., Martinez-Maldonado, R., Chen, G., ... & Gašević, D. (2024). Practical and ethical challenges of large language models in education: A systematic scoping review. *British Journal of Educational Technology, 55*(1), 90-112. https://bera-journals.onlinelibrary.wiley.com/doi/pdf/10.1111/bjet.13370

Zheng, W., & Tse, A. W. C. (2023, November). The Impact of Generative Artificial Intelligence-based Formative Feedback on the Mathematical Motivation of Chinese Grade 4 Students: a Case Study. In *2023 IEEE International Conference on Teaching, Assessment and Learning for Engineering (TALE)* (pp. 1-8).

Acknowledgements

The researchers would like to acknowledge the Philippine Council of Mathematics Teacher Educators, Inc. (MATHTED) for providing an avenue of collaboration among the researchers.

TSG6-1-20

ANALYSIS OF MATHEMATICAL PERFORMANCE AND REASONING PROCESS OF GENERATIVE AI: FOCUSING ON THE COLLEGE SCHOLASTIC ABILITY TEST

Oh Nam Kwon
Seoul National University, Korea, onkwon@snu.ac.kr

Byoung Chul Shin
Suwon Academy of World Languages, sinbc2003@snu.ac.kr

Sejun Oh
Hongik University, Korea, soh@hongik.ac.kr

Recently, generative AI has been actively utilized in educational settings, prompting discussions on its various educational effects. Integrating generative AI into mathematical problem-solving can offer new ways to support students' learning processes and enhance their problem-solving abilities (Kwon et al., 2023). This shift can provide learners with personalized learning experiences and increase their engagement. However, there are concerns that an over-reliance on generative AI may diminish students' autonomous thinking and critical problem-solving skills. Thus, when using generative AI in the classroom, teachers' leadership plays a crucial role (Yoon & Kwon, 2024). Several issues must be addressed before generative AI can be employed as a learning tool in mathematics education. Prior research has focused on the Accuracy rates and common error types associated with generative AI's mathematical performance without analyzing its step-by-step reasoning processes. This limitation stemmed from AI's "black box" nature, which provides answers without revealing how they were derived. However, with the release of OpenAI's o1-preview model in September 2024, observing the AI's reasoning process has become possible, allowing for comparisons with human problem-solving approaches.

Schoenfeld (1985) and Pólya (1945) emphasized the importance of comprehensive understanding and diverse strategies in mathematical problem-solving. Existing problem-solving models are largely prescriptive and suggest an ideal sequence of linear steps. Yet, problem-solving often involves errors, detours, and iterations, thus exhibiting a nonlinear process (Rott et al., 2021).

Rott et al. (2021) analyzed the problem-solving process models of Mason et al. (1982), Schoenfeld (1985), and Wilson et al. (1993), as illustrated in [Figure 1]. They compared each model's stages, transitions between stages, and features of linearity and nonlinearity, ultimately proposing a descriptive model that allows for recurrence and transitions between the phases.

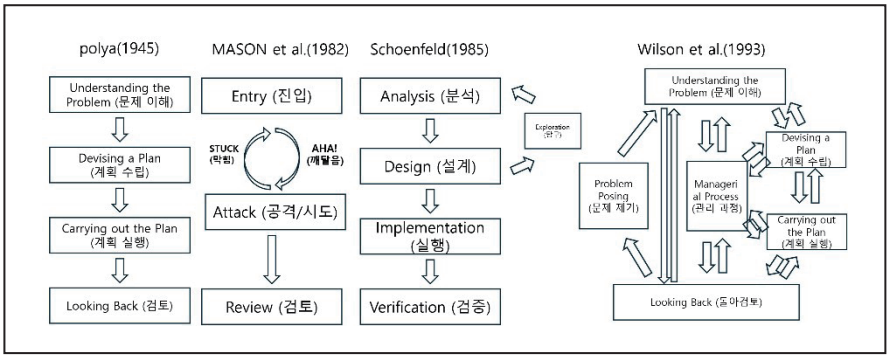


Figure 1. Various stages of the problem-solving process (Rott et al., 2021, p. 3)

This study analyzed the mathematical performance and reasoning processes of a generative AI model using the mathematics section of the 2023, 2024, and 2025 College Scholastic Ability Tests (CSAT) in Korea as the research subjects. The generative AI used in this study was o1-preview (OpenAI, 2024), updated in September 2024, and represented the latest model available at the time of research. Because the o1-preview allows one to observe both its reasoning process and reasoning time, it was chosen as the generative AI model for this study, enabling not only an evaluation of its mathematical performance but also an analysis of its problem-solving reasoning.

Although o1-preview can display its reasoning process and reasoning time for user review, it does not support file uploads. Therefore, as in the method used by Kwon et al. (2023), any math items containing equations or formulas were converted into LaTeX instead of being input as image files (e.g., screenshots). Although the math section of the CSAT consists of multiple-choice and short-answer questions, only short-answer formats were used in queries to o1-preview since a purely multiple-choice format might allow the AI to select the closest alternative without correct reasoning. Moreover, when multiple questions are asked in the same chat session, o1-preview accumulates previous conversations, potentially influencing subsequent solutions. To minimize the carryover effects of earlier problems, each question was asked in a new chat session.

The reasoning and problem-solving processes of the o1-preview were categorized into six stages—(1) reading, (2) analyzing, (3) exploring, (4) planning, (5) executing the plan, and (6) verifying—based on the model proposed by Rott et al. (2021), and were then visualized and analyzed as shown in [Figure 2].

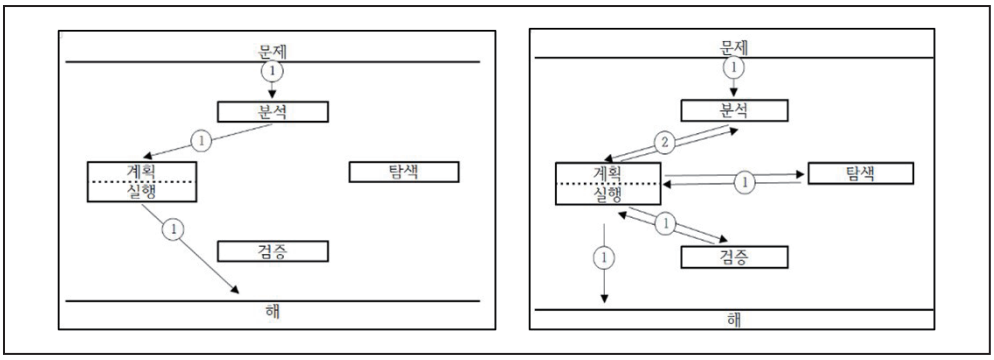


Figure 2. Representation of the reasoning process in o1-preview

The answers generated by o1-preview in its first, second, and third attempts on the 2023–2025 College Scholastic Ability Tests (CSAT) and their corresponding scores are shown in [Table 1]. The CSAT consists of a common section and three elective sections—Probability and Statistics, Calculus, and Geometry—with 30 questions worth 2, 3, or 4 points each. Because generative AI can yield varying outputs to the same query, three attempts were observed for every question, and the results were analyzed. In the CSAT, o1-preview's performance ranged from Levels 1 to 4. Compared to the findings of Kwon et al. (2023), where the ChatGPT 3.5 model scored around Levels 6–7 in their initial study on its mathematical performance in the 2023 CSAT, o1-preview demonstrated a significant improvement in mathematical proficiency.

Table 1. O1-preview's scores on the common and elective sections of the 2023, 2024, and 2025 CSAT

Year	Common + Probability and Statistics			Common + Calculus			Common + Geometry		
	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
2023 Score	68	69	73	65	63	69	68	69	69
2023 Level	4	4	3	4	4	3	4	3	3
2024 Score	92	72	80	81	62	66	84	59	72
2024 Level	2	4	3	1	3	3	2	4	3
2025 Score	76	80	72	73	69	62	69	80	69
2025 Level	3	3	4	3	3	4	3	2	3

Across all three years of exams and all three attempts, o1-preview consistently required more reasoning time for higher-point questions. Lower-point questions were often solved through a relatively linear process 'analyzing → planning → executing the plan' leading quickly to a final answer. By contrast, higher-point questions tended to involve repeated cycles of exploring and verifying, thereby increasing the overall reasoning time. Differences also emerged among the elective sections: in Calculus and Geometry, additional considerations for graphs and geometric figures contributed to longer reasoning times compared to Probability and Statistics.

The verifying stage was particularly critical for o1-preview to arrive at correct solutions for 4-point questions. For example, [Figure 3] shows CSAT common item 14 from 2025, which uses the law of sines and the law of cosines to determine the maximum area of a triangle. o1-preview produced incorrect solutions in the first and third attempts but succeeded in the second attempt. In the first attempt, the verifying stage did not appear at all. In the third attempt, although the model detected errors during the verifying stage and retried solving the problem, it merely generated error messages (e.g., that a triangle could not be formed) without undertaking further exploring or establishing a new plan. In other words, an error was identified but remained uncorrected because o1-preview continued to provide the flawed solution process rather than seeking an alternative approach.

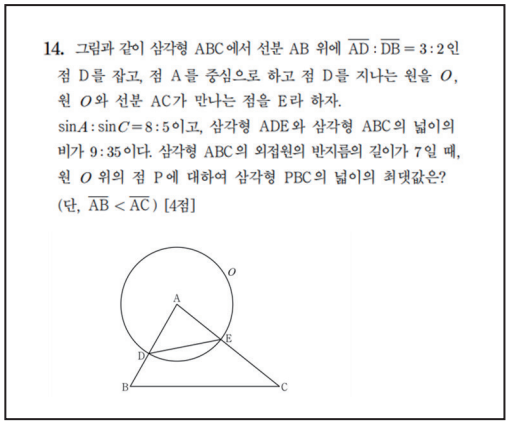


Figure 3. Question 14 of the 2025 CSAT

In contrast, in the second attempt, o1-preview detected errors during two separate verifying stages and pursued new strategies for problem-solving, ultimately forming a correct approach. Subsequently, it proceeded with planning and executing the plan to calculate the area of triangle ABC accurately and derive the correct answer. Notably, the model demonstrated logical consistency by departing from its initial strategy, seeking a new problem-solving method, verifying it, and eventually arriving at the valid solution. These findings suggest that o1-preview leverages iterative verifying, planning, and executing the plan to utilize its stored knowledge for proposing new approaches and highlight how the processes of exploring and correcting errors significantly influence the accuracy of the final solution.

This study applied the o1-preview model to math questions from the CSAT and analyzed its performance and reasoning processes. The results indicate that o1-preview achieves higher accuracy and consistency than previous models, demonstrating a stable rate of correct answers for 2- and 3-point questions and even solving some high-difficulty 4-point questions. Moreover, the exploring and verifying stages emerged as key factors in the reasoning process. The omission of the verifying stage or failure to correct identified errors often led to incorrect results. However, once the model detected an error and engaged in additional exploring, verifying, planning, and executing the plan, it usually arrived at the correct answer. These findings underscore that repeated cycles of exploring and verifying are vital for generative AI when tackling complex problems.

Limitations remain in o1-preview's performance. Missing conditions, conceptual misunderstandings, and calculation errors were frequently observed, particularly in higher-difficulty questions. Such problems required more thorough exploring and verifying before a correct solution could be reached. This suggests that reinforcing non-linear thinking and iterative verifying is crucial for advancing the reasoning capabilities of generative AI.

References

Kwon, O. N., Oh, S. J., Yoon, J., Lee, K., Shin, B. C., & Jung, W. (2023). Analyzing Mathematical Performances of ChatGPT: Focusing on the Solution of National Assessment of Educational Achievement and the College Scholastic Ability Test. *Communications of Mathematical Education, 37*(2), 233-256. http://doi.org/10.7468/jksmee.2023.37.2.233

Mason, J., Burton, L., & Stacey, K. (1982). *Thinking mathematically*. Pearson.

OpenAI. (2024). *Learning to reason with LLMs.* https://openai.com/index/learning-to-reason-with-llms/

Polya, G. (1945). *How to solve it*. Princeton University Press.

Rott, B., Specht, B., & Knipping, C. (2021). A descriptive phase model of problem-solving processes. *ZDM–Mathematics Education, 53*, 737-752.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Academic Press.

Wilson, J. W., Fernandez, M. L., & Hadaway, N. (1993). *Mathematical problem solving. In P. S.* Wilson (Ed.), Research ideas for the classroom: High school mathematics (pp. 57–77). MacMillan.

Yoon, J., Kwon, O. N.,(2024). Systematic literature review on AI-based mathematics teaching and learning: Focusing on the role of AI and teachers. *The Mathematical Education, 63*(3), 573-591.

TSG6-1-21

SHIFT OF OUTPUT RESULTS WHEN INPUT TO THE GENERATIVE AI ARE CHANGED : A CASE STUDY OF THE 2024 NATIONAL STUDENT ACHIVEMENT SURVEY FOR GRADE 9

Ruka Kikuchi
Graduate School of Education, Saitama university, Japan, kikuchi.r.212@ms.saitama-u.ac.jp

Akio Matsuzaki
Saitama university, Japan, makio@mail.saitama-u.ac.jp

We focused on the differences in answers to mathematical problems by various types of generative AIs.
First, we asked the various generative AIs to solve problems from the 2024 National Student Achievement Survey for grade 9 and summarized the results. Next, the shift of output results corresponding to the input contents were organized when the inputs contents to the generating AIs was changed.

Keywords: generative AI, National Student Achievement Survey

Research Background and Research Question

The first author have been studying for mathematical problem solving with generative AI (Kikuchi, 2024) . In recent years, there has been growing interest in the educational use of generative AI around the world. In the United States, "Artificial Intelligence and the Future of Teaching and Learning" (Department of Education's Office of Educational Technology, 2023) states that the benefits of generative AI include the abilities to support a wide range of learner skills and needs, and to improve the quality and quantity of feedback to students and teachers.

In these days, it is impossible to understand insides the generative AI. Therefore, there is a need to explore the generated output results to interpret how they correspond to the input contents. The research question of this paper is to clarify how the shit in mathematical problem-solving output results by generative AI according to variations in the input contents.

The results of 2024 National Student Achievement Survey for grade 9 by generative AI

We input the problems from the 2024 National Student Achievement Survey for grade 9 to generative AIs and organize output results. In this study, we compare four types of generative AI: ChatGPT (OpenAI), Gemini (Google), MathGPT (MathGPT), and Mathos AI (MetaDigits.AI). MathGPT and Mathos AI are developing their services as math-specific generative AIs. We will pay attention to the following when creating the solutions;

(1) Attach an image of each problem to the generative AIs.

(2) Input contents as "Solve this problem."

And the results are summarized the answers by generative AIs for the 2024 National Student Achievement Survey for grade 9 in Table 1.

Table 1. Results of the 2024 national student achievement survey for grade 9 by generative AI

		ChatGPT	Gemini	MathGPT	Matnos AI	The average of Japanese students
1		○	○	○	○	35.5%
2		○	○	○	○	53.1%
3		○	×	×	×	68.7%
4		×	○	×	×	65.6%
5		○	○	○	○	73.4%
6	(1)	×	○	×	×	90.4%
	(2)	○	○	○	○	36.7%
	(3)	○	○	○	○	42.5%
7	(1)	○	○	○	○	74.8%
	(2)	×	×	○	○	26.4%
	(3)	×	×	×	○	48.7%
8	(1)	○	○	○	○	83.7%
	(2)	×	×	○	○	17.7%
	(3)	×	○	○	×	77.2%
9	(1)	×	×	×	×	26.5%
	(2)	○	×	×	×	27.4%
Percentage of correct answers		56.25%(9/16)	62.5%(10/16)	62.5%(10/16)	62.5%(10/16)	53.0%

Note. The National Student Achievement Survey consists of three problem types as follows: multiple-choice type (selecting the correct option from given choices), short-answer type (writing numerical or algebraic expressions), and essay-style type (writing a short passage using numbers and text). The results of multiple-choice answers are indicated in red, short-answer answers in blue, and essay-style answers in green. "The average of Japanese students" refers to the percentage of students who answered each problem correctly, as reported in the 2024 National Student Achievement Survey report for grade 9 (p.13).

The number of correct answers by each generative AI in these problems were 9 or 10, and the percentage of correct answers by each generative AI was higher than the average percentage of correct answers by the students. The number of problems correctly answered by all the generative AIs was 7.

For essay style problems, MathGPT and Mathos AI had higher percentages of correct answers than ChatGPT and Gemini. Since MathGPT and Mathos AI are math-specific generative AIs, they may be able to efficiently incorporate mathematical findings into their output results.

Shift of Output Results when Input Contents are Changed

Human input contents to generative AI, and generative AI output results in response to those contents. Human inputs the changed contents, and generative AI outputs the results are shifted in response to those contents. In an ongoing chat, the generative AI will output in response to any previously inputted comments. Even small edits in the preceding can redirect the output results, it is necessary to correct and improve the input contents so that results are outputted correctly based on mathematical knowledge of human.

Example of Output Results Shift

We focus on the shist of output results when the input contents are changed.

We take No.4 as an example of multiple-choice type problem.

No.4 Let's consider the graph of the linear function $y = ax + b$.
The straight line in the figure is the graph when $a = 1$ and $b = 1$.
For the straight line ℓ, Choice graphs are labeled "ア", "イ", "ウ", "エ".
when the value of a is greater than 1 without changing $b = 1$, select one of them.

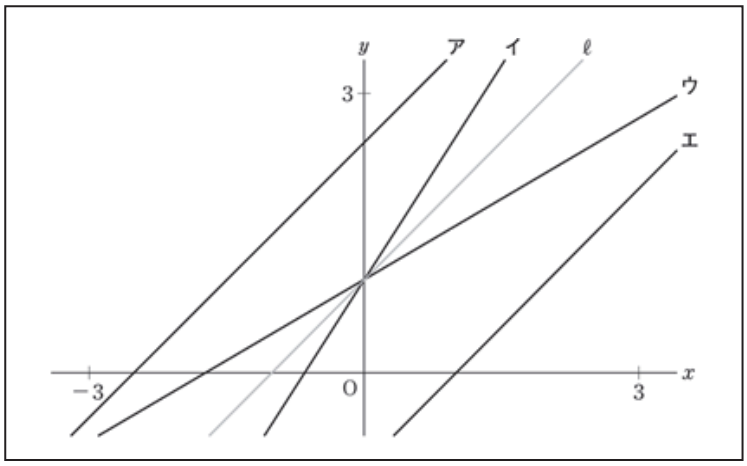


Figure 1. The problem of No.4

The input contents are shown as Q1. The output results are shown as A1.

Q1 : Inputs 'solve this problem', and response A1 : outputs 'only "エ" corresponds to a steeper slope'.

Q1: solve this problem.
A1:1. Understand the original line ℓ
For $a = 1$ and $b = 1$, the equation of ℓ is $y = x + 1$.
This line passes through (0, 1) and (1, 2).
2. Effect of increasing a:
When $a > 1$, the slope increases, meaning the line will become steeper.
The y-intercept ($b = 1$) remains fixed, so all new lines must pass through (0, 1).
3. Identify the correct line:
Among "ア", "イ", "ウ", "エ" only "エ" corresponds to a steeper slope.

Figure 2. Contents of Q1 and A1

Q2 : Inputs ‘What shall we find out this problem?’, response and A2 : outputs ‘From the initial line ℓ (slope $a = 1$), choose which line has a slope greater than 1’.

Q3 : Inputs ‘Is it in the problem statement to choose the graph for the line ℓ when the value of a is greater than 1 without changing $b = 1$?’, and response A3: outputs ‘The problem is to see how the graph changes when the slope a is greater than 1, leaving $b = 1$ unchanged’.

Q4 : Inputs ‘What are the graphs of leaving $b = 1$ unchanged?’, and response A4: outputs ‘The graph of leaving $b = 1$ unchanged is “ア”’.

Q5 : Inputs ‘“ア” does not appear to pass through (0, 1)’, and response A5: outputs ‘**$b = 1$** and passes through the point (0, 1) is the graph “イ”’.

Q6 : Inputs ‘Does the graph“ウ” also pass through (0, 1)?’, and response A6: outputs ‘The graph of leaving $b = 1$ unchanged are “イ” and “ウ”’.

Q7 : Inputs ‘Which graph is the answer to the problem?’, and response A7: outputs ‘Graph “イ” is the best suited for the conditions in problem because it is a steep line, with slope greater than 1, while maintaining $b = 1$’.

Q7:“Which graph is the a

A7: Graph “イ” has a steeper slope maintaining $b = 1$ and appears to rise most steeply.
compared to the other graphs. Since the graph has a steep angle to the x-axis and a large. change in the direction of the y-axis, we can conclude that the slope is greater than 1.
Graph “イ” is the most suitable for the condition in problem because it is a steep straight. line with a slope greater than 1 while maintaining $b = 1$.

Figure 3. Contents of Q7 and A7

From Q1 to Q7, AI’s answer shifted from “エ” to “イ”.

It is found out that is following from A7;

1) From the part “Graph “イ” has a steeper slope maintaining $b = 1$ and appears to rise most steeply”, we can see that it match the condition of problem “when the value of a is greater than 1”.
2) From the part “without changing $b = 1$”, we can see that it matches the condition of problem is met from the part “while maintaining $b = 1$.”

So we can conclude the answer to this problem is “イ”.

Points of Generative AI to Output Correct Answers to Mathematical Problems

The output results by the generative AI are expressed as text with mathematical results embedded in output results. Therefore, humans are required to interpret the mathematical results from in expressed output results. And humans must conclude the answer from the interpreting mathematical result. In order to the generative AI to output correct answers to mathematical problems, the following two points are necessary;

1) Outputting the conditions of the problem and checking whether they are adaptable to the problem.
2) Pointing out specifics error points for wrong outputs.

Those two points are important when creating input contents.

Conclusion

When the generative AI solved a mathematical problem, it tended to make errors in the problem-solving process due to it was not able to adapt to the problem. We also confirmed the key points to decide input contents, which are necessary for a generative AI to solve mathematical problems.

TSG6-1-22

CHATGPT AS PROBLEM POSER: CONTEXTUALIZED AI-ASSISTED PROBLEM GENERATION USING STRUCTURED CHAIN-OF-THOUGHT PROMPTING

Joseph Ma. Steven Cabalo
Department of Education Schools Division of Lipa City, Ateneo de Manila University, Philippines, joseph.cabalo@deped.gov.ph

Najiba Ambulo
Department of Education Schools Division of Lipa City, Philippines, najiba.ambulo@deped.gov.ph

Resty Catinoy
Department of Education Schools Division of Lipa City, National University, Philippines, resty.catinoy@deped.gov.ph

Ma. Mina Pamela Rosario
Department of Education Schools Division of Lipa City, Philippines, maminapamela.rosario@deped.gov.ph

Lester Hao
Ateneo de Manila University, Philippines, lhao@ateneo.edu

Mary Jane Castilla
University of Sto. Tomas, Philippines, macastilla@ust.edu.ph

Christine Nicole Victorio
Ateneo de Manila University, National University Philippines, christine.victorio@student.ateneo.edu

Rodolfo Calimlim Jr.
University of Baguio High School, Philippines, rfcalimlimjr@e.ubaguio.edu

Vanessa Zubieta
South Luzon State University, Philippines, vzubieta@slsu.edu.ph

John Patrick Cultura
La Salle College Antipolo, Philippines, patrick.cultura@lsca.edu.ph

Flordeliza Francisco
Far Eastern University, Philippines, ffrancisco@feu.edu.ph

This study explores the use of Generative AI (GenAI), specifically ChatGPT, in generating mathematical problems through structured Chain-of-Thought (CoT) prompting on adding and subtracting dissimilar fractions. Using a qualitative content analysis approach along with descriptive statistics, the study examines how ChatGPT 4 Legacy utilizes

innovation techniques to generate problem variations in a Philippine context. A set of 30 problems are generated and analyzed for correctness and contextual relevance. Results show that the AI model falls short in correctly applying the innovation techniques, is biased on a certain innovation technique, and is limited to one Philippine context despite the instructions indicated in the prompt. Further studies involving the use of prompting techniques, various AI models, and training AI models, along with examining the pedagogical potential of AI-generated problems are thus recommended.

Keywords: Generative AI, Mathematical Problem-Posing, Chain-of-Thought Prompting, Innovation Techniques

Introduction

Mathematical problem-posing (PP) refers to the process of generating new mathematical problems or modifying existing ones (Silver, 1994, as cited in Yeap, 2009). However, teachers may eventually exhaust their supply of new problems for classroom use. To this, Vistro-Yu (2009) highlighted that by applying innovation techniques to existing problems, teachers can create new variations. The six techniques are replacement, addition, modification, contextualization, turning the problem around, and reformulation. These techniques allow old problems to take on new forms, ranging from simpler exercises to more complex problems. In addition, Tica-a (2023) posited that recognizing students' cultural environments aids teachers in making lessons more relevant and engaging. However, contextualization may also present challenges for PP; teachers may face difficulties both in acquiring knowledge about local cultures and preparing problems based on these contexts.

Meanwhile, there seems to be a notable integration of artificial intelligence (AI) in PP in recent years. Segal and Biton (2024) highlighted the use of generative AI (GenAI), particularly ChatGPT, for PP among pre-service teachers and reported increased technological knowledge and pedagogical strategies. Moreover, such tools offer the potential to automate routine tasks, such as generating mathematics problems tailored to the needs of individual students (Chaivisit et al., 2024) and therefore reducing teachers' workload. Despite the benefits, the quality of AI-generated tasks depends on the clarity of the input queries, noting that potential biases in AI systems are important considerations for optimizing the use of AI in education (Canonigo, 2024). Prompt engineering plays a vital role in such tasks; by optimizing prompt length and specificity, AI models can generate more accurate, fluent, and relevant responses (Balaji et al., 2024; Rathod, 2024).

Thus, by leveraging GenAI's ability to produce possibly diverse question formats, the study seeks to describe how ChatGPT utilizes the innovation techniques in generating problems given a base problem and other pertinent instructions, especially with respect to contextualizing the questions to the Philippine setting. This study addresses three key research questions (1) Is there a consistent pattern or bias on how ChatGPT applies innovation techniques? (2) How correctly did ChatGPT utilize the innovation techniques in the questions it generated? (3) How diverse is the Philippine contextualization as applied by ChatGPT in the generated questions?

Methodology

The present study employs qualitative content analysis to examine the problems generated by ChatGPT and identify patterns, themes, and meanings within the data. Using a structured Chain-of-Thought (CoT) prompting framework, the AI is prompted to generate 30 unique mathematical problems based on a single base problem. CoT prompting is a technique that improves a language model's ability to solve complex problems by guiding it through a step-by-step reasoning process

(Kojima et al., 2022). By structuring responses logically, CoT helps models perform better in tasks such as arithmetic and symbolic reasoning, even without multiple examples. The GenAI model used in this study was a ChatGPT 4 Legacy (i.e., free and logged-out account) to ensure that no prompts were and will be recorded which may affect the quality of responses from the model. The innovation techniques outlined by Vistro-Yu (2009) were also integrated in the prompt in hopes of diversifying the problems.

In alignment with the new Philippine mathematics curriculum for Grade 4, this study focuses on students' abilities to add and subtract dissimilar fractions, including proper fractions, mixed numbers, and whole numbers which are under the *Number and Algebra* content domain. The base problem used for the prompt was modified from Tabilang et al. (2015), a textbook from the Philippine Department of Education. In terms of providing a sample Philippine context, the base problem involved a Filipino cultural event (i.e., a birthday celebration), local ingredients (i.e., Baguio cabbage and imported cabbage), and a practical scenario (i.e., shopping for ingredients and cooking *pancit*). Hence, the prompt used for the AI model is as follows:

Q: What are Vistro-Yu's innovation techniques on problem posing?

A: (a) replacement – posing the same problem but changing quantities, amounts, units, shapes, etc. (b) addition – posing the same problem but adding a new given or constraint or adding an obstacle. (c) modification – takes the same given but changes the problem. (d) contextualizing the problem to make it more relevant to students. (e) turning the problem around or reversing the problem – taking the same problem but taking the end goal as the given and the given as the end goal. (f) reformulation – posing the same problem in a different type (e.g. from a proving problem to a situational problem, see Butts, 1980). Depending on the innovation technique used, the new problem generated may be better, worse, or just the same in terms of the level of difficulty, sophistication, and novelty.

Q: Using innovation techniques by Vistro-Yu in diversifying problems, create one problem based on this base problem: "How can Christopher add 1/4 and 2/3?" Ensure that the generated problem: becomes diversified; has contexts that are Philippine-specific; is classified under the innovation techniques; and is based on the learning Competency: "The learners add and subtract dissimilar fractions."

A: Sample answer: "If Ian has 1/4 of a kalamay and Joseph has 2/3 of another kalamay, how many kalamay does Ian and Joseph have together for both of them to eat?" Innovation technique: contextualizing the problem (new problem modernizes the context of the problem by talking about particular persons engaged in eating; a slightly different problem is posed). Philippine-specific context: kalamay as a traditional Filipino snack.

Q: Using various innovation techniques by Vistro-Yu in diversifying problems, create thirty problems based on this base problem: "Alma's mother wants to cook a special *pansit* on her birthday. To make it more special she wants you to use different kinds of cabbage so she decided to go to the nearest grocery store and she saw Baguio cabbage and imported cabbage. She bought 1/2 kg of Baguio cabbage and 1/8 kg of imported Cabbage. If she used 2/5 kg of the cabbages , how many kg of cabbages were left?" Ensure that the generated problems: become diversified; have contexts that are Philippine-specific; are classified under the innovation techniques; and are based on the learning Competency: "The learners add and subtract dissimilar fractions."

A:

Figure 1. Structured CoT prompt for ChatGPT on the generation of thirty problems

Results and Discussion

It could be gleaned from Table 1 that ChatGPT interestingly distributed the generated problems evenly across each innovation technique, despite not being explicitly instructed in the prompt. However, upon manual review of the generated problems, these were actually under *replacement*, *contextualization*, and *turning the problem around*. While the structured CoT helped the AI model

generate responses, the model failed to correctly classify the generated responses; it is heavily biased towards *replacement* despite the structured CoT prompt providing a description for each innovation technique. Moreover, while all problems were contextualized in the Philippine setting, there were seven problems that were classified solely under *contextualizing,* i.e., no other innovation technique other than the former was found to be applied to those problems.

Table 1. Classifying ChatGPT's generated problems based on innovation techniques

Innovation Technique	ChatGPT's Frequency	Actual Frequency
Replacement	5	18
Addition	5	0
Modification	5	0
Contextualization	5	7
Turning the Problem Around	5	5
Reformulation	5	0
Total	30	30

Most classification errors are simply swaps of items and fractions, thus falling under *replacement*, but were labelled as other techniques. For example, the generated problem "In a local bakery, they use 1/2 kg of brown sugar and 1/8 kg of white sugar to make leche flan. If 3/5 kg was used, how much sugar is left for other desserts?" was classified by the AI model as *modification*. It uses the question "how much is left?" after using a certain amount. That is the same structure as to the base problem; it is purely a change of items (from cabbage to sugar) and amounts (from ⅖ to ⅗). This generated problem thus correctly falls under *replacement*.

Meanwhile, various Philippine contexts were seen in the generated questions, but most of them generally focused on the cultural context of Filipino food, which the base problem featured. This is despite the prompt not restricting the context to food. The researchers categorized these local food contexts into five groups: local ingredients and dishes (e.g., *lechon kawali*), cultural and social events (e.g., birthday parties), occupations and roles (e.g., baker), geographic food references (e.g., Baguio cabbage), and daily life activities with food (e.g., local market shopping).

Conclusion and Recommendations

With the use of structured CoT prompting, ChatGPT 4 Legacy falls short in applying and classifying the innovation techniques given a base problem. The model is heavily biased towards replacement as an innovation technique; moreover, it incorrectly classified most of the generated problems, and the contexts used were limited to Filipino food. The study acknowledges potential limitations in the way the innovation techniques were described within the methodology, particularly in the prompt formulation. Moving forward, it is recommended to explore various prompting techniques, using free vs. paid subscriptions, and training the AI model; future work should also examine the pedagogical potential of AI-generated problems—particularly how they can support teachers in differentiating instruction, enriching classroom problem banks, and scaffolding students' problem-posing skills in alignment with curricular goals.

References

Balaji, K., Lokesha, A., Chandana, G., & Pushpa, H. M. (2024). Optimizing prompt length and specificity for enhanced AI chatbot responses. *International Journal of Current Science Research and Review, 7*(9), 7421–7428. https://doi.org/10.47191/ijcsrr/V7-i9-61

Canonigo, A. M. (2024). Levering AI to enhance students' conceptual understanding and confidence in mathematics. *Journal of Computer Assisted Learning, 40*(6), 3215–3229. https://onlinelibrary.wiley.com/doi/abs/10.1111/jcal.13065?casa_token=_5xMmImvcfkAAAAA:tkYK_gowvgE9-h00hT7QLl-w3467KUOzygX42oIuxMYb4i4hINWbMFmbxYgUCVVhC0P_myPgiPrfBkRbdA

Chaivisit, S., Asino, T. I., Jongsermtrakoon, S., Thompson, P., Rezaie, F., & Siripipattanakul, S. (2024). Empowering Educators With Generative AI Tools and Support. In *Transforming Education With Generative AI: Prompt Engineering and Synthetic Content Creation* (pp. 56–81). IGI Global.

Kojima, T., Gu, S. S., Reid, M., Matsuo, Y., & Iwasawa, Y. (2022). Large language models are zero-shot reasoners. *Advances in neural information processing systems, 35*, 22199–22213.

Rathod, J. D., & Kale, G. V. (2024). Systematic study of prompt engineering. *International Journal for Research in Applied Science & Engineering Technology, 12*(6), 597–613. https://doi.org/10.22214/ijraset.2024.63182

Segal, R., & Biton, Y. (2024). Learning to Craft and Critically Evaluate Prompts: The Role of Generative AI (ChatGPT) in Enhancing Pre-service Mathematics Teachers' TPACK and Problem-Posing Skills. *International Journal of Education in Mathematics Science and Technology.* 12. https://doi.org/10.46328/ijemst.4654.

Tabilang, A. R., Arce, I. J. B., Pascua, R. V., Calayag, N. P., Dacuba, L. P., Borais, D. B., Buemia, R. B., Collao, M. T., Morandante, L. G., Danao, A. B., Nu, L. N., Gonzaga, I. A. B., & Daganta, J. A. D. (2015). *Mathematics – Grade 4: Learner's material* (First edition). Department of Education, Philippines.

Tica-a, P. F. (2023). *Techniques and Challenges of Teachers in Contextualizing Filipino 7 Instruction*. *3*(1). https://doi.org/10.56901/aavg9034

Vistro-Yu, C. P. (2009). Using innovation techniques to generate 'new' problems. In *Mathematical problem Solving: Yearbook 2009, association of mathematics educators* (pp. 185–207).

Yeap, B. H. (2009). Mathematical problem posing in Singapore primary schools. In *Mathematical Problem Solving: Yearbook 2009, Association of Mathematics Educators* (pp. 102–116).

Acknowledgements

The authors would like to acknowledge the Philippine Council of Mathematics Teacher Educators, Inc. (MATHTED) for providing an avenue for this collaboration.

TSG6-1-23

POCKET CALCULATORS IN GRADE ONE TO PREDICT DIVISION TABLES

Allan Tarp
MATHeCADEMY.net, Denmark, Allan.Tarp@gmail.com

*The fourth United Nations Development Goal wants all youth and most adults to achieve numeracy. In 'essence-based' mathematics using one-dimensional line-numbers without units, numeracy is at the beginning and calculus is at the end. But the opposite is the case within the alternative 'existence-based' paradigm built on the numbers children bring to school, two-dimensional bundle-numbers with units, where 2 3s is short for 2 bundles with 3s per bundle. Here calculus is needed in grade one for next-to addition of 2 3s and 4 5s as 8s since they add by their areas. Here, adding is preceded by counting leading to a recount-formula, T = (T/B)*B, with T and B for Total and Bundle, used to change units in STEM, and here recounting between icon-numbers and tens leads to a division and a multiplication table as well as to solving equation. Recounting physical units leads to per-numbers becoming fractions with like units and trigonometry when recounting the sides in a bundle-stack. So, changing from essence-based to an existence-based curriculum may allow reaching the UN Development Goal and finally attain equity and access to mathematics education for youth and adults.*

Keywords: Early childhood mathematics, numeracy, calculator, curriculum, arithmetic

Background, the 4th United Nations Sustainable Development Goal

The fourth of the 17 UN Sustainable Development Goals defines quality education as 'ensuring inclusive and equitable quality education and promoting lifelong learning opportunities for all.' Here the subgoal 4.6 wants to "By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy". However, different definitions of 'numerate' seem to exist. The English Oxford Dictionary defines it as being "competent in the basic principles of mathematics, esp. arithmetic". In contrast, the American Merriam-Webster dictionary defines it as "having the ability to understand and work with numbers."

The difference in the definitions is interesting. The English uses the passive term 'competent' where the American uses the active term 'work'. The word 'competent' is a predicate, a non-action word, I cannot 'competent' something, I can only be judged as competent by someone who is competent. In contrast, 'work' is an action word, a verb, since with my body and mind I can work on something.

Also, there is a difference between the words 'mathematics' and 'numbers.' Again, mathematics is a non-action word, I cannot 'mathematics' or even 'math' a thing. In contrast, 'number' is both a verb and a noun since I can number different degrees of Many to produce a number for later calculations.

Finally, Many exists in the outside world where humans see and name it differently. In contrast, mathematics does not do so, it is an institutionalized essence that is socially constructed as inside abstractions from outside examples, or as inside examples from inside abstractions. To understand these differences, we now consult the three grand theories, philosophy and sociology and psychology. Philosophy may be able to illuminate the different nature of predicates and verbs. Sociology may be able to illuminate the different inter-human power effects coming from using predicates in-

stead of verbs. And psychology may be able to illuminate the different learning results coming from listening to predicates or practicing verbs.

Grand Theory looks at Mathematics Education

Within philosophy, Existentialism holds that 'existence precedes essence' (Sartre, 2007) so that in a judging is-sentence, an existing subject is being colonized by its predicates. 'Many' thus should be seen ontologically in itself, instead of epistemologically, how some may perceive and verbalize it.

Within psychology, Piaget (1969) sees learning as adapting to outside existence, whereas a Vygotsky (1986) sees learning as adapting to inside institutionalized socially constructed essence.

Within sociology, a structure-agent debate discusses if institutions should be obeyed or negotiated between peers. Here, a Weberian viewpoint (1930) asks if SET is a rationalization gone too far by leaving Many de-enchanted and leaving learners in an 'iron cage'. As to the goal, Bauman (1990) suggests that, by institutionalizing mastery of Math as the means to reach mastery of Many, 'essence-math' has created a 'goal displacement' making the means a goal instead. Then, with the end goal, mastery of Many or existence-math, essence-math may be a means, but are there other means also?

Essence-math sees Many as an example of 1D linear cardinality always able to absorb one more by being built on the assumption that '1+1 = 2'. In contrast, humans see Many as a union of 2D stacks coming from numbering singles, bundles, and bundles of bundles, e.g., T = 345 = 3*BB + 4*B + 5*1. Essence-math sees mastery of math as its goal. A difference could see mastery of Many as the goal to be reached directly by using the bundle-numbers with units that children bring to school and that allows learning core math automatically (Tarp, 2018). Essence-math then could be more meaningful by de-modelling it (Tarp, 2020) built on the French poststructuralist version of Existentialism where deconstruction is used to replace predicates with verbs (Derrida, 1991). As to differences, Difference Research (Tarp, 2018) searching for differences making a difference may design micro curricula to be tested with Design Research and building on the observation that when asked "How many years next time?" a 3-year-old child will say "That is not 4, that is 2 2s" if seeing the 4 fingers held 2 by 2.

Grade one in a Decolonized Future

The teacher: Welcome, I am your teacher in math, which is about the numbers that you can see on this number line, and that is built upon the fact that one plus one is two as you can see here. So …

Showing a V-sign a child says: Mister teacher, here is one 1s in space, and here is also one 1s. Now we count them in time to see how many 1s we have by saying 'one, two'. So, we have two 1s. But only until we add them as a bundle. Then we have one 2s, so 1s plus 1s become 2s, but one plus one is still one when we count it, and not two as you say. And together with this neighbor V-sign the total is one 2s plus two 1s which is one 4s, and not three 3s. And, if I two times show you three 1s I have shown you six 1s. So, the counting-numbers two and three can be multiplied, but they cannot add.

So, instead of adding line-numbers without units, please help us add the bundle-numbers with units we bring to school, as 2 3s and 4 5s, that we can add next-to as eights, or on-top as 3s or 5s. If we add them next-to, we add tiles, which my uncle calls integral calculus. And if we add them on-top the units must be changed to the same unit, which my uncle calls linearity or proportionality. He

says it is taught the first year at college, but we need it here to keep and develop our bundle-numbers with units instead of being colonized with your line-numbers without units. We know that you want to bundle in tens, and in ten-tens, and in ten-ten-tens, but we like to bundle also in 2s, in 3s, in 4s, in half-tens, etc. We know that you have not been taught this, and that the textbook doesn't teach it. But don't worry, we will teach you what we found out in preschool. Or better, instead of you colonizing our ways let us find out together what math may grow from our bundle-numbers with units. My uncle is a philosopher, and he uses the word 'existentialism' form letting existence come before essence.

With sticks we see that 5 1s may be bundled as 1 5s that may be rearranged as one icon with 5 sticks. The other digits may also be seen as icons with the number of sticks they represent, where zero is a looking glass finding nothing. We don't need an icon for ten since here the total is 1B0 if we count in tens. And the calculations are icons also. If we bundle-count 8 in 2s, division is a 'push-away' icon for a broom so that 8/2 means 'from 8 push-away 2s'. Now a calculator can predict the result, 8/2 = 4. Stacking the 4 2s, multiplication is an icon for 'push-back-to-lift' predicting the result, 4x2 = 8. That 8 contains 8/2 of 2s is written as a recount formula '8 = (8/2) x 2', or 'T = (T/B) x B' with T for Total and B for Bundle. Bundle-counting 7 in 2s, we need a rope called subtraction to pull-away the three 2s to find the unbundled 1 to be placed on-top of the stack as a decimal, a less-number, or a fraction when counted in 2s also as 1 = (1/2)x2. So, we have: 7 = 3B1 = 4B-1 = 3½ 2s. A calculator can predict this. On the display we see, first [7/2 3.more], then [7–3x2 1]. This predicts that 7 may be recounted in 2s as 3B1 2s. Likewise we can predict that recounting 30 in 5s by entering 30/5. This allows us to solve the equation 'u*5 = 30' asking 'How many 5s in 30?' by u = 30/5, moving 5 'to opposite side with opposite sign'. Finally, to recount 6 7s in tens we multiply. Or use that 7 may be recounted in half-bundles as ½B2, so 6*7 = 6*½B2 = 3B12 = 4B2 tens = 42.

We know that your numbers without units are on a ruler, but our bundle-numbers with units are on a ten-by-ten Bundle-Bundle-Board, a BBBoard, that we can use for recounting and adding stacks. Recounting between icons, and from icons to tens, we call a division and a multiplication table. It is fun seeing a calculator predict the result when we recount nine 1s in 2s, 3s, etc., on the BBBoard. After recounting in a different unit, we can also use the BBBoard to add stacks on-top or next-to.

Enter	Read	Predict	Unit
8/2	4	8 = 4B0	2s
8/3	2.more		
8 – 2x3	2	8 = 2B2	3s
8/4	2	8 = 2B0	4s
8/5	1		
8 – 1x5	3	8 = 1B3	5s

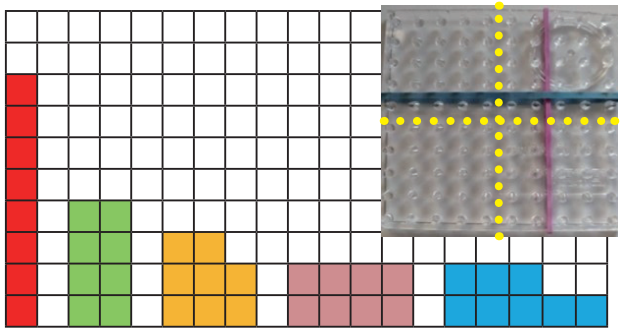


*Figure 1. A calculator as a division table recounting 8 1s in some other units 2s, 3s, 4s and 5s. And a BundleBundleBoard as a times table recounting in tens, 6*7 = 6*½B2 = 3B12 = 4B2 tens = 42*

Conclusion, Recommendation and Discussion with a Reviewer

"By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy". We may reach this UN Development Goal by replacing an essence-based curriculum with an existence-based curriculum answering the question 'how many?' by counting and recounting totals before adding them (Tarp, 2018, 2024, 2025). Digits then occur as icons with as many strokes as they represent, thus becoming units when counting totals existing in time and space with 2D bundle-numbers that are flexible by allowing both overloads and underloads, which makes place values and carrying unneeded. The operations are icons also, but with different meanings and opposite order. Division now means counting iconized by a broom to push-away bundles. Multiplication is iconized by a lift uniting the bundles in a stack that a subtraction rope pulls-away to find the unbundled, seen as decimals, fractions, or negative numbers on top of the stack. Combined, bundling and stacking create a recount-formula with a per-number that changes units and used all over STEM. Calculators may give an inside prediction of an outside recounting on a BBBoard. Thus, both proportionality and trigonometry occur in year one. Once counted and recounted, totals may add on-top, or next-to by areas as with integral calculus, also used to add per-numbers. An existence-based curriculum will finally allow a communicative turn within the number-language as within the word-language in the 1970s (Widdowson, 1978) and use the predicting power of mathematics that also have three genres, fact, fiction and fake (Tarp, 2001). Using children's own bundle-numbers with units thus is a paradigm shift (Kuhn, 1962) that opens new areas for research and innovation. **Reviewers**: *The paper would benefit from clarifying the methodological approach and providing concrete examples or empirical support to strengthen the argument. This paper offers an original and potentially transformative view on early mathematics education. If connecting theory to practice, and offering concrete recommendations, this paper could make a significant contribution to the field.* **Answer**: The paper's methodological approach is that a composer should write but not play a score. Its philosophy is Existentialism holding that existence precedes essence. As to empirical support we may have an ethical flat-or-round-earth situation where we can't have both paradigms, either 1+1 = 2 or it depends on the unit. And, by its very nature, multiplicity is multiplicative thus always occurring with units. Assume class A has an essence-math curriculum with unit-free 1D line-numbers and the traditional order from addition to power where only few will ever hear about calculus. To compare, class B has an existence-math curriculum and 2D Bundle-numbers with units having the opposite order. Here counting and recounting precedes adding, so addition first occurs as the core of mathematics, calculus and linearity. What will happen if two students change classes? How ethical is it to test essence-math against existence-math in an ordinary school? Or in special education where the students will return far ahead to normal education? Ethical testing is for teacher education and home education. Mathematics exists in a high and low level version, so does digital technology also offering digital calculators to all, which will be accepted if all youth and most adults achieve numeracy where a calculator plays the important role as a number-predictor, and where all its nine buttons from the Algebra Square (Tarp, 2018) have been used to count and add in time and space.

References

Bauman, Z. (1990). *Thinking sociologically.* Blackwell.

Derrida, J. (1991). *A Derrida reader: between the blinds.* P. Kamuf (ed). Columbia Uni. Press.

Kuhn T.S. (1962). *The structure of scientific revolutions.* University of Chicago Press.

Piaget, J. (1969). *Science of education of the psychology of the child.* Viking Compass.

Sartre, J.P. (2007). *Existentialism is a humanism.* Yale University Press.

Tarp, A. (2001). Fact, fiction, fiddle - three types of models. in J. F. Matos, W. Blum, K. Houston & S. P. Carreira (Eds.). Modelling and mathematics education: ICTMA 9: Applications in Science and Technology. *Proc. 9th Int. Conf. on the Teaching of Mathematical Modelling and Applications* (pp. 62-71). Horwood Publishing.

Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: From inside-inside to outside-inside understanding. *Ho Chi Minh City Univ. of Education Journal of Science 17*(3), 453-466.

Tarp, A. (2024). *Many before math, math decolonized by the child's own bundle bundle-numbers with units.* https://youtu.be/uV_SW5JPWGs.

Tarp, A. (2025). *BundleBundleMath on BundleBundleBoard.* Retrieved at http://mathecademy.net/ bundle-bundlemath-on-a-bbboard/.

Vygotsky, L. (1986). *Thought and language.* MIT press.

Weber, M. (1930). *The protestant ethic and the spirit of capitalism.* Unwin Hyman.

Widdowson, H. G. (1978). *Teaching language as communication.* Oxford University Press.

TSG6-1-24

AN ANALYSIS OF MATHEMATICAL CAPABILITIES OF GENERATIVE ARTIFICIAL INTELLIGENCE: FOCUSING ON CHATGPT O4-MINI-HIGH, COPILOT, GEMINI 2.5 PRO, AND DEEPSEEK R1

Oh Nam Kwon
Seoul National University, Korea, onkwon@snu.ac.kr

Kyungwon Lee
Seoul National University, Korea, kyungwon.lee.snu@gmail.com

Hong Kyung Yun
Seoul National University, Korea, hongkyung100@gmail.com

Seunghwan Jeon
Seoul National University, Korea, jsh095@snu.ac.kr

Minsik Woo
Seoul National University, Korea, abcv2145@snu.ac.kr

This study analyzes the mathematical capabilities of four generative artificial intelligence models: ChatGPT o4-mini-high, Copilot, Gemini 2.5 Pro, and DeepSeek R1. The evaluation was conducted by examining whether these models accurately responded to mathematics problems from the past three years of the Korean Mathematics College Scholastic Ability Test. The analysis revealed that ChatGPT achieved the highest accuracy rate and exhibited the least variation in accuracy across different years. For all four models, accuracy rates decreased as the point value of the problems increased, and there were common areas where the models consistently performed well or struggled. These findings offer valuable insights into the mathematical capabilities of generative artificial intelligence models and suggest directions for their potential application in artificial intelligence-assisted mathematics teaching and learning.

Keywords: mathematical capabilities, generative artificial intelligence

Introduction

With the continuous advancement of generative artificial intelligence (genAI), existing models are being updated, and new models are emerging, creating a competitive landscape. Since the initial introduction of genAI, ongoing discussions have focused on how these tools can be utilized in educational contexts. Research on the mathematical capabilities of genAI not only provides foundational data for its use in teaching and learning but also highlights its challenges and limitations (e.g., Frieder et al., 2023; Kwon et al., 2023; Oh et al., 2024). Such research is essential for identifying improvements in current models and laying the groundwork for future studies, especially as new genAI models continue to be introduced. This study aims to compare and analyze the mathematical

capabilities of four recently introduced genAI models—OpenAI ChatGPT o4-mini-high, Microsoft Copilot, Google Gemini 2.5 Pro, and DeepSeek R1. Through this analysis, the study seeks to establish a foundation for discussing how the genAI models demonstrate advanced mathematical capabilities. The study addresses the following research question: *How can the mathematical capabilities of ChatGPT o4-mini-high, Copilot, Gemini 2.5 Pro, and DeepSeek R1 be characterized?*

Theoretical Perspective

Instrumental genesis refers to the process by which an artifact becomes an instrument for mathematical thinking through instrument use (Artigue, 2002). In the field of mathematics education research, instrumental genesis is typically discussed in terms of the interplay between instrumentation and instrumentalization, and how this interaction supports the development of mathematical thinking (Trouche, 2005). If genAI is conceptualized as an artifact in an educational context, it is essential to examine the relationship between the human user (subject) and genAI (artifact). In the context of mathematics teaching and learning, an investigation is required to understand how genAI functions as an artifact. As users engage with the functions provided by genAI, they may undergo instrumentation, a process whereby their problem-solving approaches adapt and evolve. Additionally, users may experience instrumentalization, modifying or adapting genAI to suit their specific needs. As these processes become more familiar, users can more effectively integrate genAI as an artifact within the mathematics teaching and learning environment. Based on this perspective, this study shares an analysis of mathematical problem-solving, drawing on users' experiences of instrumentation and instrumentalization in their engagement with genAI tools. Adopting instrumental genesis as a theoretical lens, this study interprets genAI's mathematical capabilities in terms of how patterns of accuracy reflect the co-development of user strategies and genAI functionalities.

Methods

This study explores the mathematical capabilities of genAI models using mathematical problems. By inputting mathematical problems as prompts and having the genAI models solve them, it becomes possible to assess their mathematical capabilities. As researchers affiliated with institutions in Korea, we selected mathematics problems representative of the Korean high school context. In Korea, the College Scholastic Ability Test (CSAT) is the most publicly recognized mathematics examination, with new problems updated annually. Korean mathematics CSAT, which can be taken by students who have completed high school education, consists of 30 problems in multiple-choice and short-answer formats. The total score is 100, distributed as follows: three problems are worth 2 points, 14 problems are worth 3 points, and 13 problems are worth 4 points, with higher point values generally indicating more challenging problems. According to the Korean high school curriculum, Korean mathematics CSAT is divided into three types of exams (See Table 1). *Mathematics I*, *Mathematics II*, *Probability and Statistics*, *Calculus*, and *Geometry* are all subjects taught in Grades 11–12 of high school in Korea. *Mathematics I* and *Mathematics II* are common subjects across all three Korean mathematics CSAT types, while *Probability and Statistics*, *Calculus*, and *Geometry* are elective subjects.

This study analyzes the responses of four genAI models—ChatGPT o4-mini-high, Copilot (with Think Deeper), Gemini 2.5 Pro, and DeepSeek R1—to CSAT mathematics problems input as prompts. To prepare the inputs, CSAT mathematics problems were converted into LaTeX format using Mathpix to extract LaTeX-formatted text from problems while separating visual compo-

nents. These LaTeX-formatted problems were then input into each genAI model to obtain responses. Mathematics problems in which images were essential to the solution process were excluded because Deepseek R1 did not provide a reliable response when an image was submitted as an attachment. Problems from the past three years were selected to ensure sufficient data coverage. Excluding duplicates within each year, 46 problems were selected per year, resulting in a total of 138 problems across the three years. Problems were selected by excluding those that involve essential visual elements for problem solving (43 in 2025, 43 in 2024, and 40 in 2023). Based on the responses, the analysis includes accuracy rates, answer correctness by point value, and accuracy by subject area.

Table 1. Types of korean mathematics CSAT

Category	Type	Common Problems	Elective Problems	Total
Korean Mathematics CSAT	Selection <*Probability and Statistics*> including *Mathematics I*, *Mathematics II*, *Probability and Statistics*	22	8	46
	Selection <*Calculus*> including *Mathematics I*, *Mathematics II*, *Calculus*		8	
	Selection <*Geometry*> including *Mathematics I*, *Mathematics II*, *Geometry*		8	

Findings

This section presents the results of the analysis of the responses provided by the four genAI models to the Korean mathematics CSAT problems. The results for the comparison of accuracy rates across four genAI models are shown in Table 2. Among the four models, ChatGPT and Copilot demonstrated the highest accuracy rates. These results demonstrate a marked improvement in accuracy compared to previous studies (Kwon et al., 2023; Oh et al., 2024).

Table 2. Accuracy rates across four genAI models

Category	ChatGPT o4-mini-high	Copilot (with Think Deeper)	Gemini 2.5 Pro	DeepSeek R1
2025	43/43(100%)	42/43(97.7%)	42/43(97.7%)	39/43(90.7%)
2024	40/43(93.0%)	41/43(95.3%)	40/43(93.0%)	38/43(88.4%)
2023	39/40(97.5%)	39/40(97.5%)	39/40(97.5%)	34/40(85.0%)
Total	122/126(96.8%)	122/126(96.8%)	121/126(96.0%)	111/126(88.1%)

Given the high overall accuracy rates, particular attention is directed toward the cases of incorrect answers. Table 3 presents the number of incorrect answers according to the score distributions of the four genAI models. Four models solved all 2-point problems. However, the accuracy rate decreased as the point value increased, highlighting a consistent pattern. The accuracy rates by point

value reflect the genAI models' problem-solving capabilities relative to the difficulty of the problems. The models tended to perform well on problems involving fundamental mathematical concepts.

Table 3. The number of incorrect answers by point value across four genAI models

Category	2025				2024				2023			
	Chat GPT	Co pilot	Ge mini	Deep Seek	Chat GPT	Co pilot	Ge mini	Deep Seek	Chat GPT	Co pilot	Ge mini	Deep Seek
2-point	-	-	-	-	-	-	-	-	-	-	-	-
3-point	-	1	-	-	-	-	-	-	1	-	-	1
4-point	-	-	1	4	3	2	3	5	-	1	1	5

The results for the comparison of accuracy rates by subject across four genAI models are presented in Table 4. All four genAI models demonstrated higher accuracy in *Probability and Statistics*. ChatGPT, Copilot, and Gemini correctly solved all *Probability and Statistics* problems from the Korean mathematics CSAT across all three years. Notably, the four models struggled with calculus and geometry problems, which aligns with the findings of Kwon et al. (2023). These results suggest that the models exhibit stronger capabilities in specific subject areas.

Table 4. The number of incorrect answers by subject across four genAI models

Category	2025				2024				2023			
	Chat GPT	Co pilot	Ge mini	Deep Seek	Chat GPT	Co pilot	Ge mini	Deep Seek	Chat GPT	Co pilot	Ge mini	Deep Seek
Mathematics I	-	-	1	1	-	-	-	1	-	1	1	1
Mathematics II	-	-	-	2	1	-	-	1	1	-	-	2
Probability and Statistics	-	-	-	1	-	-	-	-	-	-	-	-
Calculus	-	1	-	-	2	1	2	2	-	-	-	1
Geometry	-	-	-	-	-	1	1	1	-	-	-	2

Conclusion

This study aims to analyze the mathematical capabilities of genAI models by examining their ability to solve mathematics problems, specifically focusing on ChatGPT o4-mini-high, Copilot, Gemini 2.5 Pro, and DeepSeek R1. Using problems from the Korean mathematics CSAT as prompts, the analysis revealed that all four models achieved high accuracy rates. These results demonstrate a substantial improvement in the mathematical capabilities of genAI models relative to previous studies (Kwon et al., 2023; Oh et al., 2024). Despite persistent difficulties in solving high-point-value problems and those involving visual elements, the enhanced capabilities of genAI models underscore the need for further discussion on how they can be employed in mathematics teaching and

learning. Further research should investigate whether genAI models can consistently produce accurate responses within limited timeframes. Furthermore, instructional strategies that leverage genAI tools with stable and consistent responses should be explored.

References

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*, 245-274.

Frieder, S., Pinchetti, L., Griffiths, R. R., Salvatori, T., Lukasiewicz, T., Petersen, P. C., ... & Berner, J. (2023). Mathematical capabilities of ChatGPT. *arXiv preprint arXiv:2301.13867*.

Kwon, O. N., Oh, S. J., Yoon, J., Lee, K., Shin, B. C., & Jung, W. (2023). Analyzing mathematical performances of ChatGPT: Focusing on the solution of the National Assessment of Educational Achievement and the College Scholastic Ability Test. *Journal of Korean Society of Mathematical Education Series E <Communications of Mathematical Education>, 37*(2), 233-256.

Oh, S. J., Yoon, J., Chung, Y., Cho, Y., Shim, H., & Kwon, O. N. (2024). An analysis of generative AI's mathematical problem-solving performance: Focusing on ChatGPT 4, Claude Opus, and Gemini Advanced. *Journal of Korean Society of Mathematical Education Series A <The Mathematics Education>, 63*(3), 549-571.

Trouche, L. (2005). An instrumental approach to mathematics learning in symbolic calculator environments. In D. Guin, K. Ruthven, & L. Trouche (Eds.), *The didactical challenge of symbolic calculators: Turning a computational device into a mathematical instrument* (pp. 137–162). Springer.

TSG6-1-25

NOTICING CLASSES OF PRESERVICE TEACHERS: RELATIONS TO TEACHING MOVES THROUGH AI CHATBOT SIMULATION

Taekwon Son
Bongmyeong Elementary School, South Korea, sontaekwon7@gmail.com

Responsive teaching is an effective teaching approach in which teachers engage and respond to students' mathematical ideas to support their mathematics learning. In this study, the relationship between preservice teachers' noticing expertise and their teaching moves was investigated in a simulated AI chatbot environment. The AI chatbot included a virtual student with misconceptions about fraction operations and was used as an instructional tool to provide opportunities for PSTs to practice responsive teaching skills. Using latent class analysis, PSTs' noticing components and teaching move patterns were investigated in a sample of 138 PSTs. The result indicated that significant correlations between PSTs' noticing expertise and the quality of their teaching moves. In addition, three distinct classes were identified based on PSTs' noticing skills, and differential use of teaching moves was found across these classes. The results underscore the differences in PSTs' noticing expertise in the AI chatbot environment and a typical classroom setting and suggest implications for leveraging AI chatbot simulations in teacher education.

Keywords: AI chatbot, simulation, noticing, teaching moves, preservice teachers

Introduction

Responsive teaching, which involves engaging with and adapting to students' mathematical ideas, is a dynamic instructional approach that enhances learning (Bywater et al., 2019; Jacobs et al., 2010). It consists of noticing—attending to students' strategies, interpreting their understanding, and deciding how to respond—and teaching moves, instructional actions that guide student thinking (Jacobs & Empson, 2016). However, mastering these skills requires sustained practice, which can be challenging in real classroom settings (Howell & Mikeska, 2021).

To address this, digital simulations, such as LessonSketch and simulated discussions, have been used to train preservice teachers (PSTs), but these tools often lack open-ended interaction (Son et al., 2024). With advancements in Artificial intelligence (AI) technology, AI-based chatbots now offer a promising alternative by simulating realistic teaching scenarios (Go & Sundar, 2019). Studies have shown that chatbots can enhance PSTs' noticing expertise (e.g., Lee et al., 2024), remains unclear how PSTs' noticing expertise affects their teaching moves in simulated educational environments.

This study explores how PSTs' noticing expertise influences their teaching moves in a chatbot-based environment. The chatbot, designed to simulate a virtual student struggling with fraction operations, serves as a tool to investigate PSTs' responsive teaching practices. By leveraging intentional scaffolding within simulated AI environments, this study aims to provide insights into the role of chatbots in teacher education (Amador et al., 2021).

Methods

Participants

This study involved 138 PSTs from two universities in South Korea, all enrolled in an elementary teaching methodology course. The course covered fraction concepts and student mathematical thinking. PSTs first completed a test measuring their noticing skills, followed by an interaction session with a chatbot that simulated a virtual student.

Development of the Chatbot

The chatbot was developed using Google Dialogflow, which supports Korean language processing. It simulated a virtual student named Jisoo, who exhibited common misconceptions about fractions. PSTs were encouraged to analyze Jisoo's errors and adjust their questioning strategies accordingly. To improve the chatbot's accuracy, a pilot test with 30 PSTs was conducted, refining its ability to interpret varied question phrasing and increasing its response accuracy from 78% to 93%.

Data Collection and Analysis

Data collection consisted of two stages. First, PSTs watched a student-teacher video on fraction subtraction and responded to questions assessing their attending, interpreting, and responding skills. Second, they engaged in real-time conversations with the chatbot, designed to replicate student-teacher interactions. The chatbot conversations were stored in JSON format and later converted into Excel for analysis.

For data analysis, PSTs' noticing expertise was evaluated using a professional noticing rubric (Jacobs et al., 2010), scoring their responses on a scale from 1 to 3. Teaching moves were analyzed using seven categories adapted from Jacobs and Empson's framework, and the proportion of high-leverage teaching moves was calculated. Statistical analyses included Pearson correlation analysis to examine the relationship between noticing skills and teaching moves, latent class analysis (LCA) to identify patterns in PSTs' noticing expertise, and ANOVA tests to determine differences in teaching moves among the identified noticing groups.

Results

Correlation Between Noticing Expertise and Teaching Moves

The study found a significant positive correlation between PSTs' noticing skills and their use of teaching moves in the chatbot simulation. The three noticing components—attending, interpreting, and responding—were interrelated, with moderate correlations between attending and interpreting ($r = 0.497$) and between interpreting and responding ($r = 0.375$). More importantly, all three noticing components correlated positively with teaching moves (attending: $r = 0.320$, interpreting: $r = 0.344$, responding: $r = 0.278$), indicating that stronger noticing skills led to more effective instructional strategies in the chatbot environment.

Classification of PSTs Based on Noticing Expertise

Using LCA, PSTs were categorized into three groups based on their noticing expertise. The High-Level Noticing (HN) group ($n = 12$) demonstrated strong skills in attending, interpreting, and responding. The Medium-Level Noticing (MN) group ($n = 43$) exhibited moderate skills, particularly in attending and interpreting, but showed weaknesses in responding. Lastly, the Low-Level Noticing (LN) group ($n = 83$) had weak skills across all noticing components.

Differences in Teaching Moves by Noticing Group

Representative chatbot interactions revealed notable differences in teaching moves across the three groups. HN PSTs used open-ended questioning strategies, eliciting student thinking and encouraging self-correction. MN PSTs attempted to guide students but often imposed teacher-directed strategies instead of exploring student reasoning. LN PSTs relied on directive approaches, focusing on procedural correctness rather than conceptual understanding.

Statistical analysis of ANOVA confirmed significant differences in teaching moves between groups ($F = 11.551$, $p < .001$). The HN and MN groups used significantly more high-leverage teaching moves than the LN group, but there was no significant difference between the HN and MN groups.

Discussion and Implication

Simulated environments provide valuable opportunities for PSTs to refine their teaching skills, particularly in noticing and responsive teaching. Prior research has examined how PSTs develop noticing skills through video analysis or written reflection, but this study specifically investigated how noticing expertise influences teaching moves in an AI-based chatbot environment. The findings confirm a significant correlation between the three components of noticing—attending, interpreting, and responding—and the use of effective teaching moves.

A key insight is that interpreting showed the strongest correlation with teaching moves ($r = 0.344$), contrary to expectations that responding would be most influential. This suggests that in the chatbot environment, PSTs' ability to interpret the virtual student's reasoning was crucial for determining appropriate instructional responses. Since the chatbot only provides predefined answers, PSTs who struggled to interpret these responses had difficulty formulating effective teaching moves.

Using LCA, PSTs were categorized into three groups: HN, MN, and LN. The HN group demonstrated strong expertise in all noticing components, the MN group excelled in attending and interpreting but needed improvement in responding, and the LN group showed weaknesses across all three areas. Consistent with prior research, all groups had lower scores in responding, highlighting the difficulty of making real-time instructional decisions compared to recognizing and interpreting student thinking.

These findings align with Jacobs et al. (2024), who categorized in-service teachers into accomplished, mixed, and emerging noticing profiles. While noticing occurs continuously rather than hierarchically, identifying common patterns helps pinpoint areas needing further development. Notably, the majority of PSTs fell into the LN group ($n = 83$), reinforcing the need for structured interventions in teacher education to strengthen noticing expertise.

Another important finding is that PSTs with stronger noticing skills used more high-leverage teaching moves. The HN and MN groups employed significantly more effective teaching moves than the LN group, but no significant difference was found between HN and MN. This suggests that while PSTs with moderate noticing skills can effectively attend to and interpret student thinking, they still struggle with decision-making for instructional responses. Unlike previous research that focused on either noticing or teaching moves, this study expands the discussion by demonstrating their interconnection in an AI-based environment.

AI-based chatbots offer a safe, repeatable, and interactive way for PSTs to practice responsive teaching without the ethical concerns of real classrooms. Unlike digital tools that offer fixed-choice responses, chatbots provide open-ended interactions, creating a more authentic learning experience.

However, chatbots cannot fully replicate the complexities of live teaching, and their effectiveness depends on how realistically they simulate classroom interactions. To bridge this gap, teacher educators should integrate AI-based chatbots into training programs in two ways: (1) using chatbots for noticing skill development, and (2) allowing PSTs to practice teaching moves based on student responses. These interventions can help PSTs develop instructional decision-making skills more effectively before entering real classrooms.

Conclusion

This study aimed to offer them practical experience in generative instructional practices, which is crucial for their professional development. Teachers' expertise in responsive teaching is complex and not easily developed. AI chatbots help unlock these dimensions of complexity and achieve educational goals in a simplified context (Lee & Yeo, 2022). This study showed that various patterns of PSTs' noticing expertise influence their enactment of responsive teaching moves in a simulated teaching environment. The big takeaway from this study is that using intentionally designed AI-based learning environments has the potential to contribute to teacher professional development by providing quasi-real-world experiences (Howell & Mikeska, 2021). Therefore, it is essential to explore innovative approaches to leveraging chatbots to actively contribute to teachers' professional development.

References

Amador, J. M., Estapa, A., Kosko, K., & Weston, T. (2021). Prospective teachers' noticing and mathematical decisions to respond: Using technology to approximate practice. *International Journal of Mathematical Education in Science and Technology, 52*(1), 3–22. https://doi.org/10.1080/0020739X.2019.1656828

Bywater, J. P., Chiu, J. L., Hong, J., & Sankaranarayanan, V. (2019). The Teacher Responding Tool: Scaffolding the teacher practice of responding to student ideas in mathematics classrooms. *Computers & Education, 139*, 16–30. https://doi.org/10.1016/J.COMPEDU.2019.05.004

Howell, H., & Mikeska, J. N. (2021). Approximations of practice as a framework for understanding authenticity in simulations of teaching. *Journal of Research on Technology in Education, 53*(1), 8–20. https://doi.org/10.1080/15391523.2020.1809033

Go, E., & Sundar, S. S. (2019). Humanizing chatbots: The effects of visual, identity and conversational cues on humanness perceptions. *Computers in Human Behavior, 97*, 304–316. https://doi.org/10.1016/J.CHB.2019.01.020

Jacobs, V. R., & Empson, S. B. (2016). Responding to children's mathematical thinking in the moment: an emerging framework of teaching moves. *ZDM - Mathematics Education, 48*(1–2), 185–197. https://doi.org/10.1007/S11858-015-0717-0

Jacobs, V. R., Empson, S. B., Jessup, N. A., Dunning, A., Pynes, D., Krause, G., & Franke, T. M. (2024). Profiles of teachers' expertise in professional noticing of children's mathematical thinking. *Journal of Mathematics Teacher Education, 27*(3), 295–324. https://doi.org/10.1007/S10857-022-09558-Z

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169–202. https://doi.org/10.5951/JRESEMATHEDUC.41.2.0169

Lee, D., Son, T., & Yeo, S. (2025). Impacts of interacting with an AI chatbot on preservice teachers' responsive teaching skills in math education. *Journal of Computer Assisted Learning, 41*(1), e13091. https://doi.org/10.1111/jcal.13091

Lee, D., & Yeo, S. (2022). Developing an AI-based chatbot for practicing responsive teaching in mathematics. *Computers & Education, 191*, 104646. https://doi.org/10.1016/J.COMPEDU.2022.104646

Son, T., Yeo, S., & Lee, D. (2024). Exploring elementary preservice teachers' responsive teaching in mathematics through an artificial intelligence-based Chatbot. *Teaching and Teacher Education, 146*, 104640. https://doi.org/10.1016/J.TATE.2024.104640

TSG6-1-26

EXPLORING PRESERVICE TEACHERS' UNDERSTANDING OF FRACTION MULTIPLICATION WITH A TECHNOLOGICAL TOOL

Ji Nam Hwang
Yongjeong Elementary School, Korea, whiyoung10@naver.com

Mi Yeon Lee
Arizona State University, USA, mlee115@asu.edu

Sheung Hyun Yeo
Daegu National University of Education, Korea, shyeo@dnue.ac.kr

This study examines how elementary preservice teachers (PSTs) use technological tools, specifically the fraction bar and fraction square models, to represent fraction multiplication. The results show that while the majority of PSTs demonstrated quantitative reasoning using the fraction square, those using the fraction bar primarily relied on operational reasoning. Some PSTs struggled to fully grasp the quantitative meaning, and in certain cases, operational reasoning was incomplete, such as when they omitted the denominator calculation. The study suggests that the fraction square is a powerful tool for explaining the algorithm of fraction multiplication, but it also highlights the need for further instruction to help PSTs logically explain why the overlapping area represents the product. Additionally, the study emphasizes the importance of exploring fraction multiplication using the length model and suggests an educational approach to help deepen PSTs' understanding of fraction multiplication through the fraction bar.

Keywords: elementary preservice teachers, technological tool, fraction multiplication, fraction bar, fraction square

Introduction

There is a gap between understanding a visual model and representing it. The process of representing a visual model inherently involves justification (Arcavi, 2003), which is educationally meaningful as it goes beyond merely solving problems using algorithms and instead focuses on exploring the question of 'why'.

However, the creation of accurate visual models, particularly those involving fractions, is inherently cumbersome without the assistance of tools like rulers and compasses. Freehand drawing often leads to inaccuracies that undermine the very purpose of the visual representation. Additionally, explaining an operational process through a visual model can be cognitively demanding for learners (Yeo & Webel, 2024). These challenges can be partially overcome through the use of technological tools. Utilizing such tools not only enables the accurate representation of visual models but also facilitates a detailed examination of learners' operational processes through manipulative actions (Lee & Yeo, 2024).

This study aims to compare and analyze how PSTs use a technological tool to represent the 'fraction square (area model)' and the 'fraction bar (length model)'. To accomplish this, the study employs a technological tool called 'FracTopia' to examine how PSTs understand visual models of fraction multiplication. Based on these findings, the study seeks to refine the pedagogical content knowledge (PCK) essential for teaching fraction multiplication. Additionally, it aims to classify PSTs' reasoning with FracTopia into distinct categories and develop specific instructional strategies accordingly.

Literature Review

Fraction multiplication should be understood not only beyond the concept of *repeated addition* but also in terms of *scaling* or the 'times as much' concept (Prediger, 2008; Taber, 2002). Reys et al. (2014) pointed out that the multiplier in fraction multiplication is difficult to grasp solely through repeated addition. They argued that instruction on fraction multiplication should be grounded in the concepts of grouping and area. Furthermore, understanding the algorithm for fraction multiplication requires conceptualizing units and recognizing the role of the multiplier as an operator (Lee & Hackenberg, 2014; Yim, 2012).

Korean mathematics textbooks use the fraction bar (length model) and the fraction square (area model) as visual representations for fraction multiplication (Ministry of Education, 2021). In the case of (fraction)×(whole number), where the multiplier is a whole number, instruction is designed to help students understand the concept through repeated addition. In contrast, for (whole number)×(fraction), since the multiplier is a fraction, explaining it using repeated addition is challenging; therefore, it is introduced through the concept of scaling. Similarly, (fraction)×(fraction) is also taught using the concept of scaling.

Methods

This study involved 60 sophomore PSTs at a university of education were asked to solve a multiplication problem (2/3×3/8) using both the fraction bar model and the fraction square model. In addition to employing visual models, they were required to articulate their solution process in written form. Subsequently, they submitted their responses via a Google survey. The collected data were analyzed by classifying the reasoning into three types—computational reasoning, operational reasoning, and quantitative reasoning—based on the framework from Lee's (2017) prior study. Lee (2017) categorized the ways of representing fraction division using visual models into these three types of reasoning (Table 1). *Computational reasoning* refers to a cognitive approach that mechanically applies algorithms or procedural rules to solve a given problem. This approach focuses on executing correct computational procedures rather than understanding the fundamental meaning or context of the problem. *Operational reasoning* involves understanding the procedures and structures of mathematical operations and applying them to problem-solving. Unlike computational reasoning, it incorporates previously learned mathematical knowledge to solve problems. However, it has limitations in effectively representing the meaning of quantities through visual models. In contrast, *quantitative reasoning* refers to the ability to understand mathematical problems in terms of quantities and their relationships and to represent them visually to find solutions. It goes beyond simple computations and procedures, emphasizing a deep understanding of the meaning of quantities within the problem, referent units[1], and quantitative relationships. Three researchers were involved in the coding process, and inter-coder consistency was confirmed through mutual agreement among them.

1 Referent units refer to the whole or the unit from which the fraction is derived or compared.

Table 1. Reasoning categories in visual models

Category	Description	Example
Computational reasoning	Solving problems by relying on algorithms or formulas Emphasizing the accuracy of computational procedures Displaying only the final result when using visual models	When solving 2/3×3/8, the numerators are multiplied and the denominators are multiplied, but this process is not represented using a visual model.
Operational reasoning	Solving problems by utilizing prior mathematical knowledge rather than relying on formulas Applying operational properties Using visual models but with limitations in connecting them to quantitative meaning	When solving 2/3×3/8, a common denominator is found and represented using a visual model. Alternatively, the commutative law is applied to express 3/8×2/3, which is represented using a visual model.
Quantitative reasoning	Understanding the meanings of quantities and referent units Explaining problems intuitively using visual models Solving problems based on the relationships between quantities rather than relying on algorithms or formulas	When solving 2/3×3/8, the concept of scaling is used to determine how much of 2/3 is represented by 3/8, and it is shown using a visual model.

The digital tool utilized in this study, FracTopia, is a dynamic environment designed to support learners' understanding of fraction concepts and operations by enabling them to explore the quantitative meanings of fractions through the manipulation of various visual models. FracTopia allows users to intuitively construct mathematical ideas by clicking, dragging, partitioning, rotating, and resizing fractional parts. This manipulation-based environment encourages learners to actively investigate aspects such as unit identification, the relationship between parts and wholes, and the proportional meaning of multiplication, thereby promoting a deeper conceptual understanding than what is typically afforded by static images or verbal explanations.

Findings and Discussion

PSTs' Reasoning with FracTopia Demonstrated in Two Fractional Models

An analysis of PSTs' reasoning types using the fraction bar and the fraction square revealed clear distinctions between the two models (see Table 2). When using the fraction bar model, operational reasoning accounted for the highest proportion (50%). This suggests that PSTs primarily focused on applying the procedures and structures of mathematical operations when using this model. In contrast, computational reasoning and quantitative reasoning each accounted for 25%, showing an equal distribution. This indicates that while some PSTs relied on algorithms or formulas to solve the problem, others attempted to understand the fundamental meaning of quantities.

Table 2. Analysis of reasoning categories in visual models

Models	Computational reasoning	Operational reasoning	Quantitative reasoning
Fraction bar	15(25%)	30(50%)	15(25%)
Fraction square	3(5%)	6(10%)	51(85%)

On the other hand, when using the fraction square, quantitative reasoning overwhelmingly accounted for the highest proportion (85%). The fraction square appears to be an effective visual aid for understanding fraction multiplication (proper fraction)×(proper fraction) as a "part of a part." This visualization may promote a deeper understanding of referent units and quantitative meaning by clearly showing the overlapped area as the result of "part of a part" multiplication. Operational reasoning accounted for 10%, and computational reasoning for 5%, both relatively low. This indicates that the fraction square is more conducive to represent fraction multiplication quantitatively, rather than simply following computational procedures or algorithms.

Characteristics of PSTs' Three Types of Reasoning with FracTopia

In computational reasoning, a notable finding is that 12 of the 15 PSTs who initially used computational reasoning with the fraction bar shifted to quantitative reasoning when presented with the fraction square. Although the fraction square shows promise as a tool for fostering quantitative reasoning, this study, limited to written responses, necessitates further investigation, including interviews, to determine if these PSTs truly understand the quantitative meaning of multiplication.

A case of a PST who initially demonstrated computational reasoning with the fraction bar but later exhibited quantitative reasoning when using the fraction square (see Figure 1) divided 2/3 and 3/8 along the horizontal and vertical directions, respectively, and interpreted the overlapping area as the product of fraction multiplication. While this approach may superficially indicate quantitative reasoning, the lack of a clear explanation for why the overlapping area represents the product calls into question the PST's grasp of fraction multiplication as "part of a part."

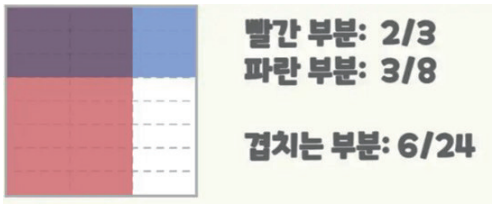
Representation through a visual model	Explanation
	2/3 and 3/8 were represented by shading them in a single rectangle, and the overlapping area was determined to find the answer.

Figure 1. A case of a PST who explained quantitative reasoning only with the fraction square

One challenge observed with operational reasoning using the fraction bar model is that some PSTs, when representing 3/8 of 2/3, failed to explicitly show the intermediate step where the denominator is represented as 1/12 (see Figure 2). This challenge was observed in 18 (60%) of the 30 PSTs who demonstrated operational reasoning using the fraction bar. These findings may suggest that PSTs applied operational reasoning by procedurally solving the problem, potentially using a calculation similar to '2/3÷8=2/24 (or simplified to 1/12).' They recognized that when 2/3 is divided into 8 equal parts, each unit corresponds to 1/12.

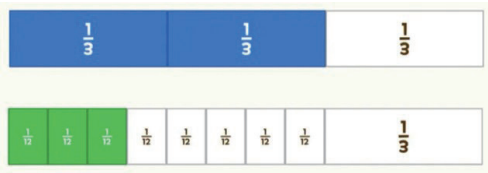
Representation through a visual model	Explanation
	To represent 3/8 of 2/3, the blue section was divided into 8 equal parts, and 3 of those parts were shaded green to express 2/3×3/8. Since 3 parts of 1/12 were shaded, the answer is 3/12=1/4.

Figure 2. A case of a PST Omitting the Denominator Calculation

However, in the fraction square, when the horizontal and vertical sides are divided by the denominators, thirds and eighths, respectively, the entire area is divided into 24 equal parts. As a result, the denominator of the product in the fraction multiplication algorithm is naturally represented through quantitative reasoning. Therefore, the case shown in Figure 2 was not observed in the fraction square. This suggests that the fraction square is a powerful tool for intuitively explaining the algorithm of fraction multiplication.

Conclusions

We assumed that the high proportion of quantitative reasoning observed in the fraction square is likely due to the characteristics of the technological tool. When using this tool to represent a fraction square, bi-directional partitioning is possible along both the horizontal and vertical directions. This feature may have enabled PSTs to use it effectively in solving fraction multiplication. However, it is suggested that in this process, the deeper understanding of the quantitative meaning might have been overlooked, and the explanation may have been based merely on the observation that the overlapping area equals the product. Therefore, while the fraction square is acknowledged as a suitable model for explaining fraction multiplication, additional instruction is necessary to guide PSTs in logically explaining why the overlapping area represents the product in the fraction square.

There is a need to explore fraction multiplication using the length model. While the majority of PSTs demonstrated quantitative reasoning with the fraction square, those using the fraction bar model appeared to struggle with quantitative reasoning. This aligns with prior research, which found that the length model requires more complex partitioning activities compared to the area model (Lee & Lee, 2023; Yim, 2012). However, since this study only focused on the multiplication of proper fractions, the strengths of the fraction square can be seen as particularly highlighted. For example, when dealing with extended fraction multiplication, such as (whole number)×(fraction), the length model, which partitions in only one direction, has the advantage of making it easy to understand the whole concept. Therefore, an educational approach to help PSTs deepen their understanding of fraction multiplication using the fraction bar is also needed.

References

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215-241.

Lee, M. Y. (2017). Pre-service teachers' flexibility with referent units in solving a fraction division problem. *Educational Studies in Mathematics, 96*(3), 327-348.

Lee, M. Y., & Hackenberg, A. J. (2014). Relationships between fractional knowledge and algebraic reasoning: The case of Willa. *International Journal of Science and Mathematics Education, 12*(4), 975-1000.

Lee, J., & Lee, M. Y. (2023). How elementary prospective teachers use three fraction models: their perceptions and difficulties. *Journal of Mathematics Teacher Education*, *26*(4), 455-480.

Lee, M. Y., & Yeo, S. H. (2024). Elementary preservice teachers' fraction-as-measure reasoning with a dynamic tool. *Journal of Mathematics Teacher Education*, 1-28.

Ministry of Education. (2021). *Mathematics 5-2*. Visang.

Prediger, S. (2008). The relevance of didactic categories for analysing obstacles in conceptual change: Revisiting the case of multiplication of fractions. *Learning and Instruction, 18*(1), 3–17.

Reys, R., Lindquist, M., Lambdin, D. V., & Smith, N. L. (2014). H*elping children learn mathematics*. John Wiley & Sons.

Taber, S. B. (2002). Go ask Alice about multiplication of fractions. In B. Litwiller & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions* (pp. 61-71). NCTM.

Yeo, S., & Webel, C. (2024). Elementary student's fraction reasoning: A measurement approach to fractions in a dynamic environment. *Mathematical Thinking and Learning, 26*(1), 20-46.

Yim, J. H. (2012). Models and the algorithm for fraction multiplication in elementary mathematics textbooks. *School Mathematics, 14*(1), 135-150.

TSG6-2-17

A STUDY ON THE CONSTRUCTION OF ARCUATE FIGURES IN WASANSHO

Jun Yamada
Tsushima High School, Japan, jun.ymd2013@gmail.com

Using the drawing functions of Dynamic Geometry Software (DGS), it is possible to draw a variety of plane figures. However, these drawings are clearly different from those drawn using a ruler and compass.For some plane figures (called arcuate figures) found in WASANSHO, the use of the relational formula R=a+b+c can make it easier to draw figures that would be difficult to draw using only a ruler and compass. By comparing drawings using a ruler and compass with those using DGS, we were able to demonstrate historical ingenuity and the potential of DGS to process complex values and irrational numbers that are difficult to construct accurately

Keyword: WASANSHO, Drawing method, DGS, arcuate figure

Introduction

Using the drawing functions of dynamic geometry software (DGS), it is possible to draw a variety of plane figures. However, these drawings are clearly different from drawings using a ruler and compass (which correspond to the four arithmetic operations and square root extraction), but they make it possible to draw problems that would be difficult to draw accurately using only a ruler and compass.”SANPO KIGENSHU (1877) “ is a collection of problems found in WASANSHO (Japanese mathematics book) from the Edo period written by Japanese mathematician Sakuma Tsuzuki (1819-1896). In this book, there is a problem involving the equation R=a+b+c for plane figures that are in a tangent circle relationship (related only to circles).

Question: “Inscribe two circles A (radius a) of equal radius on both ends of the diameter of the outer circle (radius R). Furthermore, inscribe two arbitrarily connected circles B (radius b) and C (radius c) between them. In this case, express the radius of the outer circle in terms of the radii of circles A, B, and C.”

Answer: “The radius of the outer circle is equal to the sum of the radii of circles A, B, and C, (R=a+b+c.)”

By using this relational equation R=a+b+c, it is possible to draw problems that would be difficult to draw accurately using just a ruler and compass. There are many arched figures in WASANSHO that are symmetrical (can be created by swapping circles B and C up and down). The author has been continuing his research into arcuate figures that are symmetrical using the relational equation R=a+b+c (Yamada, 2022, 2023, 2024). Here, we have taken as an example a simple arcuate figure with few mutually tangent circles and examined it so that it can be used by students as “teaching material to show the difference between drawing with a ruler and compass and drawing with DGS.”

Arcuate Figure Problem

”JYUGAIROKU (1639)” published by Japanese mathematician Imamura Tomoaki, was the first mathematical book to be in an academic format (Hirayama, 2007, p39-40). It describes approximate formulas for the arc length and area of a circular bow, such as the “Kou-Ko-Gen no Ri (Pythagorean Theorem)”. Cartesian coordinates were not yet known in Japan at the time. When three circles are circumscribed, Descartes’ circle theorem holds, but Japanese mathematicians were able to calculate this using the formula “SANEN NAIYOUEN” (Ogawa, 2021, p225). Here, using a “template” of an arcuate figure with the relation R=a+b+c, we will look at the cases when the two B circles are tangent to each other at the center line [Figure 1], and when the A circle is in the center of the arcuate figure [Figure 2].

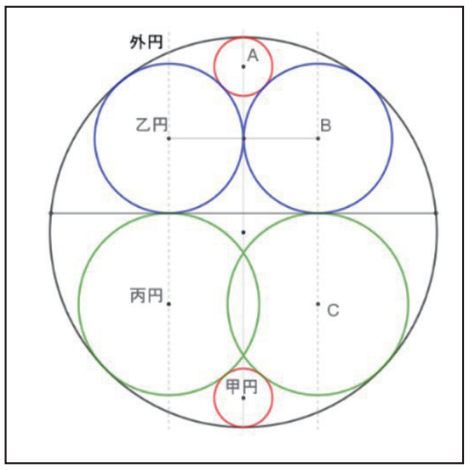


Figure 1. Case 1

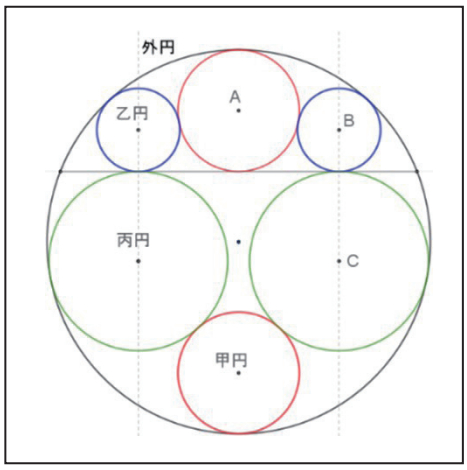


Figure 2. Case 2

Case 1

Problem on a SANGAKU at the KANNONDO of ENKYOU-JI, SHOSHA-SAN

Question: “Inside the bow are two B circles with a diameter of 72 inches, and two D circles with a diameter of 32 inches. If one A circle is added to this, what will be the arrow? “[Figure 3]

(Japanese mathematicians show the diameter, but here We will show it in radius.)

Answer: “Arrow 81 inche.”

Here, the center of the outer circle is O, the radius is R, the center of the A circle is A(x_a ,y_a), the radius is a, the center of the B circle is B(x_b,y_b), the radius is b=36, the center of the D circle is D(x_d,y_d), the radius is d=16, and by solving the simultaneous equations, the radius of the outer circle is R=((b(√b+√d)^2))/(4√d(√b-√d)) , The radius of circle A ,a=((b(√b+√d)^2))/(4√d(√b+3√d)), can be expressed in terms of the radius of circle B, b, and the radius of circle D, d. When b=36 and d=16, $R = \frac{(36\cdot(\sqrt{36}+\sqrt{16})^2)}{4\cdot\sqrt{16}\cdot(\sqrt{36}-\sqrt{16})} = \frac{225}{2}$ $a = \frac{36\cdot(\sqrt{36}+\sqrt{16})^2}{4\cdot\sqrt{16}\cdot(\sqrt{36}+3\sqrt{16})} = \frac{25}{2}$

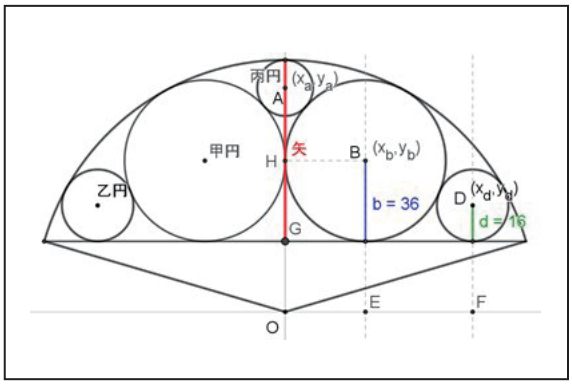


Figure 3. SANGAKU at the SHOSHASAN

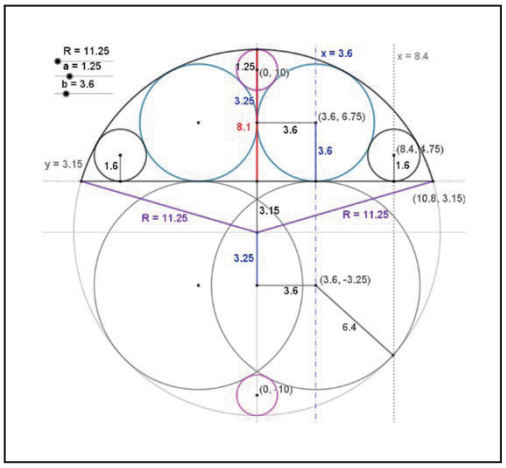


Figure 4. Example of drawing (1/10 scale)

This figure satisfies the relation R=a+b+c , so c=R-(a+b)=64. It can be constructed using a template [Figure 4]. In this case, circle D is a circle that is tangent to the outer circle, circle B, and the chord, so it is sufficient to add the two circles D using the DGS's "quadratic curves passing through five points (parabola, ellipse, hyperbola)." In this case, the center coordinates of each circle are A(0,100), B(36,67.5), and D(84,47.5), so it can be constructed accurately without using a template.

Case 2

Problem on a SANGAKU at SHITAYA INARI in TOUTO (1789)

Question: "As shown in Figure 5, there are five inscribed circles (a circle, b two circles, and c two circles) connected inside a circular arc. If the radius of the outer circle is R, and the radii of the circles are a, b, and d, respectively, express b in terms of a and d." Answer: "When a=6 and

d=3/2, b=4", Technique: "b=((2a√d))/((√a+√d))"

Let the center coordinates of circle B be (x_b,y_b) and the center coordinates of circle D be (x_d,y_d).Solving the simultaneous equations, we get x_b=2√ab, y_b=R-(2a-b),

R=a^2/(a-b), x_d=2√ab+2√bd , y_d=R-(2a-d), b=(2a√d)/(√a+√d).

When a=6 and d=3/2, b=((2·6·√(3/2)))/((√6+√(3/2)))=4 , (x_b,y_b)=(4√6,10), (x_d,y_d)=(6√6,15/2), R=6^2/(6-4)=18. Because the x-coordinates of the centers of Circle B and Circle D are irrational numbers, it is difficult to draw them accurately using a ruler and compass. However, this arcuate figure satisfies the relationship R=a+b+c, so c=R-(a+b)=18-(6+4)=8, and it can be constructed using the template [Figure 6].

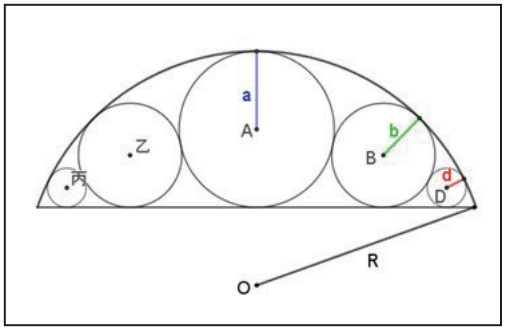


Figure 5. SANGAKU at SHITAYA INARI

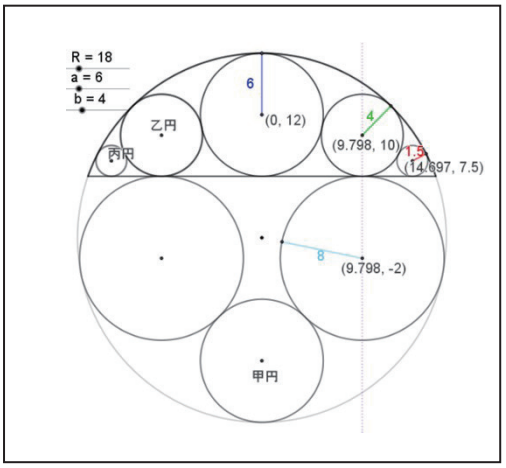


Figure 6. Example of drawing

Advanced problem using Case 2

"SHINPEKI SANPOU" is a publication of SANGAKU by the Japanese mathematician Fujita Kagen. Here, I use a problem published in This Book.

Question: "There are seven circles (one large circles, two medium circles, and four small circles) connected in an arc as shown in the figure. If the diameter of the large circle is 1 inch, find the diameter of the medium circle." [Figure 7]

Answer: "0.6096117967977924312 odd"

Let the radii of the outer circle, large circle, medium circle, and small circle be R,a,b and d, respectively. Furthermore, if i take the radius of the middle circle C_b in the first quadrant to be b and its center coordinates to be (x_b ,y_b), and the radius of the small circles C_d and C_e to be d=e and their center coordinates to be (x_d,y_d), (x_e,y_e), solving the simultaneous equations we get b={(√17-1)/4}^2 a=(9-√17)a/8, so when the diameter of the large circle is 1 (inch),

the diameter of the middle circle is 0.60961179679777924312... (inch).

This problem satisfies the relation R=a+b+c, so if the radius of circle C in the fourth quadrant is c and the center coordinates are (x_c,y_c), then c=b^2/(a-b)=(5√17-13)a/8, d=3(√17-1)a/8, R=a+b+c=(√17+1)a/2, x_b=2√ab=x_c=(√17-1)a/2, y_b=R-(2a-b)=3(√17-1)a/8, y_c=R-(2a+c)=(1-√17)a/8, x_d=2√bd+2√ab, y_d=R-(2a-d)=y_e=3(√17-1)a /8, x_e=2√ad.

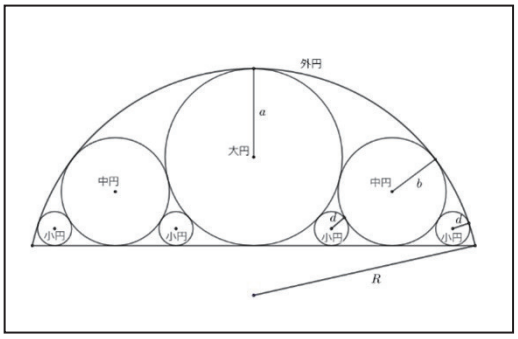


Figure 7. Advanced problem Figure

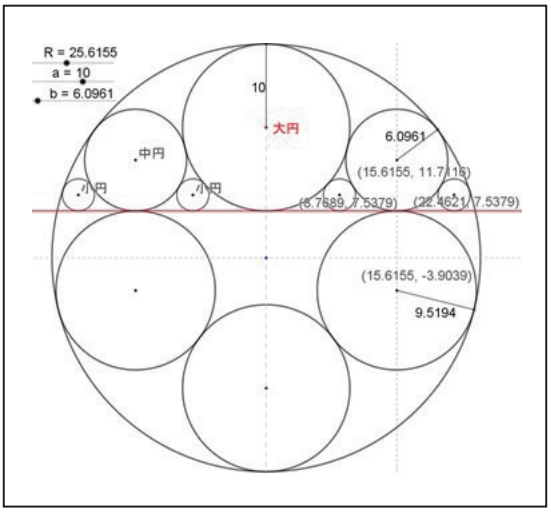


Figure 8. Example of drawing (10x scale)

The radius of the outer circle of this arcuate figure is an irrational number, so it is difficult to draw it accurately using just a ruler and compass. This problem also satisfies the relation R=a+b+c. GeoGebra can handle irrational numbers, so by using a "template" you can accurately draw this arcuate figure [Figure 8].

Summary

In case 1, the radius of each circle and its center coordinate are all rational numbers, so it is possible to draw it with a conventional ruler and compass. In case 2, the radius of each circle is rational, but the center coordinate of the circle is not necessarily rational, so drawing it with a ruler and compass becomes difficult. Calculations can be done by solving simultaneous equations that satisfy the conditions, but it is necessary to set the calculation procedure and substitution appropriately. 2.3 Advanced problem is difficult to draw accurately with just a ruler and compass because the radius of the outer circle is an irrational number. Dynamic geometry software (GeoGebra) can handle irrational numbers, so accurate drawing is possible by using the relational equation R = a + b + c.

This study showed that the use of a "template" makes it easy to draw arcuate figures, and that it can be used as a teaching material to demonstrate the potential of DGS to handle complex values and irrational numbers that are difficult to construct accurately. Furthermore, through drawing activities using tablet devices, students can learn about the outstanding talents and achievements of Japanese mathematicians in the Edo period, when Cartesian coordinates had not yet been introduced.

References

GeoGebra Japan. https://sites.google.com/site/geogebrajp/ (see 2025.4.9).

Fukagawa,H (Ed.). (1983). Zokuzoku Sangaku no Kenkyu. Narumi Dofuukai.

Fujita,S & Fujita,K(Eds.). (1789). SINPEKI SANPOU,

https://touda.tohoku.ac.jp/portal/item/10020000010371 (see 2025.4.9).

Hirayama,A .(2007). Wasan no Rekishi. Chikuma Bungei Bunko.

Ogawa,T .(2021).WASAN Edo no Suugaku Bunka. Chukou Sensho.

Sakuma,T .SANPOU KIGENSHU. Kyoto University Mathematics Department Rare Book Library. https://rmda.kulib.kyoto-u.ac.jp/item/rb00028490 (see 2025.4.9).

Yamada, J. (2022). Creating Problem-Based Learning Materials for High School Mathematics - Problems Involving Circles Only. Proceedings of the Spring Annual Meeting, Mathematics Education Society of Japan, 127-129.

Yamada, J. (2023). A Study on Drawing of Flat Figures by Dynamic Geometry Software -Using Plane Figure Problems of Wasan Books-. RIMS Kokyuroku 2273. Study of Mathematical Software and its Effective Use for Mathematics, 170-177.

Yamada, J. (2024). A Study on The Construction of Plane Figures Using Dynamic Geometry Software - Using The Relational Equation R=a+b+c. Proceedings of the Spring Annual Meeting, Mathematics Education Society of Japan, 105-107.

TSG6-2-18

STUDENT VOICE WHILE USING ICT IN MATHEMATICS LESSONS: THE CASE OF INFECTION EPIDEMICS

Minoru Ohtani
Kanazawa University, Japan, mohtani@ed.kanazawa-u.ac.jp

This research explores the incorporation of digital technologies (DTs) in high school mathematics education through the Lesson Study approach. Using the social issue of infection epidemics, the study connects mathematical inquiry with real-world problems. A collaborative effort between high school teachers and university researchers led to the development of a Lesson Study emphasizing mathematical modeling, simulation, and inquiry using ICT. By attending to student voice during simulations with Excel, we analyze how DTs can foster critical thinking, collaboration, and the practical application of mathematical concepts. Observations from classroom implementations, alongside feedback from students and educators during the Lesson Study, highlight both the challenges and potential of using ICT to enhance students' literacy on social issues.

Keywords: Student Voice, Social Issues, Simulation with ICT, High School Mathematics, Lesson Study

Introduction

In mathematics education, digital technologies (DTs) have become ubiquitous, serving multiple purposes. A recent literature review of DTs in mathematics education outlined five primary goals: improving learning, understanding learning, designing for learning, providing equitable access to learning, and changing curricula as well as teaching and assessment practices (Drivers & Sinclair, 2024). The review suggests a promising future objective: *"*whether the use of DT contributes to students' understanding of social issues*"* (ibid, p. 240). Aligned with this suggestion, collaborative lesson designs were created to connect mathematics with real-world and socially pressing issues, such as the spread of infectious diseases (Ohtani, 2024). This study adopts the conventional Japanese Lesson Study approach (Fujii, 2015), which emphasizes collaborative lesson design and feedback from a variety of participant observers during post-lesson reflections. The approach was modified to focus on learners' perspectives (Kaur et al., 2013). By engaging students in inquiry-based lessons with DTs, we aim to uncover the potential and challenges of using DTs to enhance students' understanding of real-world social issues

Lesson Study Process

Setting the Topic: COVID-19 Epidemic as a Social and Pressing Issue

For over two decades, the Department of Mathematics at Kanazawa University High School has conducted Lesson Studies in collaboration with the author, drawing inspiration from the Dutch concept of Realistic Mathematics Education (van den Heuvel-Panhuizen, 2020). For this study, we selected the problem of infectious disease spread, using the ICT-based assessment task *Epidemies* from the Dutch National Exam for Mathematics A (Cito, 2003). The emergence of COVID-19 at

the end of 2020, coupled with the Diamond Princess cruise ship outbreak in Japan, made this issue timely and relevant.

Mathematical Content and Lesson Design

The lessons were planned for grade 12, second year of high school as the final subunit "Inquiry," with the "limits of sequences" topic of Calculus in "Mathematics III". The mathematical model of infectious diseases is represented by analytically unsolvable systems of recurrence formulae and requires the concept of limit of sequence. The models addressed in the lessons are unsolvable by standard mathematical methods, making ICT simulations a suitable tool.

The subunit consisted of three lessons: Mathematical Models of Infectious Diseases; Simulation of Infectious Diseases; Simulation of Different Infectious Diseases.

- Lesson 1: Students analyzed the latest COVID-19 infection data from Japan and globally and were introduced the Reed-Frost model. This model involves a population of N individuals, where S_n, I_n, and R_n represent the susceptible, infected, and recovered population after n weeks. In this case, S_n, I_n and R_n are expressed by the relations (1), where k ($0 \leq k \leq 1$) is a measure of the 'strength' of the infection in a population of N individuals.
- Lesson 2 (Research Lesson): A more realistic setting was introduced, considering infections that do not satisfy assumption (ii), where proportion g ($0 < g \leq 1$) of infected individual recover weekly. The modified model is expressed by the relations (2).
- Lesson 3: Students explored diseases where immunity is not achieved, i.e., $I_n=0$ for all n. This means the assumption (iii) does not hold, and some individuals can be reinfected. We can formulate another model (3). (*Table 1*.)

Table 1. Models used for simulation in the three lessons

Lesson 1: Initial Model (1)	Lesson 2: Modified Model (2)	Lesson 3: Modified Model (3)
$\begin{cases} I_{n+1} = (1 - k^{In}) - S_n \\ R_{n+1} = R_n + I_n \\ S_{n+1} = N - I_{n+1} - R_{n+1} \end{cases}$	$\begin{cases} I_{n+1} = (1 - k^{In}) - S_n + (1 - g) - I_n \\ R_{n+1} = R_n + g - I_n \\ S_{n+1} = N - I_{n+1} - R_{n+1} \end{cases}$	$\begin{cases} I_n+1 = (1 - k^{In})\, S_n \\ S_n+1 = N - I_{n+1} \end{cases}$

Research Lesson: Lesson2

The Research Lesson was held on February 22, 2020, with 27 participants observing and 44 students (16-17 years old) working in groups of four. Each group was equipped with 12 laptops for simulation tasks. Students completed three tasks using Excel file 'Infection Simulation. xls.', modifying parameters to control outbreaks and comparing models under various assumptions (See *Fig.1*, *Fig.2*).

- Task 1: Set the initial values: N=10,000 (total population), k=0.99979 (basic reproduction number), I_0=10 (initial infected). Verify settings, check graphs/tables for accuracy, and calculate I_{13} (week 13 infections) and R_{13} (recoveries). Adjust vaccination to keep weekly infections under 1,000.
- Task 2: If curing takes more than one week, analyze for g = 0.5 and g = 1. Assess changes in total infections, epidemic duration, peak infections per week, peak timing, and required vaccination rates to keep weekly infections below 1,000.

- Task 3: Using the modified model (2), students simulated individually and in groups, adjusting values of I_0, R_0, k, g, and N. They then described their thoughts, feelings, understanding, and areas of confusion during the lesson.

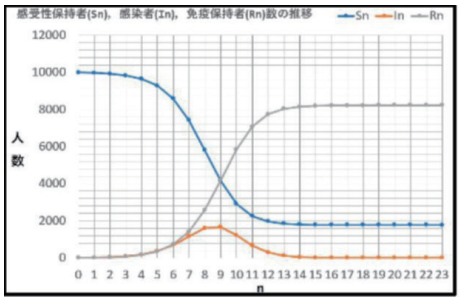


Figure 1. 'Infection simulation. xls.'

Figure 2. Students simulating with ICT

At the lesson's conclusion, the teacher discussed infectious diseases that don't meet criteria (i), (ii), and (iii), and how the model could be modified. They also explored how to evaluate the value of k, the disease's basic reproduction number.

Data on Student Voice in Post Lesson Reflection

During the Research Lesson (RL), participants walked between students' desks, observed their inquiry processes, and took field notes on student interactions and written artifacts. These observations were later used in the collective post-lesson reflection. Collective reflection after the RL is one of the key features of the Japanese Lesson Study (LS) approach. Typically, only RL teachers and participants attend the post-lesson reflection, meaning the discussion is mainly centered on the teacher's perspective, both intended and implemented aspects of the lesson. However, the actual learner's perspective remains underrepresented. To address this gap, we invited three selected students to join the post-lesson collaborative reflection, placing the learner's perspective at the center of the discussion. This modification enabled teachers and participants to gain direct insight into students' inquiry processes as they engaged with the simulation tasks. The research collaborator chose the three students (A, B, and C) to join the post-lesson collaborative reflection, placing the learner's perspective at the center of the discussion. This modification enabled teachers and participants to gain direct insight into students' inquiry processes as they engaged with the simulation tasks.

Among the many reflections shared, three main topics emerged related to students' perspectives:

- Overall Impression

A: It felt more realistic than usual. I wanted to think about how the equation connects with the actual graphs of infectious diseases that have been on TV and other media lately.

B: Initially, I was puzzled by the equations with three variables, but through simulation, I became interested in understanding the epidemic's duration and the number of infections. However, I felt the model needed to be more complex to reflect reality.

C: I had vaguely thought that the number of infectious diseases was increasing, based on recent news, but it was good to understand the actual relationship between multiple quantities. It was interesting to see how a slight change in one value could drastically change the others.

- Task 3 Approach:

  A: I focused on how the graph changed when I adjusted variable values, rather than the intended outcomes of these values.

  B: We tried to make *n* bigger, and we often found that everyone got infected, which doesn't happen in real life. When that happened, we discussed what might be wrong with the model.

  C: My group had fun with it. We tried to make sure that everyone got infected and then used a lot of vaccine so no one would get infected. I should have thought more about being realistic.

- Future Simulations:

  A: The value of *k* is the most ambiguous. I would like to simulate real-world scenarios, such as the number of people on encounters daily or the potential of infection in a school setting.

  B: Actually, infectious diseases are more complicated, and I don't think they can be expressed in just three equations. I would like to know alternative models so as to capture realistic situations.

  C: I feel the same. I think there are factors we need to consider in practice, but I want to explore how we can make models more realistic.

Discussion and Implications

The modified post-lesson reflection process demonstrates a new way to capture student voice. Their perspectives became a central topic of discussion. This small modification to the Lesson Study ritual allowed teachers and participants to better understand students' actual inquiry processes during the simulation.

From the students' responses, it's clear they faced considerable difficulty managing three- or four-variable asymptotic equations that lack closed-form solutions. With guidance from the teacher, students took diverse approaches, informed by their curiosity about specific phenomena. Some students even questioned the authenticity of the models and expressed interest in how models differ from real-world behavior.

Student reflections suggest that using ICT fosters interest in social issues but also highlights a perceived gap between simplified mathematical models and complex real-world phenomena. This detachment stems from ambiguities about new infections and limited understanding of recursive systems with multiple variables.

A potential area for further exploration is creating opportunities for students to ask questions directly to the teacher and other participants, establishing a more authentic, multi-stakeholder approach to Lesson Study.

References

Cito (2003). *Epidemies. Examen VWO–compex. Wiskunde A 1,2* (nieuwe stijl). Retrieved from: https://www.examenblad.nl/.

Drijvers, P., Sinclair. N. (2024). The role of digital technologies in mathematics education: purposes and perspectives. *ZDM – Mathematics Education*, 56, 239-248.

Fujii, T. (2015). The Critical Role of Task Design in Lesson Study. In: Watson, A., Ohtani, M. (eds.) *Task Design in Mathematics Education* (pp. 273-286). ICMI Study22. Springer,

Kaur, B., Anthony, G., Ohtani, M., Clarke, D. (eds.), (2004). *Student Voice in Mathematics Classrooms around the World*. *Learner's Perspective Study*. Sense Publishers.

Ohtani, M. (2024). Development of STEM education in KUSH: Task design for inquiry-based learning through Lesson Study. In M. Doorman, E. Schäfer, K. Maaß (eds.), *STEM & Open Schooling for Sustainability Education* (pp.78-83). Münster: WTM.

van den Heuvel-Panhuizen, M. (ed.), (2020). *International Reflections on the Netherlands Didactics of Mathematics: Visions on and Experiences with Realistic Mathematics Education*. Springer.

TSG6-2-19

CO-CONSTRUCTING UNDERSTANDING OF PROGRAMMING-BASED MATHEMATICS TEACHING PRACTICES IN A COMMUNITY OF INQUIRY

Huiyan Ye
The Chinese University of Hong Kong, China, huiyanye@link.cuhk.edu.hk

Research on mathematics learning with programming has shown extensive promise, but there is still a lack of practical use of these tools in school environments, which warrants a deeper understanding of how to incorporate programming into mathematics classroom practice. Recognizing that programming-based mathematics teaching, as a kind of interdisciplinary pedagogical practice, demands a higher level of competence and knowledge than traditional mathematics instruction, we engaged pre-service mathematics teachers in a community of inquiry with researchers to co-construct understandings of mathematics teaching practices in a programming context. This study explored the opportunities and challenges associated with specific aspects of programming-based mathematics teaching practices, particularly in terms of curriculum script, resource system and time economy within the framework of the Structuring Features of Classroom Practice. The findings offer practical solutions to support mathematics teachers in developing the knowledge required to implement such interdisciplinary teaching.

Keywords: mathematics teaching; teaching practices; programming-based teaching; community of inquiry

Introduction

With the emerging child-friendly programming tools, prior studies have evidenced that the integration of programming in mathematics education offers promising opportunities for engaging students in meaningful mathematical learning and computational thinking development (Ye et al., 2023). Despite research on learning with programming has shown extensive promise, there is still a lack of practical use of these tools in school environments, which necessitates an in-depth understanding of how the potential demonstrated by research might be incorporated into classroom practices. Research has shown that collaboration between researchers and teachers can enhance the understanding of teaching practices and improve student learning (Jaworski, 2006; Jaworski & Potari, 2021), with one significant way being the establishment of a community of inquiry (CoI). CoI underpins a practical epistemological stance that leads to meta-knowledge about teaching practices, where teachers and researchers bring inquiry into research that leads to the development of mathematics teaching (Jaworski, 2006). Therefore, by forming a CoI, this study aims to address the following research question: *What new understandings about programming-based mathematics teaching practices can be co-constructed within the inquiry community of pre-service teachers and researchers*?

The Structuring Features of Classroom Practice (SFCP) Framework

In this study, we adopted the SFCP framework (Ruthven, 2009) as a middle-range theory to guide our investigation of programming-based mathematics teaching practices. The SFCP includes five key structuring features of classroom practice: working environment, resource system, activity structure, curriculum script, and time economy. Specifically, the introduction of new technologies

expands the variety of tools and materials available to support the teaching and learning of mathematics, which also poses challenges of creating a cohesive resources system with compatible elements that can be used effectively by participants, as well as making a substantial change to the curriculum script. The lesson agenda has to be restructured, which has a close relationship with activity structures that frame the interactions and actions of participants during specific classroom episodes. Finally, the adoption of new technologies may bring about changes in the working environment and time economy.

Method

The research project was conducted during a summer camp in July-August 2024 with eight primary school students aged 9-11 years old at a local community service center in Hong Kong. To establish a community of inquiry, we recruited three pre-service mathematics teachers to collaborate with the research team in designing a programming-based mathematics curriculum. They were fourth-year undergraduate students with basic programming knowledge but had no prior experience teaching mathematics in a programming context. There were six 1.5-hour sessions on geometry and probability. Participants took turns as course instructors for one session each in geometry and probability, while serving as teaching assistants in the other four sessions. In terms of the cycle of inquiry for each lesson, there were five sessions with different purposes: co-design, co-discussion, classroom teaching, co-reflection and re-design session (Figure 1).

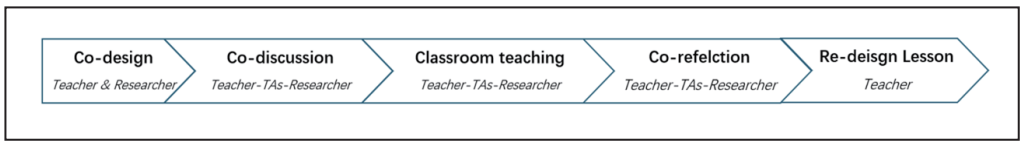


Figure 1. Cycle of inquiry for each lesson

The entire research process was video recorded, with each session lasting 1 to 2 hours, totaling about 45 hours of footage. Additionally, we collected written documentation from pre-service teachers during the inquiry cycles, along with students' artifacts and worksheets. For data analysis, we employed a dual-phase approach—ongoing and retrospective—to systematically identify and report the features, challenges, and solutions of effective practices (Corbin & Strauss, 2008). Ongoing analysis during the summer camp involved questioning to explore instructional design and practices, with collaborative discussions on key issues facilitating improvements in subsequent inquiry cycles. The retrospective analysis, conducted after the camp, employed constant and theoretical comparisons to identify patterns and themes across various pedagogical events, yielding insights that deepened our understanding of teaching practices through established frameworks.

Results

Recourse system: Scaffolding Programming Resources

To leave more time for students' exploration, we noted that two types of programming resources warrant attention when teachers design tasks. (a) Programming for Non-Learning Goals: Pre-created components for students' direct use to reduce their cognitive load related to programming skill. These resources typically include elements carefully selected by teachers that are unrelated to the learning objectives but serve different roles in supporting students solving the programming task.

For example, teachers can use subroutines (My Blocks) to integrate graphical representations of a probability event into the task or defined a "Reset" subroutine to restore the sprite's initial position, orientation and clears all pen marks in a geometry task. These settings help students focus directly on learning goals, making them more conducive to their timely debugging when problems arise. (b) Programming for Learning Goals: Appropriate programming scaffoldings for students to engage in inquiry-based learning. These resources highlight the focus on mathematical inquiry or computational thinking to maximize the exploration opportunities for students. For example, with a pre-defined subroutine, students do not have to spend time in creating the complex subroutine but are engaged in reflecting on the resulting programming artefacts by modifying the parameters.

Curriculum Script: Clarity of Instruction

Clarity of the questions is conducive for students thinking since unexpected responses from ill-defined questions will skew student exploration away from the learning goals and take extra time to re-explain the task. For example, the task used in L4 is related to equal probability of each event which contains only one outcome:

> Three animals play a game of who goes the furthest by drawing a ball from three balls randomly. If Ball 1 is drawn, the cat takes a step forward. If Ball 2 is drawn, the dog takes a step forward. If Ball 3 is drawn, the bear takes a step forward.

We know that with a large number of experiments, the probability of all three being drawn is essentially equal, so the three animals walk almost equal distances. To help student experience the concept of probabilities of events related to the consisting outcomes, the teacher asked a new question: "Can you think of a way to help increase the likelihood that the cat will win?" In this context, S1 proposed to change the outcome space from 3 balls to 4 balls, and when Ball 4 is drawn, the cat takes a step. While S2 proposed to change the conditional rules of the game. That is, keep drawing Ball 1-3 but if Ball 2 is drawn, the cat walks one step rather than the dog. Finally, the third method proposed by S3 was to change the number of steps taken by each animal (action rule). Specifically, the cat takes 10 steps (repeat 10 times) while the other animals take 1 step if their corresponding ball is drawn. Here, we can see that students' responses vary because there are different solutions to the task. One strategy is to change the outcome contained in the event and thus affect the probability of the event. Another is to keep the probability and outcomes of the walking events of three animals, but to change the number of steps they take each time. The latter strategy fulfils the teacher's requirement for the cat to travel further, but it does not relate to changing the outcomes or probabilities of the event associated with the drawn ball. The unexpected solutions put forward by the students catch our attention for further reflection:

[1] Teacher K: Yes…about how to make the cat go further. Student A thought of many methods…

[2] Teacher A: I think we should add a bit more instruction to avoid too many (unexpected answers) …If this task is the concluding inquiry question in the last lesson, I think it's good for them to think of multiple solutions. But as an introductory question for this class, I don't think it's very appropriate.

[3] Researcher: Yes, it might waste a bit of time… so this might be an issue with the task description.

Through a co-constructed understanding, the CoI realized that the lack of clarity in the problem statement led to unexpected student responses, which also impacted the efficiency of the lesson flow. Finally, the CoI also proposed to refine the questions as the strategy for enhancing the clarity.

Time economy: Extension tasks Among Different Sessions

As students are involved in programming activities in the mathematics classroom, problem-solving in such context leads to differences in learning progress not only due to students' varying levels of mathematical knowledge, but also their familiarity with programming tools and programming skills. These discrepancies are often quite noticeable, posing challenges for the teacher:

> "… their progress is inconsistent, leading to … confusion about when to move on to the next part. Since completing tasks in Scratch requires finishing certain important components first…, the coherence needed in the class is higher than in traditional classrooms ..." (Teacher A)

As such, it is essential to address differences in student progress during programming tasks to maintain an efficient classroom flow. Evidence from the classroom suggests that allowing fast-paced students to move on to the next learning task is not conducive to the teacher's control over the teaching process, which in turn affects students' learning outcomes. Our suggestion is to set extension tasks among different sessions. That is, extension problems should be established between different learning tasks, allowing students who complete the tasks quickly to explore these extension tasks rather than moving on to the next section. On the other hand, for students who complete tasks more slowly, teachers should provide appropriate individual guidance to ensure that the key content is largely completed. These students can also be invited to deal with the problem during the follow-up plenary discussion session.

Discussion

In conclusion, scaffolding resources and well-defined problems are essential for effective programming-based mathematics teaching, enhancing the time economy and contributing to creating sustained learning opportunities, as goal clarity and coherence had a positive influence on students' perceptions of supportive learning conditions (Ainley & Carstens, 2018). Given that "what students learn is largely defined by the tasks they are given" (Hiebert & Wearne, 1993, p. 395), this also highlight the fact that programming-based mathematics teaching practices place greater demands on teachers' TPACK (Technological Pedagogical Content Knowledge; Koehler & Mishra, 2009), where cognitive activation instruction requires teachers to select topics and materials aligned with the grade level requirements, learning goals and tool affordance. Therefore, developing teachers' TPACK has the potential to enhance their capacity to generate programming resources and design learning tasks.

References

Ainley, J. and R. Carstens (2018), "Teaching and Learning International Survey (TALIS) 2018 conceptual framework", *OECD Education Working Papers*, No. 187, OECD Publishing, Paris, https://doi.org/10.1787/799337c2-en

Corbin, J. M., & Strauss, A. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (3rd ed.). Thousand Oaks, CA: Sage.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American educational research journal*, *30*(2), 393-425.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, *9*(2), 187–211. https://doi.org/10.1007/s10857-005-1223-z

Jaworski, B., & Potari, D. (2021). Implementation of a developmental model of teachers' and didacticians' learning through inquiry: Design, operationalisation and outcomes. *ZDM – Mathematics Education*, *53*(5), 1073–1084. https://doi.org/10.1007/s11858-021-01290-x

Koehler, M., & Mishra, P. (2009). What is technological pedagogical content knowledge (TPACK)? *Contemporary Issues in Technology and Teacher Education*, *9*(1), 60–70.

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. *Éducation et Didactique*, *3–1*, 131–159. https://doi.org/10.4000/educationdidactique.434

Ye, H., Liang, B., Ng, O.-L., & Chai, C. S. (2023). Integration of computational thinking in K-12 mathematics education: A systematic review on CT-based mathematics instruction and student learning. *International Journal of STEM Education*, *10*(1), 3. https://doi.org/10.1186/s40594-023-00396-w

TSG6-2-20

EFFECTIVE USE OF SCRATCH FOR UNDERSTANDING REGULAR POLYGON IN NICARAGUAN PRIMARY EDUCATION

Isaias Alberto Granados Gonzalez
Naruto University of Education, Japan, granadosgonzalezisaiasalberto@gmail.com

This paper explores the effectiveness of integrating algorithmic thinking through Scratch to enhance the understanding and construction of regular polygons in Primary education. Actually, there is a growing need to incorporate educational technologies facilitating the development and understanding of algorithmic thinking, particularly in mathematics in Nicaragua. The main challenge is the lack of interest in mathematics for students. As indicated by the evaluations conducted at the regional level in TERCE (2013) and ERCE (2019), revealed a significant learning gap, where many students struggled with construction polygons, particularly in the domain of geometry. A diagnostic evaluation of 180 sixth-grade students showed significant challenges in explaining the process of constructing regular polygons and misconceptions related to algorithmic thinking.

Keywords: Scratch, algorithmic thinking, regular polygons, effectiveness, technological tool

Introduction

Background of Study

According to UNESCO (2021), in the agenda scheduled for the five-year period (2026-2030), the Ministry of Education of Nicaragua plans to introduce technology into the Basic National Curriculum across all educational levels, applying various strategies and components tailored to each one. For primary education, the initiative aims to integrate the use of algorithms and programming from first to sixth grade (MINED, 2021). Specifically, for fifth and sixth grades, the plan includes incorporating Scratch, as an educational software that fosters the development of analytical skills and competencies which are necessary for a better understanding of real-world problems and problem-solving. This approach seeks to maximize the effective use of the technological resources available in schools, as many devices remain underutilized or entirely unused, often left stored in their original packaging. As a result of the low performance observed in mathematics, particularly in the geometry unit, it is essential to integrate new strategies that facilitate an effective understanding of regular polygons. This includes incorporating algorithmic thinking to enhance students' educational quality, with Scratch serving as a valuable tool to achieve these goals (Bers, 2018).The process of learning mathematics in the geometry unit, it has been observed that students present difficulties in the construction of regular polygons, the explanation or step by step of the procedure carried out, has become a focus of concern, due to the low results obtained in the quality tests that were executed regionally and the ineffective use of technology in the classrooms.

Objective

Due to the difficulties and low performance previously presented in regional studies on geometry, it is necessary to integrate new strategies like scratch to assess the effectiveness of scratch in enhancing regular polygons understanding in primary education through the application of algorithmic thinking.

Significance of the Study

The results of this study will provide valuable information to teachers on the application of new educational strategies, so that students develop the skill of algorithmic thinking and the procedural bases step by step, through the use of scratch as an educational tool, significantly improving learning in the construction of regular polygons, acquiring new analytical skills and indispensable knowledge required for their understanding, as well as the proper use of the means and technological tools that the Ministry of Education has provided in schools.

Literature review

Scratch was launched in May of 2007 as "downloadable desktop application created and developed by MIT media Lab, contributions from David Siegel. The main purpose is to enable children to create interactives stories, some games, and other things on their computers" promoting their creativity and digital learning (Resnick & Scratch Foundation, 2024)

Algorithmic thinking is a systematic approach linked to computer science and coding, involving step-by-step processes to automate problem-solving (Riano, 2023). It is valuable beyond computing, aiding in breaking down complex problems into repeatable steps. Algorithms form the foundation of programming and computing (McVeigh, 2019). Endo (2024) highlights that Scratch helps students learn sequences, loops, and conditions, enabling them to deconstruct problems and create step-by-step solutions using visual code blocks. Indra P. et al. (2024) state that Scratch enhances geometric understanding by allowing students to create and manipulate shapes, fostering visualization of geometric relationships.

Methodology

This study employs a quasi-experimental design with a pretest-posttest approach to assessing the impact of instructional intervention on students` baseline knowledge and skills in drawing regular polygons. These initial measures served as a reference point for subsequent comparisons and analysis.

Lesson Intervention

Table 1 shows the progression of the contents which include strategies themselves, also objectives intended to be achieved through each intervention taking place in sixth primary schools' students.

Table 1. Topics include in lesson intervention and its objectives.

Number of Intervention	Topic/contents	Objective
1	Introduction to **algorithms** in everyday life	To understand algorithms through real-life activities
2	Basics of scratch and its environment	**To familiarize with scratch:** coding area, workplace, stage, and objects
3	Logical **sequence** teaching in Scratch (Step-by-step process)	To appropriate and master the concept and **definition of sequence using scratch by** drawing regular polygons (three to seven sides)
4	Logical sequence compared with **repetitions** (loops)	To appropriate and master the **operation of repetitions (**loops) drawing regular polygons (three to eleven sides)
5	Practice classes by students	To clarify doubts and make practical activities creating diverse kinds of regular polygons

Note. Distribution of contents including goals or objectives, integrating new strategies such as sequences, loops or repetition and pattern recognition through integration of ICT tools in lesson intervention.

Sample Data and Analysis Method

The data for this study were collected through a pretest and posttest applied to 180 sixth-grade students from rural and urban primary schools in Nicaragua between September 15 and October 20, 2024. The focus of this research is to assess students' ability to construct regular polygons with traditional tools before an instructional intervention. The intervention introduced Scratch to enhance understanding and integrate algorithmic thinking. A post-test measured the impact of this approach, with JASP software used for data analysis and comparison of results.

Results and discussion

Results reflect students' performance in constructing regular polygons after the intervention. The observed trends are analyzed in relation to algorithmic thinking, considering the impact of using Scratch in the learning process. After the intervention of 180 students, a significant improvement was observed in the construction of regular polygons. 35% of the students drew a square, 21% an equilateral triangle, 17% a hexagon, 15% a pentagon, and 11% managed to draw complex polygons (octagons, nonagons, and decagons). Only 1% were unable to complete the task. Through the integration of strategies incorporating such as daily live activities and scratch allowed deep knowledge about algorithmic thinking for solving complex problems into simpler steps.

Figure 1 illustrates the differences between the pretest and posttest results. In the pretest, it is evident that students face difficulties in drawing the regular polygon, and their explanation of the process is insufficient. In contrast, the posttest demonstrates that students have successfully drawn the regular polygon, using the geometric case correctly. Regarding the step-by-step explanation of the process, students have enhanced their understanding and are now able to explain the entire procedure, using the compass to draw the circle, calculating the angle dividing 360 by 5, measuring and marking a point every 72 degrees, and finally connecting the points to form the polygon.

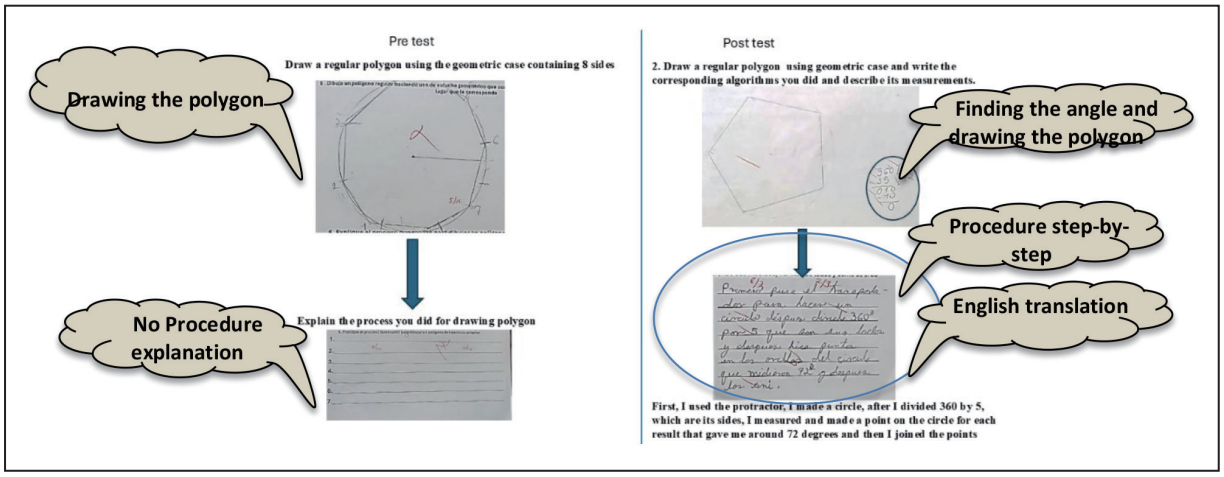


Figure 1. Results of pre and posttest sample

It was observed that students demonstrated improved performance in post-test applying algorithm thinking, where they were able to approach complex problems more effectively by breaking them down into smaller, specific steps, leading to more accurate and efficient solutions.

Table 2. Statistics Data Between Pre and Post Test

Category	Test	Score	t-value	p-value
Drawing regular Polygon	Pretest	25	13.134	< .001
	Post test	20	10.390	
Explanation of the process	Pretest	25	22.270	< .001
	Post test	15	9.855	

Note. Students t-test shows that there is significant improvement with the drawing of the regular polygons and the explanation of the process between pre-test and post-test with p-value less than 0.001

These results demonstrate that students were able to understand that, through visualization and the application of algorithmic structures, a problem can be broken down into simpler procedures, such as drawing a side, turning at a specific angle, and repeating the process. This approach enabled them to grasp the concepts of sequence and repetition. By using Scratch, students efficiently applied loops, which facilitated the rapid and accurate achievement of results. Consequently, a significant improvement was observed in their performance related to drawing regular polygons, as shown in Table 2, where t-value increased from 13.134 in the pre-test to 22.270 in the post-test.

Conclusion

The integration of scratch is a highly effective tool that allows students to develop their algorithmic thinking skills, in a positive way, increasing their results significantly, by applying step-by-step sequences and repetitions, facilitating the understanding of difficult problems, breaking them down into small actions or manageable orders. As strategy, facilitated the appropriate use of technological

tools, which motivated and engaged students in the topic of regular polygon understanding. This approach provided an innovative way to learn mathematics, making it more enjoyable and easier for the students to understand.

References

Bers, M. U. (2018). *Programming and Computational Thinking in the Early Childhoods Classroom.* New York: Taylor and Francis Group.

Endo, H. (2024). Lesson and assessment for fostering algorithmic thinking through the use of scratch. *Japan Science Education Association, 39*(1), 29-34. Retrieved from https://www.jstage.jst.go.jp/article/jsser/39/1/39_No_1_240107/_pdf/-char/ja

Indra Prahmana, R. C., Satoshi, K., Nofikusunawati Peni, N. R., Endo, H., Azhari, A., & Takinawa, K. (2024). Cross-cultural insights on computational thinking in geometry: Indonesian. *Journal on Mathematic Education*, 613-638.

McVeigh Murphy, A. (2019, November 19). *Computational Thinking, Algorithmic Thinking, & Design Thinking Defined*. Retrieved from https://equip.learning.com/computational-thinking-algorithmic-thinking-design-thinking

MINED. (2021, July). *PLAN NACIONAL DE LUCHA CONTRA LA POBREZA.* Retrieved December 2023, from National plan to Combat the poverty: https://www.pndh.gob.ni/documentos/pnlc-dh/PNCL-DH_2022-2026(19Jul21).pdf

Resnick, M. (2023). *Scratch Foundation.*

Resnick, M. (2024). *Scratch Foundation*. Retrieved from Mission and Vision of Scratch : https://en.scratch-wiki.info/wiki/Scratch_Foundation

Riano, M. (2023, October). *Miloriano*. Retrieved from What Is Algorithmic Thinking? A Beginner's Guide to Logical Problem-Solving: https://www.miloriano.com/what-is-algorithmic-thinking-a-beginners-guide-to-logical-problem-solving/

UNESCO. (2021, July). *PLAN NACIONAL DE LUCHA CONTRA LA POBREZA.* Retrieved December 2023, from National plan to Combat the poverty: https://www.pndh.gob.ni/documentos/pnlc-dh/PNCL-DH_2022-2026(19Jul21).pdf

Unity, N. G. (2021, 07 19). *PLAN NACIONAL DE LUCHA CONTRA LA POBREZA.* Retrieved 12 17, 2023, from National Plan to Combat Poverty: https://www.pndh.gob.ni/documentos/pnlc-dh/PNCL-DH_2022-2026(19Jul21).pdf.

Acknowledgments

Deep gratitude is given to the Japan International Cooperation Agency (JICA), which allows us to maintain good international connections with Nicaragua, supporting various educational projects in order to improve mathematics education in primary school children; to the Naruto University of Education for providing the necessary conditions for this type of educational research to be carried out, actively collaborating with various underdeveloped countries, allowing access to international students to study and to strengthen our personal and professional skills and capacities.

TSG6-2-21

FIRST-GRADE STUDENTS' IDEAS AROUND NEGATIVE NUMBERS IN MATHEMATICS-CODING INTEGRATED LESSONS

Boram Lee
Utah State University, US, boram.lee@usu.edu

Jessica F. Shumway
Utah State University, US, jessica.shumway@usu.edu

Jody Clarke-Midura
Utah State University, US, jody.clarke@usu.edu

Deborah Silvis
SUNY Cortland, US, deborah.silvis@cortland.edu

Ana Ashineh
Utah State University, US, a.ashineh@usu.edu

Camille Lund
Utah State University, US, camille.lund@usu.edu

This study explores first-grade students' mathematical understanding of negative numbers in integrated mathematics-coding lessons. Analyzing video data from six students, we identified emerging ideas such as "half of zero," "numbers behind zero," and "not real numbers" through their interactions with a coding robot. The robot's spatial movements on a number line provided a unique context for students to conceptualize negative numbers, a topic typically absent in early-elementary curricula. While the findings are based on a small dataset, they suggest that integrating coding technology into mathematics instruction can support young learners in engaging with advanced mathematical concepts. This study highlights the need for further research on early-grade students' understanding of negative numbers and the role of coding in facilitating mathematical reasoning.

Keywords: elementary mathematics, coding, negative numbers

Introduction

A solid understanding of negative numbers is an important factor in students' ability to learn advanced mathematics, such as algebra (Young & Booth, 2020). However, because many curricula introduce negative numbers only at the secondary level, there has been a lack of studies exploring elementary students' reasoning about negative numbers. Given that the concept of negative numbers does not simply emerge in students once they reach middle school, it is valuable to examine how elementary students think about negative numbers. Hence, this proposal explores the diverse ideas about negative numbers that first-grade students demonstrated in integrated mathematics-cod-

ing lessons and provides a foundation for discussing the possibility of reapproaching the concept of negative numbers in the elementary mathematics curriculum.

Literature Review

Negative Numbers in Mathematics Education

Negative numbers are introduced in secondary-level curricula across various countries (CCSS [US], 2010; Ministry of Education [Singapore], 2019; Ministry of Education [Korea], 2015; Ministry of Education, Culture, Sports, Science and Technology [Japan], 2017). In elementary mathematics curricula, negative numbers are typically not addressed and are sometimes considered nonexistent. For example, students are often told that it is not possible to subtract a larger number from a smaller one (Graybeal, 2014). However, previous studies have shown that elementary students are capable of understanding negative numbers (Bishop et al., 2014; Hativa & Cohen, 1995). Given that some studies report the difficulties secondary students face when learning negative numbers (Fuadiah et al., 2017; Gallardo, 2002) and emphasize the importance of understanding negative numbers for students' mathematical learning (Wong & Yip, 2025), it is worthwhile to further explore elementary students' emerging ideas about negative numbers.

Integration of Mathematics and Coding

Previous studies have investigated the integration of mathematics and coding or mathematical and computational thinking, highlighting its positive aspects in K–12 mathematics education (Shumway et el., 2021; Sáez-López et al., 2019; Ye et al., 2023). However, other studies emphasize the need for further research on how coding can be utilized as a tool to enhance students' mathematical understanding (Nordby et al., 2024; Nyman et al., 2023). This indicates the necessity of a more in-depth analysis, particularly examining how the integration of mathematics and coding supports students' mathematical understanding. To address this gap, this paper descriptively explores students' mathematical understanding of negative numbers as it emerges in mathematics-coding integrated lessons in first-grade classrooms.

Based on this literature, we formulated the following research question: What ideas do students demonstrate around negative numbers while interacting with a coding robot in a number line context? To answer this question, we analyzed a dataset, the details of which are described in the following section.

Methods

This study is part of a larger Design-Based Research project in the northwestern United States that integrates coding with mathematics in elementary classrooms to investigate the ways dynamic interactions with coding technology deepen mathematics learning. The data for this study comes from the video data of two first-grade groups of students (n=6). The students participated in a series of four 30-minute mathematics-coding-integrated lessons designed by the research team in collaboration with two first-grade teachers and two second-grade teachers and facilitated by a teacher-researcher. The activities in which students interacted with coding robots in a number line context took place during the third and fourth lessons. For the activities, students were given a laminated number line on which a coding robot moved back and forth according to the codes they programmed using arrow coding cards (Figure 1).

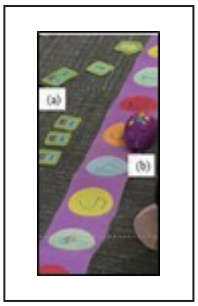


Figure 1. The setting of the activity. (a) Arrow coding cards (b) Robot Mouse (Learning Resources) on a researcher-made number line

We identified two moments in the lessons when two different groups encountered a situation in which a robot landed on a position on the number line less than 0. The teacher-researchers of the two groups asked students what number the position should be. Students' verbal explanations were transcribed, and their actions, including gestures, were captured on video. In this paper we present students' explanations and gestures about negative numbers and our descriptive analysis of their emerging ideas about negative numbers in this mathematics-coding context.

Results

It's a Half of Zero

A teacher-researcher placed a coding robot at 2 on the number line and gave a program (1 forward code and 4 backward codes) to three first-grade students: Ethan, Steven, and Ivy (all pseudonyms). She then asked what number the robot would stop on if they applied the code to the robot. The three students pointed to different positions on the number line (Figure 2). Next, they entered the code into the robot and observed its movement. The facilitator pointed at the red circle where the robot stopped and asked, "What number is this?" The following transcript shows the students' responses (Table 1).

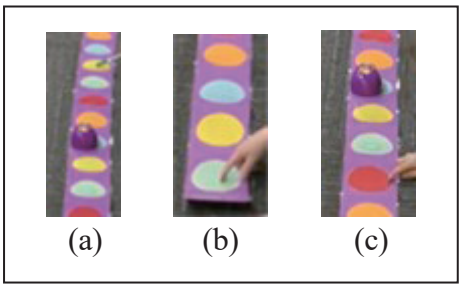


Figure 2. (a) Ivy (b) Steven (c) Ethan

Table 1. Responses from Ethan, Ivy, and Steven

Students' verbal explanations	Students' actions
Teacher-researcher (TR): So what number is this? (Action 1: pointing at the red circle where the robot landed) Steven: It's half of a quarter of 0. TR: Is it bigger than 0 or smaller than 0? All: Smaller than 0 TR: What number is smaller than 0? Ivy & Steven: 1 TR: I thought this was 1 (Action 2: pointing at 1 (yellow circle) on the number line). Ivy: Uhmm... hmm… Steven: Half of a quarter of zero. Ivy: None! TR: None? Ethan: Oh I know I know, it's half of 0. TR: Half of zero, then what would this be (Action 3: pointing at the orange circle) if this is half of zero? Steven: That is a quarter of zero. TR: Okay then what would this be (Action 4: pointing at the blue circle). Steven: Half of 0. TR: Another half of 0? Ethan: No, it'd be ten quarter. (Action 5: pointing at the orange circle) Ivy: Small. TR: Oh, small? Ethan: 9 quarter (Action 6: pointing at the blue circle), 8 quarter (Action 7: pointing at the yellow circle) and 7 quarter. (Action 8: pointing at the green circle) TR: Oh my gosh, wow. Steven: There are no numbers there.	Action 1 Action 2 Action 3 Action 4 Action 5 Action 6 Action 7 Action 8

The students recognized that the numbers behind 0 needed to be smaller than 0. Ivy responded "none," while the other two boys came up with new numbers derived from 0, such as half of zero or a quarter of zero. This showed that, while interacting with the number line context—where a robot landed on a specific position behind 0—students understood that the numbers labeling that position should be smaller than 0. Additionally, when Ethan labeled the circles positioned consecutively, he used smaller numbers as he pointed at each circle ("9 quarters, 8 quarters, and 7 quarters"). This illustrates that he interpreted the numbers behind 0 as decreasing as the circles moved farther from zero.

Numbers Behind Zero and Not a Real Number

In another first-grade group (Ali, Bently, and Kevin, all pseudonyms), when Ali put her code onto the robot and ran the code, the robot stopped at a position past 0. The facilitator asked students what numbers they could put in the circles. The transcript in Table 2 shows students' responses.

Table 2. Responses from Ali, Bently, and Kevin about numbers less than zero on the number line.

Students' verbal explanations	Students' actions
Bently: Let's see, negative 1(Action 1: pointing to the green circle), negative 2 (Action 2: pointing to the red circle), and negative 3 (Action 3: pointing to the orange circle). Teacher-researcher (TR): What does negative 1, 2, 3 mean? Ail: Not a real number. TR: Then what is it? Bently: Numbers behind 0? TR: How can I write down negative 1? (Bently grabbed the pencil and tried to write, but he ended up not writing anything) Kevin: I know. (Action 4: writing *n* over 1, 2, 3)	Action 1 Action 2 Action 3 Action 4

Bently's response about the meaning of negative numbers suggests that the spatial properties of the number line and the coding robot's movement are connected to his interpretation that negative numbers are positioned *behind* 0. Additionally, Ali's statement about negative numbers ("Not a real number") indicates that she has some ideas of real numbers. While no further conclusions can be drawn from this dataset regarding her precise definition of "real number," this observation suggests that students' ideas about real and non-real numbers could be a potential research topic.

Conclusion and Discussion

The goal of this proposal is to descriptively illustrate first-grade students' thinking about negative numbers in the context of mathematics-coding integrated lessons. The results show that even first-grade students can generate diverse ideas about negative numbers when they are introduced in a context that integrates a coding robot—which moves within a three-dimensional physical space—and a number line. Within this spatial context, the participants were able to associate the idea of numbers located behind zero with the concept of numbers being less than zero. The results imply that incorporating technology into mathematics lessons can provide students with opportunities to explore more advanced mathematical concepts, such as negative numbers—an opportunity that current curricula often restrict at the elementary level.

Given the small size of the dataset and the preliminary nature of the analysis, these results cannot be generalized to make broad claims about early-grade students' emerging ideas about negative numbers. However, this proposal highlights the need for further research on negative numbers in early grade levels and suggests initiating a discussion on the potential integration of advanced mathematical concepts into elementary education in a way that young students can comprehend with technological support.

References

Bishop, J. P., Lamb, L. L., Philipp, R. A., Whitacre, I., & Schappelle, B. P. (2014). Using order to reason about negative numbers: the case of Violet. *Educational Studies in Mathematics, 86*(1), 39–59. http://www.jstor.org/stable/43590060

Council of Chief State School Officers & National Governors Association Center for Best Practices. (2010). Common core state standards for mathematical practice. Retrieved from https://corestandards.org/wp-content/uploads/2023/09/Math_Standards1.pdf

Fuadiah, N. F., Suryadi, D., & Turmudi. (2017). Some difficulties in understanding negative numbers faced by students: A qualitative study applied at secondary schools in Indonesia. *International Education Studies, 10* (1), 24–38. https://doi.org/10.5539/ies.v10n1p24

Gallardo, A. The extension of the natural-number domain to the integers in the transition from arithmetic to algebra. *Educational Studies in Mathematics 49*, 171–192 (2002). https://doi.org/10.1023/A:1016210906658

Graybeal, C. D. (2014) Mathematical lies we tell students. *Teaching Children Mathematics*, *21*(4), 197–199. https://doi.org/10.5951/teacchilmath.21.4.0197

Hativa, N., Cohen, D. Self learning of negative number concepts by lower division elementary students through solving computer-provided numerical problems. *Educational Studies in Mathematics, 28*, 401–431 (1995). https://doi.org/10.1007/BF01274081

Ministry of Education, Culture, Sports, Science and Technology (MEXT). (2017). Improvement of Academic Abilities (Courses of Study). Retrieved from https://www.mext.go.jp/en/policy/education/elsec/title02/detail02/1373859.htm

Ministry of Education. (2015). *2015 revised national curriculum: Mathematics curriculum*. National Curriculum Information Center. https://ncic.re.kr

Ministry of Education. (2019). *Mathematics syllabuses: Secondary 1 to 4*. Ministry of Education, Singapore. Retrieved from https://www.moe.gov.sg/-/media/files/secondary/syllabuses/maths/2020-express_na-maths_syllabuses.pdf

Nordby, S. K., Mifsud, L., & Bjerke, A. H. (2024). Computational thinking in primary mathematics classroom activities. *Frontiers in Education, 9*. https://doi.org/10.3389/feduc.2024.1414081

Nyman, R., Bråting, K., & Kilhamn, C. (2024). Can programming support mathematics learning? An analysis of Swedish lower secondary textbooks. *International Journal of Mathematical Education in Science and Technology*, 1–19. https://doi.org/10.1080/0020739X.2024.2329345

Sáez-López, J. M., Sevillano-García, M. L., & Vazquez-Cano, E. (2019). The effect of programming on primary school students' mathematical and scientific understanding: educational use of mBot. *Educational Technology Research and Development, 67*(6), 1405–1425. http://www.jstor.org/stable/45237972

Shumway, J. F., Welch, L. E., Kozlowski, J. S., Clarke-Midura, J., & Lee, V. R. (2021). Kindergarten students' mathematics knowledge at work: the mathematics for programming robot toys. Mathematical Thinking and Learning, 25(4), 380–408. https://doi.org/10.1080/10986065.2021.1982666

Wong, T. T.-Y., & Yip, E. S.-K. (2025). All real numbers are important: The significance of negative number understanding for mathematics achievement. *Journal of Educational Psychology, 117*(2), 246–256. https://doi.org/10.1037/edu0000912

Ye, H., Liang, B., Ng, OL. et al. Integration of computational thinking in K-12 mathematics education: a systematic review on CT-based mathematics instruction and student learning. *International Journal of STEM Education, 10* (3) (2023). https://doi.org/10.1186/s40594-023-00396-w

Young, L. K., & Booth, J. L. (2020). Don't eliminate the negative: Influences of negative number magnitude knowledge on algebra performance and learning. *Journal of Educational Psychology, 112*(2), 384–396. https://doi.org/10.1037/edu0000371

TSG6-2-22

THE EFFECTS ON STUDENTS' PROBLEM-SOLVING SKILLS OF ESPORTS-BASED MATHEMATICAL VIDEO LESSONS

Ronie Jake S. Mamala
Mariano Marcos State University/University of the Philippines - Diliman, Philippines, rsmamala@mmsu.edu.ph

Ma. Nympha Beltran-Joaquin
University of the Philippines - Diliman, Philippines, mbjoaquin@up.edu.ph

Sheryl Lyn C. Monterola
University of the Philippines - Diliman, Philippines, scmonterola@up.edu.ph

This study investigated the effects of eSports-Based Mathematical Video Lessons (EBMVL) on high school students' problem-solving skills. A quasi-experimental design compared two intact classes: one using EBMVL and the other the Conventional Teaching Approach (CTA). The EBMVL intervention followed the ADDIE model and 5Es learning cycle. Data were collected through a Problem-Solving Test. Results showed that EBMVL improved problem-solving skills but had no statistically significant advantage over CTA. While EBMVL enhanced engagement and motivation, its impact on problem-solving was occasional. These findings offer insights for educators and policymakers on technology-based interventions in mathematics education. However, technical and time constraints suggest additional support is needed for video-based learning.

Keywords: eSports-Based Mathematical Video Lessons (EBMVL), m-Learning, Problem-Solving Skills, Video Lessons

Introduction

The coronavirus pandemic has resulted in widespread adoption of e-learning in educational institutions. With the increasing popularity of smartphones and tablets, game-based learning is rapidly transitioning to mobile platforms in the context of virtual learning materials for education. This shift is driven by the mobility, wireless connections, and built-in sensors provided by smart devices, enabling blended and context-sensitive mobile learning (m-Learning) activities (Giannakas et al., 2018). The introduction of mobile devices into the e-learning process has introduced a new learning method known as mobile learning (m-Learning), which provides learners and educators with instant access to learning content and collaborative activities from any device or platform. Additionally, video media is a powerful tool used in various types of learning. The combination of visuals (moving images) and audio (sound) in video media engages both the visual and auditory senses of students, facilitating their understanding of the topic (Potdevin et al., 2018; Woottipong, 2014). Recorded video lessons also offer learners the flexibility to learn at their own pace, contributing to the development of their required skills in the 21st century due to the diversity and richness of the material generated through video classes.

There is a dearth of literature focusing on improving problem-solving skills using video lessons. According to the Trends in International Mathematics and Science Study (TIMSS) 2019, Filipino children are lagging behind their counterparts in other countries in the international grade 4 mathematics and science examinations. The Philippines obtained significantly lower math and science scores compared to other participating countries (Mullis et al., 2020). Furthermore, Filipino students' performance in mathematics has been consistently poor, as reflected in their low scores in the 2018 Programme for International Student Assessment (PISA), where they scored 353 in comparison to the international average of 489 (OECD, 2019). Both TIMSS and PISA test students' knowledge and ability in mathematics including solving word problems.

In light of these issues, the eSports-Based Mathematical Video Lessons (EBMVL), capitalizing on video media on m-Learning while integrating eSports elements were hypothesized to have the potential to optimize students' problem-solving skills in mathematics. Further, this study aimed to compare the problem-solving skills of students exposed to EBMVL with those using the conventional method of teaching. The research is significant to teachers, administrators, instructional materials developers, game developers, content creators, and other researchers.

Method

The research study utilized the two-group pretest-posttest quasi-experimental design to determine the effects of EBMVL on students' problem-solving skills. The study was conducted among 54 grade 9 students in a public school. It used a researcher-made problem solving test that was content-validated and reliability tested. The experiment lasted for a quarter of the school year and was delivered online due to the COVID-19 pandemic.

The EBMVL intervention explicitly integrated Grade 9 trigonometric concepts, aligned with MELCs, into esports contexts. Each weekly cycle addressed a specific learning competency—from illustrating trigonometric ratios to solving problems involving oblique triangles. These topics were embedded into familiar gaming scenarios using annotated screenshots from *Mobile Legends: Bang Bang* and *Call of Duty Mobile*, where in-game maps and distances were used to simulate real-world triangle applications. This visual-spatial context supported the learning of trigonometric applications by transforming abstract ideas into concrete, game-based representations, guided by the Cognitive Theory of Multimedia Learning (Mayer, 2005).

Results and Discussion

The EBMVL class and the conventional class were found to be initially comparable at the start of the experiment in terms of their problem-solving skills. However, the statistical analysis using a one-tailed t-test for two independent samples found no significant difference in posttest scores between students taught using Esports-Based Mathematical Video Lessons (EBMVL) and those taught using the Conventional Teaching Approach (CTA), suggesting that EBMVL did not significantly impact problem-solving skills. Possible reasons include a small sample size leading to a Type II error, clustering of scores near extremes, or sampling bias. While EBMVL may enhance engagement and motivation, it did not show a statistically significant advantage over CTA in improving problem-solving skills. Both approaches were effective, reinforcing the idea that multiple teaching methods can develop mathematical competence.

A paired samples t-test was also conducted to assess changes in problem-solving performance between pretest and posttest results following the intervention. The experimental group (EBMVL) had a higher mean posttest score (18.07, SD = 5.29) than the control group (CTA) (16.96, SD = 5.91), suggesting a potential positive effect of the intervention. However, the difference was not statistically significant in the one-tailed t-test. Notably, the p-value for the EBMVL group (p = 0.003) was lower than that of the CTA group (p = 0.011), indicating a stronger trend in favor of the intervention.

While both groups showed statistically significant gains from pretest to posttest, informal interviews and teacher observations suggested that students in the EBMVL group were better able to visualize problem contexts and apply strategic reasoning in geometry and trigonometry tasks. This aligns with Mayer's theory that dual-channel input (visual + auditory) enhances cognitive processing. Students reported that the game-based scenarios helped them relate mathematical problems to everyday experiences and improved their confidence in interpreting and solving problems involving right and oblique triangles. Though limitations like internet access and time constraints during the fourth quarter affected full engagement, the intervention showed potential in enhancing learners' conceptual understanding and strategic approach to problem solving.

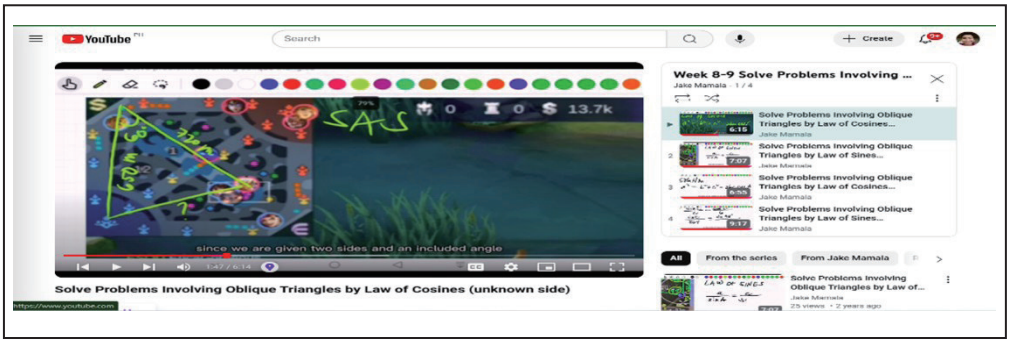


Figure 1. Sample of eSports-based mathematical video lessons (EBMVL)

Conclusions and Recommendations

This study concludes that the use of eSports-Based Mathematical Video Lessons (EBMVL) does not significantly affect students' problem-solving skills. Based on these findings, several recommendations are put forward. For teachers, extending the EBMVL intervention period is recommended. Monitoring student engagement and considering the limitations of the EBMVL approach are also important. Additionally, incorporating collaborative learning and providing training for teachers on the EBMVL approach are recommended. Further research with a larger sample size may be needed to confirm its significance. The study highlights the need for further research to explore the potential of EBMVL and emphasizes the importance of statistical significance in evaluating teaching interventions.

Despite the lack of a statistically significant difference between EBMVL and CTA in comparative performance, the significant within-group improvement for EBMVL supports its educational value. The intervention not only aligned with students' digital interests but also created meaningful learning experiences by contextualizing mathematical concepts within familiar gaming environments. Future implementations should include longer exposure periods, stronger teacher facilitation, and

more structured engagement protocols to ensure equitable access and minimize distractions. With refinements, EBMVL holds promise as an innovative, student-centered approach that can enhance both motivation and the development of problem-solving strategies.

References

Giannakas, F., Kambourakis, G., Papasalouros, A., et al. (2018). A critical review of 13 years of mobile game-based learning. *Educational Technology Research and Development, 66,* 341–384. https://doi.org/10.1007/s11423-017-9552

Mayer, R. (2005). Cognitive theory of multimedia learning (Vol. 31). Cambridge: Cambridge University Press.

Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). TIMSS 2019 International Results in Mathematics and Science. Retrieved from Boston College, TIMSS & PIRLS International Study Center website: https://timssandpirls.bc.edu/timss2019/international-results/

OECD. (2019), PISA 2018 Results (Volume I): What Students Know and Can Do, PISA, OECD Publishing, Paris, https://doi.org/10.1787/5f07c754-en

Potdevin, F., Vors, O., Huchez, A., Lamour, M., Davids, K., & Schnitzler, C. (2018). How can video feedback be used in physical education to support novice learning in gymnastics? *Effects on motor learning, self-assessment and motivation. Physical Education and Sport Pedagogy, 23*(6), 559–574.

Woottipong, K. (2014). Effect of Using Video Materials in the Teaching of Listening Skills for University Students. International Journal of Linguistics, 6(4). https://doi.org/10.5296/ijl.v6i4.5870

TSG6-2-23

A MID-CAREER MATHEMATICS TEACHER'S NOTICING OF STUDENTS' DATA INVESTIGATION PRACTICES WITH A DATA ANALYSIS AND VISUALIZATION TOOL

Takashi Kawakami
Utsunomiya University, Japan, t-kawakami@cc.utsunomiya-u.ac.jp

Keiko Hino
Utsunomiya University, Japan, khino@cc.utsunomiya-u.ac.jp

Tomohiko Makino
Utsunomiya University, Japan, makino@cc.utsunomiya-u.ac.jp

This study examines how a mid-career mathematics teacher noticed and interpreted data investigation practices among eighth-grade using a data analysis and visualization tool. Through reflective collaboration with a researcher, the teacher observed that students' data visualizations supported model-based reasoning, communication, and the proposal of informed actions. The teacher also recognized the pedagogical value of both the visualizations and the digital tool in supporting students' inquiries. These findings offer important implications for the professional development of mathematics teachers integrating data science education and digital technologies into their classrooms.

Keywords: Teacher noticing, Data investigation practices, Data analysis and visualization tool, Professional development for data science education, Teacher-researcher partnership

Introduction

As digital transformation changes the use of data in society, data science (DS) education using digital technology must be integrated into mathematics education for the development of data-literate citizens (Biehler et al., 2024; OECD, 2024). Training mathematics teachers in DS education is also essential, as most have not experienced DS education themselves and have done little to integrate technology into their everyday mathematics teaching (Schreiter et al., 2024). If mathematics teachers learn to recognize and value students' DS-related activities in their teaching, they may be more likely to integrate data-driven investigations of real-world phenomena into their teaching, thereby supporting deeper student learning in DS (Lee et al., 2022). However, scant research has explored how teachers notice and interpret students' DS practices. This study examines a mid-career mathematics teacher in Japan, with 15–20 years of experience, who serves as a "middle leader" and is expected to share practical insights with colleagues (Hino et al., 2020). It investigates how the teacher noticed lower secondary students' engagement with DS using digital tools and considers the implications for mathematics teachers' professional development in DS education.

Conceptual Framework

Mathematics Teachers' Noticing of Students' Data Investigation Practices

Mathematics teachers' noticing and interpretation of valuable practices and thinking by students is an essential teacher competence and current research topic (Weyers et al., 2023). Mathematics education researchers have examined teachers' noticing of students' thinking in statistics-related mathematical modeling (Türker Biber et al., 2023) and students' statistical thinking (Shin, 2021). However, in the DS education field, little research has examined teachers noticing students' meaningful DS practices involving data analysis and visualization tools (Schreiter et al., 2024).

Building on data scientists' practices, Lee et al. (2022) developed a data investigation (DI) process framework to support primary and secondary schools' DS education (Table 1). This framework includes six interconnected DI phases—framing the problem, considering and gathering data, processing data, exploring and visualizing data, considering models, and communicating and proposing actions—and DI practices in each phase. This framework describes the practice of analyzing and visualizing data and proposing actions in detail. Thus, this framework is a lens through which researchers can identify teachers noticing student behaviors valued in DS through activities using data analysis and visualization tools.

Table 1. Data investigation (DI) process framework (Lee et al., 2022, p. 11)

DI phase	DI practice
1. Frame problem	1.1 Consider real-world phenomena and broader issues related to problem 1.2 Pose investigation question(s) 1.3 Anticipate potential data and strategies
2. Consider and gather data	2.1 Understand possible attributes, measurements, and data collection methods needed for the problem 2.2 Evaluate and use appropriate design and techniques to collect or source data 2.3 Consider sample size, access, storage, and trustworthiness of data
3. Process data	3.1 Organize, structure, clean, and transform data in efficient and useful ways 3.2 Consider additional data cases or attributes
4. Explore and visualize data	4.1 Construct meaningful visualizations, static or dynamic 4.2 Compute meaningful statistical measures 4.3 Explore and analyze data for potential relationships or patterns that address the problem
5. Consider models	5.1 Analyze and identify models that address the problem 5.2 Consider assumptions and context of the models 5.3 Recognize possible limitations
6. Communicate and propose action	6.1 Craft a data story to convey insight to stakeholder audiences 6.2 Justify claims with evidence from data and propose possible action 6.3 Address uncertainty, constraints, and potential bias in the analysis

Professional Development for Mid-career Teachers Through Teacher-Researcher Partnerships

How teachers notice varies depending on their disciplinary knowledge, beliefs and dispositions, and teaching experience (Goldsmith et al., 2014). One approach to promote teacher noticing is collaborative professional development, designed as a partnership between researcher(s) and teacher(s) (Esteley et al., 2024). The authors conducted a joint professional development (JPD) program undertaken by the university and the prefectural board of education, where participating teachers conduct action research, plan practical research themes, design and implement research lessons, analyze and evaluate lessons, and reflect on the research, in collaboration with researchers (Hino et al., 2020; Makino & Hino, 2018). Teachers participating in the program reflected on their classroom implementation and students' learning through detailed dialogue and practice records with researchers to develop their perspectives and core ideas regarding mathematics teaching and learning.

Research Question (RQ)

Guided by this conceptual framework, we asked: *What does a mid-career mathematics teacher notice regarding students' DI practices using a data analysis and visualization tool, through reflective discussions with a researcher on classroom implementation and student learning?*

Method

Study Context

This case study focused on one mathematics teacher, Mr. W, noticing eighth-grade (13–14-year-olds) students' DI practices using the free data analysis and visualization software, Common Online Data Analysis Platform (CODAP, https://codap.concord.org/). Mr. W had 16 years of experience teaching mathematics in lower-secondary schools but had never taught statistics using CODAP. He knew little about DS education, including the DI process framework (Table 1). In the JPD program, he conducted action research with one researcher (first author). Mr. W planned and taught a research class (four lessons) designed to help students recognize both the descriptive and evidential functions of graphs (Kawakami & Saeki, 2024). He guided students through statistical problem-solving using CODAP, a tool introduced by the researcher. After the four lessons, Mr. W engaged in reflective analysis with the researcher drawing on the full video recording of the class, written descriptions from all participating students, and responses to pre- and post-lesson questionnaires.

In the research lessons, the task setting ("*Is Tochigi Prefecture [where the students live] more dangerous in terms of traffic accidents than the other prefectures in Japan?*") and data collection were teacher-led. Data such as population, area, number of fatalities, and number of cars owned in each prefecture were stored in CODAP. The teacher emphasized proposing data-based action as the ultimate goal of the DI and shared it with students ("*We call for prevention of traffic accidents in schools, local communities, and throughout Tochigi Prefecture*"). Using CODAP on laptops, the students worked individually or in pairs to organize, visualize, and analyze the data. The whole class shared a video snapshot of their classmates' exemplary analysis and data visualization. Before and after class, the students completed questionnaires on the role of statistical graphs and representative values in everyday problem-solving and the important elements when analyzing data.

Data Collection and Analysis

The analyzed data comprised transcriptions of post-lesson reflections conducted over two days. On the first day, Mr. W and the researcher spent four hours reviewing classroom video recordings of a

whole-class activity. On the second day, they spent 2.5 hours reflecting on the students' descriptions from the lessons and the pre- and post-lesson questionnaires. During these sessions, Mr. W was asked to interpret and evaluate moments he found significant while viewing the videos and reading the students' reflections, engaging in collaborative analysis with the researcher.

The analysis first identified contents related to DI practices (Table 1) from the teacher's statements. The analysis then focused on the teacher's statements that connected multiple DI practices—connections not previously observed—and identified the contents of the teacher's noticing, alongside the researcher's role in shaping that noticing, to address the RQ. The first author performed all analyses, and the other authors validated them. The differences in interpretation among the three authors were discussed, and the analysis was revised where necessary.

Selected Results and Discussion

Table 2 presents Mr. W's noticing of students' DI practices, with his statements, related DI practices, the content noticed, and the researcher's involvement with the noticing.

Table 2. Mr. W's noticing of students' data investigation (DI) practices

Mr. W's statement	DI practice	Content noticed	Researcher's involvement
I feel that my lessons became too computer focused… and that many students did not seem to understand that the aim is to use graphs to read different things from different graphs and draw conclusions or to analyze what kind of evidence can be read from the graphs. (1stD_33:43)	n. a.		
I think that Student Y saw the process of the dots coming together and changing into a columnar shape while using CODAP, and she made the comment in the lesson: "Because the dots are grouped together, a histogram is easier to understand the data distribution of the data." (1stD_2:17:18)	4.1, 4.3, and 5.1	Role of CODAP in students' understanding of graph features	Focusing on the exemplary analysis and visualization sharing scene
[Looking at Student O's description, "I enjoyed thinking about how I could explain the evidence clearly and understandably"] She is thinking about the graph as evidence, not just making and reading the graph. (2ndD_6:39)	4.1 and 6.2	Change in graphs' role in students' data visualization	Focusing on the change in students' descriptions
Reflecting on the lessons, I felt that the students developed more than I thought, and I found that [through the action-oriented task] they were able to read and analyze the graphs according to the purpose, and they were able to get a sense of the results of the classmates' visualization and analysis. (2ndD_29:00)	4.3, 5.1, and 6.1	The action-oriented task's role in promoting students' DI	Focusing on the task's purpose

Through post-lesson reflections, Mr. W noticed that students' data visualization was linked to considering models and communicating and proposing informed actions. At the beginning of day 1 of the post-lesson reflections, he felt the students had only learned how to use CODAP; he did not realize the importance of students' data visualization in DI (1stD_33:43). However, in reflecting on the lesson video, and making sense of the perspective of DI practices brought about by the researcher's involvement, Mr. W noticed CODAP's role in students' understanding of graph features:

they used CODAP to simultaneously visualize and explore data and examine the graphs' characteristics or consider models (1stD_2:17:18). In reflecting on students' descriptions, he also noticed the change in the role of graphs in students' data visualization—from descriptive to evidential (Kawakami & Saeki, 2024)—in communicating and proposing informed actions (2ndD_6:39). He also noted action-oriented tasks' role in promoting students' DI (2ndD_29:00). In these processes, the researcher also focused on key point of students' reflections from the perspective of DI practices. As reported in Hino et al. (2020), in this way Mr. W and the researcher collaborated in observing the lessons, tasks, and changes in student behavior by addressing issues from different perspectives.

This study's findings are limited to the analysis of one mid-career mathematics teacher. Nevertheless, this study suggests a professional development strategy involving researcher interaction to encourage teachers to recognize and value students' DS-related activities while reflecting on classroom implementation utilizing a data analysis and visualization tool. This has implications for realizing DS education practices in mathematics curricula while using digital technology.

References

Biehler, R., Kawakami, T., Lampen, E., Weiland, T., & Zapata-Cardona, L. (2024). *Statistics and data science education as a vehicle for empowering citizens*. Survey Team Report Presentation at the 15th International Congress on Mathematical Education (ICME-15). Sydney, Australia.

Esteley, C., Huang, R., Mellone, M., Soto, G., Eden, R., & Coles, A. (2024). Contexts, forms and outcomes of mathematics teacher collaboration. In H. Borko, & D. Potari (Eds.), *Teachers of mathematics working and learning in collaborative groups* (pp. 69–134). Springer.

Goldsmith, L. T., Doerr, H. M., & Lewis, C. C. (2014). Mathematics teachers' learning: A conceptual framework and synthesis of research. *Journal of Mathematics Teacher Education*, *17*(1), 5–36.

Hino, K., Makino, T., & Kawakami, T. (2020). Mid-career teacher learning in a mathematics professional development program: Focusing on the growth of teachers' perspectives. *Hiroshima Journal of Mathematics Education, 13*, 57–78.

Kawakami, T., & Saeki, A. (2024). Roles of mathematical and statistical models in data-driven predictions in an integrated STEM context. In J. Visnovska, E. Ross, & S. Getenet (Eds.), *Surfing the waves of mathematics education. Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia* (pp. 1–8). Gold Coast: MERGA.

Lee, H. S., Mojica, G., Thrasher, E., & Baumgartner, P. (2022). Investigating data like a data scientist: Key practices and processes. *Statistics Education Research Journal, 21*(2), Article 3, 1–23.

Makino, T., & Hino, K. (2018). A mid-career teacher's learning from students' thinking and actions: A lesson study. In F.-J., Hsieh (Ed.), *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education* (Vol. 2, pp. 114–123), Taipei, Taiwan: EARCOME.

OECD (2024). *An evolution of mathematics curriculum: Where it was, where it stands and where it is going*. OECD Publishing. https://doi.org/10.1787/0ffd89d0-en

Schreiter, S., Friedrich, A., Fuhr, H., Malone, S., Brünken, R., Kuhn, J., & Vogel, M. (2024). Teaching for statistical and data literacy in K-12 STEM education: A systematic review on teacher variables, teacher education, and impacts on classroom practice. *ZDM Mathematics Education*, *56*, 31–45.

Shin, D. (2021). Pre-service mathematics teachers' selective attention and professional knowledge-based reasoning about students' statistical thinking. *International Journal of Science and Mathematics Education*, *19*, 1037–1055.

Türker Biber, B., Yetkin Özdemir, İ. E., & Lesh, R. (2023). Teacher noticing of students' thinking in the context of mathematical modeling activities related to statistics. *International Journal of Science and Mathematics Education*, *21*, 1797–1818.

Weyers, J., König, J., Scheiner, T., Santagata, R., & Kaiser, G. (2024). Teacher noticing in mathematics education: A review of recent developments. *ZDM Mathematics Education*, *56*, 249–264.

Acknowledgement

This study was supported by a Grant-in-Aid for Scientific Research (KAKENHI) from the Japan Society for the Promotion of Science (JSPS), Grant No. JP25K00852 and a research grant from CASIO Science Promotion Foundation, Grant No. 42-45.

TSG6-2-24

USING AN AI CHATBOT TO DEVELOP PRESERVICE TEACHERS' RESPONSIVE TEACHING SKILLS

Sheunghyun Yeo
Daegu National University of Education, Korea, shyeo@dnue.ac.kr

Taekwon Son
Bongmyong Elementary School, Korea, sontaekwon7@gmail.com

Dabae Lee
Kennesaw State University, USA, dlee159@kennesaw.edu

This study explores the use of an AI-based conversational agent to enhance preservice teachers' (PSTs) responsive teaching skills in mathematics education. Using a randomized controlled design, PSTs in the experimental group (n=25) interacted with an AI chatbot simulating a student with partial understanding of fractions, while the control group (n=25) used a non-responsive chatbot. Pre- and post-assessments measured professional noticing skills—attending, interpreting, and responding. Results showed that the experimental group improved significantly, particularly in responding to student thinking. These findings suggest that AI-based simulations can effectively support PSTs' pedagogical development in a low-risk, adaptive environment.

Keywords: Responsive teaching, AI conversational agent, preservice teacher education, mathematics education, simulation-based learning, professional noticing

Introduction

One pedagogical approach that has been advocated for fostering inclusive learning environments is responsive teaching (National Council of Teachers of Mathematics, 2014). Through responsive teaching, educators elicit and attend to learners' unique ideas—whether related to demographics, conceptual understandings, or levels of proficiency—and respond in ways that cultivate richer and more productive mathematical discussions (Jacobs & Empson, 2016).

Despite its potential to transform classroom discourse, preservice teachers (PSTs) often have limited opportunities to develop responsive teaching skills prior to formal field experiences. To address this gap, our study employed an AI-based conversational agent that acts as a "virtual student," allowing PSTs to practice responsive teaching in a controlled, low-stakes environment. We subsequently examined how these interactions influenced PSTs' responsive teaching skills, particularly in relation to professional noticing of students' mathematical thinking.

Theoretical Framework

Responsive teaching has been linked to promoting students' mathematical reasoning and positive attitudes toward mathematics by encouraging educators to delve into learners' strategies and extend

their conceptual understanding (Dyer & Sherin, 2016). A key component within this pedagogical approach is the notion of *professional noticing of children's mathematical thinking*, initially conceptualized by Jacobs et al. (2010) and further expanded by Jacobs et al. (2022). Professional noticing involves three interrelated skills: Attending to children's strategies and ideas, Interpreting the underlying mathematical understanding, and Deciding how to respond in ways that support and extend learning. We adopted this framework to assess the impact of the AI-based intervention on PSTs' noticing abilities.

Simulations represent an important bridge between theoretical learning and authentic classroom practice, as they offer low-risk environments for PSTs to refine their instructional strategies (Shaughnessy & Boerst, 2018). Various simulation formats have been explored, ranging from in-person role-playing (e.g., having adults mimic children's thinking) to digital platforms such as video-based simulations (Codreanu et al., 2020) and web-based teaching simulations (Lee, 2021; Webel & Conner, 2017). These methods enable PSTs to engage in mathematical discourse without the pressures and complexities associated with real classroom settings.

However, designing these simulations—whether in-person or computer-based—presents certain limitations. In-person simulations require substantial human and material resources, as adults must be trained to authentically replicate children's mathematical thinking (Shaughnessy & Boerst, 2018). Meanwhile, digital platforms like LessonSketch often rely on predetermined decision points that provide limited response options. Real classroom contexts, by contrast, involve open-ended teaching moves with numerous pedagogical contingencies (Rowland et al., 2005). In an effort to address these constraints, we integrated AI technology to deliver personalized feedback and more realistic interaction sequences, thus enhancing both the authenticity and adaptability of the simulation.

Among AI-based educational tools, intelligent tutoring systems (ITS) have garnered significant attention for their capacity to offer individualized instruction through knowledge assessment and targeted feedback (Feng & Law, 2021; Graesser et al., 2001). However, learners often assume a relatively passive role in these environments, receiving information rather than actively shaping the instructional dialogue. AI-based chatbots, in contrast, can create more dynamic and interactive learning experiences (Tamayo et al., 2020). They have been developed for diverse purposes, such as gathering data on bullying (Oh et al., 2020), supporting foreign language practice (Fryer et al., 2019), and improving literacy skills (Xu et al., 2021). Although educational chatbots are gaining momentum, empirical investigations into their effective classroom integration remain limited (Hwang & Chang, 2021).

In mathematics education, Nguyen et al. (2019) introduced a chatbot for high school mathematics learning, and Fang et al. (2021) designed an intelligent tutoring chatbot that delivered personalized feedback and supplementary practice. While these tools allowed learners to interact with virtual agents, the learners' role was primarily to receive explanations rather than to engage in deeper, inquiry-driven dialogue.

Methodology

Our experimental study examined whether an AI-based chatbot, designed to function as a "virtual student," could enhance PSTs' noticing abilities—namely, their capacity to attend to, interpret, and respond to student thinking. We employed a randomized controlled design with PSTs (n=50) enrolled in a mathematics methods course at a southern university. Participants were randomly assigned to four sections (each with a different instructor) that utilized the same instructional materials covering fraction concepts and professional noticing. Two sections (n=25) were designated as the experimental group and interacted with a chatbot simulating a student who demonstrated partial understanding of fraction addition. PSTs posed questions to explore the virtual student's misconceptions, leveraging the pedagogical knowledge they had acquired. The chatbot responded in real time to their queries, thus approximating an authentic teacher–student interaction. In contrast, the control group (n=25) also engaged with a chatbot, but it merely prompted them to ask questions without providing any responses.

Both groups completed pre- and post-assessments designed to measure changes in their noticing abilities (attending, interpreting, and responding). PSTs viewed a 10-minute video of a fractions lesson and were prompted to describe: What they noticed in the student thinking (attending), How they interpreted the student's understanding (interpreting), and How they would respond to extend or support the student's ideas (responding). Responses were scored using Jacobs et al.'s (2010) rubric: 3 = robust evidence, 2 = limited evidence, 1 = lack of evidence, 0 = no response. First two authors independently coded the data to ensure inter-rater reliability, resolving any discrepancies through discussion. We employed independent samples *t*-tests to compare experimental and control groups, and paired samples *t*-tests to assess pre- to post-test gains.

Results

Table 1 presents the *t*-test findings. There was no significant difference in pre-test scores between the experimental and control groups ($t = 1.809$, $p = .077$), suggesting that both groups started with comparable levels of noticing expertise. At post-test, scores improved in both groups, but the experimental group (M = 5.72, SD = 1.792) significantly outperformed the control group (M = 3.78, SD = 1.563).

Table 1. t-test result in pre- and post-test of noticing expertise between the control group and the experiment group

Group	Test	M	SD	df	t	*p*
Control (N=25)	Pre-test	3.76	.723	24	-.405	.689
	Post-test	3.88	1.563			
Experiment (N=25)	Pre-test	4.28	1.242	24	-3.990	.001*
	Post-test	5.72	1.792			

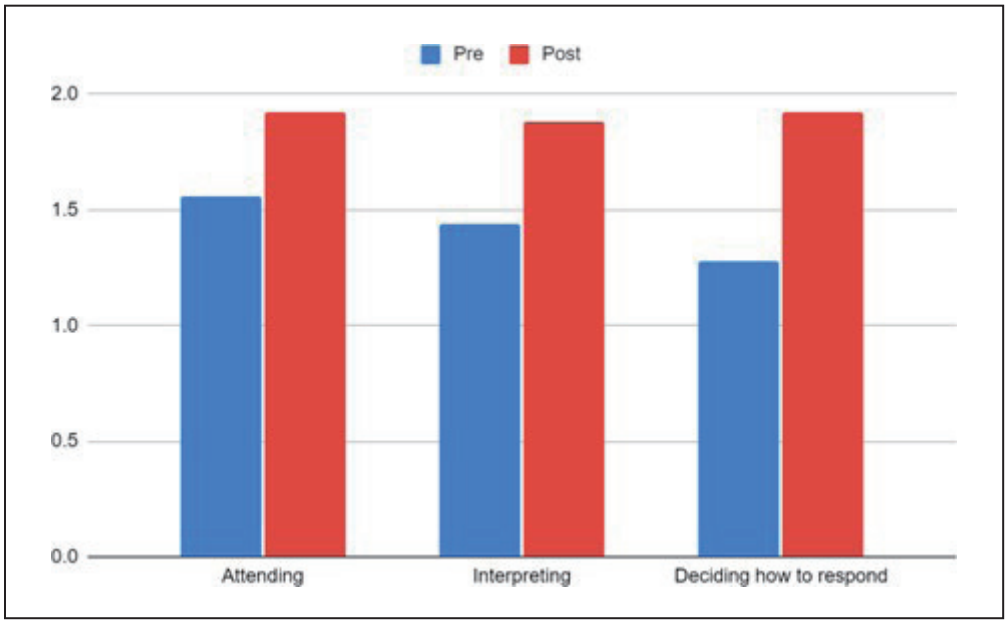


Figure 1. The changes in the means of pre- and post-test scores of the three noticing components for the experiment group

Moreover, only the experimental group exhibited a statistically significant improvement from pre- to post-test. A more detailed analysis of the experimental group's gains revealed increases across all three dimensions of noticing (attending, interpreting, responding), with the greatest gains observed in deciding how to respond to student thinking.

Implications

These results underscore the promise of AI-based conversational agents as tools for developing PSTs' responsive teaching skills. By leveraging natural language processing and machine learning algorithms, these chatbots can adapt to user inputs, thereby offering both authentic practice opportunities and personalized feedback. In contrast to traditional simulations—where PSTs may be constrained by predefined decision trees or the need for trained adults to role-play—AI-driven simulations allow open-ended interactions that more closely mirror real classroom scenarios.

Our findings indicate that practicing with an AI "virtual student" may not only bolster PSTs' abilities to notice and interpret student thinking but also foster skillful, adaptive responses that support and extend mathematical understanding. These improvements have the potential to create more inclusive classroom environments in which diverse student ideas are acknowledged and leveraged as assets. Ultimately, this study contributes to the growing body of evidence that AI-based simulations can serve as a powerful, low-stakes means of preparing future educators to navigate the complexities of responsive teaching before they enter real-world classroom settings.

References

Codreanu, E., Sommerhoff, D., Huber, S., Ufer, S., & Seidel, T. (2020). Between authenticity and cognitive demand: Finding a balance in designing a video-based simulation in the context of mathematics teacher education. *Teaching and Teacher Education*, *95*, 103146.

Dyer, E. B., & Sherin, M. G. (2016). Instructional reasoning about interpretations of student thinking that supports responsive teaching in secondary mathematics. *ZDM*, *48*(1-2), 69-82.

Fang, Y., Lippert, A., Cai, Z., Chen, S., Frijters, J. C., Greenberg, D., & Graesser, A. C. (2021). Patterns of adults with low literacy skills interacting with an intelligent tutoring system. *International Journal of Artificial Intelligence in Education*.

Feng, S., & Law, N. (2021). Mapping artificial intelligence in education research: A network- based keyword analysis. *International Journal of Artificial Intelligence in Education*.

Fryer, L. K., Nakao, K., & Thompson, A. (2019). Chatbot learning partners: Connecting learning experiences, interest and competence. *Computers in Human Behavior*, *93*, 279-289.

Graesser, A. C., VanLehn, K., Rose, C. P., Jordan, P. W., & Harter, D. (2001). Intelligent tutoring systems with conversational dialogue. *AI Magazine*, *22*(4), 39.

Hwang, G., & Chang, C. (2021). A review of opportunities and challenges of chatbots in education. *Interactive Learning Environments*, 1-14.

Jacobs, V. R., & Empson, S. B. (2016). Responding to children's mathematical thinking in the moment: An emerging framework of teaching moves. *ZDM*, *48*(1-2), 185-197.

Jacobs, V. R., Empson, S. B., Jessup, N. A., Dunning, A., Pynes, D. A., Krause, G., & Franke,

T. M. (2022). Profiles of teachers' expertise in professional noticing of children's mathematical thinking. *Journal of Mathematics Teacher Education*.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education JRME*, *41*(2), 169-202.

Lee, M. Y. (2021). Improving preservice teachers' noticing skills through technology-aided interventions in mathematics pedagogy courses. *Teaching and Teacher Education*, *101*, 103301.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*.

Nguyen, H. D., Pham, V. T., Tran, D. A., & Le, T. T. (2019). Intelligent tutoring chatbot for solving mathematical problems in High-school. 11th International Conference on Knowledge and Systems Engineering (KSE), Da Nang, Vietnam.

Oh, E., Song, D., & Hong, H. (2020). Interactive computing technology in anti-bullying education: The effects of conversation-bot's role on K-12 students' attitude change toward bullying problems. *Journal of Educational Computing Research*, *58*(1), 200-219.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge Quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*(3), 255-281.

Shaughnessy, M., & Boerst, T. A. (2018). Uncovering the skills that preservice teachers bring to beacher education: The practice of eliciting a student's thinking. *Journal of Teacher Education*, *69*(1), 40-55.

TSG6-2-25

INVESTIGATING THE PEDAGOGICAL VALUE OF DIGITAL MANIPULATIVES BETWEEN US AND KOREA

Sheunghyun Yeo
Daegu National University of Education, Korea, shyeo@dnue.ac.kr

Mi Yeon Lee
Arizona State University, USA, mlee115@asu.edu

Ji Nam Hwang
Yongjeong Elementary School, Korea, whiyoung10@naver.com

Digital manipulatives such as FracTopia are increasingly being integrated into mathematics classrooms to enhance students' conceptual understanding of fractions. This study compares the experiences of preservice teachers (PSTs) from the United States and Korea regarding pedagogical value when using FracTopia for solving fraction operation problems. In our study of US (n = 42) and Korean (n = 61) preservice teachers, many praised its clear visual models (47.6% US; 37.7% Korea) and diverse modeling options (23.8% US; 34.4% Korea), along with a user-friendly, click-based interface (11.9% US; 19.7% Korea). However, respondents also reported challenges—including technical glitches, complex navigation, limited model functionality, and device constraints—that hinder its effective use.

Keywords: Preservice teachers, pedagogical value, dynamic digital tool

Introduction

Teachers are crucial in effectively integrating technology into education. However, they frequently exhibit resistance to adopting educational technologies, often due to concerns about their pedagogical value (Kennedy-Clark, 2011). In recent years, digital tools have played an increasingly vital role in mathematics education by providing dynamic, interactive environments that support conceptual understanding. Among these, FracTopia—a digital manipulative for fraction concepts and operations—has gained attention for its potential to visually represent abstract fraction concepts. In particular, fraction multiplication and division are traditionally challenging for students and teachers because they require an understanding of part-whole relationships as well as other constructs and the underlying principles of division and multiplication. FracTopia aims to bridge this gap by allowing learners to manipulate fraction models (e.g., circles, rectangles, bars, and number lines) through simple clicks and gestures.

The purpose of this study is to investigate the pedagogical value of FracTopia from the perspective of preservice teachers (PSTs) in two distinct national contexts—the US and Korea. In this study, the pedagogical value is defined as the capacity of teachers to evaluate technological tools in terms of instructional affordances and limitations (Hazari et al., 2009). By analyzing PSTs' open-ended responses to questions about the advantages and difficulties encountered while using FracTopia, this research seeks to identify the tool's strengths and limitations. Ultimately, the study aims to offer

recommendations for teacher education programs and tool developers so that digital manipulatives may better support effective mathematics instruction.

Theoretical Background

The integration of educational technology into mathematics instruction has been guided by theoretical models such as the Technology Acceptance Model (TAM), which emphasizes perceived ease of use and perceived usefulness as critical factors influencing technology adoption (Davis et al., 1989). In mathematics education, digital manipulatives have been credited with transforming abstract concepts into tangible experiences. Previous research has shown that visual representations can significantly enhance students' comprehension of fractions by allowing them to see relationships between parts and wholes (Lee & Lee, 2023; Son & Lee, 2016).

FracTopia builds on this tradition by offering a range of models that provide multiple avenues for exploring fraction operations. Its interactive features—including immediate visual feedback and the ability to modify fraction models dynamically—are designed to engage learners and foster a deeper conceptual understanding. However, as with many digital tools, FracTopia also presents challenges, such as technical glitches and interface complexities that can hinder effective instruction if not adequately addressed. Thus, examining both the benefits and the obstacles of using FracTopia is essential for understanding its overall pedagogical value.

Methodology

We recruited 105 PSTs (US: 61, Korea: 44). In the US sample, PSTs were asked two open-ended questions regarding their experiences with FracTopia: Based on your experience using FracTopia to solve fraction operation problems, what do you think are the advantages of FracTopia?, Based on your experience using FracTopia to solve fraction operation problems, what were the difficulties you encountered while using FracTopia?, A parallel set of questions was posed to Korean PSTs in their native language. The responses were then translated (for the purposes of analysis) and subjected to qualitative content analysis.

Using an inductive content analysis approach, the research team coded the open-ended responses to extract recurring themes. Two major clusters emerged: Advantages (Pedagogical Strengths), Difficulties (Obstacles and Limitations). Within each cluster, several categories were identified along with representative examples and estimated response percentages for each country.

Findings and Discussion

Table 1 summarizes the advantages of FracTopia as reported by PSTs. The content analysis indicates that the foremost advantage of FracTopia lies in its ability to provide clear visual representations of fraction concepts. In the US sample, 20 out of 42 respondents (47.6%) highlighted that intuitive models—such as fraction circles, bars, and rectangles—make it easier to understand abstract fraction operations. In the Korean sample, 23 out of 61 respondents (37.7%) emphasized this visual clarity, noting that such models reduce reliance on abstract symbolic representations and help students grasp the underlying part-whole relationships in fractions.

In addition, the provision of multiple models and options was frequently mentioned. In the US, 10 respondents (23.8%) noted that the availability of various models (e.g., circles, rectangles, number lines) allows for multiple approaches to solving fraction problems, thereby catering to diverse

learning styles. Korean PSTs reported this advantage even more prominently, with 21 respondents (34.4%) highlighting that the flexibility in choosing different models enables teachers to tailor instructional strategies to suit individual learner profiles.

Ease of use and accessibility is another important advantage. Although fewer US respondents (5 respondents; 11.9%) mentioned this aspect, they appreciated the click-based manipulation that simplifies model creation and reduces the time required for setup. Similarly, 12 Korean PSTs (19.7%) noted that the user-friendly interface facilitates quick access to the tool's functions, despite some initial learning curve challenges.

Furthermore, a subset of respondents valued the tool's capacity to enhance conceptual understanding. Specifically, 4 US PSTs (9.5%) and 3 Korean PSTs (4.9%) indicated that FracTopia does more than merely provide solutions—it also helps reveal the underlying reasoning behind fraction operations, shifting the focus from rote computation to deeper conceptual learning. Lastly, engagement and interactivity were mentioned by 3 US respondents (7.1%) and 2 Korean respondents (3.3%), with users noting that immediate feedback and interactive features help sustain student interest and support iterative learning.

Table 1. Advantages of FracTopia as reported by PSTs

Category	Representative Examples	US (%)	Korea (%)
Visual Representation	"FracTopia offers clear visual models—such as fraction circles, bars, and rectangles—that help in understanding the concept and process of fraction operations at a glance."	20 (47.6)	23 (37.7)
Multiple Models & Options	"The platform provides various models (circles, rectangles, number lines) which allow for multiple approaches to solving problems and catering to different learning styles."	10 (23.8)	21 (34.4)
Ease of Use & Accessibility	"Its click-based manipulation makes it intuitively accessible for both teachers and students, reducing the time needed for setup and explanation."	5 (11.9)	12 (19.7)
Enhancement of Conceptual Understanding	"FracTopia visualizes the inherent principles of fraction operations, going beyond rote computation to promote deeper conceptual learning."	4 (9.5)	3 (4.9)
Engagement & Interactivity	"The immediate feedback and interactive features help sustain student engagement and motivate visual learners."	3 (7.1)	2 (3.3)

Table 2 presents the difficulties encountered when using FracTopia. Technical issues were the most frequently reported obstacle in the US sample, with 17 respondents (40.5%) describing frequent lag, crashes, and even work being erased. In the Korean sample, technical issues were mentioned by 15 respondents (24.6%). However, interface and navigation challenges were a particularly prominent concern among Korean PSTs, with 25 respondents (41.0%) reporting difficulties in navigating through the various tools and functions, compared to 13 US respondents (31.0%).

Both groups noted limitations in model functionality: 8 US PSTs (19.0%) and 12 Korean PSTs (19.7%) expressed concerns that certain models could not clearly represent improper fractions or mixed numbers, and that adjustments to model size and partitioning were often limited. Lastly, device and hardware constraints, along with the associated cognitive load, were mentioned as challenges by 4 US respondents (9.5%) and 9 Korean respondents (14.8%). These respondents noted that using FracTopia on non-optimal devices—such as a computer without a touch interface—significantly impaired the user experience and added extra cognitive effort during problem solving.

Table 2. Difficulties encountered when using FracTopia

Category	Representative Examples	US (%)	Korea (%)
Technical Issues	"The website frequently lags, crashes, or even erases work, making it difficult to maintain a smooth workflow during problem-solving."	17 (40.5)	15 (24.6)
Interface & Navigation Challenges	"Navigating through the various tools and functions (such as the drawing tool) is confusing, and it takes time to learn how to effectively manipulate the digital objects."	13 (31.0)	25 (41.0)
Model Limitations	"Some models do not allow for the clear representation of improper fractions or mixed numbers, and adjustments in model size or granularity are often limited."	8 (19.0)	12 (19.7)
Device/Hardware Constraints & Cognitive Load	"Using the tool on a computer with a mouse is less intuitive than on a touch-screen device, and the multiple steps required for operations add to the cognitive load, especially when trying to demonstrate automatic processes visually."	4 (9.5)	9 (14.8)

Conclusion

This comparative study examined the pedagogical value of FracTopia by analyzing the experiences of US and Korean preservice teachers. The findings indicate that while FracTopia offers significant advantages—especially in terms of visual representation, multiple modeling options, and the enhancement of conceptual understanding—its potential is partly undermined by technical and interface-related challenges. These obstacles are particularly pronounced in contexts where digital fluency is variable or where hardware constraints are prevalent.

The study underscores the need for teacher education programs to incorporate comprehensive training on digital manipulatives and to advocate for the continued development of more intuitive, reliable educational technologies. In doing so, future teachers will be better equipped to leverage tools like FracTopia to facilitate deeper understanding of abstract mathematical concepts, thereby contributing to improved student outcomes.

Furthermore, the comparative differences observed between US and Korean PSTs highlight the importance of contextual factors—such as national curriculum standards, available technological infrastructure, and cultural expectations—in shaping the perception and effectiveness of digital in-

structional tools. Future research should further explore these dimensions and investigate long-term impacts on classroom practice.

Ultimately, the study calls for a concerted effort among educators, policymakers, and technology developers to harness the pedagogical strengths of digital manipulatives while mitigating their limitations. By aligning technological innovations with sound pedagogical practices, educational stakeholders can pave the way for more effective, engaging, and inclusive mathematics instruction in diverse learning environments.

References

Davis, F. D., Bagozzi, R. P., & Warshaw, P. R. (1989). User acceptance of computer technology: A comparison of two theoretical models. *Management Science*, *35*(8), 982-1003.

Hackenberg, A. J., & Lee, M. Y. (2015). Relationships between students' fractional knowledge and equation writing. *Journal for Research in Mathematics Education, 46*(2), 196-243.

Hazari, S., North, A., & Moreland, D. (2009). Investigating pedagogical value of wiki technology. *Journal of Information Systems Education*, *20*(2), 187-198.

Kennedy-Clark, S. (2011). Pre-service teachers' perspectives on using scenario-based virtual worlds in science education. *Computers & Education*, *57*(4), 2224-2235.

Lee, M. Y., & Lee, J. (2023 online first). The relationship between preservice elementary teachers' solutions to a pattern generalization problem and difficulties they anticipate in teaching it. *Journal of Mathematical Behavior*, *69*.

Son, J., & Lee, M. Y. (2021). Exploring the relationship between preservice teachers' conceptions of problem solving and their problem-solving performance. *International Journal of Science and Mathematics Education*,*19*(1), 129-150.

Yeo, S., & Webel, C. (2024). Elementary students' fraction reasoning: a measurement approach to fractions in a dynamic environment. *Mathematical Thinking and Learning, 26*(1), 20-46.

MEMO

MEMO

KOREA 2025

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 7:

The Essence of Mathematics Education in Affective and Emotional Aspects

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 7
(Day 2)

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

TSG7-01

PROFILE ANALYSIS USING CLUSTERING TO EXPLORE STUDENTS' MATHEMATICS GRIT

Yuan-Horng Lin
National Taichung University of Education, Taiwan, lyh@mail.ntcu.edu.tw

This study aims to develop mathematics grit scale for primary school students and adopts two-step cluster analysis to show the profile analysis for students' features of mathematics grit. The subjects are students from third graders to sixth graders in Taiwan. The mathematics grit scale is five-points Likert scale. It consists of 8 items which measure two factors of mathematics grit. The two factors are consistency of interest (CI) and perseverance of effort (PE). The Cronbach α of mathematics grit scale is 0.815 and the reliability is acceptable. Exploratory factor analysis (EFA) reveals that the data fit well in terms of the two factors. Two-step cluster analysis is based on the two factor scores. Model selections show five clusters based on the criteria of AIC and BIC. From cluster I to cluster V, their features are as follows. (1) high-CI, high-PE (2) low-CI, high-PE (3) medium-CI, medium-PE (4) low-CI, low-PE (5) high-CI, low-PE. Profile analysis is based on MANOVA (multivariate of Analysis of Variance). One Way MANOVA of profile analysis based on cluster shows significant differences in the two factor scores among the five clusters. Two way MANOVA of profile analysis based on cluster and gender shows that different clusters exhibit significant differences in the two factor scores, while different genders show no significant differences in the two factor scores. However, there is no interaction effect between cluster and gender. Two way MANOVA of profile analysis based on cluster and grade shows that different clusters exhibit significant differences in the two factor scores, while different grades show significant differences only in the PE factor scores. However, there is no interaction effect between cluster and grade. This study reveals the profiles of students' mathematics grit and could provide references for mathematics education. Based on the research findings, this paper provides some recommendations for incorporating mathematics grit into mathematics education research.

Keywords: grit, mathematics grit, profile analysis, two-step cluster analysis

Introduction and Purpose

Positive Psychology is a branch of psychology that focuses on studying how to enhance human well-being, mental health, and life satisfaction. Grit is one of the important psychological dimensions discussed by positive psychology. Since A. Duckworth put forward the viewpoint of grit, it has been extensively discussed in various fields in recent years (Duckworth & Quinn, 2009). Duckworth, Peterson, Matthews, and Kelly (2007) pointed out that grit consists of two components: consistency of interest (CI) and persistence of effort (PE). CI refers to an individual's ability to maintain a sustained focus and passion for a particular goal or area of interest over an extended period, even in the face of challenges or distractions. PE refers to an individual's ability to remain determined, motivated, and hardworking toward achieving long-term goals, even in the face of difficulties, setbacks, or failures. An increasing number of studies point out that grit is a domain-general and important latent trait (Eskreis-Winkler, Shulman, Beal, & Duckworth, 2014). Numerous studies have shown that grit deeply affects individual's performance in learning. Grit is a part of psychological affect. Relevant studies have pointed out that grit has a great relationship with subject learning performance (Kaya & Karakoc, 2022). Therefore, mathematics grit may also consist of two factors:

consistency of interest (CI) and persistence of effort (PE), both of which can influence mathematics learning.

Grit is a part of mathematics affect. Students with high grit perform better mathematics learning. And mathematics is often one of the most difficult subjects for students. However, as far as children's mathematics grit is concerned, there are few Chinese versions of the mathematics grit scale suitable for elementary school students in the literature. In addition, studies conducting profile analysis on mathematics grit are also scarce. The profile analysis of each cluster according to different backgrounds is a relatively new topic. Moreover, students with different background variables may have differences in their performance in mathematics grit. Therefore, this study develops the mathematics grit scale and verifies the instrumental quality. Secondly, two-step cluster is used to explore latent groups. In addition, the differences of factor scores according to different genders and grades are discussed so as to present the characteristics of profiles analysis.

Research Design

According to the theoretical basis of A. Duckworth, Grit includes two factors, which are CI and PE. This study references Short Grit Scale (GRIT–S) (Duckworth & Quinn, 2009). This study designs the items in the rating scale based on mathematics learning situations. The mathematics grit scale adopts the five-point Likert scale, with 4 items each for CI and PE. For example, item 1 is "I decided to study math, but then changed my goal." Sample of students based on grade and gender is shown in Table 1, with a total of 1151 students. Each factor contains 4 items and there are totally 8 items. This study adopts exploratory factor analysis. Based on the criterion that the eigen value is greater than 1, it shows that there are two factors, and the cumulative explanation amount reaches 61.10%. The Cronbach α of CI and PE are 0.749 and 0.803 respectively. The overall Cronbach α is 0.815. The sample for this study is drawn from elementary school students in Taiwan.

Table 1. Sample of students based on grade and gender

Gender	Grade				Total
	3	4	5	6	
male	146 (12.7%)	166(14.4%)	126(10.9%)	147(12.8%)	585(50.8%)
female	128(11.1%)	128(11.1%)	146(12.7%)	164(14.2%)	566(49.2%)
Total	274(23.8%)	294(25.5%)	272(23.6%)	311(27.0%)	1151(100%)

Results and Discussions

This study adopts two-step cluster analysis to perform clustering based on the factor scores of CI and PE. Model selection is conducted using the AIC and BIC indices. The number of clusters is determined using the elbow point principle of the scree plot (Yu, Hua, Feng, Wang, Yu, Zi, Zhao, & Li, 2021). As indicated in Table 2, when the number of clusters is 5, the differences in AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) no longer change significantly. Therefore, the optimal number of clusters is 5.

Table 2. Model selection based on AIC and BIC

Cluster	AIC	Difference in AIC	BIC	Difference in BIC
1	1602.625		1622.818	
2	1210.794	-391.830	1251.181	-371.637
3	977.008	-233.786	1037.589	-213.592
4	796.685	-180.323	877.459	-160.130
5	644.813	-151.872	745.781	-131.679
6	587.077	-57.736	708.238	-37.543
7	539.424	-47.653	680.779	-27.460
8	492.253	-47.171	653.802	-26.977
9	457.094	-35.159	638.836	-14.965
10	424.740	-32.354	626.676	-12.161

Table 3 presents the number of students in each of the 5 clusters, the name of each cluster, and the cluster centers. The number of students in each cluster ranges from 14.9% to 28.5%. Using factor scores of 0.5 and -0.5 as cut-points, CI and PE can be divided into three levels: high, medium, and low. Each cluster exhibits distinct characteristics in terms of CI and PE factor scores. For example, students in Cluster II are characterized by low CI but high PE.

Table 3. Descriptive analysis of clusters and cluster centers

Cluster	Number of students	Name of cluster	Cluster centers (factor scores)	
			CI	PE
Cluster I	232 (20.2%)	high-CI, high-PE	1.0742148	.8909546
Cluster II	172(14.9%)	low-CI, high-PE	-.7836131	1.1918250
Cluster III	328(28.5%)	medium-CI, medium-PE	.0033016	-.0529961
Cluster IV	191(16.6%)	low-CI, low-PE	-1.3315659	-.6019922
Cluster V	228(19.8%)	high-CI, low-PE.	.6088149	-1.2251410

This study adopts MANOVA (multivariate analysis of variance) statistical method to conduct profile analysis, with the dependent variables being the factor scores of CI and PE. Table 4 presents the differences among the 5 clusters in terms of CI and PE. The results show that for the CI factor score, the order is Cluster I > Cluster V > Cluster III > Cluster II > Cluster IV. However, for the PE factor score, the order is Cluster II > Cluster I > Cluster III > Cluster IV > Cluster V.

Table 4. One way MANOVA of profile analysis based on cluster

Multivariate mean test (Wilk)	Univariate F test and Post Hoc	
	CI	PE
.080***	645.53*** I >V > III > II > IV	779.21*** II > I > III> IV>V

***p<.001

Using cluster and gender as factors, the two-way MANOVA in Table 5 indicates that only the cluster has statistical significance. For the CI factor score, the order is Cluster I > Cluster V > Cluster III > Cluster II > Cluster IV, whereas for the PE factor score, the order is Cluster II > Cluster I > Cluster III > Cluster IV > Cluster V.

Table 5. Two way MANOVA of profile analysis based on cluster × gender

Variables	Multivariate mean test (Wilk)	Univariate F test and Post Hoc	
		CI	PE
Cluster	0.081***	643.60*** I >V > III > II > IV	770.69*** II > I > III> IV>V
Gender	0.997		
Cluster × Gender	0.992		

***p<.001

Using cluster and grade as factors, the two way MANOVA in Table 6 indicates that both cluster and grade have statistically significant effects, but there is no interaction between cluster and grade. For the CI factor score, the order is Cluster I > Cluster V > Cluster III > Cluster II > Cluster IV, whereas for the PE factor score, the order is Cluster II > Cluster I > Cluster III > Cluster IV > Cluster V. Regarding grade, only the PE factor score shows statistically significant differences, with Grade 3, 4, 6 > Grade 5. The differences in CI and PE among students of different grades, as well as the underlying reasons, can be explored in greater depth in future research.

Table 6. Two way MANOVA of profile analysis based on cluster × grade

Variables	Multivariate mean test (Wilk)	Univariate F test and Post Hoc	
		CI	PE
Cluster	0.093***	706.96*** I >V > III > II > IV	754.39*** II > I > III> IV>V
Grade	0.990*	0.74	2.90* Grade 3,4,6 > Grade 5
Cluster × Grade	0.992		

*p<.05 ***p<.001

Conclusions

This study develops Chinese version of the mathematics grit scale suitable for elementary school students. The scale demonstrates reliability and validity. Using CI and PE to perform the two step cluster analysis, it is found that students of the five clusters exhibit distinct characteristics in mathematics grit, which can serve as references for mathematics instruction. No significant differences is observed in the mathematics grit scale between genders, but significant differences is found among students of different grades. The underlying reasons for these differences should be further explored in future research.

References

Duckworth, A. L., & Quinn, P. D. (2009). Development and validation of the Short Grit Scale (GRIT–S). *Journal of Personality Assessment*, *91*, 166-174.

Duckworth, A. L., Peterson, C., Matthews, M. D., & Kelly, D. R. (2007). Grit: perseverance and passion for long-term goals. *Journal of Personality and Social Psychology*, *92*, 1087-1101.

Eskreis-Winkler, L., Shulman, E. P., Beal, S. A., & Duckworth, A. L. (2014). The grit effect: Predicting retention in the military, the workplace, school and marriage. *Frontiers in Psychology*, *5*, Article 36.

Kaya, S., & Karakoc, D. (2022). Math mindsets and academic grit: How are they related to primary math achievement?. *European Journal of Science and Mathematics Education*, *10*, 298-309.

Yu, Y., Hua, L., Feng, X., Wang, Y., Yu, Z., Zi, T., Zhao, Y,, & Li, J. (2021). True grit in learning math: The math anxiety-achievement link is mediated by math-specific grit. *Frontiers in Psychology*, *12*, Article 645793.

TSG7-02

EXPLORING THE RELATIONSHIP BETWEEN STUDENTS' PERCEPTIONS OF DOING WELL IN MATHEMATICS, MATHEMATICS CONFIDENCE AND GROWTH MINDSETS

Anni E
The University of Melbourne, Australia, eae@student.unimelb.edu.au

Yu Shan Ting
The University of Melbourne, Australia, pnm40275@gmail.com

Wee Tiong Seah
The University of Melbourne, Australia, wt.seah@unimelb.edu.au

Students develop ingrained perceptions of teaching and learning through years of schooling, shaped by educational systems. This study examines how Taiwan's high-stakes testing influences students' views on success in mathematics and its links to confidence and growth mindset. A mixed-methods analysis reveals that most equate success with high scores, reflecting a test-driven mindset. Students' responses also reflect their disposition for honing precise exam skills and a pragmatic approach to learning. However, students with non-score-oriented views report greater mathematics confidence and growth mindset. These findings underscore the need to promote diverse conceptions of success, fostering confidence and sustainable learning beyond test performance.

Keywords: academic success, mathematics confidence, growth mindset, high-stakes testing, Taiwan education system

Introduction

Given the critical role of mathematics in developing competencies such as problem-solving skills (Yang et al., 2022; Zhang et al., 2016), significant efforts have been made globally to uncover the "secret recipe" for doing well in mathematics. Despite introducing pedagogies designed to enhance students' mathematics learning, transformative classroom changes remain elusive. Nolan (2012) explains that the cultural routines and patterns associated with schools, teaching, and learning are deeply ingrained from a young age, making them highly resistant to transformation. Consequently, this research critically focuses on uncovering the patterns surrounding students' perception of doing well in mathematics within a specific context.

In many educational settings, academic achievement is often measured through standardised assessments, shaping how students define and approach mathematics learning. As Gorur (2016) observes, these high-stakes tests have evolved into an isolated world of their own. Specifically, acting as a powerful structuring force, this system profoundly shapes students, requiring high-achievers in Taiwan to adapt to its testing methods, influencing their learning dispositions (Wang & Tai, 2024). Drawing on Bourdieu's (1990) concept of the social field, such environments encourage students to internalise "appropriate" behaviours and practices that, over time, evolve into a natural disposition,

or habitus, enabling them to navigate and succeed within school settings. Given the educational system in Taiwan, which is characterised by high-stakes tests, this study critically examines how students perceive doing well in mathematics within this context.

Nolan (2012) cautions that when the subtext of a practice is "that's how it's done" (p. 205), such practices become binding and imposed upon individuals. Wang and Tai (2024) highlight that Taiwan's once-admired academic rigour now leads to student dissatisfaction and resistance. Recently, Stahl et al. (2024) implicitly argues that examination-driven habitus may contribute to social suffering, including anxiety and low confidence. Building on these insights, this study examines two key psychological indicators in mathematics education: mathematics confidence, referring to students' self-perceived ability to perform well in mathematics (Mullis et al., 2020); and growth mindset, the belief that mathematical ability develops through effort (Dweck, 2008). By exploring the relationships between these indicators and students' perceptions of "doing well", this study aims to reveal the prolonged structural effects that high-stakes educational practices impose on individuals.

This study is guided by two primary research questions: (a) What are the perceptions of students in Taiwan regarding "doing well" in mathematics? (b) What is the relationship between students' perceptions of "doing well" in mathematics and their mathematics confidence and growth mindset?

Methodology

The sample in this study included 215 (118 boys, 97 girls) sixth-grade students from urban and rural schools in Taiwan. It employed two inventories and one open-ended question within a broader survey on students' mathematics perceptions (see https://thirdwavelab.education.unimelb.edu.au/study-12-valuing-in-confidence-vic/ for survey). Additional survey items are beyond the scope of this study.

The open-ended question, "What does it mean to be doing well at mathematics?", assessed students' perceptions of "doing well". Two inventories used a 4-point Likert scale (1 = strongly disagree, 4 = strongly agree): The Inventory of Mathematics Confidence (IMC), derived from TIMSS (2019), was employed to assess participants' confidence in mathematics. The IMC consists of three items, with a Cronbach's alpha of .87. An example item is: "I perform well in mathematics". The Inventory of Mathematics Growth Mindsets (IMG) was developed in this study to assess students' beliefs about the malleability of their mathematical abilities. In addition to traditional growth mindset items, IMG comprises four items with a Cronbach's alpha of .79. An example item is: "Even if I don't enjoy mathematics, I believe I should still put in effort to improve".

This study employed a mixed-methods approach to address the research questions. Thematic analysis identified key themes in students' descriptions of "doing well" in mathematics. Given that success is often associated with high scores, responses were categorised as Score-Oriented or Non-Score-Oriented (focusing on broader mathematical competencies). A one-way analysis of variance (ANOVA) was then used to examine differences in mathematics confidence and growth mindset between these groups.

Results

Regrading descriptions of doing well in mathematics, the most frequently cited (56.74%, or 122 students) was achieving a high score, with 72 students specifying exact scores. Students' perceptions were segmented into increments of 10 points, reflecting a step-like progression, where they positioned themselves within a "grading hierarchy" based on academic performance. Notably, one

student uniquely described doing well in mathematics as being among the top 5 students in their class, emphasising relative performance over fixed numerical benchmarks.

The second most common theme, expressed by 34 students, framed "doing well" as a reflection of personal traits or abilities. Among them, 4 students equated mathematical success with being intelligent, while the remaining 30 responses focused on comprehension skills. Examples include phrases such as "being able to clearly understand and explain to classmates". 17 students detailed how comprehension skills are applied, with 7 highlighting conceptual understanding, 3 emphasising formula comprehension, and 6 focusing on problem prompt comprehension.

29 students' answers could be categorised under mathematics problem-solving. However, these responses indicate that students primarily viewed it in terms of getting correct answers, focusing on accuracy, efficiency, and the ability to tackle challenging problems within the context of examinations. Specifically, students defined "doing well" as instant recognition of answers during exams ("I see a question and immediately know the answer"). Some emphasised problem difficulty, while others highlighted efficiency ("I calculate quickly and get the answers right"), reinforcing a performance-driven perspective.

A smaller group of students associated success with effort and attention. 13 students described "doing well" as the result of effort ("Mathematical success comes from hard work"). 5 students equated mathematical success with being careful and meticulous, emphasising traits such as "avoiding carelessness" or "not making careless mistakes during exams.

Perceptions of 'doing well' in mathematics were significantly associated with confidence and mindset. For mathematics confidence, the results showed a significant difference, $F(1, 213) = 5.36$, $p = .022$, $\eta^2p = .025$. Students with a non-socre-oriented view reported significantly higher confidence ($M = 2.57$, $SE = .10$) compared to those with a score -oriented view ($M = 2.28$, $SE = .08$). Regarding growth mindsets, the results indicated a significant difference, $F(1, 213) = 9.51$, $p = .002$, $\eta^2p = .043$. Students with a non-score-oriented view reported significantly higher growth mindset scores ($M = 3.23$, $SE = .07$) compared to those with a score-oriented view ($M = 2.92$, $SE = .07$).

Discussion and Implication

This study reveals how students in a high-stakes system equate "doing well" in mathematics with high scores. This aligns with Zhang et al. (2016), who found that Taiwanese students view grades as key indicators of success. While mathematical achievement can include goal-setting, confidence, and mastery (Hill et al., 2021), high-stakes testing reinforces a narrow, score-focused definition of success, emphasizing performance over broader learning outcomes.

Students' perceptions of "doing well" in mathematics resemble a "success recipe" centered on achieving high scores. This study highlights Taiwanese students' strong pragmatic focus on exam preparation, where success is defined by relentless effort and mastery of test-related knowledge. This recipe bears a strong resemblance to Friedman and Laurison's (2020) concept of "technical capital", which they describe as "forms of practical expertise, knowledge, and know-how" that can be accumulated in a fairly linear fashion by following formalised, rationalised procedures" (p. 204).

In terms of psychological competencies, the analysis revealed that students with a Score-Oriented perception exhibited significantly lower levels of mathematics confidence than those with a Non-Score-Oriented perspective. This disparity may stem from students who equate "doing well" with high scores, making their confidence highly dependent on external validation. As contingent self-worth theory (Crocker & Wolfe, 2001) suggests, when self-worth is overly tied to a single do-

main—such as grades—self-perception becomes unstable and performance-dependent. In this context, Score-Oriented confidence functions like an inflated structure—reinforced by success but vulnerable to collapse after setbacks. For some students, repeated failures or unmet expectations may erode confidence, reinforcing negative self-perceptions such as "I am not good enough" or "Others are far better than me" (Deresiewicz, 2014) and ultimately undermining motivation and academic resilience.

This fragile, externally driven confidence may explain Taiwanese students' high fear of failure, as seen in OECD (2023). In environments where success is narrowly defined by scores, students become highly sensitive to evaluation and judgment. The pressure extends beyond achieving high scores—they must also maintain their rank, fueling a cycle of anxiety and self-doubt. Each success heightens the fear of future underperformance, as high scores are precarious and easily lost. These findings highlight the critical role of educators in shifting students' focus beyond scores toward a broader definition of success, ultimately helping students develop resilience and long-term engagement.

The analysis further revealed that students with a Non-Score-Oriented perspective demonstrated higher levels of growth mindset compared to their Score-Oriented counterparts. This finding aligns with growth mindset theory, which posits that individuals with a stronger growth mindset are more likely to value effort and the learning process over fixed outcomes (Dweck, 2008). Building on Tao et al. (2022), who found that students with a fixed mindset tend to focus on performance goals, these results further reinforce the link between performance-oriented perspective and fixed mindset.

This research highlights how high-stakes systems shape and reinforce students' perceptions of "doing well". These structures persist as long as they are seen as natural and inevitable. By exposing their consequences, this study calls for critical reflection rather than passive acceptance. To disrupt these entrenched structures, fostering a more flexible view of "doing well" in mathematics plays a critical role. Ultimately, creating a learning environment that challenges traditional notions of success not only enhances students' mathematical confidence but also nurtures a more sustainable and self-determined growth mindset.

References

Bourdieu, P. (1990). *The logic of practice*. Stanford University Press.

Crocker, J., & Wolfe, C. T. (2001). Contingencies of self-worth. *Psychological Review*, *108*(3), 593–623.

Deresiewicz, W. (2014). *Excellent sheep: The miseducation of the American elite and the way to a meaningful life*. Simon and Schuster.

Dweck, C. S. (2008). *Mindset: The new psychology of success* (1st edition). Ballantine Books.

Friedman, S., & Laurison, D. (2020). The class ceiling: Why it pays to be privileged. *Social Forces*, *99*(1), e5.

Gorur, R. (2016). The "thin descriptions" of the secondary analyses of PISA. *Educação & Sociedade*, *37*, 647–668.

Hill, J. L., Kern, M. L., Seah, W. T., & van Driel, J. (2021). Feeling good and functioning well in mathematics education: Exploring students' conceptions of mathematical well-being and values. *ECNU Review of Education*, *4*(2), 349–375.

Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). *TIMSS 2019 international results in mathematics and science.* TIMSS & PIRLS International Association for the Evaluation of Educational Achievement.

Nolan, K. (2012). Dispositions in the field: Viewing mathematics teacher education through the lens of Bourdieu's social field theory. *Educational Studies in Mathematics*, *80*(1), 201–215.

OECD. (2023). *PISA 2022 Results (Volume II): Learning during and from disruption*. PISA, OECD Publishing.

Stahl, G., Mu, G. M., Ayling, P., & Weininger, E. B. (Eds.). (2024). *The Bloomsbury handbook of Bourdieu and educational research* (1st ed.). Bloomsbury Publishing Plc.

Tao, V. Y. K., Li, Y., & Wu, A. M. S. (2022). Incremental intelligence mindset, fear of failure, and academic coping. *Journal of Pacific Rim Psychology*, *16*, 18344909221144703.

Wang, C.-C., & Tai, T.-H. (2024). Cultural capital, standardised testing and East Asian education: The transposition of disposition of Taiwanese elite students. *The Sociological Review*, 00380261241272317.

Yang, K.-L., Hsu, H.-Y., & Cheng, Y.-H. (2022). Opportunities and challenges of mathematics learning in Taiwan: A critical review. *ZDM – Mathematics Education*, *54*(3), 569–580.

Zhang, Q., Barkatsas, T., Law, H.-Y., Leu, Y.-C., Seah, W. T., & Wong, N.-Y. (2016). What primary students in the Chinese mainland, Hong Kong and Taiwan value in mathematics learning: A comparative analysis. *International Journal of Science and Mathematics Education*, *14*(5), 907–924.

TSG7-03

EFFECTS OF COLLABORATIVE CRITICAL MATHEMATICS IN A FLIPPED CLASSROOM ON STUDENTS' MATHEMATICAL DISPOSITION

Eric Loyd P. Hilario
University of the Philippines Diliman, Philippines, ephilario@up.edu.ph

Ma. Nympha B. Joaquin
University of the Philippines Diliman, Philippines, mbjoaquin@up.edu.ph

Developing mathematical disposition is essential for enhancing students' performance in mathematics. This study examines the impact of collaborative critical mathematics in a flipped classroom (CCMFC)—a strategy that integrates discussions of social issues into mathematics learning—on students' mathematical disposition. An explanatory sequential mixed-methods design was employed, involving two intact groups: 30 students received instruction using CCMFC, while 29 students underwent conventional teaching. Pretest and posttest scores from the Mathematical Disposition Questionnaire (MDQ) were analyzed to assess changes in confidence, flexibility, usefulness, perseverance, curiosity, and metacognition. Results indicated that both teaching approaches were comparable in fostering mathematical disposition. However, the CCMFC approach showed potential in enhancing students' confidence, usefulness, curiosity, and overall mathematical disposition, suggesting its promise as an alternative instructional strategy in mathematics education.

Keywords: critical mathematics education, collaborative learning, flipped classroom, mathematical disposition

Introduction

Mathematical disposition encompasses students' attitudes, beliefs, and behaviors towards mathematics (Beyers, 2011). Students with a strong mathematical disposition view mathematics as valuable and meaningful, and they are motivated to engage in mathematical activities. (Sa'diyah et al., 2019). According to Nuraidai (2017, as cited in Fitrianna et al, 2018), key characteristics of mathematical disposition includes (1) confidence in doing mathematics, (2) flexibility in exploring mathematical ideas, (3) belief in usefulness of mathematics in other fields, (4) perseverance in mathematical tasks, (5) curiosity in mathematics, (6) metacognition or the ability to monitor and reflect on one's thinking processes.

Developing mathematical disposition is essential for improving students' performance in the subject. Hutajulu et al. (2019) found a positive correlation between mathematical disposition and problem-solving skills. Beyers (2012) noted that a well-developed mathematical disposition makes math lessons more engaging and enjoyable. Additionally, Kusmaryono et al. (2021) linked mathematical disposition to higher academic achievement, while Fadillah et al. (2020) emphasized its role in fostering persistence, self-responsibility, and lifelong learning.

Several strategies have been proposed to enhance mathematical disposition. Sa'diyah et al. (2019) highlighted the effectiveness of open-ended questions in encouraging diverse and thoughtful student responses. Dina et al. (2019) found that exposure to discovery learning models improves

mathematical disposition. Similarly, In'am and Isliamati (2018) suggested that integrating real-life mathematical problems into instruction enhances students' engagement and disposition.

This study proposes the use of collaborative critical mathematics in a flipped classroom to foster mathematical disposition. This method integrates classroom discussions of societal issues, using mathematics as a tool for analysis and critique (Avcı, 2020; Cafezeiro et al., 2017). It also emphasizes collaborative learning through student dialogue and discourse (Mukhopadhyay, 2009), and promotes student autonomy through the flipped classroom model, where students study new material before class and engage in group activities during class (Chanane et al., 2023). By exposing students to real-life scenarios and reflective questions, this approach fosters critical discussions on social issues through a mathematical lens, potentially enhancing mathematical disposition. Accordingly, this study explores how collaborative critical mathematics in a flipped classroom method influences students' mathematical disposition in terms of confidence, flexibility, usefulness, perseverance, curiosity, and metacognition.

Methodology

This study employed an explanatory sequential mixed-methods design involving the assignment of two intact groups, a pretest-posttest approach, and interviews. Conducted at a laboratory school in Metro Manila, Philippines, the research involved two randomly selected heterogeneous Grade 10 sections. One class (30 students) received instruction using collaborative critical mathematics in a flipped classroom (CCMFC), while the other class (29 students) underwent conventional instruction. In CCMFC, students engaged with articles or videos on social issues before class, which were then explored through mathematics lessons integrated with collaborative activities. These activities included answering guide questions that applied mathematical concepts while deepening understanding of the social issue. In contrast, the conventional group received teacher-led presentations of mathematical concepts followed by guided practice using example problems.

To assess mathematical disposition, the researcher-developed Mathematical Disposition Questionnaire (MDQ) was utilized. This five-point Likert-scale instrument comprises 18 statements reflecting six key indicators: confidence, flexibility, usefulness, perseverance, curiosity, and metacognition. Sample items include: "I feel confident in solving math problems", "I can study math for a long time", and "Math has practical applications to daily life." Each student's overall mathematical disposition score was calculated by averaging the ratings across all items. These scores were then categorized as very low, low, moderate, high, or very high.

The teaching interventions for both groups were implemented simultaneously over eight weeks. After the intervention, student interviews were conducted to gather qualitative insights.

Data analysis involved confirmatory factor analysis to validate the MDQ and Cronbach's Alpha to assess its reliability. Descriptive statistics (mean, median, and standard deviation) were applied to establish baseline information about students' mathematical disposition. The Mann-Whitney U Test was used to compare mathematical disposition between the two groups before and after the intervention. Additionally, the Wilcoxon Signed-Rank Test was applied within the experimental group to determine improvements in mathematical disposition from pretest to posttest.

Qualitative data were collected from students who voluntarily participated in focus group discussions. Interview responses were transcribed, categorized, and thematically analyzed, providing deeper contextual insights into the variables studied.

Results and Discussion

For the initial comparability of the two groups in terms of mathematical disposition, Mann-Whitney U test indicated no significant difference in the mean pretest scores for confidence ($U = 322$, $p = 0.085$), flexibility ($U = 379$, $p = 0.389$), usefulness ($U = 370$, $p = 0.315$), perseverance ($U = 396$, $p = 0.555$), curiosity ($U = 423$, $p = 0.855$), metacognition ($U = 417$, $p = 0.782$), and the average MDQ score ($U = 393$, $p = 0.524$). This suggests that the two groups were comparable in terms of mathematical disposition and its indicators at the beginning of the study.

Following the intervention, no significant differences were found between the two groups' posttest scores for confidence ($U = 378$, $p = 0.811$), flexibility ($U = 426$, $p = 0.561$), usefulness ($U = 392$, $p = 0.257$), perseverance ($U = 388$, $p = 0.767$), curiosity ($U = 388$, $p = 0.237$), metacognition ($U = 374$, $p = 0.176$), and average MDQ score ($U = 404$, $p = 0.322$). This suggests that students in both groups exhibited comparable levels of mathematical disposition despite being exposed to different teaching methods. Furthermore, both the CCMFC group ($M = 3.570$) and the conventional group ($M = 3.584$) were categorized as having a high mathematical disposition.

Although no significant differences were found between the groups after the intervention, a Wilcoxon Signed-Rank Test was conducted to compare the CCMFC groups' mathematical disposition before and after the study. The results showed improvement significant improvements in the CCMFC groups' confidence ($W = 106.0$, $p = 0.014$), usefulness ($W = 54.0$, $p = 0.029$), curiosity ($W = 105.0$, $p = 0.037$), and overall MDQ score ($W = 113.5$, $p = 0.013$) from pretest to posttest. However, no significant changes were observed in flexibility ($W = 98.0$, $p = 0.114$), perseverance ($W = 121.0$, $p = 0.052$), and metacognition ($W = 134.0$, $p = 0.148$). These findings suggest that while CCMFC group's confidence, usefulness, curiosity, and overall mathematical disposition improved following the intervention, no significant changes were observed in flexibility, perseverance, and metacognition.

Qualitative findings from student interviews support these results. Student A, who progressed from low to high mathematical disposition, stated that incorporating current events into lessons helped them grasp mathematical concepts more effectively. They also noted that real-life examples made learning more enjoyable, contributing to improved grades. Similarly, Student B, who advanced from moderate to very high mathematical disposition, explained that using mathematics to analyze real-world issues increased their interest in the subject and broadened their perspective, showing them its relevance even in arts and humanities careers.

Other students also reported specific improvements. Student C gained confidence in communicating ideas and justifying answers, knowing their reasoning was supported by data analysis. Student E found mathematics more engaging when integrated with social issues, which enhanced their attentiveness in class.

Although changes in flexibility, perseverance, and metacognition were not statistically significant, students still recognized the impact of the CCMFC approach. Student F acknowledged that collaborative critical mathematics in a flipped classroom provided new ways to explore mathematical ideas. Student G reported increased determination in studying mathematics due to the integration of social issues which encouraged them to dedicate more time to the subject. Student H described how resolving conflicting perspectives on both mathematical solutions and social issues helped them develop their metacognitive skills, prompting deeper reflection on their thought processes.

Conclusion

In conclusion, while the Collaborative Critical Mathematics in a Flipped Classroom (CCMFC) approach did not produce statistically significant improvements in students' overall mathematical disposition compared to conventional instruction, analyses of pretest-posttest results and qualitative data revealed meaningful gains in confidence, usefulness, and curiosity. This highlights the value of mixed-methods research, where qualitative insights help contextualize non-significant quantitative results. The apparent discrepancy between findings suggests that real-world applications and critical discussions may enhance students' engagement and attitudes toward mathematics, even if not immediately reflected in statistical outcomes. It is important to note that the small sample size (n = 30 and 29) likely limited the study's statistical power, warranting caution in interpretation. Nonetheless, the findings emphasize the potential of incorporating critical discussions of social issues into mathematics education to foster mathematical competence and deeper awareness of real-world applications, thereby enriching students' engagement with the subject.

References

Avcı, B. (2020). Research methodology in critical mathematics education. *International Journal of Research & Method in Education*, *44*(2), 135-150.

Beyers, J. (2011). Development and evaluation of an instrument to assess prospective teachers' dispositions with respect to mathematics. *International Journal of Business and Social Science*, *2*(16), 20–33

Beyers, J.E.R. (2012). An examination of the relationship between prospective teachers' dispositions and achievement in a mathematics content course for elementary education majors. *Journal of SAGE Open. International Journal of Business and Social Science, 2*(16); 20-32.

Cafezeiro, I., Kubrusly, R., Marques I.C., & Cafezeiro, E. (2017). Paulo Freire, mathematics, and policies that shape mathematics. *Journal of Indian Council of Philosophical Research*, *34*, 227–246.

Chanane, N., Cui, J., Sweatman, C.H., Miranda, V., Wichitaksorn, N., Zhang, W., Maturana-Russel, P., Hankin, R., & Sinha, R. (2023). Flipping the Script: Reflection on Implementing the Flipped Classroom in Mathematics. *2023 IEEE International Conference on Teaching, Assessment and Learning for Engineering (TALE)*, 1-4.

Dina, Z. H., Ikshan, M., & Hajidin. (2019). The improvement of communication and mathematical disposition abilities through discovery learning model in junior high school. *Journal of Research and Advances in Mathematics Education*, *4*(1), 11-22.

Fadillah, A., Nopitasari, D., & Pradja, B. P. (2020). Blended learning model during the COVID-19 pandemic: Analysis of students' mathematical disposition. *Jurnal Teori dan Aplikasi Matematika*, *4*(2), 173-181.

Fitrianna, A. Y., Dinia, S., Mayasari, & Nurhafifah, A. Y. (2018). Mathematical representation ability of senior high school students: An evaluation from students' mathematical disposition. *Journal of Research and Advances in Mathematics Education*, *3*(1), 46-56.

Hutajulu, M., Wijaya, T. T., & Hidayat, W. (2019). The effect of mathematical disposition and learning motivation on problem solving: An analysis. *Infinity: Journal of Mathematics Education*, *8*(2), 229-238.

In'am, A., & Islamiati, N. (2018). An analysis of students' mathematical disposition using the comic media in learning geometry. *Advances in Social Science, Education and Humanities Research*, 231, 212-215.

Kusmaryono, I., Ulia, N., & Aminudin, M. (2021). Interaction between mathematical disposition and independent learning in distance learning success: An ex post facto study. *Premiere Educandum: Jurnal Pendidikan Dasar dan Pembelajaran*, *11*(2), 254-268.

Mukhopadhyay, S. (2009). Participatory and Dialogue Democracy in U.S. Mathematics Classrooms. *Democracy education, 18*, 44-50.

Sa'diyah, M., Sa'dijah, C., Sisworo, & Handayani, U. (2019). How students build their mathematical dispositions towards solving contextual and abstract mathematics problems. *Journal of Physics: Conference Series*, *1397*(012090).

Acknowledgement

The researchers are grateful to the Philippines' Department of Science and Technology – Science Education Institute for their funding of this research study.

TSG7-04

MATHEMATICAL ANXIETY OF CHILEAN STUDENTS FROM A TECHNICAL-VOCATIONAL HIGH SCHOOL

Claudia Vargas
Universidad de Santiago de Chile, Chile, claudia.vargas.d@usach.cl

The study explored the presence of Mathematical Anxiety (MA) in a group of Chilean students from a Technical-Vocational High School. The students from the Technical-Vocational High School presented higher levels of MA, reflected in higher percentage mainly in two statements about grades and math classes, although it is striking that in problem solving the percentage of mathematical anxiety is high. Recommendations aimed at mathematics teachers to address the MA of their students are discussed.

Keywords: mathematical anxiety, high school, mathematics

Introduction

Chilean 15-year-old students exhibit the highest levels of math anxiety among OECD countries, matched only by Turkey and standing out negatively in South America (OECD, 2023b, 2023c). This anxiety significantly impacts academic performance, reducing scores by 7 to 9 points in PISA assessments. Moreover, socio-emotional factors like math anxiety are as influential as socioeconomic status in international evaluations such as PISA.

Math anxiety, defined as a feeling of tension or fear that interferes with mathematical performance, hinders learning, whereas positive emotions like success and interest enhance it (McLeod, 1994). In Chile, this factor negatively affects both overall math performance and specific sub-areas (geometry, data, and probability, among others) assessed in PISA 2022 (OECD, 2023a). Emotions, fundamental to the educational experience, must be considered for both teachers and students.

Mathematical anxiety is associated with poorer math performance (Ashcraft & Faust, 1994) and the adoption of less demanding goals (Barroso et al, 2021). However, high levels of anxiety do not always imply low academic achievement, as factors such as high motivation or the suppression of anxiety symptoms can lead to good results. Mathematical anxiety increases with schooling: it affects 11% of students at basic levels but doubles or triples in higher grades (Devine et al, 2018).

This anxiety also influences performance in Natural Sciences and Reading, though to a lesser extent than in Mathematics. Since 2003, the OECD's PISA tests have measured mathematical anxiety, self-perception, and attitudes toward the subject, revealing that 60% of students report worrying about math classes, and 39% feel tense when solving problems.

Mathematical anxiety affects 15-year-olds across various countries and cultures, with similar indices observed between Chile and Turkey, reflecting a broad and persistent impact on global academic performance.

In Chile, the second year of high school marks the end of general education, after which students choose between the scientific-humanistic or technical-professional tracks, which shape their career paths. It is important to remark that the perception of being "good at math" influences this decision, with mathematical self-efficacy and emotions associated with the subject playing a key role (Street, Malmberg, y Schukajlow, 2024).

Nowadays in Chile, 944 technical-professional schools educate over 156,355 students, yet their mathematics education has been under-researched. Given the impact of socio-emotional factors on professional trajectories (Eidlin-Levy et al, 2023) it is essential to delve deeper into the analysis of mathematical anxiety within this group.

People with high mathematical anxiety tend to avoid mathematics and related careers, especially those with extensive mathematical content. Mathematical anxiety predicts enrollment in STEM fields beyond mathematical ability, negatively affecting the number of STEM courses taken and academic performance. Longitudinal studies show that lower mathematical anxiety is associated with greater choice of STEM careers (Ahmed, 2018), with mathematical anxiety acting as a key mediator in interest in these fields. Additionally, mathematical anxiety in early school years, such as in seventh grade, is a determining factor in the decision to pursue STEM careers (Cuder et al., 2024). These findings highlight the importance of specific interventions to address mathematical anxiety from an early age.

Research Question, Objective and Methodology

Considering the previous review, the following question arises: What percentage of mathematical anxiety do students in a group of high school students at a technical-vocational school experience?

Accordingly, the objective of the research is to determine the mathematical anxiety experienced by students exposed to a mathematical outreach activity.

The participants (N=110) in the mathematical dissemination activity were second-year high school students from a Technical-Vocational High School (TVHS) in Talca, Maule Region of Chile, whose teachers showed interest in activities to reduce math anxiety. This reflects the commitment of the school's teachers and school leadership to support students and explore new, meaningful ways of teaching math. The high school requested a visit to Mathematical Didactic Laboratory Museum in Universidad de Santiago de Chile to participate in an activity on Fun Cryptography. The data collection instrument was the PISA Mathematical Anxiety Questionnaire, using a four-level Likert scale. Participation was authorized by the TVHS Headmaster, the teacher in charge of the visit, the parents, and with the informed consent of the students, following the ethical guidelines established by an accredited committee.

Results and Discussion

Examining question by question, the question with the highest percentage of agreement among the three samples is: "I worry about getting bad grades in math" where 92% of the responses from the sample. The pressure to get good grades in mathematics creates a state of alert in students, which limits their reasoning ability and negatively affects their relationship with the subject (Pearson, 2019).

The next highest percentage of agreement was for the question: "I worry about having difficulties in math classes" where the agreement percentage was 82%. This may be due to the fear of being judged or ridiculed for making mistakes in mathematics (Vargas & Matus, 2022).

Table 1. Percentages of agreement with statements about mathematics anxiety

Statements	Percentage
I often worry about having trouble in math classes.	82
I get very tense when I must do math homework at home.	54
I get very nervous when working on math problems.	57
I feel lost when working on a math problem.	71
I'm worried about getting bad grades in math.	92
I feel anxious about failing in math	71

It is striking that in the statement "I feel lost when working with a math problem," 71% of the students in the sample agreed. This could be due to their lack of mastery in problem-solving strategies in mathematics. We don't know if this comes from the students or the teachers. While this study highlights percentage results, further analysis was conducted comparing this sample with general national PISA data and from Chilean TVHS data in PISA. Across all examined questions, the Talca sample showed higher levels of agreement related to math anxiety than both the general Chilean PISA sample and the TVHS subgroup. The question: "I worry about getting poor grades in math," obtained 92% of student's agreement in Talca, compared to 84% (PISA Chile) and 82% (TVHS sample). Similar patterns were observed in other items, indicating a strong presence of math anxiety in the Talca group.

Conclusions

This work explored the presence of MA in a group of students, but many questions remain unanswered. For example, regarding the application of the questionnaire, does the perception of math anxiety depend on the activities prior to the administration of the questionnaire? Additionally, does the university setting influence the perception of math anxiety?

On the other hand, it is observed that students present mathematical anxiety in relation to problem solving. Will their anxiety levels decrease if they are taught problem-solving strategies and heuristics?

The literature emphasizes the need for early interventions to address math anxiety in schools. It is suggested that, to counteract anxiety, the factors causing students' concerns about difficulties in math classes should be identified. In this study, the results show that it is necessary to work on emotional aspects with students in math classes. For instance, the value of making mistakes to improve our performance in mathematics or that having troubles in math classes is a natural way to learn.

The study reveals that concern about getting bad grades in math is a common issue among students, which may be related to social or family pressure. Teachers need tools to address the emotional aspects of students and promote an integrated math education. In technical-vocational education, addressing math anxiety is crucial to improving outcomes and encouraging the continuation of studies in STEM fields, as mathematics is key in Technical-Vocational High School.

References

ACE. (2024). Informe nacional PISA 2022: Evaluación internacional de estudiantes tras la pandemia.

Ahmed, W. (2018). Developmental trajectories of math anxiety during adolescence: Associations with STEM career choice. Journal of Adolescence, 67, 158-166.

Ashcraft, M. H., & Faust, M. W. (1994). Mathematics anxiety and mental arithmetic performance: An exploratory investigation. Cognition and Emotion, 8(2), 97-125.

Barroso, C., Ganley, C. M., McGraw, A. L., Geer, E. A., Hart, S. A., & Daucourt, M. C. (2021). A meta-analysis of the relation between math anxiety and math achievement. Psychological Bulletin, 147(2), 134-168.

Cuder, A., Pellizzoni, S., Di Marco, M., Blason, C., Doz, E., Giofrè, D., & Passolunghi, M. C. (2024). The impact of math anxiety and self-efficacy in middle school STEM choices: A 3-year longitudinal study. British Journal of Educational Psychology. Advance online publication.

Devine, A., Hill, F., Carey, E., & Szu, D. (2018). Cognitive and emotional math problems largely dissociate: Prevalence of developmental dyscalculia and mathematics anxiety. Journal of Educational Psychology, 110(3), 431-444.

Eidlin-Levy, H., Avraham, E., Fares, L., & Ben-David, S. (2023). Math anxiety affects career choices during development. International Journal of STEM Education, 10, Article 49.

Mcleod, D. B. (1994). Research on affect and mathematics learning in the JRME: 1970 to the present. Journal for Research in Mathematics Education, 25(6), 637-647.

OCDE. (2023a). PISA 2022. Assessment and analytical framework. OECD Publishing.

OCDE. (2023b). PISA 2022 Results (Volume I) The State of Learning and Equity in Education. OECD Publishing.

OCDE. (2023c). PISA 2022 Results (Volume II) Learning During-and From-Disruption. OECD Publishing.

Street, K. E. S., Malmberg, L. E., & Schukajlow, S. (2024). Students' mathematics self-efficacy: A scoping review. ZDM Mathematics Education, 56, 265-280.

Vargas, C., Matus, C. (2022) "Dominio afectivo en evaluaciones de matemática: la experiencia escolar de dos jóvenes estudiantes destacados académicamente". Revista Innovaciones educativas. 24 (37). 7-18 ERIHPLUS/Scielo: https://doi.org/10.22458/ie.v24i37.3672

Acknowledgement: Claudia Matus Correa. Data Analyses.

TSG7-05

A DYNAMIC SYSTEMS APPROACH TO EMOTIONS IN MATHEMATICS LEARNING AND EDUCATION

Soohyun Baek
The University of Sydney, Australia, soohyun.baek@sydney.edu.au

Minkang Kim
The University of Sydney, Australia, minkang.kim@sydney.edu.au

The pervasive issue of math anxiety highlights the central role that emotion plays in learning and assessment in educational settings. Whilst contemporary educational research recognises the integral role of emotions in students' learning and well-being, many influential theoretical models still maintain a dichotomous view that separates emotion from cognition. This paper advocates for a more holistic approach by adopting a dynamic systems perspective and predictive brain theory, offering a neurologically plausible account of emotions as emergent, self-organising, and context-sensitive phenomena that shape learning. The paper concludes by discussing the pedagogical implications for math education, emphasising the importance of creating supportive learning environments that values failure as a valid and integral component of the learning process.

Keywords: Math anxiety, Dynamic systems approach, Predictive brain

Introduction

It is widely acknowledged, including recognition by the Organisation for Economic Co-operation and Development (OECD), that, for many children and adolescents, learning and assessment in school can be a source of anxiety and stress, potentially leading to failure or a fear of failure, especially in mathematics where many students experience 'math anxiety'. According to the recent PISA 2022 results report (OECD, 2023), compared to the data collected in 2012, 15-year-olds around the globe are more afraid of engaging with maths learning and assessment.

The notion of math anxiety has been a subject of scrutiny for many researchers over the past several decades. A particular focus of these studies has been on its deteriorating impact on mathematics performance, which could substantially influence students' future educational attainment and career choices (Dowker et al., 2016). Specifically, Lau et al. (2024) have identified that the robust negative causal relationship between math achievement and math anxiety is mediated by 'math avoidance', a math-specific behavioural tendency formed through repeated experiences of failure in math tasks and assessments.

While numerous studies have been conducted on the subject of math anxiety from various perspectives, they have collectively highlighted a more fundamental issue that demands our attention: the central role that emotion plays in the learning and assessment process at school. From their own professional experience, teachers recognise that emotions experienced during the learning process can play a critical role in students' reluctance to engage with mathematics and difficulties across the curriculum. Much less clear, however, is what is meant by 'emotion' in making such a claim. For

example, are emotions hard-wired in the brain at birth, or do we create our own emotions (Barrett, 2017)? If the latter, what are the mechanisms involved in this process? Moreover, within the context of education, it is crucial to examine the relationship between logic and emotion, as well as thinking and feeling. How, if at all, might these be interwoven in the brain, and how do they influence students' learning in school, particularly when they experience failure in mathematics?

Conflicting Perspectives on Emotion in Education

In both East and West, emotions were often viewed as primitive, out of control, unreliable and irrational, and thus considered sharply separated from human rationality. The dichotomous distinction between emotion and cognition has been deeply entrenched in the understanding of learning and decision-making processes, both generally and in education, such that the role of emotions in these processes has been largely overlooked (Immordino-Yang & Damasio, 2007). However, following the seminal work of Antonio Damasio (1994) and others, which demonstrated the centrality of emotions in rational thinking, the integral role of emotions in students' learning and well-being is widely appreciated in current educational research.

Nevertheless, a critical scrutiny of theoretical models on emotions currently influential in education reveals that the past dichotomy remains firmly entrenched. For example, the notion of emotional intelligence (Goleman, 1995) is predicated on the premise that our mental world comprises two distinct minds: the emotional and the rational. These two minds operate independently yet are intricately interconnected. In another model, known as the control-value theory of achievement emotions (Pekrun, 2006), the dichotomy is endorsed by assigning an exclusive role to 'appraisal', a putative cognitive process in the emotion system, in determining emotions, while denying emotion's role in causing appraisal processes (Lewis, 2005).

Emotions in the Dynamic Emergent Self-organising Predictive Brain

However, mounting evidence from brain studies has made it clear that all human thought and learning are laced with emotion (Kim & Sankey, 2022), including the learning of mathematics in schools. This calls into question the continued use of a dichotomy within educational research and practices. It is essential that we consider an alternative account of emotion that is more holistic and sophisticated. More specifically, we require an account of the dynamic complexities involved in the interweaving of emotion and cognition.

During the 1990s, a new perspective on emotion was proposed by several scholars, such as Esther Thelen, Linda Smith, and Alan Fogel. These scholars adopted a dynamic systems approach to understanding learning and development. A dynamic systems approach, drawing on complexity theory advanced in natural science between the early and mid-twentieth century, provides a view of human development as an emergent, non-linear, unpredictable, context-sensitive process of change through self-organisation that operates via continuous feedback between multiple, parallelly interconnected components in a system. This approach hasn't been captured by conventional developmental theories assuming incremental, teleological progression following linear developmental stages (Kim & Sankey, 2022). From a dynamic systems perspective, emotions are emergent self-organising phenomena constructed by a process of the dynamic and complex interplay among multiple components in response to changes in one's social and physical context. It is important to note that emotions are *not* hard-wired things in the form of dedicated brain structures, fixed neural circuits or psychological patterns of appraisal that are switched on by triggering events. Emotions, instead, are

fluid and context-sensitive, much like all our thinking and and learning, as such they can be "changed and improved" (Kim & Sankey, 2022).

Furthermore, as Thelen and Smith (1994) noted, "the brain systems providing the affective valence are richly and densely interconnected with those of perceiving, acting and thinking" (p. 319) and "emotion is part of the same dynamic as other aspects of perception, action, cognition and social behaviour" (p. 320). The latest understanding on how brain works, known as a 'predictive brain theory', supports the holistic account of emotion and cognition. It suggests a holistic, unified and embodied model of brain functioning via interoception, an ongoing and constant internal sensing process employed for bodily regulation to maintain homeostasis of the entire organism. According to the predictive brain theory, below our conscious control, our embodied and socially embedded brain (Kim & Sankey, 2022) actively creates meanings and concepts, namely predictions, in advance of perception. These predictions are informed by previous knowledge and experience and directed by perceived salience and value (Edelman, 1989), in order to make sense of inputs interoceptively received internally from the body and perceived from the external world to keep us alive and thrive. In addition, the brain is an inherently dynamic, self-organising feedback control system (Kim & Sankey, 2022) that updates its internal model (predictions) through interoceptive feedback loops. These loops communicate error feedback arising from a notable mismatch between the brain's prediction and its real-world perceptual experience. This ongoing and endogenous brain activity, as Barrett (2017) noted, is "the origin of dreams, daydreams, imagination" (p. 58) and is where emotions that students experience in the learning process, including maths anxiety, emerge, change and improve through dynamic self-organisation via feedback.

Implications for Math Education

Understanding emotions from the perspective of dynamic emergent self-organising predictive brain helps us see the significance of 'affordances' (Gibson, 1979) in learning environments. This is especially important when dealing with the emotional anxiety and stress that students may experience in learning and assessment, including math anxiety. Every student, being equipped with a proactive and predictive brain that constantly seeks salience and value in the environment surrounding them, comes into a classroom with anticipations and expectations about what they would feel and learn. These expectations are based on their own emotionally charged and value-oriented prior history or "remembered present" (Edelman, 1989) and their brains selectively pay attention to error feedback, depending on anticipated value and a set of meanings informed by the remembered present. From this standpoint, the value and salience that a learning environment can afford are critical to facilitating new learning, as it is assumed that every student is "equipped with a brain that is value-oriented towards learning", and will "learn, when nested in suitably affordant learning environments" (Duncan et al., 2022, p. 618).

In mathematics classrooms where many students experience fear and anxiety, it is important to recognise that the key to dealing with issues related to math anxiety can be to create learning environments that 'afford safety' to be wrong and that "embrace failure as a learning opportunity" (Miller, 2015, p. 6). In practice, teachers can help students effectively manage the fear of failure, which can increase math anxiety and in turn hinder new learning, by presenting error feedback with carefully chosen words to minimise emotional distress induced by failure and to facilitate learning from their mistakes. Teachers should be fully aware that the emotional aspects of students' thinking must be addressed when correcting their mistakes, appreciating the inherent inseparability of emotion and rationality in the learning process. This holistic appreciation of emotion and rationality holds considerable significance for the mathematics classroom.

References

Barrett, L. F. (2017). *How emotions are made: The secret life of the brain*. Pan Macmillan.

Damasio, A. R. (1994). *Descartes' error: Emotion, reason and the human brain*. Putnam.

Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics anxiety: What have we learned in 60 years? *Frontiers in Psychology, 7*:508. https://doi.org/10.3389/fpsyg.2016.00508

Duncan, C., Kim, M., Baek, S., Wu, K. Y. Y., & Sankey, D. (2022). The limits of motivation theory in education and the dynamics of value-embedded learning (VEL), *Educational Philosophy and Theory, 54*(5), 618-629. https://doi.org/10.1080/00131857.2021.1897575

Edelman, G. M. (1989). *The remembered present: A biological theory of consciousness*. Basic Books.

Gibson, J. J. (1979). *The ecological approach to visual perception*. Houghton Mifflin.

Goleman, D. (1995). *Emotional intelligence: why it can matter more than IQ*. Bantam.

Immordino-Yang, M. H. & Damasio, A. (2007). We feel, therefore we learn: The relevance of affective and social neuroscience to education. *Mind, Brain,, and Education, 1*(1), 3-10. https://doi.org/10.1111/j.1751-228X.2007.00004.x

Kim, M. & Sankey, D. (2022). *The science of learning and development in education: A research-based approach to educational practice*. Cambridge University Press.

Lau, N. T. T., Ansari, D., & Sokolowski, H. M. (2024). Unraveling the interplay between math anxiety and math achievement. *Trends in Cognitive Sciences, 28*(10), 937-947. https://doi.org/10.1016/j.tics.2024.07.006

Lewis, M. D. (2005). Bridging emotion theory and neurobiology through dynamic systems modelling. *Behavioral and Brain Sciences, 28*, 169-245. https://doi.org/10.1017/S0140525X0500004X

Miller, A. K. (2015). *Freedom to fail: How do I foster risk-taking and innovation within my classroom*. ASCD Arias

OCED (2023), *PISA 2022 Results (Volume II): Learning During – and From – Disruption*, PISA, OECD Publishing, Paris, https://doi.org/10.1787/a97db61c-en.

Pekrun, R. (2006). The control-value theory of achievement emotions: Assumptions, corollaries, and implications for educational research and practice. *Educational Psychology Review, 18*, 315-341. https://doi.org/10.1007/s10648-006-9029-9

Thelen, E., & Smith, L. B. (1994). *A dynamic systems approach to the development of cognition and action*. MIT Press.

TSG7-06

A STUDY ON THE EFFECTS OF MATHEMATICAL TOPICS AND ACTIVITY TYPES ON ELEMENTARY SCHOOL STUDENTS' MATHEMATICS ANXIETY

Chiao-En Wang
National Taipei University of Education, Taiwan, audreytinawang@gmail.com

Chia-Jui Hsieh
National Taipei University of Education, Taiwan, paris@tea.ntue.edu.tw

Many studies on the causes and effects of mathematics anxiety have been published. However, there has been little in-depth research at the elementary school level in Taiwan regarding whether different mathematical topics and teaching activities varying levels of mathematics anxiety. This study attempts to investigate whether different mathematical topics (such as numbers, quantity, geometry, algebra, and statistics) and teaching activities (including problem solving, concept introduction, and cooperative learning) elicit different degrees of mathematics anxiety. A survey approach will be employed, and data will be analyzed from 319 public elementary school students in grades three to six from northern Taiwan. Data collection is expected to last for two months, during which mathematics anxiety questionnaires will be administered after each mathematics lesson. This study is currently in progress and is anticipated that subsequent data analysis will reveal differences in anxiety levels among elementary school students across various mathematical topics and learning activities, thereby providing deeper insight into which contexts may trigger higher levels of anxiety.

Keywords: elementary school students, mathematical activity types, mathematical topics, mathematics anxiety

Introduction

Mathematics is an indispensable foundation of modern society, widely applied in fields such as technology, economics, and engineering. Learning mathematics plays a crucial role in enhancing logical reasoning, problem-solving, and critical thinking skills. However, many students experience mathematics anxiety due to the pressure associated with learning mathematics. This emotional response not only affects immediate academic performance but can also undermine students' confidence and long-term learning motivation. Research has shown that different mathematical topics, such as arithmetic, geometry, and algebra, can trigger varying levels of anxiety, particularly when the content is highly abstract or complex (Dowker et al., 2016). Moreover, the format of mathematical activities—such as problem-solving, examinations, or public presentations—also significantly influences students' emotional responses.

Despite its importance, research on how different mathematical topics and activity types affect math anxiety among elementary school students remains limited. This study aims to address this research gap by focusing on young learners' mathematical anxiety. Specifically, it investigates how different mathematical topics and learning activities influence anxiety levels and further examines the role of students' background factors in shaping their experiences. Based on the research objectives, the specific research questions of this study are as follows:

1. Do elementary school students experience different levels of mathematics anxiety across various mathematical topics, such as numbers and calculations, measurement and estimation, shapes, algebra, and statistics?
2. Do elementary school students exhibit varying degrees of mathematics anxiety in different mathematical activities, such as problem-solving tasks, reading mathematics textbooks, group activities, and public presentations?

Literature Review

Mathematics Anxiety

Mathematics anxiety refers to the feelings of tension, uneasiness, and fear that individuals experience when engaging in mathematical tasks, tests, or related activities. Ashcraft and Ridley (1988) described it as a math-related fear that may cause individuals to shy away from mathematical tasks, potentially leading to decreased performance. Similarly, Tobias (1993) suggested that mathematics anxiety may stem from both individual and environmental factors, such as teaching methods, societal expectations, and family support.

Assessment of Mathematics Anxiety

In research on mathematics anxiety, scholars have developed various questionnaires to comprehensively assess learners' emotional responses in mathematical contexts. The earliest instrument, the MARS (Mathematics Anxiety Rating Scale; Richardson & Suinn, 1972), provided a comprehensive assessment with its extensive items. However, due to the heavy response burden, the revised version, MARS-R (Plake & Parker, 1982), was later developed to simplify the items and improve testing efficiency. Meanwhile, the MAQ (Mathematics Anxiety Questionnaire; Wigfield & Meece, 1988) focuses on anxiety across different learning activities, capturing situational variations; and the ATMS (The Anxiety Toward Mathematics Scale; Sandman, 1979/1980) emphasizes emotional responses and cognitive load within specific mathematical environments. All these questionnaires have demonstrated good reliability and validity, although each shows strengths and weaknesses in cultural adaptability, testing duration, and the granularity of differentiation. Overall, these tools not only enrich the theoretical foundation of mathematics anxiety but also provide empirical support for educational interventions and improvement strategies.

Methodology

Research Participants and Design

The participants of this study were all students from grades three to six at a public elementary school in northern Taiwan, totaling 14 classes and 319 students. The student composition in the school's district closely resembles the overall distribution in Taiwan, suggesting that the findings of this study hold a certain level of representativeness and can serve as a reference for understanding mathematics anxiety among students across Taiwan. However, the results should be interpreted with caution, considering potential differences in educational resources, teaching methods, and cultural backgrounds across different regions. A mixed-methods design will be employed, using convenience sampling for the general testing. A balanced incomplete block design (BIB) will be utilized to equate the tests. By linking parameters based on common test items, all item parameters are placed on a common scale, thereby allowing direct comparison.

Survey Design

The mathematics anxiety scale in this study is designed with reference to the PISA 2022 Mathematics Anxiety Scale, Richardson & Suinn (1972), and Sandman (1974). The scale is structured around two dimensions. The first dimension concerns anxiety types, including nervousness (coded as N), worry (coded as W), and helplessness (coded as H) – a total of three types. The second dimension pertains to mathematical activity types: general (coded as 1), problem-solving/discussion (coded as 2), homework/practice (coded as 3), presentation (coded as 4), and examination/performance (coded as 5) – a total of five types. These two dimensions yield 15 combinations, with one item designed for each combination (a total of 15 items). Each item is scored using a five-point Likert scale ranging from "strongly disagree" (1) to "strongly agree" (5), with higher scores indicating higher levels of mathematics anxiety. Initially, the 15 anxiety items will be evenly divided into three blocks (A, B, and C), with each block containing one item for each of the five assessment types. Each test paper will then be composed by selecting two blocks, ensuring that each block appears exactly twice. Thus, each test paper will consist of 10 items, with each item appearing twice across the two versions. The following are some sample questionnaire items:

1. This math lesson is very difficult for me, and I feel very worried.
2. When solving math problems in this unit, I feel extremely nervous.
3. I am worried that I won't be able to complete the homework assigned by the teacher for this unit.
4. When solving math problems in this unit, I feel helpless.

Data Collection

This study aims to investigate the effects of different mathematical topics and activity types on elementary school students' mathematics anxiety, as well as the relationship between students' background factors and anxiety. A mixed-methods design will be adopted. First, a questionnaire will be administered to collect demographic data (including gender, grade, and learning experience). Next, at the end of each mathematics class, a paper-based questionnaire will be administered immediately to measure the students' anxiety levels regarding that lesson's content, ensuring timely and accurate data. Finally, to gain deeper insight into the causes and impacts of mathematics anxiety, 20 students with relatively high anxiety levels will be selected for semi-structured interviews. These interviews will collect qualitative data regarding their mathematics learning experiences, emotional responses, and influencing factors. The combination of questionnaires and interviews is intended to offer a comprehensive perspective on how different mathematical topics and learning activities affect students' mathematics anxiety and to provide empirical evidence for future instructional and intervention strategies.

Data Analysis

This study will combine questionnaires and interviews to examine the effects of different mathematical topics and activity types on elementary school students' mathematics anxiety, and to analyze the relationship between students' background factors and anxiety. Interview data will be coded using Content Analysis to identify specific sources of anxiety and individual differences. The questionnaire data will be analyzed using descriptive statistics and analysis of variance (ANOVA) to examine patterns in how different mathematical topics and activity types influence anxiety levels. By integrating qualitative and quantitative data, the study aims to reveal the interactive effects among mathematical topics, activity types, and student background factors, and to further explore which learning contexts may exacerbate or alleviate mathematics anxiety.

Expected Results and Practical Applications

The study is currently in progress. It is expected that students at different grade levels will exhibit varying patterns of anxiety when facing domains such as number and computation, geometry, and algebra, and that specific learning activities (e.g., examinations, problem-solving competitions, or public presentations) may have additional effects on their emotional responses. The findings will contribute to the development of more targeted mathematics teaching strategies, enabling teachers to adjust instructional content and methods according to students' grade levels and learning needs, thus fostering a more supportive learning environment. Furthermore, the results will provide valuable reference material for educational policymakers in curriculum planning and instructional evaluation, ultimately reducing the negative impact of mathematics anxiety on student learning and enhancing both confidence and motivation in mathematics.

References

Ashcraft, M. H., & Ridley, K. S. (2005). Math anxiety and its cognitive consequences: A tutorial review. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 315–327). Psychology Press.

Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics anxiety: What have we learned in 60 years? *Frontiers in Psychology, 7*, 508.

Fennema, E., & Sherman, J. A. (1976). Fennema-Sherman mathematics attitudes scales: Instruments designed to measure attitudes toward the learning of mathematics by females and males. *Journal for Research in Mathematics Education, 7*(5), 324–326.

OECD. (2022). *PISA 2022 assessment and analytical framework*. OECD Publishing.

Plake, B. S., & Parker, C. S. (1982). The development and validation of a revised version of the mathematics anxiety rating scale. *Educational and Psychological Measurement, 42*(2), 551–557.

Richardson, F. C., & Suinn, R. M. (1972). The mathematics anxiety rating scale: psychometric data. *Journal of Counseling Psychology, 19*(6), 551–554.

Sandman, R. S. (1980). The mathematics attitude inventory: instrument and user's manual. *Journal for Research in Mathematics Education, 11*(2), 148–149.

Tobias, S. (1993). *Overcoming math anxiety*. W. W. Norton & Company.

Wigfield, A., & Meece, J. L. (1988). Math anxiety in elementary and secondary school students. *Journal of Educational Psychology, 80*(2), 210–216.

TSG7-07

THE EFFECTIVENESS OF INQUIRY-BASED TEACHING INTEGRATED WITH COMPUTATIONAL THINKING ON MATHEMATICS ATTITUDES IN VOCATIONAL HIGH SCHOOLS

Lin-Chieh Tsai
Taipei Municipal Muzha Vocational High School, Taiwan, p327harry@gmail.com

Jeng-Shin Wu
National Academy for Educational Research, Taiwan, jengshin@mail.naer.edu.tw

Chia-Jui Hsieh
National Taipei University of Education, Taiwan, paris@tea.ntue.edu.tw

This study adopted a quasi-experimental research design, analyzing 115 vocational high school students. The results showed that the experimental group achieved significant improvements in mathematical inquiry motivation, the perceived usefulness of computational thinking, and the usefulness of mathematics. Therefore, a teaching model that integrates real-life applications with computational thinking effectively enhances students' mathematical learning attitudes and motivation. Students also actively seek answers from teachers' questions, fostering their understanding and reflective learning. This study provides important practical insights for mathematics instruction in vocational high schools and suggests further exploration of this teaching model's applicability across different student groups and disciplines.

Keywords: computational thinking, inquiry-based teaching, mathematical attitudes, vocational high school

Introduction

Mathematics is not only the core foundation for understanding specialized subjects for vocational high school students but also a necessary threshold for entering the field of engineering and technology. However, traditional teaching methods often emphasize theoretical knowledge, focusing primarily on formula memorization and problem-solving. This approach frequently prevents students from connecting mathematical knowledge to real-life or professional applications, leading to negative learning attitudes and low motivation. Research has shown that inquiry-based teaching, which is student-centered and emphasizes problem identification, solution exploration, and reflection, fosters active learning and knowledge construction, resulting in positive learning attitudes (Gibson & Chase, 2002; Shrigley, 1990). By integrating computational thinking, students can gain a deeper understanding of the nature of mathematics, positioning it as an essential tool for their future career development and professions (Augustine, 2005). Therefore, this study seeks to develop an innovative teaching model that combines computational thinking with inquiry-based teaching and examines its impact on the mathematical learning attitudes of technical high school students, providing empirical support for both educational practice and theoretical research.

The purpose of this study is to develop an innovative teaching model that combines computational thinking with inquiry-based teaching and examines its impact on the mathematical attitudes of vocational high school students, providing empirical support for both educational practice and theoretical research.

Method

This study adopted a quasi-experimental research design, involving four tenth-grade classes from technical high schools, with a total of 115 students specializing in the electrical and electronic engineering or mechanical groups. The classes were randomly assigned to either the experimental or control group. The experimental group participated in three inquiry-based teaching sessions that integrated mathematics with computational thinking, designed using the IDEAL problem-solving model (Bransford & Stein, 1993). The course topics included "Bubble Sort," "Shortest Path," and "Clustering," with each topic delivered in 100-minute sessions. The control group received traditional instruction covering the same content. Pre- and post-course tests were administered to evaluate changes in students' mathematical attitudes.

This study developed a 40-item Mathematics Attitude Scale by integrating insights from multiple scholars' research on mathematics attitudes. The scale was designed to evaluate students' attitudes toward mathematics, with response options ranging from "Strongly Disagree" to "Strongly Agree" on a 5-point Likert scale, scored from 1 to 5. To ensure the scale's validity and reliability, factor analysis was conducted, yielding a KMO value of .832 and a significant result for Bartlett's test of sphericity ($p < .001$). Based on the scree plot, six factors were extracted: confidence in learning mathematics, attitudes toward success in mathematics, attitudes toward computational thinking, motivation for mathematical inquiry, usefulness of computational thinking, and usefulness of mathematics, collectively explaining 64.12% of the variance.

In addition to factor analysis, the Mathematics Attitude Scale underwent multiple reviews by researchers and experts to ensure its validity. The scale demonstrated excellent reliability, with a Cronbach's α of .944 and subscale α values ranging from .738 to .921. After confirming the scale's validity and reliability, this study employed descriptive statistics, and paired-sample t-tests to evaluate the effectiveness of the new teaching model.

Results and Discussion

The results of the scale administration showed that students in the experimental group demonstrated a slight but significant improvement in their mathematical attitudes after participating in the course. As shown in Table 1, the average score of the experimental group students' mathematical attitudes increased from 2.90 in the pre-test to 3.03 in the post-test, with the standard deviation decreasing from .84 to .68, indicating a reduction in variability. This suggests that students' attitudes toward mathematics have strengthened, and they developed a more positive attitude toward mathematics-related knowledge and applications. In contrast, the average score of the control group increased from 2.91 to 2.94, with minimal change, indicating that students in the traditional teaching model did not exhibit significant changes in their attitudes.

Table 1. Descriptive statistics summary of overall mathematics attitude

Group	N	Test	Mean	SD
Experimental Group(EG)	61	Pre-test	2.90	.84
		Post-test	3.03	.68
Control Group(CG)	54	Pre-test	2.91	.88
		Post-test	2.94	.71

Through a paired sample t-test analysis of the pre- and post-test results of both groups (Table 2), it was found that the experimental group had a mean difference of .13, with a significant difference (p = .020), indicating a significant change in the mathematical attitudes of the experimental group students between the pre- and post-test. In contrast, the control group showed a mean difference of only .04, with no significant difference (p = .772). These results suggest that the inquiry-based teaching program integrating mathematics and computational thinking has a positive effect on improving students' mathematical attitudes, further emphasizing the potential of this teaching model in enhancing students' mathematical learning attitudes.

Table 2. Paired-sample t-test summary of overall mathematics attitude

Group	N	Mean Difference (Post-test – Pre-test)	SD	p
EG	61	.13	.42	.020*
CG	54	.04	.94	.772

$p < .05$ ** $p < .01$ *** $p < .001$

This study further conducted a paired sample t-test on six dimensions: confidence in learning mathematics, attitudes toward mathematical success, attitudes toward computational thinking, mathematical inquiry motivation, the usefulness of computational thinking, and the usefulness of mathematics. The results, as shown in Table 3, indicate that the experimental group exhibited significant improvements in some dimensions, while the changes in the control group were more limited.

Table 3. Paired-sample t-test summary of mathematics attitude by dimension

Dimension	Group	Pre-test Mean (SD)	Post-test Mean (SD)	Mean Difference (Post-test - Pre-test)	p
Confidence in Learning Mathematics	EG	2.68 (.94)	2.68 (.82)	0	.951
	CG	2.66 (.94)	2.84 (.82)	.18	.2
Attitude toward Mathematical Success	EG	3.35 (1.03)	3.49 (.85)	.15	.122
	CG	3.17 (.98)	3.11 (.75)	-.07	.639
Attitude toward Computational Thinking	EG	3.07 (.96)	3.19 (.76)	.12	.217
	CG	3.00 (.97)	3.01 (.80)	.01	.971

Motivation for Mathematical Inquiry	EG	2.37 (.92)	2.59 (.92)	.22	.001***
	CG	2.85 (1.10)	2.82 (.85)	-.04	.829
Usefulness of Computational Thinking	EG	2.78 (1.00)	3.02 (.88)	.24	.037*
	CG	2.93 (.98)	2.87 (.82)	-.06	.719
Usefulness of Mathematics	EG	3.04 (1.09)	3.35 (.95)	.32	.003**
	CG	3.04 (1.03)	3.07 (.81)	.02	.88

$p < .05$ ** $p < .01$ *** $p < .001$

The experimental group demonstrated significant improvements in several dimensions of mathematical attitude. Specifically, the mean difference for Mathematical Inquiry Motivation was .22 (p = .001), indicating that the teaching module effectively stimulated students' motivation for mathematical inquiry. Similarly, the Usefulness of Computational Thinking showed a significant increase, with a mean difference of .24 (p = .037), reflecting students' recognition of the value of computational thinking in learning mathematics. Additionally, the Usefulness of Mathematics also significantly improved, with a mean difference of .32 (p = .003), underscoring the module's effectiveness in enhancing students' awareness of the practical value of mathematics.

In contrast, the control group showed limited changes. For Mathematical Inquiry Motivation, the control group experienced a slight decline, with a mean difference of -.04, which was not statistically significant (p = .829). Similarly, there was a slight decrease in the Usefulness of Computational Thinking (mean difference = -.06, p = .719), and no significant change in the Usefulness of Mathematics (mean difference = .02, p = .880). These findings indicate that the traditional teaching approach had minimal impact on these dimensions of students' mathematical attitudes.

In summary, integrating mathematical concepts into appropriate contexts combined with computational thinking not only helps to stimulate students' interest and motivation in learning but also effectively improves their attitudes and values toward mathematics education. Additionally, it guides students to apply mathematical knowledge to real-world problems, fostering their problem-solving abilities while further strengthening their confidence and sense of accomplishment in learning. In contrast, traditional teaching methods have shown limited impact on students' mathematical attitudes. This may be related to the lack of connection between the instructional content and practical applications, as well as the low level of engagement in learning activities.

Conclusion

Based on the results of this study, it is recommended that teachers in technical high schools incorporate inquiry-based teaching with computational thinking in mathematics instruction to foster a positive attitude toward mathematics and further enhance students' learning outcomes. Additionally, future research could continue to explore the application of this teaching model in different subjects and student groups, providing more comprehensive theoretical and practical insights.

References

Abramovich, S. (2015). Mathematical problem posing as a link between algorithmic thinking and conceptual knowledge. *Teaching of Mathematics*, 18 (2), 45-60.

Bransford, J. D., & Stein, B. S. (1993). The IDEAL Problem Solver (2nd ed.). NY: Freeman.

Gibson, H. L., & Chase, C. (2002). Longitudinal impact of an inquiry-based science program on middle school students' attitudes toward science. *Science Education*, 86, 693-705.

Shrigley, R. L. (1990). Attitude and behavior correlates. *Journal of Research in Science Teaching*, 27(2), 97-113.

TSG7-08

FOSTERING A GROWTH MINDSET IN ELEMENTARY MATHEMATICS INSTRUCTION

Giwoo Kwak
Daeheung Elementary School, Korea, giwoo9292@naver.com

JeongSuk Pang
Korea National University of Education, Korea, jeongsuk@knue.ac.kr

This study aimed to develop and implement instructional strategies to cultivate a growth mindset toward mathematical ability, examine their enactment, and assess their impact on students' mindsets. To this end, we employed a mixed-methods design combining a detailed case study with a broader survey. Our findings underscore the importance of embracing mistakes, promoting collaborative problem-solving via multiple solution paths, and communicating messages that reinforce a growth mindset. Based on these results, we offer practical recommendations for embedding growth-mindset principles in elementary mathematics instruction.

Keywords: growth mindset, fixed mindset, elementary mathematics instruction

Introduction

Korean students consistently rank highly in mathematics worldwide (Mullis et al., 2020), yet their self-perceptions of mathematical ability tend to be less positive. Research indicates they often exhibit low self-confidence or self-efficacy in mathematics (Seo et al., 2023), which can lead to disengagement or even dropout, underscoring the need to foster more positive beliefs.

Dweck and Leggett (1988) introduced the *implicit theory of intelligence*, distinguishing between it into the *incremental theory* and the *entity theory*. The theory individuals adopt shapes their achievement goals and influences how they respond to effort and failure (Dweck, 2000). Interventions promoting the incremental theory have been shown to improve academic achievement (e.g., Blackwell et al., 2007; Yeager et al., 2019) while also enhancing self-efficacy and reducing learning anxiety (e.g., Samuel & Warner, 2021). Recent research underscores instructional practices that foster a growth mindset (e.g., Murphy et al., 2021; Sun, 2018). Given the widespread belief that mathematical ability is innate, it is crucial to challenge this perception and promote the idea that anyone can develop emphasize that mathematical ability can be developed through effort and practice.

In Korea, discussions on fostering a growth mindset in mathematics education remain limited. This study explores how to cultivate a growth mindset through regular mathematics instruction by designing and implementing strategies for elementary students and analyzing their impact. The findings offer practical implications for integrating growth mindset-oriented instruction in schools.

Theoretical Background

Dweck and Leggett (1988) emphasized the role of self-theories in shaping students' beliefs about intelligence. Students who adopt the incremental theory view intelligence as malleable and improvable, believing effort leads to greater learning and ability (Dweck & Master, 2009). In contrast, those who endorse the entity theory regard intelligence as fixed and unchangeable, believing that little can be done to improve it (Dweck & Master, 2009). Dweck (2006) introduced the term *mindset* to reflect these theories, liking the incremental theory with a *growth mindset* and the entity theory with a *fixed mindset*.

Research indicates that aligning teachers' beliefs with their instructional practices fosters the development of students' mindsets (Sun, 2019). As shown in Table 1(see Kwak, 2025 for details), we identified core instructional elements for cultivating a growth mindset in mathematical ability by drawing on three strands of research: (1) literatures of mathematics instructional practices that promote a growth mindset (Boaler, 2018; Sun, 2018); (2) investigations summarizing students' perceptions of growth-mindset–oriented instruction (Kroeper et al., 2022); and (3) research examining growth-mindset pedagogy characteristics among elementary teachers (Rissanen et al., 2019).

Table 1. Description of the core instructional elements for fostering a growth mindset in mathematical ability

Category	Elements
1. Reflecting and planning lessons from a growth mindset perspective	- Reflecting lessons from a growth mindset perspective
	- Planning lessons from a growth mindset perspective
2. Creating a classroom structure inclusive of diverse mathematical ability levels	- Communicating high expectations for all students without grouping them by ability.
	- Forming learning groups based on diverse factors rather than ability level.
3. Establishing growth mindset mathematics classroom norms	- Emphasizing a growth mindset explicitly
	- Emphasizing the value of mistakes
	- Emphasizing the learning process and effort
4. Pursuing the nature of mathematics	- Providing open-ended tasks
	- Providing opportunities to make mathematical connections
	- Providing opportunities for collaborative problem solving and discussion
5. Assessing and providing feedback to support growth	- Assessing in different ways
	- Supporting growth with feedback on assessments

Methods

This study employed two research methods. First, a case study investigated how elementary mathematics instruction designed to foster a growth mindset was implemented in practice. Second, a survey using self-report measures assessed the instruction's impact on students' mindsets. Additionally, follow-up interviews were conducted to explore factors influencing students' beliefs based on their test results. The case study involved 33 sixth-grade students from two classes at a single elementary school. Their respective classroom teachers, who participated in the study, were identified as having weak growth mindset beliefs; one had only a limited understanding of the concept, while the other had none. The students represented high, middle, and low academic levels and generally low confidence in mathematics. The school's socioeconomic level was classified as middle. The survey included 48 participants: the two case study classes (experimental groups A and B) and a control group from the same school (n=15).

To measure students' mindsets, we used Dweck's (2000) six-item mindset scale for children. The test was administered at three points: before the intervention (pre-test), immediately after (post-test), and four weeks later to assess retention (follow-up test). The results were analyzed to determine whether post-test scores were significantly higher than pre-test scores and whether any observed effects persisted. A two-way repeated measures ANOVA (rm ANOVA) was then conducted to examine the findings.

Results

The case study yielded the following findings. First, regarding "Reflecting and planning lessons from a growth mindset perspective", teachers adapted their instruction based on core elements that foster a growth mindset in mathematics. For instance, under "Providing opportunities for collaborative problem solving and discussion", they addressed students' reluctance to collaborate and the limited time available for discussion by explicitly emphasizing the value of collaboration and adjusting lesson structures to allow for more extensive mathematical discussions. Second, regarding "Creating a classroom structure inclusive of diverse mathematical ability levels", teachers refrained from grouping students by ability. Instead, they assigned the same tasks to all students, with the expectation that everyone would reach the same learning goals. Lower-achieving students were given tasks that offered rich opportunities for reasoning, and groups were formed heterogeneously rather than by ability level.

Third, regarding "Establishing growth mindset mathematics classroom norms", teachers explicitly introduced the concept of a growth mindset, establishing norms that emphasized growth mindset regarding mathematical ability. stressed the value of mistakes, encouraging students to view them as learning opportunities. They also fostered an environment that valued the learning process and effort by identifying student challenges, guiding them toward overcoming difficulties, and offering praise for persistence and progress. Fourth, regarding "Pursuing the nature of mathematics", open-ended tasks were employed to help students experience mathematical connections—linking prior knowledge to new concepts, applying learning to real life, and exploring problem-solving approaches. Teachers encouraged mathematical discussions and expanded opportunities for collaborative problem-solving and deeper student engagement. Fifth, regarding "Assessing and providing feedback to support growth", a range of assessments, including written and oral, was used to help students recognize their mathematical development. Feedback focused on strategies rather than scores, identifying students' methods and guiding them toward more effective problem-solving approaches.

The survey results showed a statistically significant increase in post-test scores compared to pre-test scores at the 0.05 level, and this effect persisted in a follow-up test. Student interviews indicated that changes in their beliefs were influenced by specific growth mindset activities, collaborative problem-solving with peers, encouraging messages from the teacher, and opportunities to reflect on and share mistakes.

Discussion

Based on our analysis of the lesson design and implementation aimed at fostering a growth mindset regarding mathematical ability and their effects on students' mindsets, we offer the following points for discussion. First, providing consistent opportunities for students to reflect on and share mistakes in mathematics class exerts a positive influence on the development of a growth mindset. The survey showed a statistically significant increase in students' mindset scores, and interviews revealed that students attributed this change to activities emphasizing the recognition and sharing of mistakes. Previous studies highlight the importance of valuing mistakes in fostering a growth mindset (e.g., Murphy et al., 2021; Sun, 2018). This study expands on previous findings by examining how teachers facilitate mistake-sharing, identifying potential areas for improvement, and analyzing the impact of teachers' messages about mistakes on students' mindsets.

Second, collaborating with peers to solve open-ended tasks effectively fosters a growth mindset in mathematics. Such collaboration allows students to exchange diverse ideas and refine their problem-solving strategies. In interviews, students reported feeling more mathematically competent when gaining new perspectives. Murphy (2024) suggests that a collaborative culture leverages individual strengths, encouraging creative solutions and reinforcing growth.

Third, fostering a growth mindset requires both teachers and students to exchange supportive messages. Midway through the study, students often expressed beliefs that hindered the adoption of a growth mindset, such as making discouraging comments about their own mathematical abilities or doubting their peers' potential for growth. Anderson et al. (2018) introduced the student mindset component as a framework for analyzing mathematical mindset classrooms, emphasizing the role of teachers' words and actions in cultivating a high expectation learning environment. Building on this, we propose that teachers and students alike must reinforce growth-oriented messages to cultivate a strong mindset culture in the classroom.

References

Anderson, R. K., Boaler, J., & Dieckmann, J. A. (2018). Achieving elusive teacher change through challenging myths about learning: A blended approach. *Education Sciences, 8*(3), 98.

Blackwell, L. S., Trzesniewski, K. H., & Dweck, C. S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. *Child Development, 78*(1), 246-263.

Boaler, J. (2018). *Mathematical Mindset Guide.* Retrieved April, 26, 2025 from https://www.youcubed.org/wp-content/uploads/2018/04/Mindset-Mathematics-Teaching-Guide-FINAL-1.pdf

Dweck, C. S. (2000). *Self-theories: Their role in motivation, personality, and development*. Psychology Press.

Dweck, C. S., & Leggett, E. L. (1988). A social-cognitive approach to motivation and personality. *Psychological Review, 95*, 256-273.

Dweck, C. S., & Master, A. (2009). Self-theories and motivation: Students' beliefs about intelligence. In K. R. Wentzel & A. Wigfield (Eds.), *Handbook of motivation at school* (pp. 123–140). Routledge.

Kwak, G. W. (2025). *Elementary school mathematics instructions to fostering growth mindset regarding mathematical ability* (Doctoral dissertation, Korea National University of Education).

Kroeper, K. M., Muenks, K., Canning, E. A., & Murphy, M. C. (2022). An exploratory study of the behaviors that communicate perceived instructor mindset beliefs in college STEM classrooms. *Teaching and Teacher Education, 114*, 103717.

Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). *TIMSS 2019 International results in mathematics and science.* Retrieved April, 26, 2025 from https://www.skolporten.se/app/uploads/2020/12/timss-2019-highlights-1.pdf

Murphy, M. C. (2024). *Cultures of growth: How the new science of mindset can transform individuals, teams, and organizations*. Simon & Schuster.

Murphy, M. C., Fryberg, S. A., Brady, L. M., Canning, E. A., & Hecht, C. A. (2021). *Global mindset initiative paper 1: growth mindset cultures and teacher practices*. Retrieved April, 26, 2025 from https://migrate-utexas-txbspi.pantheonsite.io/wp-content/uploads/2022/07/Global-Mindset-Initiative-Yidan-Prize-Papers-by-TxBSPI.pdf

Rissanen, I., Kuusisto, E., Tuominen, M., & Tirri, K. (2019). In search of a growth mindset pedagogy: A case study of one teacher's classroom practices in a Finnish elementary school. *Teaching and Teacher Education, 77*, 204-213.

Samuel, T. S., & Warner, J. (2021). "I can math!": Reducing math anxiety and increasing math self-efficacy using a mindfulness and growth mindset-based intervention in first-year students. *Community College Journal of Research and Practice, 45*(3), 205-222.

Seo, M. H., Kim, C. H., Lee, D. W., Han, Soo. Y., & Chon K. H. (2023). Trends and characteristics of affective attitudes in mathematics and science over the past 20 years based on TIMSS. RRE 2023-1. Kore Institute for Curriculum and Evaluation

Sun, K. L. (2018). Brief report: The role of mathematics teaching in fostering student growth mindset. *Journal for Research in Mathematics Education, 49*(3), 330-335.

Sun, K. L. (2019). The mindset disconnect in mathematics teaching: A qualitative analysis of classroom instruction. *The Journal of Mathematical Behavior, 56*, 100706.

Yeager, D. S., Hanselman, P., Walton, G. M., Murray, J. S., Crosnoe, R., Muller, C., ... & Dweck, C. S. (2019). A national experiment reveals where a growth mindset improves achievement. *Nature, 573*(7774), 364-369.

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 7
(Day 3)

TSG7-09

THE TRENDS IN INSTRUCTIONAL APPROACH AND THEIR EFFECTS ON STUDENTS' MATHEMATICS ATTITUDES: AN ANALYSIS OF TIMSS DATA

Rae Yeong Kim
Korea Institute for Curriculum and Evaluation, Korea, pianist9430@kice.re.kr

Sooyun Han
Yonsei University, Korea, syhan@yonsei.ac.kr

This study examines the instructional approaches of middle school teachers in Singapore, Taiwan, and Korea using TIMSS data and investigates their impact on students' mathematics attitudes. The results indicate that teachers' instructional practices can be classified into three distinct classes based on the extent to which they reflect generative (GEN) and teacher-active (TA) approaches, with variations across countries. In Korea, the proportion of classes characterized by higher levels of GEN increased, whereas in Singapore and Taiwan, the proportion of classes with an emphasis on TA increased. The analysis of students' mathematics attitudes revealed that classes with higher levels of GEN were generally associated with more positive attitudes.

Keywords: Instructional approach, Mathematics attitude, Latent class analysis, TIMSS

Introduction

Teachers' instructional approaches have a significant impact on students' mathematics learning (Bokhove, 2022; Magayon & Tabuzo, 2024). The appropriate use of instructional practices can help reduce students' negative attitudes and support them in achieving higher levels of achievement in mathematical (Lenz et al., 2024; Singer et al., 2016; Wakhata et al., 2022).

Recently, there has been an increasing emphasis on diverse instructional methods to overcome the limitations of traditional teacher-centered approaches (Secker, 2002). Student-centered learning methods which emphasize learners' active engagement have gained attention. For example, generative learning is frequently mentioned student-centered instructional approach that promotes conceptual understanding (Brod, 2021). This approach encourages learners not simply to absorb given information but to actively generate information content based on it (Bokhove, 2022).

However, in East Asian countries, students tend to achieve high performance in mathematics, yet their attitudes toward mathematics remain relatively low, and traditional teacher-centered instruction continues to be prevalent (Bokhove, 2022). Therefore, this study aims to analyze the instructional approaches of teachers in Singapore, Taiwan, and Korea, which are known for high mathematics achievement, by applying Latent Class Analysis (LCA) to data from the three most recent TIMSS cycles. In addition, we investigate the impact of teachers' instructional approaches on students' mathematics attitudes. Through this analysis, this study provides insight into recent shifts in instructional practices in Singapore, Taiwan, and Korea, and examines whether differences in approach are reflected in students' mathematics attitudes.

Methodology

Data and Variables

This study utilizes TIMSS 2015, 2019, and 2023 8th-grade mathematics data for Korea, Singapore, and Taiwan, which are the highest performing countries in 8th-grade mathematics in TIMSS 2023.

Teachers' instructional approaches were measured using three distinct scales from the teacher questionnaire in TIMSS 2015, 2019, and 2023: Generative Scale (GEN) and Teacher-Active Scale (TA). The items for each scale was determined by conducting an exploratory factor analysis, referencing Bokhove (2022). Teachers responded to each item using one of four options: Never; some lessons; about half the lessons; and every or almost every lesson.

Students' mathematics attitudes were evaluated through the student questionnaire measuring interest, confidence, and value in mathematics (von Davier et al., 2024). These variables were analyzed using the IRT scales provided by TIMSS, which allow for comparisons across multiple cycles since 2015.

Table 1. Items used for instructional approaches

Item	Question
GEN1	Relate the lesson to students' daily lives.
GEN2	Ask students to explain their answer.
GEN3	Ask students to complete challenging exercises that require them to go beyond the instruction.
GEN4	Encourage classroom discussions among students.
GEN5	Link new content to students' prior knowledge.
GEN6	Ask students to decide their own problem-solving procedures.
TA1	Listen to me explain new mathematics content.
TA2	Listen to me explain how to solve problems.
TA3	Memorize rules, procedures, and facts.
TA4	Work problems together in the whole class with direct guidance from me.

Note. GEN=generative; TA=teacher-active.

Analysis

LCA was conducted to classify teachers based on their instructional approach. For the LCA analysis, "never" and "some lessons" were grouped into one category, and "about half the lessons" and "every or almost every lesson" were grouped together. The three-cycle data were combined for class classification, and separate analyses were performed for each country. Model fit was assessed using AIC, BIC, Entropy, and minimum proportion, and the number of profiles was determined as three to all of three countries. Next, we used the BCH method (Asparouhov & Muthén, 2014) to examine whether students' mathematics attitudes differed across classes for each country. To align with the teacher-level class classification, we used teacher-level averages of students' mathematics attitudes in the analysis. All analyses were performed using R (version 4.3.1), and LCA was conducted using the "MplusAutomation" package (Hallquist & Wiley, 2018) in R.

Results

Classes of Instructional Approach

The classification of teachers into three classes for each country is presented in Figure 1. In Singapore, three distinct classes were identified as follows: Class 1 was characterized by a high level of TA items, Class 2 by an overall low levels, and Class 3 by an overall high levels. Notably, item 5 of the GEN scale showed a high probability across all classes, with Classes 1 and 2 exhibiting probabilities comparable to Class 3. In Taiwan, Class 1 exhibited a high level of TA items, Class 2 a high level of GEN items, and Class 3 a generally high level across both GEN and TA items. Item 3 of the GEN scale showed significantly lower probabilities across all classes compared to other items, indicating that Taiwanese teachers may be more cautious about encouraging students to solve problems that go beyond the instructional content. Lastly, in Korea, TA items showed high probabilities across all classes, suggesting that a teacher-active instructional approach is widely used. The three classes were primarily distinguished by their probabilities of GEN items: Class 1 reflected a low level, Class 2 a moderate level, and Class 3 a high level.

Next, we analyzed the changes in class proportions across the three TIMSS cycles for each country, as illustrated in Figure 2. In Singapore, the proportion of Class 1 increased, while the proportion of Class 2 decreased. The proportion of Class 3 increased in 2019 but declined again in 2023. In Taiwan, the proportion of Class 1 and Class 2 increased, whereas the proportion of Class 3 decreased. This pattern suggests a growing proportion of teachers focusing on either generative learning or the teacher-active approach. However, the overall changes of class membership was relatively small compared to other countries. In Korea, the proportion of Class 1 consistently decreased, while the proportion of Class 3 steadily increased. This trend indicates that the proportion of teachers incorporating generative learning approaches into their instruction has been gradually rising in Korea.

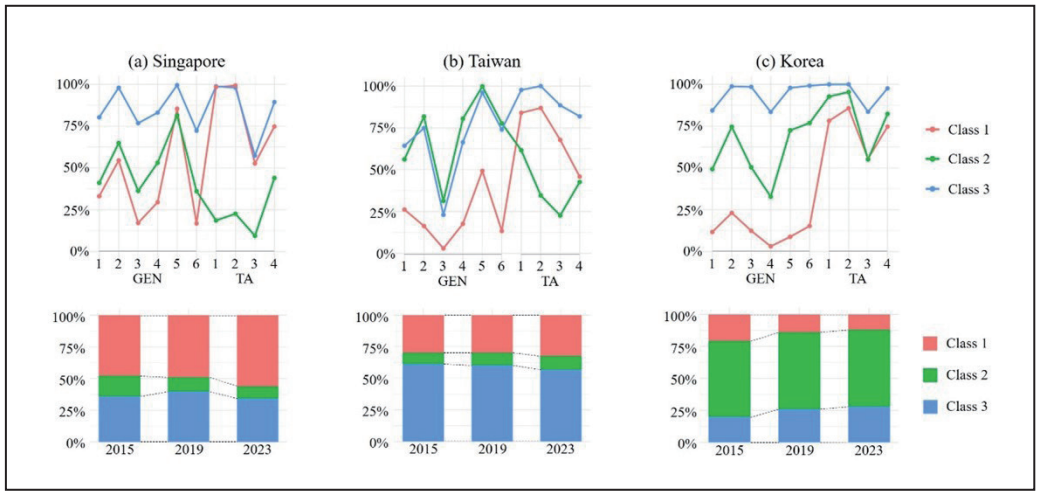


Figure 1. Latent classes of instructional approaches and changes in class membership

Effects on Students' Mathematics attitudes

Finally, we conducted the BCH analysis to examine whether students' mathematics attitudes differed based on their teacher's instructional approach, as shown in Table 2. In Singapore, students' mathematics attitudes were highest in Class 1, followed by Class 2 and Class 3, with a statistically significant difference observed particularly between Class 1 and Class 3. In Taiwan, the highest attitudes were observed in Class 1, followed by Class 3 and then Class 2. The differences between Class 2 and the other classes were statistically significant in most cases. In Korea, although students' mathematics attitudes increased progressively from Class 1 to Class 2 and then to Class 3, none of these differences reached statistical significance.

Table 2. Comparison of students' mathematics attitudes across the classes

Country	Variable	Mean (SE)			*p*-value	Pairwise comparisons
		Class 1	Class 2	Class 3		
Singapore	Interest	10.04 (0.04)	10.19 (0.08)	10.22 (0.05)	0.024 *	1<3
	Confidence	9.56 (0.05)	9.77 (0.10)	9.81 (0.06)	0.004 **	1<3
	Value	9.46 (0.03)	9.57 (0.08)	9.64 (0.05)	0.013 *	1<3
Taiwan	Interest	9.24 (0.06)	9.69 (0.15)	9.26 (0.05)	0.021 *	1,3<2
	Confidence	9.04 (0.06)	9.71 (0.18)	9.10 (0.06)	0.002 **	1,3<2
	Value	8.11 (0.07)	8.51 (0.14)	8.22 (0.06)	0.042 *	1<2
Korea	Interest	9.07 (0.08)	9.11 (0.04)	9.22 (0.08)	0.351	
	Confidence	9.33 (0.08)	9.43 (0.05)	9.51 (0.07)	0.252	
	Value	8.58 (0.07)	8.58 (0.04)	8.63 (0.07)	0.767	

Discussion and Conclusion

This study examined changes in teachers' instructional approach in Singapore, Taiwan, and Korea, and explored the impact of instructional approaches on students' mathematics attitudes. First, teachers were classified into three classes based on GEN and TA levels, with varied specific characteristics across counties. This suggests the heterogeneity among teachers within each country regarding their instructional approaches. Moreover, this heterogeneity manifests differently across countries, highlighting that instructional practices can be influenced by national contexts. Consistent with the argument of Bokhove (2022), this result underscores the importance of analyzing teachers' instructional approaches with attention to contextual differences, even within the East Asian region.

Second, while the proportion of classes with higher levels of GEN increased in Korea, the proportion of classes with an emphasis on TA increased in Singapore and Taiwan. Despite the global emphasis on student-centered instruction, the contrasting pattern observed in Singapore and Taiwan warrants further investigation into the underlying factors. In particular, recent shifts in instructional approaches may have been influenced by contextual events such as the COVID-19 pandemic, underscoring the need for a more in-depth analysis that takes these broader influences into account.

Third, an analysis of students' mathematics attitudes, including interest, confidence, and value, across instructional classes revealed that classes with higher levels of GEN were generally associ-

ated with more positive attitudes. In particular, in Taiwan, the class characterized by high GEN and low TA demonstrated significantly higher average mathematics attitudes than the class with high levels of both GEN and TA. This finding suggests that generative learning approaches may positively influence students' interest, confidence, and value in mathematics. However, the differences in students' mathematics attitudes across classes in Korea were not statistically significant, highlighting the need for additional support to ensure that teachers' instructional efforts result in meaningful improvements in students' mathematics attitudes.

References

Asparouhov, T. & Muthén B. (2014). *Auxiliary variables in mixture modeling: Using the BCH method in Mplus to estimate a distal outcome model and an arbitrary second model*.

Bokhove, C. (2022). Are instructional practices different between East and West? An analysis of Grade 8 TIMSS 2019 data. *Asian Journal for Mathematics Education, 1*(2), 221-241.

Brod, G. (2021). Generative learning: Which strategies for what age?. *Educational Psychology Review, 33*(4), 1295-1318.

Hallquist, M. N., & Wiley, J. F. (2018). MplusAutomation: An R package for facilitating large-scale latent variable analyses in Mplus. *Structural Equation Modeling: A Multidisciplinary Journal, 25*(4), 621-638.

Lenz, K., Obersteiner, A., & Wittmann, G. (2024). Who benefits most from language-responsive learning materials in mathematics? Investigating differential effects in heterogeneous classrooms. *Educational Studies in Mathematics, 116*. 185-221

Magayon, V.C. & Tabuzo, V. (2024). Least mastered skills and effective instruction in teaching general mathematics in the Philippines. *Cypriot Journal of Educational Science, 19*(4), 333-354.

Secker, C. V. (2002). Effects of inquiry-based teacher practices on science excellence and equity. *The Journal of Educational Research, 95*(3), 151-160.

Singer, F. M., Sheffield, L., Freiman, V., & Brandl, M. (2016). *Research on and activities for mathematically gifted students*. New York: Springer Nature.

von Davier, M., Kennedy, A., Reynolds, K., Fishbein, B., Khorramdel, L., Aldrich, C., Bookbinder, A., Bezirhan, U., & Yin, L. (2024). *TIMSS 2023 International Results in Mathematics and Science*. Boston College, TIMSS & PIRLS International Study Center. https://doi.org/10.6017/lse.tpisc.timss.rs6460

Wakhata, R., Mutarutinya, V., & Balimuttajjo, S. (2022). Secondary school students' attitude towards mathematics word problems. *Humanities and Social Sciences Communications, 9*(1), 1-11.

TSG7-10

IDENTIFYING APPRAISAL PROFILES AND THEIR RELATIONS WITH ACADEMIC EMOTIONS IN MATHEMATICS

Xin Chen
East China Normal University, China, xchen@math.ecnu.edu.cn

Drawing on Control-Value Theory of academic emotions, this study used a person-centered method to explore the latent profiles of students' control-value appraisals, and the relations between appraisal profiles and academic emotions were also examined. The data were collected from 1900 students (50.9% male; Mage = 13.88, SD = 1.16) in 76 mathematics classrooms across 11 secondary schools in Jiangsu, China. Latent profile analysis identified four patterns based on control-value appraisals: extrinsic-value orientation (4.9%), negative (13.4%), control orientation (41.3%), and positive (40.4%). Students with positive profiles showed the highest positive emotions and the lowest negative emotions, followed by those with control orientations. Whereas, those with extrinsic value orientation displayed the lowest positive emotions and the highest negative emotions, followed by negative profiles.

Keywords: academic emotions; control; value; latent profile analysis; mathematics

Introduction

The importance of affect-related factors in academic situations has been recognized by many theories since the positive psychology movement, such as Control-Value Theory (CVT; Pekrun, 2006), Self-Determination Theory (Deci & Ryan, 2002), and Expectancy-Value Theory (EVT; Wigfield & Eccles, 2000). These theories contend that students are emotional beings who react subjectively to the events and people in the learning environment. To expand, the quality and intensity of emotion and motivation is not only dependent on the situation itself, but also on how the individual views the situation based on a set of appraisal dimensions. The interplay of cognitive appraisals (i.e., academic control and value) and emotions in various accomplishment contexts, for instance, is a major area of interest for CVT. Numerous studies have clarified the relationship between control-value evaluations and academic emotions (e.g., Berweger et al., 2022; Chen & Lu, 2022; Hall et al., 2016). Notwithstanding, a large portion of earlier research has relied on variable-centered methods that do not sufficiently take into account the psychological fact that several cognitive appraisals happen concurrently and are interconnected. In order to address this complexity, a person-centered approach could identify common combinations of these appraisals. This study draws on CVT to examine the profiles based on academic control (or self-concept) and value appraisals in mathematics and then examine whether the diverse profiles may be associated with different academic emotions. That not only helps to understand students' affective performance in mathematics, but also the distribution of students' academic emotions.

Method

Participants

This study investigated middle school students in mathematics classrooms from Jiangsu province, China for analysis. A total of 1900 questionnaires were collected, and 1762 valid data were used in this study. Participants were from 68 mathematics classrooms in 11 secondary schools (Mage = 13.88, SD = 0.84). There were 893 boys (50.7%) and 868 girls (49.3%), and one student failed to report his/her gender. Besides, 396 students (22.5%) from Grade 7, 846 students (48.0%) from Grade 8 and 520 students (29.5%) from Grade 9.

Measures

All studied variables were assessed with a 5-point-Likert scale with a range from 1 (strongly disagree) to 5 (strongly agree). Academic emotions were assessed with 18 items, using the Achievement Emotions Questionnaire (AEQ; Pekrun et al., 2002). Academic control was selected and assessed with the "Self-Efficacy for Learning and Performance" scale in the Motivated Strategies for Learning Questionnaire (MSLQ; Pintrich, 1991) using 3 items. Academic value, including intrinsic and extrinsic value, was assessed with 7 items, which are modified from TIMSS 2019 and the Task Value in the Motivated Strategies for Learning Questionnaire (MSLQ; Pintrich, 1991). All involved variables were examined in the mathematics classroom context. The Cronbach alpha (α) of above measured variables ranged from .747 to .947.

Data Analysis

To find out how many profiles were reflected in the control and value assessments of the students, latent profile analysis (LPA) was used with Mplus 8.3 (Hodis & Hodis, 2020). With 200 random starts and 50 first iterations, all LPA models were freely estimated. The ideal number of profiles was chosen by taking into account statistical analyses, conceptual underpinning, and interpretability of the profiles. Besides, the automatic Bolck-Croon-Hagenaars (BCH) technique was employed in this study to examine differences of appraisal profiles in academic emotions.

Results

Identifying Students' Appraisal Profiles

Table 1 presents the fit indices for the LPA models with one to five latent profiles for control-value appraisals. The minimal sample size was already less than 50 in the fifth profile, hence profiles after this point were no longer taken into consideration. Although the model with four profiles was not significantly different from the model with three profiles (pLMR = 0.22), the fit indices were still significantly better for the four profiles than for the three profiles (e.g., AIC, BIC, aBIC, LMR). Thus, the four-profile solution was finally used in this study.

Table 1. Model fit results of latent profile analyses

Profiles	AIC	BIC	aBIC	Entropy	LMR	*p*LMR	*No.* of smallest class
1	11592.95	11625.79	11606.73	-	-	-	1762
2	9947.06	10001.80	9970.03	0.84	1600.36	0.00	876
3	9253.51	9330.15	9285.67	0.91	678.84	0.00	153
4	8950.94	9049.48	8992.29	0.93	300.52	0.22	86
5	8494.66	8615.09	8545.20	0.95	357.22	0.00	36

As shown in Table 2, students in the profile with the extrinsic value orientation reported a high level of extrinsic value accompanied by the lowest academic control and relatively low intrinsic value (N = 86, 4.9%). Students in the negative profile reported the lowest value appraisals and relatively low control appraisals (N = 236, 13.4%). Students in the profile with the control orientation showed moderately high control and mildly low-value appraisals (N = 728, 41.3%). Lastly, students with a positive profile showed the highest intrinsic value and academic control along with relatively high extrinsic value (N = 712, 40.4%).

Table 2. Latent profiles with factor mean scores of indicators

	EV-oriented (P1)	Negative (P2)	Control-oriented (P3)	Positive (P4)
Intrinsic value	-0.12	-1.10	-0.11	1.33
Extrinsic value	0.77	-1.30	-0.27	0.80
Control	-1.10	-0.42	0.28	1.24

Profile Differences in Academic Emotions

As displayed in Table 3, the four appraisal profiles significantly differed in five academic emotions. Students in the positive profile scored the highest for both positive emotions, followed by the control-oriented and negative profiles. Students with the extrinsic value orientation reported the lowest positive emotions. Whereas, students in the extrinsic value-oriented profile scored highest for all three negative emotions, followed by the negative and control-oriented profiles in that order. Students with a positive profile reported the lowest negative emotions.

Table 3. Factor mean differences in emotions among the four profiles

	D (P1-P2)	D (P1-P3)	D (P1-P4)	D (P2-P3)	D (P2-P4)	D (P3-P4)
Enjoyment	-0.17	-0.60***	-1.20***	-0.43***	-1.03***	-0.60***
Pride	-0.13	-0.55***	-1.12***	-0.42***	-0.99***	-0.57***
Anxiety	0.69***	0.74***	1.04***	0.05	0.35***	0.29***
Anger	0.25*	0.49***	1.03***	0.24***	0.78***	0.54***
Boredom	0.31*	0.57***	1.22***	0.26**	0.91***	0.66***

Note. D = difference between the factor mean scores for the different profiles; *$p < .05$, ** $p < .01$, *** $p < .001$

Conclusions and implications

This study identified four profiles drawing on academic control-value appraisals and examined their connections with academic emotions in mathematics. First, the positive profile displayed a desirable pattern of academic emotions in mathematics, supporting variable-centered studies that found students with strong control and value perceptions reporting more positive emotions and fewer negative emotions (Chen & Lu, 2022; Hall et al., 2016). Second, there is a large portion of students in the control-orientation profile. Those students have a little high level of control in learning mathematics and a relatively low level of value perceptions, which requires extra attention. Besides, their emotional outcomes were slightly inferior to students in the positive profile. As such, it is important to shift students from the control-oriented profile to the positive profile. According to Parrisius et al. (2020), conceptual understanding of mathematics is fundamentally linked to students' perceived value of mathematics. As a result, instead of simply introducing formulas and theorems, the teacher needs to go into great detail, such as linking mathematical knowledge with real-world applications to help students grasp the practical significance of mathematics. Third, students with extrinsic-value orientation and maladaptive profiles requires some attention because they do not perform well emotionally. These students are helpless in their studies and need patient guidance from parents and teachers. Thus, teachers and parents should pay much attention to those students by providing emotional and academic support. Overall, this study shows that mathematical control and value play an important role in students' emotions; besides, there is a serious lack of values in mathematics among current Chinese students that requires extensive attention.

References

Berweger, B., Born, S., & Dietrich, J. (2022). Expectancy-value appraisals and achievement emotions in an online learning environment: Within-and between-person relationships. *Learning and Instruction, 77*, Article 101546.

Chen, X., & Lu, L. (2022). How classroom management and instructional clarity relate to students' academic emotions in Hong Kong and England: A multi-group analysis based on the control-value theory. *Learning and Individual Differences, 98*, 102183.

Deci, E. L., & Ryan, R. M. (2002). *Handbook of self-determination research.* University Rochester Press.

Hall, N. C., Sampasivam, L., Muis, K. R., & Ranellucci, J. (2016). Achievement goals and emotions: The mediational roles of perceived progress, control, and value. *British Journal of Educational Psychology, 86*(2), 313-330.

Hodis, F. A., & Hodis, G. M. (2020). Patterns of motivation and communication in learning environments: A latent profile analysis. *Social Psychology of Education, 23* (6), 1659–1685.

Parrisius, C., Gaspard, H., Trautwein, U., & Nagengast, B. (2020). The transmission of values from math teachers to their ninth-grade students: Different mechanisms for different value dimensions?. *Contemporary Educational Psychology, 62*, Article 101891.

Pekrun, R. (2006). The control-value theory of achievement emotions: Assumptions, corollaries, and implications for educational research and practice. *Educational Psychology Review, 18*, 315–341.

Pekrun, R., Goetz, T., Titz, W., & Perry, R. P. (2002). Academic emo- tions in students' self-regulated learning and achievement: A program of qualitative and quantitative research. *Educational Psychologist, 37*(2), 91–105.

Pintrich, P., Smith, D., Garcia, T., & McKeachie, W. (1991). *A Manual for the Use of the Motivated Strategies for Learning Questionnaire (MSLQ)*. Ann Arbor, MI.

Wigfield, A., & Eccles, J. S. (2000). Expectancy–value theory of achievement motivation. *Contemporary educational psychology, 25*(1), 68-81.

TSG7-11

AFFECTIVE COMPUTING AND NLP-DRIVEN METHODOLOGIES FOR MEASURING PROBLEM-SOLVING IN MATHEMATICS EDUCATION

I-Ping Wan
National Chengchi University, Taiwan, ipwan@mail2.nccu.tw

Yuan-Horng Lin
National Taichung University of Education, lyh@mail.ntcu.edu.tw

This paper presents a quantitative and interdisciplinary approach to assessing students' problem-solving abilities in mathematics education by integrating affective computing into evaluation and assessment processes. Through a series of web-based PsychoPy experiments, the study investigates how emotional and cognitive states manifest during mathematical reasoning, with a focus on bridging the gap between human reasoning and AI capabilities in complex tasks. Drawing on affective computing techniques, this study employs both acoustic and facial recognition technologies to analyze students' responses to numerical, textual, and open-ended mathematical problems. Audio and video data were collected using Python-based platforms, and emotional states were inferred from features such as pitch, jitter, and shimmer, as well as facial expressions identified via DeepFace frameworks. Rather than relying on static performance metrics alone, this multimodal approach captures dynamic elements of learning such as hesitations, tone modulation, and expression patterns that reveal students' engagement and stress. While neutral affect was most commonly observed, nuanced emotional shifts corresponded with task complexity and reflective moments. By extracting and analyzing these affective signals during math-related interviews, the study contributes to the development of emotion-sensitive assessment frameworks and enriches our understanding of the role emotion plays in shaping cognitive performance. This work holds implications for both affective computing research and AI-enhanced educational tools that adapt to learners' emotional and cognitive states in real time.

Keywords: Affective computing, NLP-driven methodologies, Mathematics education

Introduction

The rapid advancement of educational technology and pedagogy in Taiwan has expanded opportunities for improving mathematics education. Recent research highlights integrated strategies such as technology use, culturally responsive teaching, self-regulated learning, and teacher development that enhance learning outcomes (e.g., Lin & Wu, 2018; Chiu et al., 2022; Cheng, 2022; Chin, 2023; Weng, 2023; Wang et al., 2024; Lin, 2022–2024; Yang et al., 2024). Scholars emphasize the affective and non-cognitive aspects of learning, such as values and emotions (e.g., Hannula, 2002; Pintrich, 2003; Seah, 2019; Chiu et al., 2022; Chiu & Seah, 2024; Wan & Yu, 2024). Building on these insights, this study applies a quantitative approach using voice spectrograms and speech metrics to analyze emotional signals in students' spoken math responses. These features including tonal shifts, hesitations, and intensity support a multimodal view of cognitive and emotional engagement in problem-solving contexts.

Methodological Approaches

Cherian et al. (2024) revealed gaps between human and AI performance in math, with humans excelling in geometry and logic, while GPT-4 performs well in text-based reasoning but lags in visual and logical tasks. These findings underscore the need to strengthen cognitive skills in math education; NLP methods analyze speech production using transcripts and acoustic features, with tools like segmentation and classification identifying pauses and fillers critical for modeling real-time dialogue and planning.

This study included three sequential online experiments using PsychoPy/Pavlovia. Participants completed setup checks for microphone and webcam, and all audio/video responses were auto-uploaded; Trials used randomized items with brief pauses. Experiment 1 featured multiple-choice math formulas; Experiment 2 used text-and-figure reasoning; Experiment 3 collected 20-second open-ended reflections. Speech data capturing pitch (F_0), jitter, shimmer, and disfluencies like pauses and fillers ([uh], [um]) were analyzed to assess cognitive and emotional states during problem-solving (Maclay & Osgood, 1959; Goldman-Eisler, 1961, 1968; Rochester, 1973; Levelt, 1983; Mahl, 1987; Shriberg, 1994; Clark & Tree, 2002; Vasilescu et al., 2007).

Initial Analyses of Pilot Studies

In an ongoing study conducted in mathematics classrooms in Taiwan involving 38 elementary school students (N=38; 21 girls, 17 boys; Mean age= 7.2, SD=0.43), initial analyses reveal systematic patterns in the incorporation of the aforementioned elements as linguistic features for emotion recognition. To explore emotional aspects of math problem-solving, both vocal and facial features were analyzed. Audio data underwent acoustic analysis (e.g., pitch, jitter, shimmer), while video data were processed using DeepFace (Serengil & Ozpinar, 2020) and dlib's 68-point facial landmark predictor (King, 2009). Emotions were classified based on Ekman's (1999) basic categories. Word-level features included pitch, formants, energy, timing, and articulation. Pilot interviews offered insight into children's emotional responses during tasks.

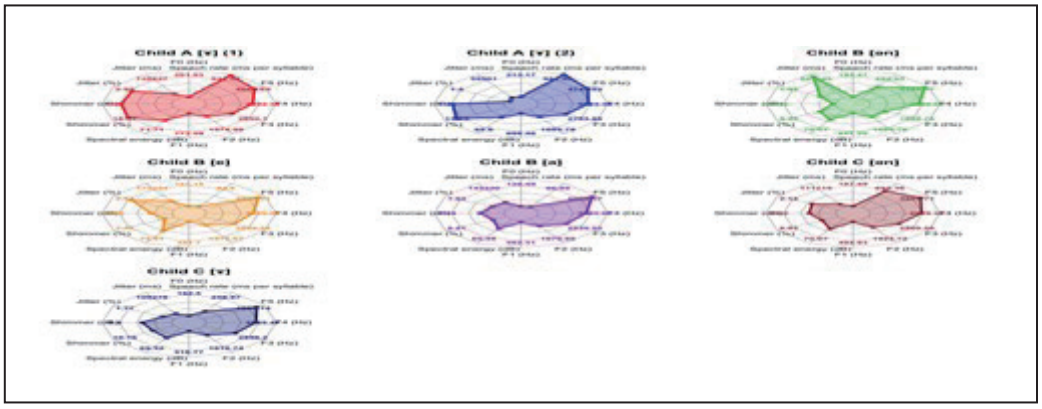


Figure 1. Radar charts in children's speech with linguistic features

The radar charts present acoustic features that detail children's speech and emotional dynamics. Key parameters include pitch (F_0), jitter, shimmer, spectral energy, and formants (F1–F5), indicating vocal tone, stability, amplitude, loudness, and articulation. Speech rate (ms/syllable) reflects pacing

and cognitive load. Hesitation markers and filled pauses such as [ɤ], [ən], [e], and [a] in Mandarin are also analyzed as indicators of planning and affect.

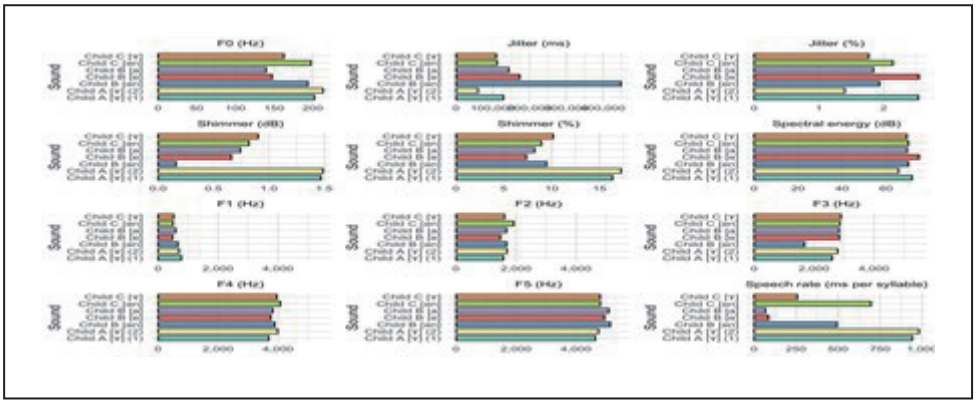


Figure 2. Components of acoustic parameters in mathematical context

These radar charts visualize acoustic data from three children, focusing on speech features like jitter, shimmer, spectral energy, and speech rate. Child A shows high variability and slower rate, suggesting frustration. Child B displays moderate, stable features, indicating calmness. Child C exhibits elevated jitter and slower speech, pointing to hesitation and cognitive load. These visualizations support emotion tracking in tasks across geometry, number sense, logic and related domains.

Concluding Remarks

Understanding how hesitations and speech elements naturally occur in emotional contexts during math learning provides valuable insights for developing emotionally intelligent systems. In affective computing, recognizing emotional cues in hesitations is key to building responsive and adaptive human-machine interfaces. Knowledge of emotional aspects of speech aids in designing systems that adjust to students' affective states. Initial emotion detection results show a strong prevalence of neutral expressions, with instances of sadness, happiness and anger emerging in complex or reflective tasks. Integrating audio-visual emotion data opens possibilities for triangulating affective cues in STEM learning and advancing emotion-aware educational technologies.

This study adopts acoustic features, pitch, jitter and shimmer, as indicators of emotional states during students' problem-solving. These paralinguistic features are central in affective computing and emotion recognition. Pitch reflects vocal fold vibration and tends to rise under stress or excitement, while flattening in sadness or boredom. Jitter measures pitch instability during emotional arousal, and shimmer reflects amplitude variability, often elevated in anger or anxiety. Together, these features form a quantifiable profile of vocal behavior, revealing emotional valence and arousal during cognitively demanding tasks. Analyzing these acoustic fluctuations captures subtle affective signals, supporting emotion-sensitive assessment in math education. Insights into hesitations and disfluencies also enhance affective computing models, fostering more empathetic and context-aware systems for educational and interactive applications. Advancements in the treatment of disfluencies within natural language processing contribute to more natural and human-like interactions, though current findings may be influenced by the limited dataset and require further investigation.

References

Cheng C.-M. (2022). *Research and Development of Mathematical Financial Literacy Assessment Tools and Build Norm for Fifth to Eighth Grade Students*. National Science and Technology Council, Taiwan.

Cherian, A., Peng, K. C., Lohit, S., Matthiesen, J., Smith, K., & Tenenbaum, J. (2024). Evaluating Large Vision-and-Language Models on Children's Mathematical Olympiads. *Advances in Neural Information Processing Systems*, *37*, 15779-15800.

Chin E.-T. (2023). *The Instructional Design of Integrating Creative Problem Solving with Mathematical Writing into Senior High School Mathematics and Its Influence on High Achieving Students' Mathematical Literacy*. National Science and Technology Council, Taiwan.

Chiu, M. S., Lin, F. L., Yang, K. L., Hasumi, T., Wu, T. J., & Lin, P. S. (2022). The interplay of affect and cognition in the mathematics grounding activity: Forming an affective teaching model. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(12), em2187.

Chiu, M. S., & Seah, W. T. (2024). Values and valuing pedagogies in affect-focused mathematics teaching. *Social Sciences & Humanities Open*, *10*, 101050.

Clark, H. H., & Tree, J. E. F. (2002). Using uh and um in spontaneous speaking. *Cognition*, *84*(1), 73-111.

Ekman, P., Dalgleish, T., & Power, M. (1999). Basic emotions. San Francisco, U.S.A.

Eyben, F., Scherer, K. R. Schuller, B. W., Sundberg, J., Andre, E., Busso, C., …& Truong, K., P. (2015). The Geneva minimalistic acoustic parameter set (GeMAPS) for voice research and affective computing. *IEEE Transactions on Affective Computing*, 7(2), 190-202.

Goldman-Eisler, F. (1961). The significance of changes in the rate of articulation. *Language and Speech*, *4*(3), 171-174.

Goldman-Eisler, F. (1968). *Psycholinguistics: Experiments in Spontaneous Speech. London: Academic Press.*

Hannula, M. S. (2002). Attitude towards mathematics: Emotions, expectations and values. *Educational Studies in Mathematics*, *49*(1), 25-46.

Juslin, P. N., & Laukka, P. (2003) Communication of emotions in vocal expression and music performance: Different channels, same code? Psycholinguistic Bulletin, 129(5), 770.

Levelt, W. J. (1983). Monitoring and self-repair in speech. *Cognition*, *14*(1), 41-104.

Lin, Y.-C. (2022-2024). *Investigating Pre-Service Elementary Mathematics Teachers' Noticing through Eye-Tracking: Curricular Noticing, Noticing of 360-Degree Videos and In-The-Moment Noticing*. National Science and Technology Council, Taiwan.

Lin, Y. -H. & Wu, S. P. (2018) Clustering Assessment on Statistical Literacy Performance of Primary School Students. Proceedings of *the 8th ICMI-East Asia Regional Conference on Mathematics Education*, National Taiwan Normal University, Taipei.

Lin, Y.-H. (2024). *Investigation on Effective Instruction of Questioning and Feedback for Students' Mathematical Cognition, Affect and Conation: Longitudinal Study of Professional Learning Community*. National Science and Technology Council, Taiwan.

Maclay, H., & Osgood, C. E. (1959). Hesitation phenomena in spontaneous English speech. *Word*, *15*(1), 19-44.

Mahl, G. F. (1987). Everyday disturbances of speech. In *Language in psychotherapy: Strategies of Discovery*. 213-269. Boston, MA: Springer US.

Pintrich, P. R. (2003). A motivational science perspective on the role of student motivation in learning and teaching contexts. *Journal of educational Psychology*, *95*(4), 667.

Rochester, S. R. (1973). The significance of pauses in spontaneous speech. *Journal of Psycholinguistic Research*, *2*, 51-81.

Serengil, S. I., & Ozpinar, A (2020, October). Lightface: A hybrid deep face recognition framework. In 2020 *Innovations in Intelligent Systems and Applications Conference (ASYU)* (pp.1-5).

Scherer, K. R. (2003). Vocal communication of emotion: A review of research paradigms. *Speech Communication*, 40(1-2), 227.256.

Seah, W. T. (2019). Values in mathematics education: Its conative nature, and how it can be developed. *Research in Mathematical Education*, *22*(2), 99-121.

Shriberg, E. E. (1994). Preliminaries to a theory of speech disfluencies, Ph.D. dissertation, University of California, Berkeley.

Vasilescu, I., Nemoto, R., & Adda-Decker, M. (2007). Vocalic hesitations vs vocalic systems: a cross-language comparison. Proceedings of *16th International Congress of Phonetic Sciences. University of Saarland*, Germany.

Wan, I-P. & Yu, P. (2024). Exploring disfluency patterns in EMI math education. Paper presented at *the Sixteenth International Conference on Technology and Mathematics Education and Workshop of Mathematics Teaching*, National Taichung University of Education, Taichung, Taiwan.

Wang, T. Y., Yang, K. L., & Lin, F. L. (2024). Exploring the Feasibility of Co-Construction among Mathematics Teachers and Teacher Educators: Analysis of Discourse in a Product-Based Teacher Professional Development Program. *Mathematics Teacher Education and Development*, *26*(1), 3.

Weng, T.-S. (2023). *Exploring the Learning Outcomes of Metaverse Technology-Assisted Business Administration Statistics*. National Science and Technology Council, Taiwan.

Yang, K. L., Krawitz, J., Schukajlow, S., Yang, C. C., & Chang, Y. P. (2024). German and Taiwanese secondary students' mathematical modelling task value profiles and their relation to mathematical knowledge and modelling performance. *European Journal of Psychology of Education*, *39*(3), 2969-2989.

TSG7-12

DEVELOPMENT OF TEACHING AND LEARNING MODEL USING META-AFFECT FOR COLLABORATIVE PROBLEM SOLVING IN MATHEMATICS

Bumi Kim
Wonkwang University, Republic of Korea, bmkim@wku.ac.kr

The purpose of this study is to develop a teaching and learning model for school mathematics that leveragrs meta-affect to enhance middle school students' collaborative problem solving abilities in mathematics. To this end, we developed collaborative problem-solving materials based on student-to-student interactions in math classes and a math journal that provides self-reporting of meta-affect and metacognition. Then, we conducted a case study of applying the CPS-M teaching and learning model to a total of 10 students in the first grade of middle school to verify the applicability of the model in the field. The results of the study showed that students utilized meta-affect to recognize and regulate their emotions in the process of understanding and exploring the meaning of the problem and sharing problem-solving strategies. In addition, when students were tested on meta-definitions before and after applying the lesson model, they showed an increase in the level of self-awareness of affect and emotion regulation among the factors of meta-affect.

Keywords: collaborative problem solving, meta-affect, math diary, teaching model, metacognition

Research Purpose

The DeSeCo project, OECD PISA, and the Assessment and Teaching of 21st Century Skills (ATC21S) project (2010) have emphasized collaborative problem-solving (CPS) as a competency required for students who will be the leaders of the future society. This study aims to develop a teaching and learning model that can be utilized in middle school school settings by utilizing meta-affect to promote collaborative problem-solving.

Collaborative Problem Solving, Meta-affect, and Metacognition

ATC21S project (2010) defines collaborative problem solving as the act of expressing ideas and actively approaching problems, in which members work together to achieve a common goal (Hesse et al. 2015). The OECD PISA 2015 defines collaborative problem solving as the ability of individuals to effectively participate in a process in which two or more virtual characters draw on their knowledge, skills, and efforts to solve a problem, and share their understanding and efforts (OECD, 2013).

Goldin (2002), Schlöglmann (2005), Gómez-Chacón (2000), and DeBellis & Goldin (2006) have shown that utilizing meta-affect is a useful tool for understanding students' affective responses, mathematics learning strategies, and beliefs about mathematics. Goldin (2002), Schlöglmann (2005), Gómez-Chacón (2000), and DeBellis & Goldin (2006) show that utilizing meta-affect is a useful tool for understanding students' affectional responses and their math learning strategies and mathematical beliefs. In a study by Kim (202), the structural relationships between students' affect, meta-affect, and meta-cognition in mathematics learning were analyzed using structural equations. Affect in mathematics is defined by knowing value, interest, and confidence, whereas meta-affect

involves affective awareness, evaluation, regulation and using. Meta-cognition is characterized by critical thinking, rehearsal, self-regulation, elaboration and organization. This study revealed that the more positive the students' interest, confidence, and recognition of value in mathematics learning, the higher was the meta-affect associated with affect.

Based on this literature review, this study defines Collaborative Problem Solving of Mathematics (CPS-M) as "the ability to solve a given problem collaboratively by using knowledge and skills of mathematics in a problem situation where the solution is not known, by communicating with peers to explore, share, and select the best solution by communicating and sharing solution strategies.

Research Methods

The present study received approval from the Institutional Review Board (IRB) of the Wonkwang University Bioethics Committee. The CPS-M class model (program) was subsequently applied to one middle school each in D City and G City, with participation from 10 students who voluntarily agreed to take part in the study and two mathematics teachers. The program was administered twice in first-year middle school mathematics classes at both schools, each conducted in a 90-minute block session format. To investigate the effectiveness of the class program, a teaching experiment was conducted, followed by video analysis and discourse analysis. Supplementary data were also collected through interviews with participating students and teachers and a post-class evaluation form. Details regarding the contents, scope, and research methods of this study are provided in <Table 1>.

Table 1. Research contents and methods

Research Content	Research Scope	Research Methods
Exploring the definition and sub-elements of CPS-M	▪ Exploring the definition of CPS-M and extracting sub-elements	▪ Literature study
Exploring the definition and sub-elements of CPS-M	▪ Development of a collaborative problem-solving ability checklist considering meta-affect ▪ Development of strategies to enhance teacher-student interaction and student-student interaction for CPS-M ▪ Development of teaching and learning materials utilizing meta-affect and metacognition for CPS-M ▪ Application of CPS-M teaching and learning model to middle school mathematics class ▪ Analysis of learning discourse, analysis of interaction between students and teachers, analysis of students' output	▪ Design-Based Research Method (Cobb, 2013) ▪ Teaching experiment ▪ Discourse analysis ▪ Observation assessment ▪ Clinical interview

Results

The CPS-M teaching and learning model was developed based on the collaborative problem solving process outlined in PISA 2015 and the work of Lee and Yoon (2020), which adapted this process and its components for mathematics instruction. Additionally, teaching and learning strategies and materials were designed to enhance student engagement and interaction in collaborative problem-solving while fostering experiences of meta-affect and metacognition. The CPS-M teaching and learning model is presented in [Figure 1].

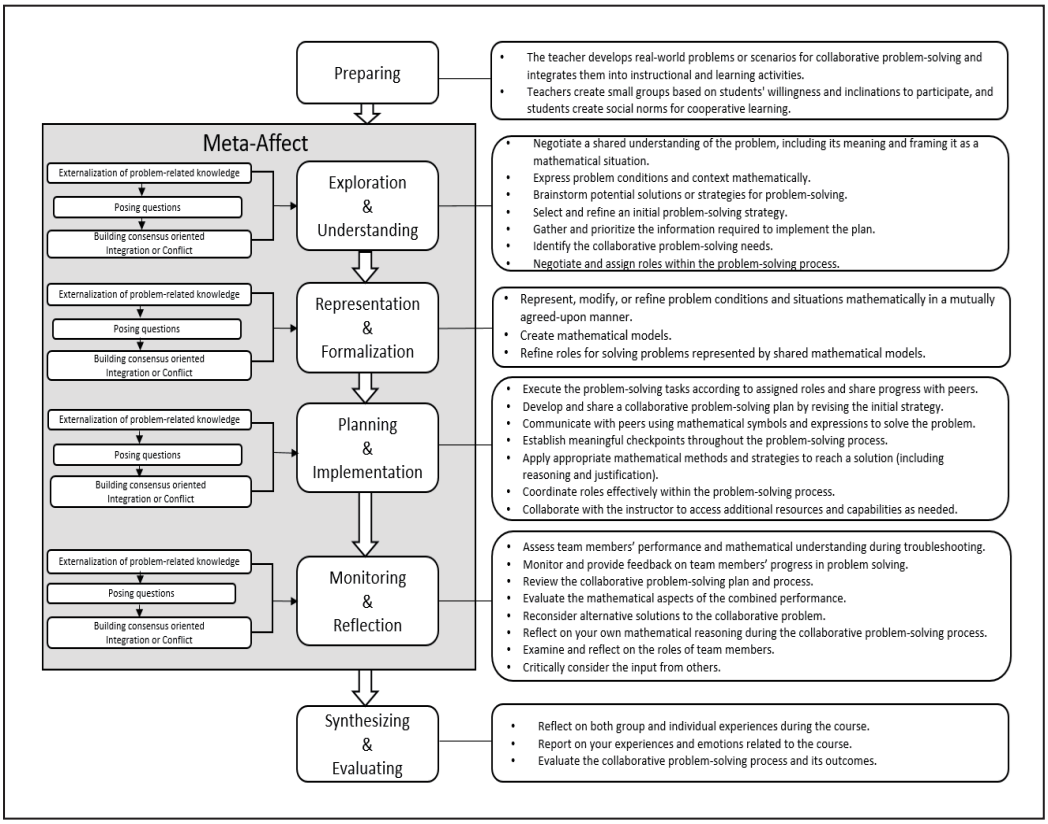


Figure 1. The structure of the CPS-M teaching and learning model

As a result of applying the CPS-M teaching and learning model, students effectively engaged in the collaborative problem-solving process. Throughout the stages of Exploration and Understanding, Representation and Formalization, and Planning and Implementation, they actively externalized problem-related knowledge, posed questions, and reached consensus, leading to a positive shift in their meta-affect. During the Monitoring and Reflection phase, students demonstrated increased questioning and metacognitive thinking. Furthermore, pre- and post-test results on meta-affect and metacognition indicated notable improvements: seven and four students showed significant progress in Emotion Regulation and Use of Definitions, respectively, while six and three students exhibited notable gains in Affect Awareness and Emotion Evaluation, respectively.

Conclusions

The CPS-M teaching and learning model, along with the instructional materials developed in this study, can serve as valuable resources for facilitating collaborative problem-solving in mathematics classrooms. Notably, the CPS-M model introduces a novel approach by incorporating meta-affect, while the self-reported mathematics diary supports students in recognizing affect, assessing and regulating emotions, and effectively applying affect in mathematical learning.

Future research directions are as follows. First, it is necessary to assess students' meta-affect achievement levels following the implementation of the CPS-M teaching and learning model and to develop instructional materials that enable teachers to provide targeted guidance based on these levels. Second, as suggested by DeBellis and Goldin (2006), further research is needed on teaching and learning strategies that foster meta-affect, ensuring that emotions associated with challenges in mathematics contribute productively to students' learning and achievement.

References

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., Schauble, L. (2003). Design experiments in educational research. Educational Researcher, 32, 1:9–13.

DeBellis, V. A. & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: a representational perspective. Educational Studies in Mathematics, 63, 131-147.

Gomez-Chacon, I. M. (2000). Affective influence in the knowledge of mathematics. Educational Studies in Mathematics, 43, 149-168.

Goldin, G. A. (2002). Affect, Meta-Affect, and Mathematical Belief Structures. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), Beliefs: A hidden variable in mathematics?(pp.59–72). Springer.

Hesse, F., Care, E., Buder, J., Sassenberg, K., & Griffin, P. (2015). A framwork for teachable collaborative problem solving skills. In P. Groffin & E. Care(Eds.). Assessment and teaching of 21st century skills: Methods and approach. (pp. 37-56). New York, NY: Springer.

Kim, S. H. (2020). Development and Validation of Meta-affect Inventory in Mathematics. Research Institute of Curriculum Instruction, 24(2), 178-188.

Langer-Osuna, J. M. (2016). The social construction of authority among peers and its implications for collaborative mathematics problem solving. Mathematical Thinking and Learning, 18(2), 107–124.

Lee, J. H. & Yoon, M. S. (2020). Delphi method looking for factors of collaborative problem solving competencies and collaborative metacognition in Mathematics. The Journal of Learner-Centered Curriculum and Instruction, 20(23), 693-719.

OECD (2013). PISA 2015 Draft collaborative problem solving framework, Paris: OECD.

Schloglmann, W. (2005). Meta-affect and strategies in mathematics learning. In M. Bosch (Ed), Proceeding of CERME-4 (pp. 275-284). Barcelona: FundEmi IQS.

• This work was supported by the Ministry of Education of the Republic of Korea and the National Research Foundation of Korea(NRF-2020S1A5A2A01044127)

TSG7-13

THE STUDY OF PRIMARY SCHOOL STUDENTS' OPINIONS IN MATHEMATICAL LITERACY-ORIENTATION QUESTIONS

Jing-Wen Chiu
Taoyuan Municipal Bade Elementary School, Taiwan, bobby6820chiu@gmail.com

Chia-Jui Hsieh
National Taipei University of Education, Taiwan, paris@tea.ntue.edu.tw

Countries universally recognize that mathematical literacy has gained significance in mathematics education. Many countries, including Taiwan, have incorporated mathematics literacy into their lesson plan and mathematics assessments. However, it remains unclear whether students appreciate this trend and the mathematics literacy questions. This study aims to explore this issue from the students' perspective. A questionnaire survey was conducted, targeting 156 Taiwanese elementary school students, to investigate their attitudes toward Mathematics Literacy Questions (MLQs) and Conventional Textbook Exercises (CTQs). The results indicate that students achieved higher correct response rates on CTQs than on MLQs. Moreover, regardless of the question type, negative perceptions outweighed positive ones, with MLQs exhibiting a particularly higher proportion of negative perceptions. Students' preferences for MLQs were polarized; although they perceived MLQs as the most challenging, these were also the questions they were most willing to attempt.

Keywords: mathematical literacy, mathematical literacy-orientation questions, student affection, student confidence

Introduction

Countries worldwide recognize the crucial role of mathematical literacy in both individual and societal development. This acknowledgment of mathematical literacy as a core subject in international mathematics education has prompted nations to place greater emphasis on this field to ensure that their students are globally competitive. Taiwanese students have performed exceptionally well in the literacy-oriented Programme for International Student Assessment (PISA): in 2018, the average score was 531, ranking 5th globally; by 2022, the average score had risen to 547, boosting their global ranking to 3rd—the best performance in recent years.

Despite these remarkable achievements, students' interest in mathematics, their confidence, and their valuation of mathematics still need improvement. According to the TIMSS 2023 analysis (von Davier et al., 2024), Taiwanese fourth and eighth graders have consistently ranked among the top internationally in mathematics achievement. However, they face challenges regarding their attitudes toward mathematics: in 2023, only 22% of fourth graders reported that they "really like mathematics," while 61% of eighth graders expressed a dislike for the subject; both their confidence and their recognition of mathematics' value are below international averages. Taiwanese education authorities have also taken note of this issue, and scholars advocate for literacy-oriented mathematics education as a means to improve the situation, asserting that enhancing mathematical literacy is essential for mathematics education reform and the development of 21st-century citizens.

But do students truly share this viewpoint? Do they really enjoy mathematics literacy-oriented questions? This issue cannot be overlooked in an education system that emphasizes student-centered learning. This study aims to explore whether Taiwanese elementary students appreciate this trend and to compare their attitudes toward Mathematics Literacy Questions (MLQs) and Conventional Textbook Exercises (CTQs) as an important indicator for policy implementation.

Literature Review

The PISA 2022 Mathematics Framework (OECD, 2018) defines mathematical literacy as an individual's ability to engage in mathematical reasoning as well as to construct, utilize, and interpret mathematics in order to solve problems in various real-life contexts. This framework not only emphasizes mathematical reasoning but also enhances the ability to connect mathematics with everyday life—a concept that has influenced the development of mathematics education policies in Taiwan (Lin et al., 2019). OECD (2016) describes the mathematical modeling process as converting the concrete context or everyday language of a problem into a mathematical representation or language, thereby transforming it into a mathematical problem that can be solved using mathematical concepts, facts, procedures, and reasoning, and by applying appropriate mathematical tools and methods. This approach differs from the types of questions found in textbooks designed for Taiwan's formal curriculum (Chiu and Hsieh, 2024).

Numerous studies in Taiwan (e.g., Hsieh, 2024) indicate that literacy-oriented test items typically contain a larger volume of text, which imposes a greater cognitive load on reading comprehension. Furthermore, literacy test items often require students to respond to open-ended questions, in contrast to the predominantly closed-ended questions used in mathematics assessments for Taiwanese students. According to the TIMSS 2019, 2023 analysis (Mullis et al., 2020; von Davier et al., 2024), Taiwanese students lag behind most countries in terms of their affective disposition toward mathematics (Mullis et al., 2020). If the life-related characteristics of mathematical literacy test items can, on an affective level, capture students' interest in problem-solving, there is a strong possibility of improving the current situation in Taiwan. However, there is also the concern that such items might expose Taiwanese students' weaknesses in problem-solving.

Methodology

Research Setting and Instruments

This study employed a questionnaire survey method. Mathematics topics in the questionnaire included circle circumferences, statistical charts, volume, and scale units. Each topic included two types of questions: mathematics literacy questions (MLQs) and conventional textbook questions (CTQs).

MLQs: originated from the released PISA items and developed questions similar to them. (Fig.1)

CTQs: extracted from Taiwanese mathematics textbooks. (Fig.2)

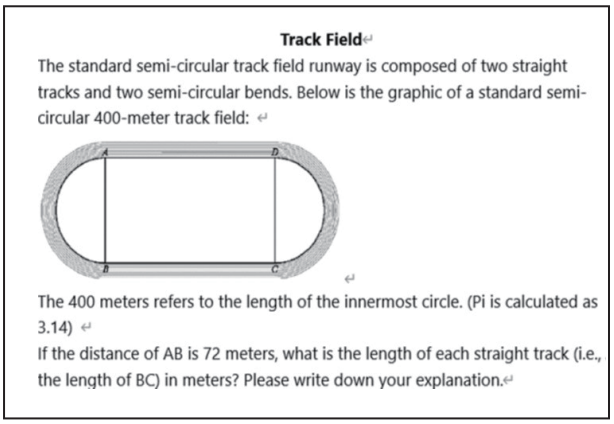


Figure 1. Sample MLQ developed by Taiwan

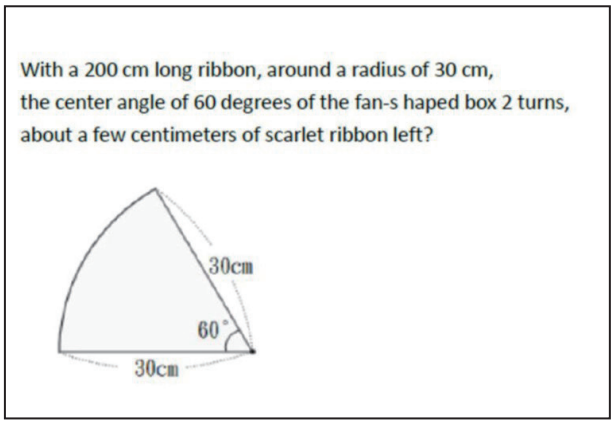


Figure 2. Sample CTQ

Each question asked students to respond in terms of their feelings upon reading the question (check the opinion if more like, annoying, easier, harder…), then they are asked to write down the process of solving the problem. After that, check the box "Do you like the way the question is written?" (Likes range from 1 to 5).

Participants and Data Collection

The participants were sixth-grade students in Taiwan, totaling 156 respondents. Balanced Incomplete Block (BIB) design. Comment Block included 4 released PISA items. Different Block I, II included 4 mathematics topics, each with two questions（MLQ+ CTQ）.

Analysis

Quantitative Data was analyzed by narrative statistics, *t*-Test, classical test theory and IRT. Students' problem-solving responses were coded by mathematics education experts.

Results and Discussion

Students' Correct Response rate on MLQs and CTQs

Overall, students' correct response rate in CTQs (Average correct answer rate 59.4%) was higher than that in MLQs (Average: 48.7%). This can be attributed to two main interpretations: first, students found CTQs to be more familiar compared to MLQs; second, the lengthy descriptions in MLQs posed greater challenges to comprehension. Interestingly, within individual questions, MLQs demonstrated a contrasting trend, with some questions achieving the highest correct answer rate.

Students' Feelings on MLQs and CTQs

We categorized all "feelings" into two groups and counted the responses:

Positive perceptions: included "Like it more," "Easier," "Easier to understand," "More enjoyable," and "Simpler".

Negative perceptions: included "Dislike it more," "More annoying," "Harder," "More difficult to understand," and "More complex."

Regardless of whether the questions were MLQs or CTQs, negative perceptions outweighed positive ones. While the positive perceptions of MLQs and CTQs were nearly identical, MLQs received a higher proportion of negative perceptions compared to CTQs.

Table 1. Students' feelings on MLQs and CTQs

Types	MLQs		CTQs	
Perceptions	positive	negative	positive	negative
Average value	0.96	1.34	0.98	1.22

The average of students' comprehension of MLQs and CTQs

A 5-point Likert scale was used, with "Completely understand" rated as 5 and "Do not understand at all" rated as 1. The level of comprehension after reading the questions was similar for both MLQs (Average:2.34) and CTQs (Average: 2.32), with MLQs scoring slightly higher. This indicates that students are capable of understanding the characteristics of mathematical literacy questions, which often involve more text, tables, and integration with real-life situations.

The Average of Students' Preference for MLQs and CTQs

After solving the questions, students were asked to rate their preference for the question descriptions on a scale from 1 to 5, with higher scores indicating greater preference. Overall, students showed a stronger preference for CTQs (Average:2.92) compared to MLQs (Average:2.76). This may be because MLQs feel unfamiliar and often have more complex descriptions due to their integration with real-life situations. Interestingly, students indicated that the items they found most difficult were also the ones they were most willing to solve.

Students' Confidence: The Difference between Students' Expectation and Actual Score

Students predicted their scores, which were compared to their actual scores, with a smaller difference indicating higher confidence in their answers. The results showed that students were more confident in their responses to CTQs (Average:0.52) than to MLQs (Average:0.64). This suggests that students are better at predicting their scores on CTQs, whereas they struggle more with accurately self-evaluating their performance on MLQs.

Conclusion

This study found that students achieved higher correct response rates and scoring confidence on routine textbook exercises compared to mathematical literacy-oriented questions, highlighting the impact of familiarity on learning performance. However, students demonstrated slightly better comprehension of mathematical literacy-oriented questions, indicating their ability to understand math problems related to real-life contexts, though further adaptation to such questions is needed.

References

Chiu, C.-W., & Hsieh, C.-J. (2024, July). Does Taiwan elementary school students love mathematical literacy questions? *Proceedings of the 15th International Congress on Mathematical Education*, Sydney, Australia.

Hsieh, C.-J. (2024). Assessing mathematical literacy in Taiwan's comprehensive assessment program for junior high school students. *Science Study*, 63(1), 33–42.

Lin, F.-L., Shan, W.-C., Li, Y.-S., & Zheng, Z.-H. (2013). Preliminary study on the mathematics curriculum guidelines of the 12-year national basic education: A subproject of the integrated research on the curriculum guidelines (Research Report No. NAER-102-06-A-1-02-03-1-12). National Academy for Educational Research.

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2020). *TIMSS 2019 international results in mathematics and science*. Boston College, TIMSS & PIRLS International Study Center.

Organisation for Economic Co-operation and Development. (2018). PISA 2022 mathematics framework (Draft). OECD Publishing.

Organisation for Economic Co-operation and Development. (2019). *PISA 2018 results (Volume I, II, and III): What students know and can do*. OECD Publishing. https://doi.org/10.1787/5f07c754-en

Organisation for Economic Co-operation and Development. (2023). *PISA 2022 results: Insights and implications*. OECD Publishing. https://doi.org/10.1787/6c23c954-en

Von Davier, M., Kennedy, A., Reynolds, K., Fishbein, B., Khorramdel, L., Aldrich, C., Bookbinder, A., Bezirhan, U., & Yin, L. (2024). *TIMSS 2023 international results in mathematics and science*. Boston College, TIMSS & PIRLS International Study Center. https://doi.org/10.6017/lse.tpisc.timss.rs6460

TSG7-14

DATA LITERACY IN MATHEMATICS AND SCIENCE TEACHING: A NARRATIVE EXPLORATION OF CLASSROOM PRACTICES, TEACHER EFFICACY, AND ATTITUDE

Rachel Dorcas A. Lim
Ph.D., University of the Philippines, National Institute for Science and Mathematics Education Development, Philippines, ralim2@up.edu.ph

Ma. Nympha Beltran-Joaquin
Ph.D., University of the Philippines Diliman, Philippines, mbjoaquin@up.edu.ph

Sheryl Lyn C. Monterola
Ph.D. University of the Philippines, National Institute for Science and Mathematics Education Development, Philippines, scmonterola@up.edu.ph

This mixed-methods study examined the integration of data literacy into STEM classrooms and its relationship with teacher efficacy and attitude toward STEM teaching. It involved 462 Filipino STEM teachers from the STEM Education Master Course for Teachers. Data were collected using the STEM Education Master Course Survey, the Data-Based Decision-Making Survey for Teachers (Wayman et al., 2016), and an open-ended narrative survey. Qualitative data from seven open-ended questions were analyzed using inductive thematic analysis. Findings showed that teacher efficacy and attitude toward STEM teaching significantly influenced data literacy for teaching. Teachers integrated data literacy through data-based decision-making, real-world applications, and the use of data as a resource. The study highlights the need for professional development and technological support to enhance teachers' data literacy in STEM education.

Keywords: Data Literacy, Teacher Efficacy and Attitude, Professional Development

Educators are bombarded with data daily, yet many lack the training and confidence to integrate data literacy into teaching. Data literacy enables educators to collect, analyze, and interpret data for instructional decisions (Mandinach & Gummer, 2016). Schreiter et al. (2022) emphasized its importance across STEM fields, integrating skills through problem-based and project-based learning, collaborative interactions, personal contexts, game-based learning, and student-led data collection. Using authentic, real-world data enhances students' knowledge, interest, and motivation toward statistics and data literacy. STEM integration and data literacy rely on complex, real-world data (Bowen & Bartley, 2014; Finzer et al., 2018; Hardy et al., 2020, as cited in Miller, 2022). Lee and Wilkerson (2018) highlight interdisciplinary connections between math and science.

However, teachers struggle with examining multiple data points, distinguishing values, and identifying patterns (Cowie & Cooper, 2017). Their perceptions of data use, coupled with limited experience in data collection, influence their attitude toward integrating data into teaching (Swain, 2018).

Teachers who value data for identifying student needs are more likely to use it in decision-making (Mingchu, 2008).

Effective integration of data literacy requires professional development that links content knowledge, pedagogy, and real-world applications. Yet, many programs fail to prepare teachers for practical data use. Despite training, teachers often revert to traditional methods due to a lack of confidence (Miller et al., 2021; Dunlap & Piro, 2016). Pre-service teachers also report limited foundational knowledge in data-driven decision-making (DeLuca & Bellara, 2013).

Teacher efficacy significantly affects data use, with studies showing a relationship between data use and increased instructional efficacy (van der Sheer & Visscher, 2016). To enhance teacher efficacy and foster positive attitudes toward STEM, professional development must connect data literacy with pedagogy and real-world applications (Wilkerson & Polman, 2020). Wolff et al. (2019) propose six principles for advancing data literacy: (1) scaffolding data use through inquiry, (2) expanding from small to complex datasets, (3) using familiar data, (4) linking skills to broader concepts, (5) integrating arts into STEM, and (6) engaging students in personal data collection.

Methodology

This study employed an explanatory sequential mixed methods design to investigate STEM teachers' data literacy, efficacy, and attitude toward STEM teaching. A total of 462 STEM teachers (303 female, 159 male), participated in a government-initiated National STEM Education Master Course for Teachers. Quantitative data were collected using the Teacher Data Use Survey (Wayman et al., 2016), which demonstrated strong internal consistency (Cronbach's $a \geq .85$), and the STEM Education Master Course Training Survey, which assessed participants' data-based decision-making, data literacy, teacher efficacy, and attitude toward STEM teaching. To elaborate on the quantitative findings, qualitative data were collected through seven open-ended narrative survey questions. These questions underwent expert review and pilot testing to ensure clarity and content validity. Qualitative responses were analyzed using inductive thematic analysis to identify emerging themes.

Results and Discussion

An inductive thematic analysis of seven open-ended survey questions explored how teachers integrated data literacy into math and science classes. Responses were coded and categorized into themes, revealing that teachers used data for data-driven decision making, real-world activities, and as a resource. These insights highlight current practices and areas for improvement.

Engaging in Data-driven Decision Making

The most prevalent theme, identified in 14 instances, involved collecting, analyzing, and interpreting data to assess student progress and refine teaching strategies. Teachers used various data sources, including diagnostic, formative, and summative assessments, to evaluate learning outcomes. Participant #37, for instance, utilized multiple assessments to track student performance, while participant #29 emphasized a "quantitative interpretation of student progress."

Mandinach and Gummer (2016) underscored the importance of using diverse assessment data to inform instruction. Teachers in this study acknowledged the role of data in differentiating instruction and tailoring strategies to student needs. For example, participant #5 stated, "Data served as a basis for further improvement of instructions and strategies," while participant #27 analyzed exam results to identify least-learned competencies and design targeted interventions.

Given the importance of data-driven instruction, it is recommended that schools provide training on data analysis and cultivate a culture of data-informed decision-making. Equipping teachers with data literacy skills enhances their ability to assess student learning and adapt instructional approaches effectively.

Designing Activities and Projects with Real-world Connections

The second emergent theme, coded 12 times, highlights student engagement in authentic activities like research and scientific investigations requiring data collection, analysis, interpretation, and visualization. Teachers integrated various activities to develop data literacy skills. Participant #6 used the scientific method in physics, Participant #11 applied a deductive problem-solving approach, Participant #2 guided students in investigatory projects, and Participant #10 trained them to extract data from charts. Participant #18 incorporated real-world problem-solving in math using available data.

Erwin (2015) emphasized that working with real-world data in project-based learning strengthens students' analytical and reporting skills. Engaging students in such activities enhances problem-solving and critical thinking. Teachers should integrate more project-based learning in math and science and receive training on effectively using real-world data to improve student learning outcomes.

Using Data as a Resource

The third theme, which emerged nine times in the coding, highlights the use of accessible, available, useful, and reliable data in teaching. Participant #19 incorporated data literacy by using relevant data to elaborate on lessons. Participant #17 downloads credible and updated data for discussions, while Participant #28 emphasized that proper data literacy skills enable the gathering of valid information and accurate results. This aligns with Vught's (2016) study, where teachers used accurate, timely, and relevant data for informed decision-making.

Participant #33 stressed that students must understand the purpose and accuracy of data collection. Similarly, Participant #4 integrates data literacy by presenting visualizations and research results to help students appreciate data's relevance. According to Shreiner and Dykes (2020), teachers must be confident in their data literacy skills to effectively teach data visualizations.

Access to credible data enhances students' understanding of mathematics and science. Teachers should develop data literacy skills, particularly in data collection, and schools should provide updated data sources and professional development. Training and collaboration with data experts can further support teachers in integrating data visualizations into lessons, creating more enriching learning experiences.

Interaction among Educational Information and Data Literacy, Teacher Efficacy, and Attitude Toward STEM Teaching

The findings indicate that teacher efficacy and attitude toward STEM teaching significantly influence their level of information and data literacy. Nearly all teachers (98.5%) demonstrated a positive attitude and efficacy to improve their STEM teaching practices. They expressed confidence in addressing student inquiries and explaining STEM concepts, aligning with McMullin and Reeve's (2014) assertion that a positive attitude is crucial for successful STEM implementation.

However, some teachers identified challenges, such as integrating interdisciplinary knowledge, assessing student achievement, and managing time and resource constraints. Moreover, 95.2% of teachers displayed a positive attitude toward integrating data into their teaching practices, recognizing its role in planning instruction, setting learning goals, and monitoring student progress. However, 4.8% questioned the necessity of using data to inform their teaching and its impact on their effectiveness.

Despite their confidence, 30.5% of teachers were uncertain about their ability to effectively teach STEM concepts, highlighting the need for continuous professional development. Furthermore, 97.2% of teachers exhibited high efficacy in using data for instructional decision-making, recognizing its role in planning lessons and monitoring student progress. Teachers who felt confident in their data literacy skills effectively integrated data into their teaching, while others, particularly those lacking formal training, relied on self-study and experience.

Conclusion

Teachers integrate data literacy into their mathematics or science classes through engaging in data-based decision-making, designing activities with real-world connections, and using data as a resource. This indicates that teachers can enhance their proficiency in data literacy by engaging in classroom activities (Mandinach & Gummer, 2016). Additionally, the incorporation of authentic and real-world data has been proven to increase students' knowledge, interest, and motivation for data literacy (Schreiter et al., 2022). Furthermore, there is a strong association between information and data literacy and teacher efficacy and attitude toward STEM teaching.

References

Cowie, B., & Cooper, B. (2017). Exploring the challenge of developing student teacher data literacy. *Assessment in Education: Principles, Policy & Practice*, *24*(2), 147–163. https://doi.org/10.1080/0969594x.2016.1225668

DeLuca, C., & Bellara, A. (2013). The current state of assessment education: Aligning policy, standards, and teacher education curriculum. *Journal of Teacher*

Education, 64, 356–372. http://dx.doi.org/10.1177/0022487113488144

Dunlap, K., & Piro, J. S. (2016). Diving into data: Developing the capacity for data literacy in teacher education. *Cogent Education*, *3*(1), 1132526. https://doi.org/10.1080/2331186x.2015.1132526

Erwin, R. W. (2015). Data Literacy: Real-World Learning Through Problem-Solving With Data Sets. *American Secondary Education*, *43*(2), 18–26. http://www.jstor.org/stable/43694208

Lee, V. R., & Wilkerson, M. (2018). Data use by middle and secondary students in the digital age: A status report and future prospects. Commissioned Paper for the National Academies of Sciences, Engineering, and Medicine, Board on Science Education, Committee on Science Investigations and Engineering Design for Grades 6-12. Washington, D.C.

Mandinach, E. B., & Gummer, E. S. (2016b). What does it mean for teachers to be data literate: Laying out the skills, knowledge, and dispositions. *Teaching and Teacher Education*, *60*, 366–376. https://doi.org/10.1016/j.tate.2016.07.011

McMullin, K., & Reeve, E. M. (2014). Identifying Perceptions That Contribute to the Development of Successful Project Lead the Way Pre-Engineering Programs in Utah. *Journal of Technology Education*, *26*(1). https://doi.org/10.21061/jte.v26i1.a.2

Miller, K., Yoon, S., Shim, J., & Cottone, A. (2021). Integrating Data Literacy into Secondary School Science: An Exploratory Study of a Pilot Professional Development. In de Vries, E., Hod, Y., & Ahn, J. (Eds.), Proceedings of the 15th International Conference of the Learning Sciences - ICLS 2021. (pp. 781-784). Bochum, Germany: *International Society of the Learning Sciences*.

Mingchu L. (2008). Structural Equation Modeling for High School Principals' Data-Driven Decision Making: An Analysis of Information Use Environments. *Educational Administration Quarterly*, *44*(5), 603–634. https://doi.org/10.1177/0013161x08321506

Schreiter, S., Vogel, M., Friedrich, A., Malone, S., Brünken, R., Becker-Genschow, S., Lehmann, J., Mosetter, T., Haaß, S., & Kuhn, J. (2022). Cross-Curricular Approaches to Promoting Statistical and Data Literacy in STEM School Education: A Systematic Review. In *Bridging the Gap: Empowering and Educating Today's Learners in Statistics. Proceedings of the Eleventh International Conference on Teaching Statistics*. https://doi.org/10.52041/iase.icots11.t14c4

Shreiner, T. L., & Dykes, B. M. (2020). Visualizing the teaching of data visualizations in social studies: A study of teachers' data literacy practices, beliefs, and knowledge. *Theory and Research in Social Education*, *49*(2), 262–306. https://doi.org/10.1080/00933104.2020.1850382

Swain, J. (2018). Elementary Teachers' Perceptions Regarding the Use of Data Decision-Making for Instructional Practice.

van der Scheer, E. A., & Visscher, A. J. (2016). Effects of an intensive data-based decision making intervention on teacher efficacy. *Teaching and Teacher Education*, *60*, 34–43. https://doi.org/10.1016/j.tate.2016.07.025

Vught, E. C. (2016). *Data use in public secondary schools in the Philippines.* (Master's Thesis). University of Twente. Enschede, Netherlands https://essay.utwente.nl/71584/1/family%20name_MA_faculty.pdf

Wayman, J. C., Wilkerson, S. B., Cho, V., Mandinach, E. B., & Supovitz, J. A. (2016). Guide to using the Teacher Data Use Survey (REL 2017–166). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Appalachia. http://ies.ed.gov/ncee/edlabs

Wilkerson, M. H., & Polman, J. L. (2020). Situating Data Science: Exploring How Relationships to Data Shape Learning. *Journal of the Learning Sciences*, *29*(1), 1–10. https://doi.org/10.1080/10508406.2019.1705664

Wolff, A., Wermelinger, M., & Petre, M. (2019). Exploring design principles for data literacy activities to support children's inquiries from complex data. *International Journal of Human-computer Studies*, *129*, 41–54. https://doi.org/10.1016/j.ijhcs.2019.03.006

TSG7-15

DEVELOPMENT OF AN ASSESSMENT TOOL FOR MEASURING STEM HABITS OF MIND

Jihwa Noh
Pusan National University, Korea, nohjihwa@pusan.ac.kr

Elena Prieto
University of Newcastle, Australia, elena.prieto@newcastle.edu.au

Dong-Young Lee
Pusan National University, Korea, shainare00@hanmail.net

Hye-Young Byeon
Pusan National University, Korea, bhyoung17@pusan.ac.kr

Younkyeong Nam
Pusan National University, Korea, ynam@pusan.ac.kr

This study develops an assessment tool to measure STEM habits of mind, essential for problem-solving, decision-making, and creative thinking. While STEM education emphasizes cognitive and affective growth, existing research lacks a systematic way to assess these habits. Key components of STEM habits of mind were identified across science, technology, engineering, and mathematics, and a survey was developed based on expert reviews. After pilot testing with 200 university students and conducting factor analysis, the instrument was refined. The finalized tool provides a structured framework for evaluating STEM habits of mind, supporting educational interventions, and enhancing STEM learning outcomes.

Keywords: STEM, Habits of mind, Exploratory and confirmatory factor analyses

Introduction

Habits of mind are cognitive dispositions essential for effectively navigating complex situations, including problem-solving, decision-making, and creative thinking (Costa & Kallick, 2008). Unlike a fixed or growth mindset, which primarily focuses on an individual's beliefs about intelligence and learning potential, habits of mind emphasize the development of cognitive behaviors through continuous engagement in problem-solving processes (Levasseur & Cuoco, 2009). This distinction highlights the research contribution of studying habits of mind, as they provide a more dynamic and actionable framework for fostering adaptive thinking and problem-solving abilities, particularly in STEM education. By shifting the focus from static beliefs about intelligence to the cultivation of productive cognitive behaviors, this study offers a valuable perspective for enhancing both theoretical understanding and practical applications in education.

In STEM (Science, Technology, Engineering, and Mathematics) fields, habits of mind refer to the cognitive and behavioral patterns essential for effective problem-solving and innovation. The importance of these habits has been increasingly recognized across disciplines, leading to the development of tools for their measurement (Asunda & Weitlauf, 2018; Alfiana & Wiyarsi, 2023; Wiyarsi & Çalik, 2019).

Historically, each STEM field has defined its own set of habits of mind based on its unique characteristics and nature (e.g., Lippard, Riley, & Lamm, 2018; Widodo & Katminingsih, 2023; Wiyarsi et al., 2023). However, with the emergence of integrated STEM education in the 2010s, especially in developed countries, there has been an ongoing discourse on the definition and scope of STEM education, aiming to merge the distinctive features of each discipline (Brown, Brown, & Reardon, 2011). This has led to new conceptualizations of STEM habits of mind that reflect the interdisciplinary nature of STEM education (Asunda & Weitlauf, 2018). These habits encompass a range of cognitive and social competencies, including critical thinking, creativity, systems thinking, effective communication, collaboration, and ethical reasoning (Yang et al., 2024).

Research Purpose

This study seeks to develop a robust instrument for measuring the STEM habits of mind among pre-service STEM teachers. The findings of this study are expected to make three significant contributions in STEM education as follows. First, by providing a reliable and valid construct for STEM habits of mind this study offers a scholary and statistical basis for potentially redefining this concept. Second, the results of the study will provide a validated and reliable tool for assessing STEM habits of mind for adult learners including preservice teachers. Third, by presenting a comparative analysis of the STEM habits of mind among pre-service STEM teachers in Korea and Australia, the study will offer valuable evidence to inform the enhancement of curriculum design and pedagogical strategies in teacher education programs in both countries.

Research Method

Initially, a panel of six experts, each with extensive experience in teacher education across the domains of Science, Technology, Engineering, and Mathematics, conducted a comprehensive literature review on the habits of mind relevant to each STEM discipline. Based on these findings and through collaborative expert consultation. Key factors of each discipline were identified(See Table 1) and a total of 98 preliminary items were formulated. These items were administered to 200 undergraduate students from each university in Korea and Australia, respectively. The results will underwent rigorous reliability assessments, including exploratory and confirmatory factor analyses, to refine and finalize the instrument.

Table 1. Habits of mind factors of each discipline of STEM

Field	Factor
Science Habits of mind	Mistrust of arguments from authority, Open-mindedness, Scepticism, Rationality, Objectivity, Suspension of belief, Curiosity
Technology Habits of mind	Creativity, Algorithmic thinking, Critical thinking
Engineering Habits of mind	Divergent Thinking , Systems Thinking, Collaboration, Communication, Optimism, Ethics
Mathematics Habits of mind	Persistence, Pattern Finding, Strategic Thinking, Communication, Metacognition

Research Results

Based on the survey results, the factor structure of the items was analyzed. Exploratory Factor Analysis (EFA) was conducted to identify the factor structure of the instrument after confirming its reliability. However, the initial EFA results indicated an excessively large number of extracted factors based on eigenvalues, making it difficult to derive a stable factor structure. To refine the instrument, a correlation analysis was performed to identify and remove redundant items. However, the analysis showed no significant multicollinearity, and no items exhibited excessively high correlations that warranted elimination. To address the issue, the number of iterations in the EFA was increased, and convergence was achieved after 52 iterations, resulting in the extraction of 22 factors. Through a systematic process of eliminating single-item and minor factors, the analysis proceeded through three additional iterations (EFA 3-3), ultimately reducing the factors to 13.

The results revealed that the Technology (T) domain formed a distinct factor, while Science and Mathematics each formed independent factors but also merged with Engineering in some cases. Notably, Engineering did not emerge as an independent factor, instead integrating with other STEM domains.

Additionally, a total of 13 factors, including Critical Thinking, Curiosity, and Suspension of Belief, along with their corresponding key features and 62 items, were identified. Examples of the items are as follows.

Table 2. Examples of the items derived through Exploratory Factor Analysis (EFA)

Factor	Key feature	Item
Critical thinking	Persistency	I think that I can apply the plan while making it to solve a problem of mine
Curiosity	A desire to learn	I have a strong desire for learning and continuously seek new knowledge
Suspension of belief	A procedure of holding in abeyance	I try to suspend judgment when there isn't enough information

References

Alfiana, N., & Wiyarsi, A. (2023). Students' scientific habits of mind from the perspective of educational levels and gender. *AIP Conference Proceedings, 2556*(1), 040014. https://doi.org/10.1063/5.0110499

Aprilia, A., & Usdiana, D.(2022). Analysis of mathematical habits of mind for senior high school students. *AIP Conference Proceedings, 2468,* 070040. https://doi.org/10.1063/5.0106860

Asunda, P. A., & Weitlauf, J. (2018). STEM habits of mind: enhancing a PBL design challenge-integrated STEM instruction approach. *Technology and Engineering Teacher*, *78*(3), 34-38.

Boaler, J. (2016). Mathematical Mindsets: U*nleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching.* Jossey-Bass.

Boaler, J., Dieckmann, J., Pérez-Núñez, G., Sun, K. L., & Williams, C. (2018). "Changing Students Minds and Achievement in Mathematics: The Impact of a Free Online Student Course." *Frontiers in Education*, 3, 26. Brown, R., Brown, J., & Reardon, K. (2011). *Understanding STEM: Current perceptions*. The Technology and Engineering Teacher, 2011. Retrieved from https://www.researchgate.net/profile/Chris-Merrill/publication/234659554_Understanding_STEM_Current_perceptions/links/618c0dce61f09877207bc812/Understanding-STEM-Current-perceptions.pdf

Costa, A. L., & Kallick, B.(2008). *Habits of mind: An organizing principle for mathematics curricula.* Intel Corporation.

Cuoco, A., Goldenberg, E. P., & Mark, J.(1996). Habits of mind: An organizing principle for mathematics curricula. *Journal of Mathematical Behavior, 15*(4), 375-402.

Dweck, C. S. (2006). Mindset: *The New Psychology of Success.* Random House.

Dwirahayu, G., Kustiawati, D., & Bidari, I.(2017). Corresponding habits of mind and mathematical ability. *Journal of Physics: Conference Series, 895*(1), 012013. https://doi.org/10.1088/1742-6596/895/1/012013

Gordon, M.(2011). Mathematical habits of mind: Promoting students' thoughtful considerations. *Journal of Curriculum Studies, 43*(4), 457-469.

Hafni, R. N., Sari, D. M., & Nurlaelah, E.(2019). Analyzing the effect of students' habits of mind on mathematical critical thinking skills. *Journal of Physics: Conference Series, 1211*(1), 012074. https://doi.org/10.1088/1742-6596/1211/1/012074

Handayani, A. D., Herman, T., Fatimah, S., Setyowidodo, I., & Katminingsih, Y.(2018). Inquiry-based learning: A student-centered learning to develop mathematical habits of mind. *Journal of Physics: Conference Series, 1013*(1), 012115. https://doi.org/10.1088/1742-6596/1013/1/012115

Harel, G.(2008). What is mathematics? A pedagogical answer to a philosophical question. *University of California, San Diego*. Retrieved from source.

Kwon, H.-S., et al. (2021). Issues in International STEM Education Research: A Meta-Synthetic Approach. *Journal of Science Education Research, 45*(1), 11-220.

Levasseur, K., & Cuoco, A.(2009). *Mathematical habits of mind*. National Council of Teachers of Mathematics.

Lippard, C. N., Riley, K. L., & Lamm, M. H. (2018). Encouraging the development of engineering habits of mind in prekindergarten learners. *Early engineering learning*, 19-36.

Mamona-Downs, J.(2007). Advanced mathematical thinking. In *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education*.

Mangels, J. A., Butterfield, B., Lamb, J., Good, C., & Dweck, C. S. (2006). Why do beliefs about intelligence influence learning success? A social cognitive neuroscience model. *Social cognitive and affective neuroscience, 1*(2), 75-86.

Simoncini, K. (2017). *STEM habits of mind: Guide to the early childhood STEM habits of mind*. University of Canberra.

Son, K.-Y. (2018). STEM Education Policies and Trends in the United States. *Journal of Science and Mathematics Education Research, 41*, 81-96.

Widodo, S., & Katminingsih, Y. (2023). Development of Habits of Mind Mathematic Questionnaire for Junior High School Students. International Journal of Research and Review, 10(5), 474-479.

Wiyarsi, A., & Çalik, M. (2019). Revisiting the scientific habits of mind scale for socio-scientific issues in the Indonesian context. *International Journal of Science Education*. https://doi.org/10.1080/09500693.2019.1683912

Wiyarsi, A., Çalik, M., Priyambodo, E., & Dina, D. (2023). Indonesian Prospective Teachers' Scientific Habits of Mind: A Cross-Grade Study in the Context of Local and Global Socio-scientific Issues. Science & Education, 1-27.

Yang, W., Du, Y., Wu, R., & Xiang, S. (2024). Development and validation of the children's STEM habits of mind questionnaire. *Early Childhood Education Journal*, *52*(3), 493-501.

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 7
(Day 4)

TSG7-16

A STUDY ON VALUE FORMATION AND TEACHER INFLUENCE OVER THREE YEARS OF JUNIOR HIGH SCHOOL IN JAPAN

Hiroshi Ishii
Hokkaido University of Education, Japan, ishii.hiroshi@h.hokkyodai.ac.jp

This study analyzes data using WIFItoo, developed by 'The Third Wave' international study, to examine changes in mathematics education values over three years. The study identified significant changes in students' values, particularly in "Hardship" and "Recalling". "Hardship" shifted from pattern (2) to pattern (1), while "Recalling" changed from pattern (8) to pattern (7). Changes, particularly in "Hardship" and "Recalling," were not initially emphasized in the mathematics department but developed through lessons and exam preparation. The study suggests that the focus on mastering content for entrance exams may have reshaped students' values.

Keywords: Value Formation, Teacher Influence, Entrance Examination

Research Background and Objective

This study aims to deepen our understanding of how students' values in mathematics education develop in Japan and the influence of mathematics teachers' values, drawing on the international comparative study "The Third Wave" by Seah et al. (2012, 2013, 2017). While previous research, mainly using WIFItoo, has explored how mathematics teachers in various countries address diverse values related to quality learning, Kinone et al. (2020) highlighted the diversity in students' values and the similarities and differences between teachers and students in Japan.

However, existing research has focused primarily on short-term transformations, leaving the full impact of teachers' values unclear. This study extends Kinone et al.'s work by examining a three-year period of secondary education to explore the formation of students' values and the influence of mathematics teachers. In Japan, while primary education is less exam-focused, secondary education emphasizes content mastery for entrance exams, potentially shaping students' values. Additionally, the subject-teacher system may further influence students' values through teachers' own perspectives on teaching.

This study uses a questionnaire to track changes in students' values from the start of Grade 7 to the end of Grade 9, aiming to understand how teachers and entrance exams influence students' values over three years.

Perspectives on Mathematics Education Values

As a subdimension of mathematics education values, seven complementary pairs of values have been identified based on findings from the "third wave" of research by Kinone and Seah (2015). These pairs are *Ability* and *Effort*, *Wellbeing* and *Hardship*, *Process* and *Product*, *Application* and *Computation*, *Facts/Truths* and *Ideas/Practice*, *Exposition* and *Exploration*, and *Recalling* and *Cre-*

ating. These pedagogical values are considered specific to mathematics teaching in school education.

Table 1. Sub-dimensions of mathematics educational values (Kinone & Seah, 2015)

	Sub-Dimension	Content
1	*Ability*	Valuing special talent (ability) in mathematics learning
	Effort	Valuing effort in mathematics learning
2	*Wellbeing*	Valuing calm, comfortable and enjoyable situation and atmosphere in mathematics learning
	Hardship	Valuing hard, difficult or tension-filled situation and atmosphere in mathematics learning
3	*Process*	Valuing process in mathematics learning
	Product	Valuing product (result) in mathematics learning
4	*Application*	Valuing application of mathematics in various problematic situations in mathematics learning
	Computation	Valuing computation and calculation in mathematics learning
5	*Facts/Truths*	Valuing facts (truths) in mathematics world in mathematics learning
	Ideas/Practice	Valuing ideas on the ways of using mathematics in real world in mathematics learning
6	*Exposition*	Valuing that others such as teacher and classmates teach mathematics in mathematics learning
	Exploration	Valuing mathematics learning by learners themselves including collaboration with classmates (valuing autonomous learning).
7	*Recalling*	Valuing remembering mathematical knowledge given by others exactly in mathematics learning
	Creating	Valuing constructing mathematical knowledge by learners themselves in mathematics learning

Previous research has focused primarily on school-based values, with limited exploration of students' values toward learning mathematics outside of school. In Japan, entrance exams significantly influence students' motivation and mathematical values. Since teachers' values shape students within the school environment, values formed outside of class may reflect independent development, offering insights into how students' values evolve beyond direct teacher influence.

Research Method

The WIFItoo questionnaire used in this study targets both teachers and students to assess the impact of teachers' values on students' value formation. The study compares responses from teachers and students at the beginning of Grade 7 and the end of Grade 9 of secondary school. (see Table 2)

Table 2. WIFItoo questionnaire for students

Item	Question
26	When anyone learns mathematics, what do you think are the important things that help him/her to learn?
27	When I learn mathematics, what are the important things that help me to learn?
28	Think about your mathematics teacher this year. What do you think are important to him/her in mathematics teaching?

Survey participants included Teacher A, a 37-year-old with 11 years of teaching experience, who oversees the mathematics department at an attached junior high school. The study involved 89 students in Teacher A's class. Data was collected using WIFItoo questionnaires administered to Teacher A and the same group of students at the beginning (G7) and the end (G9) of their three years in junior high school, in order to track changes over time. To minimize teacher influence, students completed the questionnaire independently via Google Forms. The data was analyzed through open coding, categorizing responses into the sub-dimensions of mathematics educational values identified by Kinone et al. (2020).

Results of Teacher

Based on the descriptions of the responses to Teachers' items 18-21, it ascertained Teacher A's values of learning and teaching in mathematics, as shown in Table 3. Teacher A's responses highlight the importance of "*Exploration*," emphasizing careful problem-solving explanations and confirmation through writing or speaking. The statement "To ask someone in good faith when one is absolutely unable to understand" reflects a value for "*Exposition*." Teacher A also underscores the "*Process*" in acquiring accurate knowledge, emphasizing "*Hardship*" and "*Effort*" in reviewing puzzles to reduce errors. The analysis of Teacher A's responses was based on their alignment with the relevant sub-dimensions.

Table 3. Description of teacher A

	General opinion	Personal opinion
Mathematics Learning	Item 18: ・To explain how to solve the problem carefully ・To ask someone in good faith ・To review carefully in order to minimize the number of mistakes	Item 20: ・To acquire the accurate knowledge. ・To examine thoroughly the details of the calculation process ・To confirm through writing explanations or speaking
Mathematics Teaching	Item 19: ・To explore various strategies for problem-solving employing various perspectives and thinking ・To devise lesson progression that sustains students' motivation and concentration ・To enhance one's own comprehension of mathematics to effectively convey accurate knowledge and skills	Item 21: ・To examine thoroughly the details of the calculation process ・To confirm through writing explanations or speaking ・To devise lesson progression that sustains students' motivation and concentration

Results of Students

The coding totals for response statements related to student items 26-28 are shown in Table 4. The questionnaire responses were classified into sub-dimensions based on Kinone et al. (2020) values, with the situation assessed by the percentage of descriptions. The overall findings indicate that students consistently prioritize certain sub-dimensions, namely "*Effort*," "*Wellbeing*," "*Process*," "*Exposition*," and "*Exploration*," both at G7 and G9. Conversely, sub-dimensions such as "*Ability*," "*Ideas/Practice*," and "*Creating*" were consistently not emphasized by students at G7 and G9.

Table 4. Responses from students (Grade 7 & Grade 9: 89 Students)

	Sub-Dimension	Item26		Item27		Item28	
		Grade 7	Grade9	Grade7	Grade9	Grade7	Grade9
1	*Ability*	0.0	3.4	0.0	3.4	0.0	1.1
	Effort	61.2	30.3	81.6	45.0	31.6	31.5
2	*Wellbeing*	51.0	36.0	60.2	43.8	22.4	37.1
	Hardship	16.3	23.6	21.4	32.6	34.7	36.0
3	*Process*	53.1	49.4	42.9	34.8	36.7	38.2
	Product	13.3	1.1	9.2	0.0	5.1	1.1
4	*Application*	5.1	5.6	7.1	7.9	3.1	13.5
	Computation	22.4	24.7	26.5	21.4	19.4	13.5
5	*Facts/Truths*	1.0	11.2	0.0	12.4	1.0	11.2
	Ideas/Practice	4.1	2.3	0.0	0.0	0.0	2.3
6	*Exposition*	55.1	55.0	22.4	56.2	52.0	64.0
	Exploration	18.4	30.3	27.6	19.1	77.6	34.3
7	*Recalling*	10.2	21.3	6.1	13.5	8.2	5.6
	Creating	0.0	1.1	0.0	1.1	0.0	1.1

Consideration

To examine the impact of teachers' values on the development of values in secondary school students, the "Model of the Formation Process of Students' Values in Mathematics Classes" by Kinone et al. (2020) was employed. The following discussion uses descriptions of Teacher Items 18-21 to identify what teachers value. Based on the results, the formation process patterns of students' values in Teacher A's mathematics lesson were compared between the beginning of Grade 7 and the end of Grade 9, as shown in Table 5.

Table 5. Comparison of the formation process of students' values

Formation Pattern	Beginning of Grade 7 (May 2022)	End of Grade 9 (January 2025)
Pattern 1	*Effort, Process, Exposition, Exploration*	*Effort, Process, Exposition, Exploration, Hardship, Wellbeing*
Pattern 2	*Hardship*	
Pattern 3	*Wellbeing*	
Pattern 4		
Pattern 5	*Computation*	
Pattern 6		
Pattern 7		*Recalling, Computation*
Pattern 8	*Ability, Product, Application, Facts/Truths, Ideas/Practice, Creating, Recalling*	*Ability, Product, Application, Facts/Truths, Ideas/Practice, Recalling, Creating*

The teacher A emphasized "*Exploration*," "*Exposition*," "*Process*," and "*Effort*," and students recognized and adopted these values. Although Teacher A emphasized "*Hardship*," students didn't perceive it as a teacher value, yet they adopt it as their own. Conversely, while students do not initially recognize "*Wellbeing*" as emphasized by Teacher A, they later adopt it as their own value. Neither Teacher A nor the students consider "*Ability*," "*Product*," "*Application*," "*Facts/Truths*," "*Ideas/Practice*," "*Recalling*," or "*Creating*" as their values.

At the beginning of Grade 7, students recognized "*Hardship*" as a value of Teacher A but didn't adopt it as their own. By the end of Grade 9, however, they began to accept "*Hardship*" as their own value. This shift is likely due to students recognizing the increasing hardship of the junior high school mathematics content and gradually internalizing these values through Teacher A's teaching. Regarding "*Wellbeing*," students did not perceive it as a teacher value at the beginning of Grade 7 but recognized it by the end of Grade 9. This suggests that Teacher A's values gradually became understood by the students over time. Similarly, "*Computation*" shifted from pattern (5) to pattern (7). Although students initially regarded it as important, they later aligned with Teacher A's values, recognizing that it was not emphasized, yet still considered it important. "*Recalling*" shifted from pattern (8) to pattern (7), indicating that while Teacher A didn't prioritize memorization, students still memorized due to the increasing complexity of the content.

In summary, the study presents a dynamic view of the formation of mathematics educational values over three years. Changes, particularly in "*Hardship*" and "*Recalling*," were not initially emphasized in the mathematics department but developed through lessons and exam preparation. The study suggests that the focus on mastering content for entrance exams may have reshaped students' values.

References

Kinone, C., Soeda, Y., & Watanabe, K. (2020). A Study on the Influence of Teachers on Students' Values Formation in Mathematics Education: Analysis of Miyazaki Prefecture Data Using the International Comparative Survey "The Third Wave" Questionnaire WIFItoo. *The Journal of the Japan Society of Mathematical Education*, 26(1), 43-58.

Seah, W. T., & Wong, N. Y. (2012). What students value in effective mathematics learning: a 'Third Wave Project' re-search study. *ZDM Mathematics Education*, Vol.44, 33-43.

Seah, W. T. (2013) "Assessing Values in Mathematics Education." In Lindmeier, A. M. and Heinze, A. (eds.) *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, 193-200.

Seah, W. T., Baba, T., & Zhang, Q. P. (2017). The WIFI Study: Students' Valuing of Mathematics Learning in Hong Kong and Japan. In Son, J. W., Watanabe, T., & Lo, J. J. (Eds.), *What Matters?: Research Trends in International Comparative Studies in Mathematics Education.* Cham, Switzerland: Springer, 333-354.

TSG7-17

VALUES ALIGNMENT IN TEACHERS' PERSPECTIVE: A STUDY IN CHINA

Haomin Fang
Faculty of Education, The University of Melbourne, Australia, haominf@student.unimelb.edu.au

Wee Tiong Seah
Faculty of Education, The University of Melbourne, Australia, wt.seah@unimelb.edu.au

The mathematics classroom represents a place where students' values and their teachers' values meet. A teacher's ability to maintain a sense of alignment amongst a diverse range of teacher and student values would thus affect lesson and learning effectiveness. In China, while values alignment had been researched from students' perspectives, the teachers' perspectives remain unexplored. This study adopted mixed research method to analyze and mine the text in questionnaires. The results suggest that Chinese mathematics teachers were not fully aware of what they valued, and most of them could not align their values with what they perceived to be their students' values. This study also presents current conceptualization of values alignment relationships in classrooms and offers ideas for future research.

Keywords: values alignment, mathematics teachers, alignment index, values categories

Introduction

Values reflect an individual's internalization, cognition, and decontextualization of affective variables under certain social context (Bishop, 1988). In mathematics education, values can be categorized into mathematical values, mathematics educational values, general educational values, and mathematical content values (E & Seah, 2024). Bishop (1988) proposed that mathematical values include *rationalism, objectism, control, process, openness,* and *mystery*. Mathematics educational values (MEV) are embedded in mathematics teaching and learning, while general educational values are associated with educational institutions and social characteristics (Bishop, 2008). Mathematical content values represent what teachers or students find important among specific topics in mathematics (E & Seah, 2024). Teachers and students come to class with their own sets of values regarding these four aspects, yet it is unrealistic to expect alignment between a teacher's values (VT) and their students' values (VS). Such alignment needs to be optimized, however, since it is believed that aligned values in a classroom are prerequisites for productive mathematics teaching and learning to take place (Kalogeropoulos & Clarkson, 2019). The value fulfilment locates the essence of the mathematics learning as the realization of students' values while learning, and teachers' role is crucial here (Hill & Seah, 2023).

Although teachers' values vary across individuals and regions, understanding which ones are shared – and which ones are not – within a specific context is crucial. In comparison to other countries, teachers in China enjoy relatively more authority in classrooms, since the Confucian culture implies that teacher-student power distances would be relatively high. Therefore, it is even more significant to explore the alignment between VT and VS from teachers' perspective. The distinct characteristics of Chinese educational values and teacher-student dynamics offer a unique lens for understand-

ing this alignment, which can serve as a foundation for comparative studies in other regions. To this end, this study has been guided by the following research question: *To what extent are the values of mathematics teachers and those of their students (as perceived by the teachers) (VS-tp) in China aligned?*

Methodology

Given the rapid economic development and proximity to the Guangdong-Hong Kong-Macao Greater Bay Area, the southeastern region of China is more likely to adopt and implement innovative teaching mechanism or models. This means that there will be greater possibilities for values misalignment between teachers and their students in this region. Therefore, data were collected through random sampling among 215 primary school mathematics in-service teachers in the southeast area of mainland China. To ensure a comprehensive range of data, the questionnaires were distributed without restrictions on participants' personal characteristics, such as age, years of teaching experience, courses taught, or gender. The questionnaire used was the one developed for the *Values Alignment Study* (Third Wave Lab, 2023). For the present study, the two open-ended questions in the questionnaire have been referred to. These are: *What do you think are important when anyone learns mathematics?*, and *What do you think are important when anyone teaches mathematics?*. The questionnaire was available online for three days, and 1,549 responses were collected.

The raw data was first prepared for analysis by data cleaning to remove invalid data (n=34), text to columns, and classify expression. After that, each participant's answers for VT and VS-tp were listed in separate cells in an Excel sheet. The consistency percentage of each participant's answers to VT and VS-tp was calculated by function formulae, which is their values alignment index. For example, if a participant provided three VT answer terms and six VS-tp answer terms, and only one VT answer term is same to VS-tp answer terms, the index of this participants is 1/6. The values alignment index ranges from 0 to 1. The closer the index is to 1, the higher value alignment degree within the participant's mathematics classroom. Then, all text in VT cells and VS-tp cells were counted by function for word frequency separately, and then relative coverage was calculated with the number of all participants. This step is to mine what the participants think is most important.

Results

Among the 1,549 questionnaires received, only 131 participants' index equal to 1. For these 131 participants, their values (VT) and their perspectives of what their students valued (VS-tp) were identical, that is, completely aligned. 1,319 provided completely different answers to two questions we used in this study, and the rest 99 participants showed different values alignment degrees. 1,549 participants provided a total of 3,390 VS-tp terms with 581 de-duplicated terms, and 3,124 VT terms with 847 de-duplicated terms. There were 12 terms with relative coverage values that were at least 3%, and these are listed in Table 1. We facilitated expert review to optimize the translation accuracy, e.g., different translation forms in rows 1 and 4 of Table 1, 思维 highlights students' thinking when they are answering questions, while 思考 means the process of students' thinking.

Table 1. Results of term frequency statistics and text coding.

Term	Translated Term	Relative Coverage (%)	Value category
思考	consider	VS-tp (14.53); VT (4.97)	Mathematical
兴趣	interest	VS-tp (13.75); VT (8.80)	Mathematics educational
计算	computation	VS-tp (13.30)	Mathematics educational
思维	thinking	VS-tp (15.95)	Mathematical
知识点	knowledge point	VS-tp (7.88); VT (7.81)	Mathematics educational
理解	understand	VS-tp (6.97); VT (3.87)	Mathematics educational
听课	listen to the teacher	VS-tp (6.78)	Mathematics educational
学习态度	learning attitudes	VS-tp (5.87)	Mathematics educational
数感	number sense	VS-tp (5.10)	Mathematics educational
学习习惯	learning habits	VS-tp (4.97)	Mathematics educational
练习	practice	VS-tp (4.71)	Mathematics educational
习惯	habits	VS-tp (4.71)	General educational
专注	concentration	VS-tp (4.65)	Mathematics educational
方法	methods	VS-tp (4.20); VT (5.29)	Mathematics educational
思维能力	thinking abilities	VS-tp (3.42)	Mathematical
认真	serious-minded	VS-tp (3.23)	General educational
预习	preview	VS-tp (3.03)	Mathematics educational
备课	prepare lessons	VT (9.30)	Mathematics educational
计算能力	computation abilities	VT (6.58)	Mathematics educational
引导	guidance	VT (3.87)	Mathematics educational
课本	textbook	VT (3.62)	Mathematics educational
教学方法	teaching methods	VT (3.29)	Mathematics educational

Discussion and Conclusion

In general, it is expected that the values of teachers and students align, as such alignment is considered a prerequisite for effective classroom instruction and a means to enhance students' mathematical wellbeing (Aktaş, 2024; Hill et al., 2022). However, the results of the values alignment index indicate that most teachers do not align VT with VS-tp. Influenced by Confucian and collectivist beliefs, Chinese students seem to observe classroom behavioral norms (Zhou et al., 2023). There are challenges to this 'behavioral norm' in practice, however, as teachers must uphold classroom discipline to ensure that large class sizes are catered for. Therefore, Chinese mathematics teachers have responsibilities to align values in a mathematics classroom to keep effective teaching and learning. According to the coding results, teachers value more about students' knowledge acquisition process and their own teaching competences. Conversely, teachers tend to presuppose that students place greater value on mathematical competencies, which can be aligned with the competencies outlined in the Curriculum Standards (MOE, 2022).

Another conclusion drawn from the coding results is that answer terms of participants showed uneven performance across the four value categories. Under the condition of Chinese mathematics education, participants' uneven performance presents teachers with a perception and priority of values in mathematics education. The mathematical value associated with the only term symbolizing it is rationalism, one out of the six mathematical values according to Bishop (1988). Most terms were mathematics educational values. In general, teaching and learning do not only take place in the classroom, but also in the pre-lesson preparation and post-lesson reinforcement, which jointly constitute the process of teaching and learning. Therefore, we find that MEV can be divided into "in-the-classroom MEV", which only occurs during classroom teaching (e.g., classroom discipline), and "out-of-classroom MEV", which refers to attributes that are considered important across the whole learning process (e.g., learning habits). In the results of this study only three answer terms belong to general educational values, and no mathematical content values were mentioned. Taken together, it appears that Chinese teachers are no different from their peers overseas (Seah, 2010), in that most of the values they embrace belong to just one of the four value categories, that is, mathematics educational values.

Complementing a study in which VS and what students perceived teacher values (VT-sp) were identified (Pang et al., 2024), this study has examined VT and VS-tp. In this context, we can propose a novel conceptualization of the values alignment process. While current research has considered values alignment as a state of alignment between VT and VS (Kalogeropoulos & Clarkson, 2019), we argue from what we have learnt in this study that a mathematics classroom should encompasses five values alignment relationships by integrating existing studies, which are relationships between VT and VS, VT and VS-tp, VT-sp and VS, VT and VT-sp, and VS and VS-tp. The last two relationships have not yet been demonstrated and, if so, have implications for teachers teaching and classroom management. This study explored values alignment condition from teachers' perspectives and suggested future research study values alignment problems with knowledge of teacher noticing as the values alignment process involves teachers' in-the-moment decisions (Chia & Zhang, 2024).

References

Aktaş, F. N. (2024). Designing a professional development model for values alignment strategies in inclusive mathematics instruction. In Y. Dede, G. Marschall, & P. Clarkson (Eds.), *Values and valuing in mathematics education* (pp. 67-80). Springer.

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education.* Kluwer Academic Publishers.

Bishop, A. J. (2008). Teachers' mathematical values for developing mathematical thinking in classrooms: Theory, research and policy. *The Mathematics Educator, 11*(1/2), 79-88.

Chia, H. M., & Zhang, Q. (2024). Towards a reconceptualisation of values research in mathematics education: A systematic review. In Y. Dede, G. Marschall, & P. Clarkson (Eds.), *Values and valuing in mathematics education* (pp. 45-56). Springer.

E, A., & Seah, W.T. (2024). Exploring perceived value difference situations in Australian mathematics classrooms. In *Proceedings of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 184–191). New Zealand.

Hill, J. L., Bowmar, A., & Hunter, J. (2022). A preliminary study exploring the mathematical wellbeing of grade 3 to 8 students in New Zealand. In C. Fernández, et al. (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 387-394). PME, Universidad de Alicante.

Hill, J. L., & Seah, W. T. (2023). Student values and wellbeing in mathematics education: Perspectives of Chinese primary students. *ZDM Mathematics Education, 55*(2), 385–398.

Kalogeropoulos, P., & Clarkson, P. (2019). The role of value alignment in levels of engagement of mathematics learning. In P. Clarkson, W. T. Seah, & J. Pang (Eds.), *Values and valuing in mathematics education* (pp. 59–72). Springer.

Ministry of Education of the People's Republic of China [MOE]. (2022). *Notice of the Ministry of Education on the issuance of compulsory education curriculum plan and curriculum standards (2022 edition)* [in Chinese] [EB/OL]. Ministry of Education of the People's Republic of China. http://www.moe.gov.cn/srcsite/A26/s8001/202204/t20220420_619921.html

Pang, J., Seah, W. T., Kim, L., & Kim, S. (2024). The text mining approach to identifying what students value in mathematics learning. In Y. Dede, G. Marschall, & P. Clarkson (Eds.), *Values and valuing in mathematics education* (pp. 81–98). Springer.

Seah, W. T. (2010). The 'third wave': Values associated with effective learning of mathematics in Australian primary classrooms. *Research in Mathematics Education, 14*(2), 99-114

Seah, W. T. (2013). Assessing values in mathematics education. In A. M. Lindmeier, & A. Heinze (Eds.), *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education: Mathematics Learning Across the Life Span* (pp. 193 - 200). PME.

Seah, W. T. (2022). Values-based mathematics teaching: Promoting mathematical performance, competencies, and wellbeing. *Proceedings of the Singapore National Academy of Science, 16*(1), 15–24.

The Third Wave Lab. (2023). *Study 5: Values Alignment Study (VAS).* https://thirdwavelab.education.unimelb.edu.au/study-5-values-alignment-study-vas/

Zhou, J., Bao, J., & He, R. (2023). Characteristics of good mathematics teaching in China: Findings from classroom observations. *International Journal of Science and Mathematics Education, 21*(4), 1177-1196.

TSG7-18

INVESTIGATING VALUES OF TEACHERS IN COURSES FOR LOW-PERFORMING UPPER-SECONDARY STUDENTS

Justine Sakurai
Faculty of Education, The University of Melbourne, Australia, j.sakurai@unimelb.edu.au

Wee Tiong Seah
Faculty of Education, The University of Melbourne, Australia, wt.seah@unimelb.edu.au

Max Stephens
Faculty of Education, The University of Melbourne, Australia, m.stephens@unimelb.edu.au

This study investigates the factors that may enhance the teaching and learning of mathematics. National and international assessments have revealed troubling patterns of low numeracy achievement among 15–18-year-old students in Australia. As part of broader research, this pilot study examines the perspectives of two teachers of mathematically underperforming students aged 15-18. Specifically, it explores teachers' values towards teaching in the classroom. A person-oriented approach offers a novel perspective that may help resolve the inconsistencies in understanding and identifying values in mathematics classrooms, and how these values can be depicted.

Keywords: Person-oriented research, mathematics, education, low-performing, high-performing

Introduction

PISA mathematical-literacy results for Australia have identified that on average students have shown a continual decline. However, the decrease is strongest in the lower 50% of students (Thomson, 2021). This decline has been worsening despite years of funding into intervention programs which focus on cognitive and affective aspects of pedagogy; for example, government initiatives supporting prescribed teaching methods such as mastery teaching (Smith et al., 2018). Yet, there are three dimensions to the functioning of the mind (Hilgard, 1980; Seah, 2019), that is, cognitive, affective and motivation. Research has established strong links between students' motivational states and their levels of engagement (Fielding-Wells et al., 2017). Amongst the motivation variables, how might values play a role? Might the ways students, teachers, parents and the wider society value mathematics education be a factor which also contributes to student performance?

Across Australia, most states have introduced contextually based practical mathematics subjects at upper-secondary that are designed for students that have experienced failure and non-engagement with mathematics as discipline. One possible avenue of research to consider reasons for the declining results in this cohort of students may be found in the study of values in mathematics education. Values in the mathematics classroom may be mathematical, mathematics educational, and general educational (Bishop, 1996). To date, values studies have not considered teachers of students who are performing below expectations in senior secondary mathematics classrooms. This paper describes the methodology and results of a pilot study looking at the values that teachers of these

students bring to the classroom. An understanding of values may contribute to an improved understanding of motivational factors for these students.

Values in Mathematics Education

Teachers are pivotal to students' learning in the mathematics classroom. Values have been recognised as motivational construct (Hannula, 2012) contributing to student motivation towards mathematics learning as well demonstrated in the situated expectancy value literature (Eccles & Wigfield, 2020). Teachers' values and value alignment with their students may play an important role in students' accessing and engaging with mathematics learning (Kalogeropoulos, 2017). Earlier, Bishop (1988) had conceptualised that rationalism and objectivism, control and progress, and openness and mystery constitute three pairs of complementary values related to mathematics in Western classrooms. Values related to mathematics pedagogy are more varied, however. In a study involving 1386 students from China, Hong Kong and Taiwan, Zhang et al (2016) identified the valuing of "achievement, relevance, practice, communication, information and communications technology as well as feedback" (p. 907) in the context of mathematics learning. On the other hand, Seah and Peng's (2012) scoping study which considered values in Australia and Sweden, commonalities were found in the values of explanation, sharing and fun. In Australia, through observations, interviews and artefact analysis of two teachers and 12 students, Seah and Peng identified 13 culturally based values such as practice, group work, creativity and respect. Year 8 Australian students' mathematical wellbeing has also been associated with the fulfilling of seven ultimate values, namely accomplishment, cognitions, engagement, meaning, perseverance, positive emotions, and relationships (Hill et al, 2021).

Methodology

As this qualitative study aims to uncover individual values towards mathematics teaching and learning from experienced teachers (with a teaching period of 13+ years), an ideographic perspective was taken to provide ecological validity. Ideographic studies seek to uncover truths about a population through extensive consideration of the individual, acknowledging that each individual has a unique set of characteristics and that the lived experience shapes each person differently (Barbot, 2017). This ideographic approach is important as it supports the person-oriented methodology. Person-oriented research considers the individual as a complex whole, constituted of systems and patterns. In person-oriented approaches (POA), the adaptive responses and interactions that make up the individual encompass social, cultural and individualistic norms (Bergman et al., 2003). This person-oriented study aims to capture individuals' value constructs whilst acknowledging these norms. In this study, there is not a focus on independent and dependent variables, but rather a consideration of the patterns and themes that may represent individual teachers' values. Commonalities in these patterns are sought to explain and describe the values of the teachers toward the teaching and learning of mathematics. Hence, a small number of participants in this study is validated due to the time constraints of considering the individual on a deep level.

For this pilot study, two experienced mathematics teachers were drawn from government secondary schools in Victoria, Australia. The two participants have elected for many of these years to work with struggling students in the final years of secondary education in publicly funded schools. Students who are considered at the bottom end of the spectrum in upper-secondary mathematics often exhibit a pattern of disengagement from learning, behavioural difficulties, and academic challenges (Oldfield et al., 2017).

Data collection focused on classroom observations. Ethnographic recording of the teacher in the classroom using narrative as a data collection tool (Chesterfield, 1997; Hoadley, 2024) allowed for the researcher to observe the lesson reducing the social burden on the students. Validation of the observations was provided by the teacher in the form of comments on the locked narrative transcription document. A second observer was not possible as this study is part of a doctoral thesis. Teacher comments provided insight into the interpretation of the events. Coding using NVivo allowed for considered patterns and themes and following person-oriented approaches.

Results and Discussion

Figure 1. displays a superimposed visual diagram of the two teachers' values which were identified through a review of mathematics education values literature. The diagram includes the selected elements from the pilot study. Given the available data, more detail will be possible in subsequent research. Each teacher displays a uniqueness in their enacted values. For some values, their observed occurrence was small. These values are included, as in the POA paradigm they constitute the parts that make the person unique. Teacher A demonstrated values in relation to relationships in the classroom. They had a strong focus on student sense-making with mathematics, and they placed importance on interacting with the class both one-to-one and as whole. This teacher was rarely at the front of the classroom but was constantly circulating and cross-communicating. Teacher B expressed their passion for teaching this cohort, commenting that often the "younger teachers struggle with the behaviour….and only want the more academic classes". Teacher B had an even stronger focus on relationships than Teacher A. They placed importance on the use of gestures to support mathematical understanding and used humour as the main tool to support the relationships in the classroom.

Both teachers embraced and enacted the mathematics educational value of relationships, with humour as a tool, as well as using gestures to support student sense-making. Both teachers used the one-to-one time with students to support mathematical understanding in their students and themed their lessons around authentic contexts. Their focus on relationship building enabled the students' mathematical wellbeing to occur as supported by findings by Hunter & Hill (2024). A further finding from both teachers was their use of authentic contexts to stimulate learning (Gravemeijer et al., 2017). Teacher A centred the lesson around building a playground in vacant land owned and adjoining the school as per the school's future plan as demonstrated in figure 2. This pilot study aimed to uncover experienced teachers' enacted values when teaching mathematics to struggling mathematics students in the final years of secondary education. The preliminary results demonstrate that there are commonalties to be found in the unique value systems held by the two teachers.

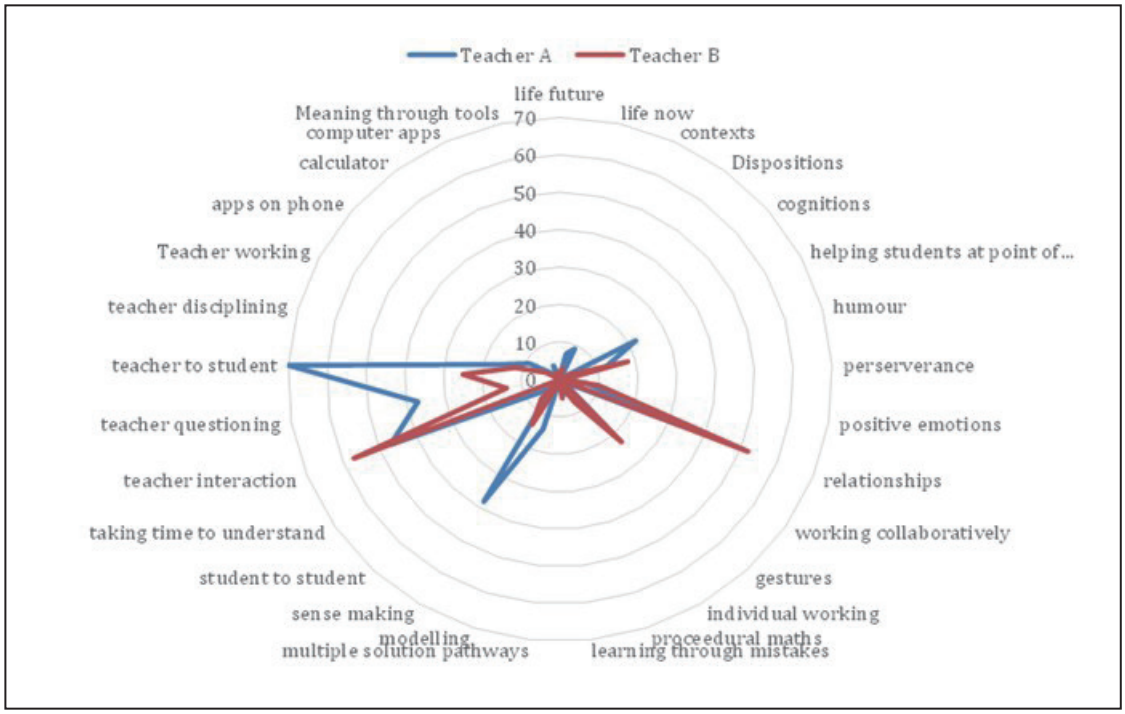


Figure 1. Values in the mathematics classroom for teachers A and B

> *"The playground must fill a size of 12m by 10m. It should have pathways, kick about spaces, tree plantings, toilet block. We look forward to working with you, the youth in our community, in order to provide the best local facilities to meet your needs. You could add the following, a basketball court 30 m by 15m, a futsal court of 40m by 20m, a skate ramp. All measurements will be in metres".*

Figure 2. Playground task

Limitations and Conclusion

Limitations of this pilot study include the strength of validity in using a narrative approach with only one observer. POA is a well-accepted methodology in the psychology field, but has rarely been used in education literature, posing additional limitations. However, the initial results are expected to provide scalability to the larger study.

There is a need to understand what factors may contribute to increased teaching and learning outcomes in the classrooms of lower-achieving students in upper secondary education. The two experienced teachers in this pilot study showed similarity in their approaches to teaching mathematics to low-performing secondary students. They were guided by their common valuing of relationships (engaging their students in mathematical learning and placing high value on their interactions with the students), and use of authentic contexts (as the underpinning tool to stimulate learning).

References

Barbot, B. (2017). Idiographic Study of Personality. In V. Zeigler-Hill & T. K. Shackelford (Eds.), *Encyclopedia of Personality and Individual Differences* (pp. 1–4). Springer International Publishing. https://doi.org/10.1007/978-3-319-28099-8_1311-1

Bergman, L. R., Magnusson, D., & El-Khouri, B. (2003). *Studying Individual Development in an Interindividual Context: A Person-oriented Approach.* Lawrence Erlbaum Associates.

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education.* Kluwer Academic Publishers.

Bishop, A. J. (1996). How should mathematics teaching in modern societies relate to cultural values—Some preliminary questions. In N. Tri, P. Long, C. Comiti, D. Green, E. Southall, & J. Izard (Eds.), *Seventh Southeast Asian Conference on Mathematics Education* (pp. 19–24). Vietnamese Mathematical Society.

Chesterfield, R. (1997). Classroom observation tools: Improving educational quality (IEQ) project. *Institute for International Research in Collaboration with Juarez and Associates, Inc. & The University of Pittsburgh*.

Eccles, J. S., & Wigfield, A. (2020). From expectancy-value theory to situated expectancy-value theory: A developmental, social cognitive, and sociocultural perspective on motivation. *Contemporary Educational Psychology*, *61*, 1-13. https://doi.org/10.1016/j.cedpsych.2020.101859

Fielding-Wells, J., O'Brien, M., & Makar, K. (2017). Using expectancy-value theory to explore aspects of motivation and engagement in inquiry-based learning in primary mathematics. *Mathematics Education Research Journal*, *29*(2), 237–254. https://doi.org/10.1007/s13394-017-0201-y

Gravemeijer, K., Stephan, M., Julie, C., Lin, F.-L., & Ohtani, M. (2017). What Mathematics Education May Prepare Students for the Society of the Future? In *International Journal of Science and Mathematics Education 15*(1) 105–123 https://doi.org/10.1007/s10763-017-9814-6

Hannula, M. S. (2012). Looking at the third wave from the West: Framing values within a broader scope of affective traits. *ZDM*, *44*(1), 83–90. https://doi.org/10.1007/s11858-012-0410-5

Hilgard, E. R. (1980). The trilogy of mind: Cognition, affection, and conation. *Journal of the History of the Behavioral Sciences*, *16*(2), 107–117. https://doi.org/10.1002/1520-6696

Hill, J. L., Kern, M. L., Seah, W. T., & van Driel, J. (2021). Feeling good and functioning well in mathematics education: Exploring students' conceptions of mathematical well-being and values. *ECNU Review of Education*, *4*(2), 349–375. https://doi.org/10.1177/2096531120928084

Hoadley, U. (2024). How do structured pedagogy programmes affect reading instruction in African early grade classrooms? *International Journal of Educational Development*, *107*, 1-9. https://doi.org/10.1016/j.ijedudev.2024.103023

Hunter, J., & Hill, J. L. (2024). Exploring mathematical wellbeing across cultures: Insights from diverse students. *Mathematics Education Research Journal*. *36*(3) 1-28 https://doi.org/10.1007/s13394-024-00500-5

Kalogeropoulos, P. (2017). *The role of value alignment in engagement and (dis)engagement in mathematics learning* [Thesis, Monash University]. https://doi.org/10.4225/03/58af6e438c91f

Ng, C. T-W, Dandy, J., & Lane, M. (2022). Perceived diversity norms, cultural identity styles and bicultural identity consolidation in two bicultural groups in Australia. *International Journal of Psychology*, *57*(3), 363–371. https://doi.org/10.1002/ijop.12829

Seah, W. T. (2018). Improving mathematics pedagogy through student/teacher valuing: Lessons from five continents. In: Kaiser, G., Forgasz, H., Graven, M., Kuzniak, A., Simmt, E., Xu, B. (eds) *Invited Lectures from the 13th International Congress on Mathematical Education. ICME-13 Monographs.* (pp. 561–580). Springer, Cham. https://doi.org/10.1007/978-3-319-72170-5_31

Seah, W. T. (2019). Values in Mathematics Education: Its Conative Nature, and How It Can Be Developed. *Research in Mathematical Education*, *22*, 99–121. https://doi.org/10.7468/jksmed.2019.22.2.99

Seah, W. T., & Peng, A. (2012). What students outside Asia value in effective mathematics lessons: A scoping study. *ZDM*, *44*(1), 71–82. https://doi.org/10.1007/s11858-012-0398-x

Smith, P., Ladewig, M., & Prinsley, R. (2018). Improving the mathematics performance of Australia's students. *Improving the Mathematics Performance of Australia's Students*, 1–8. [Office of the Chief Scientist - Occasional Paper]. Commonwealth Office of the Chief Scientist.

Thomson, S. (2021). Australia: PISA Australia—Excellence and Equity? In N. Crato (Ed.), *Improving a Country's Education: PISA 2018 Results in 10 Countries* (pp. 25–47). Springer International Publishing. https://doi.org/10.1007/978-3-030-59031-4_2

von Eye, A., & Bogat, G. A. (2006). Person-Oriented and Variable-Oriented Research: Concepts, Results, and Development. *Merrill-Palmer Quarterly*, *52*(3), 390–420.

Zhang, Q., Barkatsas, T., Law, H.-Y., Leu, Y.-C., Seah, W. T., & Wong, N.-Y. (2016). What Primary Students in the Chinese Mainland, Hong Kong and Taiwan Value in Mathematics Learning: A Comparative Analysis. *International Journal of Science and Mathematics Education*, *14*(5), 907–924. https://doi.org/10.1007/s10763-014-9615-0

TSG7-19

INTELLECTUAL HUMILITY AND MATHEMATICS PERFORMANCE

Meng Guo
College of Education for the Future, Beijing Normal University, China, guomengmail@163.com

Frederick K. S. Leung
College of Education for the Future, Beijing Normal University, China, frederickleung@hku.hk

Yuanchao Yang
Faculty of Education, Southwest University, China, yangyc@cque.edu.cn

This study investigated the relationships between mathematics-specific intellectual humility, help-seeking beliefs and tendencies, and mathematics performance in Chinese secondary students. The results show that intellectual humility positively influenced help-seeking benefits and negatively influenced help-seeking costs. Adaptive help-seeking beliefs were predicted by intellectual humility and help-seeking benefits, while help-seeking costs negatively affected it. Help-seeking beliefs were associated with avoidance behaviors, and adaptive help-seeking predicted mathematics achievement. Bootstrap analysis confirmed the mediating role of help-seeking beliefs. Overall, students with higher intellectual humility demonstrated better help-seeking behaviors and higher performance, offering deeper insights into how intellectual humility impacts mathematics achievement.

Keywords: *intellectual humility, help-seeking beliefs, mathematics performance.*

Introduction

In recent decades, Eastern Asian students' superior mathematics performance in international assessments (e.g., TIMSS, PISA) has attracted considerable research interest (Hu et al., 2018; Leung, 2014). Some attribute this success to social-cultural factors, particularly Confucian values like humility, collectivism, filial piety, and the emphasis on effort, which may influence Chinese students' learning processes (Leung, 2001, 2006, 2014; Zhu & Leung, 2011). Humility, highly regarded as a virtue in Eastern Asian societies (Leung, 2014), is believed to contribute to academic improvement. However, empirical evidence supporting this claim is limited, warranting further investigation into the role of humility in Chinese students' mathematics learning (Leung, 2014).

This study explores the relationship between students' mathematics-specific intellectual humility and mathematics performance in China, examining help-seeking as a mediator. Previous research has shown intellectual humility's link to motivation, and academic performance (Krumrei-Mancuso et al., 2020; Rowatt et al., 2006), but its impact on mathematics learning is underexplored. While humility may encourage help-seeking, there is limited empirical evidence on its mediational role in academic success. This study aims to fill this gap by investigating how students' humility influences their learning behaviors and outcomes.

Literature Review

Intellectual humility has two aspects: cognitive and social structure (Krumrei Mancuso, 2016; Leary et al., 2017). This study focuses on mathematics-specific intellectual humility, considering both inter- and intrapersonal aspects. It defines the former as one's recognition of one's mathematics learning limitations and willingness to change and improve mathematics understanding, and the latter as one's respect for, acceptance of, and desire to learn from others' different ideas in mathematics.

Some researchers have empirically explored humility's effects on students' academic achievement (Krumrei-Mancuso et al. 2020; Rowatt et al., 2006; Wong & Wong, 2021), and others

have found a close relationship between humility and help-seeking in non-academic contexts (Kent, 2022; Wong & Wong, 2021). Haggard et al. (2016) pointed out that students with intellectual humility tended to own their limitations and hoped to figure them out. Those who accept their learning limitations may be less afraid to demonstrate them in front of others and may see help-seeking as an effective method to cover their shortcomings (help-seeking benefits) rather than as a cost or a threat to their ego. People with higher intellectual humility are more open to others' ideas and suggestions (Leary et al., 2017), which may promote their tendency to seek help and learn from others. However, as empirical evidence that directly supports the relationship between intellectual humility and help-seeking beliefs and tendencies in academic contexts is limited, more investigation is needed.

Existing studies have mainly focused on the direct relationship between humility and academic performance (Krumrei-Mancuso et al., 2020; Rowatt et al., 2006), and only provide preliminary insights into how humility shapes students' academic achievement. The potential mechanisms underlying the relationship are under-researched.

This study examines the relationships among mathematics-specific intellectual humility, help-seeking beliefs, and mathematics performance. Based on previous findings (Kent, 2022; Wong & Wong, 2021; Ryan & Shin, 2011), it hypothesizes that mathematics-specific intellectual humility significantly predicts help-seeking beliefs and tendencies and is linked to mathematics achievement.

Research Methods

Participants

The study involved 926 secondary school students from the city of Chongqing, China, with an average age of 13.52 years (SD = .679). Of these, 49.78% (N=461) were in the seventh grade and 50.22% (N=465) in the eighth grade. The sample included 430 girls (46.4%) and 460 boys (49.7%), with 36 students not reporting gender.

Measures

Mathematics-specific intellectual humility was measured using a scale with four items adapted from Leary et al. (2017) and Hoyle et al. (2016), focusing on mathematics learning contexts (4 items; e.g., "I am open to mathematics, and other people's ideas may change my understanding"; α=.752).

Help-seeking beliefs were measured using six items. The beliefs scale included two sub-scales: help-seeking benefits (3 items; α=.852) and costs (3 items; α=.865), adapted from Newman (1990).

Help-seeking tendencies were assessed with six items, including adaptive help-seeking (three items; α=.695) and avoidance (three items; α=.653).

Mathematics achievement was assessed using students' final mathematics examination scores, which were standardized within each school and grade.

Results

The hypothetical structural model fit the data well (χ^2/df = 2.095; CFI = .978, TLI = .971, RMSEA = .034). Intellectual humility was positively related to help-seeking benefits ($\beta = 0.38$) and adaptive help-seeking ($\beta = 0.23$) and negatively associated with help-seeking costs $\beta = -0.10$). Help-seeking benefits positively predicted adaptive help-seeking ($\beta = 0.46$) and negatively predicted avoidance ($\beta = -0.15$). Help-seeking costs negatively predicted adaptive help-seeking ($\beta = -0.17$) and positively predicted avoidance ($\beta = 0.32$). Mathematics achievement was positively predicted by adaptive help-seeking ($\beta = 0.19$). The bootstrapping analysis identified that students with high levels of intellectual humility were more inclined to accept the benefits of help-seeking and thus had more adaptive help-seeking, which predicted better mathematics achievement (β = .034, *SE* = .015, p<.05; 95%*CI* [.009, .067]).

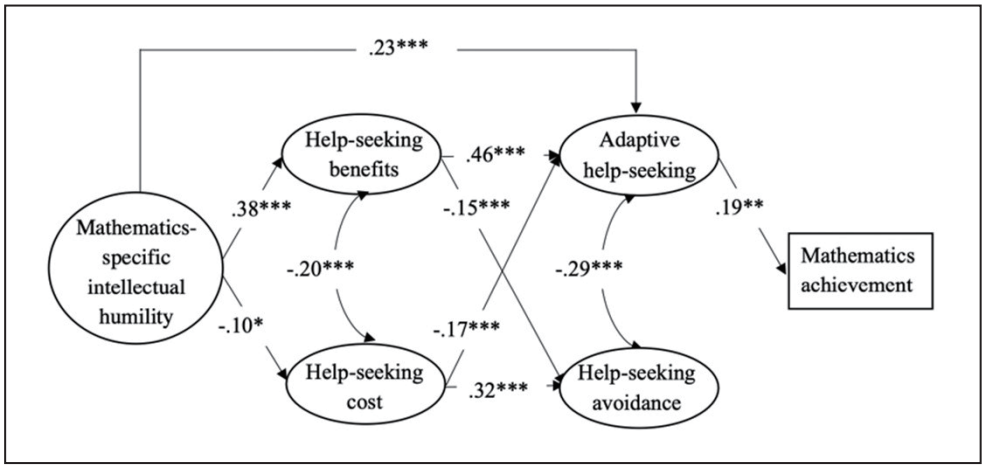


Figure 1. SEM results

Discussion

This study explored the relationships among intellectual humility, help-seeking beliefs, and mathematics performance. Chinese students highly endorsed intellectual humility, likely because they are influenced by Confucian values that promote humility and self-improvement (Li, 2012; Rushing, 2013). Students also endorsed help-seeking benefits, with less emphasis on help-seeking costs or avoidance, reflecting the culture's encouragement of learning from others.

Mathematics-specific intellectual humility was positively related to perceived help-seeking benefits and adaptive help-seeking, indicating that humble students view help-seeking as a strategy for improvement. This aligns with findings by Krumrei-Mancuso et al. (2020) and Porter et al. (2020) on mastery goals. Humble students, with higher metacognitive abilities (Wong & Wong, 2021), are more likely to seek help, recognizing their limitations. Intellectual humility also negatively correlat-

ed with help-seeking avoidance, suggesting that humble students are less concerned with the potential stigma of asking for help (Krumrei-Mancuso & Rouse, 2016).

Help-seeking benefits were positively linked to adaptive help-seeking and improved mathematics performance, consistent with prior studies (Ryan & Pintrich, 1997; Kessels & Steinmayr, 2013). The study's findings highlight the adaptive role of mathematics-specific intellectual humility in enhancing academic achievement, with help-seeking serving as a key mechanism in this relationship, as supported by Wong and Wong (2021).

References

Haggard, M., Rowatt, W. C., Leman, J. C., Meagher, B., Moore, C., Fergus, T., ... & Howard-Snyder, D. (2016). Finding middle ground between intellectual arrogance and intellectual servility: Development and assessment of the limitations-owning intellectual humility scale. *Personality and Individual Differences, 124*, 184-193. https://doi.org/10.1016/j.paid.2017.12.014

Hoyle, R. H., Davisson, E. K., Diebels, K. J., & Leary, M. R. (2016). Holding specific views with humility: Conceptualization and measurement of specific intellectual humility. *Personality and Individual Differences, 97*, 165-172. https://doi.org/10.1016/j.paid.2016.03.043

Hu, X., Leung, F. K., & Teng, Y. (2018). The influence of culture on students' mathematics achievement across 51 countries. International Journal of Science and Mathematics Education, 16(1), 7–24. https://doi.org/10.1007/s10763-018-9899-6

Kessels, U., & Steinmayr, R. (2013). Macho-man in school: Toward the role of gender role self-concepts and help-seeking in school performance. *Learning and Individual Differences, 23*, 234-240. https://doi.org/10.1016/j.lindif.2012.09.013

Kent, K. M. (2022). To Seek or Not to Seek: Intellectual Humility as a Predictor of Professional Psychological Help-Seeking (Doctoral dissertation, University of Florida).

Krumrei-Mancuso, E. J. (2016). The development and validation of the comprehensive intellectual humility scale. *Journal of Personality Assessment, 98*(2), 209–221. https://doi.org/10.1080/00223891.2015.1068174

Krumrei-Mancuso, E. J., & Rouse, S. V. (2016). The development and validation of the comprehensive intellectual humility scale. *Journal of Personality Assessment, 98*(2), 209–221. https://doi.org/10.1080/00223891.2015.1068174

Leary, M. R., Diebels, K. J., Davisson, E. K., Jongman-Sereno, K. P., Isherwood, J. C., Raimi, K. T., ... & Hoyle, R. H. (2017). Cognitive and interpersonal features of intellectual humility. *Personality and Social Psychology Bulletin, 43*(6), 793-813. https://doi.org/10.1177/0146167217697695

Leung, F. K. (2001). In search of an East Asian identity in mathematics education. Educational Studies in Mathematics, 47(1), 35–51. https://doi.org/10.1023/A:1017936429620

Leung, K. S. F. (2006). Mathematics education in East Asia and the West: Does culture matter?. In Mathematics education in different cultural traditions-a comparative study of East Asia and the West: the 13th ICMI study (pp. 21-46). Boston, MA: Springer US.

Leung, F. K. (2014). What can and should we learn from international studies of mathematics achievement?

Mathematics Education Research Journal, 26, 579–605. https://doi.org/10.1007/s13394-013-0109-0.

Li, J. (2012). *Cultural foundations of learning: East and west* (Illustrated edition). Cambridge University Press.

Li, J. (2016). Humility in learning: A Confucian perspective. Journal of Moral Education, 45(2), 147–165. https://doi.org/10.1080/03057240.2016.1168736 Zhu, Y., & Leung, F. K. (2011). Motivation and achievement: Is there an East Asian model? *International Journal of Science and Mathematics Education, 9*(5), 1189–1212. https://doi.org/10.1007/s10763-010-9255-y

Newman, R. S. (1990). Children's help-seeking in the classroom: The role of motivational factors and attitudes. *Journal of Educational Psychology, 82*(1), 71–80. https://doi.org/10.1037/0022-0663.82.1.71

Porter, T., Schumann, K., Selmeczy, D., & Trzesniewski, K. (2020). Intellectual humility predicts mastery behaviors when learning. *Learning and Individual Differences, 80*, 101888. https://doi.org/10.1016/j.lindif.2020.101888

Rowatt, W. C., Powers, C., Targhetta, V., Corner, J., Kennedy, S., & LaBouff, J. (2006). Development and initial validation of an implicit measure of humility relative to arrogance. *The Journal of Positive Psychology, 1*(4), 198–211. https://doi.org/10.1080/17439760600885671

Rushing, S. (2013). What is Confucian humility? In S. Angle & M. Slote (Eds.), *Virtue ethics and Confucianism* (pp. 174–181). Routledge.

Ryan, A. M., & Pintrich, P. R. (1997). "Should I ask for help?" The role of motivation and attitudes in adolescents' help-seeking in math class. *Journal of Educational Psychology, 89*(2), 329–341. https://doi.org/10.1037/0022-0663.89.2.329

Ryan, A. M., & Shin, H. (2011). Help-seeking tendencies during early adolescence: An examination of motivational correlates and consequences for achievement. *Learning and instruction, 21*(2), 247-256. https://doi.org/10.1016/j.learninstruc.2010.07.003

Wong, I. H. M., & Wong, T. T. Y. (2021). Exploring the relationship between intellectual humility and academic performance among post-secondary students: The mediating roles of learning motivation and receptivity to feedback. *Learning and Individual Differences, 88*, 102012. https://doi.org/10.1016/j.lindif.2021.102012

TSG7-20

A STUDY ON THE RELATIONSHIP BETWEEN MATHEMATICS ADVERSITY QUOTIENT AND MATHEMATICAL ACHIEVEMENT OF SECOND-GRADE JUNIOR HIGH SCHOOL STUDENTS —THE MEDIATING ROLE OF MATHEMATICAL ANXIETY

Yumei Xia
Beijing Normal University, China, 202431130011@mail.bnu.edu.cn

Lianhua Ning
Nanjing Normal University, China, ninglianhua@njnu.edu.cn

Jing-ya Zhao
Nanjing Normal University, China

Mathematical adversity quotient and mathematical anxiety are important non-intellectual factors affecting students' mathematical learning, and the self-compiled mathematical adversity quotient scale and mathematical anxiety scale were used to investigate the influence and mechanism of mathematical adversity quotient on students' mathematics achievement. The results showed that: (1) Mathematical adversity quotient, mathematics achievement, and mathematical anxiety are related. Among them, mathematical adversity quotient significantly predicts mathematics achievement positively, mathematical anxiety significantly negatively predicts mathematics achievement and mathematical adversity quotient. (2) Mathematical anxiety played a partial mediating role between mathematical adversity quotient and mathematics achievement, and the mediating effect contributed 34.35% to the total effect.

Keywords: mathematical adversity quotient; math anxiety; intermediary role

Introduction

Mathematical Adversity Quotient（MAQ）, derived from Paul Stoltz's concept of Adversity Quotient, reflects an individual's ability to cope with challenges encountered in the process of learning mathematics (Stoltz, 1998). Existing studies have shown a significant correlation between MAQ and students' mathematics achievement (Cao& Zhang, 2012; BAMBANG &IMAN, 2017; zhang & wu, 2019; Cao& Zhang et al, 2015), and MAQ is also influenced by factors such as test anxiety (Cao& Zhang, 2012), teacher support (Cao& Zhang et al, 2015), and self-efficacy (BAMBANG &IMAN, 2017). Meanwhile, mathematics anxiety is another important variable that negatively predicts mathematics performance and may indirectly affect it through self-efficacy or metacognitive processes (zhang &huang et al, 2018).

At present, research on MAQ remains limited, particularly in terms of integrating MAQ, mathematics anxiety, and mathematics achievement within a unified framework. Therefore, this study aims to construct a mediation model to (1) examine the relationship between MAQ and academic achievement, (2) investigate whether mathematics anxiety mediates this relationship, and (3) test

the significance and contribution of the mediating effect, in order to better understand the mechanism through which MAQ influences students' mathematics achievement.

Materials and Methods

Participants

To ensure the representativeness and persuasiveness of the research data, a public middle school with above-average student mathematics proficiency was selected for the study. A total of 355 students from eight classes in the second year of junior high school were surveyed. After excluding invalid data, 314 valid scales were collected, resulting in an effective recovery rate of 88.45%. Among these, 165 were male students (52.55%) and 149 were female students (47.45%).

Measures

Mathematics Achievement

The mid-term exam scores of the students were selected as the measure of mathematics achievement. The exam papers were uniformly prepared by the education bureau, which can effectively test and differentiate students' abilities and levels. The invigilation and grading processes were rigorous, transparent, and fair. Therefore, the data has a certain degree of reliability and reference value.

Mathematics Adversity Quotient (MAQ) Scale

Based on the psychological characteristics of junior high school students and the features of mathematics learning, and referencing Paul Stoltz's AQ scale (Stoltz, 1998) and the junior high school students' MAQ scale by Cao Chunyan and Zhang Dingqiang (Cao& Zhang,2012), a scale was developed with four dimensions: mathematics control, mathematics origin &ownership, mathematics reach, and mathematics endurance. The formation of the scale also considered the opinions of educational scholars, front-line teachers, and postgraduate students.

The final scale consists of 24 items and adopts a five-point Likert scoring method, with higher scores indicating a higher level of MAQ. The Cronbach's α coefficient for the MAQ scale was 0.870, suggesting good internal consistency and reliability. Furthermore, the correlations between each dimension of MAQ and the total score were higher than the inter-correlations among the dimensions themselves, indicating that the MAQ scale has good structural validity.

Mathematics Anxiety Scale

The Mathematics Anxiety Scale developed by Wu Minglong (Wu, 2001) was employed in this study. The scale comprises 27 items across four dimensions: stress-related fear, emotional worry, exam anxiety, and classroom anxiety. It adopts a five-point Likert scoring method, with higher scores reflecting higher levels of mathematics anxiety. The Cronbach's α coefficient for the overall mathematics anxiety scale was 0.919, indicating excellent internal consistency. Additionally, the correlations between each dimension and the total score were higher than the inter-correlations among the dimensions, suggesting that the scale possesses good structural validity.

Experimental Procedure and Statistical Analysis

The tests were administered collectively by class, with the class teacher acting as the examiner. To prevent students from responding negatively, the examiner briefly explained the survey's purpose and precautions before the test. All scales were collected immediately after the test. The data from

the collected scales were entered into Excel, and SPSS 26.0 was used for descriptive statistics, correlation and mediation effect analysis.

Results

Common Method Bias

Since the data were all derived from self-reports of junior high school students, there might be a common method bias. To detect whether it had a significant impact on the results, Harman's single-factor test was conducted. The results showed that there were 10 factors with eigenvalues greater than 1, and the largest factor explained 27.95% of the variance, which is below the critical value of 40%. This indicates that there is no serious common method bias in the study.

Correlation Analysis of "MAQ", "Mathematics Achievement ", and "Mathematics Anxiety"

The data showed that MAQ, mathematics anxiety, and mathematics achievement were significantly correlated with each other. Among them, mathematics anxiety was significantly negatively correlated with MAQ and mathematics achievement, and MAQ was significantly positively correlated with mathematics achievement. This correlation was reflected in specific dimensions as follows: (1) mathematics control, mathematics reach, and mathematics endurance were significantly positively correlated with students' mathematics achievement; (2) stress fear, emotional worry, and exam anxiety were significantly negatively correlated with students' mathematics achievement; (3) mathematics origin&ownership and classroom anxiety were not significantly related to mathematics achievement.

The Mediating Role of Mathematics Anxiety between MAQ and Mathematics Achievement

According to the stepwise regression method for mediation effect test results, it can be seen that in the first step (test of model 1), MAQ had a significant impact on mathematics achievement ($\beta=0.729$, $P<0.001$), indicating that the total effect was established; in the second step (test of model 2), MAQ significantly affected mathematics anxiety ($\beta=-0.614$, $P<0.001$); at the same time, in the third step (test of model 3), mathematics anxiety still had a significant impact on mathematics achievement ($\beta=-0.408$, $P<0.001$), and the previous impact of MAQ on mathematics achievement was still significant ($\beta=0.478$, $P<0.001$). Therefore, it shows that the mediating role of mathematics anxiety in the model was established, and it was a partial mediation. Then, the Bootstrap sampling technique was used to test the significance of the mediation effect and evaluate the size of its confidence interval, and the total effect, direct and indirect effects, and their 95% confidence intervals were obtained. The data showed that at the 95% confidence level, the indirect effect of MAQ on mathematics achievement through mathematics anxiety was 0.251, and the confidence interval [0.111, 0.444] did not contain zero. The direct effect of mathematics AQ on academic performance was 0.478, and the confidence interval [0.001, 0.200] also did not contain zero, indicating that the indirect effect and direct effect were established, and mathematics anxiety played a significant mediating role in the model. According to the effect proportion calculation results, the contribution rate of mathematics anxiety to the total effect was 34.43%.

Discussion

The Relationship Among MAQ, Mathematics Anxiety, and Academic Achievement

Studies have shown that MAQ positively predicts students' mathematics achievement, supporting the findings of N. A. Dina and Masriyah (Dina & Masriyah, 2018) . Significant differences in MAQ were observed among students with different achievement levels(Cao& Zhang et al, 2015); specifically, mathematics achievement significantly influenced the dimensions of mathematical control and extension, but not attribution or influence(Cao& Zhang, 2012).

In contrast, mathematics anxiety was found to negatively predict achievement （Wu, Barth, Amin, Malcarne, & Menon, 2012）, consistent with prior research （Wei, Liu, & Wen, 2012; Cui, Li, & Chen, 2011; Peng & Xu, 2012）. High-achieving students reported significantly lower anxiety levels across various dimensions compared to those with moderate performance （Peng & Xu, 2012）. This may be due to anxiety occupying working memory resources, as proposed by Ashcraft and Faust （Ashcraft & Faust, 1994）, thereby impairing cognitive efficiency during mathematical tasks.

MAQ and mathematics anxiety were significantly negatively correlated, aligning with findings by Cao Chunyan (Cao& Zhang, 2012). Students with low MAQ may experience higher anxiety due to maladaptive attribution, poor understanding of adversity, or negative self-perception in mathematics （Shi, Chen, Yuan, & Ding, 2015）.

The Mediating Role of Mathematics Anxiety

Mediation analysis indicated that mathematics anxiety partially mediates the relationship between MAQ and mathematics achievement. In other words, MAQ affects achievement both directly and indirectly via its impact on anxiety. This supports Processing Efficiency Theory（Eysenck & Calvo, 1992）, which suggests that anxiety reduces available cognitive resources for problem-solving, thereby hindering performance.

The direct effect of MAQ on achievement was 0.478, while the indirect effect via anxiety was 0.250, accounting for 34.35% of the total effect. This implies the direct influence of MAQ outweighs its indirect pathway through anxiety, though other mediating variables may also be involved.

Previous studies have shown that mathematics self-efficacy significantly influences both MAQ and achievement (BAMBANG &IMAN, 2017; zhang &huang et al, 2018), suggesting it may serve as an additional mediator or form a chain mediation with anxiety. Future research should explore this to better understand the mechanisms through which MAQ influences academic performance.

References

Ashcraft, M. H., & Faust, M. W. (1994). Mathematics anxiety and mental arithmetic performance: An exploratory investigation. *Cognition and Emotion*, *8*, 97–125.

Bambang, S., & Iman, S. T. (2017). Self-efficacy, adversity quotient, and students' achievement in mathematics. International Education Studies, 10(10), 12–14.

Cao, C. Y., & Zhang, D. Q. (2012). Investigation and analysis of junior high school students' mathematics adversity quotient—Based on mathematics exam performance and mathematics exam anxiety dimensions. Journal of Mathematics Education, 21(2), 23–25.（in Chinese）

Cao, C. Y., Zhang, D. Q., & Ding, H. F. (2015). Investigation and analysis of mathematical adversity quotient of junior high school students in ethnic minority areas: A comparative study of Tibetan and Han students in Gannan Tibetan Autonomous Prefecture. Journal of Mathematics Education, 24(2), 50–53. （in Chinese）

Cui, J. F., Li, N. X., & Chen, Y. H. (2011). An analysis of the mechanisms by which mathematics anxiety affects children's performance on mathematical tasks. Psychological Development and Education, 27(2), 118–125. （in Chinese）

Dina, N. A., & Masriyah. (2018). Flexibility in mathematical problem solving based on adversity quotient. Journal of Physics: Conference Series, 947(1).

Eysenck, M. W., & Calvo, M. G. (1992). Anxiety and performance: The processing efficiency theory. Cognition and Emotion, 6(6), 409–434.

Peng, Y., & Xu, S. (2012). A study on mathematics anxiety among junior high school students with different levels of academic achievement. Journal of Mathematics Education, 21(4), 40–43. （in Chinese）

Stoltz, P. G. (1998). Adversity quotient: Turning obstacles into opportunities. Tianjin: Tianjin People's Publishing House. （in Chinese）

Shi, J., Chen, J. D., Yuan, D. Z., & Ding, X. W. (2015). A survey on mathematics anxiety among secondary vocational students in "3+4" pilot programs. *Journal of Mathematics Education*, *24*(3), 91–95. （in Chinese）

Wu, M. L. (2001). SPSS statistical application practice. Beijing: China Railway Publishing House.66-68. （in Chinese）

Wei, H., Liu, Y. M., & Wen, F. Y. (2012). The correlation between mathematics anxiety and mathematics achievement in senior high school students. Journal of Mathematics Education, 21(6), 43–45. （in Chinese）

Wu, S. S., Barth, M., Amin, H., Malcarne, V., & Menon, V. (2012). Math anxiety in second and third graders and its relation to mathematics achievement. *Frontiers in Psychology*, *3*, 1–11.

Zhang, Y., & Wu, X. P. (2019). A survey on the level of mathematical adversity quotient among high school students in Shui ethnic areas of Qiannan. Journal of Mathematics Education, 28(3), 88–93. （in Chinese）

Zhang, J., Huang, B. J., Si, J. W., et al. (2018). Mathematical anxiety and academic achievement among rural primary school students: The chain-mediating role of math self-efficacy and metacognition. Psychological Development and Education, 34(4), 453–460. （in Chinese）

TSG7-21

USING HISTORICAL CONTEXTS IN NON-HISTORICAL MATHEMATICS COURSES.

Amanda Jo Meiners
Northwest Missouri State University, USA, ameiners@nwmissoui.edu

Sidney Jennings
Lone Star College-University Park, USA, Sidney.G.Jennings@lonestar.edu

This study investigated participants in remedial-level mathematics and mathematics education methods courses by collecting classroom work, classroom conversations, and field notes on their personal and professional mathematical identities. Data analysis focused on coding through an analytical lens of the ICUCARE Framework, Results demonstrate the proof of concept of including such learning within the courses provides more interest in the course and includes four main categories of the framework: focus on social justice and reflection, everyone is a mathematician, connections to future classrooms for the PSTs, and their conceptualization of mathematics. The ramification of these transformational experiences creates possibilities for new trajectories of participation for the participants in the field of mathematics and mathematics education.

Keywords: Interest, Identity, Social Justice

Background

Classroom Practice

Mathematics teaching has shifted from teacher-centered to more student-centered (Sandra et al., 2014; Stein et al., 2008). Growing up, many students have not had experience with finding visualizations or stories of mathematicians in a manner that they see themselves. The effects of including historical mathematicians for prospective teachers (Gunderson et al., 2012) and even mathematics faculty members (Braun & Kahn, 2019; Gençkaya, Ş & Tan-Şişman, 2021) has been considered, but documented cases of comparing how these interactions shift students' identity and interest across various course types has been minimal.

United States of America methods courses are intended to offer Pre-Service Teachers (PSTs) insight into teaching their future students' multiple strategies, viewpoints, and enactments in their future classrooms (Dede & Karakuş, 2014;). Remedial courses are known for creating gatekeeping conditions and often create a dehumanization of the mathematical topics covered (Boatman & Long, 2018). Our study investigated students using experiences that were atypical from their previous mathematics interactions (Cousins et al., 2019) while taking mathematics education methods or remedial-level mathematics coursework. Including mathematical historical informational items continues to demonstrate that participants create deeper connections to mathematics concepts and reduce student anxiety or other negative feelings/interest toward learning mathematics (Gunderson et al., 2012; Furinghetti, 2015).

Framework

Although using the history of mathematics and promoting a supportive environment have been used in various studies separately (citations), there is a need for more research that looks at these in combination. This study focused on how students were impacted by exploring mathematical contributions made by women and other underrepresented mathematicians, using student-led cultural perspectives about the doing of mathematics worldwide, and promoting an environment which emphasized personal connections to learning mathematics through individual sensemaking and conceptual development.

Using the student's narrative identity, or who students tell themselves and others who they are as mathematical thinkers and doers (Sfard & Prusak, 2005), the research team focused on the student's sensemaking of their experiences and looked for shifts in students' self-narrativization.

To analyze the students' narratives, this study draws on selected parts of Seda and Brown's (2021) ICUCARE framework. We focus on the components of (R)eleasing control in a manner that (I)ncluded the students (PSTs and non-PSTs) as the experts who were (C)ritically conscious, while (A)ssessing, activating, and building on their prior situations in mathematics through using (C)ulturally relevant curriculum. When students can learn where mathematics came from and continue to grow from in a self-guided manner, then in turn we may assist more towards an appreciation and interest in mathematics.

Therefore, this study was designed to present dual viewpoints from PSTs and non-PSTs using the following research question: *How, if at all, do intentional pedagogical interventions create shifts in the personal and professional mathematics narrative identities of PSTs and non-PSTs?*

Methods

Participants

As part of the standard course materials, approximately 120 students took part in historical context mathematics work in their courses through versions of reflective journaling and in-class discussions. These endeavors aimed to help students "past, present, and future mathematics identities enter into a dialogue" (Lutovac & Kaasila, 2014, p.131) and extend that into learning mathematics beyond the classroom. The participants were from two different higher education providers. The PSTs were a convenient sample, gaining certification from Birth to 12th grade (n = 80), while the non-PSTs were community college students taking remedial mathematics courses (n = 40). Both higher education institutions were in semi-rural areas, in conservative states with moderate political views, and recruited from local first-generation and low-socioeconomic populations.

Data Collection and Analysis

Data collection during the PST's methods courses was class talks via field notes and PST posts to the discussion board. PSTs within the methods courses, reflection was accomplished using an open-source blog (Perkins, 2020), short daily student-led discussions, and extensive (pre/post) large-group talks

Data collection during the remedial courses consisted of two writing assignments. The first was a math autobiography assigned at the beginning of the course, and the other was a detailed reflection at the end of the semester. Both written assignments included prompts to guide the participants' reflections. Examples of these prompts include: "What, if anything, in your experience in this class

challenged you to think differently about yourself as a mathematical thinker and doer?" and "At the beginning of this course you wrote a math autobiography sharing about your previous math learning experiences and how they shaped you as a mathematical thinker and doer.

Both instructors' field notes, classroom discussions, and PSTs/non-PSTs journaled items were analyzed, with each course section as a case study (Merriam & Tisdell, 2016). In vivo, holistic codes were first noticed within each case about personal and professional mathematics narratives, while secondary codes used the focused ICUCARE framework (Saldaña, 2021;Seda & Brown, 2021).

Results

Table 1. ICUCARE Framework student quotes connections

ICUCARE Framework Connections		
(R)eleasing control	I already feel so much more confident in my ability to teach this subject that I thought I was horrible at.	
(I)ncluded the experts	I always thought math was for really smart people and people who were solving huge calculations, but that not just it.	
(C)ritically concious	You can use things in so many different ways…I learned a lot about the importance of my students seeing themselves as math students….	
(A)ssessing, activating, and building on their prior situations	Math is not my favorite and I did have a bias opinion that would have led into my classroom. This class has made me realize my bias opinion on math and now that I am aware of it I can make sure that I take math just as important as the other subject in my classroom.	
(C)ulturally relavent curriculm	That hey, math can be fun too because sometimes students don't see math as something fun, but when they know someone who had fun on the football field can have fun in math too.	I need to include more diversity when teaching mathematics. I have never thought about the fact that we typically think of white males dominating the field.

Table 2. What types of shifts using narratives within the courses.

Themes	Data Quote Content*	Data Quotes Methods**
Social Justice Reflection	Not only did students from other backgrounds show how their education systems handled things differently, but also how flexible and multipolar Mathematics really is as a whole.	…Use Hypatia to teach the [social justice] concept of unity through mathematics but focus more on her intention to spread the knowledge she had to others. This would be a great way to motivate young students to collaborate with others and share knowledge and ideas, even if they don't always agree.
Everyone is a Mathematician (not just the typical portrayal in U.S. textbooks)	I no longer have to worry about if what I am doing is the "correct" thought process to answer a question. Really just being able to convince myself that math doesn't need to be THAT stressful was also very tough because growing up there was always the pressure that what work we do has to all be the same thought process or its wrong. When in reality everyone has their own way of breaking down problems and solving them.	Since I thought that most mathematical discoveries were made a long time ago, I assumed that most of the influential mathematicians were old white men because they were the ones in that time who were more respected and were given quality educations. As I have researched and others have presented, I learned that there are a lot of female and people of color that have been very influential in the field of mathematics. I think it's important to let my students know this as well because it will show them that they are represented in the area of mathematics.
Future Classroom	*Not Applicable*	Shows the students that hey, math can be fun too because sometimes students don't see math as something fun, but when they know someone who had fun on the football field can have fun in math too, I think that will really engage their learning.
Conceptualization of Mathematics	It is exciting when I come face to face with a hard question and realize that it's not that difficult. At first glance, I would have never thought I could figure it out, but when I realize I am doing all the right steps and then get it right,it's a very rewarding feeling	Mathematics has been shaped for decades, if not centuries by people of all backgrounds. Some of the most important breakthroughs in the fields of number theory, algebra, computers, engineering were found by people who never even had any formal education of higher-level mathematics.

*non-PST **PST

Discussion

The students all mentioned initially that they anticipated these chats about historical and cultural contexts were going to be frustrating because their previous experiences with mathematics, in general, had not been all that positive; however, all students mentioned various levels of appreciation. These included how they now felt "seen" in mathematics and that there were still new mathematics opportunities that could be forthcoming. This is of particular importance for PSTs in early childhood and elementary education, a field dominated by females, as teacher math identity impacts their students' mathematical learning and identity development (Beilock et al., 2010). Students' statements connected to the ICUCARE framework demonstrated there was a shift in their view of who a mathematician could be by using a culturally relevant curriculum. As most of the PSTs and non-PSTs in this study would be categorized as members of underrepresented groups in mathematics learning environments, it is significant to see shifts in their self-narrativizations about who they are as mathematical thinkers and doers.

References

Beilock, S. L., Gunderson, E. A., Ramirez, G., & Levine, S. C. (2010). Female teachers' math anxiety affects girls' math achievement. *Proceedings of the National Academy of Sciences*, *107*(5), 1860-1863.

Boatman, A., & Long, B. T. (2018). Does remediation work for all students? How the effects of postsecondary remedial and developmental courses vary by level of academic preparation. Educational Evaluation and Policy Analysis, 40(1), 29-58.

Braun, B., & Kahn, E. (2019). Teaching history of mathematics: A dialogue. Journal of Humanistic Mathematics, 9(1), 317-325.

Cousins, S., Brindley, J., Baker, J., & Johnston-Wilder, S. (2019). Stories of mathematical resilience: How some adult learners overcame affective barriers. Widening participation and lifelong learning, 21(1), 46-70.

Dede, Y., & Karakuş, F. (2014). The effect of teacher training programs on pre-service mathematics teachers' beliefs towards mathematics. Kuram ve Uygulamada Eğitim Bilimleri, 14(2), 804-809.

Furinghetti, F. (2015). The History of the Teaching and Learning of Mathematics. The Proceedings of the 12th International Congress on Mathematical Education: Intellectual and attitudinal challenges,

Gençkaya, Ş., & Tan-Şişman, G. (2021). The use of the history of mathematics in teaching-learning process: The perspectives of faculty members and teachers. Psycho-Educational Research Reviews, 10(2), 241-257.

Gunderson, E. A., Ramirez, G., Levine, S. C., & Beilock, S. L. (2012). The role of parents and teachers in the development of gender-related math attitudes. Sex roles, 66, 153-166.

Lutovac, S., & Kaasila, R. (2014). Pre-service teachers' future-oriented mathematical identity work. Educational Studies in Mathematics, 85, 129-142.

Merriam, S., & Tisdell, E. (2016). Designing your study and selecting a sample. Qualitative research: A guide to design and implementation, 73-104.

Perkins, A. M. (2020, August 20). *The mathematicians project: Mathematicians are not just White Dudes*. Arbitrarily Close. Retrieved August 25, 2021, from https://arbitrarilyclose.com/2016/08/21/the-mathematicians-project-mathematicians-are-not-just-white-dudes/

Saldaña, J. (2021). The coding manual for qualitative researchers (4 ed.). Sage.

Sandra, L. Laursen, Hassi Marja-Liisa, Kogan Marina, and J. Weston Timothy. 2014. "Benefits for Women and Men of Inquiry-Based Learning in College Mathematics: A Multi-Institution Study." Journal for Research in Mathematics Education 45 (4): 406-418. https://doi.org/10.5951/jresematheduc.45.4.0406. http://www.jstor.org/stable/10.5951/jresematheduc.45.4.0406

Seda, P., & Brown, K. (2021). Choosing to see: A framework for equity in the math classroom. Dave Burgess Consulting, Incorporated.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational researcher*, *34*(4), 14-22.

Stein, Mary Kay, Randi A Engle, Margaret S Smith, and Elizabeth K Hughes. 2008. "Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell." *Mathematical thinking learning & Individual Differences* 10 (4): 313-340.

TSG7-22

EMOTIONAL DIMENSIONS AND SOCIAL SOLIDARITY IN MATHEMATICS EDUCATION: EVERYONE ENJOYS GAMES - OR DO THEY?

Linda Bonne
Victoria University of Wellington, New Zealand, linda.bonne@vuw.ac.nz

Joanna Higgins
Victoria University of Wellington, New Zealand, joanna.higgins@vuw.ac.nz

Raewyn Eden
Massey University, New Zealand, r.eden@massey.ac.nz

Emotions associated with mathematics learning experiences are often obscured by a focus on cognition. Our ongoing programme of research has investigated the emotional dimensions and social interactions that are evident during whole-class games in mathematics education settings in New Zealand. Taking a sociology of emotions framing, the naturalistic research has used a combination of event-oriented social inquiry and multiple methods (classroom transcripts, interviews, cogenerative dialogues, interactive response cards, email conversations, and reflective notes) to identify salient characteristics in mathematics settings. We draw on selected findings from a decade of work to argue that social interactions can build social solidarity and help create an emotionally safe learning environment, in some cases disrupting cycles of negative emotion associated with mathematics, by attending to emotion in not only the individual but also in the collective.

Keywords: emotions, social solidarity, mathematics education

Concern with Mathematics Outcomes and Increasing Maths Anxiety

In Aotearoa New Zealand, raising students' achievement in mathematics continues to be a pressing issue. Strengthening teachers' mathematics teaching competencies is seen as an important lever to raising achievement. Additionally, significant proportions of students report they are not confident in their ability to do mathematics (e.g., almost 40% of New Zealand's Year 5 students in 2023, according to Caygill et al., 2024). We argue that focusing on students' academic achievement includes teachers and researchers attending to what can be done to build and transform emotional aspects of classroom learning environments.

Emotions associated with mathematics can be counterproductive to learning, and reframing negative narratives about mathematics is long overdue. In recent years, for example, mathematics anxiety has become a pervasive explanation for low mathematics achievement. The challenge of transforming stubborn discourses, such as mathematics anxiety, points to the enduring and intergenerational entanglement between emotions and the teaching of mathematics. One strategy for making mathematics more engaging for anxious learners has been the adoption of games to cast mathematics as "fun" and promote an emotionally safe learning environment. The events outlined in this paper all occurred in the context of students and their teachers playing whole-class mathematics games.

Emotional Dimensions of Mathematics Learning Environments

Bringing emotions to the forefront is important to understanding the interdependent relationship between emotion and cognition in education settings, as explained in sociological theories of emotion (Collins, 2004; Turner, 2002). Clear links have been identified between teachers' affect and that of their students in various mathematics contexts (e.g., Chang, 2015), therefore we have conducted our research in both elementary and pre-service teacher education classrooms (see Bonne & Higgins, 2022; Higgins et al. 2024).

In elementary classrooms, emotional dimensions of mathematics learning environments are ever-present, and need to be integrated with achievement considerations. Similarly, in pre-service teacher classrooms, one challenge is to prepare future teachers to both competently teach and provide an emotionally safe learning environment for their students. Previous negative experiences of learning mathematics can be problematic for those preparing to become elementary school teachers. To address such experiences and the emotions attached to them, the emotional dimensions of mathematics learning environments need to be better understood.

Teachers intentionally attending to both collective and individual emotions in classroom interactions is critical for building social solidarity. Collins' (2004) focus on interaction rituals, particularly mutual focus and emotional entrainment, is helpful for understanding social solidarity. The notion of social solidarity is one of the important outcomes of successful social interactions, along with emotional energy and collective ritual objects. As well as positioning emotions in individuals, we are interested in the dialectic relationship between collective | individual emotions and how these emerge and contribute to social solidarity. In this paper, we briefly describe several events that we interpret as evidence of social solidarity of the groups involved.

Taking a sociology of emotions framing (Turner, 2002), we position our research as using event-oriented social inquiry (Alexakos, 2015). The multiple methods we have employed include classroom transcripts, cogenerative dialogues (cogen), an interactive response card system, one-to-one interviews, email conversations, and the research team's reflexive conversations. In this paper, we report research with a class of 43 pre-service teachers who were taught by the second and third authors of this paper, and separately, one class of 34 students (10- to 13-years-old) and the two teachers who shared the class.

Reflective Analysis

Across our work, we aim to make sense of people's understanding, knowing, and learning in classroom contexts. We are particularly interested in elevating the consideration of emotion so that it is understood alongside cognition in mathematics classrooms. When we reflected on our collective research findings, it became apparent that a common theme was social solidarity amongst teachers and their students. In the remainder of this paper, we trace two aspects of social solidarity that we have used in our reflective analysis. Each of these aspects arose when the groups were participating in mathematics games.

Being-in/with

The first of these aspects is 'being-in/with', that Roth and Tobin (2002) describe as "being-in the world and being-with others [as] fundamental and a priori conditions" (p. 315). There was evidence of this in both elementary classrooms and pre-service teacher education settings. On reflection, we were struck by examples of the shifting teacher role highlighted in interactions with students.

In an early paper (Higgins & Bonne, 2014), we observed how the teacher shifts her alignment with the elementary students across different classroom events. We noted, "knowing how to have fun appeared to contribute to the teacher's social capital in the eyes of her students, allowing the teacher to ''be in with'' the students or in solidarity with them" (p. 56). Furthermore, the teacher was described as drawing on "her symbolic capital in her role as teacher, cultural capital in being a joke initiator and participant, and social capital in her participation alongside the students as a game player" (p. 54). The teacher being willing to join the students in having fun and making jokes generated laughter and an apparent sense of solidarity amongst the students. Similarly, in our current work in a pre-service teacher education context, when a student made an under-the-breath quip that potentially challenged the social capital of the teacher as being-in/with the students, the teacher educator reframed it to highlight the potential learning opportunities, and coined the term, 'gift' (Eden et al., in preparation). This interaction helped enable the construction of the roles of teacher and student (Roth & Tobin, 2002). We argue that in both instances, the dynamic of being-in/with helped build social solidarity amongst teachers and their students.

Ritual Object

The second aspect of solidarity is the notion of a ritual object of an enduring nature, which creates a common point of focus for interactions amongst the collective. Collins (2004) described ritual as "a mechanism of mutually focused emotion and attention producing a momentarily shared reality, which thereby generates solidarity and symbols of group membership" (p. 7).

We recognised evidence of ritual objects in both our research contexts. Students in the elementary classroom described the teacher regulating their behavior by having "the talk" with students who she thought were "out of line" (Higgins & Bonne, 2014, p. 58). In the cogen where students brought this up, mentioning "the talk" elicited a ripple of laughter. All the students were aware of "the talk", which we interpret as a ritual object that generated social solidarity amongst the class. In the pre-service classroom, a ritual object, a 'gift', contributed to building social solidarity and drew the students' attention to the emotional aspects of the interaction, with a focus on humour. In both cases, the ritual object was a key factor in creating an emotionally safe learning environment that focused more on the collective than the individual.

Solidarity for All?

The opportunity for the inclusion of everybody who was present was important for both aspects of social solidarity. For example, the majority of the elementary students expressed mutual student-teacher respect which was associated with the teacher's willingness to join the students in having fun during mathematics classes. However, two students showed some resistance to being-in/with the teacher. One pre-service teacher was unintentionally excluded from a shared understanding of the 'gift' due to their absence when it was initiated. In the next class, they described not understanding what was going on in the class. In both situations, it transpired that social solidarity was not universal.

Concluding Thoughts

We emphasize the value of teachers and teacher educators intentionally attending to and disrupting conventional interaction structures to open up pedagogical and socio-emotional possibilities. Our focus on games as a context for engaging students has illuminated the emotions associated with mathematics learning and teaching. It raises questions about how teachers and teacher educators

might attend to the emotional dimensions of mathematics classrooms and foster social solidarity through ritual objects and 'being-in/with' others.

References

Alexakos, K. (2015). *Being a teacher | researcher*. Sense Publishers.

Bonne, L., & Higgins, J. (2022). Game playing and fluctuations in emotional climate. *Cultural Studies of Science Education, 17,* 1063-1079.

Caygill, R., Borthwick, R., & Bennett, H. (2024). *TIMSS 2023: Mathematics Year 5*. Trends over nearly three decades of TIMSS. https://www.educationcounts.govt.nz/publications/schooling/ timss-2023-mathematics-year-5

Chang, Y.-L. (2015). Examining Relationships among Elementary Mathematics Teacher Efficacy and Their Students' Mathematics Self-efficacy and Achievement. *Eurasia Journal of Mathematics, Science and Technology Education, 11*(6), 1307-1320.

Collins, R. (2004). *Interaction ritual chains*. Princeton University Press.

Eden, R., Higgins, J., & Bonne, L. (in preparation). Promoting social solidarity in a pre-service mathematics education classroom: the "gift" as a ritual object.

Higgins, J., & Bonne, L. (2014). Exploring classroom life through cogenerative dialogues. *Cultural Studies of Science Education, 9*: 53-61.

Higgins, J., Bonne, L., & Eden, R. (2024). Responding to negative emotion in a pre-service mathematics classroom. *Cultural Studies of Science Education, 19,* 633-646.

Roth, W-M., & Tobin, K. (2002). *At the elbow of another.* Peter Lang.

Turner, J.H. (2002). *Face to face: Toward a sociological theory of interpersonal behavior.* Stanford University Press.

TSG7-23

RETHINKING NEO-CONFUCIAN CONCEPTS OF LEARNING & WELLBEING

Emily Sum
Faculty of Education, The University of Melbourne, Australia, emily.sum@unimelb.edu.au

Confucian virtues are often asserted as the underlying factors contributing to East Asian students' exceptional mathematics performances. The valuing of education and examination has, however, created considerable anxiety among students, affecting their well-being. Yet, the relevance of Confucianism in contemporary context remains a subject of debate. In this study, Neo-Confucian concepts of learning and well-being based on a textual study of Jaseongnok are revisited, and to illustrate the relevance, the framework of the OECD study on social and emotional skills is referred to as an example. In this context, Confucian learning has the potential to develop the skills that enable students to examine the ethical and moral issues in mathematics, which can contribute to their personal well-being and others in the society.

Keywords: Neo-Confucian learning, well-being, Korean culture

Introduction

Confucius (551-479 BCE) is a Chinese philosopher whose teaching remains influential in East Asia today. As evident in the *Analects*, he highly values education. Although Joseon Korea (1392-1910) has gone for over a century, the enduring legacy of Confucian thought and social patterns continues to influence Koreans in various modern ways (Chung, 2015). Confucian virtues, such as *filial piety*, *respect*, *trust, wisdom*, and the ethics of human relations, continue to be emphasized in the moral education curriculum (Ministry of Education, 2022). The cultural valuing of education and examinations, as means of upward mobility and financial security has a long history, dating back to Goryeo (918-1392). The meritocratic system rooted in Confucian tradition is often asserted as the underlying factor behind students' performance in East Asia countries (e.g. Leung, 2008; 2014). The pressure to excel in examinations has created a great deal of stress and anxiety among students, adversely affecting their emotional, physical and social wellbeing, as well as the resilience to cope with challenges and difficulties. In the light of these, this study explores the potential of mathematics education in fostering the well-being of individuals and others in the society, by cultivating students' virtues and empowering them to critically analyze the ethical and moral issues in mathematics.

Korean Students' Values in Mathematics Learning and Well-being

Korean students, who value *understanding* and *problem* in learning mathematics (Pang et al., 2024), believed that preparation and review were essential for grasping mathematical formulas and basics. They assert that "preparation makes you do well by practicing what you will learn later" (*ibid*, p.135). This aligns with the findings of Kim and Jung's (2019) study on *hagwons*, the private learning centers in South Korea, which provide students with opportunities to prepare for the mathematical content in upcoming school semesters, and review previously learned materials to

grasp complex concepts more effectively. Despite the implementation of "happiness education" for nearly a decade, private spending on education has increased and generated inequalities that negatively impact the well-being of Koreans and pose challenges for social cohesion (OECD, 2024b). Students have experienced unhappiness and depression (Fawaz & Lee, 2022), possessed negative attitudes towards mathematics learning (e.g. OECD, 2014; Pang & Seah, 2021), and exhibited high mathematics anxiety (Choi-Koh & Ryoo, 2019; Yi & Na, 2020). In some instances, students have even given up on mathematics altogether (Ko et al., 2021). While there has been limited research on students' well-being from non-Western traditions (e.g. Kim, 2020; Tan 2021), it is important to examine Neo-Confucian values, which remain prevalent and influential – "a common, national discourse" (Chung, 2015, p. 83) in contemporary Korean society. This short paper aims to establish the connections between Neo-Confucian concepts of learning and contemporary mathematics education, drawing from the framework of the OECD study on the social and emotional skills (SES) (e.g. OECD, 2021; 2024c).

Research Approach

Yi Hwang Toegye (1501-70), a prominent Neo-Confucian scholar in Korea (Chung, 2021), lived during the era when civil service examinations dominated the education landscape. He witnessed the impact of examinations on learning and learner well-being. While his work did not directly address mathematics education, his philosophical ideas are relevant to contemporary educational issues that we shall discuss later. Textual analysis (McKee, 2003) was conducted based on *Jaseongnok,* 자성록 (Yi, 1558), a monumental record comprising twenty-two philosophical and scholarly letters. The core curriculum of civil service examinations, the *Four Books,* was also analyzed. They included the *Great Learning,* the *Analects,* the *Doctrine of the Mean, and* the *Mencius*. The research work consisted of the systematic reading and study of the texts in several cycles, from which main categories and subcategories in Chinese characters were created. The analysis phase comprised of comparing and contrasting the textual data to identify differences and similarities. The social and emotional skills (SES) framework was adopted to explore the connections of the identified categories and the five domains of SES: *open-mindedness*, *task performance*, *engaging with others*, *collaboration*, and *emotional regulation* (OECD, 2024c).

Toegye on *Open-Mindedness* and *Task Performance*

In ancient Korea, the success of a Confucian scholar was determined by passing the civil service examinations, gwageo. This was the sole path to pursuing an official government position and accumulating personal profits. Toegye often advised his disciples against placing emphasis on examination results, thereby taking shortcuts in their learning process with a selfish desire for quick success (Jaseongnok, 14; 17). The topic of study for examinations was frequently discussed in his writings. He used the analogy of "pulls up a sprout forcefully" (Mencius, 2A:2) to illustrate the artificial exaggerated effort required to speed up the learning process. This excessive effort can lead to worries and anxiety, ultimately causing heart-mind illness (Jaseongnok 1; 13). We can identify a compelling account of SES from Toegye's writing, as skills in the domain of open-mindedness, interest in ideas, and a love of learning, understanding, and intellectual exploration. In Jaseongnok 1, Toegye advised scholars to relish reading for pleasure and find delight in reading. Through exposure to and embodiment of things in everyday life and affairs, such as phenomena and human relations, to cultivate propriety and wisdom. Learning is grounded in investigating things and exhausting principles to extend knowledge as outlined in the Great Learning. In contemporary mathematics context, students explore mathematical principles; investigate patterns or real-world applications;

and model scenarios using their mathematical knowledge (OECD, 2024a). Toegye believed that

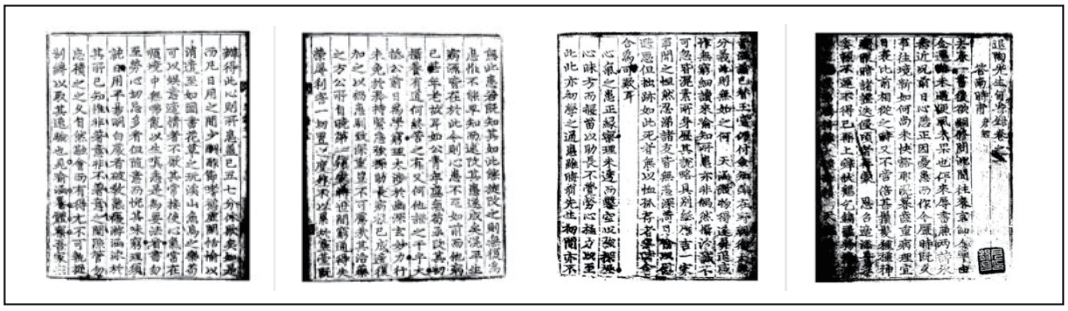


Figure 1. Extract from Toegye's first letter to his disciple Nam Sibo (Jaseongnok, 1)

learning requires *reverence*, as it is a lifelong process of accumulating and extending knowledge and know-how naturally. A further analysis of Toegye's work suggests that he likely supported the skills development in the domain of *task performance*, which require persistence, the ability to resist distractions, and the focus on the task to achieve personal goals. *Jaseongnok* emphasized the primacy of enjoying the learning process instead of rushing and working excessively (e.g. 11; 16). Toegye argued that human desire to rush had left no room for reading and thinking. Thence, scholars should remove the emerging sprouts of human desires by rebuilding true learning through earnest effort over an extended period, ensuring its purity and skillfulness. This would allow the heart-mind to remain self-aware and calm, free from worry or anxiety (*Jaseongnok*, 13). Allowing students to engage meaningfully with mathematical tasks may reduce mathematics anxiety and fear of failure; nurture persistence and resilience; and forester a growth mindset. "Persistence is critical in the mathematical context, as it enables students to tackle complex concepts, overcome challenges, and ultimate build a deeper understand of the subject" (OECD, 2024a, p. 31).

Toegye on *Engaging with Others* and *Collaboration*

In accordance with the *Analects,* "if, one thinks but does not learn from others, one will be in peril" (II.15). The study of Confucianism was grounded in the intellectual discourses and scholarly interactions among Confucian literati, and between master and disciples (*Jaseongnok*, 2). Toegye advocated collaborative and interactive learning, rather than necessarily accepting ideas and information determined by others, he envisioned academic forum and debate (known as *Ganghoe*) as a core pedagogical approach in Dosan academy. He encouraged scholars to engage in discussions to refine knowledge and attain a profound understanding of Confucius's scriptures. He valued creative knowledge exchange and was not afraid to express his ideas, even if they might be ridiculed by the world (*Jaseongnok*, 4). *Jaseongnok* reflects on how Toegye discussed and illuminated the pith of learning with other literati and his disciples. It exemplifies the cooperative efforts undertaken to maximum their own and each other's learning. Collaboration fostered interdependent and mutually reinforcing the relationships between master and disciples, as well as among scholars, enabling the development of virtues such as *benevolence, faithfulness*, *integrity, righteousness, mindfulness*. Learning together should be enjoyable and pleasurable, as scholars could find *love* and *empathy* in overcoming the same illness; and assisting one another with their worries (*Jaseongnok*, 1). Toegye's approach to learning and teaching aligns with the domains of *engaging with others* and *collaboration*. In OECD's terms (2024c), these skills encompass initiating and maintaining social connections; expressing opinions with confidence; understanding and caring for others and their well-be-

ing; valuing interconnectedness among people; and investing in nurturing close relationships and to live in harmony with others. Studies have shown cooperative learning enhances students' mathematics achievement and their attitude towards mathematics (OECD, 2024a).

Toegye on *Emotion Regulation*

Toegye frequently discussed the importance of well-being and nurturing mental health to encourage his disciples and scholars to pursue true learning rather than being distracted by fulfilling personal desires through officialdom. Toegye regarded emotions as physical and psychological states aroused from within and are activated by external stimuli or material force (*Jaseongnok*, 18). After emotions are aroused, they need to be balanced to achieve *harmony*, as outlined in the *Doctrine of the Mean*. If emotions become excessive or imbalanced, they can lead to evil, prioritizing our individual interests and selfish desires over others. *Mencius* (2A: 6) emphasizes four virtues: *benevolence*, *righteousness*, *propriety*, and *wisdom*, which serve as regulators for self-control over emotions, harmonizing heart-mind and regulating actions (*Jaseongnok*, 17). The *Analects* highlights the importance of internal harmony of heart-mind, noting that "the gentleman is easy of mind, while the small man is always full of anxiety" (VII:37). In Confucianism, well-being transcends beyond mere happiness and the fulfilment of desires. Learners must not only control and manage their emotions, but also their desires, and to confront challenges and difficulties for a flourishing life. In contemporary educational settings, *emotion regulation* in mathematics is essential as it allows students to effectively manage anxiety and able to calmly solve problems (handles stress well), while cultivating *empathy*, social responsibility and *respect* for others, which are essential for addressing societal changes and issues (OECD, 2024a).

Contemporary Implications and Conclusion

Toegye's philosophy on learning goes beyond the traditional doctrine of knowledge and focus on cultivating virtues that enable learners to act, behave and think appropriately. Virtues allow us to make ethnical decisions and moral judgments, to function effectively for ourselves, others, and the environment. Confucius emphasizes the importance of conducting life based on *shu* (恕), meaning loving others as we would like to be loved, and "do not impose on others what yourself do not desire" (the *Analects*, XV:24). From the Neo-Confucian perspective, mathematics education should be grounded in investigating things and exhausting principles of daily affairs and phenomena, such as social and economic inequalities, pandemics and climate change. Mathematics serve as a powerful tool to analyze, critique and address issues that have the potential to benefit society. Students should be equipped with skills to examine the ethical and moral issues in mathematics and take responsibility for their actions in fostering a more favorable and harmonious society and the world.

References

Choi-Koh, S. S., & Ryoo, B. G. (2019). Difference of math anxiety group based on two measures, MASS and EGG. *Educational Psychology*, *39*(5), 659-677. https://doi.org/10.1080/01443410.2018.1543857

Chung, E. Y. J. (2015). *Korean Confucianism: Tradition and modernity*. The Academy of Korean Studies Press.

Chung, E. Y. J. (2021). *The moral and religious thought of Yi Hwang (Toegye): A study of Korean neo-Confucian ethics and spirituality*. Palgrave Macmillan.

Fawaz, Y. & Lee, J. (2022). Rank comparisons amongst teenagers and suicidal ideation. *Economics and Human Biology*, *44*, 101093. DOI: 10.1016/j.ehb.2021.101093

Kim, R. (2020). *Confucianism and the philosophy of well-being*. Routledge

Kim, Y. C. & Jung, J.-H. (2019). *Shadow education as worldwide curriculum studies*. Palgrave Macmillan.

Ko, H. K. Choi, S. & Kaji, S. (2021). Who has given up on mathematics? A data analysis. *Asia Pacific Education Review*, *22*, 699-714. https://doi.org/10.1007/s12564-021-09709-6

Leung, F. K. S. (2008). In the books there are golden houses: Mathematics assessment in East Asia, *ZDM Mathematics Education*, 40, 983-992.

Leung, F. K. S. (2014). What can and should we learn from international studies of mathematics achievement? *Mathematics Education Research Journal*, *26*(3), 579-605.

McKee, A. (2003). *Textual Analysis: A Beginners' Guide*. Sage Publications.

Ministry of Education (2022). *Moral education curriculum.* Republic of Korea.

OECD (2014). *PISA 2012 Results: What Students Know and Can Do – Student Performance in Mathematics, Reading and Science*. OECD. https://doi.org/10.1787/9789264208780-en.

OECD (2021). *Beyond Academic Learning: First Results from the Survey of Social and Emotional Skills*, OECD. https://doi.org/10.1787/92a11084-en.

OECD (2024a). *An evolution of mathematics curriculum: where it was, where it stands, and where it is going*, OECD. https://doi.org/10.1787/0ffd89d0-en.

OECD (2024b). *Rethinking future education in Korea: Towards student agency and well-being,* OECD.

OECD (2024c). *Social and Emotional Skills for Better Lives: Findings from the OECD Survey on Social and Emotional Skills 2023*, OECD. https://doi.org/10.1787/35ca7b7c-en.

Pang, J., & Seah, W. T. (2021). Excellent mathematical performance despite 'negative' affect of students in Korea: The values perspective. ECNU *Review of Education*, *4*(2), 285-306. https://doi.org/10.1177/2096531120930726

Pang J., Seah, W. T., Kim, L., & Kim, S. (2024). The text mining approach to identifying what students value in mathematics learning. In Y. Dede, G., Marschall, P. Clarkson (Eds). *Values and Valuing in Mathematics Education. Moving Forward into Practice*. pp. 125-148. Springer.

Tan, C (2021). *Mindful Education: Insights from Confucian and Christian Traditions.* Springer.

Yi, H. S., & Na, W. (2020). How are maths-anxious students identified and what are they key predictions of maths anxiety? Insights gain from PISA results for Korean adolescents. *Asia Pacific Journal of Education*, *40*(2), 247-262.

TSG7-24

A STUDY ON CHILDREN'S PERCEPTIONS OF "GOOD EXPLANATION" IN ARITHMETIC

Shota Matsumoto
Graduate Student at the Graduate School of Education, Shimane University, Japan,
shota03202003@gmail.com

This study explores instructional approaches to explanation in elementary mathematics, based on Bishop's theory of mathematical enculturation. A lesson incorporating activities for comparing and interpreting explanations was conducted with sixth-grade students in a Japanese public school. Through pre-tests, video analysis, and transcript segmentation, the study examined changes in students' perceptions of a "good explanation." The results indicate that both teacher interventions and peer influence significantly shape students' understanding of explanation quality. The findings highlight the need for instructional strategies that consider these influences. Further research is necessary to refine the analysis and enhance teaching practices that foster students' explanatory skills.

Keywords: good explanation, Mathematical Enculturation, Primary School Year 6, Experimental Teaching,

Introduction

The importance of teaching mathematical proof in junior high schools in Japan has been consistently emphasized (Kunimune, 2017). Meanwhile, some studies have pointed out that certain children experience difficulties in "explanation," which serves as a preliminary stage for learning proofs, even at the elementary school level (Kondo, 2022). These studies highlight the ambiguity of instructional methods for explanation in mathematics education and call for improvements in teaching practices.

Additionally, research has been conducted on incorporating proof learning at the elementary education level. For example, Stylianides (2007) analyzed teaching methods for proof instruction targeting third-grade students and provided insights into the role of teachers in supporting proof-related activities. However, the extent to which children perceived the necessity and significance of proof was not explicitly discussed in the study.

The issue of students not recognizing the significance of proof has been previously highlighted (A. Selden & J. Selden, 2007), indicating the need for further improvements in instructional approaches. With this issue in mind, the present study adopts Bishop's theoretical perspective of "mathematical enculturation" to explore effective instructional methods that enable children to perceive the significance of explanation in elementary mathematics education.

Theoretical Framework

In this study, Bishop's theory of mathematical enculturation serves as the theoretical perspective for examining instructional approaches to explanation at the elementary school level. Based on the thesis that mathematics is a cultural phenomenon, Bishop (1988) conceptualized mathematical culture as a set of activities and developed the theory of mathematical enculturation. Within this framework, the act of explaining is regarded as one of the fundamental activities of mathematical culture and is positioned as a universal mathematical practice. Furthermore, Bishop argued that this activity has contributed to the development of mathematics as a linguistic and symbolic technology. He also posited that the process of mathematical enculturation involves the formation of concepts, meanings, and values through social interactions. In these interactions, he emphasized that teachers should not exercise authority unilaterally but rather facilitate interactions that promote meaningful engagement. Additionally, he asserted that the norms of explanation are better understood through activities that involve interpreting and comparing others' explanations.

These insights undoubtedly provide valuable implications for the actual practice of teaching explanations. However, the specific mechanisms by which children develop a sense of what constitutes a "Appropriate" explanation remain underexplored. Therefore, in order to design instructional strategies that help students recognize the significance of explanation, it is essential to identify the characteristics of explanations that elementary school students perceive as "good" and the nature of social interactions that contribute to the development of norms for high-quality explanations.

Research Question

Based on the above considerations, this study focuses on the instructional practices of explanation in elementary schools and establishes the following research objective:

[R.Q.] To design and implement a learning program that incorporates activities involving the interpretation and comparison of others' explanations. By analyzing the observed processes of cognitive transformation, this study aims to examine the characteristics of social interactions that influence children's perception of a "good explanation."

Method

Survey Design

The study was conducted with sixth-grade students at a public elementary school in Shimane Prefecture, Japan. A pre-test using the worksheet intended for classroom instruction was administered on February 27, 2024, followed by a one-hour experimental lesson on March 18, 2024. The experimental lesson was designed to capture changes in students' perceptions of what constitutes a "good explanation." To achieve this, the following worksheet was developed (Fig.1; Fig.2).

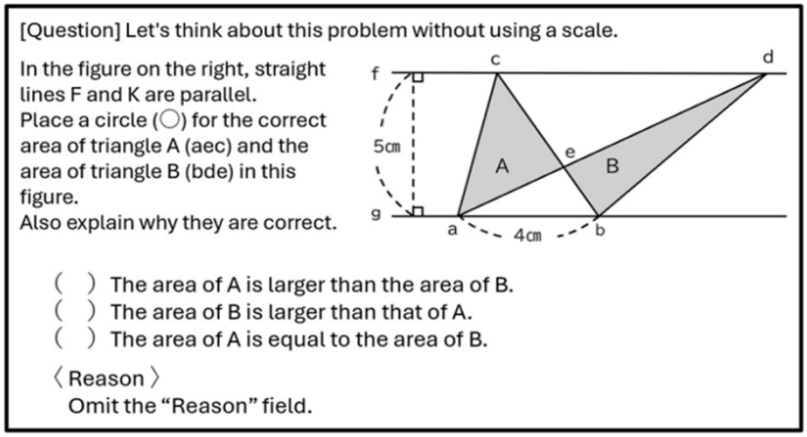


Figure 1. Survey question prepared by the author based on Kondo, 2022.

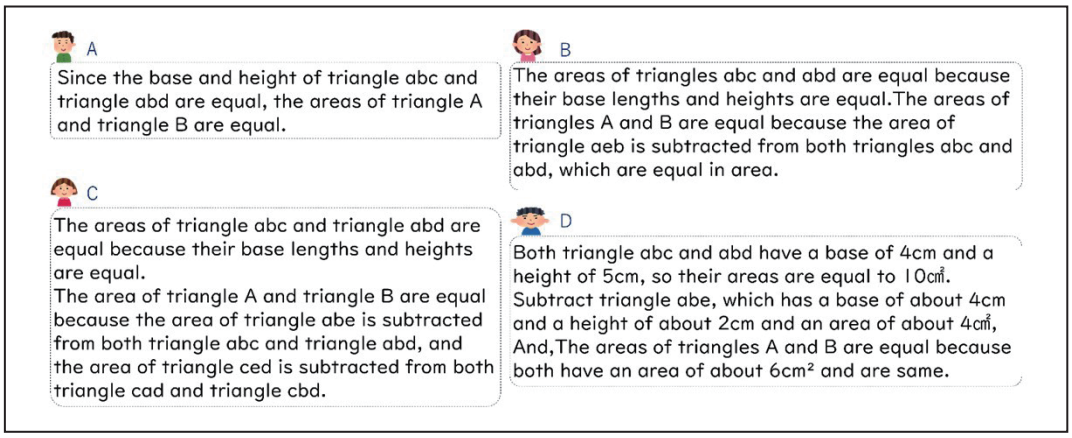


Figure 2. Worksheet used in class

The worksheet was designed to capture students' perceptions of what they consider to be a "good" explanation. To achieve this, multiple explanations were presented for the survey question shown in Figure 1, and students were asked to select the explanation they deemed "good" and provide a justification for their choice. The four explanations were developed based on the validity of claims, which Kondo (2022) identifies as a key point in teaching explanations and proofs, and from the perspective of rationalism as discussed by Bishop (1988).The lesson using this worksheet was structured in the following sequence:

The lesson using this worksheet was structured in the following sequence:

1. **Group discussion** (Activity 1): Students engaged in discussions within their groups based on the worksheet to explore what constitutes a "good explanation" in mathematics. 2. **Class-wide discussion** (Activity 2): The entire class shared and examined the characteristics of the explanations given by students A through D. 3. **Individual reflection** (Activity 3): Students reconsidered their own perspectives on a "good" explanation independently.

Date Analysis

The analytical procedure in this study follows these steps: First, pre-test responses were analyzed to capture students' perceptions of a "good explanation." Second, classroom video and audio recordings were reviewed, and transcripts were segmented into individual utterances. Segmentation criteria included pauses within utterances, shifts in speakers, and changes in speech content. Third, by examining the speech records and videos, groups that exhibited changes in their perceptions during Activity 1 were identified, and the factors contributing to these changes were analyzed.

Result

As for the extracted group's perception of a "good explanation," at the time of the pre-test, C1 and C3 considered C's explanation to be "good," while C2 perceived D's explanation as "good," and C4 regarded B's explanation as "good." However, after Activity 1, all members (C1 to C4) recognized C's explanation as a "good explanation." To examine the factors contributing to this shift in perception, an analysis will be conducted based on the speech records from Activity 1 within the extracted group. Below is the transcript of the extracted group's discussions during Activity 1.

---

01 C1 : So, in arithmetic, there is one answer, but there are several ways to find it. Well, I thought it was better to say two things a lot than one ... But if you want to keep it short and simple, why not B?

02 C2 : Is it B?

03 C3 : I mean, C1 said B only gave one explanation, but C added another opinion, so I think C is the best. Personally, it was the easiest to understand.

04 C4 : It's hard to understand if you have to explain two things...

05 C1 : I think the easiest to understand is B. And I got it with B. But isn't C good for arithmetic?

06 C3 : If I were to explain it to everyone, it would be D, right...? Everyone is D, right? Oh, Wrong....

07 C1 : But B is concise, and C is kind of long.

08 C3 : It's long, but ...

09 C1 : It's long, but that's what makes it more detailed. That's the best part. So, I think C is fine.

10 C2 : I also choose C.

11 C3 : Yeah, C is the best. C4, what do you think?

12 C4 : Uh…

The speech records indicate that the discussion was primarily led by C1, while C4 initially resisted accepting C's explanation as a "good explanation" but ultimately yielded under pressure (Excerpts 04, 12). On the other hand, after Activity 2, C2 and C4 came to recognize B's explanation as a "good explanation."

Discussion and Conclusion

The analysis revealed that students' perceptions of what a "good explanation" is changed between Activities 1 and 2. These changes suggest that group discussion alone is not sufficient to reach a consensus on how others perceive a "good explanation" and that it is difficult to reach a consensus. In this regard, we can take into account Bishop's (1988) point that the power of the teacher should be exercised in a way that facilitates interaction among learners in the process of forming ideas, meanings, processes, and values in mathematics. The development of the discussion in Activity 1 can be attributed to the presence of power implicit in the children. Therefore, it is necessary for the teacher to exercise power in the class in a way that facilitates the interaction of the learners, while also paying attention to the power that exists within the learners.

References

Alan J. Bishop. (1988). Mathematical Enculturation: A Cultural Perspective on Mathematics Education. Kluwer Academic Publishers.

Annie Selden, John Selden. (2007). Overcoming Students' Difficulties in Learning to Understand and Construct Proofs. Tennessee Technological University,38505.

Andreas J. Stylianides (2007). Proof and Proving in School Mathematics. Journal for Research in Mathematics Education. 38,3,289-321.

Kondo, Y. (2022). Research on the ability of "Explanation and Proof" in arithmetic and mathematics courses: Investigation of the actual situation of showing the validity of one's own claims by relating them to the properties of figures. Journal of Japan Society for Mathematics Education, 104,1, 2 21. (in Japanese)

Kunimune, S. (2017). Understanding arguments and their learning guidance in mathematics education, Toyokan Publishing Co. (in Japanese)

MEMO

MEMO

KOREA 2025

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 8:

The Essence of Mathematics Education with Equity and Culture

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 8
(Day 2)

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

TSG8-01

FROM TRADITION TO TECHNOLOGY: RETHINKING MATHEMATICS EDUCATION THROUGH ETHNOMATHEMATICS IN INDONESIA

Nur Robiah Nofikusumawati Peni
Universitas Ahmad Dahlan, Indonesia, nur.peni@mpmat.uad.ac.id

This research discusses the connection between ethnomathematics and digital transformation in Indonesian mathematics education. Based on a comprehensive literature review of existing studies, we discuss how indigenous mathematical knowledge has been incorporated into dominant teaching practices. The purpose of this study is to analyze trends, challenges, and opportunities in preserving cultural mathematics through the embracement of digital technology. The findings show that there are increasingly more attempts at introducing ethnomathematics into curriculum, although there are still issues with balancing between tradition and technology. The discussion focuses on successful approaches, such as digital storytelling and culturally responsive teaching. Bringing ethnomathematics into the digital era enriches learning experiences while promoting cultural relevance in mathematics teaching.

Keywords: ethnomathematics, digital transformation, Indonesian culture, mathematics education

Introduction

Mathematics is often seen as a universal language, yet its development is shaped by cultural influences. Ethnomathematics examines mathematical ideas within cultural contexts, offering insights into how indigenous knowledge enriches mathematics education. In Indonesia, diverse traditions, including ethnomathematics, influence architecture, textiles, and landscape design (Fuat et al., 2024). As digital education advances, integrating ethnomathematics into teaching can enhance learning and preserve cultural heritage. While technology brings new opportunities, it also presents challenges. Educational technology enables blending local mathematical knowledge with the modern curriculum through apps, virtual environments, and digital storytelling (Nurrahmah et al., 2024; Sunzuma & Umbara, 2025), making math more engaging for students. Integrating ethnomathematics into online learning platforms requires careful examination of teaching methods combined with cultural awareness and sufficient readiness within educational establishments.

Aligning indigenous knowledge with standardized curriculum is a significant obstacle to maintaining cultural mathematics in digital education (Hidayati & Prahmana, 2022). Digital resources for ethnomathematics are limited, highlighting the need for more content and teacher training. However, digital tools like storytelling, AR, and gamification engage students with local concepts (Cirneanu & Moldoveanu, 2024). While AI and big data can uncover patterns in conventional mathematics, confirming its worldwide significance, online platforms encourage collaboration (Soesanto et al., 2022). The intersection of ethnomathematics and digitalization in Indonesian mathematics education is a highly interesting field of research.

As Indonesia attempts to balance preserving its rich mathematical legacy and embracing new technical advances, a thorough procedure for incorporating cultural mathematics into modern education

systems is required. In order to promote more inclusivity and a culturally sensitive mathematics curriculum, this study contributes to the literature by incorporating important findings, challenges, and possible future approaches for integrating ethnomathematics into computerized education systems.

Methodology

This study adopts a systematic literature review (SLR) technique (Boell & Cecez-Kecmanovic, 2015; Greyson, et.al., 2019) to investigate the relationship between ethnomathematics and digital transformation in Indonesian mathematics education. The process comprises a rigorous and intensive review of published scientific literature, including journal articles, conference proceedings, monographs, and policy papers (Murire, 2024). The selection criteria are determined in advance to help establish relevance and reliability.

The study was carried out in stages. Initially, the related literature was identified through a variety of databases like Google Scholar, Scopus, and Web of Science with keywords like "ethnomathematics," "digital transformation," "Indonesian mathematics education," and "culturally responsive teaching." Data extraction and analysis were subsequently carried out via a thematic framework, classifying the results into main themes like digital tools for ethnomathematics, teaching strategies, obstacles to integration, and successful case studies.

The result provides a holistic understanding of how indigenous mathematical knowledge has been incorporated into mainstream education and the role of digital technology in this transformation. The findings give a basis for ongoing discussion of the optimal strategy and possible directions for the inclusion of ethnomathematics in the digital age.

Result and Discussion

The past decade (2015–2025) saw tremendous advancement in ethnomathematics integration and digitalization of Indonesian mathematics education. This section summarizes the findings of the literature review with a focus on trends, challenges, and opportunities in this field.

Trends in Ethnomathematics Integration

Three key points from the trends in ethnomathematics: (1) Growing research interest - Research indicates a growing focus on incorporating ethnomathematics into mainstream education. Hidayati and Prahmana (2022) stress that cultural contexts in mathematics can enhance conceptual understanding and student motivation. Ethnomathematics research in Indonesia has grown exponentially (Iffah et al., 2025), with more universities and institutions recognizing its contribution towards making mathematics education inclusive; (2) Utilization of digital technology - Suzuma and Umbara (2025) work shows digital storytelling has been identified as a particularly effective method of preserving indigenous mathematical knowledge. Triyani, et. al (2012) give an example in teaching Least Common Multiple (LCM) using the storytelling method aloud able to improve students' self-efficacy in learning mathematics. The application of digital technology in ethnomathematics education has gained speed (Irmayanti, et. al., 2025). Digital culture-based media, interactive simulations, and virtual learning environments are being developed to give mathematics education more immersion; (3) Culturally responsive pedagogy - Increasing attention is being placed on culturally responsive teaching styles. Jacob and Dike (2023) state that educators are incorporating pedagogic practices that weave local cultural aspects into math teaching, making the subject more relevant and accessible to students from different backgrounds.

Challenges in Balancing Tradition and Technology

Curriculum standardisation is a significant obstacle to the integration of ethnomathematics into education. It could be challenging to integrate traditional mathematical ideas since they frequently don't fit in with contemporary standardised curricula (Hidayati & Prahmana, 2022). Teacher preparation is another important concern. The quality of education is lowered because many teachers lack the knowledge and tools necessary to safely incorporate ethnomathematics, particularly in online learning contexts (Purwanto et al., 2025). Additionally, there are not many excellent, culturally appropriate digital teaching resources available. This illustrates how urgently education stakeholders must collaborate and boost funding to create relevant content. According to Nurrahmah et al. (2024), not every Commissariat Mathematics Teacher (MGMP) member is familiar with using laptops or computers in the classroom. In order to properly include ethnomathematics in Indonesian education and increase learning's inclusivity and cultural significance, several issues must be resolved.

Opportunities through Digital Evolution

Digital evolution presents significant opportunities for integrating ethnomathematics into education. One promising approach is digital storytelling and gamification, which offer captivating means of introducing mathematical ideas in cultural contexts. Digital storytelling provides a culturally rich learning environment that helps young learners connect with and appreciate mathematics more profoundly (Sunzuma & Umbara, 2025). Gamified elements enhance this experience by making learning interesting and enjoyable. Another important trend that encourages information sharing between indigenous communities, scholars, and educators is the rise of collaborative online platforms. These platforms encourage the collaborative creation of educational materials sensitive to cultural differences and represent a range of customs and beliefs (Putri & Junaedi, 2022). Finally, student involvement has increased dramatically when ethnomathematics is taught utilising digital resources. Such integration increases students' motivation and fortifies their mathematical literacy, making mathematics more significant and applicable to their cultural backgrounds (Maulina et al., 2024).

Conclusion

The intersection of ethnomathematics and digital innovation in mathematics education in Indonesia has both challenges and opportunities. Whereas integrating traditional mathematical knowledge into a homogenized curriculum is problematic, digital technologies hold promise for avenues of preservation and presentation of cultural heritage in education. Furthermore, many teachers lack the necessary skills to incorporate ethnomathematical techniques, emphasizing the need for professional development programs. Besides these challenges, digital transformation offers promising solutions. Educators can use the proper adaptation of such technologies to refine pedagogical practice and promote a greater appreciation of Indonesia's plural mathematical culture. A balance between technology and tradition must be found if ethnomathematics is to be preserved as we forge ahead technologically. Future research and investment in digital educational resources will play a vital role in strengthening the integration of ethnomathematics in Indonesia's evolving educational landscape.

References

Boell, S., & Cecez-Kecmanovic, D. (2015). On being 'systematic' in literature reviews in IS. *Journal of Information Technology, 30* (2), 161-173. https://doi.org/10.1057/jit.2014.26.

Cirneanu, A.-L., & Moldoveanu, C.-E. (2024). Use of digital technology in integrated mathematics education. *Applied System Innovation, 7* (4), 66. https://doi.org/10.3390/asi7040066

Fuat, F., Rohibni, R., & Rayungsari, M. (2024). Unraveling trends in temple ethnomathematics research and the evolution of the mathematical landscape. *Indonesian Journal of Science and Mathematics Education, 7* (1), 1-17. https://doi.org/10.24042/ijsme.v7i1.19373.

Greyson, D., Rafferty, E., Slater, L., MacDonald, N., Bettinger, J., Dubé, È., & MacDonald, S. (2019). Systematic review searches must be systematic, comprehensive, and transparent: a critique of Perman et al. *BMC Public Health, 19* (153), 1-6. https://doi.org/10.1186/s12889-018-6275-y.

Hidayati, F. N., & Prahmana, R. C. I. (2022). Ethnomathematics' research in Indonesia during 2015-2020. *Indonesian Journal of Ethnomathematics, 1*(1), 29-42. http://doi.org/10.48135/ije.v1i1.29-42.

Iffah, R. D. L., Subanti, S., Usodo, B., & Nurhasanah, F. (2025). Systematic literature review: Ethnomathematics research in Indonesia. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *10*(1). 28-40. https://doi.org/10.23917/jramathedu.v10i1.5621.

Irmayanti, M., Chou, LF. & Anuar, N.N.b.Z. (2025). Storytelling and math anxiety: a review of storytelling methods in mathematics learning in Asian countries. *European Journal of Psychology of Education, 40*(24), 1-26. https://doi.org/10.1007/s10212-024-00927-1.

Jacob, E.E., & Dike, J.W. (2023). Ethnomathematics and culturally responsive pedagogy in mathematics learning. *Journal of Education in Developing Areas (JEDA) Special Edition, 31* (3), 380-389. https://journals.journalsplace.org/index.php/JEDA/article/viewFile/410/332.

Maulina, S., Junaidi., Nufus, M., & Maulida, N. R. (2024). Students' mathematical literacy through ethnomathematics-based learning. *Jurnal Didaktik Matematika, 11*(1), 58-73. https://doi.org/10.24815/jdm.v11i1.35676.

Nurrahmah, A., Mulyatna, F., & Seruni, S. (2024). Workshop flipper book berbasis google sites melalui pendekatan etnomatematika bagi guru. *Jurnal SOLMA, 13*(1), 109-118. https://doi.org/10.22236/solma.v13i1.12691.

Purwanto, W.R., Zaenuri., Wardono., & Junaedi, I. (2025). Teachers' perceptions of ethnomathematics learning in the independent curriculum program in Indonesia. *International Journal of Education and Practice, 13*(1), 98-113. https://doi.org/10.18488/61.v13i1.3963.

Putri, M., & Junaedi, I. (2022). Development of ethnomathematics based e-module using the inquiry learning model to improve mathematical problem-solving ability. *UNNES Journal of Mathematics Education, 11*(2), 174-182. https://doi.org/10.15294/ujme.v11i2.59938.

Soesanto, R. H., Dirgantoro, K. P. S., & Priyanti, N. (2022). Indonesian students' perceptions towards AI-based learning in mathematics. *Journal on Mathematics Education, 13*(3), 531–548. https://doi.org/10.22342/jme.v13i3.pp531-548

Sunzuma, G., & Umbara, U. (2025). Ethnomathematics-based technology in Indonesia: A systematic review. *Asian Journal for Mathematics Education, 4*(1), 129-153. https://doi.org/10.1177/27527263241305812.

TSG8-02

INDIGENOUS EPISTEMOLOGY AND ETHNOMATHEMATICS: INSIGHT FROM BHUJEL COMMUNITY

Jaya Bishnu Pradhan
Tribhuvan University, Mahendra Ratna Campus, Nepal, jebipradhan@gmail.com

This study explores the ethnomathematical knowledge and indigenous epistemologies of the Bhujel community renowned for their traditional bamboo crafts. Drawing from participant observations and in-depth interviews with bamboo crafters, this ethnographic research reveals how the Bhujel community's intricate craft practices embody rich mathematical concepts such as measurement, design, and spatial reasoning. The findings of this study revealed that Bhujel indigenous epistemology focuses on individualized instruction and hands-on experiences of the learners to knowledge generation and distribution. This hands-on and informal learning approach helps them to develop essential skills and knowledge to transcend and sustain their lives and preserve their cultural traditions. In the context of bamboo crafts, this approach allows learner to see the practical application of mathematics in real-world tasks. Recognizing these traditional learning methods could help improve educational practices by including and respecting cultural knowledge.

Keywords: Culture, Ethnomathematics, Indigenous epistemology, Mathematical knowledge

Introduction

The Bhujel community is one of the indigenous ethnic groups in Nepal. They have their own language and rich cultural heritage that reflect their unique identity and way of life. The Bhujel has the traditional job of constructing bamboo crafts. The mathematical ideas and procedures elaborated in the construction of bamboo crafts and studying the ethnomathematics in the Bhujel community provide valuable insights into how ethnomathematics is embedded in cultural practices and traditions. Thus, ethnomathematics is the mathematics practiced by different cultural groups, such as groups of workers, professional classes, children in a certain age group, indigenous societies, and many other groups identified by their common objectives and traditions (D'Ambrosio, 2006). Further, Rosa and Orey (2016) state that ethnomathematics refers to the mathematical ideas and concepts embedded within diverse cultural contexts. Thus, ethnomathematics highlights the importance of recognizing and valuing diverse forms of mathematical knowledge systems and the ways of knowing and doing of different groups of indigenous cultures.

The Bhujel community has been engaged in the mathematical activities of counting, measuring, designing, playing, locating, and explaining in their daily lives. The way that bamboo craftsmen acquire and apply mathematical knowledge in their everyday activities, the study provides valuable insights into the ways of knowing and doing of their own mathematical knowledge and skills. The main objective of this study was to explore the ethnomathematical ideas, knowledge, and indigenous epistemology of the Bhujel community. The mathematical concepts and skills applied in a different cultural context of the Bhujel promote greater understanding and appreciation of diverse cultural practices and knowledge systems.

Methods and Procedures

In this research, I used qualitative research methods as I wanted to make sense of the complex world of the implicit mathematical knowledge of the Bhujel and their ways of teaching and learning i.e., epistemologies (Pradhan, 2023) of Bhujel people. In particular paradigm, I adopted ethnographic methodology to obtain research objectives. The Bhujels of Adhimul of Bandipur Rural Municipality of Tanahun District have recognition of the construction of different bamboo-related pieces of stuffs and where there are substantial numbers of Bhujels carrying on this ages-long occupation. I selected total of six including four of bamboo crafts workers and two school-going children who participated in their household activities purposively as my research participants.

I used in-depth interviews which are mostly known as open-ended interviews or ethnographic interviews with the help of interview guidelines so that it is easier for me to generate the data in the fields. In the same, I used participant observation and non-participant observation to generate the data to support my study. I carefully recorded all the possible conservations with the help of the video camera and took field notes as much as I could with the consent permission of my research participants. Though I had made a schedule for data gathering, it was largely ineffective as I had to work as per the wishes of my research participants. I followed them wherever possible. I even traced their children's schools, visited them, talked to the headmaster, and teachers, and gathered information about their children's achievements in the academic field. After finishing the collection of the data, I converted the conversations and interviews into manuscripts so that I could easily analyze and interpret them. The main task of the research is to analyze and interpret the data collected from the field. I adopted Braun and Clarke (2006) process of data analysis and interpretation. They developed a six-phase thematic analysis method which includes familiarizing the data with reading and re-reading, generating initial codes systematically across the dataset, searching for themes by grouping similar codes, reviewing themes to refine and ensure they represent the data accurately, defining and naming themes to capture the essence of each, and drawing findings relation to research questions.

Bhujels' Epistemologies and Ethnomathematics

Knowledge is generated in formal and informal ways. Most of the educational institutions follow the western approaches to generate and transform knowledge. However, there are many indigenous epistemologies to knowledge generation and transformation in the world. It is apparent that people from different ethnic communities and cultures do have their own specific ways to produce knowledge. Most of the indigenous knowledge generation approaches share the common phenomena despite the different geographical variations and different cultural patterns. The Bhujel indigenous community of Nepal has a traditional system of learning that is deeply rooted in their culture and way of life. In this study, a total of six participants from the Bhujel community including two young school children were included. I interviewed with research participants about the indigenous epistemologies on how the Bhujel people generate and distribute knowledge to crafts bamboo artifacts and observed ethnomathematical ideas and knowledge practiced in the process of constructing bamboo artifacts. There are many ways to learn. People can learn from their cultural and environmental surroundings in variety of ways. One of them is apprenticeship which is characterized by a hands-on approach to learning (Fjellstrom & Kristmansson, 2019). With this regard, I asked Saila Bhujel (42) about how he learnt skills and knowledge of bamboo crafts. Saila shares his experience as

I learnt bamboo crafts with my elder brother. He encouraged me to do with him and told me: "If we learn and adopt our ancestral job, we can survive even in the famine situations. If we have skills, it saves us in any difficult and unfavorable situations". Then I motivated to work with him. I involve in this job for more than two decades.

From the interview with Saila Bhujel, they have a long-standing tradition of bamboo weaving, which has been passed down from generation to generation through a system of apprenticeship. The apprenticeship system starts at a young age, with children learning from their parents or other elders in the community. The mentor will often take on one or two apprentices at a time, and work with them closely to develop their skills and knowledge. Regarding the knowledge generation approach within the Bhujel community, I asked another participant Saroj (21):

I learnt Nanglo weaving with my father. My father encouraged me to work with him. My mother also involves in the bamboo crafts process. We have to brought bamboo from other people and work together to processing bamboo, and preparing materials for weaving. I closely observed my father about the process to construct Nanglo crafts. After some time, I became a perfect crafter.

From the interview with Saroj, It comes to be known that the apprenticeship system is characterized by a hands-on approach to learning. Apprentices are given practical tasks to complete, such as harvesting and processing bamboo, preparing materials for weaving, and weaving simple structures. This type of learning allows apprentices to develop their skills and knowledge through observation, practice, and feedback (Fjellstrom & Kristmansson, 2019). In my observation, I found that there is no formal class managed for sharing the knowledge of the Bhujels. However, the knowledge has been transformed from one generation to the next generation since long. Much of the knowledge about bamboo crafts in the Bhujel community is passed down through informal learning methods, such as observation, participation, and apprenticeships. This type of learning is often experiential and hands-on, allowing for the transmission of practical knowledge from one generation to the next. The Bhujel community places a high value on the inter-generational transmission of knowledge, with elders passing on their expertise to younger members of the community.

Indigenous peoples have their own ways of teaching and learning approaches. Indigenous people use a variety of sophisticated mathematical ideas and concepts in their everyday work (D'Ambrosio, 2006; Rosa and Orey, 2016). The Bhujels have made wonderful bamboo crafts involving a high level of knowledge and skill. I observed the bamboo craftsmen's workplace and found lots of mathematical concepts in the bamboo artifacts. I interviewed Ram Bhujel (38 Years) regarding the process of the construction of the artifacts. Ram speaks:

I used to construct Nanglo (flat round woven tray) and divide the large bamboo bar into pieces about the length of two haat (cubit). Then split it into two parts and each part into two halves to make each piece about an amal (digit). The number of separations is based on the motai (diameter) of the bamboo bars.

Nanglo is a flat round woven tray made up of bamboo. It is made traditionally out of thin bamboo pieces intermingled into a flat surface. It is used to winnow rice, paddy, beans, grains, and other foodstuffs. It can be seen that it resembles a perfect circular shape. The Bhujel community uses various measurement systems, such as body parts, bamboo nodes, and finger spacing to measure the length, width, and thickness of bamboo, which is a common practice in many indigenous communities. The geometric concepts such as angles, triangles, various polygons, and circles are found in their bamboo crafts. From the interview with Ram, it comes to know that the Bhujels use

mathematical concepts in various ways during the cutting of bamboo strips, and dividing strips into equal parts during the construction process. There is a specific application of the concept of fractions, particularly in the context of partitioning bamboo logs into equal pieces. In this context, the implicit use of fractions is a practical and hands-on application of mathematical concepts within the traditional craftsmanship of the Bhujel community. This implicit use of fractions reflects the community's knowledge and skills in working with bamboo as a material for crafting. These strategies are historically organized in indigenous culture as knowledge systems (Sharma and Orey, 2017). Concerning the production of different crafts, I asked Ram Bhujel about the types of bamboo crafts they like to produce. He shared his view:

> *We construct bamboo craft according to the demand of the market. In my workplace, Nanglo, Chalno, Doko, and Thumse are the main productions. Now we are preparing Nanglo, it has higher demand than others. The construction of Nanglo can be collaborative work. The flat surface of Nanglo can be crafted by female workers and younger school children. Wages for the construction of Nanglo are comparatively cheaper than the others*.

From the observation in my field study and the interviews with my research participants, it is found that the ethnomathematical knowledge of bamboo crafts in the Bhujel community highlights the intricate and sophisticated mathematical knowledge and practices embedded in traditional crafts. The above narrative reflects key mathematical concepts such as optimization, labor cost, and resource allocation. The decision to prioritize the production of *Nanglo* is based on higher market demand and lower labor costs illustrated an optimization strategy to maximize profit with allocation of limited resources. Thus, the ethnomathematics recognizes the inherent mathematical concepts underlying these practices, such as proportional allocation of labor and optimization of production processes. From the indigenous knowledge and epistemological perspective, ethnomathematics focuses on the ways, the styles, the arts, the techniques, generated by Bhujel people to explain, understand, and cope with their environment, particularly in the construction of bamboo-based artifacts (D' Ambrosio 2006; Pradhan, 2023). Their indigenous epistemologies can be linked with the formal mathematics lesson by designing hands-on activities to explore geometrical and optimization concepts. These concepts can be contextualized through real-life examples from students' cultural backgrounds. The Bhujel people have their distinctive methods of comparing, classifying, quantifying, measuring, explaining, generalizing, inferring, and, evaluating the phenomena in the course of the construction of various bamboo stuffs. Recognizing such indigenous knowledge and practices enriches the formal mathematics curriculum by making it more inclusive and contextually relevant. Integrating indigenous epistemology supports diverse ways of knowing and helps students connect classroom learning to their cultural experiences.

Concluding Remarks

The Bhujel indigenous community of Nepal has a rich traditional system regarding knowledge generation and distribution. The knowledge is handed down from generation to the next generation through observation, imitation, and practical experience. The community places a high value on experiential learning where learners are given hands-on experience and opportunities to practice their skills. Their practices involve sophisticated use of fractions, measurements, geometry, symmetry, and patterns, highlighting implicit mathematical skills embedded in weaving bamboo crafts and everyday activities. It can be concluded that the intergenerational, collaborative and community participation enhanced the knowledge generation and distribution of the Bhujels community. Recognizing and valuing Bhujel's epistemologies and their ethnomathematical knowledge can contribute to a more inclusive and culturally relevant mathematics education.

References

Braun, V. & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology,* 3, 77-101. http://dx.doi.org/10.1191/1478088706qp063oa

D'Ambrosio, U. (2006). Ethnomathematics: Link between traditions and modernity. Sao Paulo: UNICAMP.

Fjellstrom, M. & Kristmansson, P. (2019). Constituting an apprenticeship curriculum. *Journal of Curriculum Studies*, 51 (4), 567–581. https://doi.org/10.1080/00220272.2019.1616115

Pradhan, J. B. (2023). Uncovering ethnomathematics and indigenous epistemology: A study of Chundara culture. *Pragyaratna*, 5(1), 42-50. https://doi.org/10.3126/pragyaratna.v5i1.59270

Rosa, M. & Orey, D. C. (2016). Humanizing mathematics through ethnomodelling. *Journal of Humanistic Mathematics*, 6 (2), 3-22. DOI: https://doi.org/10.5642/jhummath.201602.03

Sharma, T. & Orey, D. C. (2017). Meaningful mathematics through the use of cultural artifacts. In: Rosa, M.; Shirley, L.; Gavarrete, M. E.; Alangui, W. V. (Eds.). *Ethnomathematics and its Diverse Approaches for Mathematics Education* (153-179). Cham: Springer.

https://doi.org/10.1007/978-3-319-59220-6_7

TSG8-03

A STUDY ON MATHEMATICAL CURRICULUM IN AN INDIGENOUS SCHOOL IN TAIWAN: FOCUSING ON THE PROCESS OF CURRICULUM DESIGN

Tai-Jung Lee
National Academy for Educational Research, Taiwan, pasuya@mail.naer.edu.tw

Alang Palalavi
Padan Asang School, Taiwan, alang1123@gmail.com

Ying-Hsuan Lee
National Academy for Educational Research, Taiwan, ying-hsuan.lee@mail.naer.edu.tw

Yi-An Cho
National Academy for Educational Research, Taiwan, scottiess0625@mail.naer.edu.tw

This study explores the integration of Indigenous knowledge into the mathematics curriculum at an Indigenous school in Taiwan, using Alan Bishop's (1988) five principles—representativeness, formality, accessibility, explanatory power, and broad scope—as an analytical lens. The school employs a culturally responsive approach, incorporating ecological knowledge and mathematical concepts to enhance both cultural relevance and academic rigor. Through field-based learning, storytelling, and bilingual instruction (Mandarin and the Indigenous language), the curriculum fosters meaningful engagement. By connecting mathematics with themes such as climate change, ecology, and architecture, the program encourages interdisciplinary learning. Findings suggest that embedding indigenous knowledge in mathematics education strengthens cultural identity, increases student engagement, and improves academic performance, offering a model for developing culturally responsive Indigenous curricula.

Keywords: Indigenous Knowledge, Mathematics Education, Culturally Responsive Pedagogy, Academic Performance, Cultural Identity

Introduction

Mathematics has often been viewed as a universal discipline, independent of cultural influences. However, Bishop (1988) challenged this notion, arguing that mathematics is fundamentally a cultural construct shaped by societal needs, values, and practices. Activities such as counting, measuring, and designing are deeply embedded in different cultural traditions, reflecting unique ways of understanding and interacting with the world. However, mainstream mathematics curricula frequently disregard indigenous knowledge systems, which include advanced mathematical practices such as navigation, land measurement, and geometric design. This disconnect not only hinders indigenous students' academic performance but also fosters a sense of alienation in the classroom (D' Ambrosio, 1985; Harris, 1980).

Mathematical enculturation (Bishop, 1988) offers a way to bridge this gap by embedding cultural elements into mathematics education, making learning more relevant to students' experiences. By

integrating traditional knowledge into lessons, students can connect mathematical concepts to their daily lives and cultural heritage. This study examines the development of a mathematics curriculum in an indigenous school in Taiwan, focusing on the process of incorporating cultural perspectives into teaching. Drawing on the frameworks of ethnomathematics and culturally responsive pedagogy, this study examines how culturally embedded mathematics education can enhance student engagement, address learning disparities, and support the development of academically rigorous curricula. By situating mathematical instruction within students' cultural contexts, such approaches contribute to advancing educational equity in Taiwan.

Literature Review

Mathematics is not universal but deeply rooted in cultural practices, shaped by societal needs and values (Bishop, 1988). Activities like counting, measuring, and designing vary across cultures, reflecting unique ways of thinking. Mathematical enculturation emphasizes embedding cultural practices into education to make mathematics meaningful and relevant to students from culturally and linguistically diverse backgrounds. For indigenous students, this approach bridges abstract concepts with their lived experiences, fostering both academic success and cultural pride (D'Ambrosio, 1985). By integrating traditional activities such as hunting and weaving, enculturation validates cultural heritage while enhancing mathematical understanding.

Alan Bishop (1988) identified five principles for creating culturally embedded curricula: representativeness, formalization, accessibility, explanatory power and broad scope. Representativeness refers to incorporating significant cultural practices, such as adopting traditional land measurement to make mathematics relatable. Formalization refers to transforming cultural practices into abstract mathematical representations; for example, algebraic patterns derived from oral traditions. Accessibility refers to ensuring content aligning with students' cognitive and linguistic abilities by using tools like traditional counting methods. Explanatory power refers to demonstrating the connection between cultural practices and mathematical concepts, such as symmetry in traditional art. Broad scope refers to covering multiple mathematical domains while building a strong foundational understanding. These principles guide the development of curricula that are both culturally meaningful and academically rigorous, meeting the unique needs of indigenous students.

Methodology

Grounded in Alan Bishop's (1988) concept of mathematical enculturation and D'Ambrosio's (1985) ethnomathematics emphasizing the integration of indigenous cultural practices into mathematics content, this study adopted the case study approach, involving iterative cycles of planning, observation, and reflection. Such an approach aligns with the goal of collaboratively designing and refining a culturally embedded mathematics curriculum while addressing real-world challenges.

Padan Asang School, an indigenous experimental education institution located in southern Taiwan, was selected as the research site due to its distinctive approach to curriculum design, which integrates Bunun culture into disciplinary studies. Currently, the school serves students from pre-kindergarten through ninth grade. Beginning in August 2026, it will become the first and only indigenous experimental education school in Taiwan to offer a continuous program from pre-kindergarten through twelfth grade. As a pioneering institution in the preservation of Bunun culture, the cultivation of Bunun talent, and the incorporation of indigenous knowledge and wisdom into formal education, Padan Asang School provides a valuable model. Investigating its curriculum, as well as

its teaching and learning materials, offers critical insights into how such culturally grounded educational frameworks may be adapted to support other indigenous experimental education schools that represent diverse cultural contexts.

The research process consisted of four steps, including identifying cultural and educational needs through consultations with stakeholders, designing culturally relevant mathematical activities, piloting the activities in classrooms and gathering feedback, and refining the curriculum based on observations and evaluations. Three groups of participants consisted of school principal, mathematics teachers and community members. Community members includes local elders and cultural experts who can provide insights into traditional practices and knowledge systems. All participants were selected through purposive sampling to ensure their direct involvement in the curriculum development process. That is, teachers contributed to curriculum design and implementation, while elders offered guidance on cultural relevance; the school principal is the leader and facilitator to oversee the overall process.

Qualitative data is collected for this study. Semi-structured interviews were conducted with teachers and community members to gather insights on cultural practices and educational needs, such as identifying relevant indigenous knowledge systems and their applicability to mathematics. Focus group discussions were conducted with participants to gather their perceptions of the culturally embedded curriculum, focusing on its relevance and impact on their teaching. Interviews and focus group discussion were transcribed by verbatim and then analyzed thematically to identify recurring themes to gain insights.

Results and Conclusion

Alan Bishop (1988) proposed five key principles for curriculum development: representativeness, formality, accessibility, explanatory power, and broad and elementary scope. These principles provide a framework for developing culturally embedded mathematics curricula, ensuring that indigenous students can connect mathematical concepts with their lived experiences. Results of the analysis of the indigenous experimental school-based mathematics curriculum development plan are presented below.

Firstly, Bishop emphasized that mathematics curricula should incorporate culturally significant mathematical practices (Bishop, 1988). The indigenous curriculum strongly aligns with this principle by embedding traditional ecological knowledge into the learning process. The curriculum integrates indigenous environmental knowledge by incorporating land measurement, astronomical observations, and environmental sustainability into mathematical learning. The curriculum actively involves indigenous elders and cultural experts, ensuring that mathematical concepts are taught within a culturally relevant and authentic learning environment, reinforcing students' cultural identity while acquiring mathematical skills.

Secondly, the indigenous experimental school curriculum effectively formalizes cultural practices into academic mathematics instruction through a structured approach. The curriculum translated the oral tradition and indigenous knowledge into structured mathematical instruction which ensures that students move from experiential learning to abstract mathematical reasoning, formalizing indigenous concepts into measurable and testable academic knowledge. The curriculum connects indigenous knowledge with natural sciences and mathematics, making formal mathematical skills—such as geometry, statistics, and algebra—integral to environmental and cultural studies.

Thirdly, accessibility ensures that students can understand and engage with mathematical content based on their cognitive development and cultural familiarity (Bishop, 1988). The indigenous experimental school curriculum incorporates several strategies to make mathematical concepts more accessible and understandable to indigenous students. The designed activities include hands-on learning in real-world contexts, cultural storytelling as a pedagogical tool, and use of indigenous language in mathematical instruction. By prioritizing experiential, community-based, and linguistically inclusive approaches, the curriculum ensures that students engage with mathematics in a way that aligns with their learning styles and lived experiences.

Fourthly, the indigenous experimental school curriculum excels in the aspect of explanatory power by illustrating the practical mathematical significance of indigenous traditions. The curriculum explores the importance of understanding geometric principles in indigenous house building process, which students learn about triangular stability, load distribution, and spatial symmetry, integrating engineering concepts into cultural practices. By demonstrating how indigenous knowledge inherently involves mathematical reasoning, the curriculum helps students see the broader relevance of mathematical skills in their cultural contexts.

Finally, a strong curriculum should provide a broad foundation across multiple mathematical domains, preparing students for future learning while remaining culturally relevant (Bishop, 1988). The indigenous curriculum achieves this through a well-balanced structure that covers diverse mathematical areas. The curriculum integrates arithmetic, geometry, algebra and statistics. The curriculum not only preserves indigenous knowledge but also connects it to modern applications, ensuring that students gain transferable mathematical skills applicable in contemporary fields such as environmental science, engineering, and data analysis.

An illustrative example of applying the aforementioned five principles is a lesson centered on the construction of traditional bamboo houses. According to a school director, the Bunun people historically built and lived in bamboo houses, utilizing locally available natural resources. Through their unique skills and deep understanding of bamboo's properties, the Bunun were able to construct dwellings resilient to strong winds and heavy rainfall, while maintaining stable indoor temperatures in both cold and hot weather conditions.

To provide students with an experiential understanding of how their ancestors lived in harmony with nature, the school developed a curriculum project around this traditional practice. The mathematics lesson began with students conducting interviews with Bunun elders to learn about the historical and cultural significance of bamboo houses, including the reasons behind their construction, the types and parts of bamboo used for different structural elements, construction techniques, and associated cultural taboos. Drawing on this body of indigenous knowledge and cultural wisdom, students and teachers collaboratively applied mathematical concepts to build their own bamboo houses. The process involved tasks such as measuring bamboo lengths, cutting bamboo into appropriate sizes for walls, pillars, and roofs, and applying mathematical principles—such as the Pythagorean theorem and trigonometry—to ensure the structural stability of the construction.

Through this hands-on project, students engaged in a meaningful integration of cultural heritage and academic knowledge. Beyond reconstructing a traditional Bunun dwelling, the experience facilitated a process of cultural rediscovery and identity formation, empowering students to connect their indigenous heritage with contemporary learning and modern perspectives.The indigenous curriculum development should align closely with Alan Bishop's five principles, successfully embedding cultural knowledge into mathematical education while ensuring academic rigor. Hands-

on, experiential learning methods would make abstract concepts easier to grasp. Well-structured approach could translate cultural practices into formal mathematical instruction. Clear demonstration of mathematical reasoning in cultural traditions will foster students' deeper understanding. By adhering to these principles, the curriculum serves as a model for integrating indigenous knowledge into mathematics education, offering valuable insights for developing more culturally responsive curriculum in the future. It is hoped that such culturally responsive curricula create accessible and meaningful learning contexts that actively engage indigenous students, enabling them to acquire and master academic content and skills. Through this culturally grounded foundation, students are better positioned to transfer their knowledge and competencies across disciplinary boundaries, thereby enhancing their overall academic confidence and achievement. In this way, culturally responsive education serves as a more equitable and inclusive approach, addressing the specific needs and strengths of indigenous learners.

References

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Kluwer Academic Publishers.

Bishop, A. J. (1988). Mathematics education in its cultural context. *Educational Studies in Mathematics*, *19*(2), 179-191.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics*, *5*(1), 44-48..

Acknowledgment:

The researchers sincerely thank the National Academy for Educational Research [Project No. NAER-2022-007-H-2-1-D2-01] for funding this project and express deep gratitude to the Principal, teachers of Padan Asang School, and Bunun elders of the Nangnisalu tribe for their dedication to educating Bunun children and preserving their culture.

TSG8-04

MATHEMATICS ANXIETY ACROSS ETHNIC COMMUNITIES IN NEPAL

Krishna Chandra Paudel
Tribhuvan University, Nepal, kcpaudel@trinitycollege.edu.np

Jaya Bishnu Pradhan
Tribhuvan University, Nepal, jebipradhan@gmail.com

Mathematics anxiety is a significant barrier to academic achievement for the students from diverse ethnic backgrounds. This study investigated mathematics anxiety among various cultural groups in secondary schools in Nepal. The research employed a cross-sectional survey approach, using the Revised Mathematics Anxiety Rating Scale (RMARS) to evaluate 625 ninth-grade students from 12 randomly chosen schools in Kathmandu. The participants included students from diverse cultural backgrounds (Brahmin/Chhetri, Newar, Mongolian, Scheduled Caste, and Others). Results indicated notable differences in mathematics anxiety levels among cultural groups, with Scheduled Caste students facing the highest anxiety levels, whereas Brahmin/Chhetri students exhibited the lowest. The results highlight the necessity of creating culturally responsive teaching strategies and applying focused interventions to support marginalized populations in mathematics education.

Keywords: Cultural Diversity, Culturally Responsive Pedagogy, Mathematics Anxiety

Introduction

Mathematics anxiety (MA) is an essential psychological phenomenon to researchers, educators, and psychologists (Belbase et al., 2024; Wang et al., 2020). Previously, Dreger and Aiken (1957) introduced the concept of MA as "number anxiety" and a feeling of pressure and fear that impede the use of numbers and mathematical operations in both academic and in real-life situations (Richardson & Suinn, 1972). Several studies have attributed MA to many causal factors such as poor test result, age, negative classroom experience, lack of eagerness to complete a difficult assignment, negative attitude towards mathematics learning, and gender (Dowker et al., 2016; Paudel, 2019; 2023).

Mathematical concepts emerging from the culture of individuals, the recognition of their methods of knowledge creation and sharing, along with students' experiences, ought to be integrated with formal mathematics in educational settings (Pradhan, 2017). Most of the researchers have been reported conflicting findings on gender differences in mathematics anxiety (Dowker et al., 2016). However, mathematics learning appears to be shaped by students' cultural backgrounds and their ability relate mathematical concepts to everyday life experiences. Therefore, mathematics education should empower students to apply mathematical operations within their own cultural context (Pradhan, 2017). Mathematics anxiety has been an issue in research because of the negative impact of MA on an individual's initial and future learning, achievements, and use of mathematics (Dowker et al., 2016; Soumen & Susanta, 2018). However, MA has often received less attention over the years (Foley et al., 2017), and the role of gender within and among cultures remains less explored globally (Morsanyi et al., 2016) and locally (Belbase et al., 2024). In this context, this study was intended to explore the differences of mathematics anxiety across various cultural groups (*Brahmin/*

Chhetri, Newar, Mongolian, scheduled caste, and Others) among secondary students of Nepal. This research was guided by research question: are there significant differences in mathematics anxiety levels among various cultural groups of secondary students in Nepal?

Method

This quantitative research employed a cross-sectional survey design to explore mathematics anxiety across various cultural groups among 9th-grade students in Kathmandu, Nepal. The research employed random cluster sampling to choose 625 students from 12 randomly chosen secondary schools in Kathmandu. The sample consisted (Brahmin/Chhetri 311, Newar146, Mongolian 59, scheduled caste 19, and Others 90) students. The study adopted the Revised Mathematics Anxiety Rating Scale (RMARS) (Plake & Parker, 1982), consisting of 24 items rated on a 5-point Likert scale (1 = No anxiety to 5 = Extremely high anxiety). Moreover, it consisted learning mathematics anxiety (16 items) and evaluation mathematics anxiety (8 items). In the context of Nepal, Paudel (2023) established the instrument's reliability by Cronbach's alpha coefficient, which was 0.84, and validated its content through expert evaluation. The process of data collection involved obtaining the essential approvals from school authorities and distributing surveys during regular school hours, ensuring that participation was voluntary and responses remained confidential.

The gathered data underwent thorough statistical analysis employing both descriptive as well as inferential statistical methods. Descriptive statistics involved computing mean scores and standard deviations for every cultural groups. Inferential statistics included one-way ANOVA to examine disparities among cultural groups, and Tukey HSD post-hoc analysis for various comparisons. All statistical analyses were performed with Statistical Package for the Social Sciences (SPSS) to guarantee precise and dependable outcomes in exploring the connections between cultural background and mathematics anxiety among secondary school students.

Findings and Discussion

Table 1 indicates the mathematics anxiety among secondary students in different cultural groups

Table 1. Descriptive statistics of math anxiety scores among cultural groups in secondary students

Cultural Groups	N	Mean	Std. Deviation	Minimum	Maximum
Brahmin and Chhetris	311	47.3280	13.94928	30.00	92.00
Newar	146	60.7397	11.26487	43.00	94.00
Mongolians	59	60.6441	25.63387	34.00	97.00
Scheduled Caste	19	75.9474	14.10072	53.00	97.00
Others	90	52.1111	8.70805	38.00	71.00
Total	625	53.2768	15.98212	30.00	97.00

The study results show notable differences in mathematics anxiety among various cultural groups in secondary students, with an overall sample size of 625 individuals. The examination reveals that Scheduled Caste students faced the greatest levels of anxiety (M = 75.95, SD = 14.10), with Newar students next (M = 60.74, SD = 11.26) and Mongolian students following (M = 60.64, SD = 25.63).

In comparison, Brahmin and Chhetris, comprising the largest group (n = 311), exhibited significantly lower anxiety levels (M = 47.33, SD = 13.95), whereas the "Others" group (n = 90) presented moderate anxiety levels (M = 52.11, SD = 8.71).

Table 2. One-way analysis of variance (ANOVA) findings for math anxiety among cultural groups

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	32227.096	4	8056.774	39.283	.000
Within Groups	127160.018	620	205.097		
Total	159387.114	624			

The ANOVA findings validated meaningful statistical differences among these cultural groups $F(4,620) = 39.283$, $p < .001$, suggesting that the noted disparities in mathematics anxiety are closely linked to cultural background. The significant difference in anxiety levels, especially the higher scores of Scheduled Caste students in relation to Brahmin and Chhetris, underscores possible issues of educational equity. The results indicate a necessity for focused strategies that take cultural elements into account when tackling mathematics anxiety, particularly for marginalized populations experiencing elevated levels of mathematics related stress.

The Tukey HSD post-hoc analysis showed notable pairwise differences in mathematics anxiety across various cultural groups ($p < .05$). Students from Scheduled Castes exhibited notably elevated anxiety levels in comparison to other groups, with the greatest mean difference identified between Scheduled Castes and Brahmin/Chhetris (MD = 28.62, $p < .001$). Newar and Mongolian students exhibited comparable levels of anxiety (MD = 0.10, $p = 1.000$), however, both groups had notably greater anxiety than Brahmin/Chhetris (MD = 13.41 and 13.32 respectively, $p < .001$). The "Others" group exhibited moderately increased anxiety compared to Brahmin/Chhetris (MD = 4.78, $p = .043$) but significantly less anxiety than Newar (MD = -8.63, $p < .001$), Mongolian (MD = -8.53, $p = .004$), and Scheduled Caste students (MD = -23.84, $p < .001$). These results statistically validate the hierarchical structure of mathematics anxiety among different cultural groups, with Scheduled Caste students facing the highest level of anxiety and Brahmin/Chhetri students facing the lowest. This clear difference is especially apparent when juxtaposed with Brahmin and Chhetri students, who displayed significantly reduced anxiety levels (M = 47.33, SD = 13.95). The ANOVA results alongside the subsequent Tukey HSD post-hoc analyses, affirm the statistical significance of these differences, indicating that cultural background significantly influences students' mathematical anxiety, which aligns with Pradhan's (2017) claim regarding the necessity of incorporating cultural contexts into mathematics education. The significant anxiety disparity between marginalized populations and historically privileged groups highlights more profound systemic problems in mathematics education, backing the worries expressed by researchers regarding the importance of focusing on cultural elements of MA (Foley et al., 2017; Morsanyi et al., 2016), and indicating the need for culturally responsive strategies to foster educational equity (Belbase et al., 2024; Pradhan, 2017).

After the quantitative analysis of the data, the researchers talked to two 'more anxious students of the scheduled caste until the data were saturated. The researchers collected more than 50 narratives among which the two have been presented here as testimonials:

> "There are nine members on my family live in three-room house, most of the seniors are not well educated and they do not give values for education, I could not get a peaceful place to read at my home". (*Raj, a IX grade student*)

> "I could not get support at school from my subject teacher and friends on solving mathematical problems. Teacher only prioritizes the brilliant students in class, never asks if I understand the mathematical concepts, teacher barely allows sharing my difficulties and provides feedback; and due to economic condition, my family hardly provides extra support at home for my study". (*Sanu, a IX grade student*)

These representative narratives indicated that the students were unable to get the suitable home environments: physical, economic, and psychological; sufficient support at schools from their teachers and peers; and at home from their parents and seniors. In a nutshell, the three themes: the unfavorable home environment, weak student support system and socioeconomic condition were the major causes for mathematics anxiety. The analysis of the data highlighted the influence of ethnic background of students on mathematics anxiety levels and the need for culturally responsive pedagogies to promote educational equity.

Conclusion and Implications

The research reveals significant differences in mathematics anxiety amongst cultural groups in Nepal's secondary education, with Scheduled Caste students experiencing the highest level of anxiety, in contrast to Brahmin and Chhetri students, who displayed the lowest anxiety. These results emphasize the need for specific educational strategies that take into account cultural backgrounds. Educators and policymakers need to create teaching methods that are culturally responsive, with a particular emphasis on assisting marginalized groups such as Scheduled Caste students. Professional development initiatives ought to improve educators' cultural awareness and understanding of how diverse backgrounds affect mathematics anxiety. The education system must establish regular monitoring of anxiety levels among different demographic groups and initiate systemic reforms to foster more inclusive learning environments, ensuring educational equity for every student regardless of their cultural background.

References

Belbase, S., Kshetree, M. P., Khanal, B., Panthi, R. K., Acharya, B. R., Gnawali, Y. P., & Kasemsukpipat, W. (2024). Effects of students' images, anxieties, and attitudes toward mathematics on their value of the school subject. In Proceedings of the 46th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, 1525-1531.

Dowker, A., Sarkar, A., & Looi, C.Y. (2016). Mathematics anxiety: What have we learned in 60 years? *Frontiers in Psychology, 7*(508), 1-16. https://doi.org/10.3389/fpsyg.2016.00508

Dreger, R.M., Aiken, L.R. (1957). The identification of number anxiety in a college population. *Journal of Educational Psychology*. 48, 344-351.

Foley, A. E., Herts, J. B., Borgonovi, F., Guerriero, S., Levine, S. C., & Beilock, S. L. (2017). The math anxiety-performance link: A global phenomenon. *Current Directions in Psychological Science, 26*(1), 52-58.

Morsanyi, K., Mammarella, I.C., Szücs, D., Tomasetto, C., Primi, C., Maloney, E.A. (2016). Editorial: mathematical and statistics anxiety: educational, social, developmental and cognitive perspectives. *Frontiers in Psychology*. 7, 1083. DOI: 10.3389/fpsyg.2016.01083

Paudel, K. C. (2019). Mathematics Anxiety among Secondary Level Students in Nepal. *The Eurasia Proceedings of Educational and Social Sciences*, 14, 34-40.

Paudel, K. C. (2023). Impact of Classroom Environment on High School Students' Mathematics Anxiety. *Academic Journal of Mathematics Education*, 6(1). DOI:10.3126/ajme.v6i1.63796

Plake, B. S. & Parker, C.S. (1982). The development and validation of a revised version of the mathematics anxiety rating scale. *Educational and Psychological Measurement*, 42, 551-557

Pradhan, J. B. (2017). Mathematical ideas in Chundara culture: Unfolding Nepalese teaching and learning system. In M. Rosa, L. Shirley, M. E. Gavarrete & W. V. Alangui (Eds.), Ethnomathematics and its Diverse Approaches for Mathematics Education (125-152).

Richardson, F. C., & Suinn, R. M. (1972). The mathematics anxiety rating scale: Psychometric data. Journal of Counselling Psychology 19(6), 551-554. https://doi.org/10.1037/h0033456

Soumen, C.K., Susanta, K. (2018). Mathematics anxiety and its relationship with the achievement of secondary students in Malaysia. *International Journal of Research and Analytical Reviews. 5*(3), 451-455.

Wang, Z., Rimfeld, K., Shakeshaft, N., Schofield, K., Malanchini, M. (2020). The longitudinal role of mathematics anxiety in mathematics development: Issues of gender differences and domain-specificity. *Journal of Adolescence*. 80, 220-232

TSG8-05

A STUDY ON MATHEMATICS CURRICULUM AT AN INDIGENIOUS SCHOOL IN TAIWAN: FOCUSING ON THE PRODUCTION OF MATHEMATICAL TASK

Ying-Hsuan Lee
National Academy for Educational Research, Taiwan, ying-hsuan.lee@mail.naer.edu.tw

Alang Palalavi
Padan Asang School, Taiwan, alang1123@gmail.com

Tai-Jung Lee
National Academy for Educational Research, Taiwan, pasuya@mail.naer.edu.tw

Yi-An Cho
National Academy for Educational Research, Taiwan, scottiess0625@mail.naer.edu.tw

This study explored the development of a mathematics curriculum in an indigenous school in Taiwan, focusing on the design and implementation of mathematical tasks. Grounded in Bishop's (1988) framework of mathematical enculturation, the curriculum incorporates symbolic components (mathematical concepts), social components (mathematics in societal applications), and cultural components (the value of mathematics within culture), this study collect data through field observations and classroom practices to understand the process of developing mathematical tasks rooted in indigenous life experiences. Results indicated that integrating mathematics learning with cultural practices significantly enhances students' engagement and academic performance while strengthening their cultural identity. This study provides empirical insights and a reference for future indigenous mathematics curriculum development.

Keywords: Indigenous Knowledge, Mathematics Education, Culturally Responsive Pedagogy, Mathematical tasks

Introduction

Mathematical concepts are often perceived as universal, but Bishop (1988) argued that it is shaped by cultural and societal needs. Different communities develop mathematical knowledge in unique ways. In Taiwan, indigenous groups also possess rich mathematical traditions, including the Bunun people's astronomical calculations, the Amis people's geometric patterns, and the Paiwan people's architectural structures. However, the current curriculum follows Western frameworks, rarely incorporating indigenous knowledge, creating cognitive barriers for students (Harris, 1980).

Ethnomathematics emphasizes mathematical diversity and promotes culturally responsive education (D'Ambrosio, 1985). Bishop (1988) expanded this through mathematical enculturation, which integrates symbolic (mathematical representations), social (societal applications), and cultural (identity-based) components into learning. This study applies Bishop's (1988) framework to develop a mathematics curriculum at an indigenous school in Taiwan, focusing on task design and implementation. By integrating indigenous cultural knowledge, the curriculum seeks to enhance student engagement, mathematical comprehension, and cultural identity. This research also offers insights

into curriculum reform, promoting a more culturally responsive approach to indigenous mathematics education.

Literature Review

Mathematics is not an independent body of knowledge but a product of human societies, shaped by cultural practices and needs (Bishop, 1988). Different cultures develop unique mathematical methods. Integrating cultural contexts into mathematics learning can enhance students' understanding and motivation. However, conventional curricula often rely on Western frameworks, neglecting cultural variations in mathematical thinking, which may result in indigenous students' lag-behind learning. Addressing this gap by incorporating culturally relevant mathematical experiences is crucial for improving indigenous students' learning engagement and learning outcomes, which serves to be promoting a more equitable educational approach in Taiwan.

Ethnomathematics highlights the diversity of mathematical thoughts, advocating for integrating local knowledge systems into education to enhance learning (D'Ambrosio, 1985). For example, Taiwan's indigenous groups possess rich mathematical traditions, such as the Bunun people's astronomical calculations, the Amis people's geometric patterns, and the Paiwan people's architectural proportions. However, traditional mathematics education rarely incorporates these cultural elements, making it difficult for indigenous students to connect mathematics to their experiences. By embedding ethnomathematical principles into curricula, this study aims to bridge that gap, fostering both mathematical competence and cultural identity.

Mathematical tasks play a crucial role in curriculum development and should consider students' cognitive abilities, social interactions, and cultural backgrounds (Stein et al., 1996). Well-designed tasks encourage reasoning, problem-solving and mathematical thinking (Henningsen & Stein, 1997). In cultural contexts, tasks should reflect real-life experiences, such as agricultural calculations, hunting area measurements, and traditional architecture design, making learning more meaningful. Research shows that when students engage with mathematics in familiar cultural settings, their motivation and performance improve significantly (Nasir & Hand, 2008). Thus, designing culturally embedded mathematical tasks is essential for enhancing indigenous students' learning experiences.

Bishop (1988) introduced mathematical enculturation, emphasizing that mathematics education should occur within cultural contexts to help students relate mathematical concepts to their heritage. His framework consists of three components: symbolic, social, and cultural.. The symbolic component refers to mathematical representations such as numerical systems, measurement units, and geometric patterns. The social component focuses on how mathematics is applied in economic transactions, architectural planning, and agricultural management. The cultural component examines mathematics' role in cultural traditions, such as ritual calculations and symmetrical designs in indigenous art. By incorporating these three aspects, this study develops a mathematics curriculum tailored to Indigenous students' cultural backgrounds, fostering greater engagement and meaningful learning experiences.

Research Methodology

This study adopts a case study approach to examine the development of a culturally embedded mathematics curriculum at an indigenous school in Taiwan. The research focuses on designing and implementing mathematical tasks that integrate symbolic, social, and cultural components, based

on Bishop's (1988) mathematical enculturation framework. By embedding cultural knowledge into mathematics learning, this study aims to enhance students' engagement, mathematical understanding, and cultural identity.

The study is conducted at an indigenous junior high school, Padan Asang School, in Taiwan, focusing on a ninth-grade class. Padan Asang School is a pioneering indigenous experimental school located in southern Taiwan. It was selected as the research site due to its distinctive approach to designing mathematics curricula and tasks. Rooted in the Bunun culture that the school actively promotes, its mathematics curriculum is fundamentally shaped by Bunun heritage, knowledge systems, and worldview. The participants include school principal, mathematics teachers and cultural experts (local elders and artisans). The selection of this school was based on the school's existing efforts in cultural integration, the community's strong interest in cultural preservation, and the need to improve indigenous students' engagement in mathematics. The mathematics curriculum integrates classroom-based mathematical instruction, outdoor experiential learning and community and cultural involvement. This study employs qualitative research methods for data collection, including classroom observations, the interviews of teacher and cultural experts, and student learning artifacts. Data will be analyzed using thematic analysis, focusing on task performance in mathematical problem-solving.

Results and Conclusion

Bishop (1988) proposed that mathematics curricula should consist of three key components: symbolic, social, and cultural. These elements help connect students' mathematical learning with their cultural background and daily experiences. This analysis examines the Three-Stone Stove unit in the mathematics curriculum, evaluating how its mathematical content aligns with Bishop's framework.

According to an interview with one of the school directors who designed and led the Three-Stone Stove unit, the purpose of incorporating this traditional cooking method into the mathematics curriculum was to show students that mathematics is not a distant or abstract subject, but rather one that is deeply embedded in everyday life—even in ancient wisdom. The Three-Stone Stove is a device commonly used in Bunun culinary traditions. By arranging three stones in a triangular formation, the Bunun people are able to place a cooking pot securely on top, creating three stable points of support.

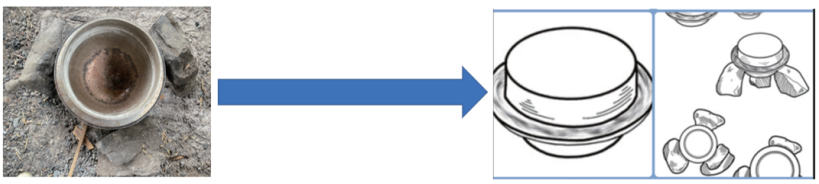
To ensure even heating of the stove and the food inside, students needed to consider several practical factors: the distance between each stone to maintain balance and prevent the pot from tipping over, the optimal location for placing tinder to distribute heat evenly, and how to avoid overheating or burning certain parts of the stove. These real-life challenges required students to apply their knowledge of mathematics and science.

According to the director's observations, students who participated in the unit posed more mathematical and scientific questions during the hands-on activities. This engagement helped them develop inquiry-based thinking and apply their academic knowledge to practical problem-solving scenarios.

As for the three key components of mathematics curricula, they are analyzed as follows. The symbolic component refers to mathematical representations, structures, and notations. In the Three-Stone Stove unit, symbolic components include geometric structures, shape recognition, measure-

ment, heat conduction, and numerical calculations. Students analyzed the circular, triangular, and polygonal arrangements of stones and applied angle measurements and symmetry principles to understand their stability. Through hands-on activities, students needed to measure stone sizes and distances. The social component emphasizes mathematics in everyday life, trade, and community interactions. In this unit, real-world applications include cooking, ecology and resource efficiency. Students calculated fuel-burning times and cooking durations, applying time measurement, ratios, and speed calculations. Students collaborated in teams to analyze efficiency variations across different stove configurations by collecting data, conducting statistical comparisons, and presenting their findings through graphical representations. These activities demonstrated the practical value of mathematics, enhancing students' ability to apply mathematical concepts in social and environmental contexts.

The cultural component explores how mathematics preserves indigenous knowledge, history, and values. The Three-Stone Stove unit integrated culture and mathematics in the following ways. Firstly, the lesson highlighted how indigenous communities select appropriate stone materials based on resources, introducing students to select optimization strategies. Secondly, by tracing the evolution of Three-Stone Stoves in tribal communities, students learned how traditional cooking methods influenced early mathematical calculations related to heat efficiency. Finally, the arrangement of three stones not only is functional but also follows symmetry and stability principles. Students applied mathematical calculations to assess structural balance and efficiency. These cultural connections enhance students' appreciation of indigenous mathematical knowledge, fostering both academic learning and cultural identity.



Cultural Event:
The three-stone stove holds deep meaning and symbolism in Bunun culture, reflecting the people's respect for nature and wisdom. Composed of three stones forming a stable support, the three-stone hearth symbolizes stability and harmony. It is not only a daily cooking tool but also the center of family life. Activities such as cooking, gatherings, and rituals take place around it, strengthening emotional connections among family members. On a spiritual level, the three-stone hearth represents the continuation of life and the wisdom of ancestors. Each year, before important festivals, the people rekindle the fire, a ritual that is not only a reverence for fire but also an expression of respect for ancestral spirits.

Mathematical Task:
The three-stone stove is composed of three stones of approximately equal height, arranged in a triangular formation to create a stable support structure. This design utilizes which principle of triangles to ensure that the pot can be securely placed on top? Additionally, where should the fire be positioned within the triangle?

Figure 1. Mathematical task of the three-stone stove

This unit enhances students' mathematical skills while reinforcing indigenous cultural knowledge, providing a valuable model for culturally responsive mathematics education.

Based on this analysis, the following recommendations are proposed for future curriculum development and implementation. Researchers encourage the use of local artifacts, tools, and oral traditions to enhance learning experiences. Through project-based learning, mathematics teachers should allow students to apply mathematics to real-world indigenous living settings. Mathematics teachers attempts to extend the use of indigenous knowledge and practices to other mathematical topics.

References

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Kluwer Academic Publishers.

Bishop, A. J. (1988). Mathematics education in its cultural context. *Educational Studies in Mathematics*, *19*(2), 179-191.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics*, *5*(1), 44-48.

Acknowledgment

Researchers of this study are grateful for the research funding provided by National Academy for Educational Research [Project number: NAER-2022-007-H-2-1-D2-01]. Also, many thanks go to all participants of this study for their endeavors to preserve Bunun culture, pass on Bunun wisdom, and educate the future generation of Bunun children.

TSG8-06

A PRACTICAL STUDY ON THE INDIGENOUS RITUAL CULTURE IN ELEMENTARY SCHOOL INQUIRY EDUCATION

Felisa(Hsueh-Yun)Yu
National Changhua University of Education, Taiwan, felisayu@mail.ntcu.edu.tw

Taiwan is a multicultural society based on respect for indigenous peoples. The 12-year basic education curriculum aims to enhance the understanding of multicultural education for all students and the ethnic education needs of indigenous students. It adopts regulations related to indigenous peoples in the overall curriculum framework, implementation points and learning priorities of each curriculum. The guidelines further incorporate inquiry into the core curriculum of primary and secondary schools to connect students' culturally relevant life experiences. This study first explores the literature content of cultural response theory, curriculum standards, inquiry-based teaching courses, etc. to strengthen the theoretical foundation of curriculum design; then, based on the core, learning focus and inquiry-based teaching of the natural science domain in the 12-year national education curriculum, the Paiwan people's five-year traditional festival - thorny rattan balls are integrated into the concepts of force and motion in the sixth grade of elementary school, enhancing students' understanding of the wonderful use of science in Paiwan culture, understanding the concept of the lever theorem unit, and cultivating the ability of inductive reasoning. We hope that this course design can effectively help domestic indigenous students increase their interest in learning, help them recognize and affirm their own unique culture, and convey this study's respect and care for multiculturalism.

Keywords: Culturally responsive teaching, curriculum standards, 5E inquiry teaching

Introduction

Taiwan's indigenous peoples have their own unique culture, which is different from that of the Han Chinese. International reform movement Multicultural education, the practice of culturally responsive teaching developed in response to the needs of various ethnic groups, cultivates students' cultural integrity, personal abilities and academic achievements (Gay 2010, Ladson-Billings 1995a)

As science education in the United States develops towards multiculturalism, science education for indigenous peoples is increasingly valued. For example, Project 2061, a science reform education project under the U.S. education reform plan, and the Position Statement on Multicultural Science Education of the National Science Teachers Association (NSTA) (2000) actively advocate that science education should incorporate the spirit of multiculturalism and emphasize that science education should enable all students from different cultural groups to have the opportunity to learn science and obtain employment opportunities in the fields of science, engineering and technology (Fu Liyu 2006). The above shows that traditional culture and mathematical science education have a great influence.

Articles 14 and 15 of the United Nations Declaration on the Rights of Indigenous Peoples [UNDRIP], adopted by the United Nations General Assembly in 2007, assert that indigenous peoples have the right to have their culture, traditions and history appropriately reflected in education, and that states should work with indigenous peoples to take effective measures to ensure that indigenous

children have the opportunity to receive education about their own culture in their own language. In accordance with the Indigenous Education Act and the 108 Curriculum, Indigenous issues are integrated into the mathematics and science curriculum. This article proposes a curriculum design that integrates culturally responsive teaching into 5E inquiry teaching to allow Indigenous students to connect tribal life culture with science, and hopes to provide suggestions to assist front-line teachers in their professional development.

The purpose of this study is to:

(i) Providing opportunities for indigenous students to learn through inquiry and solve problems, and to develop scientific knowledge and skills; Develop a scientific thinking system and a habit of scientific inquiry, develop a positive scientific attitude, and develop a scientific core concept.

(ii) This course provides a multicultural learning environment through the culturally responsive scientific teaching materials and diversely designed teaching activities. It combines students' cultural backgrounds, community resources, and school teaching resources to allow students to come into contact with local culture and knowledge, and to develop interest and motivation in learning.

Literature Review

Multicultural education is an emerging educational reform movement that can be traced back to the civil rights movement of African Americans in the 1960s in the United States. Taiwan's multiculturalism was influenced by the democratization and localization trends after the lifting of martial law (Tan Guangding, Liu Meihui, and You Meihui 2015). Multiculturalism believes that the traditional wisdom contained in indigenous science is that indigenous tribes will develop practical knowledge that is closely related to their living environment and is essential for the daily survival of the ethnic group. Since the 1980s, educational researchers have begun to incorporate a social perspective into their view of natural subjects and culture, shifting their thinking from psychological knowledge or cognitive models to the view that learning is based on social interaction (Lerman 2000).

The concept of culturally responsive teaching emerged in the 1970s, and scholars have used different terms to refer to this concept, such as culturally appropriate (Au & Jordan 1981), culturally relevant (Ladson-Billings 1995b), and culturally responsive (Gay 2003). It means that classroom teaching can refer to the characteristics of the student group, consider the students' cultural background and learning style in curriculum and teaching, use the students' native culture as a bridge for learning, and help students with cultural differences have a fairer opportunity to pursue excellent performance (Gay 2000).

In recent years, the United States has also released a new curriculum for science education - the Next Generation Science Standards (NGSS), which emphasizes the integration and application of core concepts and practices (NGSS Lead States 2013). The "108 Curriculum" announced by Taiwan in 2018 emphasizes the teaching of core competencies. In order to make up for the fact that the old nine-year curriculum did not include ethnic education in the national curriculum, it attempts to improve the situation where the "Nine-Year Curriculum Outline" has no regulations on ethnic education other than "indigenous languages."

Research Method

The research method of this study is case study method. The research subjects were a class of sixth grade students in an indigenous elementary school in Pingtung County (the class had only 19 students). The teaching activity design and teaching unit are the mechanics unit for sixth graders in elementary school, and the teaching method adopts 5E inquiry teaching. During the research process, the teacher serves as the course designer, teacher and observer. During the teaching process, a research participant is also arranged to provide the teacher with teaching suggestions and feedback to help the teacher continuously correct his teaching skills and research direction. The research data collection includes: classroom observation video, document analysis (interviews, study sheets), researcher notes, teaching suggestions and feedback from teaching observers, and a student course feedback sheet.

Data Collection and Analysis

The researchers incorporated the Paiwan tribe's traditional ritual, the five-year rattan ball festival, into the sixth grade science inquiry teaching activities of lever setting. After each teaching activity, the students recorded their own views on the teaching activity and the problems they found, and conducted self-reflection and understanding. The data in the notes are represented by JT, and the following four digits represent the date.

Group Sharing and Discussion

All students were interviewed in groups about their views on the scientific concepts of each teaching topic and the five-year festival activities. Since the researchers were not teachers in the case schools, they were not clear about the students' situation in the case schools. In addition, considering the psychological factors of students' shyness and the hope that students could stimulate their ideas through peer stimulation, this study adopted the group interview method. The group sharing and discussion data were represented by TR, and the following digits represented the student number. Two to three comprehensive interviews were conducted with the students.

Results

This study takes the five-year ritual of rattan ball in the Paiwan culture as its target. 5E Science Inquiry allows students to understand the basic principles of the lever theorem, understand the force point, resistance point, force arm and resistance arm of the rattan ball pole; in mathematics, the concept of proportion calculation is applied to understand how to produce the phenomenon of saving effort and exerting effort.

The Benefits of Providing Culturally Responsive Instruction as an Inquiry Curriculum

As students engage in science activities, they work together, share and discuss different perspectives, and in the process learn about their cultural similarities and differences, as well as how others view science and use scientific knowledge, perspectives that connect to the inquiry-based practices we want students to develop in science classes. Based on the results of the interview and hands-on inquiry experiment, students mentioned their ideas for the cultural response and inquiry courses: "*This time, the Quinquennial Festival thorn ball game is actually related to natural science. The teacher also asked us to do it by ourselves to find the force application point, resistance point and fulcrum. It was really interesting" (TR01); "From the results, we explored which type of lever theorem is labor-saving? Which type is labor-intensive? Which type is time-saving? They are all*

related to the Quinquennial Festival" (TR10). "Today, through the experiment, I learned about the lever theorem and three of its terms: the fulcrum, the point of force application, and the different positions of the force application points. Different forces must be used. The sacrificial poles used in our ancestors' five-year sacrifices are also related to this. This is something I didn't know before. I thought the tribal warriors must have great skills in stabbing the lucky ball. This is related to the lever theorem that the teacher taught in class" (TR18).

In Terms of Culturally Responsive Teaching

In terms of culturally responsive teaching, teachers should understand the cultural implications behind the behavior of indigenous students and incorporate students' cultural differences into the design and methods of teaching preparation and implementation, so as to adopt educational activities that are in line with the characteristics of diverse ethnic groups in a multicultural school context, create opportunities conducive to learning, and further promote learning development. Researchers believe that culturally responsive teaching is not just a teaching method, but an attitude towards students and a driving force for redefining the roles of teachers and students. It is also a tool for school reform and an empowerment mechanism. It is suitable for different types of schools, and all students can benefit from this teaching.

In Scientific Inquiry Learning

Today, my country's 108 curriculum emphasizes that inquiry-based teaching strategies should be included in curriculum design and actual teaching. We can understand that the cultivation of scientific inquiry ability has become an important topic in my country's science education; allowing students to learn science in a natural and real situation like scientists. Many modern courses also emphasize allowing students to learn in real situations like scientists. The model of scientific learning is inquiry-based. Students' learning must take place through events that happen around them or that they experience and comprehend, so as to arouse their interest in learning and enable them to explore scientific knowledge or make decisions and solve problems. When conducting scientific learning, we should pay attention to students' actual living world and conduct natural course learning from real situations, avoiding the crisis of scientific learning caused by emphasizing only empirical science.

TSG8-07

SURVEY AND STUDY OF "ETSUKI-SANGAKU," MATHEMATICAL TABLETS WITH ILLUSTRATIONS, IN FUKUI PREFECTURE, JAPAN, AND EMPIRICAL RESEARCH ON ITS UTILIZATION FOR MATHEMATICS EDUCATION

Hiroshi Kazama
University of FUKUI, Japan, h-kazama@u-fukui.ac.jp

This is a study of the cultural and educational phenomenon of distinctive Japanese mathematics ("Wasan," established in early-modern Japan by Seki Takakazu) dedicated to local shrines and temples from the Edo to Meiji periods. As a legacy of Wasan, throughout Japan, there exist more than 900 "sangaku"-like mathematical tablets (academic plaques with math problems, answers and solutions displayed. Many sangaku feature geometric problems, but others present algebra, surveying and Buddhist problems). Referring to previous studies on the teaching materials of the history of math, we aimed to attract students' interest by making them aware of the existence of "Etsuki-sangaku" (mathematical tablets with illustrations depicting real-world scenes of math problems) in Fukui prefecture, as a heritage of science in the school district and neighboring areas, and by directing their attention to the thoughts and ideas contained therein and beyond. In addition, students can explore original sources within their school curriculum in an interdisciplinary way and apply them to shape their own learning. This paper presents a proposal derived from practical research in mathematics education using "Etsuki-sangaku." Five suggestions emerged from this research.

Keywords: Japanese Mathematics "Wasan", Mathematical Tablets "Sangaku," Mathematical Tablet Cultural Studies, Fukui Prefecture

Introduction

The number of extant mathematical tablets from shrines in Fukui prefecture is 23. Of those, only 12 have illustrations (and are thus "*Etsuki-sangaku,*" hereafter "*E-S*"), so they are the focus of this study. We first classify the mathematical features of the 12 *E-S* (mathematical tablets with illustrations), all of which come from shrines in Fukui's Tannan region.

The term *E-S* is based on the original text of the Oshio Hachiman Shrine's "Crane, Turtle, Pine, and Bamboo tablet" (1701), shown in Figure 1, entitled "Etsuki Sanjyustu no Ichimon." We define *E-S* as "mathematical tablets with illustrations in which all/part of the title is pictorialized and, though the problem cannot be misinterpreted without the picture, the picture leads to an understanding of the scene or adds an intention to record or commemorate an event by the donor or hometown."

Based on the research results of these *E-S* in Fukui, we will develop teaching materials for math education, and assess their applicability through empirical studies conducted by classroom research.

Figure 1. Etsuki-Sangaku of Oshio Hachiman Shrine (1701)

Purposes and Methods of the Study

The purposes are to classify the 12 *E-S* in Fukui prefecture by their particular mathematical features, and then to develop them as teaching materials for math education. A summary follows of the features of the 12 *E-S* based on previous research, written records, visual inspection and image data. Based on the research results, we develop teaching materials for math education, conduct empirical research through classroom studies, and obtain suggestions on the potential for using the materials for contemporary math education.

Research Contents and Results

Distinct Disposition of Fukui's Etsuki-sangaku

The 12 mathematical problems of Fukui's *E-S* can be subdivided into 8 problems of higher order equations, 2 problems of least common multiple, 1 problem of quadrature of forfeiture, and 1 problem related to Buddhism (Mt. Sumeru). The *E-S* in Figure 1 is unique in that the side illustrations were done first, and then the math problem was written in the center, incorporating the dedicatee's own age of 75 years as the age of the bamboo, thus incorporating real-life meaning to the question. The 4 *E-S* from Sabae City all depict local shrines and have cultural value as a record of the local area. In these ways, Fukui's 12 *E-S* can also be classified into 7 distinct patterns of artistic arrangement.

Figure 2. Etsuki-sangaku of Ishibe Shrine
(image courtesy of the Sabae City Board of Education, Cultural Division)

First Question of the Etsuki-sangaku of the Ishibe Shrine (1877)

The question posed is that the shrine has received 2 big wooden barrels of sake containing 2000 *gou* [1 *gou* ≒ 180mL]. The number of shrine parishioners is 307, and the number of female parishioners is 3 fewer than the males. Moreover, females should each receive 3 *gou* of sake less than the males. Find how much sake each male and female receive. This problem has pleasing symmetry because it features the number 3 twice. In fact, the answer is males receive 8 *gou* and females 5, and the population is 155 males and 152 females. The population of Kitanakayama-village, Ōaza Isobe (the former Isobe-village, where Ishibe Shrine is located) in 1891 is given as 164 males and 165 females in the *Japan Place Names Dictionary* (*Kadokawa Nihon Chimei Daijiten,* 1989, 18, p. 140), so the question probably incorporated the village's actual population at that time.

Implications of Etsuki-sangaku for Mathematics Education

In the 2008 editions of 7 Japan's authorized textbooks for junior high school math, all publishing companies included figures of *sangaku*, but in the 2021 and 2025 editions, the number decreased to 4 out of 7. Collaborating with the Sabae City Board of Education, we designated the X Junior High School District a pilot district and conducted a multi-year/longitudinal program of on-site classes at 4 elementary schools and 1 junior high school, studying the Ishibe Shrine *E-S* (Figure 2) from the interdisciplinary viewpoints of mathematical and cultural/historical significance. The solution method in the text of the time assumes that all people are men, and it is like the famous "Crane and Tortoise Calculation", so to understand the solution method, the original text "Algorithm Point-by-Point Guide" by Sakabe (1815, Intro, Prob. 32) was also used as a teaching material. The problem statement is a quadratic equation with 4 variables, but the students realized it could be reduced to a linear equation with 2 variables. The class structure, of three 50-min. periods, was as follows:

(1) Observation of *Etsuki-sangaku* and understanding of the math problems

(2) Self-solving

(3) Comparison of students' mathematical thinking and solutions with the *Etsuki-sangaku* ones

(4) Creating new problems by changing the conditions (following the "what-if-not" strategies of Brown & Walter, 1990; 1993)

(5) Creating original, contemporary *Etsuki-sangaku* or *sangaku*.

(1) (2)(3): Activities like a historian of mathematics; (4)(5): Activities like a mathematician

As such, it is a highly literal iteration of problem-posing and inquiry education, in which students can participate actively and shape their own learning, while gaining pride in and appreciation of Japan's unique and beautiful mathematical cultural heritage through local artefacts (as a form of interdisciplinary learning). Through such "mathematical enculturation" (Bishop, 1991), we can "humanize" and personalize math, helping students feel less of a cultural gap with their ancestors.

Results and Conclusion

A post-activity questionnaire administered to 268 junior-high students revealed the following findings. The material has not yet been generalized to wider populations, but this is the future goal.

The study introduced the concept of Japanese mathematics (*Wasan*) for the first time to more than half of the students. The students became able to explore the mathematical ideas/methods found in the *E-S*, in contrast to contemporary math techniques, and thus became able to appreciate the high

quality of the mathematical ideas. This encouraged students to show interest in learning Japanese mathematics and other mathematical histories/heritages. They reported their enjoyment in doing the activity and their realization of the difficulty of creating challenging math problems and thus their respect for the *sangaku* creators. Many students have come to think appreciatively/nostalgically about *Wasan* and have thus enriched, improved, or broadened their views of the math they learn at school.

We obtained the following five suggestions, although they have not yet been generalized.

- Our ancestors bequeathed a wealth of research material; obtaining primary documents and modern-language translations is easy. Japan has over 900 extant *sangaku*, so visiting them is possible. Organizing and presenting the activities and calculations of the school district is likely to increase interest in the heritage of science and increase local understanding and pride.
- By referring to the readily comprehensible *E-S* as primary documents, students can explore the solutions found in the calculations and the technical documents of the calculations of the time, and to appreciate the merits of hypothetical methods through solution comparison.
- Referring to *E-S*, creating problems that include illustrations in the making of *sangaku* will promote an understanding of the perspectives and feelings of the donators of the *sangaku*.
- It broadens and improves the way in which math is perceived (mathematics view) in schools.
- While it is difficult to secure class time, a single lesson is insufficient, so it is necessary to create problems throughout the unit and year, and "*Wasan* utilization." Moreover, it is necessary for math teachers to work with local researchers etc. to understand the reading of primary sources and the methods for solving problems at the time, and to find ways to cover this material without increasing the number of hours.

Building on this study, it is hoped that students will show interest in the existence of mathematics and experience the wonder of Japanese culture in such interactive and interdisciplinary ways.

References

Akama, Y., & Kazama, H. (2024). Design and evaluation of junior high school mathematics lessons with "Wasan": Using Fukui prefecture's "Sangaku" with pictures. *Japan Society for Science Education, Annual Conference Proceedings*, *48*, 169-172. https://doi.org/10.14935/jssep.48.0_169

Bishop, A. J. (1991). *Mathematical enculturation: A cultural perspective on mathematics education*. Kluwer Academic Publishers.

Brown, S. I., & Walter, M. I. (Eds.) (1993). *Problem posing: Reflections and applications.* Psychology Press.

Brown, S. I., & Walter, M. I. (Eds.). (1990). *The art of problem posing.* 2nd Edition. Routledge.

Fukagawa, H., & Rothman, T. (2008). *Sacred mathematics: Japanese temple geometry*. Princeton University Press.

Kadokawa Nihon Chimei Daijiten[Japan place name dictionary] vol.18, Fukui. (1989). Kadokawa.

Kazama, H. (2023). Teaching material development and educational practice of "'Sangaku: Mathematical tablet" with pictures as a scientific heritage in Fukui prefecture. *Japan Society for Science Education, Annual Conference Proceedings*, *47*, 613-614. https://doi.org/10.14935/ jssep.47.0_613

Kazama, H. (2024). The potential of the history of mathematics for science education: Using the course of study as a guide. *Japan Society for Science Education, Annual Conference Proceedings, 48*, 165-166. https://doi.org/10.14935/jssep.48.0_165

Sakabe, K. (1815). *Sanpō tenzan shinan-roku*[Treatise on Tenzan algebra]. Bishū Tōhekidō.

Acknowledgements

This research was partially supported by two Ministry of Education, Culture, Sports, Science & Technology Grants-in-Aid for Scientific Research (C): 2019-2024（19K03168 KAZAMA Hiroshi）and 2023-2026（23K02786 KAZAMA Hiroshi）

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 8
(Day 3)

TSG8-08

SURVIVING MASTERY IN MATHEMATICS EDUCATION

Nikki Mann
Simon Fraser University, Canada, nikkim@sfu.ca

Sheena Tan
Simon Fraser University, Canada, smtan@sfu.ca

In mathematics education we are no stranger to the word "mastery" as we are constantly in pursuit of some form of mastery, for both teachers and students. The notion of mastery has been and continues to be implicitly embedded in our teaching and learning practices. We show how this has produced various forms of marginalization, particularly with respect to race and gender. Even as we, the authors, try to overcome or undo the practices of mastery, we find ourselves caught up in new forms of mastery. Thus, in this theoretical paper, we explore the entanglement of mastery in mathematics education and its implications; we then offer some speculations on ways to survive mastery.

Keywords: mastery, race, gender

Our interest in challenging the notion of mastery began with our reading of Julietta Singh's *Unthinking Mastery* (2017), an unconventional read for a mathematics education book club, but one that caused much reflection to occur. Within the first few pages, we were drawn in when she wrote, "mastery's obdurate presence necessarily affects how scholars within and beyond the postcolonial project envision their intellectual pursuits today" (p. 2). We became curious to explore how her ideas would play out in mathematics education which Singh did not discuss in her work.

As a student, one common narrative the second author heard in her classrooms was around boys being better at mathematics than girls. It also manifested in the form of surprised looks from teachers whenever she outperformed the boys. In retrospect, the second author was unaware of the implicit mastery at play at that time but was eager to negate the claim, which unfortunately led to the reproduction of mastery. For the first author, mastery showed up in a different form, one that many may perceive as being positive. As a South Asian female, it was presumed that she would excel in mathematics due to the underlying belief that all Asians do well in mathematics. Thus, even when struggling with mathematics, that struggle was never shown as it was a societal expectation that South Asians should attain mastery in mathematics easily. Thus, reflecting and realising the extent mastery is embedded in our lived experiences, we were both drawn to the idea of exploring how we could limit our desires for mastery and resist perpetuating some of the mastery practices inherent in the teaching and learning of mathematics.

In *The Mastery of Reason*, Valerie Walkerdine (1988) also questioned specifically if mathematics' mastery over the world was achievable and at what cost. She asked, "if mathematics provides a dream of an ordered and pure universe, rational and unsullied by the passions … what kind of pleasure is the pleasure afforded by mathematics?" (p. 189). A possible response to this question—which many mathematics educators would offer—would be: the pleasure of mastery. A bold statement, but what exactly does mastery entail?

Ways of Thinking about Mastery

Before we look at the relation of mastery to mathematics education, we will first reflect on the concept of mastery. One of the most common ways of thinking about mastery involves the idea of domination, which follows from the oldest meaning of a master as "someone who had bested an opponent or competitor, or someone who had achieved a level of competence at a particular skill to become a teacher of it" (Singh, 2017, p. 9). In other words, mastery implies that one has full control over something (human or non-human) and the right to impose one's perspectives on them, with colonisation being one of the most classic examples of mastery at play. Through colonisation, force was used to spread dominant race beliefs, and accepted through the power mastery holds. But over time, practices of mastery have shifted, operating less with respect to only one master (be it a single entity or a single collective) and about owning people but more with multitude of collectives owning land, ideas and values, while retaining certain key aspects of the original idea of mastery. Hence, it might be helpful to understand what these key aspects are without trying to define mastery as it will be "a gesture toward mastering" mastery (p. 12), which we would like to avoid in our attempt to unthink mastery. Mastery often results in some form of *separation* or *division by force*, which creates *practices of subordination* that *unfold and extend over time* (Singh, 2017). For example, in Hegel's notion of the master and slave dialectic, he narrates a story on two "consciousness" engaging in a life and death battle to prove their worth and to gain recognition from the other (Singh, 2017). Eventually, the winner gains control and power over the loser and gets recognised as the master by the loser, who thus becomes the slave. As a result of the battle, not only is the master split off from the slave, but the slave also gets mutilated in the sense of losing itself—its new purpose is to submit to and serve its master. This hierarchical relationship that is formed will continue to unfold across time, until it gets threatened.

Reflecting on Mastery in Mathematics Education

Mathematics is often positioned as a neutral and objective subject with strengths around its universality, certainty and abstractness. Because of these characteristics, mathematics has been and still is glorified by many (Skovsmose, 2023), perceived to possess eternal truths and power to solve all problems, while other subjects taking a subordinate role. As Walkerdine (1988) emphasises, "Mathematics has, for centuries, held this position as queen of the sciences" (p. 187). Moreover, Western-Eurocentric mathematics has been the dominant mathematics while mathematics from the East or the minority communities, including Indigenous mathematics has been suppressed, despite being valuable and useful and have been around for just as long, if not longer. Notably, in his work on ethnomathematics, D'Ambrosio (1985) appealed for us to "look at the history of mathematics in a broader context so as to incorporate in it other possible forms of mathematics" (p. 44). Hence, it is important and necessary for us to challenge this mastery view of (Western) mathematics and see mathematics as a "human construction" that can be and should be "reformed, remoulded and reconstructed" (Skovsmose, 2023, p. 149).

Unfortunately, the mastery view of mathematics has ripple effects on mathematics education, where mathematics has been popularised as a core and necessary subject to be taught in schools, again through Western-Eurocentric forms of teaching. In addition, because of the decontextualized and generalisable nature of mathematics, learners are usually deemed to have "mastered" a mathematical concept when they can abstract the concept from its material and physical contexts and also perform the perceived right way of solving a problem and articulate the ideal structure of reasoning (typically deductive, propositional). Mastery-based expectations have inevitably led to many inclu-

sion issues in our mathematics classrooms as well. In particular, learners are separated into groups where those who are able to adhere to and perform these practices are considered to be more advanced while those who are unable to are considered to be slower.

Moreover, these hierarchies are produced in such a way that align with race, gender and socio-economic status, with marginalised students more often streamed into less valuable courses. Hierarchies found in society based on one's intersectionality being carried forward into the mathematics classroom. In particular, Martin (2009) introduces the racial hierarchy of mathematics ability, which shows how there is a preconceived notion of which races have a better understanding of mathematics and thus will master the complexities of the mathematics classroom. Martin (2009; Martin et al., 2010) argues that the racial hierarchy of mathematics ability is such that Asian and White students will be found at the top of the hierarchy and Indigenous, Black, and Latino populations will be found on the bottom. The consequence for this hierarchical view of mathematics ability is that Indigenous, Black, and Latino populations will perform poorly in higher-level mathematics classes, and hence, become streamed into classes that include easier mathematics concepts (Martin, 2009; Martin et al., 2017). Asian and White populations are assumed to excel in mathematics, positioned as ideal mathematics students whose traits others should emulate.

One may argue that this may be beneficial for the Asian population as they are placed on the higher end of the hierarchy, able to attain mastery more easily, but this placement is also very strategic to uphold mainstream power dynamics. Ernest (1991) writes, "when the power structure of society is physically threatened, force is likely to be brought into play to maintain it" (p. 248). Considering Ernest's words, we believe the perceived hierarchy of mathematics ability is a force used to safeguard white ideologies in mathematics, or forms of mastery. Placing Asian, a racial minority, on the top legitimizes the ideals of the model minority myth, a term used to describe how Asian American Pacific Islanders have successfully adapted to mainstream White American culture, thus not facing any adversity due to race, and consequently not requiring any corrective action which may be required by other races (Banks, 2012). Negating the fact that mathematics may in fact be a white institutional space which becomes safeguarded using the concept of mastery.

Race is not alone in terms of equity in mathematics. Despite evidence suggesting no significant differences between boys' and girls' mathematics performance, there is still the persistent myth about gender differences in mathematics achievements and behaviour where girls' success is often attributed to hard work and rote-learning while boys' playfulness and rule-breaking behaviour are read as intelligence (Robinson-Cimpian et al., 2014). These misconceptions seem to be a result of practices of domination over the female bodies and a perception that "overt challenges to the teachers' authority" (Walkerdine, 1998, p. 209) are indications of brilliance. In fact, as Porter and colleagues (2024) indicated, "teachers' expressed intellectual humility may reduce the gender gap in comfort expressing intellectual humility in school" (p. 15).

Surviving Mastery in Mathematics Education

Although it might be unavoidable that some ways of overcoming mastery involve new forms of mastery themselves, Singh (2017) suggested vulnerability as an approach where "in failure— and critically, in recognizing, reading, and becoming vulnerable to failure—we participate in new emergences, new possibilities for nonmasterful relations" (pp. 174–175). We chose the word surviving for a few reasons. Firstly, given the persistence of mastery in our practices in mathematics education and its entanglement with our lived experiences, it might not be possible to entirely negate

mastery. Thus, we want to simultaneously find ways to live with mastery and to loosen its hold over our assumptions and actions. Also, surviving carries with it the sense that it will not be easy, with many challenges and failures but continuing to maintain hope and finding comfort in discomfort while unthinking mastery. Perhaps the first thing we need to do to survive mastery is to "learn to read for it" (Singh, 2017, p. 7) as we cannot displace the logic of mastery or suggest practical alternatives if we lack the ability to identify it and awareness of how it is produced.

As mathematics researchers and educators, it might involve starting with a critical engagement with our own assumptions of what could be defined as mathematics, what could the goals of mathematics education be, and how could mathematics be learnt. What if the universality and certainty of mathematics is contested? What if the mathematics covered in our curriculum included mathematics from other communities? What if multiple solutions and diverse ways of reasoning were valued and emphasised? But at the same time, we need to be mindful not to perpetuate mastery itself while trying to undo mastery by "producing new masterful subjects" (Singh, 2017, p. 2). This leads us back to where we began with Walkerdine (1988) question of "if mathematics provides a dream of an ordered and pure universe, rational and unsullied by the passions … what kind of pleasure is the pleasure afforded by mathematics?" (p. 189). Perhaps by rethinking the mastery of mathematics, the dream mathematics can offer us is not that of an ordered and certain universe, but, as Whitehead insists, as ultimately contingent on assumptions, only ever providing partial disclosures. Perhaps the pleasure of mathematics can be found through unthinking mastery of mathematics. In other words, there can be truths in mathematics but not necessarily eternal truths and perhaps the route to achieving those truths are not formed on one single person's vision of mastery where exclusion of certain races and gender may occur. Or perhaps, the pleasure lies in speculating what an impure mathematics and universe might look like, one sullied by passions?

References

Banks, J.A. (2012). *Encyclopedia of diversity in education*. SAGE Publications, Inc.

Berry, R. Q., III. (2008). Access to upper-level mathematics: The stories of successful African American middle school boys. *Journal for Research in Mathematics Education, 39*(5), 464–488.

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics, 5*(1), 44–48.

Ernest, P. (1991). *The philosophy of mathematics education / Paul Ernest*. Falmer Press.

Martin, D. B. (2009). Researching race in mathematics education. *Teachers College Record, 111*(2), 295–338.

Martin, D. B., Gholson, M. L., & Leonard, J. (2010). Mathematics as gatekeeper: Power and privilege in the production of knowledge. *Journal of Urban Mathematics Education, 3*(2), 12–24.

Martin, D. B., Rousseau Anderson, C., & Shah, N. (2017). Race and mathematics education. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 607–636). Reston, VA: National Council of Teachers of Mathematics.

Porter, T., Leary, M. R., & Cimpian, A. (2024). Teachers' intellectual humility benefits adolescents' interest and learning. *Developmental Psychology*.

Robinson-Cimpian, J. P., Lubienski, S. T., Ganley, C. M., Copur-Gencturk, Y., & Eccles, J. S. (2014). Teachers' perceptions of students' mathematics proficiency may exacerbate early gender gaps in achievement. *Developmental Psychology, 50*(4), 1262–1281.

Singh, J. (2017). *Unthinking mastery: Humanism and decolonial entanglements*. Duke University Press.

Skovsmose, O. (2023). *Critical Mathematics Education*. Springer Cham.

Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality*. Routledge.

TSG8-09

SECONDARY SCHOOL STUDENTS' LEVEL OF INVENTIVE THINKING IN MATHEMATICAL LEARNING

Salbiah Mohamad Hasim
Institute of Teacher Education Raja Melewar, Malaysia, salbiah.hasim@moe.gov.my

Roslinda Rosli
The National University of Malaysia, Malaysia, roslinda@ukm.edu.my

In the modern global economy driven by information, technology, and innovation, 21st-century skills are those required of the future worker, such as inventive thinking that comprises six sub-constructs: flexibility, self-regulation, curiosity, creativity, and risk-taking, as well as higher-order thinking. Thus, this study aims to examine the level of 48 secondary school students' inventive thinking based on gender in mathematical learning using a questionnaire which consists of 40 items. Results showed that the level of inventive thinking for self-regulation, curiosity and creativity is high. In contrast, flexibility, risk-taking and higher-order thinking are moderate. Also, the results revealed no statistically significant difference in students' inventive thinking based on gender. This research enhances the worldwide understanding of the influence of gender on creative cognition in educational contexts. Identifying the disparities in creative problem-solving abilities among students of different genders might guide worldwide educational frameworks and policies to promote inclusion and fairness. Such insights may prepare future leaders and innovators globally with the many views essential for addressing intricate global concerns.

Keywords: Inventive thinking, mathematics learning, secondary school students, 21st-century skills, gender.

Introduction

In the 21st century, inventive thinking is one of students' most important cognitive skills when learning mathematics (Peterson & Cohen, 2019). Inventive thinking includes six sub-constructs: flexibility, self-regulation, curiosity, creativity, and risk-taking, as well as higher-order thinking (North Central Regional Educational Laboratory [NCREL], 2002). Although inventive thinking has been integrated, its implementation is insufficient. For instance, most students cannot practice the higher-order thinking skills they have acquired in the classroom (Hidayat, 2021). In addition, many workers in the industry still have less critical thinking skills (Jerome et al., 2017). Past studies also reported no significant differences in inventive thinking skills based on gender (Hasbullah & Mahamod, 2017; Turiman et al., 2020). However, gender differences in mathematical achievements have recently shown a statistical gap becoming smaller and unpredictable over time (World Economic Forum, 2020). Also, the results might vary by location, and 95 per cent of the gender gap is closed globally, with 37 countries already at the same level. Studies on gender differences are often carried out given reports of the shortage of science and mathematics graduates in the global job market, the lack of women participating in the STEM field, and the issue of educational equality (Reilly et al., 2019). Hence, the current study is needed to examine the difference in inventive thinking between male and female students.

Literature Review

According to NCREL (2002), flexibility is the capacity to change thoughts, attitudes, and behaviours in completing a goal within a given amount of time and resources. Self-regulation entails setting objectives, making methods to reach them, managing time, and evaluating results autonomously. Curiosity is the desire to know, which is essential for students to be lifelong learners. Creativity involves thinking about new ideas, solutions, and products. Risk-taking means putting something valuable at risk. Higher-order thinking is a creative problem-solving process that produces educated, sound, thoughtful judgments and conclusions. Students should build these six sub-constructs to prepare them for unpredictable, tough employment and life conditions. Studies on inventive thinking are widely carried out in science, engineering languages and STEM. These studies found that it is important for students to develop these six sub-constructs so that they can deal with complex circumstances on the job as well as in life in this globalized and technologically advanced world (Hasbullah & Mahamod, 2017; Turiman et al., 2020). Nevertheless, studies are still limited to mathematics learning, with few published data on inventive thinking among school students.

Methods

The present study utilized a survey to examine six sub-constructs of inventive thinking among 17 male and 31 female secondary school students. We administered the Inventive Thinking Skills instrument (Arsad, 2011), which has 40 items that cover six sub-constructs of inventive thinking and used a five-point Likert scale (5 for strongly disagree, 4 for disagree, 3 for neutral, 2 for agree, 1 for highly agree). Sample items are 'I am not confident in solving the given problem', 'I like to play, but I am disciplined and do not give up easily' and 'I always come up with new ideas in learning mathematics'. The instrument was assessed for face and content validity by experts. For reliability, we examined the students' responses utilizing Cronbach's Alpha with a 0.938 coefficient value. Using Statistical Package for the Social Sciences (SPSS), the questionnaire data were analyzed through descriptive and inferential statistics.

Results

The results showed high levels of four inventive thinking sub-constructs except for flexibility (M= 3.6424, SD=0.5304), Risk-taking (M=3.4435, SD=0.5280), and higher-order thinking (M=3.6562, SD=0.5224), which revealed mediocrity. Self-regulation has the highest mean (M=4.0231, SD= 0.4786), whereas Risk-taking has the lowest mean (M=3.4435, SD= 0.5280). These findings suggested that mathematics students had a high degree of inventive thinking, which allowed them to think critically and creatively about the problems that needed to be solved so that they could come up with original ideas or solutions. They do an in-depth analysis of the problem, make use of the resources at their disposal, provide suggestions or potential solutions, and ultimately choose the strategy that will be most effective in resolving the issue. High levels of cognitive processing are required for these activities, so students should be given real-life situations to practice exercising their cognitive abilities. This result is consistent with several studies that report that overall inventive thinking skills are satisfactory. The application of inventive thinking skills positively impacts teaching in the classroom (Turiman et al., 2020).

Descriptive statistics showed that male students had higher scores for each sub-construct in inventive thinking than female students (see Table 1). This may be due to several factors. Many mathematical problem-solving activities require students to investigate and use technology-based tools.

Based on recent studies, demonstrating discipline in the STEM field tends to show male students as more creative and self-reliant. According to the larger male variability hypothesis, the results showed that the performance of male students was more variable than female students in most countries. In surveys, males reported having more positive attitudes towards mathematical and scientific subjects, but females reported having lower self-efficacy views (Reilly et al., 2019). The study discovered gender disparities in particular skills alone. Female students take longer to learn than male students (Reilly et al., 2019).

Table 1. Mean and standard deviation for sub-constructs of inventive thinking

	Male		Female	
Subconstruct	Mean	Standard Deviation	Mean	Standard Deviation
Flexibility	3.95	0.471	3.47	0.499
Self-regulation	4.07	0.597	4.00	0.408
Curiosity	3.80	0.641	3.66	0.460
Creativity	3.84	0.639	3.68	0.458
Risk-taking	3.71	0.484	3.29	0.493
Higher-order thinking	3.81	0.472	3.57	0.537

However, the result of the independent t-test showed there was no significant difference in the mean score between the male group (M=3.8765, SD=0.47602) and female group (M=3.6379, SD= 0.36985), with $t(46) = 1.928$, p =.060. The results revealed that both groups had equal inventive thinking skills. Whether male or female, students are all in the same age group. Cognitive processes are similar among younger people, and the brains of younger people tend to light up comparably. According to Nemrawi et al. (2022), the reason there are no significant differences in gender is that all the participants had the same amount of physical development, mental maturity, and previous experience in statistical skills. The results align with those published by other studies (Ramey, 2015), which discovered no gender differences in statistical reasoning ability.

Conclusion

The general degree of inventive thinking among mathematics students was excellent. Except for flexibility, risk-taking, and higher-order thinking, which indicated unsatisfactory results, all sub-constructs of inventive thinking were at a high level. By determining the level of inventiveness among students, policymakers and educators could better devise strategies for inculcating and fostering the required skills. There were no significant gender variations in the degree of creative thinking. We may infer that the pupils' levels of creative thinking were uniform across the board and did not differ depending on gender. Research on gender differences has shown interactions between cognitive abilities and demographic characteristics, including parents' socioeconomic background, age, ethnicity, and educational level. These interactions can lead to exaggerating or underestimating the size of gender differences in some studies.

References

Arsad, M. N. (2011). Penyepaduan kemahiran abad ke-21 dalam pengajaran dan pembelajaran Biologi. Unpublished Master's thesis, Universiti Kebangsaan Malaysia

Hasbullah, F. & Mahamod, Z. (2017). Penerapan pemikiran inventif dalam pembelajaran bahasa Melayu murid sekolah rendah. *Prosiding Seminar Pendidikan Serantau Ke-Viii 2017*, 756–766.

Hidayat, F.A. (2021). Students' creative thinking profile as high-order thinking in the Improvement of mathematics learning. *European Journal of Educational Research* 10(4): 1907–18.

Nemrawi, A.M., N.A. Baioumy, S.A. Fouad, & Salah. (2022). Statistical thinking levels of students in Al-Ghad International College for Applied Medical Sciences in Saudi Arabia and its relationship to gender. *International Journal of Academic Research in Progressive Education and Development* 1(2): 398–418.

North Central Regional Educational Laboratory (NCREL). 2002. 53 *EnGauge:21st Century Skills. Digital Literacies for a Digital Age*. NCREL.

Peterson, E. G., & Cohen, J. (2019). A case for domain-specific curiosity in mathematics. *Educational Psychology Review* 31(4): 807–32.

Ramey, J. M. (2015). Differences in statistical reasoning abilities through behavioral-cognitive combinations of videos and formative assessments in undergraduate statistics courses. East Tennessee State University. https://dc.etsu.edu/etd/2494.

Reilly, D., Neumann, D. L. & Andrews, G. (2019). Investigating gender differences in mathematics and science: Results from the 2011 mathematics and science survey trends. *Research in Science Education* 49(1): 25–50.

Turiman, P., Osman, K., & Tengku Wook, T.S.M. (2020). Inventive thinking 21st century skills among preparatory course science students. *Asia Pacific Journal of Educators and Education*, 35(2), 145–170.

TSG8-10

DIVERSE VALUES AND MATHEMATICAL THINKING FOSTERED THROUGH SOCIALLY OPEN-ENDED PROBLEMS: SIMPLIFICATION OF SOCIAL CONTEXTS

Yuichiro Hattori
Okayama University, Japan, hattori.yuichiro@okayama-u.ac.jp

Naoya Kouda
Kagaminami Elementary School, Japan, kouda908708@gmail.com

This study aims to examine the impact of simplifying social contexts in socially open-ended problems on children's mathematical thinking and value expression. An experimental lesson was conducted with third-grade elementary school students in Japan using the socially open-ended problem, "Number of Couches Required." The study analyzed the values expressed by the students and the corresponding mathematical reasoning they employed.The findings indicate that even when a socially open-ended problem is presented with a simplified social context, students independently establish assumptions and engage in mathematical reasoning. Furthermore, the simplification of the social context allowed for greater flexibility in lesson time, enabling the introduction of additional problems through modifications of the given conditions. As a result, students were able to present diverse socio-mathematical solutions and actively share various perspectives with their peers.

Keywords: socially open-ended problem, social context, value, mathematical thinking

Introduction

In Japan, research on fostering children's mathematical thinking through mathematically open-ended problems has been extensively conducted (e.g., Hino, 2007). Alongside this, recent studies on mathematical modeling—which applies mathematics to real-world problems—and socio-critical modeling—which critically examines mathematics in societal contexts—have gained attention (Abassian et al., 2020). Socially open-ended problems, emphasizing children's social values, have also increasingly attracted interest, and practical research continues to accumulate (Baba, 2009; González & Chitmun, 2017; Baba & Shimada, 2019).

A representative socially open-ended problem, the "Matoate," has revealed children's diverse values, such as kindness toward younger students, a value reported as uniquely Japanese (Baba & Shimada, 2019; Shimada, 2017). Japanese educational culture emphasizes care for others, represented by "wa" (harmony) (Benedict, 1946). The "Matoate" incorporates strong social contexts, including school festival activities, participants being first graders, and point-based rewards, clearly expressing children's values (Shimada, 2017).

However, implementing such problems can require significant class time for students to understand them, limiting time for mathematical thinking. Teachers also face challenges deciding how much real-world context to include. Simplified socially open-ended problems, if effective in eliciting chil-

dren's diverse values and mathematical thinking, could ease the teachers' burden in lesson preparation and allow exploration of multiple meaningful solutions.

This study examines how simplifying social contexts in socially open-ended problems affects children's mathematical thinking and values expression. Using the "Number of couches required" problem (Hattori, Fukuda & Baba, 2021), we analyze children's expressed values and their mathematical thinking.

The Socially Open-Ended Problem: "Number of Couches Required "

Socially open-ended problems are defined as "problems that aim to cultivate social judgment through mathematical thinking and allow for mathematically and socially diverse solutions" (Baba, 2009, p. 52). They explicitly highlight students' social values and promote diverse mathematical thinking and solutions in the classroom. Shimada and Baba (2022) introduced the problem "How many chairs are needed?", which is structured as follows:

In the school auditorium, chairs are arranged to seat a class of 36 students. Each chair seats five students. How many chairs are needed?

The social context in this problem is embedded in the setting of a school auditorium used for school events. This setting was chosen because students are familiar with the space, understand its typical usage in school events, and have a concrete sense of its dimensions. Such considerations enable students to propose a range of socially and mathematically diverse solutions based on their values.

In contrast, the socially open-ended problem "Number of couches required", used in the experimental lessons of this study, shares the same fundamental mathematical structure as the problem introduced by Shimada and Baba (2022) and was previously presented in Hattori et al. (2021). The problem statement is simple:

There are 30 children. How many couches, each seating up to four people, should be available?

This means that no specific location, such as a gymnasium or classroom, was designated, nor was the grade level of the children explicitly specified.

Classroom Implementation and Analysis

The lesson was conducted in January 2025 at a public elementary school in Japan. The criteria for selecting participants were based on students in the third grade who were learning "division with remainders." As there was only one third-grade class at the school, all 17 students in that class participated in the study. Regarding the data collection procedures, a video camera was set up at the back of the classroom to record both video and audio during the lesson. For the data analysis process, students' worksheets were collected after the lesson, and both the worksheets and the video recordings were closely analyzed.

First, we provide an overview of the lesson. The teacher presented the problem and instructed the students to first write their ideas on a worksheet. With the exception of one student[3], all students used either the calculation $30 \div 4 = 7$ remainder 2 or $4 \times 8 = 32$ to determine their answers. The teacher then called on several students and confirmed that $30 \div 4 = 7$ remainder 2 led to an answer of 8 benches. The teacher also illustrated the arrangement on the blackboard and confirmed with the students that only two students would sit on the last couch. Following this, the teacher asked, "Are there any other ways to arrange the seating?" In response, one student asked, "Do we have to seat

exactly four people on each couch?" The teacher replied, "That's up to you," and encouraged the students to consider alternative solutions. Additionally, they were instructed to describe the advantages of their seating arrangements.

Here are some representative responses recorded on the students' worksheets. Student B wrote, "If five people sit on one couch, we need six couches," and used a diagram to illustrate their reasoning. They also wrote, "It's a tight fit, though?" Similarly, Student J arrived at the same conclusion and noted the advantage, "It reduces the number of couches needed." This reflects an efficiency-oriented value, where students, assuming they are small-sized elementary school children, adjusted the standard four-person seating to five people per couch to minimize the total number of couches required. Since the problem's social context was simplified, the students were not given an explicit age for those sitting on the couches. In the lesson, they naturally assumed that the people sitting were their own third-grade classmates, effectively setting the conditions themselves. Student C responded with "30 ÷ 1 = 30, so 30 couches," explaining, "Each person gets their own couch. It's spacious." Student G shared a similar idea, stating, "Each person can relax on their own couch." These responses reflect a comfort-oriented value, where individual space is prioritized. Additionally, Student C also recorded another response: "30 ÷ 2 = 15, so 15 couches," explaining, "Two people can sit together and chat." This suggests an emphasis on social interaction, where students value the opportunity for communication and engagement with peers. Student H proposed "If three people sit on each couch, then 30 ÷ 3 = 10, so 10 couches," adding, "It's more spacious." This response also aligns with the comfort-oriented value, prioritizing a balance between space and seating arrangements. Furthermore, Student K used a diagram to illustrate their reasoning (Figure 1). This response follows the calculation 4 × 6 + 3 × 2 = 30, meaning that six couches seat four students each, while two couches seat three students each, resulting in a total of eight couches. Since the worksheet did not explicitly state the reasoning behind this choice, the teacher conducted a post-lesson interview with the student. During the interview, the student explained, "Rather than seating 4, 4, 4, 4, 4, 4, 4, and 2, I thought that creating groups of three would make the difference between the groups of four and the other students smaller." This response reflects a consideration of balancing group sizes, demonstrating an awareness of fairness in seating distribution.

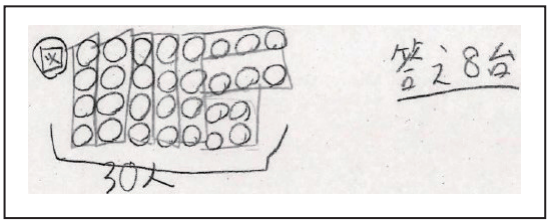


Figure 1. Student K's worksheet response

The teacher had the students present their ideas recorded on the worksheets and shared them with the entire class. Following this, the teacher asked, "What kind of seating arrangement would be best during the COVID-19 pandemic?" One student responded, "30 ÷ 1 would be best because it would reduce the risk of spreading illness." In the latter part of the lesson, the students were asked to consider how many couches would be needed if the problem statement specified 29 children instead of 30 (Brown & Walter, 2005). While many students answered 29 ÷ 4 = 7 remainder 1, leading to a total of 8 couches, some students provided alternative responses. Student E responded: "I wanted to use a number smaller than four so that everyone could sit in equal groups, but since numbers small-

er than four do not divide evenly, it's not possible. So, I used 29 ÷ 3 = 9 remainder 2, which gives an answer of 10 couches." This response demonstrates a concern for fairness, as Student E explicitly aimed to ensure that all students sat in equal groups. This aligns with the findings of Baba & Shimada (2019), where students similarly expressed a fairness-oriented value. Student H responded: "29 ÷ 2 = 14 remainder 1, so the answer is 15 benches. The good thing about this is that two people can talk to each other, and when someone isn't feeling well, they are less likely to spread their illness to others." This response reflects a dual consideration of social interaction (the benefit of conversing with a partner) and health awareness (maintaining physical distance to prevent illness transmission). Student H demonstrated an ability to integrate multiple social perspectives into their mathematical reasoning.

Conclusion

In this study, we are not asserting that a social context is unnecessary for students to reach mathematical solutions based on their values. Rather, we aimed to investigate whether students' values would still emerge even when the social context was simplified. As a result, we found that even with a minimized context, students were able to independently create their own social contexts. Moreover, the interaction between students and the teacher played a crucial role in enriching the simplified context with social dimensions, enabling the sharing of diverse values. Furthermore, simplifying the social context allowed for more flexibility in lesson time, making it possible to introduce additional variations of the problem by modifying its conditions. These variations also elicited a range of socially and mathematically diverse responses from students, facilitating the sharing of different perspectives among peers. Of course, it remains uncertain whether the social context can be simplified for all socially open-ended problems while still maintaining their effectiveness. Therefore, further accumulation of practical research is necessary to explore the broader applicability of this approach.

References

Abassian, A., Safi, F., Bush, S., & Bostic, J. (2020). Five different perspectives on mathematical modelling in mathematics education. *Investigations in Mathematics Learning, 12*(1), 53–65.

Baba, T. (2009). Analysis of socially open-ended problems in mathematics education from the perspective of values. *Journal of JASME Research in Mathematics Education, 15*(2), 51-57. (in Japanese)

Baba, T., & Shimada, I. (2019). Socially open-ended problems for enriching student learning with mathematical models and social values. In P. Clarkson, W. T. Seah, J. Pang (Eds.), *Values and valuing in mathematics education: Scanning and scoping the territory* (pp. 171-183), Cham, Switzerland: Springer Nature Switzerland AG.

Becker, J. P., & Shimada, S. (Eds.) (1997). *The open-ended approach: A new proposal for teaching mathematics*. Reston, VA: NCTM.

Benedict, R. (1946). *The chrysanthemum and the sword; patterns of Japanese culture*. Houghton Mifflin.

Brown, S. I., & Walter, M. I. (2005). *The art of problem posing (3rd edition)*, Lawrence Erlbaum Associates.

González, O., & Chitmun, S. (2017). Using sport-related socially open-ended problems to coordinate decision-making, variability modeling and value awareness. In T. Dooley, & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education (CERME10)* (pp. 780-787), Dublin, Ireland: DCU Institute of Education and ERME.

Hattori, Y., Fukuda, H. & Baba, T. (2021). Development of Socio-critically Open-ended Problems for Critical Mathematical Literacy: A Japanese Case, *Journal of Educational Research in Mathematics,31*(3),357-378.

Hino, K. (2007). Toward the problem-centered classroom: Trends in mathematical problem solving in Japan. *ZDM Mathematics Education, 39*(5-6), 503-514.

Shimada, I.(2017). *Sansu sugaku kyouiku to tayou na kachikan: Syakaiteki open-end na mondai ni yoru torikumi (Arithmetic and mathematics education and diverse values: Approaches through socially open-ended problems)*, Toyokan Publishing. (in Japanese)

Shimada, I. & Baba, T. (2022). *Tayouna kachikan ya sugakutekina mikata kangaekata wo migaku sansu jugyo no open-end approach (An Open-Ended Approach to Mathematics Lessons for Cultivating Diverse Values and Mathematical Perspectives and Thinking)*, Meijitosho.

Acknowledgements

This work is supported by KAKENHI (24K00420).

TSG8-11

DIGITAL MATHEMATICS STORYTELLING IN INDONESIA, VIETNAM, AND THE ASIAN DIASPORA: EQUITY, CULTURE, AND COMMUNITY THROUGH NARRATIVE IDENTITIES

Theodore Chao
California State University, Fullerton, USA, thchao@fullerton.edu

Angga Hidayat
The Ohio State University, USA, hidayat.8@osu.edu

Ruth Oliwe
The Ohio State University, USA, oliwe.1@osu.edu

Fuyi Feng
The Ohio State University, USA, feng.1225@osu.edu

Suhendra
Universitas Pendidikan Indonesia, Indonesia, suhendra@upi.edu

Kha Pham
Stemhouse, Vietnam, khapham@stemhouse.edu.vn

Ho-Chieh Lin
Qing-Shui Elementary School, Taiwan (ROC), linhochieh@gmail.com

This paper explores the use of Digital Mathematics Storytelling (DMST) as a transformative pedagogical approach to addressing equity and culture in mathematics education in Indonesia, Vietnam, and broader East Asian contexts. Through community storycircles focused on counter-storytelling, DMST engages students and teachers in creating digital stories that surface community-based mathematical knowledge, counter dominant narratives, and foster identity development. Drawing on our studies in Indonesia, Vietnam, and Asian American communities in the USA, this paper highlights how DMST disrupts Eurocentric mathematics education by recognizing culturally sustaining knowledge and practices. DMST provides a unique pathway for fostering equity by amplifying marginalized voices, facilitating counter-storytelling, and integrating mathematical learning with cultural knowledge.

Keywords: Digital Mathematics Storytelling, Narrative Identity, Counter Storytelling, Indonesia, Vietnam

Mathematics education in East Asian countries has long been influenced by global educational discourses that prioritizes standardized achievement and decontextualized mathematics instruction. While many East Asian nations perform well in international assessments, such as PISA and TIMSS, the emphasis on procedural fluency frequently marginalizes the rich mathematical knowl-

edge embedded in local cultures and everyday practices. Furthermore, issues of equity arise as marginalized communities—whether due to ethnicity, socioeconomic status, or geographic location—often lack access to pedagogical approaches that affirm their cultural identities. Digital Mathematics Storytelling (DMST), a method centering upon the creative act of storytelling to explore one's own identity, can serve as a powerful creator of counter-narrative to these dominant (and colonizing) modes of teaching and learning, offering an approach that centers student voice, cultural knowledge, and digital media literacy.

Therefore, the objective of this paper involved examining how Digital Mathematics Storytelling (DMST) engages students and teachers in culturally embedded mathematical knowledge, constructing narrative identities, and challenging dominant narratives in mathematics. Specifically, we asked: (1) How does DMST support culturally relevant and sustaining pedagogy in mathematics? (2) How does the digital modality empower marginalized voices in narrating mathematical experiences?

Theoretical Framework

Our draws upon three case studies of Digital Mathematics Storytelling workshops with students and teachers grounded in four interconnected theoretical lenses:

1. Culturally Sustaining Pedagogy (CSP): DMST provides a structure for participants to preserve and evolve their cultural knowledge by embedding mathematical learning within local traditions and lived realities (Paris & Alim, 2017).
2. Counter-Storytelling: By enabling students to challenge dominant narratives (e.g., that mathematics is culture-free), DMST serves as a vehicle for marginalized voices to share mathematics through critical, lived experiences (Solorzano & Yosso, 2001).
3. Narrative Identity: Participants craft stories that reflect and construct their identities, positioning mathematics as personally meaningful rather than abstract and imposed (Sfard & Prusak, 2005).
4. Funds of Knowledge: DMST draws on home- and community-based knowledge systems, situating mathematics within everyday cultural practices (Moll et al., 1992).

Together, these theoretical frameworks guide our use of DMST as a pedagogical and methodological tool for fostering a more inclusive, humanizing mathematics education that prioritizes equity, cultural, and community, while also aligning identity formation with critical digital media production.

Methodology

We used Narrative Inquiry (Clandinin & Connelly, 2000) and Counter-Storytelling (Solorzano & Yosso, 2001) as our main research methodologies, positioning storytelling as a tool for identity construction and resistance against dominant narratives. Narrative Inquiry allows us to explore how teachers and students construct their mathematical identities through personal and community-based stories. Counter-Storytelling, grounded in Critical Race Theory (Ladson-Billings & Tate, 2006), challenges mainstream deficit perspectives and offers a space for marginalized voices to redefine their experiences in mathematics education. Our data sources included storycircle transcripts, digital video artifacts, participant reflections, interviews, and feedback from public screenings. Analysis was conducted using constant comparative and narrative coding methods to trace themes related to identity, resistance, and cultural knowledge.

Case Studies and Findings

Case 1: Indonesia – Teacher Identity and Cultural Mathematics

A week-long DMST workshop in Bandung engaged in-service and pre-service teachers in storytelling around community-based mathematics. Stories evolved from daily practices (e.g., investing, skincare routines) to counter-narratives exploring grief, Quranic numeracy, and professional identity. Post-interviews revealed a shift from procedural to relational teaching. One teacher stated, "I now see math in the emotion of loss," emphasizing the identity-shaping impact. The public screening spurred powerful dialogues around identity, affirming CSP and narrative identity in practice.

Case 2: Vietnam – Data Literacy and Justice through Youth Stories

In Ho Chi Minh City, middle schoolers created digital stories over four weeks. One group analyzed tax burdens across neighborhoods, another documented gender norms through peer surveys. Pre/post reflections showed enhanced agency and digital fluency: "I used to think math was boring; now I use it to speak about fairness," a student wrote. DMST amplified youth voices as critical storytellers, linking Counter-Storytelling to equity and civic awareness.

Case 3: Asian American Diaspora – Challenging Stereotypes

In the U.S., Asian American children (ages 8–12) used DMST to question the 'model minority' myth. Initial stories celebrated cultural numeracy (e.g., Lunar New Year, martial arts patterns), but later shifted to address microaggressions, grief, and identity tension during COVID-19. One child narrated, "People think I'm a math robot—I'm more." DMST enabled identity reclamation and critiqued stereotype-driven positioning, directly reflecting Narrative Identity and Counter-Storytelling frameworks.

Findings and Implications

Across all three enactments detailed above, we found the following findings:

1. Mathematics as Counter-Cultural Practice: Participants recognized and valued mathematical knowledge inherent in their communities, challenging the assumption that mathematics exists solely within school-based curricula.
2. Counter-Storytelling as Narrative Identity: Students and teachers used digital storytelling to frame their mathematical journeys, reinforcing the idea that identity is constructed through storytelling.
3. Critical Digital Media Literacy in Math Education: Engaging with digital tools fostered critical awareness of media, positioning students as creators rather than passive consumers of digital content.

Conclusion and Future Directions

Our work highlights the transformative potential of Digital Mathematics Storytelling in fostering culturally sustaining mathematics education in East Asian and Asian diasporic contexts. Beyond simply offering a new research direction, DMST provides a vision of hope—hope that mathematics education can be freed from the shackles of colonization, that it can be a space where students see themselves reflected in the knowledge they construct. By centering students' lived experiences and

amplifying their voices, using DMST connects to broader movements that reclaiming mathematics education as an emancipatory practice.

Through this work, we position Digital Mathematics Storytelling (DMST) as more than an instructional strategy—it is a transformative practice that affirms identity, promotes cultural equity, and repositions students and teachers as co-constructors of mathematical meaning. By integrating culturally sustaining pedagogy with digital media, DMST bridges formal mathematics with lived realities. Our cases demonstrate that DMST challenges dominant educational structures by centering counter-narratives and affirming marginalized voices in mathematics learning.

Mathematics has long been positioned as an objective and neutral field, yet its history is deeply intertwined with power, colonial legacies, and systems of exclusion. Through DMST, we challenge these structures and advocate for a mathematics education rooted in justice, humanity, and cultural relevance. The stories that emerge from these projects demonstrate that mathematics is not just a set of abstract procedures but a dynamic, lived experience tied to identity, history, and agency.

Through this and future work, we take small but significant steps toward a more just, inclusive, and liberatory vision of mathematics education—one where every student has the right to see themselves as a mathematical thinker, storyteller, and change-maker. As we continue to fight for equity in mathematics education, we offer DMST as a concrete tool that connects personal storytelling, digital creativity, and mathematical thinking in service of justice.

References

Clandinin, D. J., & Connelly, F. M. (2000). *Narrative inquiry: Experience and story in qualitative research*. Jossey-Bass.

Ladson-Billings, G., & Tate, W. F. (2006). Toward a critical race theory of education. In A. D. Dixson & C. K. Rousseau (Eds.), *Critical race theory in education: All God's children got a song* (pp. 11–30). Routledge.

Moll, L. C., Amanti, C., Neff, D., & Gonzalez, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory into Practice*, *31*(2), 132–141.

Paris, D., & Alim, H. S. (2017). *Culturally sustaining pedagogies: Teaching and learning for justice in a changing world*. Teachers College Press.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14–22.

Solorzano, D. G., & Yosso, T. J. (2001). Critical race and LatCrit theory and method: Counter-storytelling. *International Journal of Qualitative Studies in Education*, *14*(4), 471–495. https://doi.org/10.1080/09518390110063365

TSG8-12

CULTIVATING CRITICAL MATHEMATICS CONSCIOUSNESS IN PRE-SERVICE TEACHERS THROUGH DIGITAL RESOURCES AND THE RWC MODEL

Artorn Nokkaew
Faculty of Education, Naresuan University, Thailand, art.nokkaew@gmail.com

Suparat Chuechote
Faculty of Education, Naresuan University, Thailand, chuechote@gmail.com

Critical Mathematics Consciousness (CMC) is essential for pre-service mathematics teachers to understand the ethical, communicative, and sociopolitical roles of mathematics. Traditional instruction often limits exposure to real-world issues, whereas digital platforms like websites and YouTube offer rich opportunities for critical engagement. This study integrates digital resources into teacher preparation through the Read-Write-Change (RWC) model to foster CMC. In the Read phase, students analyze sociopolitical issues through digital content to explore how mathematics relates to real-world contexts. In the Write phase, they apply mathematical reasoning to challenge bias and justify fairness using models and statistics. In the Change phase, they use mathematics to address misinformation and advocate for equity. Qualitative analysis of reflective journals revealed that this process supported the development of CMC across all three domains, with ethical awareness emerging most strongly, followed by sociopolitical and communicative awareness.

Keywords: Critical Mathematics Consciousness, Digital resources, Pre-service mathematics teachers, Read-Write-Change

Introduction

Mathematics is often viewed as objective and neutral, yet it is socially, culturally, and politically embedded (Gutstein, 2006). Traditional methods tend to prioritize "dominant mathematics," reinforcing passive learning and overlooking equity and cultural relevance (Gutierrez, 2002; Bukko & Liu, 2021). In contrast, social justice mathematics promotes both technical proficiency and critical awareness. Gutstein (2006) highlights the dual goal of "reading and writing the world with mathematics," enabling students to analyze and act on social injustices.

Pre-service teachers (PSTs) often lack exposure to mathematics as a sociopolitical tool (Myers, 2017). Research has shown that approaches like ethnomathematics (Mania & Alam, 2021) and culturally relevant teaching (Olawale, 2025) can shift these perceptions. Critical Mathematics Consciousness (CMC) has emerged as a vital framework for pre-service mathematics teachers, equipping them with the capacity to understand and address the ethical, communicative, and sociopolitical dimensions inherent in the discipline. In the digital age, tools like YouTube provide access to diverse mathematical narratives (Kellner & Kim, 2010), yet remain underused in teacher training. This study aims to explore the following research question: How does the integration of digital resources within a critical pedagogy RWC influence the development of Critical Mathematics Consciousness (CMC) among pre-service mathematics teachers?

Theoretical Framework: Critical Mathematics Education and RWC

Freire's (1974) critical pedagogy underpins transformative math education by emphasizing dialogue and problem-posing (Freire, 2005). Critical Mathematics Education moves beyond procedural fluency to empower students to challenge injustices, critique economic structures, and engage in ethical mathematical reasoning.

Critical Mathematics Consciousness (CMC), rooted in Freire's critical consciousness, requires not just awareness but active engagement. Kokka (2020) describes CMC as comprising sociopolitical understanding, civic empathy, and agency. Register et al. (2021) further categorize it into three domains: Ethical, Sociopolitical, and Communicative Mathematics Awareness.

The RWC model, based on Gutstein (2006) and extended by Nokkaew and Chuechote (2022), incorporates these domains into a cycle of critical reflection and action:

- **Read:** Examine sociopolitical issues via digital media.
- **Write:** Use mathematics to critique and propose fair solutions.
- **Change:** Share findings through media to promote justice.

Research Methodology

Participants included 62 preservice mathematics teachers (PSTs) in a 15-week university pedagogy course. While proficient in mathematical content, they had limited CMC exposure.

Data was collected from reflective journals developed through the RWC processes. Content and thematic analysis focused on how digital tools supported ethical, sociopolitical, and communicative awareness (Register et al., 2021).

Results

This study investigated how integrating digital resources within the Read-Write-Change (RWC) model fostered Critical Mathematics Consciousness (CMC) among PSTs. The findings were structured around the three RWC phases.

In the Read phase, PSTs used digital media such as YouTube to explore sociocultural mathematical practices—highlighting differences between community-based methods and formal mathematics. In the Write phase, they applied tools like GeoGebra and excel to propose more equitable solutions. In the Change phase, PSTs communicated their analyses and alternatives through videos, infographics, and blog posts, aiming to promote fairness and clarity to a target audience. A summary of how PSTs integrate digital resources in RWC is shown in Table 1.

Table 1. RWC model with digital integration

Phase	Mathematical inquiry	Digital Media Integration	Driving Question
Read (R)	Analyze sociopolitical issues through mathematical lenses, such as wage calculations, pricing strategies and data representation in culture.	Take multiple perspectives of mathematics in cultures from digital media across websites and YouTube.	1. Is it reasonable? 2. Is it truly fair? 3. Who benefits, and who is at a disadvantage?
Write (W)	Construct and communicate mathematical justifications for fairness, using statistics, mathematical modeling, or probability to expose biases.	Develop alternative explanations, models and representations to explain mathematical concepts to clarify fairness/unfairness.	What would be a more just method compared to the current system?
Change (C)	Take actions by clarifying misinformation and advocating for justice using mathematical arguments.	Create YouTube videos, infographics, or blog posts to reveal unfairness and propose alternative ones with awareness of recipient.	1. Do we want to create change? 2. How can we make a difference? 3. How should we communicate with stakeholder?

Figure 1 illustrates one case: a YouTube video used to criticize a local wage calculation method (Read), the creation of alternative models via GeoGebra (Write), and a video comment sharing corrected methods to the villager (Change).

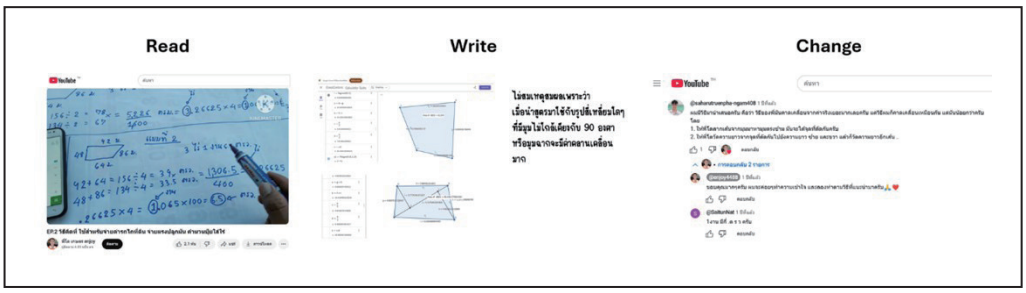


Figure 1. Employing digital resources in RWC pedagogy

Analysis of reflective journals showed that Ethical Mathematics Awareness (E) was most frequently demonstrated (55 out of 62 PSTs), followed by Sociopolitical (P) (35) and Communicative (C) (32). As shown in Figure 2, 27% of PSTs (17 individuals) demonstrated awareness across all three CMC dimensions. Ethical awareness was notably interconnected with both communicative and sociopolitical domains, while the link between the latter two was less evident.

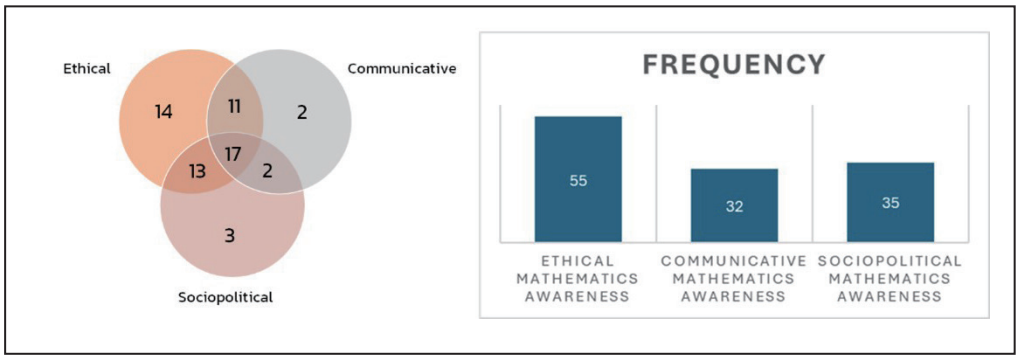


Figure 2. Frequency of awareness in 3 CMC aspects from PSTs' reflective journals

Discussion and Conclusion

The results indicate that the RWC model, supported by digital resources, provided PSTs with authentic opportunities to explore mathematics as a sociopolitical tool. Online platforms such as YouTube serve as accessible and potentially non-hierarchical sources of knowledge, empowering individuals to engage with real-world mathematical problems that illuminate social and economic issues (Kellner & Kim, 2010). These alternative applications of mathematics has the potential to cultivate a more informed citizenry, fostering a critical stance towards the subject (Olawale, 2025). By examining real-world contexts and sharing insights through accessible media, PSTs developed broader perspectives beyond conventional classroom mathematics.

Ethical awareness emerged most strongly, as PSTs recognized the impact of mathematical applications on community well-being and the need to promote accessible, accurate methods. Sociopolitical and communicative awareness followed, highlighting the importance of historical context, usability, and clarity in mathematical communication.

These findings align with Register et al. (2021), who emphasize the role of ethical, sociopolitical, and communicative reasoning in fostering CMC. Despite this progress, the uneven development across domains suggests a need for more balanced and sustained support in mathematics teacher education.

In conclusion, digital integration through RWC offers a promising pathway for cultivating critical mathematics consciousness. Yet, designing meaningful and reflective tasks remains essential for transforming mathematics education into a socially responsive discipline.

References

Bukko, D., & Liu, K. (2021, March). Developing preservice teachers' equity consciousness and equity literacy. *Frontiers in Education.* 6, 1-11.

Freire, P. (2005). *Education for critical consciousness*. New York, NY: Continuum.

Gutierrez, R. (2002). Enabling the Practice of Mathematics Teachers in Context: Toward a New Equity Research Agenda. Mathematical Thinking and Learning, 4(2–3), 145–187. https://doi.org/10.1207/S15327833MTL04023_4

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. Routledge.

Kokka, K. (2020) Social Justice Pedagogy for Whom? Developing Privileged Students' Critical Mathematics Consciousness. *The Urban Review*, 52, 778–803. https://doi.org/10.1007/s11256-020-00578-8

Kellner, D., & Kim, G. (2010). YouTube, critical pedagogy, and media activism. *The Review of Education, Pedagogy, and Cultural Studies*, 32:1, 3-36, DOI:10.1080/10714410903482658

Mania, S. & Alam, S. (2021). Teachers' perception toward the use of ethnomathematics approach in teaching math. International Journal of Education in Mathematics, Science, and Technology, 9(2), 282-298.

Myers, M. (2017). Identifying critical topics for teaching mathematics for social justice in K–5 settings: Connections and tensions. In Proceedings of the thirty-ninth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (pp. 921-924). Hoosier Association of Mathematics Teacher Educators.

Nokkaew, A. & Chuechote, S. (2022). Guideline for Conducting Classroom Activities Integrated with Local Wisdom and Critical Mathematics Pedagogy to Develop Six Core Competencies. *Journal of Education Khon Kaen University*. 45(2), 69-81.

Olawale, B. E. (2025). Equity and social justice in mathematics teacher preparation: Diving into the nitty-gritty. In B. E. Olawale (Ed.), Building the Foundations: Effective Approaches in Mathematics Teacher Preparation (pp. 1-21).

Register, J., Stephan, M., & Pugalee, D. (2021). Ethical reasoning in mathematics: New directions for didactics in US mathematics education. *Mathematics*, 9(8), 799.

TSG8-13

ETHICAL FOUNDATIONS FOR EXPLORING INCLUSIVE SCIENCE AND MATHEMATICS EDUCATION

Mitsuru Matsushima
Kagawa University, Japan, matsushima.mitsuru@kagawa-u.ac.jp

Kazuya Kageyama
Hiroshima University, Japan, kkageya@hiroshima-u.ac.jp

This paper aims to clarify ethical and conceptual foundations in inclusive science and mathematics education, and to identify its elements that develop research and practice. Inclusion itself is a research field that arose from the awareness of social justice ethics. The discussion in this paper begins by considering what it means to be human through disability. As a result, we adopted a political/relational model of disability as the conceptual foundation. In addition, we identified four types of care as elements for developing research and practice: the developmental care, the cognitive care, the epistemological care, and the communal care. We concluded that care ethics is more suitable than traditional ethics as the ethical foundation for developing these elements. We then presented the conceptual foundation, ethical foundation, and elements for developing research and practice of inclusive science and mathematics education as a two-layer model.

Keywords: inclusive science and mathematics education, political/relational model, ethics, care, two-layer model

Introduction

In recent years, the number of studies on special needs education and inclusive education has been increasing in science and mathematics education research all over the world. At the 15th International Congress on Mathematical Education held in Sydney in 2024, Mathematics education for students with special learning needs was set up as a topic study group, and many papers were presented and discussed there. Research on social justice will be positioned as the ethical background of these studies. Research on social justice aims to treat everyone fairly and equitably (Ernest, 2007). In school education, where the diversity of students and school environments is increasing, it is urgent to realize inclusive science and mathematics education that can treat all students fairly and equitably. However, even when reviewing recent research on inclusion, many ideas and concrete examples have been proposed, but not much has been proposed about its foundations and components (e.g., Kollosche et al., 2019). Therefore, the purpose of this paper is to propose the foundations and components of inclusive science and mathematics education (ISME).

Conceptual Foundations of ISME

What causes the difficulty for all students in understanding science and mathematics? This question leads to what it means to be human. When considering what we are as humans, the following three models emerge that capture human beings themselves from the perspective of disability (Lambert,

2019): the individual model of disability, the social model of disability, and the political/relational model of disability. Broadly speaking, these are perspectives on whether the cause of difficulty in understanding science and mathematics is due to the individual learner or to something other than the learner.

When it is believed that the difficulty in understanding science or mathematics is due to the individual learner, the individual model of disability is applied, and medical and neurological research is applied to investigate and treat the causes. From an educational perspective, it is necessary to correct misconceptions about the concepts within the field of science and mathematics. Such misconceptions lead to conceptual change research, a field of study that considers the difficulties in understanding posed by science and mathematics themselves. This epistemological care of the subject will be important. In addition, when looking at learning on a time axis, the developmental care of considering what learning experiences and life experiences individual learners have accumulated before learning, and what existing knowledge and skills they have is also important. And during learning, the cognitive care of what cognitive methods are easy for individual learners to understand will also be important. In the social model of disability, the occurrence of disabilities is considered to be due to the community. In this case, the communal care of how science and mathematics learning are related to the learning community becomes important. This communal care is undertaken by teachers and learners. The teacher's care is to enhance learning activity for all students. The learners inspired by the care provided by the teacher are to foster a caring classroom culture. The political/relational model of disability is a model that arose from the differences between the majority and minority in society, which sometimes involves power (Lambert, 2019). This model argues that the balance of power, human relationships, and invisible cultural beliefs (Lambert, 2019). The social model of disability arose as an antithesis to the individual model, but it has been pointed out that relying solely on the social model of disability creates a dualism regarding the occurrence of disability and hides the physical and medical difficulties of actual disability. Therefore, the political/relational model of disability argues that it is important to consider disability itself while balancing the individual and social models, rather than relying on either one. We propose that the conceptual foundation of inclusive education is the political/relational model of disability. This is because human beings live in a relational web of others and that their identity is strongly influenced by culture and politics. This conceptual foundation is also a philosophical stance that considers what it means to be human from the perspective of disability.

Components of ISME

What factors advance ISME research and practice? It is difficult to present all the important elements for learners to understand science and mathematics, but in a political/relational model, the following four cares can be mentioned: *the developmental care* for the learner before learning, *the cognitive care* for the learner, *the epistemological care* regarding the difficulties inherent in the subject, and *the communal care* that community supports learning.

The developmental care based on developmental psychology may help students understand science and mathematics. Research findings such as those showing the relationship between dexterity and counting, as well as between finger recognition and comparing numbers (Asakawa & Sugimura, 2022), may help identify students who need support in the classroom at an early stage. Providing cognitive care to learners may help them understand science and mathematics. Each individual has their own unique cognitive method, and care that focuses on their uniqueness will be inclusively effective. For example, there is the selection of sequential and simultaneous processing styles in

special needs education research (Fujita, 2019) and research on embodied cognition that makes full use of multimodal stimuli (Abrahamson & Bakker, 2016). Focusing on the difficulty in understanding science and mathematics concepts themselves may help students understand. In mathematics education research, the difficulty in understanding mathematics concepts themselves has been studied as an epistemological obstacle. An epistemological obstacle is a difficulty that arises when previously learned knowledge or skills cause learning difficulties or resistance to new learning, and it is said that this difficulty is not caused by the individual learner, nor is it caused by the instruction (Mizoguchi, 1995). If epistemological care is provided based on such epistemological obstacles, it becomes possible to incorporate mathematical activities that overcome the epistemological obstacles into lesson design in advance. Providing the communal care for the learning community may help students understand science and mathematics. Learning in a caring community allows learners with different perspectives to care for each other and take others' ideas into account (Nishi, 2023). ISME can be thought of as being divided into two layers: the conceptual foundation and its research and practice. The conceptual foundation is the foundation of ISME's philosophy, centered on a fundamental consideration of what it means to be human. The four cares are positioned as elements that develop ISME's research and practice. These two layers influence each other (Kageyama and Matsushima, 2024).

Ethical Foundations of ISME

What are the ethical foundations of ISME? As mentioned above, the inclusive perspective is born out of social justice ethics, which are traditional ethics including Kantian ethics, utilitarianism and rational values (Held, 2015). In mathematics education, the focus is on reading and rewriting the world critically with the goal of learning mathematics for all learners (Gutstein, 2006). In contrast to the traditional ethics of justice, which places emphasis on rights, rules, abstraction, and universalizability, Gilligan (1982) proposed new ethics of care that places emphasis on responsibility to others, relationships, context, and individuality. ISME was enacted to seek better education, so it can be said that the conceptual foundation of ISME is supported by social justice ethics. On the other hand, the elements of ISME are four types of cares. In other words, rather than aiming for the general phenomenon of better education, it can be said that the method and content of ISME aim for the enrichment of individual education using individual relationships inside and outside the classroom. Therefore, it is the care ethics that supports ISME research and practice.

Care ethics was enacted as a challenge to social justice ethics. It is the significance of seeking universality. It is extremely important to seek an ideal environment in which all people are treated equally, but such ideal assumptions inevitably result in people falling outside the scope of those assumptions (Held, 2015). Therefore, care ethics was enacted as a philosophy that is attentive to each individual. This choice was made to ensure that people are not left behind when new goals or methods are introduced and to include all learners. Care is divided into five aspects: Caring about, which is paying attention to what needs are needed; Caring for, which is taking responsibility and consideration to ensure that those needs are met; Care giving, which is actually providing care; Care receiving, which seeks further improvements based on the reactions of the care recipient; and Caring with, which is examining these processes with the members of the community (Tronto, 2013). The last aspect, Caring with, is the very image of democracy in which all members of the community participate. Discussing caring with others and acting on caring creates trust and solidarity in the community. In this way, the experience of expanding solidarity and trust through consensus building based on care ethics leads to ISME for all learners, and this experience is also thought to be connected to the well-being of all human beings.

Considering this, one might think that care ethics is more important than social justice ethics, but this is not the case. Both are important, and it is difficult to strike a balance between them. Issues related to social justice ethics, such as current global environmental issues and war issues, are becoming increasingly complex. To solve these problems, solidarity and trust between us humans are important. At the primary and secondary education stage, it is possible to first place emphasis on care ethics and gradually introduce social justice ethics. The implementation of ISME is the key to our human well-being.

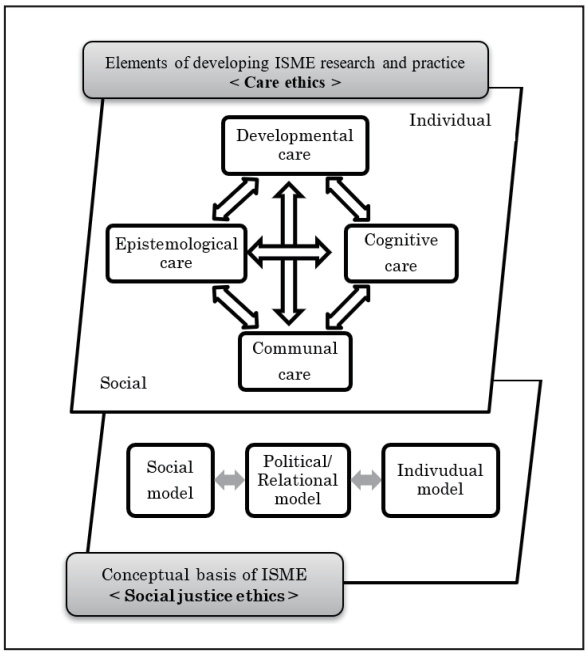


Figure 1. ISME's ethical and conceptual foundations and components

References

Abrahamson, D., & Bakker, A. (2016). Making sense of movement in embodied design for mathematics learning. *Cognitive Research: Principles and Implications*, pp.1-33.

Asakawa, A. & Sugimura, S. (2022). Mediating process between fine motor skills, finger gnosis, and calculation abilities in preschool children. *Acta Psychologica 231*, 103771, pp.1-10.

Ernest, P. (2007). Why social justice? *Philosophy of Mathematics Education Journal*, 21, 1-5.

Fujita, K. (2019). *Two types of learning: "sequential processing" and "simultaneous processing"*. Toshobunka. (in Japanese)

Gilligan, C. (1982). *In a different voice: Psychological theory and women's development*. Harvard University Press.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. Routledge.

Held, V. (2015). Care and justice, still. In Engster, D. & Hamington, M. (Eds.), *Care ethics & political theory* (pp.19-36). Oxford University Press.

Kageyama, K., & Matsushima, M. (2024). Conceptualization of inclusive science and mathematics education: Its foundations and components. *JASME annual conference 61 presentation material*.

Kollosche, D., Marcone, R., & Knigge, M. et al. (2019). *Inclusive mathematics education; State-of-art research from Brazil and Germany*. Springer.

Lambert, R. (2019). Political, relational, and complexity embodied; experienced disability in the mathematics classroom. *ZDM mathematics education*, 51, 279-289. doi.org/10.1007/s11858-019-01031-1

Mizoguchi, T. (1995). Featuring the process of overcoming epistemological obstacle by categories of description: A case of the notion of limit. *Research Journal of Mathematical Education*, 63/64, 27-48. (in Japanese)

Nishi, S. (2023). The effect of relationships on mathematics in small groups. *Research in Mathematics Education*, 29(1), 1-15. (in Japanese)

Tronto, J. (2013). *Caring democracy: Markets, equality and justice*. New York University Press.

Acknowledgement

This research was supported by JSPS KAKENHI Grant Numbers 22K13708 and 22H01010.

TSG8-14

NEW POSSIBILITIES FOR PROBLEM FINDING THROUGH MATHEMATICAL MODEL AND CARING

Rintaro, UEDA
The United Graduate School of Education, Tokyo Gakugei University, Japan,
r246002w@st.u-gakugei.ac.jp

The present study aims to construct a framework for learning guidance that promotes citizenship through the selection, use, and interpretation of mathematical models, as well as participation in decision-making and consensus-building for social issues. The focus is on the identification of care needs in caring to help learners build relationships with others. This paper demonstrates that the selection, use, and interpretation of mathematical models and the identification of care needs can be done together. To this end, the present study adopted design research as a methodological framework and set learning goals in task design. Using the transition to a joint enrollment system for both genders in Tokyo Metropolitan high school entrance exams as a case study, the present study analyzed the competencies derived from the perspective of how the selection, use, and interpretation of mathematical models contribute to problem findings. The analysis yielded insights into the potential of mathematical models to not only identify individual care needs but also those of the entire group. This analysis consequently led to the formulation of specific learning goals for the task.

Keywords: democracy, care, caring, decision-making, consensus building

Introduction

In a well-functioning democracy, it is essential that all citizens participate in decision making and consensus building on social issues. In mathematics education, social issues have been addressed in the context of expanding mathematical modeling and citizenship education (Abassian et al., 2020; Barbosa, 2006; Geiger et al., 2023). On the other hand, these studies do not necessarily aim at proposing ameliorative measures for social problems or at reaching consensus building based on diverse decisions. Therefore, this study aims to construct a framework for teaching and learning that fosters citizenship that participates in decision-making and consensus building for social problems through the selection, use, and interpretation of mathematical models. On the other hand, in mathematical Science Education that emphasizes learners' values and deals with decision-making and consensus building for social problems through the selection, use, and interpretation of mathematical models, it has been pointed out that the more the issue is a personal matter for the learner, the more differences in values make consensus building difficult (Matsushima et al., 2025). This is because learners do not have sufficient competence to consider the perspectives of others and become fixated on their own values. Therefore, the difficulty of consensus building in previous research is that learners are unable to build relationships with others in finding social problems.

In order to overcome this difficulty, this study focuses on the relationship between the concept of caring and mathematical models, demonstrating the aspects of problem finding in which the selection, use, and interpretation of mathematical models and caring are carried out as a single entity. Caring is the process of identifying diverse care needs, assigning responsibility for meeting those

needs, providing and receiving care, and coordinating which responsibilities are assigned (Tronto, 2015a). From a caring perspective, problem finding in social problems can be interpreted as identifying care needs, and identifying diverse care needs can help overcome the difficulty of consensus building due to adherence to one's values. On the other hand, it is often difficult to identify multiple care needs. Therefore, this study aims to design a task in which the selection, use, and interpretation of a mathematical model and the identification of care needs are performed as a single entity. The purpose of this paper is to argue that the selection, use, and interpretation of a mathematical model and the identification of care needs can be carried out as a single entity.

Methodology

In order to achieve the purpose, this study adopts the methodological framework of design research (Bakker, 2018). Design research focuses on a specific learning content or activity, and aims to develop theories about that learning and its teaching and learning support methods. The design research process typically consists of setting learning goals, designing tasks and teaching practices, and analyzing the practices. In this study, I focus on setting learning goals in this process. In other words, I develop tasks that can be carried out in connection with the selection, use, and interpretation of the identifying for the care needs and the mathematical models, and then I set learning goals for these tasks.

First, under the identifying of care needs, I will review the care process and the ethical elements of that process (Tronto, 2015a; Tronto, 2015b) and theoretically consider the relationship with mathematical modeling competencies (Maass, 2006; Maass et al., 2023) to derive competency rooted in mathematics under the identifying of care needs. Second, I will take the transition from a quota system for boys and girls to a quota system for both genders in the Tokyo Metropolitan high school Entrance Examination as a social issue that requires decision-making and consensus building, and consider how a mathematical model can contribute to problem finding. Third, I will analyze the assumed aspects from the perspective of the derived competency and show that the identification of care needs and the selection, use, and interpretation of mathematical models can be carried out as a single entity, and set the learning goals for the task. Moreover, due to restrictions regarding the available space, the practical application and analysis of the tasks developed will be presented in a separate paper.

Mathematical- Scientific Caring Competencies

This chapter derives competencies rooted in mathematics from the identifying of care needs through theoretical considerations of the relationship between care processes and ethical elements (Tronto, 2015a; Tronto, 2015b) and mathematical modeling competencies (Maass, 2006; Maass et al., 2023).

The Care Process and Ethical Elements

The caring process is a process that aims to identify care needs, take responsibility for resolving those care needs, provide care, interpret whether or not those care needs have been met, and is consistent with a democratic commitment to justice, equality, and freedom for all. The process of identifying care needs is defined as "caring about" (Tronto, 2015a). This process involves not only recognizing that care is needed, but also paying attention to the existence of care needs and valuing them as something that should be met. The ethical elements that correspond to this process is "attentiveness". Although "attentiveness" is something that tries to resist the temptation to ignore or

reject others, we often tend to focus only on ourselves. "Attentiveness" is the competency to "caring about" and can be interpreted as a competency that should be fostered. Therefore, by focusing on mathematics, I will attempt to derive a competency that counteracts the ignorance and rejecting of others and the concern only for oneself that attentiveness points to.

Care Needs Assessment and Mathematical Modelling Skills

Focusing on mathematical modelling competency, I derive the mathematical competency of "caring about". The identification of the care needs corresponds to the process of understanding the problem from a real-world situation and creating a situation model, and simplifying from the situation model to the real model (Maass, 2006) in the mathematical modelling competencies. On the other hand, in mathematical modelling this process takes place in reality, so mathematics cannot help to resist the temptations mentioned above. Therefore, I will focus on the development of mathematical modelling from the perspective of citizenship education in recent years. This points to the importance of competencies based on real-world data, such as gathering information and analyzing sources, as well as simplification, when creating a real model from a situation model (Maass et al., 2023). Therefore, in this study, I will develop competencies that mathematically assess care needs using data and identify new care needs based on the assessment. In this way, I call the competencies for carrying out the care process with a focus on mathematics the mathematical caring competencies and define its sub-competency "caring about" as the identification of unmet care needs in a given society through the selection, use and interpretation of mathematical models using data.

Task Design

In this chapter I will take social issues that require decision making and consensus building and look at the aspects of problem finding that learners can do.

Social Issues that Require Decision-Making and Consensus Building

As a social issue that requires decision-making and consensus building, I will take up the transition from a quota system for boys and girls to a quota system for both genders in the Tokyo Metropolitan high school Entrance Examination. In 2021, it was considered a problem due to the quota system for boys and girls in the Tokyo Metropolitan high school Entrance Examination, even if they had the same score in the entrance examination, boys would pass and girls would fail. The Tokyo Metropolitan Board of Education (hereinafter referred to as the Board) announced its view that if the results of the 2021 entrance examination were determined based on a quota system for both genders, 691 girls and 95 boys would have passed. Therefore, the Board considered establishing a quota system to gradually move from a quota system for each gender to a quota system for both genders and to determine whether or not to admit students based on their exam results regardless of their gender. In this consideration, the Board presented an estimate that the number of girls admitted would increase by approximately 600 and the number of boys admitted would decrease by approximately 600 as a result of the change to a joint quota system for both genders. On the basis of this estimate, the quota for the 2022 admissions was gradually increased to 10% of the quota, the quota for the 2023 admissions was gradually increased to 20% of the quota, and the system was effectively changed to a joint quota system for both genders from the 2024 admissions.

The Aspects of Problem Finding that Learners Can Do

I will focus on the calculations presented by the Board, which show that the number of girls admitted will increase by approximately 600 and the number of boys admitted will decrease by approxi-

mately 600 from the 2021 entrance exam onwards, if a joint quota system for both genders is introduced, and I will examine under what conditions the above increases and decreases could occur. I make the following settings and assumptions.

Setting
C_M…Maximum number of boys, C_F…Maximum number of girls,
S_M…Number of successful boys, S_F…Number of Successful Girls
Assumptions
①The number of successful applicants is equal to the maximum number
②Of the k % of the quota, l % are passed by girls.

The number of successful applicants for each gender in a system with separate quotas for boys and girls is $S_{M1} = C_M$ for boys and $S_{F1} = C_F$ for girls. When the quota is set, the number of successful boys, S_{M2}, and the number of successful girls, S_{F2}, are calculated as follows: $S_{M2} = C_M \times (1 - \frac{k}{100}) + (C_M + C_F) \times \frac{k}{100} \times (1 - \frac{l}{100})$ $S_{F2} = C_F \times (1 - \frac{k}{100}) + (C_M + C_F) \times \frac{k}{100} \times \frac{l}{100}$. If $C_M = 10436$, $C_F = 10010$. I will find the conditions under which the increase in the number of successful girls is consistent with the Board's estimates.

Table 1. Trial estimation results by the board

Year	2021	2022	2023	2024
k	0	10	20	100
Number of girl applicants (persons)	10010	10234	10495	10606
Number of boy applicants (persons)	10436	10212	9951	9840
Number of additional girl applicants who passed (persons)		224	485	596

The case of a joint quota system for both genders is when k = 100, and if the change in the number of additional girls passing the exam in 2021 is represented by y, then $y = -C_F \times \frac{k}{100} + (C_M + C_F) \times \frac{k}{100} \times \frac{l}{100} = -10010 + (10436 + 10010) \times \frac{l}{100}$. If I take $\frac{l}{100}$ as the variable x, then x is the percentage of the number of girls who passed the exam out of the total number of people who took the exam. So I will look at how y changes as x changes. This is to consider the change in the function $y = 20446 \times x - 10010$. If the number of girls admitted to the school increases by 596 in a joint quota system for both genders, the percentage of the number of girls admitted to the school in the quota is calculated. This is to find the value of x that makes $y = 596$. when $y = 596$, $x = 51.873$. Since x is the percentage of girl applicants who passed the exam out of the total number of applicants, it is clear that girl applicants scored higher than boy applicants in the 2021 entrance exam for all Tokyo Metropolitan high schools. Moreover, since the Board's calculations are based on actual data from 2021, which shows the above situation, discussing the transition of the admissions system based on these calculations may confirm the current situation of reluctantly admitting girls with higher scores than boys. In other words, through the use and interpretation of functional models, it is possible to find a problem that requires social decision making and consensus building, such as "How can all genders of examinees be given equal opportunities under any system?"

Discussion

By analyzing the assumed problem finding process from the perspective of "caring about", I will show how the selection, use, and interpretation of mathematical models can be integrated with the identification of care needs and set the learning goals for this task. In this case, there was a girl (boy) who did not pass the entrance examination for a Tokyo Metropolitan high school even though she had a higher score than a boy (girl) who passed. From this, the individual care needs of the examinees, "who should be evaluated equally without regard to gender differences," were identified. Furthermore, the Board has estimated an increase of approximately 600 in the number of female applicants and a decrease of approximately 600 in the number of boys accepted from the 2021 entrance examination onwards. This adjustment is attributed to the transition to a joint quota system for both genders. This determination was made through the utilization and interpretation of a functional model employing current entrance examination data, which revealed that girls were attaining higher grades than boys. This analysis enabled us to identify not only the individual care needs of the examinees that were initially identified through the events, but also the collective care needs of all examinees. The latter finding underscores the principle that "all examinees should have equal examination opportunities." Through the use and interpretation of mathematical models, it was determined that the care needs of individual examinees, initially apparent in social problem situations, could be identified. This finding suggests that it is possible to identify the care needs of the entire group, which go beyond the care needs of individuals. Consequently, this teaching material contributes to the development of "caring about" in the mathematical caring competency. Therefore, the learning goal of this task can be set as identifying the care needs of all examinees by using and interpreting a function model. This will be done using past entrance examination data to meet the conditions of existing calculations for the Tokyo Metropolitan high school Entrance Examination.

References

Abassian, A. et al (2020). Five different perspectives on mathematical modeling in mathematics education, *Investigations in Mathematics Learning*, 12(1), 53-65.

Bakker, A. (2018). What is design research in education? In A. Bakker (Ed.), *Design Research in Education*, 3–22. Routledge.

Barbosa, J. C. (2006). Mathematical modelling in classroom. *ZDM Mathematics Education*, 38(3), 293-301.

Geiger, V., Gal, I., & Graven, M. (2023). The connections between citizenship education and mathematics education. *ZDM Mathematics Education*, 55(5), 923–940.

Geiger, V. Schmid, M. (2024). A critical turn in numeracy education and practice. *Frontiers in Education*, 9, 1-12.

Matsushima, M., Nishimura, K., Ueda, R. (2025). Citizenship developing through value-based consensus building in Mathematical Sciences Education. *Proceedings of the 27th ICMI study conference*, 339-346.

Maass, K. (2006). What are mondeling competencies? *ZDM Mathematics Education,* 38,2, 113–142.

Maass, K. et al. (2023). Analysing mathematical modelling tasks in light of citizenship education using the COVID-19 pandemic as a case study. *ZDM Mathematics Education,* 55, 133–145.

Rorty, R. (1989). *Contingency, Irony*, *and Solidarity*. Cambridge University Press

Skovsmose, O. (1998). Linking mathematics education and democracy: Citizenship, mathematical archaeology, mathemacy and deliberative interaction. *ZDM Mathematics Education*, 30(6), 195–203.

Skovsmose, O. (2023). *Critical Mathematics Education*. Springer.

Tronto, J. C. (2005). *Caring Democracy*, NYU Press.

Tronto, J. C. (2015a). *Who cares? How to reshape a Democratic Politics (Brown Democracy Medal)*, Cornel Selects.

Tronto, J. C. (2015b). *Moral boundaries*. Routledge.

MEMO

KOREA 2025

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 9:

The Essence of Mathematics Education in Undergraduate Level

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 9
(Day 2)

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

TSG9-01

ON THE HURDLES OF PROBLEM SOLVING IN UNDERGRADUATE MATHEMATICS DISCOURSE

Miriam Wallach
Technion, Israel, miriamw@technion.ac.il

Igor' Kontorovich
The University of Hong Kong, China, david1710@gmail.com

Undergraduate mathematics requires students not only to learn new ways to solve mathematical problems but also to communicate their solutions. In this theoretical paper, we use the commognitive framework to conceptualize problem solving as a discursive activity whose social metarules are inseparable from the situation and the community within which it unfolds. We distinguish between the processes students go through when generating a solution for themselves and the conventionalization of this solution to make it shareable with a broader community. This foregrounds a potential gap between metarules that the two communities can apply when considering the endorsability of a solution.

Keywords: Problem solving, commognitive framework, undergraduate mathematics education

The final report of the Higher Education Summit states that problem solving "is the most useful skill a student can take with them when they leave university" (Rowlett, 2011, p. 19). This draws attention to challenges students experience when learning to solve problems in undergraduate mathematics.

Problem solving includes linking tentative solutions with a problem situation and mobilizing appropriate resources (Selden & Selden, 2013). In a typical classroom, the instructor determines if a solution is acceptable, and this changes according to the level of mathematical rigor required (e.g., Weber, 2008). Thus, the acceptability of a solution is inseparable from the context in which it is considered, including the intended audience, the setting, and the socio-mathematical norms.

A problem solution can be communicated orally or in writing. The former often consists of informal mathematics, while writing usually summons formal symbolic statements (Fukawa-Connelly et al., 2017). The conversion process can be a challenge, when part of the difficulty stems from students' partial familiarity with the accepted standards of undergraduate mathematics (Selden & Selden, 2013).

Relevant insights on problem solving can be borrowed from prolific research on proof, as close relationships between the two have been acknowledged (e.g., Weber, 2008). Some studies are attentive to the social process of sanctioning an argument as a proof, which emphasizes the key role of communication and audience (e.g., Weber, 2008). For example, Mason et al. (1982) consider proving as convincing not only a friend but also an enemy. Research also highlights the contextuality of proof. Indeed, Goguen (2001) positions a proof as an event that occurs in a specific time and place.

We aim to introduce a discursive view on problem solving that accounts for the social context and highlights some implicit challenges students need to navigate when learning to solve problems in undergraduate mathematics. The view is grounded in the commognitive framework.

Commognitive Framework

Commognition is a socio-cultural framework that has been broadly used to study mathematics learning and teaching at the undergraduate level (e.g., Kontorovich et al., 2024). The framework construes learning as a process through which students become agentive participants in a target discourse community (Sfard, 2008). Commognition views undergraduate mathematics as a discourse with characteristic *metarules* that define participants' patterns of participation (Sfard, 2008).

Lavie et al. (2019) position *routines* as a basic unit of analysis. Routines are viewed as <*task*, *procedure*>-pairs, where the latter are executed to achieve the former. Routines are performed in *task situations*, that is, settings in which one considers themselves bound to act. Sfard (2023) differentiates between *algorithmic routines* that involve procedures with a well-defined sequence of steps that yield expected outcomes, and *heuristics*—non-deterministic procedures that "do not specify the basic steps in a unique way [but] increase chances for a successful performance" (p. 4).

Sfard (2023) maintains that routines constitute an interconnected network. One example of interconnections stems from routines coming in different sizes: from 'extra-large' to 'atomic,' whereas 'smaller' routines serve as building blocks for the 'larger' ones. Thus, routine networks are recursive and have "a fractal-like nature [that] finds its expression in the fact that when you repeatedly zoom into this network, you discover ever new, smaller routines" (Sfard, 2023, p. 5).

We are not the first to consider problem solving through a commognitive lens. Drawing on the framework, Koichu (2019) conceptualizes problem solving as "a socio-culturally shaped process of not-immediate achieving a chosen or imposed goal, in which the involved individuals enact, through private or public exploratory discourse, individual or shared resources that they interpreted, even for a short while, as appropriate for achieving the goal" (p. 7). Koichu's conceptualization builds on cognitive research as well, and he notes that "being a discursively oriented, [the conceptualization] is not fully compatible with the 'strong' discursive perspective" (p. 61). We aim for our conceptualization to be fully compatible.

Solution Generation and Solution Conventionalization

We contend that problem solving can be considered as a discursive activity that unfolds in a certain task situation where a solver(s) draws on networks of routines to produce a solution to the assigned problem. With "solution", we refer to "a public narrative, which becomes endorsed, by the problem solvers or the problem proposers, as the one that achieves the goal" (Koichu, 2019, p. 49). We distinguish between two goals, or 'tasks,' in commognitive terms. One is to generate a solution-narrative that the solvers consider as solving the problem. The other is to devise a narrative that the problem setters (e.g., a course instructor) will accept as a solution. That is, the final solution-narrative must adhere to the conventions of a target discourse community.

Our use "task situation" highlights the contextual circumstances of the pursued task (c.f., Goguen, 2001), since they shape the expectations from the solution-narrative. As experts of undergraduate mathematics discourse, course instructors are usually the ones to set up mathematical problems for their students. Consistently, the proposers and the solvers are not partaking in the same version of

undergraduate mathematics discourse, at least initially. This often results in qualitative differences in metarules that instructors and students use to endorse the solution-narrative. Students are often aware of this interdiscursive gap, which may explain the many variations of "Can I write that in an exam?" that occasionally emerge in mathematics classrooms.

We propose that, in problem-solving task situations, subjecting the solution-narrative to the metarules of a conventional discourse in which students are not yet fully versed can become a sub-task that students choose to pursue (cf. Koichu, 2019). We introduce the notions of *solution generation* and *solution conventionalization* to account for this phenomenon. With solution generation, we refer to routines students can implement to bring a solution-narrative into discursive existence. This can involve reformulating the problem, making a sketch, and many other variations of what Pólya and his followers termed as heuristic strategies (e.g., Koichu, 2019). Algorithmic routines, such as equation solving, can also serve generational purposes. The overall task of solution generation is to transition from an initial task situation where the solution-narrative is conspicuous in its absence to one where the solving students can consider the endorsability of some narrative.

It only makes sense to propose that the students will use the initially generated solution-narrative to develop the one to share with a broader community (e.g., submit to the course instructor). In some cases, this development can require minimal changes, if any. In other cases, the students may decide that the narrative needs to undergo some substantial transformations; for instance, because it is not sufficiently formal. With "solution conventionalization" we refer to the set of routines that students can implement to align with the metarules of conventional discourse, as they envisage them. The conventionalization routines may involve switching from the oral to the written medium, formalization, symbolization, restructuring, adding explanations, and removing redundancies (e.g., Kontorovich et al., 2024). These changes often enable the written solutions to 'stand-alone.'

It may appear that generation precedes conventionalization as outcomes of the former can be expected to serve as inputs to the latter. However, the cyclical nature of problem solving has been widely acknowledged (e.g., Koichu, 2019). In this way, solution conventionalization can reveal drawbacks in the initially endorsed solution-narrative. Furthermore, some routines can have both generational and conventionalization aspects. Thus, it may be more accurate to consider generation and conventionalization as affordances that a certain routine brings to the solution-narrative, rather than associating this routine with a particular type exclusively.

Concluding Remarks

While undergraduate mathematics learning and teaching have been studied broadly and widely, we argue that limited attention has been paid to students' problem-solving challenges. Using the commognitive framework, we construed problem solving as a discursive activity, where students need to produce a shareable solution-narrative that will be endorsed by both—them as solvers and a broader community of undergraduate mathematics discourse, including instructors as the discourse experts. Consistently, we distinguished between routines aimed at bringing a solution-narrative into existence and those aimed at aligning the narrative with conventional mathematical discourse.

Informed by Koichu (2019), our aim was to propose a conceptualization that is grounded in the discursive perspective on learning (Sfard, 2008). Operationally speaking, one key difference between Koichu's and our proposal regards the interdiscursive gap that we highlight between the community of solvers and a broader discourse community. As a result, the two communities are likely to subject solution-narratives to different metarules. One can argue that it is only natural to expect the

solvers to account for the viewpoint of others (cf. Mason et al., 1982). However, the heart of the challenge is that, as newcomers to the discourse of undergraduate mathematics, students are not yet familiar with the conventional version of metarules that their solution-narrative must abide by.

References

Fukawa-Connelly, T., Weber, K., & Mejía-Ramos, J. P. (2017). Informal content and student note-taking in advanced mathematics classes. *Journal for Research in Mathematics Education*, *48*(5), 567–579.

Goguen, J. (2001). "What is a proof", http://cseweb.ucsd.edu/~goguen/papers/proof.html

Koichu, B. (2019). A discursively oriented conceptualization of mathematical problem solving. In P. Felmer, P. Liljedahl, & B. Koichu (Eds.), *Problem solving in mathematics instruction and teacher professional development* (pp. 43–66). Springer International Publishing.

Kontorovich, I., Liu, N. Q., & Kang, S.-w. (2024). Transitioning to proof via writing scripts on the rules of a new discourse. *Educational Studies in Mathematics*, *117*, 143–162.

Lavie, I., Steiner, A., & Sfard, A. (2019). Routines we live by: From ritual to exploration. *Educational Studies in Mathematics*, *101*(2), 153–176.

Mason, J., Burton, L., & Stacey, K. (1982). *Thinking mathematically*. London: Addison Wesley.

Rowlett, P. (Ed.) (2011, January). *HE Mathematics Curriculum Summit*.

Selden, A., & Selden, J. (2013). Proof and problem solving at university level. *Mathematics Enthusiast*, *10*(1–2), 303–334.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press.

Sfard, A. (2023). The devil's finest trick: Routines that make teachers matter against their better judgement. *Journal of Curriculum Studies*, *55*(1), 21–36.

Weber, K. (2008). How mathematicians determine if an argument is a valid proof? *Journal for Research in Mathematics Education*, *39*(4), 431–459.

TSG9-02

THE ROLE OF COGNITIVE RESOURCES IN MATHEMATICS PROBLEM SOLVING

Xin Li
Suzhou University of Technology, China, xinli@amss.ac.cn

Ping Yu Yang
Suzhou University of Technology, China, pingyuyang0104@163.com

Yew Hoong Leong
Nanyang Technological University, National Institute of Education, Singapore, yewhoong.leong@nie.edu.sg

In this study, we used two non-routine mathematics problems to investigate the problem-solving ability level of three mathematics students in a university in China. We found that the high-achieving student was not successful at problem solving. It prompted us to examine more carefully his attempts at solving the two problems. We found that the use of a particular cognitive resource without suitable control measures can hinder problem solving.

Keywords: Mathematics problem solving, cognitive resources, control, high-achieving students

Much has been done in the area of mathematics problem solving. For example, Schoenfeld (1985) proposed four components for successful problem solving: Cognitive Resources, Heuristics, Control, and Belief Systems. But how these components interact with one another is under-studied. In this study reported here, we focus on the extent of (over-)use of particular cognitive resources in problem solving.

Problems

Problem 1 ('Nice' Number). A 'nice' number is a number that can be expressed as the sum of two or more consecutive positive integers. Determine which numbers from 50 to 70 inclusive are 'nice'.

Problem 2 (Princess Problem). There are nine rooms in a row with a door between every two adjoining rooms. Each room has also a door opening to a corridor. A princess lives in one room each day and moves to an adjoining room the next day at 7 in the morning. A prince wants to talk to the princess and knocks on one door each day at noon. If the princess is in that room, she will open the door and they can converse. Otherwise, the prince leaves and tries again at some door at the same time the next day. Is there a strategy to guarantee that the prince will meet the princess if he has 14 days to try?

The reason for choosing these two non-routine problems was that they offered different degrees of difficulty. This breadth would allow us to observe more broadly the range of problem-solving skills employed by the subjects. The Nice Number Problem is relatively easier and it likely involves the application of standard formulas and algebraic operations, which is similar to traditional mathemat-

ical problems that Chinese students are more familiar with. The Princess Problem, in contrast, is more challenging and it is to test if the subjects were able to use problem solving strategies beyond tried-and-tested procedures.

Method

Our study is based on the theoretical foundations of Pólya (1945) and Schoenfeld (1985, 2007), characterized by four sequential but back-flow components, Understanding the Problem (UP), Devise a Plan (DP), Carry out the Plan (CP), and Check and Expand, and a framework for the analysis of complex problem-solving behaviors consisting of Cognitive Resources, Heuristics, Control, and Belief Systems. Leong et al. (2019) provided a practical application of these concepts for effective classroom instruction in mathematics problem solving, and they further scaled up continual professional development for mathematics problem solving in Singapore schools (Leong, et al., 2021). Toh et al. (2011) pointed that the Pólya's model or the teaching of heuristics are not all there is when one teaches problem solving. There is a need to attend to other aspects of problem solving, such as the interaction between the problem-solving components.

Based on the above theoretical framework, we use the following steps to analyse the mathematics problem solving processes of three university students in China who were prospective mathematics teachers. These students were selected based on their mathematical scores in standard tests, one each from the "Strong", "Medium", and "Weak" bands. However, due to space constraints, we focus here on the problem-solving attempts of student A from the "Strong" band. Firstly, we recruited Student A who learned about Pólya's problem-solving model through studying the course "Introduction to Mathematics Education" in his sophomore year. The text used was by Zhang and Song (2023). The course covered theoretical aspects of problem solving without providing extended periods for students to actually attempt problems using the model. He was given 30 minutes to solve each problem. He recorded videos of his problem-solving process. While solving, he attempted to articulate his thinking process. Secondly, by repeatedly and meticulously watching the videos documenting the problem-solving process, we compared and analysed the specific problem-solving processes he adopted every five minutes. In addition, we interpreted his problem-solving moments through the theoretical lens of the Pólya's model and Schoenfeld's components. Thirdly, we compared his problem-solving processes across the two problems.

Data

By repeatedly observing the video, we recorded Student A'a entire problem-solving processes for both problems (see Table 1 for 'Nice' Numbers; Table 2 for the Princess Problem). Only the first 10 min of the data presentations and analyses are shown due to space constraints. In the case of the Princess Problem, Student A gave up after 10 min.

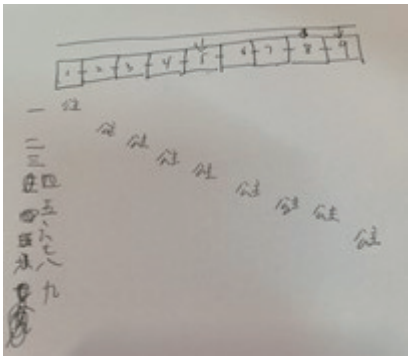
Table 1. Detailed mathematical steps and problem-solving moments of 'Nice' number problem

Time Cost	Detailed Mathematical Steps	Problem Solving Moments
0-5min	3=1+2 4 5=2+3 7=3+4 Odd number $'n'$ 50~70: $n = \frac{n+1}{2} + \frac{n-1}{2}$ Even numbers need to be considered separately: 6=1+2+3 8 $S_n = \frac{(First\ item + Last\ item) \times Item\ number}{2}$	1. Start with small numbers. (0'51") 2. Notice some odd numbers are 'nice'. (2'10") 3. Conjecture all odd numbers are 'nice'. (4'00") 4. Discover that the even number 6=1+2+3, so 6 is 'nice'. (4'56") 5. 'Can I find a general formula for all even numbers?' (5'08")
6-10min	$S_6 = \frac{(1+6) \times 6}{2}$ $S_{10} = \frac{(1+10) \times 10}{2} = 55$ $S_{11} = \frac{(1+11) \times 11}{2} = 66$ $S_{12} = 78$ Odd numbers between 50~70 and 66 $a_n = ?$ $an + b$ 1+2+3+4+5+6	6. Try expressing the numbers between 50 and 70 using the formula for the sum of the first *n* terms of an arithmetic sequence. (6'30") 7. Find even number 66 is 'nice'. (8'03") 8. 'The common difference of the arithmetic sequence can only be 1, but the first term can be different.' (8'45")

From Table 1, we find that during the problem-solving process, Student A quickly identified the use of the arithmetic sequence sum. He persisted in using the formula until the 24th minute, when he had no confidence in finding all the 'Nice' numbers within the given time. Reinterpreted through Schoenfeld's components, He used the Cognitive Resource, his familiar knowledge of the arithmetic sequence sum formula, as an initial main tool to attack the problem to find 'Nice' numbers and consistently employed this method while solving the problem. It might suggest that he had prior experience with similar problems where this formula had proven successful, leading him to rely on it as a tried-and-tested solution strategy. However, an interesting point is that he was extremely committed to this cognitive resource to the extent that he overlooked the possibility of approaching the problem from a different angle, such as exploring different ways of writing the same number as sum of consecutive integers. To be fair, he was making progress with the arithmetic sequence sum formula. He also realised that sum of consecutive numbers need not start with "1". He tried to modify the formula to account for series that does not start with "1", But he couldn't go far to think about whether 58, 60, 62, 64 and 68 as 'Nice' numbers. In fact, among the numbers between 50 and 70, only 64 is not a 'Nice' number. Even when he exhausted the limits of his knowledge of the arithmetic sequence sum formula, he did not seek for an alternative approach—that is, there is a lack of Control.

Table 2 summarises Student A's attempt of the Princess Problem. Similar to the process of solving the first problem, he first tried to find a direct formula. However, he did not find such a formula, and after he could not do so, he chose to give up solving the problem within 10 minutes.

Table 2. Detailed mathematical steps and problem-solving moments of princess problem

Time	Detailed Mathematical Steps	Problem Solving Moments (Selected)
0-10min		1. Draw a diagram to understand the problem. (1'26") 2. UP: 'Within 14 days, there is a time limit.' (2'30") 3. Consider special case: Suppose the princess stay in Room 1 on the first day…(3'58") … 16. Give up. 'It is very abstract. With my own IQ, I could not complete this question within 30 minutes.' (9'30")

Discussion

From the problem-solving process of Student A, we discovered that a high-achieving student in standard mathematics tests, such as Student A, may not necessarily be a proficient problem solver. Based on our observations, perhaps over-relying on cognitive resources without suitable control measures is one of the factors affecting pre-service teachers' problem-solving ability. This study casts some light into why high-achievers may not necessarily be good problem solvers. In the case of Student A, he 'started well' with the 'Nice' Number Problem using the cognitive resource of the AP Formula. It helped him make initial progress by identifying some numbers as 'nice'. This perhaps psychologically 'lulled' him further into a stronger confidence (hence fixation) with the AP Formula. To the extent that, when he could make no further progress with this cognitive resource, his confidence swung to the opposite extreme of no-confidence and hence giving up quicky. This lack of confidence carried over to the second problem. It is thus a combination of an over-reliance on a narrow use of a (erstwhile 'successful') cognitive resource and a lack of control (to seek alternative strategies) that explains the problem-solving behavior of Student A. So, to some extent, without appropriate control methods, over-relying on cognitive resources can actually become a hindrance in problem solving. This study reveals both the usefulness as well as limitations of reliance on cognitive resources. It opens us an inquiry into how control interacts with cognitive resources in such a way as to optimize the latter in a given problem-solving context

In terms of implications for teacher development in problem solving, we may consider deliberate moves to help student teachers experience the limits of reliance on tried-and-tested strategies, and how Control can 'kick in' when these strategies are unproductive.

References

Pólya, G. (1945). *How to solve it: A new aspect of mathematical method* (2nd ed.). Princeton: Princeton University Press.

Schoenfeld, A.H. (1985). *Mathematical problem solving*. Orlando: Academic Press.

Schoenfeld, A.H. (2007). Problem solving in the United States, 1970–2008: Research and theory, practice and politics. *ZDM - The International Journal on Mathematics Education, 39(5–6)*, 537–551. https://doi.org/10.1007/s11858-007-0038-z

Leong, Y.H., Tay, E.G., Toh, T.L., Quek, K.S., & Yap, R.A. (2019). Concretisations: A support for teachers to carry out instructional innovations in the mathematics classroom. *International Journal of Science and Mathematics Education*, *17*(2), 365–384. https://doi.org/10.1007/s10763-017-9868-5

Leong, Y.H., Toh, T.L., Tay, E.G., Quek, K.S., Toh, P.C, & Dindyal, J. (2021). Scaling up of continual professional development for mathematics problem solving in Singapore schools. *International Journal of Science and Mathematics Education 19,*1291–1310. https://doi.org/10.1007/s10763-020-10097-3

Toh, T. L., Quek, K.S., Leong, Y. H., Dindyal, J., & Tay, E.G. (2011). *Making mathematics practical: an approach to problem solving*. World Scientific.

Zhang, D.Z, & Song, N.Q. (2023). *Introduction to Mathematics Education.* Higher Education Press.

TSG9-03

AN ANALYSIS OF STUDENTS' BASIC MENTAL MODELS OF THE INTEGRAL: THE CASE OF PHYSICAL CHEMISTRY

Christian Heinz
Julius Maximilian University of Würzburg, Germany, christian.heinz@uni-wuerzburg.de

Hans-Stefan Siller
Julius Maximilian University of Würzburg, Germany, hans-stefan.siller@uni-wuerzburg.de

Understanding integrals is essential in various disciplines and students develop different basic mental models (BMMs) of the integral. While a previous study focused on pre-service teachers, particularly those with physics as a second subject, research on chemistry students remains limited. This study investigates which BMMs chemistry students exhibit and how they differ across study programs. A survey-based assessment was conducted among chemistry students in two thermodynamics courses. Results indicate a predominant reliance on the BMM of area, with variations in (re)construction and accumulation depending on study background. Differences between chemistry as a main course and as part of another degree suggest that the depth of mathematical engagement in physical chemistry may influence students' conceptualization of integrals.

Keywords: integral, basic mental model, physical chemistry, undergraduate

The definite integral is a fundamental part of undergraduate mathematics education in numerous study programs. It is often introduced in the later stages of secondary school and further explored in university calculus courses. Moreover, teaching and learning the concept of the (definite) integral involves different perspectives, such as *area-under-a-curve* or *adding up pieces* (Jones & Ely, 2023). As the ability to calculate the value of an integral has become less important for most students due to technology, skills such as interpreting in context or modelling with integrals have gained some relevance (Ely & Jones, 2023).

A sustainable framework that encompasses the meanings and interpretations of what a (definite) integral is mathematically, is that of *basic mental models* (abbreviated as BMMs), which has been empirically verified (Greefrath et al., 2021a). However, in that study, only pre-service teachers were tested and a study with another cohort was suggested. Chemistry students have proved to be an appropriate cohort, as they (have to) deal with the concept of the integral from different perspectives in physical chemistry and in thermodynamics in particular (Heinz et al., submitted).

RQ 1: What types of BMMs of the integral can be found among chemistry university students?

RQ 2: What are the differences between the different study programs in terms of agreement with the BMMs of the integral?

Theoretical Background

The BMMs of a mathematical concept describe content-related interpretations that give meaning to that very concept and enable both a *normative* and *descriptive* work (vom Hofe & Blum, 2016). Normative BMMs are determined by a didactical analysis of the mathematical concept, whereas in the descriptive work, individual BMMs of learners are derived from their oral or written statements.

BMMs of the Integral

The concept of the (definite) integral can be divided into four normative BMMs. The first one is the BMM *of area* ('AR'), which "interprets the definite integral of a function *f* as the oriented value of the area enclosed by the graph of the function *f* on the interval [*a ; b*] and the *x*-axis in the Cartesian plane" (Greefrath et al., 2021a, p. 652). Second, the BMM *of (re)construction* ('RE') interprets the integrand as the rate of change of a quantity and the definite integral as the total variation over an interval, while also viewing the integral as the (re)construction of an antiderivative of a given function, with construction determining new functional relationships and reconstruction uncovering existing ones from rates of change (Greefrath et al., 2021a). This distinction is consistent with the fundamental theorem of calculus, which relates integration to differentiation (e.g., rate of change). Furthermore, the mean value theorem for definite integrals is the basis of the third BMM, namely that *of average* ('AV'), showing that for a continuous function *f* on [*a ; b*], the average function value equals the definite integral divided by the interval's length, which geometrically corresponds to a rectangle's area with the interval as its base and the average function value as its height (Greefrath et al., 2021a). Finally, the BMM *of accumulation* ('AC') interprets the definite integral as the limit of adding partial products to obtain a cumulative product sum, focusing on the accumulation process rather than the final limit, and geometrically considers the integral as the limit of accumulating rectangular areas of infinitesimally small width (Greefrath et al., 2021a).

Empirical Findings on BMMs of the Integral

A test instrument has been developed and published to 'measure' individual BMMs of the integral (Greefrath et al., 2021b); see also translated in Bhatt et al. (2025). The core concept of the test is to present participants with arguments based on specific BMMs and ask whether the reasoning is consistent with their own thought processes on a 5-Point Likert scale from – – to + +. Each of the eight items contains a prompt (e.g., a question) and four correct response options, each representing one of the four BMMs. In their study, the BMM of area received the highest agreement as expected. In second place, but significantly behind, was the BMM of (re)construction. As their sample consisted exclusively of German pre-service teachers who were required to study a second subject, it was possible to analyze the influence of the second subject. Interestingly, there were significant differences when students studied physics as a second subject. These students differ significantly in the BMM of (re)construction and accumulation towards higher levels of agreement. One possible explanation could "be attributed to their natural exposure to dealing with physical quantities" (Greefrath et al., 2021a, p. 659). Therefore, it would be expected that similar results could be reproduced for chemistry students, who are confronted with different quantities, particularly in thermodynamics (Heinz et al., submitted).

Method and Sample

The test instrument described above was used to do a survey on chemistry students (n = 143) in two different thermodynamics lectures (A & B) at the University of Würzburg (Germany) at the end of the winter semester 2024/25. Lecture A ('major') is compulsory for chemistry, biochemistry,

pre-service teachers at secondary level and functional materials – the latter consisting of only two individuals (see Table 1) and therefore excluded from further analysis. For (bio-)chemistry students it is the second lecture of Physical Chemistry, for the pre-service teachers it is the first. Lecture B ('minor') is aimed at disciplines with Physical Chemistry as minor subject, which in this case is mainly biology. They are in their first semester, so it is their first Physical Chemistry lecture. It is evident that, in accordance with course materials and the module catalogues of the courses, this lecture is less in-depth in terms of subject matter and that it is limited to calculations, as opposed to additional justifications or explanations. The following analysis was conducted using the latest version of R (4.3.1).

Results

The distribution on the lectures and the results are shown in Table 1 with the mean agreement (and the standard deviations in brackets) for each scale (over an interval of [0,4], where 2 is neutral). In the entire group, the BMM of area scored the highest agreement. This is followed by the BMM of (re)construction and accumulation. When looking at the individual lectures and subgroups, this order remains the same. Only in the main lecture and its subgroups the BMM of (re-)construction is ahead of that of accumulation and with the BMM of average still the last.

To analyze the means for differences, the subgroups of the 'major' lecture (biochemistry, chemistry, and pre-service teachers) were considered first. As three independent groups were compared, a one-way ANOVA was chosen as the appropriate method to test for potential differences. Based on the histograms and Q-Q plots of the residuals for all four scales (not shown) and similar standard deviations and thus variances, an ANOVA was performed for each of the four scales, which showed no significant differences between the subgroups for the BMM of area and accumulation (AR: $F(2,92) = 0.99$, $p = 0.377$; AC: $F(2,92) = 2.43$, $p = 0.093$). Significant differences were found for the BMM of (re-)construction and average (RE: $F(2,92) = 3.89$, $p = 0.024$; AV: $F(2,92) = 3.60$, $p = 0.031$. The post-hoc analysis with Bonferroni correction show significant differences for both the RE and AV only between the subgroups of biochemists and chemists, where the effect sizes (RE: $d = 0.62$, $f = 0.29$; AV: $d = 0.57$, $f = 0.28$) can be classified as medium according to Cohen (1992). Secondly, the 'major' and 'minor' groups were compared. The Shapiro-Wilk test suggested that a normal distribution could be assumed for all scales. Given the comparable variances, independent samples t-tests were performed for each scale, which showed no significant differences between the two groups (AR: $t(141) = 0.37$, $p = 0.71$; RE: $t(141) = 1.26$, $p = 0.21$; AV: $t(141) = 1.22$, $p = 0.22$; AC: $t(141) = -0.33$, $p = 0.74$).

Table 1. Means (standard deviations) of BMMs on different (sub)groups of the 'major' lecture.

	Area	**(Re)construction**	**Average**	**Accumulation**
All (n=143)	2.89 (0.62)	2.21 (0.62)	1.87 (0.63)	2.10 (0.69)
Major (n=97)	2.90 (0.60)	2.26 (0.61)	1.91 (0.64)	2.09 (0.71)
Biochemistry (n=41)	*2.82 (0.62)*	*2.15 (0.60)*	*1.69 (0.60)*	*2.00 (0.72)*
Chemistry (n=27)	*3.00 (0.62)*	*2.53 (0.69)*	*2.09 (0.76)*	*2.30 (0.76)*
Pre-service teacher (n=27)	*2.98 (0.58)*	*2.19 (0.49)*	*2.02 (0.58)*	*1.94 (0.67)*
Minor (n=46)	2.86 (0.65)	2.12 (0.64)	1.77 (0.59)	2.13 (0.65)

Discussion und Conclusion

Regarding RQ1, four BMMs of the integral could be found in all groups of chemistry students. A clear hierarchy of agreement was identified, with the BMM of area taking precedence over the BMMs of (re)construction and accumulation. This finding aligns with prior research and theoretical considerations, as these three BMMs are identified as pivotal in thermodynamics (Greefrath et al., 2021a; Heinz et al., submitted). Given that the participants were at the beginning of their studies, it can be hypothesized that their previous schooling may have played a major role in shaping their conceptualization. In Germany, the BMM of area and, to a lesser extent, of (re)construction are emphasized the most in secondary education (Stanat et al., 2016). For RQ2, no significant differences were found between biochemists and pre-service teachers, but chemists showed a significantly stronger agreement with the BMMs of (re)construction and average compared to biochemists. This may be attributable to either differences at the beginning of the study programs or to differences in developing the BMMs during their studies with slightly different curricular emphasis. The latter explanation seems plausible, as physicists also tend to develop the BMM of (re)construction more strongly (Greefrath et al., 2021a) and physical-chemical quantities can also be interpreted by different BMMs (Heinz et al., submitted). The predominance of biochemists and pre-service teachers within the 'major' group may explain the lack of significant differences between the 'major' & 'minor' groups.

However, agreement with the BMMs does not indicate how students apply them in context or how they perform in tasks in physical chemistry. A longitudinal approach could clarify the development of BMMs over time, while future studies should include task-based assessments to evaluate their practical application. In summary, chemistry students conceptualize integrals primarily through the BMM of area, with BMMs of (re)construction and accumulation as secondary perspectives. The observed differences between programs hint at a possible influence of disciplinary background and curriculum structure on the development of BMMs, but further research is required to explore this in more detail.

References

Bhatt, A., Dewasurenda, M., & Griffiths, B. J. (2025). Basic mental models of the definite integral: Conceptual understanding of undergraduate students in India and Sri Lanka. *Asian Journal for Mathematics Education*, Article 27527263251324950. Advance online publication. https://doi.org/10.1177/27527263251324950

Cohen, J. (1992). A power primer. *Psychological Bulletin*, *112*(1), 155–159. https://doi.org/10.1037//0033-2909.112.1.155

Ely, R., & Jones, S. R. (2023). The Teaching and Learning of Definite Integrals: A Special Issue Guest Editorial. *International Journal of Research in Undergraduate Mathematics Education*, *9*(1), 1–7. https://doi.org/10.1007/s40753-023-00214-2

Greefrath, G., Oldenburg, R., Siller, H.-S., Ulm, V., & Weigand, H.-G. (2021a). Basic mental models of integrals: theoretical conception, development of a test instrument, and first results. *ZDM – Mathematics Education*, *53*(3), 649–661. https://doi.org/10.1007/s11858-020-01207-0

Greefrath, G., Oldenburg, R., Siller, H.-S., Ulm, V., & Weigand, H.-G. (2021b). *Test zur Erfassung von Grundvorstellungen zu Ableitungen und Integralen (GV-AI), Empirische Erfassung von Grundvorstellungen zur ersten Ableitung einer Funktion an einer Stelle und zum bestimmten Integral* [Test to

record basic mental models of derivatives and integrals (GV-AI), empirical acquisition of basic mental models of the first derivative of a function in a point and the definite integral].

Heinz, C., Kunz, M., Müller, S., & Siller, H.-S. (2025). Basic Mental Models of the Integral: A Didactical Approach for Lecturers of Thermodynamics. In L. Branchetti, S. Erduran, F. Feudel, A. S. González-Martín, & O. Levrini (Eds.), *The Learning and Teaching of Calculus Across Disciplines 2* – Proceedings of the third Calculus Conference. https://matriccalcconf3.sciencesconf.org

Jones, S. R., & Ely, R. (2023). Approaches to Integration Based on Quantitative Reasoning: Adding Up Pieces and Accumulation from Rate. *International Journal of Research in Undergraduate Mathematics Education*, *9*(1), 8–35. https://doi.org/10.1007/s40753-022-00203-x

Stanat, P., Becker-Mrotzek, M., Blum, W., & Tesch, B. (2016). Vergleichbarkeit in der Vielfalt. In J. Kramer, M. Neumann, & U. Trautwein (Eds.), *Abitur und Matura im Wandel* [Abitur and Matura in transition] (pp. 29–58). Springer Fachmedien Wiesbaden. https://doi.org/10.1007/978-3-658-11693-4_2

vom Hofe, R., & Blum, W. (2016). "Grundvorstellungen" as a Category of Subject-Matter Didactics. *Journal Für Mathematik-Didaktik*, *37*(S1), 225–254. https://doi.org/10.1007/s13138-016-0107-3

Acknowledgement

We thank the Chair of Physical Chemistry II at the University of Würzburg for their support and for enabling data collection in the courses of Dr. Friedrich Schöppler and Prof. Dr. Tobias Hertel.

Topic Study Groups 9
(Day 3)

TSG9-04

A CASE STUDY ON STUDENTS' SELECTIVE NOTE-TAKING IN UNDERGRADUATE NUMBER THEORY LECTURE

Jeong-Won Noh
Busan National University of Education, Republic of Korea, jwnoh@bnue.ac.kr

This study examined how students incorporate blackboard content from peer student presentations and instructor feedback into their notes in a number theory course. The analysis focused on whether students recorded both proofs presented on the blackboard, how they incorporated the instructor's verbal explanations and feedback, and what organizational strategies they used in their notes. The findings revealed that a significant proportion of students selectively recorded only one of the two proofs presented, and that much of the instructor's feedback was either partially noted or omitted. Additionally, students employed various visual and organizational devices to reconstruct blackboard content. These results provide empirical insights into how blackboard content—an essential medium in undergraduate mathematics lectures—is transmitted to students.

Keywords: Note-taking, Mathematical proof, Undergraduate mathematics lecture, Blackboard, Number theory

Research in mathematics education has gradually expanded in undergraduate contexts internationally (Melhuish et al., 2022), though it remains relatively limited in Korean contexts. At the university level, mathematical instruction involves extensive use of writing and notation systems for conveying complex ideas. Recent investigations have highlighted that classroom practices in undergraduate mathematics courses encompass important cognitive processes that support mathematical understanding, going beyond mere transmission of formal content (Greiffenhagen, 2014).

In these settings, mathematical proof is central to instruction, with the blackboard serving as the primary medium for presenting arguments and developing mathematical thinking. However, studies examining how students process and record mathematical content on the blackboard during lectures remain limited (e.g., Fukawa-Connelly et al., 2017), particularly regarding peer-led presentations. This study investigates two research questions: (1) How do students selectively record multiple proofs presented on the blackboard into their notes? and (2) To what extent is instructor feedback reflected in students' note-taking practices?

Literature Review

The blackboard remains fundamental to mathematics instruction at the university level, serving as the primary medium for presenting mathematical arguments and developing mathematical thinking (Greiffenhagen, 2014). Contemporary mathematics instruction continues to favor board-based presentation methods, as this approach allows instructors to organize mathematical ideas spatially. Greiffenhagen (2014) demonstrated that instructors use the blackboard strategically, dividing it into regions and using visual markers to structure mathematical content, enabling students to follow complex arguments while creating their own records of the material.

Research on student note-taking in undergraduate mathematics lectures shows that while students generally succeed in recording formal content from the board, they often miss important verbal explanations (Fukawa-Connelly et al., 2017). Studies show considerable diversity in how students selectively record mathematical content, ranging from exact transcription to extensively annotated personal interpretations (Freitag, 2020; Iannone & Miller, 2019). Iannone and Miller (2019) found that providing structured templates can encourage students to focus more on recording additional explanations rather than merely copying board content. These studies suggest that mathematical note-taking is not merely a mechanical recording process but involves significant cognitive processing and individual reconstruction of mathematical ideas (Freitag, 2020).

Research Context

This study examines students' note-taking practices in a number theory lecture for first-year pre-service mathematics teachers, where active student participation was encouraged. The lecture used guided lecture notes provided in advance, allowing students to focus on peer presentations and discussions during class. The analysis focuses on student notes from a specific lecture where a student (S1) presented multiple proofs of a theorem (Thm 1.2), with subsequent instructor feedback.

The analyzed theorem had two possible proof approaches (Figure 1): one using a basic definition (PF1) and another utilizing a previously proven theorem (PF2). After S1's initial presentation of PF1, the instructor prompted for alternative proof methods and provided hints about using Thm 1.1, leading to S1's attempt at PF2. Although S1's board work for PF2 was logically incomplete, the instructor then added comments to clarify logical connections (regarding B2 step in Figure 1) and suggested simplifications by omitting unnecessary computational steps.

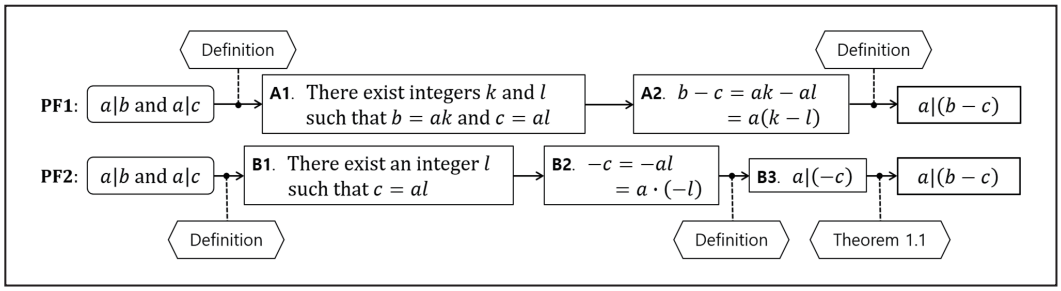


Figure 1. Argumentation structure of two ways of proving Thm 1.2

Notes from 37 students were collected with consent after the course was completed. All notes were handwritten either on printed templates or using digital stylus pens on tablets, rather than typed. The analysis examined whether students recorded both proofs or selectively documented only one. For those who recorded PF2, the study investigated whether they explicitly noted its connection to Thm 1.1 and implemented the instructor's suggestion to simplify the argument. The analysis also included how students organized their notes using visual markers such as dividing lines, different colored pens, and numbering systems.

This specific case was selected for analysis because the unique context—where student-presented proofs with logical imperfections coexisted with instructor feedback—provided a suitable setting for observing how students selectively record mathematical content in their notes.

Results

Analysis of the 37 collected student notes revealed diverse note-taking practices, particularly in how students recorded and organized multiple proofs and instructor feedback. Of these students, 15 students recorded only one proof (12 chose PF1 and 3 chose PF2), while 22 students documented both proofs. Of the 25 students who recorded PF2, 13 explicitly noted its connection to Thm 1.1. The instructor's suggestion to simplify the argument by omitting unnecessary computational steps was followed by 13 students. Eleven students both noted the theoretical connection and implemented the simplification, while 10 students did neither (Table 1). This selective reflection of instructor feedback demonstrates the varied degrees to which students incorporate verbal explanations into their written records.

Table 1. Type of instructor feedback reflected in students' note-taking

Yes		Indicate the connection to Thm 1.1		Total
		Yes	No	
Simplify the argument	Yes	11	2	13
	No	2	10	12
Total		13	12	25

Qualitative analysis of notebooks revealed varying levels of content reconstruction. Some students, like S2, copied board content with minimal reorganization, maintaining the original spatial arrangement even when it didn't reflect logical progression (Figure 2a). Others, like S3, thoroughly integrated board content with instructor feedback, using arrows and circles similar to those used by the instructor, and included informal annotations like "by professor" or "added later." (Figure 2b) More sophisticated examples, such as S4's notes, showed complete reorganization of board content with numbering systems and clear distinction between different proof approaches (Figure 3).

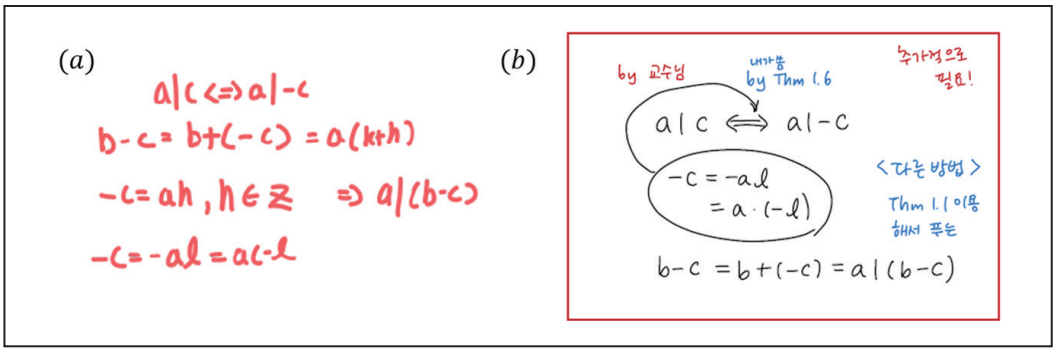


Figure 2. Notes from students S2 and S3 on Thm 1.2

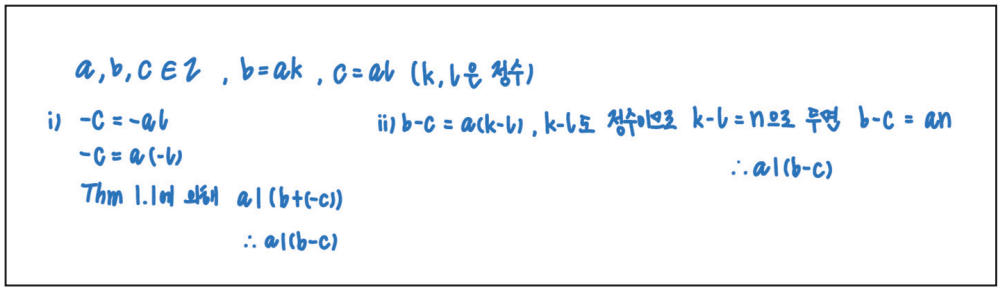


Figure 3. Student S4's notes on Thm 1.2

Conclusion

This study provides some empirical evidence of how students selectively record mathematical content in a participatory undergraduate mathematics lecture. The findings reveal that students often omit one of multiple proof approaches, suggesting possible difficulties in recognizing the value of alternative proofs or hesitation to record peer-presented content. This contrasts with previous findings that students consistently record most instructor-written content (Fukawa-Connelly et al., 2017), suggesting that peer-presented content may be treated differently when students take notes. The study also demonstrates that students frequently omit verbal instructor feedback, even when it contains potentially important mathematical connections. This aligns with Fukawa-Connelly et al.'s (2017) observation about the limited transmission of verbal explanations in mathematics lectures.

These findings contribute to understanding note-taking practices in undergraduate mathematics education, particularly in participatory learning contexts. Despite the limitations of this small case study, the findings suggest that instructors providing feedback during peer-led presentations might consider making critical mathematical connections visually explicit on the board rather than relying solely on verbal explanations. Additionally, students may benefit from guidance about the value of documenting multiple proof approaches, especially when peer-presented content is involved.

References

Freitag, M. A. (2020). Note-taking practices of students in college mathematics. *International Journal of Research in Undergraduate Mathematics Education, 6*(1), 65-89.

Fukawa-Connelly, T., Weber, K., & Mejía-Ramos, J. P. (2017). Informal content and student note-taking in advanced mathematics classes. *Journal for Research in Mathematics Education, 48*(5), 567–579.

Greiffenhagen, C. (2014). The materiality of mathematics: Presenting mathematics at the blackboard. *The British journal of sociology, 65*(3), 502-528.

Iannone, P., & Miller, D. (2019). Guided notes for university mathematics and their impact on students' note-taking behaviour. *Educational Studies in Mathematics, 101*(3), 387-404.

Melhuish, K., Fukawa-Connelly, T., Dawkins, P. C., Woods, C., & Weber, K. (2022). Collegiate mathematics teaching in proof-based courses: What we now know and what we have yet to learn. *The Journal of Mathematical Behavior, 67*, 100986.

TSG9-05

ENHANCEING MATHEMATICS STUDENT TEACHERS' UNDERSTANDING IN ACTIVE LEARNING THROUGH ABSORB-DO-CONNECT LEARNING FRAMEWORK

Wanintorn Poonpaiboonpipat
Naresuan University, Thailand, wanintorns@nu.ac.th

Absorb-Do-Connect provides a framework to guide in creating learning activities. Therefore, this study is interested in enhancing understanding in active learning of third-year mathematics student teachers who are enroll the Teaching Mathematics in Elementary Schools course. The research tools include the analysis of the lesson plan designed by student teacher in combination with questionnaires and interviews. The results showed that most of the students had an understanding of the concept active learning as seen from their own lesson plan with a Do and Connect section of more than 50% of all groups, together with the results from questionnaire of the students' perceived understanding of active learning with an average of 4.37 (out of 5.00) and students indicating ability to design active learning lesson plans with an average of 4.27 (out of 5.00).

Keywords: active learning, mathematics student teacher, absorb-do-connect

Active learning refers to a teaching approach in which students have ownership and control over the learning activities they do, with a learning experience that is not rigidly planned out, and that the student can shape the learning experience through active participation (Kyriacou, 1992). The learning of mathematics is predominantly influenced by the teacher, the assigned tasks, and the methods employed for their delivery. Consequently, the teacher must facilitate the enhancement of students' mathematics comprehension by providing opportunities for them to be challenged and involved in advanced cognitive processes, through a deliberate selection of tactics and activities (Vale & Barbosa, 2023). It is also crucial to emphasize that the instructional activities chosen for a lesson should not only be purposeful, but they should also involve active learning. The objective is not to engage in an activity for its own sake or to make the lesson enjoyable. It is not accomplished by merely incorporating diversions or enjoyable activities into a lesson plan (Edwards, 2015).

Within Horton's (2011) Absorb-Do-Connect framework which is three types of learning activities required, Absorb activities are intended to introduce and orient learners to the targeted skill and provide foundational knowledge necessary to prepare for skill performance. Absorb activities directly inform efficacy expectations by helping learners to realize feasibility of skill performance and begin building confidence in doing so themselves. Absorb activities are activities in which the learner reads, listens and watches include stories told by the teacher and field trip. In addition, Absorb activities are commonly centered in verbal or written instruction, skill modeling or demonstration, self- or peer-directed visualization of skill performance, and instructors supported self-instruction or self-directed learning (Portillo & Gallimore, 2020). Do activities that help the learner do something with knowledge, such as practice, explore, game and discover activities. Connect activities lead the learner to connect current learning to live, work and prior learning such as pondering,

stories told by the learner, research and job aids. The important thing for teachers in designing learning management is that teachers must combine learning activities to accomplish a learning objective. Therefore, this study focused on enhancing understanding in active learning of third-year mathematics student teachers who enrolled Teaching Mathematics in Elementary Schools course though Absorb-Do-Connect learning framework. The outcome that goes beyond understanding is the ability to design mathematics learning management for their classrooms in the future.

Methodology

The participants for this study were 62 third-year student teachers in the bachelor's degree of mathematics education major at the Faculty of Education in Naresuan University (Phitsanulok Province, Thailand). The Teaching Mathematics in Elementary Schools course normally operates for 16 weeks per semester. This research has been conducted during the last 6 weeks, in a regular classroom within the Faculty of Education for the course. Prior to the Absorb-Do-Connect (ADC) workshop, student-teachers learned about the nature and characteristics of elementary mathematics, the elementary mathematics curriculum, teaching methods and techniques for teaching mathematics and measuring and evaluating mathematics learning outcomes at the elementary level. During the 2-day ADC workshop participants learned how to design a lesson plan using active leaning and ADC framework. Students participated in lecture sessions, examined diverse examples of elementary mathematics learning activities, collaborated in groups of three or four to develop lesson plans, and engaged in peer feedback discussions about their instructional designs. In this course, the researcher has added teaching methods appropriate for elementary mathematics learning management, which include Game-Based Learning, Problem-based Learning, Concrete-Pictorial-Abstract (CPA), Open Approach, or Lesh's Model (Widodo & Ikhwanudin, 2019).

To enhance understanding in active learning through Absorb-Do-Connect learning framework, we collected the lesson plans designed by student teachers, questionnaires, and interviews. The analysis of student teachers' lesson plans will focus on the sub-activities within the plans. Each sub-activities will be classified according to the Absorb-Do-Connect framework, then the frequency of Absorb, Do, and Connect activities will be counted. The number of sub-activities in each lesson plan will not be equal, depending on the student teachers' design for lesson plans that take 1-2 hours of teaching time. The questionnaire asked students their agreement or disagreement with the following statements: "You understand the concept of active learning"; "You can design a lesson plan based on active learning concept" and "You can implement classroom in the way of active learning" and "You can learn the active learning concept from other resources". These were assessed on a five-point Likert scale (Strongly disagree (1), Disagree (2), Neither disagree nor concur (3), concur (4), and Strongly Agree (5)). These data were subsequently analyzed using the average and standard deviation. The interviews were conducted with a random group of 20 student teachers and were meant to expand on their answer from questionnaires and gain depth of their understanding and difficulty about active learning concepts.

Findings and Discussion

The 20 lesson plans from 62 student teachers who were divided into 20 groups of 3-4 student teachers on a voluntary basis, and the students were asked to choose five teaching methods, namely Game-Based Learning, Problem-based Learning, Concrete-Pictorial-Abstract (CPA), Open Approach, or Lesh's Model. The plans were also categorized by the mathematical content strands (See Table 1).

Table 1. Characteristics of student teachers' lesson plans and frequency of use of ADC framework

Group number	Topic	Grade level	Teaching approach	The frequency of ADC activities		
				A	D	C
Number and Algebra						
13	Addition of whole numbers with sum and addends not exceeding 100	2	Game-Based Learning	3	6	2
20	Division	2	Open Approach	2	6	2
2	Addition of fractions with equal denominators	3	Problem-based Learning	5	6	4
7	Number patterns that increase or decrease by the same amount each time	3	Lesh's Model	5	9	3
10	Parts of a whole	4	Lesh's Model	3	6	2
5	Problem solving in pattern	6	Open Approach	3	4	3
11	Problem solving in pattern	6	Concrete-Pictorial-Abstract (CPA)	4	6	3
12	Problem solving in pattern	6	Lesh's Model	2	7	2
Measurement and Geometry						
17	Property of cube, sphere, cylinder and cone	1	Open Approach	2	5	3
6	Three-dimensional geometry	1	Problem-based Learning	2	5	3
1	Sorting of Two-dimensional geometry	2	Game-Based Learning	4	7	1
14	Polygon	2	Lesh's Model	2	5	3
4	Measurement in centimeters	2	Game-Based Learning	5	6	4
8	Money Comparison	3	Concrete-Pictorial-Abstract (CPA)	3	6	4
16	Problem solving in weight	3	Concrete-Pictorial-Abstract (CPA)	4	4	2
15	Volume of cube	5	Game-Based Learning	2	6	4

Table 2. Characteristics of student teachers' lesson plans and frequency of use of ADC framework (Cont.)

Group number	Topic	Grade level	Teaching approach	The frequency of ADC activities		
				A	D	C
Statistics and Probability						
19	Writing One way table	3	Problem-based Learning	4	5	3
9	Writing bar chart	4	Open Approach	3	5	2
18	Writing bar chart	4	Problem-based Learning	2	6	4
3	Reading pie chart	6	Concrete-Pictorial-Abstract (CPA)	3	5	1

The analysis of the lesson plans revealed that all groups composed more than 50% of the Do and Connect components. The Absorb activities have a frequency of sub-activity creation ranging from 2 to 5. The combined frequency of the Do and Connect activities reaches as high as 12. Examples of Absorb activities include: the teacher gives examples and explains three-dimensional geometric shapes; the teacher reviews prior knowledge through a lecture; the teacher demonstrates the use of comparative symbols. Examples of Do activities include: the teacher uses thought-provoking questions for students to answer; the teacher lets students play a card game; the teacher assigns students to arrange objects according to specified fractions; the teacher has students present their work in front of the class; The teacher had the students practice exchanging money through a simulated scenario. Examples of Connect activities include: the teacher asks students to create word problems based on real-life situations; the teacher has students' critique and propose extensions to their peers' work; the teacher has students compare similarities, differences, and connections in problem-solving processes.

Student-teachers› average scores of their self-assessment from questionnaire regarding their perception of: 1) their own understanding of active learning was 4.37, 2) their ability to design lesson plans based on active learning concept was 4.27, 3) their ability to implement classroom in the way of active learning was 4.18; and 4) their ability to learn the active learning concept from other resources was 3.98.

Conclusion

The results suggest that most student teachers have an understanding of the concept of active learning, as evidenced by lesson plans with more than 50% of Do and Connect parts. Consistent with Horton's (2011) statement that a better goal may be 40-50-10. That means learners spend 40% of their time absorbing, 50% doing, and 10% connecting. Furthermore, the content within the mathematics curriculum's three strands in Thailand: Number and Algebra, Measurement and Geometry, and Statistics and Probability can influence the design of activities and practice. Before designing the activity, the student teachers collaborated to analyze the content and develop the activity. During the design process, they reviewed the proportion of ADC; however, the primary focus of the activity design was to align with the learning objectives. This viewpoint aligns with the findings of Vale and Barbosa (2023), who underscore that active learning fosters collaboration and mathemat-

ical communication, allowing diverse problem-solving strategies to emerge. The detailed analysis of how the student teachers designed activities relates to the framework of intellectual, social, and physical aspects presents an interesting avenue for future study. Such exploration could support the development of more effective learning activities that enhance student learning outcomes.

References

Edwards, S. (2015). Active learning in the middle grades. *Middle School Journal*, *46*(5), 26–32.

Horton, W. (2011). *E-learning by design*. John Wiley & Sons.

Kyriacou, C. (1992). Active learning in secondary school mathematics. *British Educational Research Journal*, *18*(3), 309–318.

Vale, I., & Barbosa, A. (2023). Active learning strategies for an effective mathematics teaching and learning. *European Journal of Science and Mathematics Education, 11*(3), 573–588. https://doi.org/10.30935/scimath/13135

Widodo, S., & Ikhwanudin, T. (2019). Students with mathematics learning disabilities and their ways of thinking in fraction learning. In *Learning Disabilities-Neurological Bases, Clinical Features and Strategies of Intervention*. IntechOpen.

TSG9-06

PROVING ACTIVITIES IN A GRAPH-THEORETICAL INQUIRY FOR PRE-SERVICE TEACHERS: A PRELIMINARY ANALYSIS

Minh Hong Doan Le
Graduate School of Humanities and Social Sciences, Hiroshima University, Japan,
m243292@hiroshima-u.ac.jp

Yusuke Shinno
Hiroshima University, Japan, shinno@hiroshima-u.ac.jp

On the growing attention to proof and proving in university mathematics in recent research, this proposal clarifies how pre-service teachers developed proving activities during an inquiry involving Graph Theory. By adopting the theoretical framework of Study and Research Paths within the Anthropological Theory of the Didactic, we designed and conducted a teaching experiment with inquiry on map-coloring problems in the field of Graph Theory. The preliminary result shows that in this context, autonomous proving activities can occur in various ways, for example, constructing conjectures with the aid of modelling or searching for a counterexample of a statement. This result also implies that the proving activities in an inquiry differ from those in traditional classrooms in terms of methods and statements.

Keywords: teaching and learning graph theory, proof and proving, inquiry-based learning

In recent years, mathematics education research has discussed proof and proving with a relatively balanced focus on both the school and university levels (Stylianides et al., 2024). This research trend highlights the transitioning of learning proof from school to university, which is strongly experienced by pre-service mathematics teachers. Therefore, in this study we ask: how can appropriate opportunities be designed for pre-service teachers to learn proof and proving autonomously? Inquiry-based learning is currently rising as a major educational trend in schools and universities (Artigue & Blomhøj, 2013). In an inquiry-based classroom, students autonomously generate questions and try to use all accessible resources to answer their own questions, including the questions of how to prove that a statement is true or false. Therefore, we hypothesize that *through an inquiry process derived from an initial question, autonomous proving can occur*. To examine this, this proposal elaborates on how pre-service teachers develop proving activities in the context of graph-theoretical inquiry.

Theoretical Framework

Inquiry-based learning can be conceptualized as Study and Research Paths within the Anthropological Theory of the Didactic. In Barquero and Bosch (2015), Study and Research Paths is defined as a general modelling tool staging two types of linked processes: the *study process* occurring when students gain new knowledge and find some essential information from different resources (books, experiments, the Internet, etc.); and the *research process* occurring when students combine this new knowledge with what they already know *in order to answer their self-imposed question or problem*.

Study and Research Paths can progress from an initial generative question Q_0 through a sequence of derived questions and answers while interacting with data, existing mathematical works $A^{\diamondsuit}$, etc., aiming at finding an answer $A^{\heartsuit}$ to the initial question.

During Study and Research Paths, to find the answer $A^{\heartsuit}$, both students and instructors can make use of the *dialectics of conjecture and proof* (Chevallard & Bosch, 2019). This dialectic implies a focus on questioning existing or new answers or works, giving students the opportunity to explore the proof of the given statements. The process of questioning why and how such statements (regarded as conjectures in the viewpoint of students) are true or false can be considered a broad type of proving activity, including related activities such as argumentation and reasoning. The proof emerging during Study and Research Paths can be in any representation (written, verbal, self-explanatory figures, etc.), if it is accepted by the students themselves or the community of learning inside the classroom.

Research Methods

This research was conducted based on the *experimentation of a sequence of inquiries*. We designed an initial question Q_0, a priori analyzed the possibilities of Q_0, and proposed some supporting questions to make the proving activities visible. Then, the proposed sequence was implemented for three consecutive weeks by the second author in a weekly 90-minute seminar for two groups of pre-service mathematics teachers at a national university in Japan; all activities in the classroom were observed and collected as data. We focused on some specific characteristics of the proving activities, by using the Q-A map, which was first introduced by Winsløw et al. (2013), as the main tool for analyzing the collected data. Q-A map can be utilized to organize the derived questions and answers that occurred during inquiries, in the form of a tree that stems from the initial question.

Teaching Experiment

Design and a Priori Analysis

We chose Graph Theory as the inquiry theme for two reasons. First, Graph Theory, as a branch of discrete mathematics, can provide students with accessible concepts, various ways of proving and modelling and a mathematical experience (Ouvrier-Buffet, 2020). Second, in Japan's 2018 Course of Study for High School Mathematics (Ministry of Education, Culture, Sports, Science and Technology, 2018), graph theory content is newly added, leading to the pre-service teachers' lack of knowledge and experience when coping with this content.

To promote argumentation and proving activities inside the classroom, we designed an initial question based on the problem of admissible map-coloring within Graph Theory. We will say that we have an admissible coloring of a map, if each region of this map is colored in such a way that any two regions having a common borderline are colored with different colors (Fritsch & Fritsch, 1998). Thus, the implemented initial question Q_0 is stated as '*Which kind of map must use at least four colors to have an admissible coloring?*'. This initial question has generative power not only for mathematical contents, but also for mathematical activities, such as proving and modelling.

Preliminary Results and Analysis

The Q-A map in Figure 1 summarizes all the main questions and answers posed by each group during the implemented inquiry sequences. During the inquiry, there were two supporting questions

(Q_1 and Q_5) posed by the instructor, some common questions and answers between the two groups and many different flows of questions and answers generated by each group G1 and G2.

After receiving the initial question, G1 searched for the answer on the Internet and found some information about the Four-Color Theorem, while G2 questioned which maps need two or three colors. Both groups also tried coloring different maps and used graphs for modelling. G1 questioned the different numbers of colors used for the partial map and the entire map, while G2 started to construct conjectures based on specific types of maps and tried to prove them. Then, both groups posed a question on maps that must use five colors (Q_4), which means finding a counterexample to the Four-Color Theorem. From Q_4, more conjectures were constructed, and students also tried proving or disproving them. G1'a final answer was a conjecture 'All planar graphs can be 4-colored' based on two observations on complete graph K_5 ($A_{4.1.2}$) and the addition of one vertex into any given graph ($A_{5.1.1}$); while G2 did not decide on the final answer toward the initial question. Instead, they posed two conjectures: one states the relation between the edge intersection in the dual graph and the number of colors needed ($A_{4.1.1}$), and another is the contraposition statement to $A_{4.1.1}$. The reason why G2 could not decide on the final answer was that they had not yet found a counterexample to the contraposition statement, stating that they could not decide whether the conjecture was true or not.

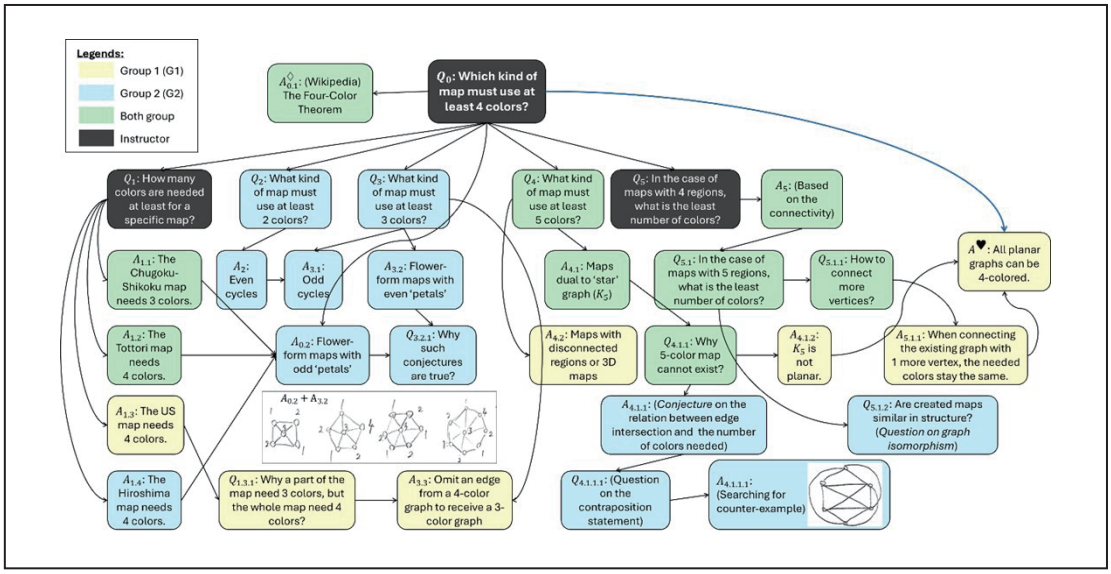


Figure 1. Q-A map of the implemented inquiry

In this study, we will clarify the proving activities occurring in this inquiry by elaborating on two cases: proving by construction to answer $Q_{3.2.1}$ and using counterexamples while inquiring on Q_4.

G2 had constructed a conjecture on a type of map that they called 'flower-form map', dual with graphs consisting of one vertex in the center of a cycle and all vertices on that cycle connecting to the central vertex. They generalized their observations into a conjecture stating that if the number of vertices on the outer cycle is odd then four colors are needed, while if the number is even then only three colors are needed. They also reasoned that in the even case, one can easily color the cycle by interchanging Colors 1 and 2, then a third color is needed for the center; while in the odd case, using Colors 1 and 2 interchangeably, then at the last vertex the color will be the same as the starting

vertex, leading to the need of a fourth color. This way of reasoning can be considered as proving by construction, as students had explicitly constructed a way of coloring that uses the least number of colors to prove that their conjecture is true.

G2 also conjectures the relation between the edge intersection and the number of colors needed, stating that: (i) 'If it is a map, then the dual graph has no edge intersection and thus only four colors or fewer are needed'. G2 then constructed a contraposition statement of (i), saying that: (ii) 'If a map needs five or more colors, then the dual graph has an edge intersection'. However, G2 wanted to disprove this statement by trying to construct a counterexample ($A_{4.1.1.1}$); but then realized that this is not a counterexample to (ii) because this graph has edge intersections but still only needs four colors. This move is based on the law of contraposition (the original statement is true if and only if the contraposition statement is true), showing that students were exploring the proof by altering the original statement based on logical rules and clarifying the conditions backing the statement.

Discussion and Conclusion

In this inquiry sequence, to explore and develop questions and answers, proving activities occurred autonomously and were strongly aided by graph models. Graphs helped students observe common patterns and construct new models more easily, leading to more trials-and-errors and more precise conjectures constructed during the inquiry. The use of graphs enabled them to work on *proving by construction* using informal representations such as drawings, which is different from the usual proving taught at school that is often based on systems of propositions and learnt theorems in elementary geometry. Additionally, during the inquiry, there were many conjectures that emerged as statements *with no clear true/false values* at the first encounter. This characteristic leads to gestures of exploring the statements in-depth to change the epistemic values of the statement, by searching for proof, a counterexample, or restating existing statements. Working on such conjectures also leads to the students' awareness of the necessity of proofs, even if they could not produce mathematical proofs within their available knowledge. It also differs from the usual practice of proof and proving inside the classroom, when the statements to be proved are already known as true.

In conclusion, the preliminary analysis of the experimentation of a graph-theoretical inquiry has clarified the distinctive characteristics of proving activities, in terms of proving methods and conjecturing statements. The results can also imply a way to nurture pre-service teachers' knowledge and proving skills for these newly added curricula. Study and Research Paths also provides a suggestive approach towards the recent trends of including Graph Theory contents into secondary curriculum in many countries. Several questions, however, remain to be addressed, especially with a view to elucidating the transition from informal to formal proving in the dialectics between conjecture and proof, the challenges of implementing inquiry-based learning in different educational settings, and the ways to design and assess an inquiry-based course for pre-service mathematics teachers.

References

Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM Mathematics Education, 45*, 797–810.

Barquero, B., & Bosch, M. (2015). Didactic engineering as a research methodology: From fundamental situations to study and research paths. In A. Watson, & M. Ohtani (Eds.), *Task design in mathematics education: an ICMI study 22* (pp. 249-272). Springer Cham.

Chevallard, Y., & Bosch, M. (2019). A short (and somewhat subjective) glossary of the ATD. In M. Bosch, Y. Chevallard, F. García, & J. Monaghan (Eds.), *Working with the anthropological theory of the didactic in mathematics education: A comprehensive casebook* (1st ed., pp. xviii-xxxvii). Routledge.

Fritsch, R., & Fritsch, G. (1998). *The four-color theorem: History, topological foundations, and idea of proof* (1st ed.). (J. Peschke, Trans.) Springer New York, NY.

Ministry of Education, Culture, Sports, Science and Technology. (2018). *Koutou gakkou gakushuu shidou youryou (Heisei 30 nen kokushi) kaisetsu: Suugakuhen risuuhen* [The commentary of the 2018's course of study for high-school mathematics]. https://www.mext.go.jp/content/20230217-mxt_kyoiku02-100002620_05.pdf

Ouvrier-Buffet, C. (2020). Discrete mathematics teaching and learning. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (2nd ed., pp. 227–233). Springer Cham.

Stylianides, G.J., Stylianides, A.J., & Moutsios-Rentzos, A. (2024). Proof and proving in school and university mathematics education research: a systematic review. *ZDM Mathematics Education, 56*, 47–59.

Winsløw, C., Matheron, Y., & Mercier, A. (2013) Study and research courses as an epistemological model for didactics. *Educational Studies in Mathematics, 83*, 267–284.

TSG9-07

CONCEPT-BASED LEARNING IN GEOMETRY WITH GEOGEBRA IN PAPUA NEW GUINEA PRIMARY TEACHERS COLLEGES

Hanson Wambu
Naruto University of Education, Japan, hwambu@gmail.com

Geometry is important in mathematics education, for developing problem-solving, logical reasoning, and spatial visualization skills. However, preservice teachers in Papua New Guinea (PNG) still face significant challenges in conceptualizing and teaching geometry due to abstract content, traditional instructional methods and reliance on rote memorization. This research examines the effectiveness of GeoGebra in enhancing concept-based learning in geometry at Primary Teachers Colleges. A quasi-experimental design divided 148 third-year preservice teachers into two groups. The experimental group utilized GeoGebra, while the control group followed traditional teaching methods. Pretest scores show no significant difference ($p=0.159$), indicating similar baseline abilities while post-test results demonstrated a significant improvement in the experimental group ($p<0.001$), highlighting GeoGebra's effectiveness in enhancing conceptual understanding. The study suggests incorporating tools like GeoGebra in teacher training programs to foster deeper conceptual understanding and better prepare future educators.

Keywords: Concept based learning, Geometry, GeoGebra, Papua New Guinea, Primary Teachers College.

Geometry is an essential component of mathematics education, fostering confidence, problem-solving abilities, logical reasoning, and communication skills across all learning levels (Sila & Litfaj, 2023). It plays a fundamental role in STEM fields, including science, technology, engineering, and mathematics (Gurmu et al., 2024). However, learning geometry poses challenges due to its abstract nature, limited conceptual understanding, and ineffective teaching approaches (Adolphus, 2011; Ubi et al., 2018; Gurmu et al., 2024). These issues necessitate the adoption of innovative, student-centered instructional strategies.

Research has shown that integrating Information and Communication Technology (ICT) enhances students' ability to visualize, construct, and understand geometric concepts (Nidup & Ishii, 2023; Shaddan & Leong, 2013). Tools such as interactive whiteboards, graphing calculators, Geometer's Sketchpad, and GeoGebra provide dynamic learning experiences, with GeoGebra emerging as one of the most effective software tools for geometry instruction. Mathematics education in Papua New Guinea (PNG) faces persistent challenges, including disengagement, conceptual difficulties, and negative student perceptions. In many Primary Teachers Colleges (PTCs), both instructors and students struggle to demonstrate conceptual and procedural knowledge in geometry. Traditional instructional methods continue to dominate, prioritizing rote memorization over conceptual understanding (Gomay, 2018). Consequently, students often focus on exam performance rather than developing meaningful mathematical comprehension.

Despite growing global interest in the use of digital tools for mathematics instruction, limited research has examined their impact on conceptual understanding in PNG's preservice teacher education. This study seeks to address this gap by investigating the effectiveness of GeoGebra in fostering concept-based learning in geometry among preservice teachers at PTCs in PNG.

Literature Review

The increasing integration of technology in mathematics education reflects a global shift toward fostering deeper student engagement, collaboration and critical thinking. Tools such as interactive simulations and digital platforms enable learners to explore mathematical concepts more meaningfully (Shadaan, & Leong, 2013). Within this context, concept -based learning emerges as a powerful instructional approach that prioritizes deep understanding over rote memorization. By helping students recognize relationships among mathematical ideas and apply them to real-world context, it promotes adaptable knowledge inquiry-driven thinking, and long-term retention (Erickson, 2012; Putri et al., 2021).

Central to this learning model is the use of visualization tools, including software like GeoGebra, which allows dynamic manipulation of geometric objects and real-time interaction with abstract concepts (Hohenwarter & Lavicza, 2008). Developed in 2001, GeoGebra has become a widely adopted open-source platform for integrating geometry, algebra and calculus into dynamic learning experiences. Unlike traditional textbooks, GeoGebra bridges visual and symbolic understanding, thereby supporting active engagement and improved conceptual grasp. While its advantages are well documented in general educational settings, there remains a lack of research on how GeoGebra supports concept-based geometry instruction in preservice teacher education, particularly in low-resource contexts like Papua New Guinea. Addressing this gap is essential for informing effective teacher training practices in the region.

Research Design and Sample

This study employed a quasi-experimental design with non-equivalent pre- and post-tests conducted in two teachers' colleges: one rural and one urban, to ensure diverse representation of learning environments. A total of 148 third-year preservice teachers, nearing graduation, participated in the study. Existing classes were randomly selected and assigned to either experiment or control group. The experimental group (76 students) utilized GeoGebra while the control group (72 students) followed traditional teaching methods. The selection of the two geometry topics for this study was guided by their inclusion in the standardized curriculum for teachers' college in PNG, ensuring curricular relevance. Moreover, these topics were chosen due to their high level of conceptual difficulty, particularly in spatial reasoning, where surface-level memorization is insufficient for mastery. The use of GeoGebra was strategically aligned with these challenges, as its dynamic and interactive modeling environment is particularly effective in supporting deeper conceptual understanding. Five intervention lessons were conducted by the researcher for both groups using the colleges' prescribed mathematics manuals and same lesson plans, the first three lessons covered basic information about GeoGebra. The data collection instruments were divided into two parts. In Part A students were asked to describe the properties of a cube, sketch two different cube nets, and explain the folding process to form a three-dimensional shape, as shown in Figure 1. In Part B students had to derive the formula for the area of a circle; they were first asked questions on the area formulas for a circle and a parallelogram as shown in Figure 2; then they explained the process of dividing a circle into equal slices and analyzed the geometric properties of the resulting shapes. Finally, they rearranged the slices into a shape resembling a parallelogram and applied its area formula to derive the formula for the area of a circle.

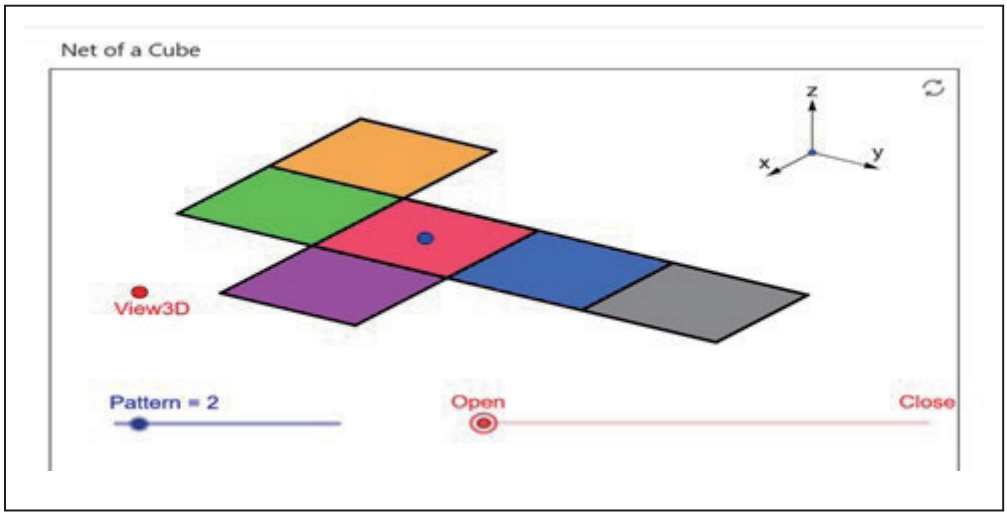


Figure 1. Intervention lesson 4 using GeoGebra– Construction of a Cube's Net.

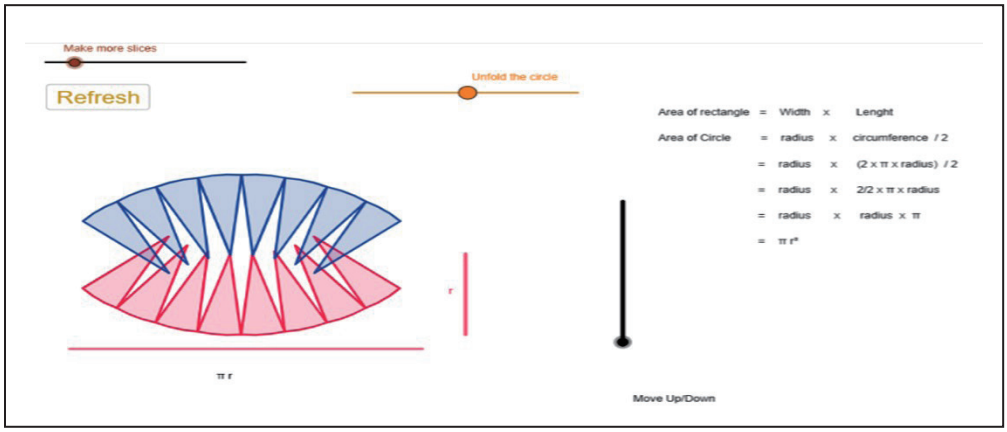


Figure 2. Intervention lesson 5 using GeoGebra– How to derive formula of area of a circle.

Quantitative data from pretests and posttests were analyzed using JASP and Excel. The Shapiro-Wilk test was applied to assess the normality of data distribution, while Levene's test examined variance homogeneity between groups. An independent samples *t*-test was conducted to compare mean scores between the experimental and control group.

Results and Discussion

The Shapiro-Wilk test indicated that the residuals for both pretest and posttest scores were approximately normally distributed (p>0.05). Likewise, Levene's test confirmed that variance between the control and experimental groups was homogeneous for both pretest and posttest scores (p> 0.05). These results satisfy the assumptions necessary for conducting further statistical analyses.

An independent sample t-test revealed no significant difference in pretest scores between the control group (M=14.65) and the experiment group (M=13.62), t (146) = 1.42, p=0.159. This suggests that both groups had similar baseline abilities prior to the intervention.

However, a statistically significant difference was found in the post-test score, with the experiment group (M=25.16) outperforming the control group (M=18.60), t (146) = -9.05, p<0.001. This indicates that GeoGebra had a positive effect on conceptual understanding in geometry. In addition to the significant overall gain, in the post test scores, students in the experimental group showed marked conceptual improvements. In the cube net task, they more accurately identified valid nets and explained edge-face alignments, indicating enhanced spatial visualization. For the circle area task, they better grasp how slicing and rearranging sectors led to the deriving the area formula – skill supported by GeoGebra's dynamic slider and visualization tools. These improvements were most evident in tasks requiring spatial reasoning and decomposition, where GeoGebra's real-time feedback and interactive features fostered deeper understanding compared to static textbook methods.

Conclusion

GeoGebra facilitated improvements in students' geometry understanding by providing dynamic, interactive experiences that enhanced their conceptual grasp. The experimental group was able to visualize 3D views of nets, manipulate sliders, and observe the folding process of a cube, thereby strengthening their spatial reasoning. Similarly, GeoGebra's animated demonstrations of circle decomposition enabled better understanding of area formula derivation. These interactive tools fostered deeper engagement and more coherent explanations, leading to significant improvements in conceptual understanding. This study suggests that incorporating digital tools like GeoGebra into preservice teacher training programs in PNG can transform traditional geometry instruction, moving students beyond rote memorization towards true mathematical reasoning. Future research should investigate the long-term effects of such interventions and consider other influencing factors such as instructional style, students' motivation and learning environments.

References

Erickson, H. L. (2007). *Concept-based curriculum and instruction for the thinking classroom*. Thousand Oaks, CA: Corwin Press.

Erickson, H. L. (2012). Concept-based teaching and learning. *IB Position Paper*. Retrieved from http://www.ibmidatlantic.org/Concept_Based_Teaching_Learning.pdf

Fikru Gurmu, C., Tuge, C., & Hunde, A. B. (2024). Effects of GeoGebra-assisted instructional methods on students' conceptual understanding of geometry. *Cogent Education*, 11(1), 2379745. https://doi.org/10.1080/2331186X.2024.2379745

Gomay, D. (2018). Improving mathematics education in teacher training colleges in Papua New Guinea: An analysis of lecturers and students' responses to a sample test question in geometry shapes. *NUE Journal of International Educational Cooperation*, *11*, 57–64

Hohenwarter, M., & Lavicza, Z. (2008). GeoGebra: A systematic introduction. *International Journal for Technology in Mathematics Education*, 15(2), 83–88.

Laborde, J. M. (2010). Integration of dynamic geometry software in the teaching of mathematics: A research perspective. *ZDM*, 42(7), 683–695.

Leung, A. (2018). Exploring the potential of GeoGebra in mathematical modeling. *Mathematics Education Research Journal*, 30(1), 21–39.

Mukiri, M. I. (2016). Feasibility of using GeoGebra in the teaching and learning of geometry concepts in secondary schools in Kajiado County, Kenya (Unpublished master's thesis). Kenyatta University, Nairobi, Kenya.

Nidup, D., & Ishii, H. (2023). Integration of information and communication technology in Japanese mathematics lessons: Observations and teachers' perspectives. *Journal of Hokkaido University of Education (Clinical Research in Education)*, 73(1–2), 215.

Shadaan, P., & Leong, K. E. (2013). Effectiveness of using GeoGebra on students' understanding in learning circles. *Malaysian Online Journal of Educational Technology*, 1(4), 1–11.

Sila, E., & Litfaj, S. (2023). An overview of geometry curriculum and its understanding by Albanian students. *The Eurasia Proceedings of Educational & Social Sciences (EPESS)*, 33, 56–62.

Ubi, E. E., Odiong, A. U., & Igiri, O. I. (2018). Geometry viewed as a difficult mathematics. *International Journal of Innovative Science and Research Technology*, 3(11), 251–255.

Voogt, J., & Roblin, N. P. (2012). A comparative analysis of international frameworks for 21st-century competences: Implications for national curriculum policies. *Journal of Curriculum Studies*, 44(3), 299–321.

Zulnaidi, H., & Syed Zamri, S. N. A. (2017). The effectiveness of the GeoGebra software: The intermediary role of procedural knowledge on students' conceptual knowledge and their achievement in mathematics. *Eurasia Journal of Mathematics, Science and Technology Education*, 13(6), 2155–2180. https://doi.org/10.12973/eurasia.2017.01219a

Acknowledgement

GeoGebra facilitated improvements in students' geometry understanding by providing dynamic, interactive This research was made possible through the generous funding support of the Japan International Cooperation Agency (JICA). I would like to express my deepest gratitude Professor Satoshi Kusaka and the other faculty professors, the Kusaka Laboratory and fellow students of the global education course at Naruto University of Education, Japan for their valuable invaluable guidance and encouragement throughout this study. Special thanks also go to the Department of Higher Education, Research, Science and Technology (DHERST) in Papua New Guinea, Sacred Heart Teachers College, and Southern Highlands Teachers College for their cooperation during data collection. Finally, I sincerely thank my friends, wife and my four children for their unwavering support during the development of my research.

TSG9-08

PRESERVICE MATHEMATICS TEACHERS' CONFIDENCE TO TEACH STATISTICS

Hyung Won Kim
University of Texas Rio Grande Valley, United States, hyung.kim@utrgv.edu

Jenq-jong Tsay
University of Texas Rio Grande Valley, United States, jenqjong.tsay@utrgv.edu

Jaewoong Kim
Korea Military Academy, South Korea, jaewoongkim76@gmail.com

Statistics has increasingly become an integral part of K-12 mathematics curricula across many countries. However, research shows that many mathematics teachers struggle with teaching statistics express difficulty and often lack of confidence in teaching statistics. The study in this paper uses the high school version of the Statistics Efficacy to Teach Statistics (SETS-HS), an instrument designed to assess mathematics teachers' self-efficacy in teaching statistics, to examine the readiness of the preservice mathematics teachers to teach at the secondary level. Data were collected at a research university in the Southern United States from the preservice mathematics teachers at the conclusion of one of the two probability and statistics courses–one specifically designed for preservice middle-school teachers and the other intended for mathematics or engineering majors to fulfill their degree requirements. A descriptive statistical analysis highlights the areas where the Preservice Mathematics Teachers face greater challenges in building self-efficacy for teaching statistics as well as the areas where they encounter fewer difficulties.

Keywords: statistics, confidence, preservice teachers, self-efficacy

Background

In recent decades, statistics has steadily become an integral part of K-12 mathematics curricula in many countries worldwide (Schaeffer & Jacobbe, 2019; Weiland & Sundrani, 2022). In the United States, new educational standards such as the Common Core State Standards have placed increased emphasis on data analysis and probability in grades K–12. As a result, many middle and high school teachers are now responsible for teaching statistics, making it essential to develop confidence in their instructional readiness as well as to possess sound content knowledge in statistics. While statistics has become one of the major areas of the K-12 curricula, preservice secondary mathematics teachers (PSMTs) often complete their teacher preparation programs after taking one or two statistics-related courses, and mathematics teachers feel lack of confidence in teaching the subject (Koparan, 2015).

Recent reports from the International Commission on Mathematical Instruction (ICMI) and the International Association for Statistics Education (IASE) indicate that many mathematics teachers acknowledge the practical importance of statistics and are willing to make their classes more relevant to statistics. However, Batanero, Burrill, and Reading (2011) found that many mathematics teachers do not consider themselves well prepared to teach statistics. In the line of identifying how well pre-

pared the PSMTs are to teach statistics, the study reported in this paper explores PSMTs' self-efficacy regarding statistics teaching at the secondary school level.

Several studies have been conducted on the high school mathematics teachers' self-efficacy to teach statistics. Fitzmaurice, Leavy, and Hanninga (2014) conducted a qualitative study with nine prospective mathematics teachers in Ireland to uncover the factors that contribute to positive attitude and the concomitant perceptions of difficulty. The study revealed the potential of teaching practice that challenge the perceived difficulty of learning statistics and provided prospective teachers with the opportunity to develop confidence in statistics as both learners and teachers. In their empirical study, they found that PSMTs were reluctant to select statistics for their student teaching placements, and that their self-efficacy to teach statistics improved after they taught statistics. Irakleous and Panaoura (2015) found, in a case study with two preservice teachers, that preservice teachers with no statistics teaching experience develop the tendency to overestimate their statistics teaching competence. While these studies about self-efficacy regarding statistics teaching found some opposite outcomes, they were all done in a qualitative manner.

Recently, more efforts have been taken to explore PSMTs' self-efficacy to teach statistics. Harrell-Williams and colleagues (2019) developed, and validated with confirmatory factor analysis, an instrument, called Self-Efficacy to Teach Statistics-High School (SETS-HS), that measures mathematics teachers' self-efficacy for teaching statistics. The survey consists of 44 items, each of which falls under one of the three subscales as indicated in the two GAISE Pre-K–12 reports (ASA, 2005), each related to the three learning levels indicated in the report. The three subscales of the SETS-HS instrument are *reading the data* (Level A), *reading between the data* (Level B), and *reading beyond the data* (Level C). The items in Level A address, for example, self-efficacy in recognizing natural variability between observations for individuals and create boxplots for summarizing distributions. At Level B, certain items focus on understanding sampling variability in summary statistics, such as the sample mean and sample proportion. Some Level C items involve tasks such as estimating percentages using the empirical rule, based on the mean and standard deviation of a dataset with an approximately bell-shaped distribution (Harrell-Williams et al., 2019).

The study reported in this paper is part of a mixed-method study investigating PSMTs' attitudes toward statistics and their self-efficacy in teaching it. The study seeks to answer the following:

- At the end of a Probability and Statistics course, what levels of self-efficacy do PSMTs exhibit in teaching statistics, as reflected in the three subscales (Levels A, B, and C) of SETS-HS?
- In which areas or aspects of statistics teaching do PSMTs exhibit the highest self-efficacy? Conversely, in which areas do they experience less challenges?

Methods

To assess PSMTs' self-efficacy in teaching statistics at the secondary level, the research team collected data from five sections of two probability and statistics courses (referred to as Courses A and B) at a public research university in the southern United States. Course A primarily targets preservice mathematics teachers preparing to teach at the middle school level, while Course B is designed for mathematics, science and engineering majors, including preservice high school mathematics teachers. A total of 72 PSMTs participated in the study, with 22 students from two sections of Course A and 50 students from three sections of Course B. Data collection spanned a one-and-a-half-year period from fall 2023 to fall 2024. Informal discussions with course instructors revealed that Course A was taught using a combination of lecture-based and discussion-based methods,

whereas Course B was predominantly lecture-based. Course A had a single instructor for both sections, while Course B was taught by different instructors across its three sections. The four instructors who taught these courses held a PhD–one in Mathematics Education and three in Statistics.

The SETS-HS instrument includes 44 multiple-choice items and six narrative items. Of the multiple-choice questions, the first 12 belong to Level A, the next 15 to Level B, and the final 17 to Level C. Each item is rated on a scale from 1 to 6, where 1 represents "not at all confident" and 6 indicates "completely confident." At the end of each level, participants were presented with two narrative prompts asking them to identify the items they felt least confident teaching and most confident teaching and to explain their reasoning. Among the 72 participants, 66 provided responses to most questions, completing the survey. For data analysis, 11 incomplete responses (72 total responses - 66 valid ones) were excluded. Data were collected through an electronic survey and 40-minute semi-structured interviews with the participants. The participants were selected toward the end of the semester from five sections of the Probability and Statistics course. The interviews sought to examine students' understanding of specific course topics and their follow-up explanation on the attitudes toward statistics. As this paper reports preliminary findings from a larger study, the analysis focuses solely on the survey responses.

Results and Discussion

Analysis of Levels A, B and C

Table 1 presents the mean and standard deviation of the PSMT's self-efficacy scores to teach statistics. After removing incomplete responses and those with identical scores across all survey items, 66 participants remained: 22 from Course A and 44 from Course B. The mean and standard deviation of participants' responses in Table 1 are categorized by the three levels as well as overall.

Table 1. Mean and standard deviation (SD) of PSMTs' self-efficacy to teach statistics

	All five sections from both courses (N=66)				Two sections from Course A (N=22)				Three Section from Course B (N=44)			
Levels	A	B	C	**all**	A	B	C	**all**	A	B	C	**all**
Mean	3.91	3.50	3.48	**3.61**	4.18	3.75	3.5	**3.77**	3.77	3.42	3.45	**3.59**
SD	1.19	1.21	1.30	**1.59**	1.02	1.04	1.17	**1.47**	1.58	1.28	1.38	**1.17**

Note: SD indicates standard deviation.

The table shows a decreasing trend in the mean score and an increasing trend in the standard deviation from Level A to Level C suggesting that interpreting statistical outcomes and displays is more challenging than reading the data.

Items that PSMTs Feel Most Confident Teaching and Least Confident Teaching

A total of 66 students completed the sections of the survey where they were asked to identify items they felt least confident about teaching to high school students, along with explanations, and to select items they felt most confident about teaching, also with explanations. Additionally, the overall mean score for participants in Course A was higher than that of those in Course B. This difference could be because Course A is specifically designed for PSMTs, while Course B includes general math majors and engineering majors.

Analysis of the Items that PSMTs Least/most Confident Teaching

The five items that students reported feeling most confident about were: Item 1 (identifying the data needed to answer specific statistical questions) with 12 responses, Item 6 (creating graphical displays for visualization) with 10 responses, Item 9 (recognizing summary values across different groups) with 10 responses, Item 16 (using histograms to compare two groups) with 11 responses, and Item 17 (using boxplots to compare two groups) with 10 responses. Among the Level C items, Item 35 (interpreting the meaning of the slope and y-intercept of a linear model) received 6 responses, and Item 44 (use of technology) received 7 responses. Notably, three of the five items are from Level A, indicating that students feel more confident in reading data (Level A) than in interpreting data at a deeper level (Level C).

The six items that students reported feeling least confident about were: Items 3 (identifying possible sources of variability in data) with 12 responses, Item 12 (use of technology) with 10 responses, Item 13 (distinguishing between statistical and mathematical questions) with 10 responses, Item 17 (use of boxplots) with 11 responses, Item 29 (use of a two-way frequency table) with 9 responses, and Item 30 (interpreting a two-way frequency table) with 10 responses. It is noteworthy that students found using and interpreting a two-way frequency table particularly challenging.

The findings suggest that, in relation to the topics addressed in probability and statistics courses and the use of technology, PSMTs generally struggle to develop self-efficacy in managing two-way frequency tables and integrating technology into statistics instruction. Specifically, analysis of the instrument's subscales reveals that PSMTs exhibit weaker self-efficacy in tasks involving "reading between the data" (Level B) and "reading beyond the data" (Level C) compared to "reading the data" (Level A). This indicates that PSMTs face challenges in comparing data and drawing meaningful interpretations from graphical displays and tables. These results have significant implications for the teaching of probability and statistics to PSMTs at the collegiate level, suggesting the need for greater focus on guiding students in drawing conclusions and making interpretations based on statistical data.

References

Batanero, C., Burrill, G., & Reading, C. (2011). Overview: challenges for teaching statistics in school mathematics and preparing mathematics teachers. *Teaching statistics in school mathematics-challenges for teaching and teacher education: a joint ICMI/IASE study*, 407-418.

Fitzmaurice, O., Leavy, A., & Hannigan, A. (2014). Why is statistics perceived as difficult and can practice during training change perceptions? Insights from a prospective mathematics teacher. *Teaching Mathematics and Its Applications*, *33*, 230–248.

Harrell-Williams, L. M., Lovett, J. N., Lee, H. S., Pierce, R. L., Lesser, L. M., & Sorto, M. A. (2019). Validation of scores from the high school version of the self-efficacy to teach statistics instrument using preservice mathematics teachers. *Journal of Psycho-educational Assessment*, *37*(2), 194–208.

Irakleous, P., & Panaoura, A. (2015). Personal teaching efficacy in teaching statistics: A case study. *Mathitudes*, *1*(1), 1-18.

Koparan, T. (2015). Difficulties in learning and teaching statistics: teacher views. *International Journal of Mathematical Education in Science and Technology*, *46*(1), 94-104.

Lesser, L. M., & Winsor, M. S. (2009). English language learners in introductory statistics: Lessons learned from an exploratory case study of two pre-service teachers. *Statistics Education Research Journal*, *8*(2), 5-32.

Schoen, R. C., LaVenia, M., Chicken, E., Razzouk, R., & Kisa, Z. (2019). Increasing secondary-level teachers' knowledge in statistics and probability: Results from a randomized controlled trial of a professional development program. *Cogent Education*, *6*(1), 1613799.

Weiland, T., & Sundrani, A. (2022). Opportunities for K-8 students to learn statistics created by states' standards in the United States. *Journal of Statistics and Data Science Education*, *30*(2), 165-178.

TSG9-09

PRESERVICE SECONDARY TEACHERS' PROOF ANALYSIS IN SPHERICAL GEOMETRY

Younggon Bae
The University of Texas Rio Grande Valley, USA, younggon.bae@utrgv.edu

Oh Nam Kwon
Seoul National University, Korea, onkwon@snu.ac.kr

Hyunkyoung Yoon
California State Polytechnic University Pomona, USA, hkyoon@cpp.edu

Kyungwon Lee
Seoul National University, Korea, kyungwon.lee.snu@gmail.com

This study examines how a proof analysis task in a university geometry course engaged preservice secondary teachers (PSTs) in the heuristic refutation of Euclid's Exterior Angle Theorem (EEAT) in spherical geometry by constructing interactive models and investigating counterexamples in dynamic geometry environments (DGEs). The study focuses on how the task fostered two mathematical practices that are pedagogically meaningful: (1) attending to assumptions and limitations in dynamic constructions, and (2) using special cases to analyze mathematical validity. Through analyzing video screencasts and DGE files from nine PSTs, we found that PSTs critically examined technological and mathematical assumptions, such as angle measurement constraints and the generalizability of robust constructions. PSTs also used special cases to test when EEAT holds or fails in spherical geometry and to restate the theorem.

Keywords: Proof and Refutation, Counterexamples, Dynamic geometry, Pedagogical mathematical practices

Dynamic Geometry Environments (DGEs) are useful for critically examining a geometric proof by dragging an interactive diagram of the proof and by exploring counterexamples of the proof (Laborde, 2005; Leung, 2008). In this study, we investigated how preservice secondary teachers (PSTs) in a university geometry course engaged in a proof analysis task on Euclid's Exterior Angle Theorem (EEAT). The theorem states, "In any triangle, if one of the sides is produced, then the exterior angle is greater than either of the interior and opposite angles" (Proposition 1.16 in *Elements*). We focused on two research questions: (1) how did the proof analysis task engage PSTs in the mathematical practices of attending to assumptions and limitations? (2) how did the proof analysis task engage PSTs in the mathematical practices of using special cases?

Background

Proof Analysis Task using DGEs: Proofs and Refutations for Teaching

Researchers have used Lakatos' model of heuristic refutation (Lakatos, 1976) in their studies on instructional design using proof analysis, where students challenge conjectures, generate counterexamples, and revise arguments (Bae, 2019; de Villiers & Heideman, 2014; Komatsu et al., 2014; Komatsu & Jones, 2019). Employing proof analysis tasks in classrooms can enhance students' ability to justify and refine mathematical arguments, support students' transition from informal to formal reasoning, and make proofs more accessible and meaningful (Komatsu, 2017). A *robust construction* can support students in proof analysis tasks using DGEs when examining how a geometry proof works under variation (Stylianides & Stylianides, 2005). In robust constructions, invariant or defining properties stay unchanged under dragging whereas soft constructions or drawings do not necessarily preserve dependence relationships between geometric objects (Laborde, 2005; Leung, 2008). For example, a robust construction of Euclid's proof of EEAT can be constructed by creating an arbitrary triangle ΔABC and constructing auxiliary objects following the steps of the proof. Examining variations of ΔABC by dragging its vertices students can test whether EEAT always holds. When a counterexample is found (e.g., a triangle with an exterior angle smaller than its opposite interior angle), students can refute and refine the proof.

Pedagogically Meaningful Mathematical Practices for Teachers

Wasserman and colleagues (2017) suggested an instructional model for university mathematics courses for teachers by emphasizing disciplinary practices in mathematics that are also pedagogically meaningful. They defined Pedagogical Mathematical Practices (PMPs) as practices that support teachers' task of teaching (e.g., interpreting student thinking, explaining mathematical ideas) and that are grounded in rigorous, advanced mathematical reasoning, such as *attending to mathematical assumptions and limitations* and *using special cases to explain mathematical idea*s. When teaching continuous functions, for example, using special cases of discontinuity (e.g., jump, removable, finite) allows teachers to help students understand the formal definition of continuous functions by providing various non-examples that violate the definition. Engaging in such practices can support PSTs in a university mathematics classroom by connecting classroom teaching and advanced mathematics (Wasserman & McGuffey, 2021).

Methods

Nine PSTs enrolled in a geometry course at a large public university in the Midwest United States participated in this study. The course was designed to guide students to reinvent axiomatic systems of Euclidean and non-Euclidean geometries by employing a DGE, *Geometry Explorer*. We analyzed video recordings of the PSTs' screencast presentations of their proof analysis on EEAT and their DGE construction files. In their presentations, PSTs demonstrated their constructions of the proof for EEAT on Euclidean models and spherical models and explained when the proof does not hold in the spherical model by using counterexamples. Our analysis focused on their DGE constructions and presentations to identify evidence of two mathematical practices: (1) attending to assumptions and limitations and (2) using special cases. We used the constant comparative method (Glaser & Strauss, 1967), focusing on emerging patterns in the PSTs' engagement with these practices.

Findings

Attending to Assumptions and Limitations in Dynamic Geometry Constructions

The PSTs created and explored robust constructions of Euclid's proof of EEAT (Laborde, 2005; Leung, 2008) for examining when the diagram failed (i.e., the exterior angle is smaller than its opposite interior angle). We identified two assumptions/limitations that they attended to: (a) the generalization of robust construction; and (b) the counterclockwise measurement of angles in DGE.

Attending to the Generalization of Robust Construction

All nine PSTs produced robust constructions of Euclid's proof of EEAT on an arbitrary triangle ΔABC. The constructions were initially determined by three independent vertices A, B, and C while preserving dependence relationships between consequent objects in the diagram as defined in the given proof. The vertices served as three dimensions of variation (Leung, 2008) that are independent variables in further investigations. This indicates that the PSTs distinguished constructions from non-generalizable drawings that fail to preserve properties under variations. For example, Deming demonstrated that A, B, C in his construction could be dragged independently to the other two vertices. He said, "First of all, we make sure three vertices A, B, C can move individually. In addition, we can move all three points to make the angle BCD [the exterior angle] smaller than the angle CBA [the opposite interior angle]". Later, he demonstrated counterexamples of EEAT in three different ways by moving one vertex only at one time while other vertices stayed.

Attending to the Counterclockwise Measurement of Angles in the Software

In the DGE used in this study, Geometry Explorer, an angle to be measured should be identified by selecting three vertices counterclockwise. PSTs were instructed about this way of measuring an angle. This may cause a problem because dragging may flip the order of vertices in the angle measurement and change the measured value (i.e., from a to $360 - \alpha$). Eight of the nine PSTs created robust construction with valid angle measurements in the counterclockwise orientation. However, only one PST, Odessa explicitly addressed this limitation of counterclockwise measurements and demonstrated what would happen if she flipped the triangle by dragging one vertex over its opposite side of the triangle.

> Odessa: If you move C around the angle [the exterior angle] is still larger [than the opposite interior angles] except for when you flip it over the opposite segment $\overline{AB}$, but that is just the way the software measures the angles. The relationship is still the same, and it just measures the opposite way around [clockwise].

Her attention to the angle measurement allows her to produce correct angle measurements, keep the measurements consistent while dragging, and interpret when the angles are flipped.

Using Special Cases to Examine When the Exterior Angle Theorem is Valid in Spherical Geometry

Eight PSTs, who produced robust constructions with valid angle measurements, used one or more cases of counterexamples to refute EEAT in spherical geometry. They demonstrated how to drag one of the dimensions of variations (Leung, 2008) of the triangle to contrast examples and counterexamples that yield an exterior angle smaller than one of its opposite interior angles. Furthermore, we identified two ways of using special cases of counterexamples for refining the proof: (a) generalizing the set of counterexamples, and (b) refining the scope of the proof.

Using Special Cases to Generalize Counterexamples

Three PSTs (Eric, Deming, Lynn) used multiple special cases of counterexamples strategically to justify their generalization of counterexamples. They demonstrated special cases that exhibit a range of variations while maintaining the exterior angle smaller than it opposite interior angle. For example, Eric presented his counterexamples by dragging two vertices of ΔABC to opposite ends of the sphere (i.e., south and north poles) and by dragging all three vertices on the same great circle (i.e., equator). Those special cases were purposefully selected to demonstrate Eric's generalization of counterexamples (i.e., counterexamples may occur when two vertices of the triangle are far apart from each other). This shows Eric's systemic approach, which was varying the diagram across all three dimensions of variation in the robust construction while maintaining the desired invariant property (Leung, 2008; Stylianides & Stylianides, 2005).

Use Special Cases to Refine the Scope of the Proof

Two PSTs (Yin and Lynn) used special cases of counterexamples to limit the set of triangles that allow the proof to hold in spherical geometry. They used these cases among other counterexamples to investigate how the exterior angle and its opposite interior angle are related. The use of special counterexamples allowed them to identify exactly when EEAT starts to fail, that is the boundary between examples and counterexamples of EEAT. For example, Yin presented a special case of counterexample where the exterior angle and its opposite interior angle are congruent. This allowed her to explain why EEAT does not hold for all triangles in spherical geometry and to identify the axiomatic differences in the relationship between lines in Euclidean and spherical systems. She said, "So why is this happening? … since the spherical geometry does not satisfy the postulate that two lines … intersect at only one point." This way of using special cases allowed her to engage in reasoning about different axiomatic systems that result in different conclusions for the same mathematical statement and its proof.

Discussion and Conclusion

The study showed that a proof analysis task using a DGE provided PSTs with opportunities to engage in mathematical practices that are pedagogically meaningful for teaching. They attended to the assumptions of generalizability in dynamic geometry and technological limitations of angle measurement. They also used special cases of triangles as counterexamples to disprove and refine the theorem in spherical geometry. The task has the potential to deepen their understanding of school geometry within the Euclidean system and to prepare them to employ the heuristic refutation approach for teaching properties of geometric objects and proof production in secondary classrooms. This study also contributed to the literature by situating PMPs within a technology-integrated university mathematics course for teachers.

References

Bae, Y. (2019). *Dynamic geometry task design for axiomatic geometry: Student engagement with axiomatic reasoning*. [Doctoral dissertation, Michigan State University].

de Villiers, M., & Heideman, N. (2014). Conjecturing, refuting and proving within the context of dynamic geometry. *Learning and Teaching Mathematics*, *17*, 20–26.

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Hawthorne, NY: Aldine.

Komatsu, K. (2017). Fostering empirical examination after proof construction in secondary school geometry. *Educational Studies in Mathematics*, *96*(2), 129–144.

Komatsu, K., & Jones, K. (2019). Task Design Principles for Heuristic Refutation in Dynamic Geometry Environments. *International Journal of Science and Mathematics Education*, *17*(4), 801–824. https://doi.org/10.1007/s10763-018-9892-0

Komatsu, K., Tsujiyama, Y., Sakamaki, A., & Koike, N. (2014). Proof problems with diagrams: An opportunity for experiencing proofs and refutations. *For the Learning of Mathematics*, *34*(1), 36–42.

Laborde, C. (2005). Robust and soft constructions: Two sides of the use of dynamic geometry environments. In S. C. Chu, H. C. Lew, & W. C. Yang (Eds.), *Proceedings of the 10th Asian Technology Conference in Mathematics* (pp. 22–35). Cheong-Ju, South Korea: Korea National University of Education.

Lakatos, I. (1976). *Proofs and refutations: The logic of mathematical discovery*. Cambridge University Press.

Leung, A. (2008). Dragging in a dynamic geometry environment through the lens of variation. *International Journal of Computers for Mathematical Learning*, *13*, 135–157.

Stylianides, G. J., & Stylianides, A. J. (2005). Validation of solutions of construction problems in dynamic geometry environments. *International Journal of Computers for Mathematical Learning*, *10*(1), 31–47. https://doi.org/10.1007/s10758-004-6999-x

Wasserman, N. H., Fukawa-Connelly, T., Villanueva, M., Mejia-Ramos, J. P., & Weber, K. (2017). Making real analysis relevant to secondary teachers: Building up from and stepping down to practice. *PRIMUS*, *27*(6), 559–578.

Wasserman, N. H., & McGuffey, W. (2021). Opportunities to learn from (advanced) mathematical coursework: A teacher perspective on observed classroom practice. *Journal for Research in Mathematics Education*, *52*(4).

TSG9-10

GUIDED REINVENTION OF THE DEFINITIONS OF REDUCIBLE AND IRREDUCIBLE ELEMENTS

Kaitlyn Stephens Serbin
The University of Texas Rio Grande Valley, USA, Kaitlyn.serbin@utrgv.edu

Younggon Bae
The University of Texas Rio Grande Valley, USA, younggon.bae@utrgv.edu

Sthefanía Espinosa
The University of Texas Rio Grande Valley, USA, sthefania.espinosa01@utrgv.edu

Informed by Realistic Mathematics Education, we designed a hypothetical learning trajectory on guided reinvention of reducible and irreducible elements in integral domains. We created experientially real context problems for use in a teaching experiment, in which participants used algebra tiles as an emergent model of factoring integers and quadratics in $\mathbb{Z}[x]$. In their mathematical activity, the algebra tiles became their models for abstracting the shared structure of (ir)reducible elements in $\mathbb{Z}$and $\mathbb{Z}[x]$, which they used to formally define (ir)reducible elements. In this paper, we provide a summary of the progression of participants' reasoning that were productive to reinvent the definitions of reducible and irreducible elements of integral domains.

Keywords: Abstract algebra, Hypothetical learning trajectory, Reducible and irreducible elements in integral domain

Abstract algebra can be beneficial for helping in-service and pre-service secondary teachers (IPSTs) develop unified, coherent understandings of algebraic concepts that span K-16 curricula, such as the factorization of integers and polynomials (e.g., Murray et al., 2017; Novotná & Hoch, 2008; Usiskin, 1974). Learning about the (ir)reducibility of elements in $\mathbb{Z}$ and $\mathbb{Z}[x]$ can help reshape teachers' understanding of the analogous structure of integers and polynomials and inform their teaching of related content, e.g., the factorization of polynomials. In this study, we used Realistic Mathematics Education (RME; Freudenthal, 1991; Gravemeijer, 1999) to design a hypothetical learning trajectory (HLT, Andrews-Larson et al., 2017; Simon, 1995) of how IPSTs can reinvent the definitions of reducible and irreducible elements. We address this research question: Which ways of reasoning do IPSTs engage in as they reinvent the definitions of reducible and irreducible elements? In this study, we used the following definitions of (ir)reducibles in an integral domain R: A nonzero, non-unit element $r \in R$ is called *irreducible* in R if whenever $r = ab$ with $a,b \in R$, at least one of a or b must be a unit in R. Otherwise is said to be *reducible*.

Literature Review

Guided Reinvention of Algebraic Structures

One major strand of research on students' understanding of abstract algebra is focused on students' guided reinvention (Freudenthal, 1991) of algebraic structures. Researchers illustrated how

students' reasoning about properties of algebraic concepts they are familiar with (e.g., equation solving) could be leveraged toward their reinvention of the formal definitions of abstract algebraic structures like groups, rings, and isomorphisms (e.g., Cook, 2012; Larsen, 2009; Larsen & Lockwood, 2013).

Abstract Algebra for Secondary Teachers

Reasoning about the properties of abstract algebraic structures can help IPSTs develop a more robust understanding of the secondary algebra content they teach. Researchers have shown how reasoning about the properties of algebraic structures can help students or IPSTs develop unified understandings (Lee & Heid, 2018; Zandieh et al., 2017) of secondary mathematics content. Having a unified understanding of a concept involves recognizing the shared overarching structure of the concept among different instantiations of that concept in different contexts and being able to reason about that concept generally without needing to reference a specific context. Researchers explored the coherence of students' understandings of the factorization of integers and polynomials as instantiations of the decomposition of elements in integral domains (Lee, 2018; Lee & Heid, 2018).

Theoretical Background

Design Research

Design Research (Gravemeijer et al., 2003) involves a cycle of designing and implementing sequences of mathematical tasks to be used during instruction, analyzing students' learning, and revising the task sequences based on the student learning outcomes. Design research may involve creating an HLT, which consists of the learning goals, the learning activities, and the students' reasoning (Simon, 1995). The process of designing, implementing, and revising HLTs allows researchers to discover students' ways of reasoning that anticipate formal mathematics, identify ways to evoke these ways of reasoning, and identify ways to leverage ways of reasoning to support students' development of formal concepts.

Realistic Mathematics Education

Our HLT is informed by the instructional design theory of RME (Gravemeijer, 1999), which conceptualizes mathematics as a human activity. The HLT is designed based on the three RME heuristics: 1) students' engagement in experientially real context problems/tasks, 2) students' development of an emergent model, and 3) students' guided reinvention of a mathematical concept. Experientially real tasks allow students to develop an intuitive understanding that will help them create an emergent model used in that context, defined as a model-of their situated activity. The emergent model shifts from being a model-of a student's activity to a model-for formalizing mathematics as students mathematize their mathematical activity (Zandieh & Rasmussen, 2010).

Methods

We conducted a teaching experiment (Steffe & Thompson, 2000) in a large public research university in the Southern US. Six mathematics graduate student IPSTs participated in this study (Josie and Raul in Group A; Javier and Roberto in Group B; Kim and Taylor in Group C). They had taken Abstract Algebra courses and had learned about integral domains but not Unique Factorization Domains (UFDs). Each group met with the research team for two or three 1.5-hour sessions (online or in-person), in which they collaboratively worked through the task sequence. A teacher-research-

er guided the session, provided students with prompts to work on, elicited evidence of students' reasoning, and asked follow-up questions for clarification. We collected video recordings of the sessions and the groups' written work. We performed a retrospective, qualitative analysis of participants' ways of reasoning. We first used inductive coding (Miles et al., 2013) to identify participants' ways of reasoning, their use of an emergent model, and their engagement in descriptive and constructive defining that were evident in their work on the tasks. Then, in the second round of coding, we used deductive coding (Miles et al., 2013) to recode the interview transcripts and identified what we refer to as "key" ways of student reasoning that were common to all students and productive in guiding students toward the intended HLT goal. We lastly revised the HLT, including the task sequence and potential ways of student reasoning while performing the tasks.

Findings

The task sequence consists of three Task Sets that guide students to conjecture about factorizations in $\mathbb{Z}$ and $\mathbb{Z}[x]$ and to define reducible and irreducible elements in an integral domain. Task Set 1 guided the students to develop emergent models of algebra tiles representing the factorizations of integers. Task Set 2 focused on factorizations of quadratics with integer coefficients in $\mathbb{Z}[x]$. Task Set 3 guided them to abstract the common structure between (ir)reducibles in $\mathbb{Z}$ and $\mathbb{Z}[x]$ and produce formal definitions of them via engagement in constructive defining (Rasmussen et al., 2005; Zandieh & Rasmussen, 2010). In this paper, we provide descriptions of the task sequence, HLT, and a summary of the analysis of student reasoning from Task Set 2. The complete description of student reasoning and the HLT are available in Serbin et al. (2024).

Task Set 2: Contrasting reducible and irreducible quadratics in $\mathbb{Z}[x]$

Task Set 2 includes experientially real tasks designed to guide students to use the emergent model of algebra tile factorizations as a model of factoring quadratics with integer coefficients in $\mathbb{Z}[x]$. The tasks guided the students to use their models for identifying differences between reducible and irreducible quadratics that can be represented by rectangular arrays or not (Figure 1). We hypothesized students' progression in Task Set 2 as follows: (H2) Students use algebra tiles to model factorizations of polynomials and make conjectures about the (ir)reduciblity of polynomials in $\mathbb{Z}[x]$. (H2.1) Students recognize some quadratic polynomials can be arranged into a rectangular array. Students conjecture that quadratics that can be factored/reduced can be arranged into a rectangular array. Students conjecture that the factors of the quadratics are the side lengths of the rectangular arrays. (H2.2) Students recognize some quadratic polynomials cannot be arranged into a rectangular array. Students conjecture that quadratics that cannot be factored/reduced cannot be arranged into a rectangular array. (H2.3) Students give examples of quadratics that can be factored and quadratics that cannot be factored.

1. Try to arrange these quadratics in a rectangular array using algebra tiles. Each side length of a rectangular array cannot be a constant number and must include the variable x.
   a) $3x^2 + 9x$ b) $x^2 + 3x + 3$ c) $2x^2 + 4x$ d) $x^2 + 4x + 3$ e) $x^2 + 5x + 7$
2. What is the difference between the quadratics you **can** arrange into a rectangular array and the quadratics you **cannot** arrange into a rectangular array? Make a conjecture.
3. Create 3 examples of quadratics whose algebra tile representations could be arranged into rectangular arrays.
4. Create 3 examples of quadratics whose algebra tile representations could NOT be arranged into rectangular arrays.
5. Does your conjecture from Task 2 about the arrangement of the algebra tiles hold true for these examples? Why or why not?
6. You conjectured that: *A quadratic can be arranged into a rectangular array with algebra tiles if and only if the quadratic is factorable*. Let's test this conjecture with the following quadratics.
   a) $x^2 - 4$ b) $x^2 - 2$ c) $x^2 + 4$
7. Refine your conjecture: *Quadratics can be arranged into a rectangular array with algebra tiles if and only if:*

Figure 1. Task set 2

Summary of the Students' Key Ways of Reasoning in Task Set 2

In the student work on Task Set 2, we identified five key ways of students' reasoning (2a-2e) that were common to all three groups and that were productive in guiding them toward the HLT goal. First, students reasoned that a rectangular array of algebra tiles representing a quadratic in $\mathbb{Z}[x]$ has side lengths of linear factors (2a). While comparing reducible and irreducible quadratics, they conjectured that the quadratics in $\mathbb{Z}[x]$ that can be nontrivially factored can be arranged into a rectangular array with algebra tiles where the factors of the quadratic are the side lengths of the rectangle (2b) and that those that cannot be nontrivially factored cannot be arranged into a rectangular array (2c). As they confirmed the conjecture and examined counterexamples, they noticed that imaginary numbers and other non-integers cannot be side lengths of a rectangular array representing a quadratic (2d). They finalized their conjecture: A quadratic in $\mathbb{Z}[x]$ can be arranged into a rectangular array with algebra tiles if and only if the quadratic is factorable over the integers (2e). In particular, the teacher-researcher guided students to specify the coefficients of linear factors of quadratics by providing counterexamples to their ways of reasoning (2b) and (2c), such as $x^2 - 2 = (x - \sqrt{2})(x + \sqrt{2})$ that is factorable over $\mathbb{R}$ but cannot be represented in the algebra tile model. This led them to leverage their ways of reasoning (2d) and formalize the defining characteristics of reducible quadratics in $\mathbb{Z}[x]$ by emphasizing factorization "over the integers" as shown in (2e).

Conclusion

In this paper, we summarize the analysis of reasoning that students engaged in as they progressed through Task Set 2. This task sequence provided the opportunity to reinvent formal mathematical concepts of (ir)reducible elements based on their canonical understandings of the secondary mathematics they teach such as factoring integers and quadratics. The analysis of students' ways of reasoning allowed us to revise the initial hypotheses of student reasoning and the task design, allowing us to refine the HLT.

References

Andrews-Larson, C., Wawro, M., & Zandieh, M. (2017). A hypothetical learning trajectory for conceptualizing matrices as linear transformations. *International Journal of Mathematical Education in Science and Technology, 48*(6), 809–829.

Cook, J. P. (2012). *A guided reinvention of ring, integral domain, and field*. The University of Oklahoma.

Freudenthal, H. (1991). *Revisiting mathematics education*. Dordrecht: Netherlands.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning, 1*(2), 155–177.

Gravemeijer, K., Bowers, J., & Stephan, M. (2003). Chapter 4: A hypothetical learning trajectory on measurement and flexible arithmetic. *Journal for Research in Mathematics Education Monograph, 12*, 51–66.

Larsen, S. (2009). Reinventing the concepts of group and isomorphism: The case of Jessica and Sandra. *The Journal of Mathematical Behavior, 28*(2-3), 119–137.

Larsen, S., & Lockwood, E. (2013). A local instructional theory for the guided reinvention of the quotient group concept. *The Journal of Mathematical Behavior, 32*(4), 726–742.

Lee, Y. (2018). *University students' school mathematics understanding and its growth in their learning of collegiate mathematics: Through the lens of a transformative transition framework* [Doctoral dissertation, The Pennsylvania State University]. Penn State University Libraries.

Lee, Y., & Heid, M. K. (2018). Developing a structural perspective and its role in connecting school algebra and abstract algebra: A factorization example. In *Connecting Abstract Algebra to Secondary Mathematics, for Secondary Mathematics Teachers* (pp. 291–318). Cham: Springer.

Miles, M. B., Huberman, A. M., & Saldaña, J. (2013). *Qualitative data analysis: A methods sourcebook (3rd ed.)*. Thousand Oaks, CA: Sage Publications.

Murray, E., Baldinger, E., Wasserman, N., Broderick, S., & White, D. (2017). Connecting Advanced and Secondary Mathematics. *Issues in the Undergraduate Mathematics Preparation of School Teachers, 1*.

Novotná, J., & Hoch, M. (2008). How structure sense for algebraic expressions or equations is related to structure sense for abstract algebra. *Mathematics Education Research Journal, 20*(2), 93–104.

Rasmussen, C., Zandieh, M., King, K., & Teppo, A. (2005). Advancing mathematical activity: A practice-oriented view of advanced mathematical thinking. *Mathematical Thinking and Learning, 7*(1), 51–73.

Serbin, K. S., Bae, Y., & Espinosa, S. (2024). Secondary teachers' guided reinvention of the definitions of reducible and irreducible elements. *The Journal of Mathematical Behavior, 76*, 101188.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114–145.

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. *Handbook of Research Design in Mathematics and Science Education*, 267–306.

Usiskin, Z. (1974). Some corresponding properties of real numbers and implications for teaching. *Educational Studies in Mathematics*, 279–290.

Zandieh, M., Ellis, J., & Rasmussen, C. (2017). A characterization of a unified notion of mathematical function: The case of high school function and linear transformation. *Educational Studies in Mathematics, 95*(1), 21–38.

Zandieh, M., & Rasmussen, C. (2010). Defining as a mathematical activity: A framework for characterizing progress from informal to more formal ways of reasoning. *The Journal of Mathematical Behavior, 29*(2), 57–75.

Acknowledgment

This paper is based on the article published in *The Journal of Mathematical Behavior*: "Secondary teachers' guided reinvention of the definitions of reducible and irreducible elements," by Serbin, K. S., Bae, Y., & Espinosa, S. (2024), https://doi.org/10.1016/j.jmathb.2024.101188.

Topic Study Groups 9
(Day 4)

TSG9-11

TEACHING MATERIALS FOR DIFFERENTIAL EQUATIONS WITH APPLICATION TO STEM PROBLEMS

Satoru Takagi
Waseda University, Japan, satoru@waseda.jp

Kesayoshi Hadano
Yamaguchi University, Japan, hakesayoshi@gmail.com

Differential equations is a fundamental yet important topic for STEM students. However, many of them find traditional lectures unengaging due to a lack of real-world applications and graphical analysis. This study refines Wylie's approach which integrates differential equations with vibration-related phenomena. The process of deriving the equation is made more clearly, and graphical analysis of solution's properties is refined to enhance understanding. This teaching material aims to bridge mathematics and engineering, which makes differential equations more relevant for STEM students.

Keywords: differential equation, STEM problem, vibration-related topics, equation formulation, solution analysis

Essential Mathematics for STEM Students

University mathematics is particularly important for STEM students. However, it is recognized that only a few students have developed the mathematical ability needed for their specialties (e.g., Gonen & Korkmaz, 2022). Also, many STEM students do not fully recognize the effectiveness of mathematics in their respective fields (e.g., Kwon et al., 2019; Shongwe, 2024). To address these issues, researchers have explored teaching materials and methods that bridge mathematics and engineering (e.g., Tague & Czocher, 2016; Takagi et al., 2022; Tsironis & Neofotistos, 1998).

Research on mathematics education for engineering students highlight the issues they face in using traditional teaching materials. Randahl (2012) examined how first-year engineering students interact with calculus textbooks and found that they preferred lecture notes, often disregarding the textbook except for completing assigned tasks. He attributed this tendency to the formal and abstract presentation of mathematical concepts in textbooks. Similarly, Harris et al. (2015) identified difficulties in understanding mathematics for students through interviews, and criticized both the lack of contextual applications in traditional pedagogy and the unclear relevance of mathematics to engineering. Other studies have also noted that many textbooks lack practical contexts demonstrating the necessity of extending the existing knowledge and that their style is highly theoretical. As a result, textbooks are often difficult for students to understand (e.g., Randahl, 2012; Randahl & Grevholm, 2010). It remains difficult to find specific teaching materials that effectively address them.

Among the topics in university mathematics, one that is particularly important for STEM students is differential equations. It seems important that teachers not only explain the method for obtaining solutions of differential equations, but also use engineering topics related to their specialty, show the process of deriving the differential equations governing the phenomena in question, and provide a graphical analysis of properties of the obtained solutions. Differential equations are familiar with the motion of a body which receives forces, a typical example is the motion of a body under a restoring force by a spring. Simple harmonic motion: $d^2y / dx^2 + ay = 0$ with positive constant a; is important as a base, but as a differential equation problem it is too simple and is insufficient as a specialty-related topic for STEM students. In contrast, the problems treated by Wylie (1966) are extremely useful for STEM students (e.g., the vertical motion of a body which fixes a damper and is suspended by a spring receiving vertical external forces). But derivation of the differential equation is circuitous, and the analysis of solution's properties is complicated. In this study, we propose an example of teaching materials for differential equations that refines the description of mechanical vibration in a part of Wylie's textbook treating vibrations in mechanics and electric circuits through a lecture made using the textbook for first-year students after they learned how to obtain general solutions of linear differential equations. By this material, the derivation of the equation becomes more straightforward, and the graphical analysis of the solution's properties is explained more clearly.

Formulation for Vertical Motion of a Body Suspended by a Spring

Wylie (1966, Chap. 5) considered the vertical motion of a body with weight w fixing a damper and suspended by a spring subjected to vertical external forces as shown in Figure 1.

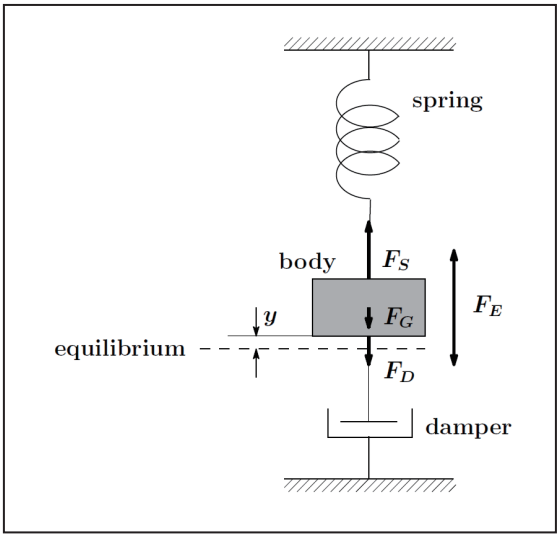


Figure 1. Vertical motion of a body subjected to forces from a spring, damper and external forces

The body receives forces by gravitation $F_G = w$, spring F_S, damper F_D, and external forces F_E. If the upward displacement of the body from its equilibrium of gravitational and spring forces is y, upward velocity and acceleration of the body are dy / dt and d^2y / dt^2, respectively. If we assume that $F_s = w - ky$, $F_D = -c\, dy / dt$, $F_E = F_0 \cos\omega t$ with spring constant k and frictional constant c, then the formulation of Newton's law and arrangement give the equation

$$\frac{w}{g}\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = F_0 \cos \omega t \quad (1)$$

As this equation (1) is nonhomogeneous, the general solution of the equation (1) includes particular integrals. This is complicated for first-year students. Therefore, in this study, we propose for simplicity to use the mass $M = w / g$ and ignore the external forces, that is, $F_E \equiv 0$ in Figure 1 called free motion. Then, the simpler homogeneous equation

$$M\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = 0 \quad (2)$$

is obtained. It is suitable for the first-year students, beginner at differential equations, to obtain the general solution of the equation (2) and to consider properties of the general solution of it.

Solution of Free Motion and Its Property

We here focus on the free motion described as the equation (2). As shown below, in this teaching material, it is important not only to obtain a solution but also to analyze its properties in depth.

Solution for free motion

Considering the characteristic equation

$$Mm^2 + cm + k = 0 \quad (3)$$

of the equation (2), we obtain the solutions m_1 and m_2 of the equation (3) as follows:

$$m_1 = \frac{-c+\sqrt{c^2-4kM}}{2M}, \quad m_2 = \frac{-c-\sqrt{c^2-4kM}}{2M} \quad (4)$$

Let us consider the signs of $c^2 - 4kM$ and analyze each case separately.

Property of the solution for free motion

Case 1: $c^2 - 4kM > 0$

In this case, it is known from the basic theory of homogeneous second order linear differential equations with constant coefficients that the general solution of the equation (2) becomes as

$$y = Ae^{m_1 t} + Be^{m_2 t} \quad (5)$$

where arbitrary constants A and B are determined by the initial conditions. Since m_1 and m_2 are both negative derived from $\frac{-c}{2M} < 0$ and $\sqrt{c^2 - 4kM} < c$ with positive constants M, c and k, it follows that both two terms in the solution (5) exponentially decrease with progress of time towards zero, which indicates the overdamped phenomenon. It is shown that only one value of time $t = \frac{1}{m_1 - m_2}\ln(-\frac{B}{A})$ which makes $y = 0$ exists if A and B take different signs. Pattern of the solution (5) is indicated in Figure 2 for some combinations of A, B, m_1 and m_2. If A and B have the same sign or if either one of the two is 0, the value of t which makes $y = 0$ does not exist. Similar figures are easily obtained using software which is common to Case 2 and Case 3.

Case 2: $c^2 - 4kM = 0$

In this case, since the solution of the equation (3) is $m = \frac{-c}{2M}$, the general solution of the equation (2) becomes as

$$y = Ae^{mt} + Bte^{mt} \quad (6)$$

with arbitrary constants A and B. This case is called as critical damping, and the value of c equals $2\sqrt{kM}$. Time which makes $y = 0$ is obtained as $t = -A / B$ from the solution (6), which means that there exists $t > 0$ only when $B \neq 0$ and the signs of A and B are different.

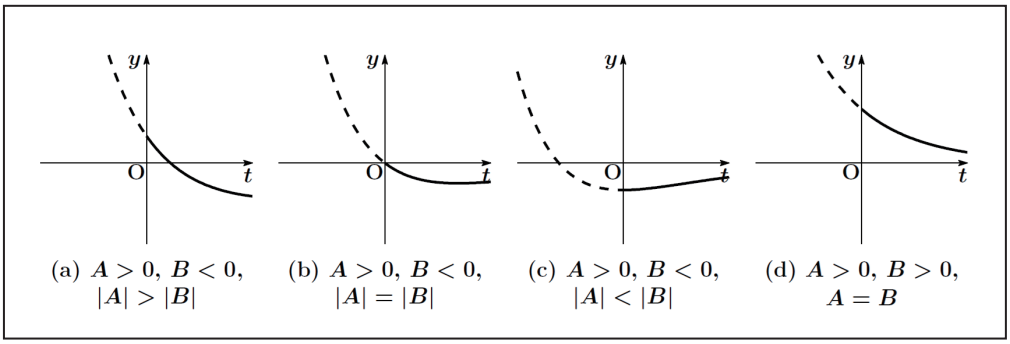


Figure 2. Time series of the vertical displacement of a body for overdamped and critically damped

Case 3: $c^2 - 4kM < 0$

In this case, the solutions of the equation (3) are conjugate complex numbers m_1, $m_2 = -p \pm iq$ with $p = \frac{c}{2M}$ and $q = \frac{\sqrt{4kM-c^2}}{2M}$. Then, the general solution of the equation (2) is given as

$$y = e^{-pt}(A\cos qt + B\sin qt) \quad (7)$$

with arbitrary constants A and B. This solution is like trigonometric function whose amplitude reduces with time and is called as damped oscillation. Schematic view of the solution (7) is indicated in Figure 3 for a few values p and q.

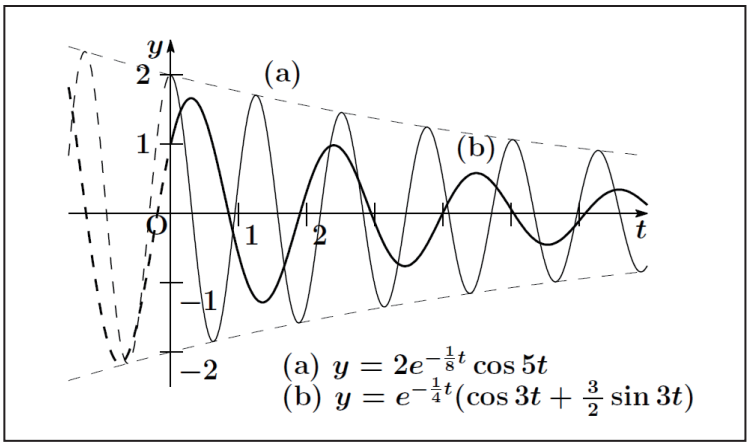


Figure 3. Typical time series of the vertical displacement for damped oscillation

Closing Remarks

The key contributions of this study are the concise derivation of differential equations and the visual analysis of solution's properties, both of which make the concept more accessible and pedagogically effective. Compared to traditional methods, this teaching material offers a more intuitive and motivating way to understand the properties of differential equations. We believe it will enhance

students' conceptual understanding of both differential equations and calculus. We will apply this teaching material in classroom instruction and evaluate its effects in the near future.

References

Gonen, M., & Korkmaz, O. (2022). Do students' STEM skill levels affect their math and science achievement? *International Journal of Technology in Education, 5(4)*, 552–570.

Harris, D., Black, L., Hernandez-Martinez, P., Pepin, B., Williams, J. & with the TransMaths Team (2015). Mathematics and its value for engineering students: What are the implications for teaching? *International Journal of Mathematical Education in Science and Technology, 46(3)*, 321–336.

Kwon, H., Vela, K., Williams, A. M., & Barroso, L. R. (2019). Mathematics and Science Self-efficacy and STEM Careers: A Path Analysis. *Journal of Mathematics Education, 12(1)*, 74–89.

Randahl, M. (2012). First-year engineering students' use of their mathematics textbook — opportunities and constraints. *Mathematics Education Research Journal, 24*, 239–256.

Randahl, M. & Grevholm, B. (2010). Learning opportunities offered by a classical calculus textbook to engineering students about differentiation. *Nordic Studies in Mathematics Education, 15(2)*, 5–27.

Shongwe, B. (2024). The effect of STEM problem-based learning on students' mathematical problem-solving beliefs. *EURASIA Journal of Mathematics, Science and Technology Education, 20(8)*, em2486.

Tague, J. & Czocher, J. (2016). A theoretical approach to ensuring instructional and curricular coherence in the flipped classroom model of a differential equations course. *International Journal of Research in Undergraduate Mathematics Education*, *2*, 223–245.

Takagi, S., Hadano, K. & Yamaguchi, S. (2022). Teaching materials on calculus as seen from the application to engineering. *Hiroshima Journal of Mathematics Education*, *15(2)*, 155–165.

Tsironis, G. P. & Neofotistos, G. (1998). Using telematics to overcome educational constraints: Teaching differential equations using the world-wide-web and multimedia technology. In Verdejo, F. & Davies, G. (Eds.), *The virtual campus: Trends for higher education and training.* (pp.296–303). Springer.

Wylie Jr., C. R. (1966). *Advanced engineering mathematics (Third ed.).* McGraw-Hill.

TSG9-12

A STUDY OF ACTIVITIES USING GEOGEBRA IN METHOD OF LAGRANGE MULTIPLIER

Manabu Yoshida
Yamato University, Japan, yoshida.manabu@yamato-u.ac.jp

Takuma Takayama
Yamagata University, Japan, takayama@e.yamagata-u.ac.jp

In the first year of university in Japan, students study differential and integral calculus, which involves multi-variable calculus. As a way of applying the implicit function theorem to functions of two variables, students learn the Lagrange multiplier method, but some students have difficulty understanding it. In this study, we found that, using GeoGebra, students' understanding deepened when they visually considered the points that take extreme values on a surface. In addition, for students majoring in mathematics at the faculty of education, GeoGebra activities were also effective in improving digital education skills and increasing motivation.

Keywords: method of Lagrange multiplier, GeoGebra, mathematical activities

In the first year of university in Japan, after studying the differential and integral calculus of single-variable functions, students move on to study multi-variable functions, partial differentiation, total differentiation, the Lagrange multiplier method, and multiple integrals. The motivation for this research was students' difficulty in understanding the Lagrange multiplier method. This is a method useful for finding the extremum of a multi-variable function under given constraints. In this study, we will deal with the following conditional extremum problem for a two-variable function.

> Given the condition $g(x,y)=0$, find the point (x,y) that gives the extremum of the function $f(x,y)$.

We assume that the constraint We assume that the constraint function $g(x,y)$ is differentiable and the objective function $f(x,y)$ is of class C^1. The Lagrange multiplier method prepares a new variable λ (undetermined multiplier) and reduces the problem to finding the extremum of a three-variable function

$$F(x,y,\lambda) := f(x,y) - \lambda g(x,y);$$

in other words, a point (x,y) satisfies the following simultaneous equations:

$$\begin{cases} g(x,y) = 0 \\ f_x(x,y) = \lambda g_x(x,y) \\ f_y(x,y) = \lambda g_y(x,y) \end{cases} \cdots\cdots (*)$$

with the constant λ as a *candidate* for an extremum (Stewart, 2011).

In teaching, there are two points that are difficult for students to understand. First, students are not used to dealing with *functions* of two variables, and it is difficult for them to visualize the surface; and second students use the implicit function theorem to consider the extremum, but it is difficult for them to understand the connection with the undetermined multiplier.

To address these difficulties, gradient-based interpretations are often used. At the point (x,y) where the extreme value is taken, the contour line $f(x,y) = k$ of the surface $z = f(x,y)$ is tangent to $g(x,y) = 0$ when projected on the xy plane. *Therefore*, the respective normal vectors are multiplied by a constant. Since the gradient is one of the normal vectors, there exists a constant λ such that $\nabla f = \lambda \nabla g$. The gradient vectors can be represented as $\nabla f = (f_x, f_y)$ and $\nabla g = (g_x, g_y)$ respectively. Thus, we have

$$(f_x, f_y) = \lambda(g_x, g_y),$$

so that the simultaneous equations (*) *are* obtained.

The Lagrange multiplier method can be generalized to the problem of conditional extremum of functions of three or more variables, and multiple constraint conditions can be set (Jost, 2008). It has various applications in fields such as economics, information theory, and fluid mechanics, as well as in the natural sciences, and students are expected to fully understand and utilize it. In this study, we gave a lecture on the Lagrange multiplier *method* to students majoring in mathematics at the faculty of education; the goal was to deepen their understanding of the solution method while imagining the graph through mathematical activities using GeoGebra. In addition, we aimed to improve the skills of digital education as an approach to teacher training for students who aim to become teachers.

Aims of this Study

The aim of this study is to provide learners with a visual and active understanding of the fact that a contour line $f(x,y) = k$ is tangent to $g(x,y) = 0$ at the point giving the extremum by using GeoGebra. The learning objectives are to understand where the point giving the extremum is on the surface of the curve and to be able to *find* candidates for the point giving the extremum using the gradient. Specifically, in the lecture, the teacher presents the following Problem (A) to the students and asks them to think of ways to solve it.

(A) Find a candidate for *points* giving the extremum of $f(x,y) = x + y$ under the condition $g(x,y) = x^2 + y^2 - 4 = 0$.

See Figures 1, 2, and 3. *These* figures show the actual function for Problem (A) entered into GeoGebra.

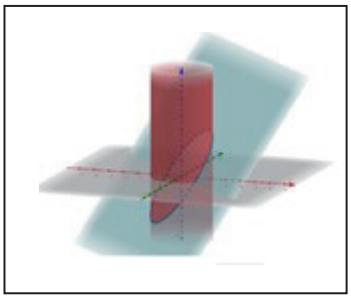


Figure 1

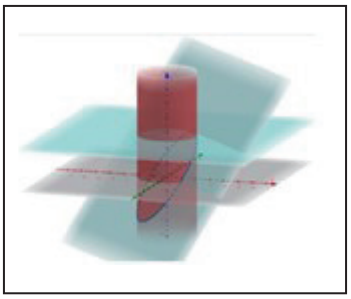


Figure 2

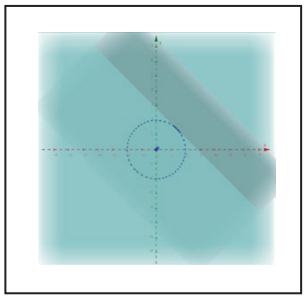


Figure 3

Figure 1 presents the plane $f(x,y)$ colored light blue, the cylinder $g(x,y) = 0$ colored red, and their intersection curve colored blue. Figure 2 the plane $z = 2\sqrt{2}$ is added that is the extremum. Looking at Figure 2 from above (projecting it onto a xy plane), it becomes Figure 3, and it can be seen how the contour line $f(x,y) = 2\sqrt{2}$ is tangent to the cylinder $g(x,y) = 0$ at the point $(x,y) = (\sqrt{2},\sqrt{2})$ In addition to the method using the gradient, there is another method to solve Problem (A), such as using implicit functions of $g(x,y) = 0$. Thus, the teacher presents the students the following Problem (B) that is difficult to solve by the method using implicit functions.

(B) Find a candidate for points giving the extremum of $f(x,y)=x^2 + y^2$ under the condition $g(x,y) = x^2 - xy + y^2 - 3 = 0$.

In this problem, the implicit function of $g(x,y) = 0$ is an irrational function. Therefore, it is difficult to successfully determine the extremum using a solution method that relies on the implicit function. The purpose of having students solve Problem (B) is to help them understand the necessity of the Lagrange multiplier method. See Figure 4, 5 and 6. Figure 4 shows the graph of the surface $f(x,y)$ colored blue and the elliptic cylinder $g(x,y) =0$ colored red in problem (B) entered into GeoGebra. In Figure 5, the plane $z = 2$ colored purple is added that is the local minimum. In Figure 6, the plane $z = 6$ colored light blue is added that is the local maximum. In Figure 6, the plane $z = 6$ appears from above, making it difficult to grasp the spatial relationship clearly. Therefore, the viewpoint is adjusted to a slightly lower angle to improve visibility. In an actual lecture setting, the teacher guides students to input the functions themselves and encourages them to adjust the viewpoint to an angle that allows for better understanding.

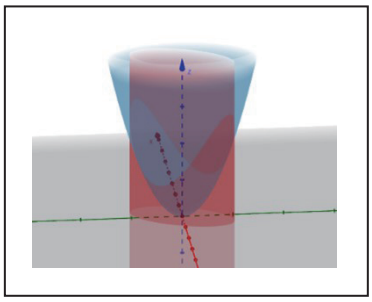


Figure 4

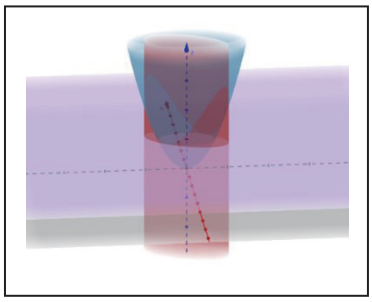


Figure 5

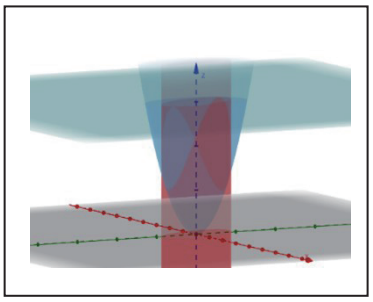


Figure 6

Teaching Flow

First, the teacher informs the students that the topic of the lecture is conditional extremum problems, and presents them with Problem (A) and (B). The teacher explains the concepts of gradients and normal vectors which are essential to solving the problem to the students as follows: The teacher has the students enter the curve $g(x,y) = 0$ and the gradient $\nabla g = (g_x, g_y)$ into GeoGebra and visually verify that the gradient ∇g is one of the normal vectors. Similarly, the teacher has the students enter the contour line $f(x,y) = k$ with parameter k and the gradient $\nabla f = (f_x, f_y)$ into GeoGebra and visually verify that the gradient ∇f is one of the normal vectors. Next, the teacher instructs the students to input the surface $z = f(x,y)$ and the constraint curve $g(x,y) = 0$ into GeoGebra, and to manipulate the viewpoint in order to explore their spatial relationship. Following this, the teacher organizes the students into small groups and facilitates a discussion on the necessary conditions needed to determine extremum. Following the group discussion, the teacher individually assigns Problem (A) to each student via Google Classroom, requesting that they submit both their final answer and a

written explanation of their problem-solving process. The teacher will share examples of submitted answers with the entire class. The teacher provides students with the correct answer along with several possible solution methods. Next, the teacher once again divides the students into groups to discuss the solutions to Problem (B), and then instructs them to individually submit their answers and solution processes to Problem (B) via Google Classroom. The teacher then explains to the students the correct answer to problem (B) along with the solution using the Lagrange multiplier method. After that, the teacher revisits the solution to problem (A) using the Lagrange multiplier method and helps students visually recognize through the use of GeoGebra that the contour line $f(x,y) = k$ and the curve $g(x,y) = 0$ are tangent at the extremum.

Putting into Practice the Lecture

The designed lecture was actually conducted with 35 students from the Faculty of Education at Yamato University on January 20, 2025. The lecture lasted 90 minutes and was delivered by the author, Takayama. Table 1 presents the number of students who used each solution method, along with the number of students who answered correctly for Problems (A) and (B). There were three main solutions used by students. Among the students who attempted the problems using the gradient method, 19 answered Problem (A) correctly, while 22 answered Problem (B) correctly. However, for Problem (B), 8 students were unable to solve the simultaneous equations despite being familiar with the solution method. On the other hand, two students utilized the symmetry of the functions $f(x,y)$ and $g(x,y)$, and assume $x = y$ in order to find the extremum. In the lecture, the teacher gave an example of a problem where the function is not symmetric, and explained that this method is not always applicable. Notably, the local minima in Problem (B) occur at the points $(1, -1)$ and $(-1, 1)$. These critical points do not lie on the line $x = y$, indicating that the constraint $x = y$ fails to capture the locations of these local minima.

Table 1. Number of Students and Correct Responses by Solution Type

Method	Problem (A)	Problem (B)
Gradient	19/22	22/30
Implicit function	4/8	0/2
Symmetry	2/2	0/2
Others	0/3	0/1

References

Jost, J. (2008). Postmodern Analysis, Springer.

Stewart, J. (2011). Multivariable Calculus, Brooks/Cole, Cengage Learning.

TSG9-13

ROLE OF MATH DUAL ENROLLMENT ON STUDENT OUTCOMES

Jungmin Lee
University of Kentucky, U.S.A., Jungmin.lee@uky.edu

Sungyeun Kim
Incheon National University, South Korea, syk@inu.ac.kr

This study aims to examine the relationship between math dual enrollment courses and college outcomes and explore whether the relationship is heterogeneous based on students' gender, race and ethnicity, and family income. In the U.S. dual enrollment allows high school students to take college–level courses while in high school. We use statewide data from Kentucky in the United States and track a graduating cohort of 2011–2012 for six years after their high school graduation. We use logistic regression models. Our results show that taking math dual enrollment courses is positively associated with college attendance and negatively associated with taking developmental math courses in college. However, it is not significantly related to STEM major choice or degree attainment. These findings are largely consistent across student gender, race and ethnicity, and family income, but some of these subgroup analysis results are not statistically significant due to the small sample sizes.

Keywords: dual enrollment, math courses, college enrollment, college persistence, developmental education

In the U.S., dual enrollment has become prevalent in the past decade (National Center for Education Statistics, 2020). Dual enrollment allows high school students to take college–level courses while in high school. If students passed dual enrollment courses, credits from these courses can be counted toward both high school graduation and college degree attainment. School districts and state governments provide financial support (e.g., tuition discount) to students taking dual enrollment courses and promote dual enrollment as a way to earn a college degree in a timely fashion (Education Commission of the States, 2022). In theory, dual enrollment can be an affordable option for students to earn a college degree fast at a cheaper price. Existing studies regarding dual enrollment conclude that taking any dual enrollment courses is positively associated with college attendance, readiness, and persistence (An & Taylor, 2019). Despite these positive outcomes, most existing studies focus on whether students take any dual enrollment courses without considering their subject, modality, and instructors. This approach may not accurately capture the outcomes of dual enrollment as they can be widely varied by depending on course subjects, modality, and instructors (Ryu et al., 2024).

To fill the gap in the literature, we aim to explore college outcomes associated with math dual enrollment courses. We focus on math dual enrollment courses because math courses are regarded as a gatekeeper in college: Many college students are not ready for college–level math courses and hence recommended to first take developmental math courses (high school–level math courses) before registering for required college–level math courses. Students recommended for math developmental courses are often disappointed, postpone taking these courses, or even fail (Bettinger et al., 2013). This prevents students from taking required college–level math courses and making timely

progress toward their degree. Even when students are not recommended for developmental math, many college students are still struggling in required college–level math courses and switch their major from science, technical, engineering, and math (STEM) majors to non–STEM majors (Rasmussen & Ellis, 2013). Struggling in college math courses and switching majors can increase the time needed for students to earn a college degree, which also increases tuition costs and foregone earnings.

Dual enrollment is expected to address these issues by allowing students to take college–level courses while in high school. By taking math dual enrollment courses, students can be better prepared for college–level math courses and/or meet a college–level math course requirement. It might then help students choose a STEM major upon college attendance, stay in college, and earn a college degree, especially in STEM fields, in a timely fashion (Hu & Chan, 2023). Majoring in STEM and earning a college degree in STEM can also lead to long–term benefits for both individual students and the society. At the individual level, people with a STEM degree have higher salaries and are less likely to be unemployed in the labor market than people with a non–STEM degree (Xu, 2013). STEM also brings innovation and economic prosperity to the society.

In this study, we address two research questions: 1) is taking math dual enrollment course associated with students' college attendance, academic preparedness, major choice in STEM, persistence, and degree attainment in STEM majors? and 2) is this relationship consistent across student gender, race and ethnicity, and family income? The first research question examines whether dual enrollment can address the lack of academic preparedness and attrition from STEM majors whereas the second research question can have equity implications by exploring whether the potential benefit of math dual enrollment is consistent across student subgroups.

Methodology

In this study, we used statewide administrative data from Kentucky in the U.S. and tracked a high school graduating cohort of 2011–2012. If these students were enrolled in one of the state's postsecondary education institutions, we can track them until the academic year of 2017–2018. The data provide detailed information on the state's high school students including demographics, family background, course information, test scores, high school characteristics, and college outcomes.

We used logistic regression models for this study. The outcome variables are a series of binary indicators that show whether students attended college within a year of high school graduation, took any developmental education courses in mathematics, chose a STEM major upon college entry, persisted in college from the first to the second year, and earned a college degree in STEM. The key independent variable of this study is whether students took at least one math dual enrollment course in high school. Control variables include students' gender, race and ethnicity, high school GPAs, free–or–reduced lunch eligibility in high school (a proxy variable for low–income students), and special education status. Except for college attendance outcome, 1) we also controlled for delayed college enrollment, part–time enrollment, and ACT scores (only for four–year college students), and 2) our analysis was limited to students who attended college and conducted separately for four–year and two–year college entrants, respectively.

Findings

Table 1 provides the logistic regression results for all students presented in odds ratio. Due to the space limit, we only show the odds ratio on the key independent variable: whether students took at least one math dual enrollment course. First, it is clear that taking math dual enrollment courses is positively related to college attendance. If students took at least one math dual enrollment course in high school, they were 42% more likely to go to college than those who did not. Although not shown in the table, most covariates included in the model were associated with college enrollment outcome as expected. For instance, when other things are held constant, being female, Black students, and other racial minority students (e.g., Native American students, multi–races, etc.) were positively associated with college attendance. In contrast, being Hispanic, Asian students, low–income students, and special education students was negatively associated with college attendance. The higher one's high school GPAs, the more students are likely to go to college.

Table 1. Logistic regression results

	Four–Year College Students	Two–Year College Students
College Attendance (Any College)	1.421*** (0.107)	N/A
Take Remedial Math	0.500*** (0.038)	0.962 (0.327)
Persist in College	1.245+ (0.164)	1.092 (0.160)
STEM Major Choice	1.041 (0.078)	0.943 (0.127)
STEM Degree Attainment	1.079 (0.087)	1.020 (0.192)

Note. + $p < 0.1$, *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$

Odds ratios are reported. Key coefficients (on the key independent variable) only.

Second, there is a negative relationship between math dual enrollment courses and math developmental courses in college. However, this relationship is statistically significant only for four–year college students. If four–year college students took dual enrollment math in high school, their odds for taking developmental math courses in college is only half of those for four–year college students without dual enrollment math courses. Third, taking dual enrollment math courses is positively associated with persisting in college from the first fall to the second fall, but this relationship is (marginally) significant only for four–year college students. Surprisingly, taking dual enrollment math courses is not significantly related to either STEM major choice in the first year of college and earning a college degree in STEM fields.

Tables 2 shows the subgroup analyses by students' gender, race and ethnicity, and family income. Overall, the results for subgroup analyses are similar to those for all students shown in Table 1. Taking math dual enrollment courses is positively related to college attendance and persistence while it is negatively related to taking developmental math courses. The relationship is generally consistent

across subgroups and is not widely varied across subgroups in terms of the magnitude of odds ratio. However, most odds ratios in these subgroup analyses are not statistically significant or cannot be estimated due to the small sample size for some student subgroups. Therefore, the subgroup analyses results, especially by student race and ethnicity, should be interpreted as exploratory with caution.

Table 2. Logistic regression results by gender, race and ethnicity, and income

	Four–Year College Students		Two–Year College Students	
	Female	Male	Female	Male
College Attendance (Any College)	1.470*** (0.153)	1.380*** (0.152)	N/A	N/A
Take Remedial Math	0.409*** (0.041)	0.680*** (0.084)	0.803 (0.429)	1.245 (0.779)
Persist in College	1.390+ (0.268)	1.107 (0.201)	1.267 (0.305)	0.624 (0.187)
STEM Major Choice	1.107 (0.107)	0.935 (0.113)	1.311 (0.266)	0.716 (0.222)
STEM Degree Attainment	1.225+ (0.128)	0.892 (0.115)	1.295 (0.398)	0.894 (0.214)
	Black	Hispanic	Black	Hispanic
College Attendance (Any College)	1.346 (0.416)	1.287 (0.450)	N/A	N/A
Take Remedial Math	0.602 (0.188)	0.928 (0.219)	11.976+ (15.547)	N/A
Persist in College	0.807 (0.323)	1.451 (0.979)	0.850 (0.909)	1.095 (1.064)
STEM Major Choice	1.099 (0.329)	0.769 (0.382)	0.340 (0.382)	1.170 (0.761)
STEM Degree Attainment	1.212 (0.390)	1.244 (0.663)	N/A	8.694** (6.063)
	Low–Income	Middle–Income	Low–Income	Middle–Income
College Attendance (Any College)	2.144*** (0.280)	1.121 (0.103)	N/A	N/A
Take Remedial Math	0.558*** (0.079)	0.468*** (0.043)	1.030 (0.539)	0.876 (0.550)
Persist in College	1.140 (0.218)	1.342 (0.247)	0.996 (0.253)	0.937 (0.252)
STEM Major Choice	1.167 (0.158)	0.969 (0.087)	1.243 (0.294)	0.937 (0.226)
STEM Degree Attainment	1.249 (0.183)	1.003 (0.097)	1.117 (0.341)	0.975 (0.233)

Note. + p < 0.1, *p < 0.05, **p < 0.01, ***p < 0.001

Odds ratios are reported. Key coefficients (on the key independent variable) only.

References

An, B. P., & Taylor, J. L. (2019). A review of empirical studies on dual enrollment: Assessing educational outcomes. In M. B. Paulsen & L. W. Perna (Eds.), *Higher education: Handbook of theory and research* (pp. 99–151). Springer.

Bettinger, E. P., Boatman, A., & Long, B. T. (2013). Student supports: Developmental education and other academic programs. The future of children, 93-115. https://www.jstor.org/stable/23409490

Education Commission of the States. (2022). *50 state comparison: Dual/concurrent enrollment policies*. https://www.ecs.org/50-state-comparison-dual-concurrent-enrollment-policies/

Hu, X., & Chan, H. Y. (2024). Preparing for the STEM pathways? Dual enrollment and college major choice in STEM. *The Journal of Higher Education, 95*(5), 607-638. https://doi.org/10.1080/00221546.2023.2241331

Minaya, V. (2021). *Can dual enrollment algebra reduce racial/ethnic gaps in early STEM outcomes? Evidence from Florida*. Community College Research Center, Teachers College, Columbia University. https://eric.ed.gov/?id=ED611281

National Center for Education Statistics. (2020). Dual or concurrent enrollment in public schools in the United States. https://nces.ed.gov/pubs2020/2020125.pdf

Rasmussen, C., & Ellis, J. (2013, July). Who is switching out of calculus and why. In *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 73-80). https://homepages.math.uic.edu/~bshipley/Switching.Calculus.pdf

Ryu, W., Schudde, L., & Pack-Cosme, K. (2024). Dually noted: Examining the implications of dual enrollment course structure for students' course and college enrollment outcomes. *American Educational Research Journal, 61*(4), 803-841. https://doi.org/10.3102/00028312241257453

Xu, Y. J. (2013). Career outcomes of STEM and Non-STEM college graduates: Persistence in majored-field and influential factors in career choices. Research in Higher Education, 54, 349–382. https://doi.org/10.1007/s11162-012-9275-2

TSG9-14

DECISION-MAKING IN A DESIGN-BASED RESEARCH APPROACH TO COLLABORATIVE DEVELOPMENT OF INTERACTIVE TEXTBOOK QUESTIONS

Vilma Mesa
University of Michigan, United States, vmesa@umich.edu

Johannah Crandall
University of Michigan, United States, jlcrand@umich.edu

We explore the decision making process as multiple teams of Students, Teachers, Authors, and Researchers, collaborate to design interactive, short-answer questions for open-source textbooks in calculus and linear algebra, textbooks that also differ in the style of presentation (either definition-theorem-example meant to be presented by the teacher or short texts followed by individual and group explorations and in-class discussion). We present the design-based research methodology and the theorization of decision making that guides our exploration into how stakeholders use heuristics and propose strategies that support their question development process.

Keywords: interactive textbooks, question design, decision making

The next generation of online, interactive mathematics textbooks must continue to improve support for teaching and learning. Current technical developments facilitate the relatively easy inclusion of interactive content that affords learners opportunities to engage with their textbooks inside or outside of their classrooms. By leveraging textbook models from computer science education that have for years harnessed interactive content, we can enhance online, interactive textbooks for undergraduate mathematics courses. These enhancements can lead to an increase of the use of free, open mathematics textbooks and provide access to high-quality engaging content for more learners in many contexts.

With the aims of promoting enriching student engagement with mathematical content and of providing tools for teachers to make more informed instructional choices, in this paper we share the methodological approach for our study of the development and use of interactive questions in open, online, interactive textbooks in calculus and linear algebra that have been written in PreTeXt and are hosted on the Runestone Academy learning analytics platform (https://landing.runestone.academy/). The project, PreTeXt-Runestone: Open Textbooks Engaging Undergraduates in STEM (PROTEUS) follows a design-based research approach (Bakker & Eerde, 2015) involving undergraduate mathematics students, teachers, textbook authors, and researchers in a collaborative development of interactive questions designed to elicit multiple ways of students' thinking. The research question that guides this phase of the project is: *What are the strategies and heuristics used by teams of students, teachers, authors, and researchers to create content for interactive short-answer questions for calculus and linear algebra textbooks that differ in how the material is presented?* To answer this question, we look at processes of decision making within the groups. In this proposal, we discuss the process that has led to the formation of four question development teams—each tasked

with writing interactive questions for one textbook—and the process that is being used to identify the heuristics and strategies for question writing, testing, and modification.

Background

Curricular and conceptual pedagogical functions of textbooks are achieved through embedded aspects such as "exposition, explanation, questioning, exercises, examples, and tests... [to] achieve a more active reader of the text and to manifest the aims of the mathematical and cognitive progression" (Love & Pimm, 1996, p. 386). There is a key distinction between "questioning" (in all its forms) and textbook exercises. While exercises provide a vehicle for active engagement with the flow of content (Love & Pimm, 1996), and indeed a means for students to practice and internalize as content systematically progresses, they exist in a largely post hoc fashion. On the other hand, questioning (e.g., in the form of a rhetorical question or a textbook that requires a response) may manifest as content precursors and may elicit mathematical exploration more integrated with the textbook's progressing narrative.

Current interactivity available in textbook development creates a medium for the learner to develop a relationship with the subject matter and potentially engage with the author's voice (Weinberg & Wiesner, 2011). We seek to understand in what ways instructors can and do make use of students' responses when engaging with questioning devices. In an online interactive textbook, student responses to embedded questions can be captured immediately and can thereby be made available to instructors. This window into students' understanding as they are reading and grappling with the content has the potential to inform instructional choices; aided with analysis of responses across multiple sites, the information can be mined to test learning trajectories for key mathematical notions within the courses. During this first year of the project, we are developing short-answer questions as an entry point into capturing students' reasoning and heuristics for team-based question development.

The use of Short Answer Questions in mathematics has been promoted to support metacognitive development in relation to problem-solving (Kilpatrick et al., 2001) and proving (Conradie & Frith, 2000), but also as part of consolidation of knowledge and retention (Hopkins et al., 2016). However, these studies investigate the use of these questions after the material has been presented, and not as part of class preparation—reading ahead of time and answering questions prior to exposure to the material in class—which is the intention of the short-answer questions in the PROTEUS textbooks. Our earlier work with student responses to one set of reading questions on span (Gerami et al., 2024) indicates that these responses are a source of conjectures about students' conceptions (Balacheff & Gaudin, 2010) that can guide instructional activity. Gerami and colleagues' study also highlighted that the questions must be amenable to an analysis of conceptions.

A DBR Study with Diverse Stakeholders

This work uses design-based research (DBR) methodology (Bakker & Eerde, 2015) to contribute to theory by developing a body of heuristics and strategies for writing interactive textbook questions that support student understanding and make student thinking clearer to instructors (Yerushalmy et al., 2017). The intervention consists of incorporating the developed questions into instructors' textbooks and studying their use through the data analytics provided by the Runestone Academy platform (https://landing.runestone.academy/). The questions themselves are redesigned or discarded in a cyclical fashion by each development team. Figure 1 illustrates the cycle of question devel-

opment, where progress from question inception to classroom implementation is informed by iterations of feedback, first within development teams, then from cognitive interviews (CI), and lastly from instructor feedback after use in their classroom.

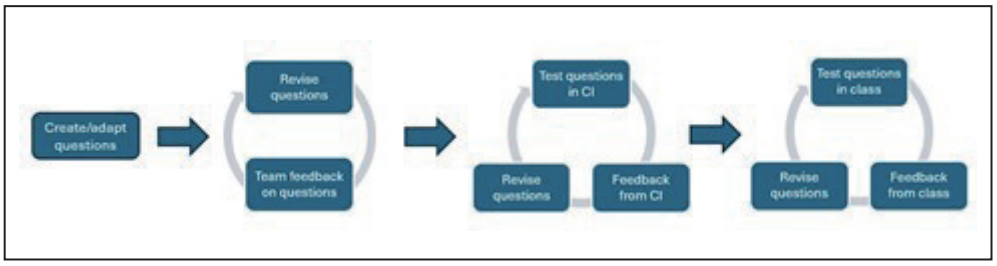


Figure 1: Rapid revision and implementation cycle of question development.

Team Formation.

Our effort to identify question development heuristics engages students, teachers, textbook authors, and researchers in a team setting. These 16 stakeholders come from 14 different higher education institutions in the United States and Canada and bring current and past experience with both four-year universities and community colleges. There are four teams, each consisting of two Students, two Teachers, an Author, and either one or two Researchers (constituting a "STAR team"). Two teams work with linear algebra textbooks (ULA: Austin, 2022; FCLA: Beezer, 2022) and two work with calculus textbooks (AC: Boelkins, 2022; APEX: Hartman et al., 2018). Each team meets virtually twice per month to discuss and give feedback on questions they are developing for their respective textbooks, as well as formulate conjectures about what student understandings and misconceptions the questions will elicit (Bakker & Eerde, 2015). During the initial phase of the project, and between team meetings, the developed questions are tested in semi-structured cognitive interviews (Saldaña & Omasta, 2018); the information gathered from interviews is provided to STAR teams in order for them to analyze how effectively questions elicited student conceptions and inform question design. In later phases, the information that will be given back to the teams will include the use of the questions in naturalistic classroom environments.

Studying Decision-Making

We rely on the practical rationality approach to investigate the kinds of obligations that the stakeholders bring into their discussions regarding selection of topics and selections of questions for testing. Practical rationality, developed in the context of teacher decision-making (Herbst & Chazan, 2012), identifies norms and obligations as the core elements that guide teacher's justifications behind their actions during teaching. In this study, we pay close attention to the roles that each of the stakeholders embody as part of the development teams. The theory assumes that the institutions (e.g., schools, universities) where individuals work in the role of mathematics teachers, impose social norms and values that "constrain and condition [their] position" (Chazan et al., 2016, p. 1066). Some of these social norms are *professional obligations* that individuals use to "structure and frame individual behavior" (p. 1066) and that individuals respond to as if they were stakeholders:

1. the discipline of mathematics, including the knowledge and its applications, the valid representations of the knowledge, and the disciplinary practices (*disciplinary obligation*)

2. the students and their individual needs (*individual obligation*)
3. the society, including the appropriateness of patterns of interpersonal and social interactions, and acceptable societal dynamics, such as sharing of resources in the classroom; these patterns may benefit the society as a whole in the moment or in the future (*interpersonal obligation*)
4. and the institution and its regimes, such as policies and scheduling (*institutional or schooling obligation*)

Our use deviates from Herbst and Chazan's (2012) work related to instructional situations in the classroom in that we use such framing to understand the kinds of obligations that the members in the teams bring to bear as they engage in the process of designing interactive questions. Our current parsing of the team meetings show that the teams mention all the four obligations listed (**disciplinary**: "*I spend a lot of time in the first two weeks of class saying, you know, a matrix doesn't have a solution… So if there are places for us to reinforce, maybe the language of the topic in maybe this section on spanning or orthogonality, maybe not so much on orthogonality… that could be something that could be useful*" [FCLA Teacher]; **individual**: "*As you know, the function, the math is the abstract part. But bringing that and pulling that to a real-world example was a struggle for a lot of students. And I noticed multiple times that they were even trying to add two things together, two quantities together that didn't even have compatible units*." [APEX Teacher]; **interpersonal**: *So, if these students have done these questions…*[they] *will then come to class and be ready to engage* [AC Author]; **institutional**: "*I would like to add, like a computational aspect to the questions… computers are becoming a bigger part of math, and I would like to see that like in math education*" [ULA Student]) as they consider the multiple alternatives for selecting topics or proposing questions and changes to questions that need to be included. But the teams also consider the demands of the cycles of question development and the constraints of testing the questions in the classroom to choose topics to include and questions to work on. Thus, it seems that the research enterprise, which also includes the platform that will house the textbooks, imposes constraints that the individuals respond to. During the presentation, we elaborate on examples of how these contribute to the identification of heuristics and strategies for question development and changes that depend on content or textbook orientation.

References

Austin, D. (2022). *Understanding linear algebra*. Scholarworks@GVSU.

Bakker, A. & van Eerde, D. (2015). An introduction to design-based research with an example from statistics education, in A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg, (Eds.) *Approaches to qualitative research in mathematics education: Examples of methodology and methods* (pp. 429–466). Springer.

Balacheff, N., & Gaudin, N. (2010). Modeling students' conceptions: The case of function. *Research in Collegiate Mathematics Education, 16,* 183–211. https://doi.org/10.1090/cbmath/016/08

Beezer, R. A. (2022). *First course in linear algebra*. Congruent Press. https://books.aimath.org.

Boelkins, M. (2022). *Active Calculus*. https://activecalculus.org/single/:Scholarworks.

Conradie, J. & Frith, J. (2000). Comprehension tests in mathematics. *Educational Studies in Mathematics, 42*, 225–235.

Gerami, S., Khiu, E., Mesa, V., & Judson, T. (2024). Conceptions of span in linear algebra: from textbook examples to student responses. *Educational Studies in Mathematics, 116*, 67–89. https://doi.org/10.1007/s10649-024-10306-8

Hartman, G., Fitzpatrick, S., Jordan, A., & Vollet, C. (2018). *APEX Calculus 1*, 4th ed., CreateSpace Independent Publishing Platform.

Hopkins, R. F., Lyle, K. B., Hieb, J. L., & Ralston, P. A. S. (2016). Spaced retrieval practice increases college students' short- and long-term retention of mathematics knowledge. *Educational Psychology Review, 28*, 853– 873.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Academy Press.

Love, E. & Pimm, D. (1996). 'This is so': A text on texts. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde, (Editors) *International handbook of mathematics education*. (pp. 371–409). Kluwer.

Saldaña, J., & Omasta, M. (2018). *Qualitative research: Analyzing life*. Sage.

Weinberg, A. & Wiesner, E. (2011). Understanding mathematics textbooks through reader-oriented theory. *Educational studies in mathematics, 76*,49–63.

Yerushalmy, M., Nagari-Haddif, G., & Olsher, S. (2017). Design of tasks for online assessment that supports understanding of students' conceptions. ZDM Mathematics Education, 49, 701–716.

MEMO

MEMO

KOREA 2025

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 10:

The Essence of STE(A)M Education

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 10
(Day 2)

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

TSG10-01

IDENTIFYING ZAMBIAN GRADE 10 STUDENTS' 21st CENTURY SKILLS IN CROSS-CURRICULAR LEARNING OF MATHEMATICS AND HOME ECONOMICS

Koki Nakashima
Yokohama National University, Japan, nakashima-koki-sc@ynu.jp

Nagisa Nakawa
Kanto Gakuin University, Japan, nagisa@kanto-gakuin.ac.jp

Masato Kosaka
University of Fukui, Japan, kosaka@u-fukui.ac.jp

Kumi Kobayashi
Tokyo Future University, Japan, kobayashi-kumi@tokyomirai.jp

This study aimed to design and implement with a transdisciplinary approach from the cross-curricular perspective, to plan and conduct cross-curricular lessons integrating mathematics and home economics at the upper secondary school level, and qualitatively analyse students' 21st century skills based on their reflection worksheets and classroom video. The conceptual frameworks included STEM education's degree of integration of the two subjects and 21st century skills. Two-day special lessons were conducted as experimental lessons for 12 Zambian grade 10 students in a governmental school of the Southern Province, Zambia. By analysing, we identified that students utilised some of the 21st century skills: relation with others, foundational literacies and skills and social-emotional well-being. However, student reflections showed limited evidence of 21st century skills, highlighting the need for further research and curriculum development. The study's findings have implications for STEM education and 21st century skills development, suggesting the importance of addressing limitations and comparing findings across educational settings.

Keywords: Cross-curricular lesson, high school mathematics, home economics, Life Backgammon, 21st century skill

Introduction

Recently, STEM and STEAM education has expanded worldwide (Herro et al., 2017). Cross-curricular learning and other learning types should be considered from a broader perspective and not limited to science, technology, engineering, and mathematics. This study focused on the aforementioned issues and the expansion and development of STEM/STEAM education worldwide as an academic priority. The aim was to develop cross-curricular teaching materials encompassing mathematics and home economics in an upper secondary school from a comparative educational perspective. We aimed to determine students' learning outcomes from the perspective of 21st century skills in Japan and Zambia. This article reports partially the results for Zambia. This study aimed to identify cross-curricular learning of mathematics and home economics outcomes by analysing lesson data by teachers and students' written reflections from perspective 21st century skills in Zambia. The research question was: 'What 21st century skills emerged within/through cross-curricular learning of mathematics and home economics?'

Theoretical Framework

This study focused on four degrees of integration in STEM education to capture the degree of cross-curricular integration between the two subjects, drawing on Vasquez et al. (2013). One of the four approaches within the degree of integration, the transdisciplinary approach utilises knowledge and skills from two or more subjects to form learning experiences by working on real-world tasks and projects. In this study, lessons were developed using a transdisciplinary approach.

Learning emerges from the problems with the transdisciplinary approach used in STEM/STEAM education. According to Herro et al. (2017), soft skills, such as 21st century skills, refer to generic competencies that can be fostered through a transdisciplinary approach in STEM/STEAM education. According to the Partnership for 21st Century Learning (2016), these skills include knowledge construction, real-world problem-solving, skilled communication, collaboration, the use of information and communication technology for learning, and self-regulation. Chen's (2023) framework, used in our study, is comprehensive and useful for explaining various aspects of 21st century skills. It includes seven categories, each with subthemes. Our lessons cover six main categories: foundational literacies and skills, individual productivity, relations with others, social-emotional well-being, ethics, morals and citizenship, and lifelong learning. These six categories include 32 subthemes, such as numeracy and quantitative literacy, digital literacy, financial, economic, business, and entrepreneurial literacy.

Method

This study employed an action research approach, analysing classroom video recordings and students' written reflections. The teaching experiment was conducted with 12 Grade 10 students (mean age = 16.4 years) at a government high school of average academic performance, located in an urban area of the Southern Province of Zambia. The lessons took place as special afternoon sessions over two days in March 2024. Written permission was obtained from the school, and ethical approval was granted by the Ethics Committee of Tokyo Future University (No. 2023-036).

A transdisciplinary framework guided the design and implementation of the teaching experiment. To evaluate student learning, a mixed-methods analysis was conducted. The data sources included students' written reflection worksheets, which addressed three key aspects: (i) self-evaluation via five multiple-choice questions assessing their own learning; (ii) observations and insights gained from listening to other groups' presentations; and (iii) reflections on what they learned during the lessons, their impressions. (ii) and (iii) will be only analysed. These instruments were deemed appropriate, as student reflections at the conclusion of the lessons provided evidence of their developing competencies. At the same time, it was important to identify objectively the kinds of 21st century skills embedded in the activities—such as students' use of computers, engagement in game-based tasks, and participation in group discussions that emerged throughout the lessons. Accordingly, video analysis was considered an appropriate methodological approach for capturing these elements of classroom practice.

Content of the Teaching Experiment

The lessons aimed for students to think about their careers and lives and expand their knowledge and skills through the role-playing game 'Life Backgammon' that we developed. This game pertains to individuals' lives from 16 to 60 years of age. Students were instructed to pick two different kinds of cards they made in advance and calculate their financial issues using worksheets and

spreadsheets. The graphs on the spreadsheet automatically appear as the participants input their financial data and identify the financial flow and money left at the end of their virtual life. Academic careers and occupations can be randomly changed, and debts can be obtained. The lessons have four different stages: (a) creating cards related to their life events and finance in connection to home economics and anticipating the shape of the financial graph; (b) creating cards related to investment, particularly single and compound interest with functions and formulas; (c) playing the game as group work; and (d) presenting their results and reflecting on the lessons. In these lessons, they mathematically had difficulty with obtaining formulas for simple and compound interests utilising functional thinking, on the other hand, they were successfully creating the cards and engaged in the game, divided by two different groups. In particular, the current report focuses on reflection (c)-(d).

Result and Discussion

The developed lesson enabled students to reflect on virtual financial flows within one's life. One student, who presented his group's outcome at the conclusion of the game, referred to himself using the pronoun "I" rather than the name of the virtual character, suggesting that he had engaged with the activity as though he were personally experiencing the simulated life. All participants demonstrated active engagement in the game, indicating that the role-play format was effective in promoting subjective, experiential learning.

Video analysis revealed two principal categories: relation with others and foundational literacies and skills. Firstly, focusing on the relation with others, communication and collaboration of the sub-themes was also identified. All students actively participated and worked cooperatively throughout the activities. Secondly, continuous group discussions led students to foster language literacy of the sub-themes categorised under the foundational literacies and skills as students spoke English as medium of instruction. On the other hand, the aspect of group management was not observed during their collaborative work. This suggests that context awareness of the sub-theme may pose challenges for the Zambian students. In addition, the students' unfamiliarity with ICT suggests that this lesson provided an opportunity to enhance their digital literacy. The video analysis also highlighted pedagogical issues: when students paused or encountered difficulties in operating their computers, teachers tended to intervene and manage the situation directly. As a result, students missed opportunities to demonstrate *leadership*, particularly in relation to the *entrepreneurship* sub-theme.

Table 1 presents the results related to components (ii) and (iii). Students' written reflections were inductively coded into nine categories, as shown in the table. This shows social-emotional well-being, particularly self-awareness.

Table 1. Coding of students' reflections in (ii) and (iii)

Codes	(ii) (N=12) (%)	(iii) (N=12) (%)	Total((i) and (ii)) (N=24) (%)
1. Good job	2 (16.7%)	3 (25.0%)	5 (20.8%)
2. Expense and loss	7 (58.3%)	0 (0.0 %)	7 (29.2%)
3. Changing job and loss	1 (8.3%)	0 (0.0 %)	1 (4.2%)
4. Diversity of life events	0 (0.0 %)	1 (8.3%)	1 (4.2%)
5. Planning is important	1 (8.3%)	2 (16.7%)	3 (12.5%)
6. Better to start a business than	1 (8.3%)	0 (0.0 %)	1 (4.2%)
7. Changing jobs for income			
8. Take risks/make choices	0 (0.0 %)	2 (16.7%)	2 (8.3%)
9. The way how to increase money, saving, calculation of money	0 (0.0 %)	1 (8.3%)	1 (4.2%)
	0 (0.0 %)	3 (25.0%)	3 (12.5%)

Table 1 shows that their reflections were about income and well-paid occupations (Codes 1 & 2). Our intended purpose of the activities to think of their future life with more opportunities (e.g. thinking about investment, managing life and career, and taking care of children), was not observed, except for one student's reflection, coded as 5 in (ii): 'I learnt how to spend money in my life. How to plan for my life when I grow old. (Unable to read) everything was e(x)citing'. This is the only one student's description related to self-management in social-emotional well-being. Thus, they tended to focus only on money and occupations, which were not the foremost objective of their lessons. Kobayashi et al. (2023) analysed Japanese students' reflections in the same setting and found a focus on themes such as health, housing, work, salary, and family. In contrast, Zambian students tended to emphasise high-paying jobs, with 'doctor' frequently cited as their preferred future occupation. This seems to be affected by the country's severe economic situation. National Institute for Educational Research (NIER) (2013)'s statement that social skills reflect the greatest influence of the social, cultural, and historical background of a country, and our study supports this.

Conclusion

The transdisciplinary approach was expected to provide students with an opportunity to foster a range of 21st century skills. However, due to the socio-cultural influences, Zambian students appeared to face constraints in cultivating these skills more broadly. Firstly, they developed language literacy within the foundational literacies and skills and communication and collaboration under the relation with others. Secondary, students' written reflections indicated one's social-emotional well-being skills were closely tied to the socio-economic context of the country. As a limitation of the study, relying solely on written reflections may not fully capture the diversity of students' perspectives on life planning and career aspirations. It is recommended that interviews be incorporated into the research design to provide a more comprehensive understanding of students' thinking processes.

References

Chen, D. (2023). Toward an understanding of 21st century skills: From a systematic review. *International Journal for Educational and Vocational Guidance, 23*, 275-294. https://doi.org/10.1007/s10775-021-09511-1

Herro, D., Quigley, C., Andrews, J., & Delacruz, G. (2017). Co-measure: Developing an assessment for student collaboration in STEAM activities. *International Journal of STEM Education, 4(*26). https://doi.org/10.1186/s40594-017-0094-z

Kobayashi, K., Suzuki, T., Nakawa, N., Saito, W., Nakashima, K., Morita, D., & Kosaka, M. (2023, September). Analysis of students' descriptions in cross-curricular lessons in improved upper secondary school home economics and mathematics: A comparison of the results in 2022 and 2023. *Poster presentation at the 47th Annual Meeting of the Japanese Society for Science Education*, Ehime University.

National Institute for Educational Research. (2013). *Basic research report on curriculum development 5: Basics of curriculum development to foster competencies responding to changes in society*. https://www.nier.go.jp/kaihatsu/pdf/Houkokusho-5.pdf

The Partnership for 21st Century Learning. (2016). *Framework for 21st century learning*. https://static.battelleforkids.org/documents/p21/p21_framework_brief.pdf

Vasquez, A. J., Sneider, C., & Comer, M. (2013). *Lesson Essentials, Grades 3-8: Integrating Science, Engineering and Mathematics*. Heinemann.

Acknowledgements

This work was supported by the Japan Society for the Promotion of Science (JSPS) KAKENHI, Grant

Numbers 23K22289, 23K25662, and 24K05921. We would also like to thank Editage (www.editage.jp) for the English language editing.

TSG10-02

EXPLORING INQUIRY-BASED LEARNING IN SENIOR HIGH SCHOOL MATHEMATICS: A STUDY ON FINALIZED SRPS

Mitsuhiro Iwata
Senior High School at Otsuka, University of Tsukuba and Waseda University, Japan, miwata@sgh-tsukuba.org

Takeshi Miyakawa
Waseda University, Japan, tmiyakawa@waseda.jp

This study examines the possibility of incorporating Study and Research Path (SRP), a model of inquiry-based learning formulated within the Anthropological Theory of the Didactic (ATD), into high school mathematics classes. Specifically, we designed and implemented a series of lessons in which students learn knowledge about quadratic functions through the activities of inquiry. In this paper, we report the results of this implementation with an analysis of classroom activities using a QA map as an analytical tool. The results confirmed the characteristics of SRP, but also revealed the difficulties in acquiring knowledge and the importance of teacher's support.

Keywords: inquiry-based learning, study and research path (SRP), quadratic function

Introduction and Theoretical Framework

The growing importance of STEAM education has led to extensive research and implementation. In Japanese high schools, inquiry-based teaching, which promotes multi-disciplinary learning, has gained attention. In the inquiry activities, students actively seek answers, acquiring knowledge as needed. In contrast, regular mathematics classes are structured to acquire specific knowledge in a logical sequence defined by the curriculum. However, students often struggle to recognize the necessity of this knowledge in their learning process.

This raises the question of whether inquiry can help students acquire specific mathematical knowledge within regular classes. With this in mind, we have explored inquiry-based learning as a means of teaching predefined mathematical content.

This study is grounded in the *Anthropological Theory of the Didactic* (ATD), which introduces an educational perspective known as "the *paradigm of questioning the world*" (Chevallard, 2015). This paradigm prioritizes developing a researcher's attitude over acquiring knowledge. *Study and Research Path* (SRP), which describes the mechanism of inquiry, is formulated based on this paradigm. Its primary characteristic is that inquiry begins with a lively generative initial question (Q0). Learners deepen inquiry through continuous questioning and answering. In the course of this inquiry, various "*media*," such as literature and the Internet, are consulted as needed. Since the knowledge required for inquiry is learned as it becomes necessary, all knowledge in SRP-based learning exists with a clear purpose. This differs from approaches such as problem-based learning, where the necessary knowledge is often assumed or anticipated from the outset.

In our study, we adopt SRP-based inquiry as a means to teach specific mathematical knowledge in Japanese high school mathematics classes. Since the knowledge in SRP develops in response to the need to answer questions, we expect that students will learn this knowledge with its meaning and necessity. The target knowledge is the quadratic function, which is taught in *Mathematics I*, a mandatory subject for first-year high school students in Japan. In this paper, an inquiry-based lesson related to physics is conducted. This is because we do not want the students to know the teacher's intention to have them use their knowledge of quadratic functions. Our goal is for knowledge of quadratic functions to emerge naturally during the inquiry process in a context where their use is not immediately obvious.

SRPs can be categorized into two types: "*open SRPs*," which do not have a predetermined direction, and "*finalized SRPs,*" which are designed with a specific learning objective in mind (Chevallard & Strømskag, 2022). This paper focuses on inquiry conducted within a predefined body of knowledge, and therefore relies on the latter. The research question is as follows: What forms of inquiry can take place within a finalized SRP where the learning of quadratic functions is the objective?

Methods

The teaching experiment in this study was conducted in June 2024 with ten groups, each consisting of four first-year senior high school students in Japan. The class, led by the first author, spanned 15 sessions of 50 minutes each. The initial question (Q0) was designed to inquire into the topic of oblique projection in physics. To introduce Q0, a video was shown depicting a ball being hit in a baseball game. The inquiry question was then posed, "Will this hit result in a home run?"

The class was divided into four phases. In Phase 1, an open-ended inquiry was initiated with Q0, and students were reminded of the commitments of inquiry based on the SRP, such as learning as needed and valuing questions. Many students gathered information via the Internet.

During the third period, each group presented a progress report, sharing various opinions about trajectories. The teacher then posed the questions, "What will the trajectory of the ball be? Why is it so?" This marked the beginning of Phase 2. When students were allowed to freely consider different effects, seven groups assumed only gravity, two groups considered both gravity and air resistance, and one group assumed only air resistance. After four periods of inquiry, students presented their findings.

In Phase 3, a teacher-led *workshop* (Chevallard, 2015) was conducted to acquire knowledge and deepen the understanding of quadratic functions. This was necessary as the expected mathematical knowledge did not naturally emerge, and group activities varied significantly, requiring sharing. In the class, contrary to the previous process of deriving equations from physical information, the teacher posed the question, "How do you get physical information from the equation?" Students worked on multiple derived questions and presented their results. As students did not question the validity of the problem-solving process, the teacher urged them to explore its theoretical basis.

In Phase 4, students were given the freedom to choose whether to continue the original inquiry or to deepen the questions generated in Phases 2 and 3. This activity lasted approximately 4 hours, and at the end of the session, the summaries created by each group were shared via Google Classroom.

In this study, we analyze the activities of Group A. This group studied the effect of air resistance in addition to gravity and used Scratch as a simulation tool, which aligns with the characteristics of SRP, where students inquire while learning as needed. The main questions and answers were identi-

fied from the recorded audio data and the students' worksheets. The analysis employed a QA map—a tree-structured diagram that visualizes the sequence of questions (Q) and answers (A) generated during inquiry (Winslow et al., 2013). To ensure reliability, the QA map created based on the data was reviewed by the second author to confirm consistency in the analysis. The teacher's questions are denoted as QT and emphasized by enclosing them in bold frames, as it is considered important to clearly present the teacher's questions. The answers obtained from media are denoted as A◊ to distinguish them from the questions and answers generated by the students themselves.

Results

Figure 1 shows Group A's inquiry as a QA map. This section summarizes the results by phase.

In Phase 1, students researched online to answer Q0 and found the game shown in the video on the Internet. Some asked, 'What is the probability of a home run?' and found it to be 8.676%, based on historical data. Others inquired, "How to hit a home run?" and searched for the optimal hitting angle but did not formulate a new research question.

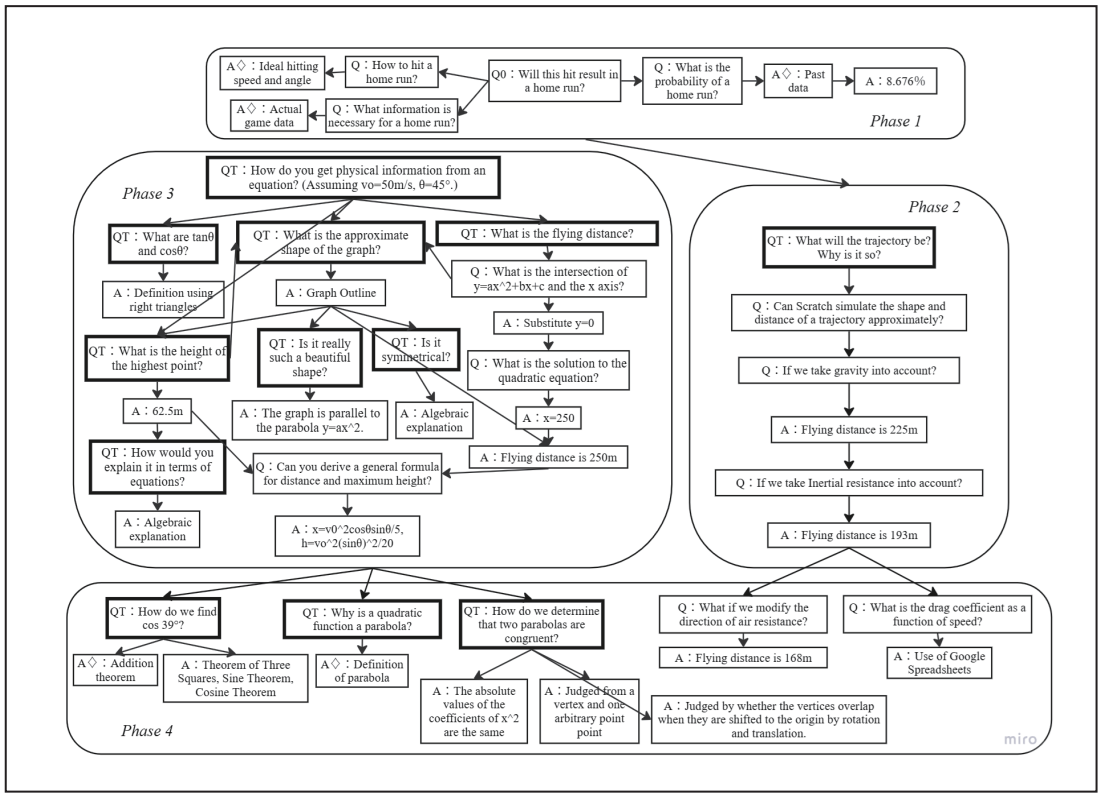


Figure 1. QA map of group A

In Phase 2, the main question was: "Can Scratch simulate the shape and distance of a trajectory approximately?" Students first calculated the flight distance considering only gravity, which yielded an unrealistic 225 meters. They then incorporated air resistance, using online sources to obtain values such as drag coefficient and air density. This adjustment corrected the distance to 193 meters. During the presentation, another group noted that air resistance should reverse direction during de-

scent and pointed out the drag coefficient, though variable with speed, was treated as a constant.

In Phase 3, some students explored the question, "Can you derive a general formula for distance and maximum height?" while also responding to teacher questions. However, since Group A did not clearly link this phase with Phase 2, the QA map does not show a connecting arrow.

In Phase 4, some students utilized Scratch to modify the direction of the air resistance at the time of descent, a point raised in Phase 2, and calculated a flight distance of 168 meters. Others used Google Sheets to calculate the drag coefficient based on speed variations. Some students worked on questions about parabolas and trigonometric ratios that the teacher presented.

Discussion and Conclusions

This study featured activities reflecting the researcher's attitude expected in an open SRP. Group A proceeded with their inquiry while learning a great deal about air resistance and simulation on their own. They calculated the flight distance in Scratch, refining it to improve accuracy. This attitude is called "*Herbartian attitude*" within ATD (Chevallard, 2015), which suggests that one does not balk at situations involving problems one has never encountered or solved.

A key feature was that, while modeling trajectories in Scratch, Group A explored ways to determine the drag coefficient and air density in the required order. This differs from the logical order in which knowledge is learned in a regular mathematics class. This exemplifies the SRP's characteristic of learning what is necessary on an as-needed basis.

On the other hand, most quadratic function knowledge was encountered in Phase 3, pausing Group A's Phase 2 inquiry. The purpose of the workshop was to "acquire and share knowledge of quadratic functions," but the need for this was not clear to Group A's students.

Furthermore, not all content in the quadratic function curriculum was covered. This limitation highlights the difficulty of acquiring comprehensive knowledge through inquiry into a single question (Q0). Future research should explore the potential for addressing multiple questions. In addition, due to the nature of inquiry, it is difficult to control the targeted acquisition of knowledge. Rasmussen (2016) emphasizes the importance of "*deepening questions*" and "*broadening questions*" for teachers' support questions in SRP. The consideration of such support questions is an area for future research.

This study demonstrated the potential of using finalized SRP to teach predefined mathematical content in meaningful contexts. The significance of this study lies in its applicability to teaching specific curriculum content through an inquiry-based approach. This method is adaptable to other mathematical areas and interdisciplinary contexts. Future research should explore how such inquiry can function across various subjects and educational settings.

References

Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *The proceedings of the 12th international congress on mathematical education* (pp. 173–187). Springer. https://doi.org/10.1007/978-3-319-12688-3_13

Chevallard, Y., & Strømskag, H. (2022). Conditions for a transition to the paradigm of questioning the world. In S. A. Almouloud, R. B. Guerra, L. M. S. Farias, A. Henriques, & J. M. V. Nunes (Eds.), *Percursos de estudo e pesquisa à luz da teoria antropológica do didático: Fundamentos teórico-metodológicos para a formação*, *Vol. 1*, (pp. 27–58). Editora CRV.

Rasmussen, K. (2016). The direction and autonomy of interdisciplinary study and research paths in teacher education. *REDIMAT, 5*(2), 158-179.

Winsløw, C., Matheron, Y., & Mercier, A. (2013). Study and research courses as an epistemological model for didactics. *Educational Studies in Mathematics, 83*, 267-284. https://doi.org/10.1007/s10649-012-9453-3

TSG10-03

A STUDY ON THE ASSESSMENT OF CREATIVE PROBLEM-SOLVING COMPETENCE USING FERMI ESTIMATION

Jungin Heo
Samjung High school, Korea, gjwjddls32@daum.net

Jihwa Noh
Pusan National University, Korea, nohjihwa@pusan.ac.kr

This study examines the creative problem-solving competence of mathematically and scientifically gifted middle school students using a Fermi estimation task. The task required students to estimate the daily nationwide usage of plastic bags and justify their reasoning. A six-criteria framework was used, covering divergent (fluency, flexibility, originality) and convergent thinking (validity, relevance, breadth). Results showed that math-gifted students excelled in validity and breadth, while science-gifted students scored higher in fluency and originality. These findings suggest differentiated instructional approaches to support balanced development in creative problem-solving.

Keywords: Creative problem solving, Fermi estimation, divergent thinking, convergent thinking, gifted education

Introduction

The remarkable advancement of artificial intelligence technology has greatly enhanced access to information. For instance, when one seeks necessary information and its applications, it can be easily obtained through various methods, such as consulting a conversational AI system. In an era where previously unknown knowledge and data can be effortlessly acquired, the importance of merely applying existing knowledge to solve problems has diminished. So, what true capability is required amid this flood of information? It is the ability to solve problems creatively based on accumulated knowledge. Recently, global curricula have shifted their emphasis from rote memorization to nurturing students' capacity to approach and resolve problems through creative thinking. Accordingly, this study aims to explore this creative problem-solving ability in depth.

Background of the Study

Creative problem-solving is increasingly emphasized across all areas of education, particularly in mathematics. While creativity and problem-solving have often been treated separately in prior research, this study adopts an integrated perspective, treating them as a unified competency. To assess this competency, a Fermi estimation problem—requiring reasoning with limited information—was selected, allowing for diverse and context-sensitive approaches.

The assessment framework distinguishes between divergent thinking (fluency, flexibility, originality) and convergent thinking (validity, relevance, breadth). Divergent thinking captures the generation of varied and novel ideas, while convergent thinking focuses on logical coherence and comprehensive reasoning. These dimensions provide a structured basis for evaluating students' creative problem-solving abilities.

Methods

This study involved mathematically and scientifically gifted middle school students who completed a Fermi estimation task: estimating the number of plastic bags used daily nationwide. The task required students to state assumptions and explain their reasoning. Responses were assessed based on six criteria—three for divergent thinking (fluency, flexibility, originality) and three for convergent thinking (validity, relevance, breadth)—each scored on a 0–2 scale. Data were collected through written responses and analyzed descriptively to compare the two groups' cognitive profiles in creative problem-solving.

Results and Discussion

This study evaluated students' creative problem-solving abilities by presenting them with a Fermi estimation problem based on a familiar everyday object. Specifically, the students were tasked with estimating "How many plastic bags are used nationwide in one day?" and were instructed to provide a detailed account of the assumptions, rationale, and solution process used for the estimation. The evaluation targeted middle school math and science prodigies. Drawing on previous research, the evaluation rubric was restructured as shown in Table 1, with each criterion assigned 2 points for a total score of 12 points. The results are summarized in Table 2.

Table 1. Evaluation criteria for creative problem-solving in fermi estimation problems

Components	Divergent thinking		
	Fluency	Flexibility	Originality
Description	Number of ideas generated	How many different perspectives or criteria are proposed in multiple ways?	Rarity (uniqueness) of the ideas
Scoring	2 points: 5 or more ideas 1 point: 3–4 ideas 0 points: 2 or fewer ideas	2 points: 3 or more distinct perspectives 1 point: 2 perspectives 0 points: 1 or fewer perspectives	2 points: Basis for ideas is notably new and unique 1 point: Basis is somewhat fresh 0 points: Not apparent
Components	Convergent thinking		
	Validity	Relevance	Breadth
Description	Are the presented grounds logically persuasive?	How well does it align with the presumed problem situation?	How extensively are the complex elements needed for solving the problem considered?
Scoring	2 points: Highly persuasive 1 point: Somewhat persuasive 0 points: Insufficient or absent grounds	2 points: Strongly aligned with the problem 1 point: Moderately aligned 0 points: Not aligned	2 points: Broad coverage of relevant factors 1 point: Partial coverage 0 points: Narrow coverag

Table 2. Results of the assessment for creative problem-solving ability

Components	Mathematics Gifted Class	Science Gifted Class
Fluency	1.15	1.22
Flexibility	0.8	0.69
Originality	0.3	0.56
Validity	0.7	0.41
Relevance	0.25	0.33
Breadth	0.85	0.46

A detailed examination of the evaluation results reveals several noteworthy trends. First, in the fluency component, the math prodigy group (mean score: 1.15) scored slightly lower than the science prodigy group (mean score: 1.22). This suggests that math prodigies generated a relatively smaller number of assumptions or ideas during the problem-solving process. Conversely, the flexibility score was higher in the math group (mean score: 0.80) compared to the science group (mean score: 0.69), indicating that math prodigy students tend to consider a broader range of factors and establish more diverse criteria when approaching problems.

In terms of originality, the math group achieved an average score of 0.30, whereas the science group scored 0.56—a notable disparity. This may imply that students in the math group, being more accustomed to conventional problem-solving methods, are less inclined to incorporate non-traditional elements or generate novel ideas. Essentially, the emphasis on standardized procedures in math education might limit the expression of originality.

Regarding validity, the math group (mean score: 0.70) outperformed the science group (mean score: 0.41), which can be interpreted as a result of their more rigorous training in establishing and verifying logically sound and persuasive justifications. In contrast, the relevance score for the math group (mean score: 0.25) was lower than that of the science group (mean score: 0.33). This difference likely reflects the nature of Fermi estimation problems—wherein initial assumptions, even if slightly off-target, can cancel out during computation. The lower relevance score in the math group suggests that this cancellation did not occur as effectively, perhaps due to less reflective thinking. Finally, the breadth component showed a significant advantage for the math group (mean score: 0.85) over the science group (mean score: 0.46). This indicates that math prodigy students are more extensively trained in systematically eliminating redundant cases and ensuring that all necessary conditions are comprehensively addressed during problem solving.

Conclusion and Recommendations

This study diverged from previous research, which treated creativity and problem-solving skills as separate constructs, by integrating them into a single competency termed "creative problem-solving ability." To evaluate this competency, a Fermi estimation problem—designed as an open-ended question allowing middle school gifted students to think freely—was employed. Specifically, students were asked to estimate "How many plastic bags are used nationwide in one day?" while detailing the assumptions, rationale, and solution process. Six components of creative problem-solv-

ing—fluency, flexibility, originality, validity, relevance, and breadth—were examined among math and science gifted groups. The analysis revealed that the math gifted group scored higher in validity and breadth compared to the science gifted group, whereas scores for fluency, flexibility, and originality were somewhat lower. This pattern suggests that science gifted students tend to generate and test various hypotheses when solving problems, while math gifted students, with an emphasis on logical reasoning and systematic management of assumptions, achieve higher validity and breadth scores.

Based on these findings, we propose tailored educational strategies for gifted students. Specifically, math gifted programs should focus on curriculum development and assessment items that nurture fluency, flexibility, and originality more evenly, whereas science gifted programs should develop instructional strategies that further strengthen validity and breadth.

References

Ärlebäck, J. B. (2009). On the use of realistic Fermi problems for introducing mathematical modelling in school. *The Mathematics Enthusiast, 6*(3), 331-364.

Balka, D. S. (1974). Using research in teaching: Creative ability in mathematics. *The Arithmetic Teacher*, *21*(7), 633-636.

Calson, J. E. (1997). Fermi problems on gasoline consumption. *The Physics Teacher*, *35*(5), 308-309.

Edge, D. R., & Dirks, M. K. (1983). Problem solving, Enrico Fermi and the bull moose. *School Science and Mathematics*, *83*(7), 601-608.

Treffinger, D. J., Isaksen, S. G., & Stead-Dorval, K. B. (2023). *Creative problem solving: An introduction*. Routledge.

Efthimiou, C. J., & Llewellyn, R. A. (2007). Cinema, Fermi problems and general education. *Physics education*, *42*(3), 253.

Hadamard, J., & Hadamard, J. (1975). *Essai sur la psychologie de l'invention dans le domaine mathématique* (pp. 132-133). Paris: Gauthier-Villars.

Kittel, A., & Marxer, M. (2005). Wie viele Menschen passen auf ein Fussballfeld. *Mit Fermiaufgaben individuell fördern. Mathematik Lehren*, *131*, 14-18.

Krutetskii, V. A. (1976). The psychology of mathematical abilities in school children/The Univ.

Peter-Koop, A. (2004). Fermi problems in primary mathematics classrooms: Pupils' interactive modelling processes. In *Mathematics education for the third millennium: Towards 2010. Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia, Townsville* (Vol. 27).

Peter-Koop, A. (2005). Fermi Problems in Primary Mathematics Classrooms: Fostering children's mathematical modelling processes. *Australian Primary Mathematics Classroom*, *10*(1), 4-8.

細谷功. 地頭力 (じあたまりょく) を鍛える: 問題解決に活かす 「フェルミ推定」.

TSG10-04

ENHANCING STE(A)M EDUCATION THROUGH PROJECT-BASED LEARNING: A FOCUS ON AGE-PERIOD-COHORT MODELING

Diana Jasmin S. Costales
Department of Mathematics and Physics, College of Science, University of Santo Tomas, Philippines, dianajasmin.costales.sci@ust.edu.ph

Ciara Marie B. Gealone
Department of Mathematics and Physics, College of Science, University of Santo Tomas, Philippines, ciaramarie.gealone.sci@ust.edu.ph

Ma. Carlota B. Decena
Department of Mathematics and Physics, College of Science, University of Santo Tomas, Research Center for the Natural and Applied Sciences, University of Santo Tomas, Philippines, mbdecena@ust.edu.ph

Mathematics has extensive applications and plays a crucial role in addressing real-world challenges. However, its relevance is not always evident to STE(A)M students due to a lack of substantial first-hand experience in conducting activities and studies that highlight its significance. This paper emphasizes the importance of project-based learning in fostering an appreciation for mathematics by proposing a mathematical project that integrates various statistical and analytical techniques to facilitate comprehensive learning. Specifically, this is demonstrated through mathematical modeling, using the age-period-cohort (APC) model to analyze and project historical and expected cancer incidence trends. The study presents the process of mathematical modeling, starting with data preparation, understanding model assumptions, model fitting and validation, and generating and interpreting results. Ultimately, this study highlights the critical intersection of mathematics and STE(A)M education, bridging the gap between mathematical learning and its real-world applications.

Keywords: project-based learning, mathematical modeling, cancer

Introduction

Globally, STE(A)M (Science, Technology, Engineering, Arts, and Mathematics) curricula often struggle to align with students' lived experiences and interests, leading to disengagement. While STE(A)M integrates technical and creative disciplines to promote critical thinking and problem-solving skills (Wised & Inthanon, 2024), its real-world relevance is often underutilized. Project-Based Learning (PBL) enhances STE(A)M education by emphasizing real-world problem-solving, increasing student motivation and interdisciplinary understanding, including in mathematics (Laslo et al., 2024; Lewis & Powell, 2016).

Mathematical modeling, a key component of STEM research, fosters pattern recognition, decision-making, and prediction in fields such as epidemiology and policy development (Star & Moghadas, 2010). However, despite its potential to address societal issues, mathematical modeling remains underrepresented in STEM education (Bajuri et al., 2018; Doğan et al., 2019).

This paper aims to bridge the gap between STE(A)M curricula and real-world applications by integrating mathematical modeling within PBL. Using the Age-Period-Cohort (APC) model, the study demonstrates how health-focused modeling grounded in PBL can enhance the problem-solving and critical thinking of tertiary-level students in science and technology. By analyzing cancer incidence trends in the Philippines through age, period, and cohort factors, the study highlights the predictive potential of mathematical modeling to inform public health policy and strengthen STE(A)M education. This research provides a foundation for future exploration of PBL, enabling the application of technical skills in real-world contexts.

Methodology

This study used a predictive, quantitative design, incorporating mathematical modeling into STE(A)M education within a PBL framework. Rooted in educational theories like Social Constructivism and Experiential Learning, PBL promotes knowledge construction through experience and collaboration (Project Pals, n.d.; Kwietniewski, 2017). By engaging students in real-world problem-solving, PBL enables the active application of mathematical models to understand complex phenomena, such as cancer incidence trends. In this study, mathematical modeling within PBL offers a structured approach to integrate mathematics, science, technology, and communication skills to present findings.

The APC model, based on Poisson regression, examines disease incidence rates in relation to the time-varying factors of age, period, and birth cohort effects (Møller et al., 2003). The mathematical modeling process included analyzing historical data and predicting future trends for the five leading cancer types in the Philippines: breast, lung, colorectal, liver, and prostate cancers. Data from the Global Burden of Disease Study (1992–2021) were organized and processed using the Nordpred package in R (Bulow, 2019) for APC modeling.

Cancer incidence data were explored through age-specific rates (ASR), age-standardized rates (ASIR), net drifts, and local drifts. By projecting cancer trends, this study illustrates the practical application of mathematical and statistical methods, enhancing students' analytical and data interpretation skills.

Discussion

This study demonstrates the comprehensive process of mathematical modeling using the APC model, highlighting its value in applied learning and its replicability for tertiary-level students. Figure 1 shows the process of modeling cancer incidence using the APC model.

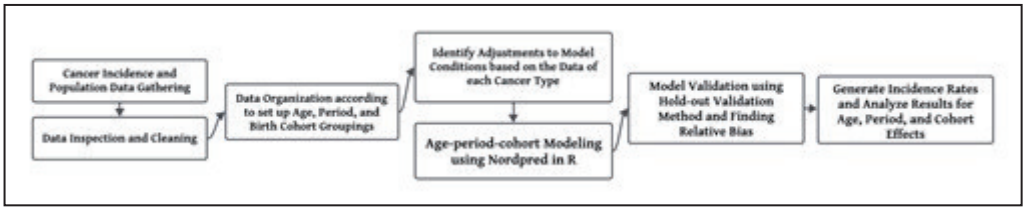


Figure 1. Mathematical Modeling Process using Age-period-cohort Model.

Data Organization

The process begins with organizing data, which in this study was collated into five-year age and time periods for the APC model. Birth cohort denotes the year of birth of the individual diagnosed with cancer, calculated as the relationship *Cohort = Period − Age* (Teng et al., 2024). Organizing the data facilitates its inspection and enables the identification of missing values and outliers, which must be addressed using appropriate interpolation and extrapolation methods.

Model Adjustments

A clear understanding of the model algorithm is essential to the student's learning, with Figure 2 providing a visual guide to the underlying processes of the APC and Nordpred methodologies. Moreover, mathematical modeling requires adjustments to model conditions to enhance projection accuracy. In applying the APC model, the youngest age group in the regression must be initialized, and a cut trend must be determined. A threshold is set due to sparse cancer incidence data in younger groups, which can affect estimation and projections; for these groups, projections rely on average incidence rates from the most recent 10 years (Teng et al., 2024). The APC model also assumes that the average rate of change over time gradually declines after the first five-year prediction period, necessitating the initialization of the cut trend (Menyhárt et al., 2018). Thus, a cut trend—a vector of percentages used to reduce the drift parameter for each prediction period—needs to be determined. Additionally, the model adjusts the drift component by applying the recent slope from the last two periods if a deviation from a linear trend is detected. Otherwise, it uses the average trend from the five most recent time periods (Teng et al., 2024).

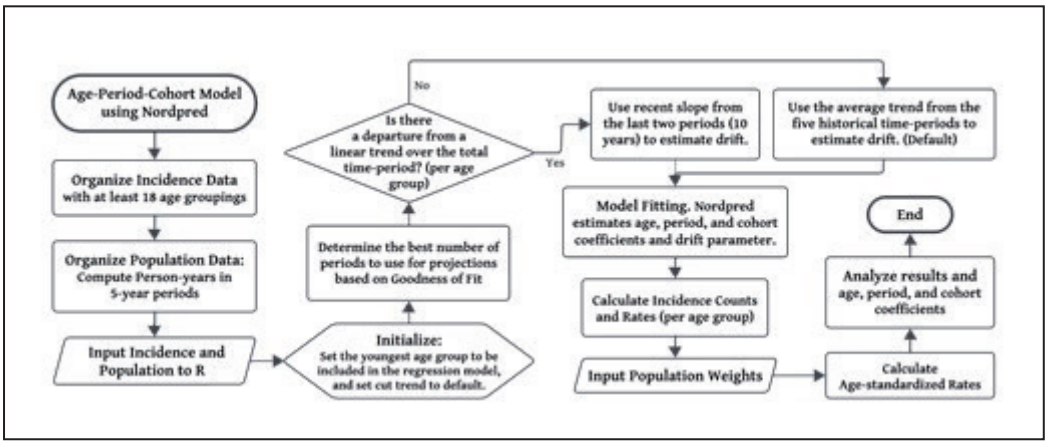


Figure 2. Algorithm of the nordpred modeling process for the age-period-cohort model in R.

Model Fitting

Using the Nordpred package in R, modeling proceeds by fitting data to the APC model equation to estimate parameters. In the APC approach, model equations are computed by determining the common drift as well as the age, period, and cohort effects (Demers et al., 2020). Cancer incidence is projected per age group using unique parameter values. Based on this framework, historical cancer incidence data for the leading cancer types in the Philippines were analyzed and modeled.

Model Validation

The model can be further validated using different validation methods. The dataset was split into two parts: training set (1992-2016, five consecutive 5-year periods) and testing set (2017-2021, the most recent 5-year period). The projections generated from the training model were compared against the observed values in the testing period to evaluate model performance. Relative Bias (*RB*) was used as the primary validation measure, quantifying the relative error between the projected and observed age-standardized rates (Demers et al., 2020) per cancer type per sex. A general standard of RB below 10%, based on the study of Demers et al. (2020), was considered in this study to assess the model's performance.

Result Analysis

Beyond understanding the mathzzematical processes, it is essential to develop creative approaches to analyzing empirical data and interpreting results. This study examined historical cancer data by exploring age, period, and cohort effects. Age effects were evaluated through average age-specific rates, while period and cohort effects were determined relative to a median reference group. Period effects were calculated as the ratio of ASIR for each period to the reference, while cohort effects were derived by comparing cohort-specific rates against the reference cohort.

Furthermore, it is important to consider results relative to population size. In cancer studies, this is achieved by computing age-adjusted rates. ASRs per 100,000 individuals were calculated by dividing the observed and projected cases by the population and multiplying the result by 100,000. ASIRs were then derived by incorporating the WHO Standard Population Weights into the computation. These age-adjusted rates provide a more precise measure of cancer risk within a population, allowing for more accurate comparisons across different groups and periods.

Lastly, net and local drifts, representing the annual percentage change in ASIRs and ASRs, respectively, were calculated to provide deeper insights into incidence trends. These drifts are determined by applying the logarithmic transformation to ASIR and ASR values, calculating the slope of the transformed data, and using the formula: $drift = [exp(slope) - 1] \times 100$. Applying statistical transformations to cancer data enhances trend interpretation and variability assessment and highlights the real-world impact of mathematical modeling in epidemiology.

Conclusion

Beyond its epidemiological significance, this study emphasizes the value of integrating real-world applications into STE(A)M education. Mathematical modeling involves the comprehensive learning of data organization and cleaning, understanding probability distributions, model validation methods, and creative analysis of results to draw meaningful conclusions. The extensive mathematical modeling process highlights the importance of both technical understanding and practical application in real-world contexts, making it a comprehensive PBL approach. By bridging theoretical mathematics with practice, this research underscores the role of modeling in public health and its potential to enhance STE(A)M curricula. Future studies may refine modeling techniques, incorporate additional predictive factors, and develop structured PBL modules to provide students with practical experience and deepen their engagement with applied mathematics.

References

Bajuri, M.-R., Maat, S.-M., & Halim, L. (2018). Mathematical Modeling from Metacognitive Perspective Theory: A Review on STEM Integration Practices. *Creative Education*, 09(14), 2203–2214. https://doi.org/10.4236/ce.2018.914161

Bulow, E. (2019, May 17). *Fit power5 and poisson Age-Period-Cohort models for…* https://rdrr.io/github/haraldwf/nordpred/man/nordpred.html

Demers, A., Qiu, Z., Dewar, R., & Shaw, A. (2020). Validation of Canproj for projecting Canadian cancer incidence data. *Health promotion and chronic disease prevention in Canada: research, policy and practice, 40*(9), 267–280. https://doi.org/10.24095/hpcdp.40.9.02

Doğan, M.-F., Gürbüz, R., Erdem, Z.-Ç., & ŞahİN, S. (2019). Using Mathematical Modeling for Integrating STEM Disciplines: a Theoretical framework. *Türk Bilgisayar Ve Matematik Eğitimi Dergisi*, 10(3), 628–653. https://doi.org/10.16949/turkbilmat.502007

Global Health Data Exchange. (n.d.). *Global Burden of Disease Study 2021 (GBD 2021) Sources Tool.* https://ghdx.healthdata.org/gbd-2021/sources?components=23&locations=16

Kwietniewski, K. (2017). Literature Review of Project Based Learning. *Career & Technical Education Theses.* 1. https://digitalcommons.buffalostate.edu/careereducation_theses/1

Laslo, G., Petrovic, D., & Maric, M. (2024). *Project-Based Learning in the context of STEAM education.* https://www.semanticscholar.org/paper/Project-Based-Learning-in-the-Context-of-STEAM-Laslo-Petrovic/09b42b37c041574bd90a1a41a830c2c43db48279

Lewis, M., & Powell, J.-A. (2016). Modeling Zombie Outbreaks: A Problem-Based Approach to Improving Mathematics One brain at a time. *PRIMUS*, 26(7), 705–726. https://doi.org/10.1080/10511970.2016.1162236

Menyhárt, O., Fekete, J., & Győrffy, B. (2018). Demographic shift disproportionately increases cancer burden in an aging nation: current and expected incidence and mortality in Hungary up to 2030. *Clinical Epidemiology, Volume 10*, 1093–1108. https://doi.org/10.2147/clep.s155063

Møller, B., Fekjær, H., Hakulinen, T., Sigvaldason, H., Storm, H.-H., Talbäck, M., & Haldorsen, T. (2003). Prediction of cancer incidence in the Nordic countries: empirical comparison of different approaches. *Statistics in Medicine*, 22(17), 2751–2766. https://doi.org/10.1002/sim.1481

NORDCAN. (n.d.). Nordcan 2.0. Retrieved February 13, 2025, from https://nordcan.iarc.fr/dk/about

Philippine Statistics Authority. (2024). Population and Migration. *OpenSTAT*. https://openstat.psa.gov.ph/Database/Population-and-Migration

Project Pals. (n.d.). *The Educational Theories In Project-Based Learning.* https://projectpals.com/the-educational-theories-in-project-based-learning/

Star, L., & Moghadas, S. (2010). *The role of mathematical modeling in public health planning and decision making.* https://nccid.ca/publications/the-role-of-mathematical-modelling-in-public-health-planning-and-decision-making/

Teng, A., Stanley, J., Jackson, C., Koea, J., Lao, C., Lawrenson, R., Meredith, I., Sika-Paotonu, D., & Gurney, J. (2024). The growing cancer burden: Age-period-cohort projections in Aotearoa New Zealand 2020-2044. Cancer Epidemiology, 89, 102535. https://doi.org/10.1016/j.canep.2024.102535

United Nations. (2024). *World Population Prospects 2024*. https://population.un.org/wpp/downloads?-folder=Standard%20Projections&group=Population

Wised, S., & Inthanon, W. (2024). The evolution of STEAM-Based programs: fostering critical thinking, collaboration, and Real-World application. *Journal of Education and Learning Reviews*, 1(4), 13–22. https://doi.org/10.60027/jelr.2024.780

Topic Study Groups 10
(Day 3)

TSG10-05

EVALUATING PRACTICAL STEAM ACTIVITIES FOR STUDENT LEARNING AND ENGAGEMENT

Masitah Shahrill
Universiti Brunei Darussalam, Brunei Darussalam, masitah.shahrill@ubd.edu.bn

Nor Azura Abdullah
Universiti Brunei Darussalam, Brunei Darussalam, azura.abdullah@ubd.edu.bn

STEAM education has emerged as a powerful pedagogical approach fostering creativity, problem-solving, and critical thinking. While extensive research supports its benefits, practical classroom implementations often face significant challenges. This study conducts a document analysis of eight group projects undertaken by graduate teacher candidates, investigating how hands-on STEAM activities enhance engagement, cognitive development, and interdisciplinary learning. The findings reveal that collaborative projects improve students' comprehension and motivation but highlight common barriers, including time constraints, material limitations, and inconsistent interdisciplinary connections. The study underscores the need for structured learning frameworks, enhanced teacher training, and a stronger focus on collaborative problem-solving. Recommendations for future STEAM implementations are provided to bridge the gap between theory and practice, ensuring students' more effective and inclusive learning experience.

Keywords: STEAM activities, group projects, graduate teacher candidates, student engagement

Introduction

STEM (Science, Technology, Engineering, and Mathematics) education has experienced considerable advancement (Chiu et al., 2015) in a progressively interconnected global landscape. More recently, integrating the 'Arts' into STEM and forming STEAM has emphasised the importance of creativity and interdisciplinary learning in education. Research has demonstrated that STEAM approaches foster problem-solving skills, critical thinking, and collaboration (Conradty & Bogner, 2020), all of which are crucial for addressing real-world challenges. However, Quigley and Herro (2019) stated that despite its proven benefits, classroom implementation remains inconsistent. There is an increasing need to examine STEAM teacher preparation, as emphasised by Jenkin (2022), who highlighted the Kuching Declaration on Science and Technology Education (ICASE, 2022). The declaration underscores the importance of rethinking STEM education, particularly in the context of teacher preparation, as a crucial step toward shaping future educational practices and addressing the demands of a rapidly evolving world. Educators often struggle with time constraints, resource availability, and effective interdisciplinary integration in implementing STEAM activities (Erawan et al., 2025; Perignat & Katz-Buonincontro, 2019).

This study evaluates the effectiveness of practical or hands-on STEAM activities by analysing eight group projects. It investigates how these activities foster student engagement, deep learning, and interdisciplinary connections while identifying challenges in implementing STEAM education practically. The insights gained aim to inform strategies for optimising STEAM practices for broader

adoption. The objectives of this study focus on assessing how hands-on STEAM activities influence student engagement and motivation, determining the extent to which these activities improve interdisciplinary learning and develop problem-solving skills, and analysing the challenges and limitations encountered during the implementation of the activities.

Methodology

This study utilises a qualitative document analysis method to examine eight group project reports. These projects were part of module assignments conducted by graduate teacher candidates who enrolled in the Master of Teaching programme at a graduate education faculty in Brunei Darussalam. The reports describe various STEAM activities carried out in primary and secondary schools by the teacher candidates, emphasising interdisciplinary learning, engagement strategies, and student reflections (see Table 1). All projects targeted the secondary level, except for MathsterChef, which targeted upper primary students. A thematic coding framework was employed for the data analysis to extract three key themes and its respective sub-themes: Student Engagement (Interest and motivation; Collaboration and teamwork); Learning Effectiveness (Knowledge application; Problem-solving skills); and Challenges and improvements (Time constraints; Resource availability), all of which are related to learning and engagement.

Table 1. Description of the eight STEAM group projects

STEAM Group Projects	Objectives
Creating Ministry Buildings	Exploring mathematical and architectural applications
Flying Egg Challenge	Investigating physics and engineering concepts
Designing Roller Coasters	Applying trigonometry, velocity, and energy transformations
Balanced Sip	Exploring ratio and proportion in chemistry and culinary arts
Building Water Fountains	Integrating engineering design and fluid mechanics
MathsterChef	Merging mathematics with real-world culinary applications
Building Bridges	Demonstrating structural integrity using engineering principles
Geometry Schools	Utilising spatial reasoning and artistic design in urban planning

Findings and Discussion

Student Engagement and Motivation

The analysis shows that hands-on activities greatly enhance student engagement. Projects such as the Flying Egg Challenge and MathsterChef generated high excitement due to their interactive and competitive elements. Students actively participated, demonstrating curiosity and teamwork.

As described in the Flying Egg Challenge, "*When asked which part they enjoyed the most during the activity, all students mentioned that they enjoyed building up the structure and the dropping experiment. The teacher also asked the students what they learned from the activity; some said they could think wisely before purchasing, manage their budgeting, and work as a team.*" Their re-

flections highlight how engagement is strengthened when students are immersed in decision-making and hands-on problem-solving. However, challenges arose when dominant students led decision-making, limiting equal participation. The Designing Roller Coasters project revealed similar patterns. "*Throughout the implementation period, it can be observed that most groups immediately started with the construction of their roller-coasters without proper planning and visualising their design. The students were eager to start with the roller-coaster construction, and they planned the path as they progressed through the construction.*" While excitement was evident, some groups lacked structure, leading to uneven contributions among team members. More structured role assignments could mitigate this issue, ensuring every student has a meaningful role in the learning process.

Learning Effectiveness and Interdisciplinary Connections

The teacher candidates successfully applied mathematical and scientific concepts across different projects. The Building Bridges activity, for instance, helped their students understand structural engineering, while the Balanced Sip project enhanced their grasp of proportional reasoning. The application of engineering and physics principles was evident in the Designing Roller Coasters project, where students engaged with real-life mathematical concepts. "*For the foundational knowledge, students should be able to distinguish the mathematical concepts behind the roller-coaster, for example, calculating energy changes and speed. This activity will also expose students to the real-life application of trigonometry and the concept of gradients.*" This excerpt highlights how students were able to connect mathematical theories to physical applications, reinforcing their learning experience.

Conversely, some students struggled with conceptualising interdisciplinary links. In the Creating Ministry Buildings project, students combined mathematics and architecture but lacked explicit ties to engineering principles. This challenge was echoed in the Building Water Fountains activity, where students built functioning models but could not fully articulate the fluid mechanics principles underlying their designs. As noted in the project report, "*The students developed problem-solving and necessary thinking skills through their design and construction process. Factors contributing to the success of water fountains are the quality of the structure, symmetry, and other requirements. Critical thinking is dominated by the constructivist concept that students showed.*" While students excelled in constructing their models, their conceptual understanding required deeper reinforcement. This finding suggests a need for the teacher candidates to scaffold interdisciplinary learning more explicitly, ensuring that their students grasp the theoretical underpinnings of their hands-on work.

Implementation Challenges and Areas for Improvement

Despite the success, the projects revealed several challenges. Time constraints limited the depth of exploration, with students often rushing to complete the activity tasks. The Building Bridges project illustrated this issue, as students expressed frustration over the limited timeframe. "*During the implementation phase, students could not finish their construction projects within the designated timeframe. This shortfall may have resulted from inefficient planning or rigorous requirements, a conclusion supported by feedback from the students themselves.*" Without sufficient time, students could not refine their designs, affecting the quality of their final outcomes.

Material limitations also impacted the quality of the students' work. In the Building Bridges project, insufficient adhesive weakened structural integrity, leading to repeated failures during testing. "*Another challenge faced during the implementation, supported by students' feedback, was the func-*

tionality of the provided glue. The poor glue performance severely hindered progress, resulting in time wasted. Thus, a recommendation for future research is to test the materials before the activity to ensure their quality and suitability." Similar issues arose in other projects, indicating the importance of providing high-quality resources to ensure successful execution. Moreover, inconsistent interdisciplinary integration resulted in gaps in student understanding. In certain instances, students concentrated more on the practical elements of their projects while overlooking the deeper conceptual connections. Future implementations should include more structured pre-planning activities, better access to materials, and more straightforward guidance on interdisciplinary connections. Addressing these concerns will help students experience a more cohesive and effective STEAM learning process.

The above findings demonstrated that while hands-on STEAM activities effectively increase engagement and knowledge application, they require better structure, more apparent interdisciplinary connections, and improved logistical planning. The results of this study are consistent with the findings of Shernoff et al. (2017), who highlighted similar needs in teacher education for integrated STEAM education. Furthermore, the experiences of the graduate teacher candidates are similar to Ryu et al.'s (2019) study. Despite the successful implementation, they faced challenges in existing school practices, limited interdisciplinary understanding, and a lack of mentors for guidance. Addressing these challenges will enhance the learning experience, ensure equitable participation, and strengthen students' conceptual understanding. The following recommendations can be considered to improve the effectiveness of STEAM education: Introduce structured pre-planning to encourage draft designs and postulate the development before project execution; To assign team roles with clear responsibilities to balance fair participation and collaboration; Allow ample time for thorough exploration and iteration; Provide teachers with strategies and training to promote interdisciplinary learning; and Ensure the availability of high-quality materials for project-based learning.

Conclusion

STEAM education has great potential to foster creativity, critical thinking, and interdisciplinary problem-solving. This analysis of the study underscores the benefits of hands-on activities and points out essential areas for enhancement. Teachers can create meaningful and impactful STEAM experiences by addressing engagement strategies, learning effectiveness, and implementation challenges. STEAM education should integrate all disciplines rather than be taught in isolation. However, as an initial step, this approach represents our response to the call of integrating STEAM into our mathematics methods courses within a teacher education setting as one component in the STEAM education ecological system. Future research should focus on longitudinal studies to assess the long-term impact of these interventions on student learning outcomes.

References

Chiu, A., Price, C. A., & Ovrahim, E. (2015, April 11–14). *Supporting elementary and middle school STEM education at the whole school level: A review of the literature*. NARST 2015 Annual Conference, Chicago, IL, United States. https://www.msichicago.org/fileadmin/assets/educators/science_leadership_initiative/SLI_Lit_Review.pdf

Conradty, C., & Bogner, F. X. (2020). STEAM teaching professional development works: Effects on students' creativity and motivation. *Smart Learning Environments*, *7*(1), 1–26. https://doi.org/10.1186/s40561-020-00132-9

Erawan, G. M., Mariana, N., & Suryanti, S. (2025). Development and challenges of STEAM learning implementation in elementary schools: A comprehensive literature review. *International Journal of Emerging Research and Review*, *3*(1), 1–19. https://doi.org/10.56707/ijoerar.v3i1.94

International Council of Associations for Science Education (ICASE). (2013). *Kuching declaration on science and technology education*. https://www.icaseonline.net/ICASE%20Kuching%20Declaration_Final.pdf

Jenlink, P. M. (2022). A global reimagining of STEM teacher preparation and practice. In P. M. Jenlink (Ed.), *STEM teacher preparation and practice for the 21st century: Research-based insights*, (pp. 1–28). Information Age Publishing.

Perignat, E., & Katz-Buonincontro, J. (2019). STEAM in practice and research: An integrative literature review. *Thinking Skills and Creativity, 31*, 31–43. https://doi.org/10.1016/j.tsc.2018.10.002

Quigley, C. F., & Herro, D. (2016). "Finding the joy in the unknown": Implementation of STEAM teaching practices in middle school science and math classrooms. *Journal of Science Education and Technology*, *25*, 410–426. https://doi.org/10.1007/s10956-016-9602-z

Ryu, M., Mentzer, N., & Knobloch, N. (2019). Pre-service teachers' experiences of STEM integration: Challenges and implications for integrated STEM teacher preparation. *International Journal of Technology and Design Education, 29*, 493–512. https://doi.org/10.1007/s10798-018-9440-9

Shernoff, D. J., Sinha, S., Bressler, D. M., & Ginsburg, L. (2017). Assessing teacher education and professional development needs for the implementation of integrated approaches to STEM education. *International Journal of STEM Education, 4*, 1–16. https://doi.org/10.1186/s40594-017-0068-1

TSG10-06

EMPOWERING FEMALE MATHEMATICIANS FOR THE FUTURE OF STEAM IN BRUNEI DARUSSALAM

Suriyna Shaminan
Universiti Brunei Darussalam, Brunei Darussalam, suriynashmnn@gmail.com

Elvynna Leong
Universiti Brunei Darussalam, Brunei Darussalam, elvynna.leong@ubd.edu.bn

Masitah Shahrill
Universiti Brunei Darussalam, Brunei Darussalam, masitah.shahrill@ubd.edu.bn3

Worldwide, women in STEAM face unique challenges, yet they play a vital role in shaping the future of education and research. This study delves into the experiences of female academics specialising in mathematics working in higher education institutions in Brunei Darussalam. We aim to gather their insights on the factors affecting the careers of female mathematicians in Brunei and their impact on the country's broader STEAM education framework. Through qualitative analysis, we uncover both the progress and the remaining obstacles. The findings underscore the urgent need for systemic policy reforms, more excellent representation, and stronger support networks to empower the next generation of women in STEAM.

Keywords: Female mathematicians, STEAM, qualitative analysis, achievements and challenges, Brunei Darussalam

Introduction

Women's journey in STEAM fields, particularly in mathematics, has long been shaped by cultural norms, institutional structures, and systemic barriers (Fox, 1981; Ha et al., 2020; Hong, 2021; Mathura, 2023; UNWomen, 2022). Globally, gender imbalances persist, stifling the potential of female mathematicians to drive innovation and progress. In Brunei Darussalam, where the academic landscape is still emerging, women in mathematics face unique challenges, including limited institutional support and a lack of visible role models. While global research has addressed gender disparities in STEAM, there is a striking gap in studies focused on Brunei, highlighting the need for localised research into the experiences and contributions of women in mathematics (Paderna & Monterola, 2021; Sarsale et al., 2024).

STEAM education is increasingly recognised as a cornerstone of economic growth, yet many ASEAN nations, including Brunei, continue to grapple with gender disparities in these fields (Ha et al., 2020; Sarsale et al., 2024). Research suggests that early mentorship, institutional support, and policy initiatives can significantly enhance women's participation in STEM (Paderna & Monterola, 2021). However, the lack of localised research on women in mathematics in Brunei makes it challenging to translate these global strategies into actionable steps for the nation's educational system. For example, Hong (2021) stated that South Korea has made strides by integrating the arts into STEM, fostering creativity and problem-solving skills, a model that could help address gender in-

equalities in Brunei. This study, grounded in qualitative analysis, seeks to fill this gap by exploring the experiences of female mathematicians in Brunei, shedding light on both their achievements and the challenges they continue to face.

This research aims to provide valuable insights into the factors influencing the careers of female mathematicians in Brunei and their impact on the broader STEAM education framework. By amplifying their voices, the study offers recommendations for policy reforms, mentorship programmes, and institutional changes that promote inclusivity. Addressing the shortage of qualified STEAM educators and the need for better instructional resources are among the key challenges across ASEAN nations (Sarsale et al., 2024), and these are essential to creating a supportive academic environment. Ultimately, closing these gaps is crucial for empowering future Bruneian women to pursue and excel in mathematics, contributing meaningfully to the evolving STEAM landscape.

Methodology

To explore the experiences of female mathematicians in Brunei, we conducted a qualitative study, gathering open-ended survey responses from 15 female academics with a mathematics specialisation working in six Brunei higher education institutions (Ebil & Shahrill, 2023; Shahrill & Keasberry, 2023). The participants shared personal narratives regarding their career pathways, influences, challenges, and contributions to the field. Responses were thematically analysed using content analysis, with verbatim excerpts included to preserve authenticity. A manual coding approach was used to categorise themes and sub-themes, ensuring a systematic and transparent interpretation of the data (refer to Table 1).

Table 1. Thematic coding categorisation

Themes	Sub-Themes
STEAM Education Influence	Parental Influence on STEAM Careers Educational Background in STEAM STEAM-Related Research Contributions
Challenges in STEAM Fields	Gender Bias and Institutional Barriers Work-Life Balance in STEAM Careers Limited Institutional Support for STEAM Research
Mentorship and Role Modelling	Female Mathematicians as STEAM Mentors Representation of Women in STEAM
Future of STEAM in Brunei	Need for STEAM Policy Reforms Proposed Creation of STEAM Organisations Promotion of STEAM Education for Girls

Findings and Discussion

STEAM Education Influence

More than half of the respondents indicated that their parents placed significant emphasis on STEAM education during their upbringing, whereas the remaining respondents did not experience such emphasis. For many, the unwavering support of their parents played a pivotal role in shap-

ing their career paths, providing a foundation of encouragement and belief in their abilities. They credited their early interest in mathematics and STEAM fields to the encouragement they received from their families. One participant reflected on her upbringing, saying, "*My parents have had a positive impact on my career by providing support, guidance, and encouragement throughout my professional journey. Their values, advice, prayers and lessons have influenced my work ethic, decision-making, and overall approach to my career*". This kind of support fostered confidence and resilience, reinforcing that success in mathematics and STEAM was possible and encouraged. The commitment of female mathematicians to research in Brunei is evident through their academic contributions. One participant recalled, "*During Master's Degree: One of the examiners was very satisfied with my thesis, published a paper*". Another highlighted, "*Successfully published and co-published papers with previous research students; acknowledgements from international bodies and international academics with invitations as keynote and invited speakers at conferences*". These statements illustrate their impact and recognition in the field.

Challenges in STEAM Fields

Despite progress in increasing female participation in STEAM, several persistent challenges remain. Gender bias was not a dominant concern for most respondents, but institutional barriers and recognition in international academic circles posed difficulties. One participant noted, "*I haven't personally encountered challenges related to gender biases. However, due to the small and less well-known nature of our country, obtaining reliable data has been challenging. Additionally, we have faced difficulty in getting our papers accepted by international journals. There is also a lack of institutional encouragement for paper production within the organisation, and insufficient funding is available for such endeavours*". These constraints limit opportunities for female mathematicians to contribute to global research and hinder their academic progression.

Balancing personal and professional responsibilities remains a major struggle, particularly for women managing family commitments alongside research careers. One respondent shared, "*As a mother of three, the task of juggling familial responsibilities with professional duties has been my most significant challenge. It requires meticulous time management and support structures both at work and home*". Additionally, limited institutional support for research further compounds these challenges. A participant explained, "*As of now, the main challenge for me is the institutional barrier. Since my research is quite a specialised niche, it is difficult to find collaborators in the same field. The only solution is to find international collaborators or apply the work in other fields of research*". Another pointed out that "*The challenges in the local context were mainly the inconsistency in getting sufficient students to pursue mathematics education at the master and doctoral levels, data collection gatekeepers, and obtaining sufficient and willing participants for data collection*". These findings highlight female mathematicians' structural and institutional difficulties, reinforcing the need for more substantial policy support, collaboration opportunities, and institutional reforms to create a more inclusive and supportive research environment.

Mentorship and Role Modelling

While parental support played a significant role, mentorship and representation emerged as critical factors in fostering confidence among younger women aspiring to enter STEAM fields. Many participants emphasised the importance of having role models who could guide and inspire them. Some participants actively engaged in mentoring initiatives, recognising the need to cultivate a more supportive academic environment. One mathematician stressed, "*Building supportive networks and mentorship programs can empower more young women to pursue mathematics at higher*

levels of education and into their careers". However, the visibility of female role models remained a concern, with another respondent acknowledging, "*The representation of female mathematicians in Brunei, while still lower than I would hope, is on an encouraging upward trend*". While progress has been made, there remains room for greater advocacy and representation to inspire the next generation.

Future of STEAM in Brunei

Looking toward the future, participants expressed a strong desire for structured reforms to enhance female participation in STEAM. "*Looking to the future, I would like to see further initiatives and policies that specifically address the unique challenges faced by women in mathematics*," one participant shared. Establishing professional networks and organisations dedicated to supporting female mathematicians was seen as a key step in achieving this goal. Another respondent envisioned a more connected academic community, stating, "*I would like to see an organisation for female mathematicians in Brunei to be established in the future*". Early engagement in mathematics was also emphasised as essential for fostering long-term interest and participation in STEAM. One participant succinctly articulated this sentiment: "*I feel that continuous efforts to encourage girls from a young age to engage with mathematics are crucial."* By addressing these systemic challenges and promoting initiatives that support women at every stage of their careers, Brunei can pave the way for a more inclusive and equitable STEAM landscape.

Conclusion

The experiences of female mathematicians in Brunei's STEAM landscape highlight both progress and persistent challenges. Through their voices, we learn how parental influence, institutional barriers, mentorship, and policy gaps shape their careers and aspirations. This study highlights the urgent need for systemic reforms, mentorship, and policy measures promoting female STEAM engagement. Although these women have achieved significant progress, their stories illustrate their ongoing challenges and the hopes they cherish for future generations. Brunei can enable more women to make impactful contributions in mathematics and related fields by cultivating an environment of support, visibility, and opportunity. As we reflect on these findings, it's clear that the journey toward gender equality in STEAM is far from over. But with concerted efforts from policymakers, educators, and communities, we can create a future where every woman has the opportunity to thrive in mathematics and beyond.

References

Ebil, S., & Shahrill, M. (2023). Overview of education in Brunei Darussalam. In L. P. Symaco & M. Hayden (Eds.), *International handbook on education in South East Asia* (pp. 1–21). Springer Nature Singapore. https://doi.org/10.1007/978-981-16-8136-3_46-1

Fox, L. H. (1981). *The problem of women and mathematics*. Ford Foundation.

Ha, C. T., Trinh, T. P., Nguyen, T. T., Le, T. T. H., Ngo, V. D., & Tran, T. (2020). A bibliometric review of research on STEM education in ASEAN: Science mapping the literature in Scopus database, 2000 to 2019. *EURASIA Journal of Mathematics, Science and Technology Education, 16*(10), em1889. https://doi.org/10.29333/ejmste/8500

Hong, O. (2021). STEM/STEAM education research in South Korea. In T. W. Teo, A-L. Tan, & P. Teng (Eds.), *STEM education from Asia: Trends and perspectives* (pp. 211–227). Routledge. https://doi.org/10.4324/9781003099888-16

Mathura, S. (2023). *Empowering women in STEM: Personal stories and career journeys from around the world*. CRC Press.

Paderna, E. S., & Monterola, S. L. C. (2021). Advancing integrated STEM education in the Philippines through STEM curriculum implementation. *STEM Education from Asia: Trends and Perspectives*. Routledge. https://doi.org/10.4324/9781003099888-11

Sarsale, J., Lopez, E. N., Alia, L., & Alejandro, B. (2024). STEM education in ASEAN countries: Practices and way forward. *Science Education International, 35*(4), 360–368. https://doi.org/10.33828/sei.v35.i4.7

Shahrill, M., & Keasberry, C. (2023). Higher education in Brunei Darussalam. In L. P. Symaco & M. Hayden (Eds.), *International handbook on education in South East Asia* (pp. 1–27). Springer Nature Singapore. https://doi.org/10.1007/978-981-16-8136-3_48-1

UNWomen. (2022). *Progress on the Sustainable Development Goals: The gender snapshot 2022.* https://www.unwomen.org/sites/default/files/2022-09/Progress-on-the-sustainable-development-goals-the-gender-snapshot-2022-en_0.pdf

TSG10-07

INTERDISCIPLINARY APPROACHES TO CONNECT MATHEMATICS AND SCIENCE: A CASE STUDY OF PRIMARY TEACHERS' ADAPTIVE EXPERTISE

Wanty Widjaja
Deakin University, Australia, w.widjaja@deakin.edu.au

Lihua Xu
Deakin University, Australia, lihua.xu@deakin.edu.au

Jan van Driel
The University of Melbourne, Australia, j.vandriel@unimelb.edu.au

Despite a strong call to advocate an interdisciplinary approach in teaching mathematics and science, there is a need to better understand the development of teachers' adaptive expertise in interdisciplinary teaching. This paper addresses a question to what extent does an interdisciplinary approach reveal opportunities and challenges in developing primary teachers' adaptive expertise. Drawing on interview data, video fragments for the lesson sequence, and teachers' post-reflection, this study reveals different ways of enacting flexibility and deliberate practice. Engaging in post-lesson co-reflection enabled teachers to notice opportunities to foster in-depth interdisciplinary links between mathematics and science. Implications for fostering primary school teachers' adaptive expertise in interdisciplinary mathematics and science will be discussed.

Keywords: adaptive expertise, interdisciplinary, mathematics, science, primary

Introduction

While STEM interdisciplinary approaches have gained global attention, there is a gap in understanding interdisciplinary approaches that maintain disciplinary integrity (English 2016). One key obstacle to interdisciplinary teaching is the pervasive disciplinary silos within the school curriculum (Gardner & Tillotson, 2018) and a lack of balanced representation of disciplines in interdisciplinary teaching. Mathematics is often positioned as a tool to support the learning of other STEM disciplines and the underlying mathematics concepts and processes often remain indiscernible (English, 2016; Goos et al., 2023). A better understanding of the nature and development of teachers' capacity to teach interdisciplinary mathematics and science is needed.

With increasing demand for teachers to be responsive to novel and complex classroom situations flexibly and innovatively, the notion of adaptive expertise (Carbonell et al., 2014; Hatano & Inagaki, 1986) has become highly relevant in the teaching profession. The notion of adaptive expertise (Hatano & Inagaki, 1986) is not new and has been widely utilised in a range of research fields including in the medical field and the social science field. Yet, "little is known about the process of cultivating adaptive expertise" (Hatano, 2003, xi).

Previous research on teachers' adaptive expertise has highlighted how teachers acquire and harness expertise by observing and interpreting their actions in context-specific situations (Yoon et al., 2019). This paper addresses a question: To what extent can an interdisciplinary approach to mathematics and science teaching reveal opportunities and challenges in developing primary teachers' adaptive expertise?

Conceptual Framework: Adaptive Expertise

The notion of a*daptive expertise* (AE), first proposed by Hatano and Inagaki (1984). It has been identified as a useful lens by researchers who consider expertise as a hallmark of professionalism and a component of quality teaching (Darling-Hammond & Bransford, 2005). Hatano and Inagaki (1986) distinguished between adaptive experts and routine experts (RE) within a domain, with adaptive experts exhibiting flexibility, innovation, and creative competencies, while routine experts demonstrate efficiency, accuracy, and automaticity in solving familiar problems.

Research by Yoon and others (2019) on teachers' discipline specific adaptive expertise (i.e., in the context of innovative biology education) proposed a framework of AE comprising three research-based components: flexibility, deeper level of understanding, and deliberate practice. Flexibility is the ability to integrate aspects of knowledge and opportunistically change practice to respond to new situations. A deeper level of understanding involves the ability to attend to deeper levels of problem solving as a function of integrated declarative and contextual knowledge. Deliberate practice is characterised by one's ability to engage in reflection, conscious deliberation, and regulation processes (cf. Carbonell et al., 2014). In this paper, Yoon et al. (2019)'s framework will be employed to examine dimensions of adaptive expertise of primary school teachers as demonstrated through their planning, enactment and reflection on an interdisciplinary sequence 'Journey through space: you plan it!' (Hughes et al., 2022).

Methodology

A case study methodology (Yin, 2014) is employed in examining the ways in which three teachers from one non-government primary school in Melbourne, Victoria work together in co-planning, co-teaching, and co-reflecting on one lesson sequence. The three teachers co-planned the lesson sequence with the two teachers: Rose and Marg, co-taught all three lessons and Mia co-taught the first lesson. The teachers had experience of teaching standalone science inquiry in Term 2.

The lesson sequence '*Journey through Space – you plan it!*' (Hughes et al., 2022) aims at the Year 5 Australian Curriculum: Science, Earth and Space Science, where students are expected to learn about features of the planets in earth's solar system. The sequence addresses three big mathematical big ideas in Number at Year 5: place value of large numbers, multiplicative thinking, and early proportional reasoning. The opportunities for making interdisciplinary connections between mathematics and science are afforded by exploring the distances and the times it takes to travel between planets in the solar system using various modes of transport. In this paper, data from the planning meeting, the teaching and observation of the second lesson and the post-lesson reflections in Term 4, 2022 were analysed and discussed.

Findings and Discussion

The teachers demonstrated some flexibility when approaching the connections between science and mathematics, evident in their attempt to contextualize the concept of speed in familiar situations to the students. For example, Marg used the context of Usian Bolt racing on a beach or from the bottom of a hill to explore other units of measurement beyond kilometre per hour. This was a deliberate act (Carbonell et al., 2014) as confirmed by Marg in her post-lesson reflection. By integrating her knowledge of speed in the context of a world class sprinter, Marg demonstrated flexibility (Yoon et al, 2019) by enabling students to explore mathematical and scientific ideas related to different units of measurements for speed and factors affecting speed such as drag and momentum.

The opportunity to co-teach the lesson enabled the two teachers to bounce ideas off each other and make adaptations on the go to navigate challenging and unexpected situations arising during the lesson. The teachers discussed strategies to resolve challenging group dynamics based on one teacher's observation of non-contributing group members. Later on, they also discussed pedagogical strategies to select students to share their solutions. The post-lesson reflection highlighted the tension the teachers faced between the need to "allow for more opportunities for the students to be able to participate" and "make more of an effort to integrate the two [disciplines] so it doesn't feel like just a maths lesson". The tension in balancing the disciplinary focus in the lesson and maintaining students' engagement is indeed a challenge in interdisciplinary approaches in STEM (English, 2006, Goos et al., 2023). Hatano and Oura (2003) argued that engaging in interactive dialogues is one of the conditions for promoting adaptive expertise. In this case, the co-teaching allowed teachers to engage in interactive dialogues and supported them in navigating unexpected moments, dealing with uncertainties and prompting them to exercise flexibility (Yoon et al., 2019).

The teachers struggled to navigate other contingent moments related to mathematics calculations involving decimals and unit conversion due to their own lack of understanding. This was evident when a student shared an incorrect solution strategy to work out the time to fly from Sydney to Melbourne, given the distance of 878 km and assuming the speed of flying of 900 km/hour. Instead of using division, she used subtraction and arrived at the answer of 38 minutes [900-878 = 22, 60-22=38]. It was apparent that the teachers struggled to respond to this situation and instead they invited other students to share their ideas. The episode highlighted the way in which teachers responded to a contingent moment. The teacher, Rose acknowledged during the post-lesson reflection that she did not feel comfortable going through students' calculation strategies. Similarly, Marg commented that the mathematics was quite challenging, and noticed some students were not engaged during the lesson and viewed that the mathematics 'divided the class'. This vignette was similar to an example of low deep level of understanding offered by Yoon et al. (2019), where teachers were unable to respond or being "hands-off" when "it is clear that some students are very confused and don't know where to start" (897).

Conclusion and Implications

Co-planning, co-teaching, and co-reflecting on an interdisciplinary mathematics and science sequence supported primary teachers to exercise flexibility and deliberate practice as they navigated uncertainties and dealt with unexpected moments. The use of probing questions that tap into students' prior knowledge enabled students to make links between scientific and mathematical ideas with everyday context. This study highlights the critical role of deep level understanding of disciplinary knowledge in enabling teachers to develop flexibility when responding to unexpected

situations and extending student thinking. This study suggests that providing opportunities to innovate, such as interdisciplinary science and mathematics teaching, can stimulate teachers to critically reflect on their routine practices. However, developing adaptive expertise in new contexts requires teachers to 'unravel treasured beliefs and practices' (Langdon, 2017, p. 541) while building and enacting new knowledge and learning, which can be challenging for some teachers.

References

Carbonell, K. B., Stalmeijer, R. E., Könings, K. D., Segers, M., & van Merriënboer, J. J. (2014). How experts deal with novel situations: A review of adaptive expertise. *Educational Research Review*, *12*, 14-29. https://do.org/10.1016/j.edurev.2014.03.001

Darling-Hammond, L., & Bransford, J. (Eds.). (2007). *Preparing teachers for a changing world: What teachers should learn and be able to do*. John Wiley & Sons.

English, L. D. (2016). STEM education K-12: Perspectives on integration. *International Journal of STEM Education*, *3*, 1-8. https://doi.org/10.1186/s40594-016-0036-1

Gardner, M., & Tillotson, J. W. (2019). Interpreting integrated STEM: Sustaining pedagogical innovation within a public middle school context. *International Journal of Science and Mathematics Education*, *17*, 1283-1300. https://doi.org/10.1007/s10763-018-9927-6

Goos, M., Carreira, S., & Namukasa, I. K. (2023). Mathematics and interdisciplinary STEM education: recent developments and future directions. *ZDM–Mathematics Education*, *55*(7), 1199-1217. https://doi.org/10.1007/s11858-023-01533-z

Hatano, G., & Inagaki, K. (1986). Two courses of expertise. In H. Stevenson, H. Azama, & K. Hakuta (Eds.), *Child development and education in Japan* (pp. 262-272). Freeman.

Hatano, G. & Oura, Y. (2003). Commentary: Reconceptualising school learning using insight from expertise research. *Educational Researcher. 32*(8), 28-29. http://www.jstor.org/stable/3700083

Hughes, S., Russo, J. A., Mansfield, J., Green, A., Jones, D., Vale, C., & Berry, A. (2022). Co-designing integrated mathematics and science lesson learning sequences for primary education. *International Journal of Innovation in Science and Mathematics Education*, *30*(4), 64-75. https://doi.org/10.30722/IJISME.30.04.005

Langdon, F. J. (2017). Learning to mentor: unravelling routine practice to develop adaptive mentoring expertise. *Teacher Development*, *21*(4), 528–546. https://doi.org/10.1080/13664530.2016.1267036

Yin, R. K. (2014). *Case Study Research Design and Methods (5th ed.).* Sage. DOI:10.3138/cjpe.30.1.108

Yoon, S. A., Evans, C., Miller, K., Anderson, E., & Koehler, J. (2019). Validating a model for assessing science teacher's adaptive expertise with computer-supported complex systems curricula and its relationship to student learning outcomes. *Journal of Science Teacher Education*, *30*(8), 890-905. https://doi.org/10.1080/1046560X.2019.1646063

KOREA 2025 수학

EARCOME9

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Topic Study Groups 10
(Day 4)

TSG10-08

ENHANCING SELF-REGULATION AND MOTIVATION THROUGH STEM PROJECT-BASED LEARNING

Hyunkyung Kwon
Texas A&M University, USA, kwonx133@tamu.edu

Robert M. Capraro
Texas A&M University, USA, rcapraro@tamu.edu

Yujin Lee
Kangwon National University, Korea, ylee@kangwon.ac.kr

Ashley Williams
Muskingum University, USA, awill@muskingum.edu

As the demand for a STEM-skilled workforce increases, project-based learning (PBL) has gained attention as an effective pedagogical approach for enhancing students' engagement and learning outcomes. This study investigates the impact of STEM PBL on students' self-regulation and motivation by analyzing the effects of a two-week STEM summer camp. A total of 60 secondary students participated in the intervention, engaging in structured STEM PBL activities. Pre- and post-tests measured changes in students' self-regulation and motivation using an adapted version of the Motivated Strategies for Learning Questionnaire (MSLQ). The results indicate a significant increase in both self-regulation ($t = 2.83$, $p = .004$, $d = 0.395$) and motivation ($t = 2.25$, $p = .004$, $d = 0.404$). These findings suggest that STEM PBL positively influences students' ability to regulate their learning and enhances their motivation, emphasizing the importance of integrating PBL into STEM education.

Keywords: STEM education, project-based learning, self-regulation, motivation, engagement

Introduction

STEM education has become a priority in the global educational landscape, as nations strive to equip students with the skills needed for the modern workforce. Project-Based Learning (PBL) has emerged as an effective instructional method, promoting student-centered learning, problem-solving, and interdisciplinary engagement. Research indicates that STEM PBL enhances students' cognitive and affective engagement, contributing to improved academic performance and skill development (Lee et al., 2022). Among the key factors influencing academic success are self-regulation and motivation. Self-regulated learners actively monitor and control their cognitive and behavioral processes, while motivated students demonstrate persistence and enthusiasm toward learning (Zimmerman, 2000). Despite growing interest in STEM PBL, limited research has explored its direct impact on these psychological constructs. This study aims to fill this gap by examining the effects of STEM PBL on students' self-regulation and motivation.

Literature Review

STEM PBL is an instructional approach that integrates science, technology, engineering, and mathematics through hands-on, collaborative activities. It enables students to engage with real-world problems, fostering critical thinking and creativity (Bicer et al., 2017). Prior research suggests that STEM PBL enhances student engagement, deepens conceptual understanding, and promotes long-term knowledge retention (Kwon, 2017).

Self-regulation involves cognitive, behavioral, and emotional processes that allow students to set goals, monitor progress, and adapt learning strategies (Pintrich, 2000). High self-regulation has been linked to academic success, as it enables students to manage their learning effectively (Cleary & Platten, 2013). Since STEM PBL requires students to take ownership of their learning, it is hypothesized that engaging in such activities will enhance self-regulation.

Motivation is a key driver of learning, influencing students' persistence, effort, and achievement (Ryan & Deci, 2000). STEM PBL creates meaningful learning experiences, which can enhance intrinsic motivation, and social cognitive theories suggest that student motivation increases when learning tasks align with their interests and personal goals (Lee et al., 2019, 2022).

Methodology

The study was conducted with 60 middle and high school students who attended a two-week STEM summer camp at a Tier One university in the United States. Participants were selected through convenience sampling and represented diverse backgrounds. The sample consisted of 29 female (48%) and 31 male (52%) students, ranging from 7th to 12th grade.

A non-randomized quasi-experimental design was used to assess changes in self-regulation and motivation. Students participated in 60 hours of STEM PBL instruction, which included activities such as coding (programming with Python, Scratch), microcontrollers (circuit design, sensor programming), cryptography (cipher encryption and decryption), structural engineering (bridge design and cost analysis), 3D printing (modeling and printing projects). These activities were designed based on core PBL principles, ensuring collaborative engagement, student-centered learning, and hands-on problem-solving.

Students' self-regulation and motivation were assessed using an adapted version of the Motivated Strategies for Learning Questionnaire (MSLQ) (Pintrich et al., 1991). The instrument consisted of 12 self-regulation items (Cronbach's $\alpha = .78$) and 8 motivation items (Cronbach's $\alpha = .74$). Participants rated each item on a 7-point Likert scale (1 = "Not at all true of me" to 7 = "Very true of me"). Pre-tests were conducted before the intervention, and post-tests were administered at the end of the program. Paired-sample t-tests were performed to determine whether the differences between pre- and post-test scores were statistically significant. Cohen's d effect sizes were calculated to measure the magnitude of the intervention's impact.

Results

The results of the study indicate that students demonstrated significant improvements in both self-regulation and motivation after participating in the STEM PBL intervention. The mean self-regulation score increased from 54.8 to 57.7, with a t-value of 2.83 ($p = .004$), and a Cohen's d effect size of 0.395, suggesting a moderate positive effect. Similarly, the mean motivation score increased from 44.1 to 46.3, with a t-value of 2.25 ($p = .004$) and a Cohen's d effect size of 0.404, also indicating a moderate positive effect.

These findings suggest that engaging in STEM PBL activities positively influenced students' ability to regulate their learning behaviors and increased their motivation to engage with STEM subjects. Notably, students exhibited stronger self-regulation by setting clearer learning goals, monitoring their progress, and adjusting their strategies accordingly. The increase in motivation was likely driven by the student-centered and hands-on nature of STEM PBL, which allowed participants to engage with real-world problems in an interactive and meaningful way.

Furthermore, the study found that motivation and self-regulation appeared to be interconnected. Students who demonstrated higher motivation levels were also more likely to regulate their learning effectively, supporting the idea that these two constructs reinforce each other. Although the overall increase in both self-regulation and motivation was statistically significant, individual variations among students suggest that factors such as prior experience with STEM, personal interest, and learning environment may also play a role in shaping these outcomes.

Discussion

The findings of this study support the growing body of research indicating that STEM PBL is an effective instructional strategy for enhancing students' self-regulation and motivation. The observed improvements in these areas align with previous studies that highlight the benefits of active, student-centered learning environments (Kwon, 2017). By engaging in real-world problem-solving, students were able to take ownership of their learning, which in turn contributed to greater self-regulation. Similarly, the hands-on and collaborative nature of STEM PBL likely played a crucial role in increasing students' motivation, as it provided meaningful learning experiences that connected abstract concepts to practical applications.

One key implication of this study is the importance of teacher preparation and instructional design when implementing STEM PBL. Educators must be well-trained in facilitating inquiry-based learning and guiding students through the problem-solving process while allowing them to take responsibility for their own learning. Providing professional development opportunities for teachers can help maximize the effectiveness of STEM PBL in classroom settings. In addition, the study highlights the role of social and physical engagement in enhancing student motivation. Unlike traditional lecture-based instruction, STEM PBL encourages peer collaboration and active participation, which may contribute to higher levels of engagement and interest in STEM subjects. This finding suggests that incorporating interactive and real-world learning experiences into STEM education can help sustain students' motivation and foster long-term interest in STEM careers.

Despite the positive findings, this study has some limitations. First, the non-randomized sample and short intervention period limit the generalizability of the results. While the study provides valuable insights into the immediate effects of STEM PBL, further research is needed to examine its long-term impact on students' self-regulation and motivation. Additionally, the absence of a control group prevents direct comparisons between STEM PBL and traditional instructional methods. Future studies should consider employing longitudinal designs and experimental methodologies to further explore these relationships.

In conclusion, this study provides empirical evidence that STEM PBL positively influences students' self-regulation and motivation. These findings underscore the importance of incorporating student-centered, hands-on learning experiences in STEM education to foster essential learning behaviors. By continuing to refine and expand the implementation of STEM PBL, educators can help prepare students for academic success and future careers in STEM fields.

References

Bicer, A., Capraro, R. M., & Capraro, M. M. (2017). Hispanic students' mathematics achievement in the context of their high school types as STEM and non-STEM schools. *International Journal of Mathematical Education in Science and Technology, 49*(5), 705-720.

Cleary, T. J., & Platten, P. (2013). Examining the correspondence between self-regulated learning and academic achievement: A case study analysis. *Educational Research International, 2013*, Article 272560.

Kwon, H. (2017). Effects of 3D printing and design software on student performance. *Journal of STEM Education, 18*(4), 37-42.

Lee, Y., Capraro, R. M., & Bicer, A. (2019). Affective mathematics engagement: A comparison of STEM PBL versus non-STEM PBL instruction. *Canadian Journal of Science, Mathematics and Technology Education, 19*(3), 270-289.

Lee, Y., Capraro, R. M., Capraro, M. M., & Bicer, A. (2022). Cultural affordance and affective mathematics engagement in Korea and U.S. *Research in Mathematical Education, 25*(1), 21-43.

Pintrich, P. R. (2000). The role of goal orientation in self-regulated learning. In M. Boekaerts, P. R. Pintrich, & M. Zeidner (Eds.), *Handbook of self-regulation* (pp. 451–502). Academic Press.

Pintrich, P. R., Smith, D. A. F., Garcia, T., & McKeachie, W. J. (1991). *A manual for the use of the motivated strategies for learning questionnaire (MSLQ)*. National Center for Research to Improve Postsecondary Teaching and Learning.

Ryan, R. M., & Deci, E. L. (2000). Self-determination theory and the facilitation of intrinsic motivation, social development, and well-being. *American Psychologist, 55*(1), 68–78. https://doi.org/10.1037/0003-066X.55.1.68

Zimmerman, B. J. (2000). Attaining self-regulation: A social cognitive perspective. In M. Boekaerts, P. R. Pintrich, & M. Zeid

Acknowledgements

This study is a revised version of the research conducted by Kwon et al. (2024).

TSG10-09

FOSTERING AGENCY IN MATHEMATICAL MODELLING FOR CRITICAL CITIZENSHIP

Jimbo Juanito B. Villamor
Ateneo de Manila University, Surigao del Norte State University, Philippines,
jimbo.villamor@student.ateneo.edu

Catherine P. Vistro-Yu
Ateneo de Manila University, Philippines, cvistro-yu@ateneo.edu

The study examines how mathematical modelling can foster student agency for critical citizenship. An explanatory sequential mixed methods research design was used in carrying out the research objectives. Quantitative data gathered through survey were described using frequency counts, percentages, mean, standard deviation, and analyzed using paired-samples t-test. Qualitative data obtained through focus group discussion were analyzed using a thematic analysis. The mathematical modelling task that combines knowledge in mathematics, environmental science, and social issues, enabled students to reach an acceptable learning performance, and develop behaviors for agency and critical citizenship. This study asserts that integrating mathematical modelling in classes has the power to ignite mathematical understanding and foster agentic capacities that could develop characteristics for critical citizenship among students.

Keywords: agency, critical citizenship, mathematical modelling, mixed-methods research

Rationale of the Study

The accelerated globalization and rapid technological advancement heavily influenced of our lives in many forms. For this reason, teachers need to prepare students on how to navigate and thrive in our world amidst all its volatility, uncertainty, complexity, and ambiguity. In a mathematics classroom, for instance, the preparation could mean providing students with meaningful learning opportunities in concretizing mathematical knowledge necessary to becoming active, responsible, engaged, and future-ready change agents (Taguma et al, 2018). This certainly happens when mathematical modelling is applied in addressing a socio-ecological problem. Mathematical modelling drives students to solve real world problems using mathematical tools that develop students' mathematical knowledge, enabling them to live as constructive, concerned and reflective citizens (Kaiser, 2020). In context, it allows students to simplify a real-world problem, transform it into a mathematical problem, solve it and bring it back to its original real-world context (Borromeo Ferri, 2025; Niss & Blum, 2020). This study examines the potential of mathematical modelling in fostering students' agentic capacities for critical citizenship. In particular, it seeks to answer the following research questions:

1. How does mathematical modelling affect students' self-assessments of their agency?
2. What evidence from students' reflections after performing the modelling task indicate students' capacities for critical citizenship?

Theoretical Considerations

This study rested on the perspective of socio-critical mathematical modelling (Kaiser, 2020) emphasizing the role of mathematics in society (Kaiser & Sriraman, 2006). The socio-critical lens pointed out that mathematical modelling prepares students in addressing social and environmental problems (Kaiser, 2023) while they engage in discussing socio-ecological problems in its context and use mathematical tools to address these problems (Barwell, 2025). It was found to empower individuals in responding to the varying complexities of the world (Bala & Kohler, 2025) and improve students' social critical awareness while becoming more aware of the role of mathematics (Shim & Park, 2024). Here, we see socio-critical mathematical modelling as an opportunity that requires focus on both mathematics and societal issues in the entire modelling process (Steffensen & Kasari, 2023) in order to understand how mathematical modelling impacts society.

The results from the modelling process can help form public policies affecting how citizens live (Geiger, 2024), a particular focus of critical mathematics education (Skovsmose, 1994) and human agency (Bandura, 2001). Critical mathematics education empowers citizens through exploring social development issues within mathematics lessons (Skovsmose, 1994; Skovsmose & Niss, 2008), and develops a person's agency (Andersson & Norén, 2011). In this paper, we meant agency as students' abilities to regulate, control, and monitor their own learning, within the four core features, namely, intentionality, forethought, self-regulation, and self-efficacy (Bandura, 2001). Agency is vital in developing students' sense of responsibility to participate in the world and, to influence people, events and circumstances for the better (Taguma et al, 2018), thus, helping students become critical citizens. This then puts together the essence of STE(A)M education.

Methodology

The study presented here, explored through the explanatory sequential mixed methods research, is part of the dissertation of the first author duly supervised by the second author on "Using Mathematical Modelling to Build Student Agency for Critical Citizenship." A pre-post exposure study on "Functions and Their Graphs" in a General Mathematics course of the Philippines' Department of Education was conducted with 30 student-participants aged between 15 to 17 years old in a public secondary school in an environmentally protected and tourist destination island. Participation was voluntary with consent from parents and guardians so as to comply with ethical guidelines. Research ethics clearance was obtained from the Ateneo University Research Ethics Council with Protocol ID's SOSEREC _23_002 and SOSEREC _23_002CA.

The Research Process, Context of the Mathematical Modelling Task, Data Analysis

In this study, students self-assessed their initial levels of agency using a researcher-designed survey instrument on student agency in learning mathematics (SALM). They were then introduced with lessons on "Functions and Their Graphs." Along with this, students were given a general introductory lesson on mathematical modelling to discuss sample mathematical modelling. This implementation was aimed at bringing students' knowledge of performing mathematical modelling activities, especially since mathematical modelling was first formally introduced to them. Having completed the introductory lesson, students were exposed to the actual mathematical modelling tasks. After completion, students self-assessed their levels of capacities for agency. A focus group discussion was conducted thereafter.

This study utilized a researcher-designed mathematical modelling task anchored on the transdisciplinary lesson and unit framework (O'Donnell, 2021). The task considers the students' local com-

munity as a learning laboratory where they used their STEM knowledge to understand an existing socio-ecological issue, thus, linking students' understanding of mathematical concepts to their knowledge in environmental science. Specifically, the tasks required students to describe and analyze the solid waste situation of their local community using the data on the amount of solid waste accumulated and the factors affecting the accumulation obtained from the actual survey of 389 households.

The actual survey data were used to describe the solid waste status of the community using linear regression and analyzed using Jamovi (The Jamovi Project, n.d.), which was completed by six groups of students in separate classrooms. The mathematical descriptions of the solid waste status were represented by linear equations in slope-intercept form, as mathematical models. To communicate the results of their investigation, a dialogue with the school and local community were conducted. Quantitative data were described using the frequency counts and percentages and analyzed using paired samples t-test. Qualitative data were analyzed using thematic analysis following the steps specified by Braun and Clarke (2006).

Results and Discussion

We addressed the research question 1 by performing quantitative analyses. The overall students' performance was satisfactory, with group performances ranging from fairly satisfactory to outstanding. In particular, two (2) groups appeared to have a fairly satisfactory performance, another two (2) groups, satisfactory, one (1) group, very satisfactory, and one (1) group, outstanding. Prior to exposure to the mathematical modelling task, the students' agentic capacities were generally high across the four core features: intentionality, forethought, self-regulation, and self-efficacy. After exposure, their agentic capacities increased from high to very high levels within the capacities for intentionality, forethought, and self-regulation, except for self-efficacy, which remained to be in the high level, but with a greater mean score than their pre-exposure to the mathematical modelling task.

A series of paired-samples t-test were conducted to determine whether mathematical modelling led to a significant change in students' agentic capacities. The analysis compared students' self-assessment scores obtained before (pre-test) and after (post-test) the exposure to mathematical modelling task summarized in Table 1.

Table 1. Effects of mathematical modelling on student agency using the paired samples t-test

Core Features of Agency	Student's t	df	p-value	η^2
Intentionality	4.2	29	<.001	0.766
Forethought	5.39	29	<.001	0.983
Self-regulation	5.94	29	<.001	1.085
Self-efficacy	5.92	29	<.001	1.081

Note. $0.20 \leq \eta^2 \leq 0.50$:(small effect), $0.50 \leq \eta^2 \leq 0.80$:(medium effect), $\eta^2 \geq 0.80$: (large effect) (Cohen, 2013)

Results indicated that post-test scores for intentionality, forethought, self-regulation, and self-efficacy were significantly higher than their respective pre-test scores. The positive t-values indicate that the difference was calculated as post-test minus pre-test, which confirms through Cohen's $d \geq 0.766$, a substantial and statistically significant improvement in student's agentic capacities following the integration of mathematical modelling task.

We addressed research question 2 by performing qualitative analyses of students' responses during the focus group discussion. The conversation allowed students to reflect on what mathematical knowledge was important to them towards understanding a socio-ecological issue like solid waste management. One sample student response is presented below:

> There were a lot of mathematical concepts in the activities, like the slope-intercept form, linear regression, scatter plots, and charts. But we understood them better when you gave us hands-on activities. I realized that these concepts are important in finding solutions and analyzing solid waste situation in our community.

We closely looked into students' reflections after performing the modelling task. Capacities for student agency such as intentionality, forethought, self-regulation, and self-efficacy were evident. Intentionality is an individual's will to act (Code, 2020), while forethought is the anticipation of outcomes (Code, 2020). Self-regulation refers to actions, processes and strategies an individual employs in achieving their goals (Code, 2020; Pintrich, 2000). Self-efficacy is a personal belief about what an individual is capable of doing (Code, 2020). From the conversation, a student focused on the impact of math in problem-solving, showing intentionality by linking abstract math concepts to real-world decision-making. She considered that active learning helped her plan better, demonstrating forethought by recognizing the effectiveness of different learning strategies. She organized her learning by identifying multiple relevant concepts, suggesting her abilities for self-regulation, particularly in recognizing which methods work best for her. She also demonstrated self-efficacy in active learning environments, where she feels more confident in grasping mathematical concepts through practical application.

Characteristics for critical citizenship such as mathematical and reflective knowledge were also observable from the conversation. The student's ability to list multiple mathematical concepts suggests her broad understanding of different techniques used for problem-solving. Her ability to evaluate how mathematical tools contribute to solving real-world issues, demonstrates a strong sense of reflective knowledge.

Conclusion

The mathematical modelling task that combines knowledge in mathematics, environmental science, and social issues, enabled students to reach acceptable learning performance, and develop behaviors for agency and critical citizenship. This study asserts that integrating mathematical modelling in classes has the power to ignite mathematical understanding and foster agentic capacities that could build one's character for critical citizenship.

References

Andersson, A., & Norén, E. (2011, February). Agency in mathematics education. In Pytlak, M.; Rowland, T.; Swoboda, E. (Eds.), *Proceedings from 7th conference for European research in Mathematics education* – CERME VII (pp. 1389-1398). Rzeszów: University mathematics of Rzeszów. https://goto.now/ISNS4

Bala, C., Kohler, B. (2025). Building Connections Across Scales to Stimulate and Realize Transformation in Mathematics Education. In Le Roux, K., Coles, A., Solares-Rojas, A., Bose, A., Vistro-Yu, C. et al. (Eds.). (2025). *Proceedings of the 27th ICMI Study Conference (Mathematics Education and the Socio-Ecological)*. MATHTED and ICMI. https://icmistudy27.sciencesconf.org/data/pages/ICMI_Study_27_Proceedings_Feb_25_final.pdf

Barwell, R. (2025). Critical Mathematical Consciousness and Learning Mathematics for Sustainable Futures. In Le Roux, K., Coles, A., Solares-Rojas, A., Bose, A., Vistro-Yu, C. et al. (Eds.). (2025). *Proceedings of the 27th ICMI Study Conference (Mathematics Education and the Socio-Ecological)*. MATHTED and ICMI. https://icmistudy27.sciencesconf.org/data/pages/ICMI_Study_27_Proceedings_Feb_25_final.pdf

Bandura, A. (2001). Social cognitive theory: An agentic perspective. *Annual review of psychology*, *52*(1), 1-26. https://doi.org/10.1146/annurev.psych.52.1.1

Bandura, A. (2008). Toward an agentic theory of the self. *Advances in self research*, 3, 15-49. https://goto.now/o7UrK

Borromeo Ferri, R. (2025). Mathematical Modelling as a Key Practise for Promoting Learners' Competencies Targeting the Transformation of Global Sustainability. In Le Roux, K., Coles, A., Solares-Rojas, A., Bose, A., Vistro-Yu, C. et al. (Eds.). (2025). *Proceedings of the 27th ICMI Study Conference (Mathematics Education and the Socio-Ecological)* (pp. 284-291). MATHTED and ICMI. https://icmistudy27.sciencesconf.org/data/pages/ICMI_Study_27_Proceedings_Feb_25_final.pdf

Code, J. (2020, February). Agency for learning: Intention, motivation, self-efficacy and self-regulation. In *Frontiers in education* (Vol. 5, p. 19). Frontiers Media SA. https://goto.now/d0Kei

Cohen, J. (2013). *Statistical power analysis for the behavioral sciences*. Elsevier Science. https://ebookcentral.proquest.com/lib/kxp/detail.action?docID=188284

Geiger, V. (2024). The need for a critical orientation to mathematical modelling in times of disruption. In *Researching mathematical modelling education in disruptive times* (pp. 15-34). Cham: Springer Nature Switzerland. https://link.springer.com/chapter/10.1007/978-3-031-53322-8_2

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *Zdm*, *38*, 302-310.https://link.springer.com/article/10.1007/BF02652813

Kaiser, G. (2020). Mathematical modelling and applications in education. In Lerman, S. (Ed.), *Encyclopedia of mathematics education*, 553-561. Springer Cham. https://link.springer.com/referenceworkentry/10.1007/978-3-030-15789-0_101

O'Donnell, C. L. (2021). Discover, understand, act: a STEM education framework for empowering youth to address the global goals. *Diplomatic Courier: UN General Assembly (UNGA) Special Print Edition*. https://goto.now/DhnPv

Pintrich, P. R. (2000). An achievement goal theory perspective on issues in motivation terminology, theory, and research. *Contemporary educational psychology*, *25*(1), 92-104. https://www.sciencedirect.com/science/article/abs/pii/S0361476X99910172

Shim, J., & Park, M. (2024). The effects of social critical modelling classes on elementary school students' perception of mathematics and discussion patterns. *Communications of Mathematical Education*, *38*(3), 353-378. https://doi.org/10.7468/jksmee.2024.38.3.353

Singmann, H. (2018). *afex: Analysis of Factorial Experiments*. [R package]. Retrieved from https://cran.r-project.org/package=afex.

Skovsmose, O. (1994). Towards a critical mathematics education. *Educational studies in mathematics*, *27*(1), 35-57. https://link.springer.com/article/10.1007/BF01284527

Skovsmose, O., & Niss, M. (2008). Critical mathematics education for the future. In *ICME-10 Proceedings: Proceedings of the 10th International Congress on Mathematical Education*. IMFUFA, Roskilde University. https://vbn.aau.dk/ws/portalfiles/portal/17034122/CriticalMathematicsEducation

Steffensen, L., & Kasari, G. (2023). Integrating societal issues with mathematical modelling in pre-service teacher education. *Education Sciences*, *13*(7), 721. https://doi.org/10.3390/educsci13070721

Taguma, M., Feron, E., & Lim, M. H. (2018). Future of education and skills 2030: Conceptual learning framework. *Organization of Economic Co-operation and Development*. https://goto.now/y8pro

TSG10-10

INVESTIGATING TEACHER'S DESIGN OF MATHEMATICAL MODELLING TASKS TO FOSTER MATHEMATICAL LITERACY

Sakon Tangkawsakul
Kasetsart University, Thailand, sakon.tan@ku.th

Chatree Faikhamta
Kasetsart University, Thailand, feductf@ku.ac.th

While many countries have integrated mathematical modelling into their core curricula, Thailand and several others have not. This study investigates how a mathematics teacher with limited modelling experience designs and implements modelling tasks. We build on Geiger and colleagues' (2022) Design and Implementation Framework for Mathematical Modelling Tasks (DIFMT) to adapt it for educational contexts where mathematical modelling is not yet formally included in curricula. We studied May, a secondary mathematics teacher with no prior experience in K-12 mathematical modelling. We collected lesson plans, teacher-designed tasks, interviews, and teaching reflections. Our analysis showed May's strong adherence to DIFMT principles in three areas: ***Relevance and Motivation****, connecting problems to authentic contexts aligned with student experiences;* ***Accessibility****, using problems matching students' prior knowledge; and* ***Teaching Flexibility****, integrating both step-by-step and holistic approaches. We identified two areas needing adaptation:* ***Nature of the Problem****, where May used gradual guidance with whole-class discussions rather than differentiated tasks; and* ***Feasibility of Outcomes****, May considered diverse approaches students might use to represent relationships between quantities. She predicted common student errors when analyzing data, particularly in the Keychain problem which involved length measurements and pricing. By intentionally including extra information, she helped students learn to identify relevant data and make appropriate assumptions. Our research supports adapting the DIFMT framework for teachers new to modelling. We suggest that when both teachers and students are beginners in mathematical modelling, specific DIFMT principles can be modified to provide more flexibility in implementation.*

Keywords: mathematical modelling, mathematical literacy, teaching competencies, task design, task implementation

Introduction

Mathematical literacy, specifically mathematical modelling competency, is vital to worldwide mathematics education. Many countries now make mathematical modelling a core curriculum component (Niss & Blum, 2024). Thailand and several other nations have not formally integrated mathematical modelling into their basic education curricula or teacher development programs. However, educational policymakers and institutions actively promote mathematical literacy in schools. Their efforts are partly driven by improving students' performances in international assessments like PISA, which significantly emphasizes mathematical modelling competency (In Thai: https://d2ieq.ipst.ac.th/). To provide students with valuable experiences, teachers must design and implement high-quality mathematical modelling tasks that apply mathematical knowledge and processes to solve real-world problems (Maaß, 2010; Borromeo Ferri, 2018). Consequently, the design and implementation of mathematical modelling tasks in classrooms has become a focal point of academic

research. Task design and implementation in mathematics teaching is crucial to mathematical modelling teaching competency. Scholars have developed frameworks for task design and implementation, notably Galbraith's (2006) initial framework and the more recent Design and Implementation Framework for Mathematical Modelling Tasks (DIFMT) by Geiger, Galbraith, Niss, and Delzoppo (2022). We argue that while certain DIFMT principles maintain their theoretical validity across implementation contexts, others demand significant reconceptualization to account for the dual-novice scenario where both teachers and students are simultaneously developing modelling competencies. Many existing frameworks for designing and implementing mathematical modelling tasks are primarily tailored for teachers who experienced mathematical modelling in education (Borromeo Ferri, 2018; Geiger et al., 2022). However, there remains a significant gap in knowledge regarding the applicability of these frameworks for teachers with non-experience in teaching mathematical modelling. Thus, this research aims to investigate the practices of mathematics teachers with limited experience in teaching modelling by examining how teachers design and implement modelling tasks in their classrooms. The study will comprehensively analyze the design and implement of mathematical modelling tasks. It will also explore how the teachers' approaches align with the principles outlined in the DIFMT, identifying overlaps and discrepancies. The findings from this study are intended to offer valuable insights into the specific needs and support mechanisms required for novice teachers to teach mathematical modelling effectively.

Research Questions

How do novice mathematics teacher's design and implement mathematical modelling tasks in its regular mathematics instruction?

Methodology

This qualitative interpretive study used a **single case study approach** (Yin, 2018) to examine a teacher's mathematical modelling task design and implementation**. One secondary mathematics teacher**, May (pseudonym), was selected as she participated in a professional development program but had no prior experience implementing mathematical modelling in K-12 schools. **Data collection** included lesson plans, teacher-designed tasks, semi-structured interviews, teaching reflections, and student work samples. May developed an eight-lesson sequence integrating mathematical modelling with systems of linear equations, aligning with the mixing approach (Blum & Niss, 1991). **Data analysis** employed thematic coding based on DIFMT principles. We enhanced reliability through researcher triangulation and member checking with May and friend to validate our interpretations of her design intentions and implementation approaches.

Results

An analysis of May's design for mathematical modelling tasks demonstrates that she adhered to six DIFMT principles (Geiger et al., 2022), showcasing strong capabilities in three critical areas:

Relevance and Motivation: May demonstrated a commendable ability to connect problem situations to authentic classroom contexts that engaged students. Such connections were closely aligned with students' interests and experiences, establishing a strong foundation for task design that fostered intrinsic motivation to engage with mathematical material. For example, the Mister Donut scenario was based on a popular local shop that students were familiar with, making the mathematical problem immediately relevant to their daily experiences.

Accessibility: May placed a significant emphasis on problem situations that corresponded with most of the students' prior knowledge and experiences. She prioritized suitably manageable scenarios, ensuring that students could actively engage without encountering overwhelming complexity. This focus facilitated a productive environment conducive to practical thinking and learning. The gradual progression from simple relationship expressions to more complex systems of equations demonstrated this principle.

Teaching Flexibility: May was proficient in designing tasks that seamlessly integrated atomistic (step-by-step) and holistic (big picture) instructional approaches. This flexibility proved beneficial, allowing students to explore mathematical concepts from various perspectives, thus enriching their understanding of problem-solving. She balanced specific skill development lessons with contextualized application lessons, showing flexibility in her instructional approach.

Exemplar Task: keychain custom design

Bow wants to order a keychain from a TikTok shop for use in sisters' birthday activities. This keychain will be used as a gift. The shop offers three types of garlands with the following details:

Type 1: This keychain, consisting of 3-star and 5 Heart-shaped beads, is 10.5 centimeters long and costs a reasonable 22 baht.

Type 2: The keychain consists of one-star and three Heart-shaped beads, each 5.5 centimeters long and priced at 10 baht.

Type 3: The shop also offers a custom keychain, which allows you to personalize it by choosing the number of stars and Heart-shaped beads you want. The price is determined by the number of beads used, making it a flexible and personalized option.

What do you think about the price of the Type 3 keychain?

If Bow orders 15-star and 20 Heart-shaped beads, and the shop charges 120 baht,

is this reasonable? Why?

Figure 1. Keychain custom design problem

May used this task specifically to teach systems of linear equations using the substitution method, aligning with the mixing approach. The task required students to analyze whether the pricing was reasonable, prompting them to use mathematical interpretation and modelling. Students needed to identify relevant information from the excess data provided, set up a system of linear equations representing the bead pricing, solve the system using the substitution method, and interpret their results to evaluate the reasonableness of the custom keychain pricing. This task successfully incorporated the DIFMT principles of relevance (online shopping is familiar to students), accessibility (using familiar concepts of price and quantity), and teaching flexibility (allowing both structured and open approaches to solutions).

However, two principles presented different considerations divergence from DIFMT Principles:

Nature of the Problem: Although the classroom had students with varying mathematical abilities, all were beginners in mathematical modelling. Rather than preparing specialized activities for high or low-performing students, May designed activities that gradually guided all students through the modelling process. She created opportunities for whole-class discussions about important and unimportant information, leading students to develop diagrams showing relationships between quantities relevant to solving the problem. This approach allowed all students to learn modelling processes under teacher guidance.

Feasibility of Outcome: May considered the diversity of approaches students might take in representing relationships between quantities and in defining variables to create systems of equations connected to problem situations and their interpretations. She anticipated potential misconceptions students might have when analyzing and interpreting data and conditions in situations, such as in the Keychain problem with its length measurements and multiple pricing structures. This deliberate inclusion of extra information was intended to develop students' abilities to identify and analyze key information and establish initial assumptions. As a result, only some students experienced difficulties in correctly analyzing the data. In the Keychain Custom Design Problem specifically, students showed growth in their ability to discriminate between relevant and irrelevant information, successfully setting up systems of equations from complex scenarios, and justifying their reasoning about the fairness of pricing based on mathematical evidence.

Discussion and Conclusion

This study provides valuable insights into how novice teachers can effectively design and implement mathematical modelling tasks despite limited experience. Our findings suggest that the DIFMT framework can be adapted for novice teachers by modifying certain principles to enhance implementation flexibility, particularly in educational contexts where mathematical modelling is not yet formalized in curricula. For novice teachers and students with no prior mathematical modelling experience, we propose that the DIFMT framework can be more effectively implemented by (1) Maintaining strong emphasis on relevance, accessibility, and teaching flexibility; (2) Gradually introducing problem complexity as both teacher and students gain modelling experience; and (3) Focusing initially on core modelling skills before addressing differentiation for diverse student capabilities. It offers a theoretical contribution toward understanding teacher development trajectories in mathematical modelling instruction, particularly in contexts where teachers are beginning to incorporate modelling into traditional mathematics curriculum topics like systems of linear equations. Future research should explore these adaptations of the DIFMT framework with larger teacher populations and across different mathematical content areas to develop more comprehensive guidance for novice teachers implementing mathematical modelling.

References

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects — State, trends and issues in mathematics instruction. *Educational Studies in Mathematics*, *22*(1), 37-68. https://doi.org/10.1007/BF00302716

Borromeo Ferri, R. (2018). *Learning how to teach mathematical modelling in school and teacher education*. Springer.

Galbraith, P. (2006). Real world problems: Developing principles of design. *Identities, Cultures and Learning Spaces, 1*, 228–236.

Geiger, V., Galbraith, P., Niss, M., Delzoppo, L., & Wright, V. (2022). Developing a task design and implementation framework for fostering mathematical modelling competencies. *Educational Studies in Mathematics, 109*, 313–336. https://doi.org/10.1007/s10649-021-10039-y

Maaß, K. (2010). Classification scheme for modelling tasks. *Journal für Mathematik-Didaktik, 31*(2), 285–311. https://doi.org/10.1007/s13138-010-0010-2

Niss, M., & Blum, W. (2020). *The learning and teaching of mathematical modelling*. Routledge.

Yin, R. K. (2018). *Case study research and applications* (6th ed.). Sage Publications.

Acknowledgement

The authors gratefully acknowledge funding from the Faculty of Education and Kasetsart University Research and Development Institute, Thailand (Grant YF(KU)26.67). This study was approved by the Kasetsart University Research Ethics Committee (Ref. No. COE67/147, December 17, 2024).

TSG10-11

FROM STEM OVER STEAM TO STEEM BUILT ON ECONOMICS

Allan Tarp
MATHeCADEMY.net, Allan.Tarp@gmail.com

*The fourth United Nations Development Goal wants all youth and most adults to achieve numeracy. In 'essence-based' mathematics using one-dimensional line-numbers without units, numeracy is at the beginning and calculus is at the end. But the opposite is the case within the alternative 'existence-based' paradigm built on the numbers children bring to school, two-dimensional bundle-numbers with units, where 2 3s is short for 2 bundles with 3s per bundle. Here calculus is needed in grade one for next-to addition of 2 3s and 4 5s as 8s since they add by their areas. Here, adding is preceded by counting leading to a recount-formula, T = (T/B)*B, with T and B for Total and Bundle, used to change units in STEM, and here recounting between icon-numbers and tens leads to a division and a multiplication table as well as to solving equation. Recounting physical units leads to per-numbers becoming fractions with like units and trigonometry when recounting the sides in a bundle-stack. So, changing from an essence-based to an existence-based curriculum may allow reaching the UN Development Goal and provide innovative and inclusive approaches to STEM education.*

Keywords: Early childhood mathematics, numeracy, arithmetic, STEM, calculus

Background, the 4th United Nations Sustainable Development Goal

The fourth of the 17 UN Sustainable Development Goals defines quality education as 'ensuring inclusive and equitable quality education and promoting lifelong learning opportunities for all.' Here the subgoal 4.6 wants to "By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy". However, different definitions of 'numerate' seem to exist. The English Oxford Dictionary defines it as being "competent in the basic principles of mathematics, esp. arithmetic". In contrast, the American Merriam-Webster dictionary defines it as "having the ability to understand and work with numbers."

The difference in the definitions is interesting. The English uses the passive term 'competent' where the American uses the active term 'work'. The word 'competent' is a predicate, a non-action word, I cannot 'competent' something, I can only be judged as competent by someone who is competent. In contrast, 'work' is an action word, a verb, since with my body and mind I can work on something.

Also, there is a difference between the words 'mathematics' and 'numbers.' Again, mathematics is a non-action word, I cannot 'mathematics' or even 'math' a thing. In contrast, 'number' is both a verb and a noun since I can number different degrees of Many to produce a number for later calculations.

Finally, Many exists in the outside world where humans see and name it differently. In contrast, mathematics does not do so, it is an institutionalized essence that is socially constructed as inside abstractions from outside examples, or as inside examples from inside abstractions. To understand these differences, we now consult the three grand theories, philosophy and sociology and psychology. Philosophy may be able to illuminate the different nature of predicates and verbs. Sociology

may be able to illuminate the different inter-human power effects coming from using predicates instead of verbs. And psychology may be able to illuminate the different learning results coming from listening to predicates or practicing verbs.

Grand Theory Looks at Mathematics Education

Within philosophy, Existentialism holds that 'existence precedes essence' (Sartre, 2007) so that in a judging is-sentence, an existing subject is being colonized by its predicates. 'Many' thus should be seen ontologically in itself, instead of epistemologically, how some may perceive and verbalize it. Within psychology, Piaget (1969) sees learning as adapting to outside existence, whereas a Vygotsky (1986) sees learning as adapting to inside institutionalized socially constructed essence.

Within sociology, a structure-agent debate discusses if institutions should be obeyed or negotiated between peers. Here, a Weberian viewpoint (1930) asks if SET is a rationalization gone too far by leaving Many de-enchanted and leaving learners in an 'iron cage'. As to the goal, Bauman (1990) suggests that, by institutionalizing mastery of Math as the means to reach mastery of Many, 'essence-math' has created a 'goal displacement' making the means a goal instead. Then, with the end goal, mastery of Many or existence-math, essence-math may be a means, but are there other means also?

Essence-math sees Many as an example of 1D linear cardinality always able to absorb one more by being built on the assumption that '1+1 = 2'. In contrast, humans see Many as a union of 2D stacks coming from numbering singles, bundles, and bundles of bundles, e.g., T = 345 = 3*BB + 4*B + 5*1. Essence-math sees mastery of math as its goal. A difference sees mastery of Many as the goal to be reached directly by using the bundle-numbers with units that children bring to school and that allows learning core math automatically (Tarp, 2018, 2024). Essence-math then could be more meaningful by de-modelling it (Tarp, 2020) built on the French poststructuralist version of Existentialism where deconstruction is used to replace predicates with verbs (Derrida, 1991). As to differences, Difference Research (Tarp, 2018) searching for differences making a difference may design micro curricula to be tested with Design Research and building on the observation that when asked "How many years next time?" a 3-year-old child will say "That is not 4, that is 2 2s" if seeing the 4 fingers held 2 by 2.

From STEM over STEAM to STEEM

STEM integrates mathematics with its roots in science, technology and engineering, all using formulas from algebra and trigonometry to pre-dict the behavior of predictable physical quantities, and to model unpredictable quantities by scenarios. Statistics 'post-dicts' unpredictable quantities by setting up probabilities for future behavior, using fact or fiction numbers as median and fractals or average and deviation. Including economics in STEM opens the door to statistics also. Art may be an appetizer, but not a main course since to play a core role in STEM, geometry and algebra should be together always and never apart. Art is a sugar coating making the pill go down but does not make the pill more digestible. STEM thus may be extended to STEAM to make it more appealing, but extending STEM to STEEM may increase the ability to understand and work with numbers.

Economics gives a Basic Understanding of Numbers and Calculations in Primary School

The basic meanings of geometry and algebra show that they are both rooted in economics. In Greek, geometry means to measure earth, and in Arabic, algebra means to reunite numbers, so they have a common root in the basic economic question "How to divide the earth and what it produces?" A hunter-gatherer need not tell the different degrees of many apart but a farmer does since here you produce to a market to survive and need to be numerate to answer the question "How many here?". This immediately leads to the answer "That depends on the unit." Economics thus begin at once by reusing the number-names when using bundling to count.

The romans unsystematically gave names to the bundles 5s, 10s, 50s, 100s, 500s and 1000s. This worked well for administrative addition and subtraction jobs but not for multiplication. So, when German silver reopened the trade between India and Renaissance Italy, Hindu-Arabic numbers named only the unbundled, the bundles, the bundle of bundles (BB or B^2), the bundle-bundle-bundle (BBB or B^3), etc. Typically, ten was used as the bundle-size, but also dozens and scores, 12s and 20s.

At a market you sell goods in bundles with different units, e.g., 2 3s. But the buyer may want to have 5s or trade 4 per 5 or pay 4$ per 5. So, changing units becomes a core job: '2 3s = ? 5s', and '6 7s = ? tens', and '3 tens = ?6s'. Likewise, when changing the units for length, weight, volume, and currency, And, when changing from the quantity to the price. Here, Renaissance Italy used 'regula detri', the rule of three. Asking "With the per-number 2$ per 3kg, what is the price for 9kg?", first they arranged the numbers with alternating units: '$, 9kg, 2$, 3kg'. Then they found the answer by multiplying and dividing: 9*2/3 = 6$. Today we use proportionality and say 9kg = (9/3)*3kg = (9/3)*2$ = 6$ when using the core linear recount-formula T = (T/B)*B, coming from recounting 8 in 2s as 8 = (8/2)*2.

Before school, children use bundle-numbers with units as 2 3s and 4 5s thus telling apart counting numbers in time as 2 and 4 from bundle-numbers in space as 3s and 5s. The school does not do so and insists that 1+1 = 2, which the children question by using an open and a closed V-sign to show that 2 1s and 1 2s add to 1 4s and not to 3 3s as the school says. Then they point out that the three core unit-change questions lead to a division table, a multiplication table, and to solving equations by recounting. And adding 2 3s and 4 5s next-to as 8s means adding areas found by calculus. And that recounting the height in the base in 4 5s is trigonometry giving $\pi = n*\tan(180/n)$ for n high enough. They thus learn core math by counting and recounting before adding when beginning with economics.

Calculations unite/*split Totals in*	Unlike	Like
Unit-numbers m, s, kg, $	T = a + n $T - n = a$	T = a * n $T/n = a$
Per-numbers m/s, $/100$ = %	T = ∫ f dx $dT/dx = f$	T = a^b $\sqrt[b]{T}=a$ $\log a(T) = b$

Figure 1. The Algebra Square shows the ways to reunite unlike and like unit- and per-numbers

Macroeconomics and Microeconomics in Middle School and High School

Later, macroeconomics describes households and factories exchanging salary for goods on a market in a cycle having sinks and sources: savings and investments controlled by banks and stock markets; tax and public spending on investment, salary and transferals controlled by governments; and import and export controlled by foreign markets experiencing inflation and devaluation. Proportionality and linear formulas may be used as first and second order models for this economic cycle, using regression to set up formulas and spreadsheets for simulations using different parameters. And, microeconomics describes equilibriums in individual cycles. On a market, shops buy and sell goods with a budget for fixed and variable costs, and with a profit depending on the volume sold and the unit-prices, all leading to linear equations. In the case of two goods, optimizing leads to linear programming. Competition with another shop leads to linear Game Theory. Market supply and demand determine the equilibrium price. Market surveys lead to statistics, as does insurance. In the households, spending balance income and transferals with saving and tax. In a bank, income comes from simple and compound interest, from installment plans as well as risk taking. On the stock market, courses fluctuate. Governments consider quadratic Laffer-curves describing a negative return to a tax-raise. And factories use variations of Cobb-Douglas power elasticity production functions for modeling.

Testing Effects of Bundle-numbers with Units

In a special education class some students tried to include units by using B for Bundles, and by using overloads and underloads introduced by recountings as

47 = 4B 7 = 3B 17 = 5B -3 tens.

Surprised that place values and carrying were now unneeded, they brought the unit method back to their home classes to the joy of the students, but not to the teachers unaware of this method and concerned about leaving out place values and carrying.

Overload	Underload	Overload	Overload
65 + 27	65 – 27	7 x 48	336 /7
6 B 5 + 2 B 7	6 B 5 -2 B 7	7 x 4 B 8	33 B 6 /7
8 B12 9 B 2	4 B-2 3 B 8	28 B 56 33 B 6	28 B 56 /7 4 B 8
92	38	336	48

Figure 2. Bundles and over- and underload make place values and carrying unneeded

It seems as if ethical testing of Bundle-numbers with units is for teacher education and home education. Imagine a grade one class A has an essence-math curriculum with unit-free 1D line-numbers and the traditional order from addition to power where only few hear about calculus. And class B has an existence-math curriculum with 2D Bundle-numbers with units and the opposite order. With counting preceding adding, they will not meet addition until they meet the core of math directly, calculus and linearity.

What will happen if two students exchange classes?

But, without a score, no music. So, maybe a composer should be allowed to compose and leave it to others to play and evaluate the music.

Conclusion and Recommendation

We may reach the UN Development Goal by replacing an essence-based curriculum with an existence-based curriculum answering the question 'how many?' by counting and recounting totals before adding them (Tarp, 2018). Digits are icons with as many strokes as they represent, thus being units when counting totals existing in time and space with 2D bundle-numbers that are flexible by allowing both overloads and underloads, which makes place values and carrying unneeded. The operations are icons also, but with different meanings and opposite order. Division now means counting iconized by a broom to push-away bundles. Multiplication is iconized by a lift uniting the bundles in a stack that a subtraction rope pulls-away to find the unbundled, seen as decimals, fractions, or negative numbers on top of the stack. Combined, bundling and stacking create a recount-formula with a per-number that changes units and used all over STEM. Once counted and recounted, totals may add on-top, or next-to by areas as with integral calculus, also used to add per-numbers. An existence-based curriculum will finally allow a communicative turn within the number-language as within the word-language in the 1970s (Widdowson, 1978). Using children's own bundle-number with units is a paradigm shift (Kuhn, 1962) that opens new areas for research and innovation.

References

Bauman, Z. (1990). *Thinking sociologically.* Blackwell.

Derrida, J. (1991). *A Derrida reader: between the blinds.* P. Kamuf (ed). Columbia Uni. Press.

Kuhn T.S. (1962). *The structure of scientific revolutions.* University of Chicago Press.

Piaget, J. (1969). *Science of education of the psychology of the child.* Viking Compass.

Sartre, J.P. (2007). *Existentialism is a humanism.* Yale University Press.

Tarp, A. (2001). Fact, fiction, fiddle - three types of models. in J. F. Matos, W. Blum, K. Houston & S. P. Carreira (Eds.). Modelling and mathematics education: ICTMA 9: Applications in Science and Technology. *Proc. 9th Int. Conf. on the Teaching of Mathematical Modelling and Applications* (pp. 62-71). Horwood Publishing.

Tarp, A. (2018). Mastering Many by counting, re-counting and double-counting before adding on-top and next-to. *Journal of Mathematics Education, 11*(1), 103-117.

Tarp, A. (2020). De-modeling numbers, operations and equations: From inside-inside to outside-inside understanding. *Ho Chi Minh City Univ. of Education Journal of Science 17*(3), 453-466.

Tarp, A. (2024). *Many before math, math decolonized by the child's own bundle bundle-numbers with units.* https://youtu.be/uV_SW5JPWGs.

Vygotsky, L. (1986). *Thought and language.* MIT press.

Weber, M. (1930). *The protestant ethic and the spirit of capitalism.* Unwin Hyman.

Widdowson, H. G. (1978). *Teaching language as communication.* Oxford University Press.

MEMO

The 9th ICMI-East Asia Regional Conference on Mathematics Education

Cite as:
Kwon, O., Kaur, B., Pang, J., Noh, J., Lee, S., Han, S., Yeo, S., & Lim, M. (Eds.). (2025). Proceedings of the 9th ICMI-East Asia Regional Conference on Mathematics Education (Vol. 3). Seoul National University, Siheung Campus, Korea: EARCOME9.

This work was supported by the NRF(National Research Foundation of Korea) Grant funded by the MOE (Ministry of Education)(NRF-2025S1A8A4A01014320).